CENG - 424 HW - 4

Q1/

1) $\forall x . \forall y ((A(x,y) \land B(x,y)) \to e(x))$ Premise

2) $\forall z (c(z))$ Premise

3) $\exists z ((c(z) \land d(z)) \to F(z))$ Premise

4) $\forall y (F(y) \to \exists x (A(x,y) \land B(x,y)))$ Premise

5) $\forall x . d(x)$ Premise

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6) $c(alper)$ UI : 2

7) $d(alper)$ UI : 5

8) $c(alper) \land d(alper)$ AI : 6, 7

9) $(c(alper) \land d(alper)) \to f(alper)$ EI : 3

10) $f(alper)$ MP : 8, 9

11) $f(alper) \to \exists x (A(x, alper) \land B(x, alper))$ AI : 4

12) $f(alper) \to (A(hasan, alper) \land B(hasan, alper))$ EI : 11

13) $A(hasan, alper) \land B(hasan, alper)$ MP : 10, 12

14) $(A(hasan, alper) \land B(hasan, alper)) \to e(hasan)$ AI : 1

15) $e(hasan)$ MP : 13, 14

16) $\exists (x)(e(x))$ EG : 15

Q2

First write axioms as logical sentences and convert them into clausal form:

1) $\forall x\,(loves(Jane, x) \Rightarrow traveller(x))$

$\equiv \forall x\,(\neg loves(Jane, x) \vee traveller(x))$ (I)

$\equiv \neg loves(Jane, x) \vee traveller(x)$ (A)

$\equiv \{\neg loves(Jane, x), traveller(x)\}$ (O)

2) $\exists y\,((\neg earn(y) \wedge doctor(y)) \rightarrow \neg travel(y))$

$\equiv \exists y\,(\neg(\neg earn(y) \wedge doctor(y)) \vee \neg travel(y))$ (I)

$\equiv \exists y\,((earn(y) \vee \neg doctor(y)) \vee \neg travel(y))$ (N)

$\equiv \{earn(a), \neg doctor(a), \neg travel(a)\}$ (E, D, O)

3) $doctor(Jim) \equiv \{doctor(Jim)\}$ (O)

4) $\exists z\,((doctor(z) \wedge \neg work(z)) \rightarrow \neg earn(z))$

$\equiv \{\neg doctor(a), work(a), \neg earn(a)\}$ (I, N, E, D, O)

5) $\exists x\,(\neg travel(x) \rightarrow \neg traveller(x))$

$\equiv \{travel(x), \neg traveller(x)\}$ (I, E, O)

And our goal becomes:

$\neg work(Jim) \rightarrow \neg loves(Jane, Jim)$

$\equiv \{work(Jim), \neg loves(Jane, Jim)\}$

1) { ¬loves (Jane, x), traveller (x) } premise

2) { earn (a), ¬doctor (a), ¬travel (a) } premise

3) { doctor (Jim) } premise

4) { ¬doctor (a), work (a), ¬earn (a) } premise

5) { travel (x), ¬traveller (x) } premise

6) { ¬work (Jim) } Negated Goal

7) { loves (Jane, Jim) }

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8) { earn (Jim), ¬travel (Jim) } 2,3

9) { work (Jim), ¬earn (Jim) } 3,4

10) { ¬travel (Jim), work (Jim) } 8,9

11) { ¬travel (Jim) } 6,10

12) { ¬traveller (Jim) } 5,11

13) { ¬loves (Jane, Jim) } 1,12

14) { } 7,13

Empty clause means that we reached the conclusion.