CENG-424 HW-2

① a)

$S: \{ p \vee q \vee r,\ \neg p \vee \neg r,\ \neg w \vee r,\ \neg p,\ q,\ \neg w,\ p,\ q \vee r \}$

$\xrightarrow{\text{delete clauses includes } \neg w} \{ p \vee q \vee r,\ \neg p \vee \neg r,\ \neg p,\ q,\ p,\ q \vee r \}$

$\xrightarrow{\text{delete clauses includes } q} \{ \neg p \vee \neg r,\ \neg p,\ p \}$

$\xrightarrow{\text{add } r} \{ r,\ \neg p \vee \cancel{\neg r},\ \neg p,\ p \}$

$\xrightarrow{\text{delete } \neg r\text{'s from clauses}} \{ \neg p,\ \neg p,\ p \} \rightarrow \{ \Box \}$

Since we have $\neg p$ and p in this set, there is not any interpretation that makes this formula satisfiable.

b)

$S: \{ \cancel{p} \vee q \vee \neg r,\ \cancel{p} \vee \neg q,\ \neg p,\ r,\ w \}$

$\xrightarrow{\text{delete } p\text{'s from clauses}} \{ \cancel{q} \vee \neg r,\ \neg q,\ r,\ w \}$

$\xrightarrow{\text{delete } q\text{'s from clauses}} \{ \neg r,\ r,\ w \} \rightarrow \{ \Box \}$

Since we have $\neg r$ and r in this final set, this formula is unsatisfiable.

(2) a)

$A: \; a \land c \land (\neg a \lor b) \land (\neg b \lor c)$

(a) (a) (a) (a)

$\boxed{a=1}$ $\boxed{c=1}$ $\neg a \lor b = 1$ (b) $\neg b \lor c = 1$ (c)

(b) (b) (c) (c)

$\neg a = 1$, $\boxed{a=0}$ × $\quad \boxed{b=1}$ $\quad \neg b = 1$, $\boxed{b=0}$ × $\quad \boxed{c=1}$

Since there is one interpretation that satisfies $I(A)=1$ s.t. $I = \{a \leftarrow 1, c \leftarrow 1, b \leftarrow 1\}$, this formula is <u>not unsatisfiable</u>.

b) $A: \; p \Rightarrow (q \lor r) \equiv \neg p \lor q \lor r$

$B: \; \neg(s \land q) \Rightarrow ((p \land s) \Rightarrow r) \equiv (s \land q) \lor (\neg(p \land s) \lor r)$

$\equiv (s \land q) \lor \neg p \lor \neg s \lor r$

First, check both sides equal to $\perp$.

$\neg p \lor q \lor r \quad \equiv \quad (s \land q) \lor \neg p \lor \neg s \lor r$

(a) $\quad$ (c) (c) (c) (c)

(b) $\neg p \lor q \lor r = 0$ $\quad$ (d) $s \land q = 0$ $\quad$ $\neg p = 0$, $\boxed{p=1}$ $\quad$ $\neg s = 0$, $\boxed{s=1}$ $\quad$ $\boxed{r=0}$

(b) $\quad$ (d) (d)

$\neg p = 0$, $\boxed{p=1}$, $\boxed{q=0}$, $\boxed{r=0}$ $\quad$ $\boxed{s=0}$ × $\quad$ $\boxed{q=0}$ (e)

