Time Series Analysis in Python with statsmodels

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What is statsmodels?

- A library for statistical modeling, implementing standard statistical models in Python using NumPy and SciPy
- Includes:
  - Linear (regression) models of many forms
  - Descriptive statistics
  - Statistical tests
  - Time series analysis
  - ...and much more

What is Time Series Analysis?

- Statistical modeling of time-ordered data observations
- Inferring structure, forecasting and simulation, and testing distributional assumptions about the data
- Modeling dynamic relationships among multiple time series
- Broad applications e.g. in economics, finance, neuroscience, signal processing...

Talk Overview

- Brief update on `statsmodels` development
- Aside: user interface and data structures
- Descriptive statistics and tests
- Auto-regressive moving average models (ARMA)
- Vector autoregression (VAR) models
- Filtering tools (Hodrick-Prescott and others)
- Near future: Bayesian dynamic linear models (DLMs), ARCH / GARCH volatility models and beyond

Statsmodels development update

- We're now on GitHub! Join us:

    `http://github.com/statsmodels/statsmodels`

- Check out the slick Sphinx docs:

    `http://statsmodels.sourceforge.net`

- Development focus has been largely **computational**, i.e. writing correct, tested implementations of all the common classes of statistical models

Statsmodels development update

- Major work to be done on providing a nice integrated **user interface**
- We **must** work together to close the gap between R and Python!
- Some important areas:
  - Formula framework, for specifying model design matrices
  - Need integrated rich statistical data structures (**pandas**)
  - Data visualization of results should always be a few keystrokes away
 - Write a "Statsmodels for R users" guide

Aside: statistical data structures and user interface

- While I have a captive audience...
- **Controversial fact**: **pandas** is the only Python library *currently* providing data structures matching (and in many places exceeding) the richness of R's data structures (for statistics)
    - Let's have a BoF session so I can justify this statement
- Feedback I hear is that end users find the fragmented, incohesive set of Python tools for data analysis and statistics to be confusing, frustrating, and certainly not compelling them to use Python...
   - (Not to mention the packaging headaches)

Aside: statistical data structures and user interface

- We need to "commit" **ASAP** (not 12 months from now) to a high level data structure(s) as the "primary data structure(s) for statistical data analysis" and communicate that clearly to end users
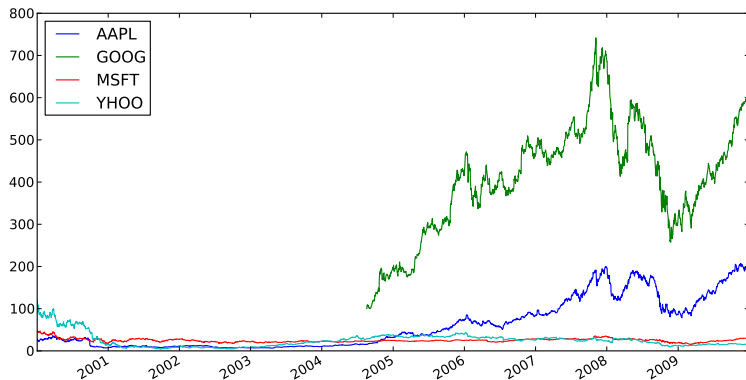 - Or we might as well all start programming in R...

