

To calculate the volume of the moving space of each hand, it easier to use spherical coordinates as the main motion is rotation. To get the maximum movement space volume, one DOF rotation around the hand base while the hand is fully stretched. The origin point is shown in the figure.

$$V = \int_{\theta=0}^{\theta\pi/2} \int_{\varphi=0}^{\varphi=\pi} \int_{\rho=9}^{\rho=85} \rho^2 \sin(\varphi) \ d\rho \ d\varphi \ d\theta$$
$$V = [\theta]_0^{\frac{\pi}{2}} \cdot [-\cos(\varphi)]_0^{\pi} \cdot [\frac{\rho^3}{3}]_9^{85}$$
$$V = 482.67 \ cm^3$$