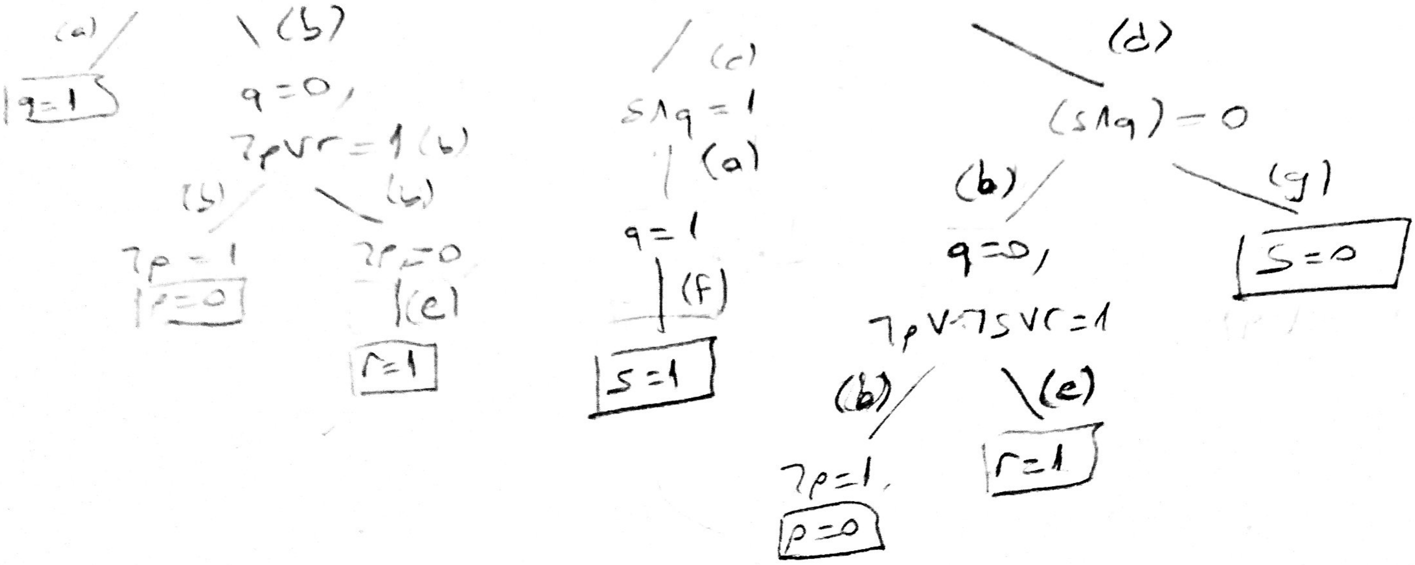
(e) (e)

$\boxed{s=0}$ × $\quad \boxed{s=1}$

There is equivalence with $I = \{p \leftarrow 1, q \leftarrow 0, r \leftarrow 0, s \leftarrow 1\}$

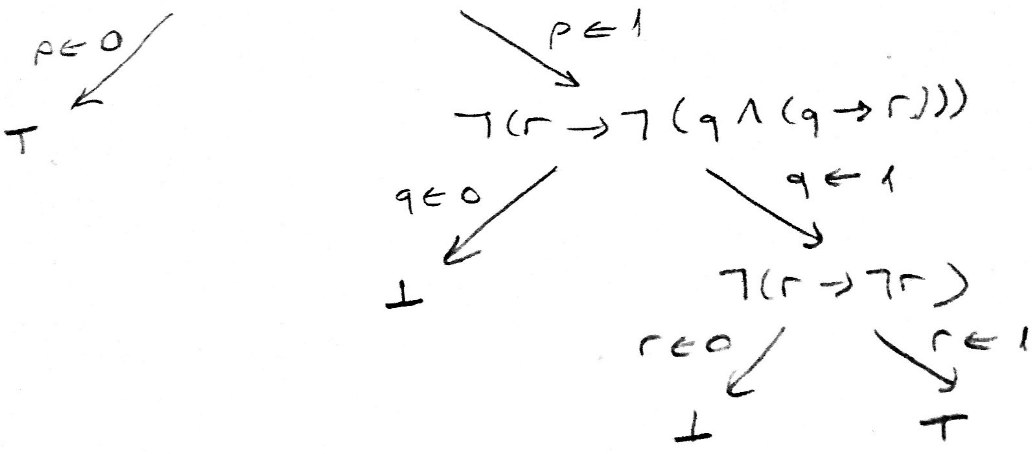
Finally, check both sides equal to T.

(a) $\neg p \vee q \vee r \equiv (s \wedge q) \vee \neg p \vee \neg s \vee r$

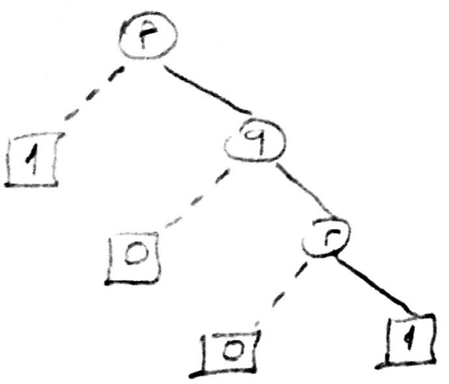


For all interpretations, both formulas are equal. So, we can say that $A \equiv B$.

(3) a) $\neg((p \rightarrow r) \rightarrow \neg((p \rightarrow q) \wedge (p \wedge q \rightarrow r)))$



b)



c) Same with part b.