Example data: EEG trace data
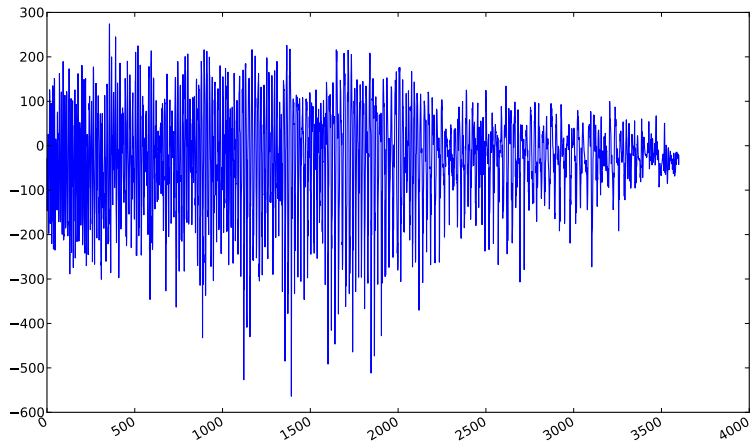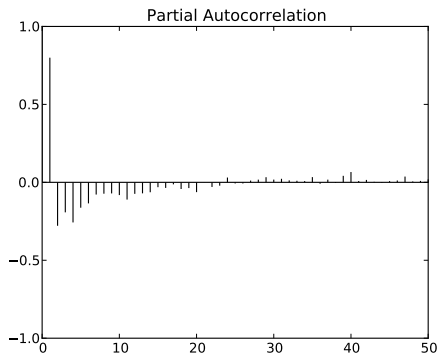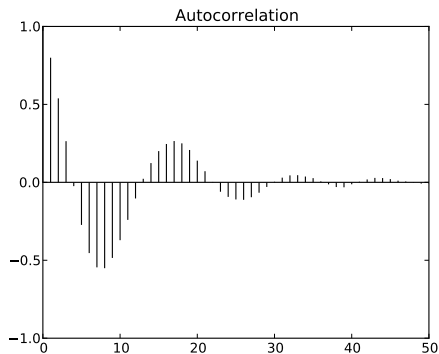
Example data: Macroeconomic data
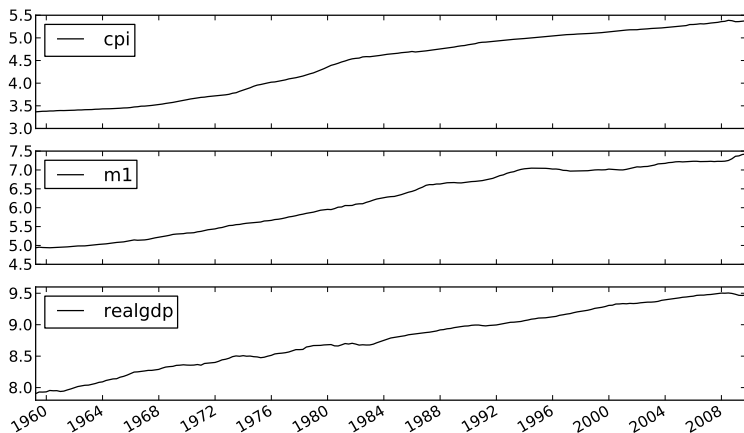
Example data: Stock data

Descriptive statistics

- Autocorrelation, partial autocorrelation plots
- Commonly used for identification in ARMA(p,q) and ARIMA(p,d,q) models

```
acf = tsa.acf(eeg, 50)
pacf = tsa.pacf(eeg, 50)
```

Statistical tests

- Ljung-Box test for zero autocorrelation
- Unit root test for cointegration (Augmented Dickey-Fuller test)
- Granger-causality
- Whiteness (iid-ness) and normality
- See our conference paper (when the proceedings get published!)

Autoregressive moving average (ARMA) models

- One of most common univariate time series models:

$$y_t = \mu + a_1 y_{t-1} + ... + a_k y_{t-p} + \epsilon_t + b_1 \epsilon_{t-1} + ... + b_q \epsilon_{t-q}$$

where $E(\epsilon_t, \epsilon_s) = 0$, for $t \neq s$ and $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$

- Exact log-likelihood can be evaluated via the Kalman filter, but the "conditional" likelihood is easier and commonly used

- `statsmodels` has tools for simulating ARMA processes with known coefficients a_i, b_i and also estimation given specified lag orders

```python
import scikits.statsmodels.tsa.arima_process as ap
ar_coef = [1, .75, -.25]; ma_coef = [1, -.5]
nobs = 100
y = ap.arma_generate_sample(ar_coef, ma_coef, nobs)
y += 4 # add in constant
```

ARMA Estimation

- Several likelihood-based estimators implemented (see docs)

```
model = tsa.ARMA(y)
result = model.fit(order=(2, 1), trend='c',
                   method='css-mle', disp=-1)
result.params
# array([ 3.97, -0.97, -0.05, -0.13])
```

- Standard model diagnostics, standard errors, information criteria (AIC, BIC, ...), etc available in the returned `ARMAResults` object

Vector Autoregression (VAR) models

- Widely used model for modeling multiple (K-variate) time series, especially in macroeconomics:

$$Y_t = A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \epsilon_t, \ \ \epsilon_t \sim \mathcal{N}(0, \Sigma)$$

- Matrices A_i are $K \times K$.
- Y_t must be a *stationary* process (sometimes achieved by differencing). Related class of models (VECM) for modeling nonstationary (including cointegrated) processes

Vector Autoregression (VAR) models

```
>>> model = VAR(data); model.select_order(8)
                 VAR Order Selection
=========================================================
            aic           bic           fpe          hqic
---------------------------------------------------------
0        -27.83        -27.78     8.214e-13        -27.81
1        -28.77        -28.57     3.189e-13        -28.69
2        -29.00       -28.64*     2.556e-13        -28.85
3        -29.10        -28.60     2.304e-13       -28.90*
4        -29.09        -28.43     2.330e-13        -28.82
5        -29.13        -28.33     2.228e-13        -28.81
6       -29.14*        -28.18    2.213e-13*        -28.75
7        -29.07        -27.96     2.387e-13        -28.62
=========================================================

* Minimum
```

Vector Autoregression (VAR) models

```
>>> result = model.fit(2)
>>> result.summary() # print summary for each variable
<snip>
Results for equation m1
=====================================================
            coefficient    std. error   t-stat   prob
-----------------------------------------------------
const          0.004968       0.001850    2.685   0.008
L1.m1          0.363636       0.071307    5.100   0.000
L1.realgdp    -0.077460       0.092975   -0.833   0.406
L1.cpi        -0.052387       0.128161   -0.409   0.683
L2.m1          0.250589       0.072050    3.478   0.001
L2.realgdp    -0.085874       0.092032   -0.933   0.352
L2.cpi         0.169803       0.128376    1.323   0.188
=====================================================

<snip>
```

Vector Autoregression (VAR) models

```
>>> result = model.fit(2)
>>> result.summary() # print summary for each variable
<snip>
Correlation matrix of residuals
              m1    realgdp        cpi
m1        1.000000 -0.055690 -0.297494
realgdp  -0.055690  1.000000  0.115597
cpi      -0.297494  0.115597  1.000000
```
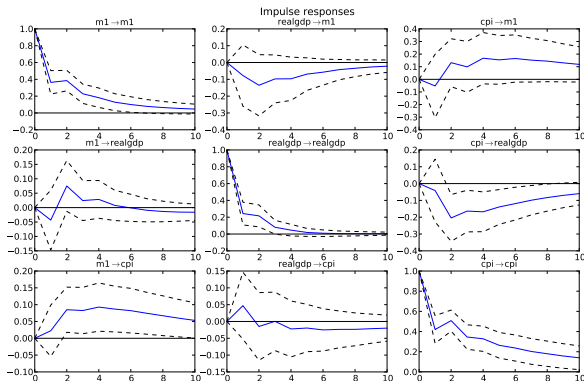
VAR: Impulse Response analysis
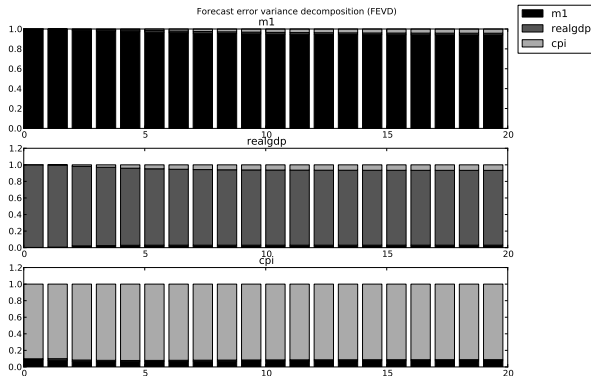
- Analyze systematic impact of unit "shock" to a single variable

```
irf = result.irf(10)
irf.plot()
```

VAR: Forecast Error Variance Decomposition

- Analyze contribution of each variable to forecasting error

```
fevd = result.fevd(20)
fevd.plot()
```


Forecast error variance decomposition (FEVD)

VAR: Statistical tests

```
In [137]: result.test_causality('m1', ['cpi', 'realgdp'])
Granger causality f-test
==============================================================
    Test statistic    Critical Value      p-value         df
--------------------------------------------------------------
          1.248787          2.387325        0.289   (4, 579)
==============================================================
H_0: ['cpi', 'realgdp'] do not Granger-cause m1
Conclusion: fail to reject H_0 at 5.00% significance level
```
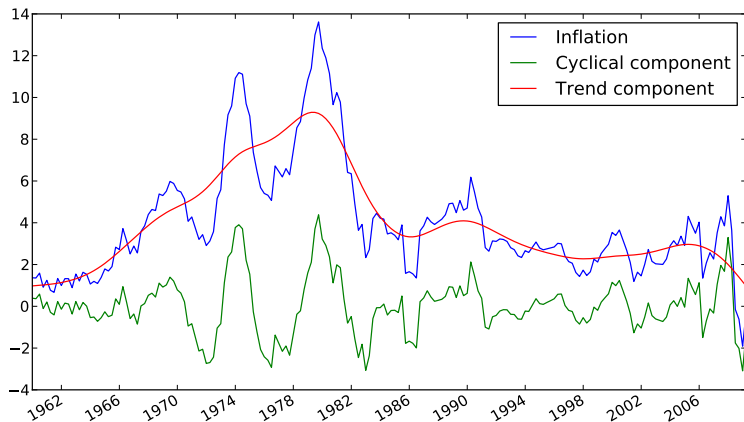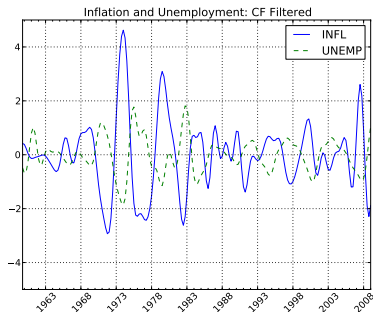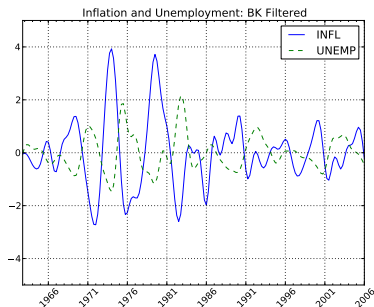
Filtering

- Hodrick-Prescott (HP) filter separates a time series y_t into a trend τ_t and a cyclical component ζ_t, so that $y_t = \tau_t + \zeta_t$.

Filtering

- In addition to the HP filter, 2 other filters popular in finance and economics, Baxter-King and Christiano-Fitzgerald, are available
- We refer you to our paper and the documentation for details on these:

Preview: Bayesian dynamic linear models (DLM)

- A state space model by another name:

$$y_t = F_t'\theta_t + \nu_t, \quad \nu_t \sim \mathcal{N}(0, V_t)$$
$$\theta_t = G\theta_{t-1} + \omega_t, \quad \omega_t \sim \mathcal{N}(0, W_t)$$

- Estimation of basic model by Kalman filter recursions. Provides elegant way to do time-varying linear regressions for forecasting
- Extensions: multivariate DLMs, stochastic volatility (SV) models, MCMC-based posterior sampling, mixtures of DLMs

Preview: DLM Example (Constant+Trend model)

```
model = Polynomial(2)
dlm = DLM(close_px['AAPL'], model.F, G=model.G, # model
          m0=m0, C0=C0, n0=n0, s0=s0, # priors
          state_discount=.95) # discount factor
```
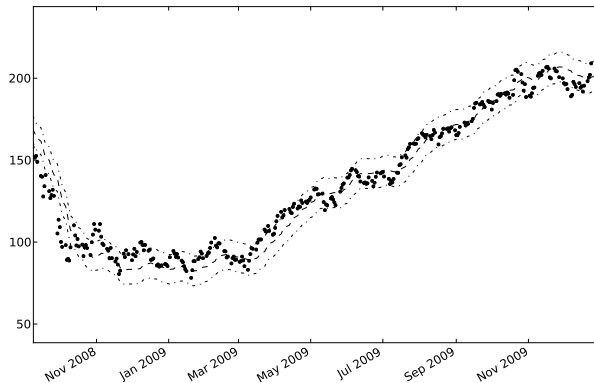
Preview: Stochastic volatility models

JPY-USD Exchange Rate Volatility Process
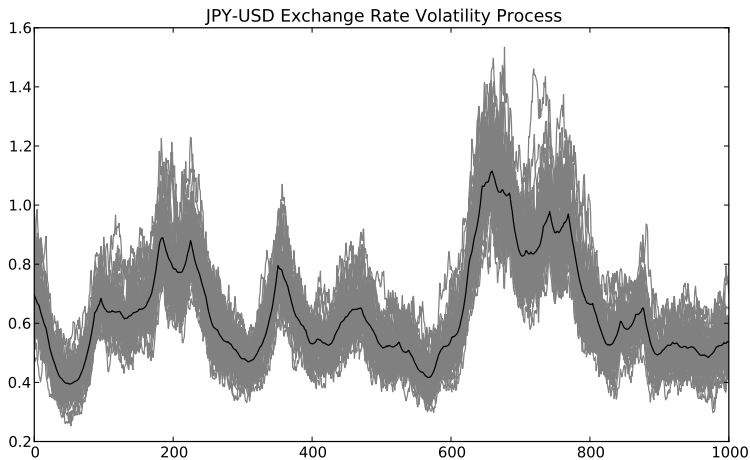
- ARCH / GARCH models for volatility
- Structural VAR and error correction models (ECM) for cointegrated processes
- Models with non-normally distributed errors
- Better data description, visualization, and interactive research tools
- More sophisticated Bayesian time series models

Conclusions

- We've implemented many foundational models for time series analysis, but the field is very broad
- User interface can and should be much improved
- Repo: `http://github.com/statsmodels/statsmodels`
- Docs: `http://statsmodels.sourceforge.net`
- Contact: `pystatsmodels@googlegroups.com`