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CSE 150 - Foundations of Computer Science: Honors
Professor Bender

Homework 1A

Problem 1

Write $\mathbf{P} \implies \mathbf{Q}$ using $\wedge$ and $\neg$. Show that your two representation are equivalent.

Response

P $\implies Q \iff \neg(P \wedge \neg Q)$

P	Q	$P \implies Q$
T	T	**T**
T	F	**F**
F	T	**T**
F	F	**T**

P	Q	$P \wedge Q$	$P \wedge \neg Q$	$\neg(P \wedge \neg Q)$
T	T	T	F	**T**
T	F	F	T	**F**
F	T	F	F	**T**
F	F	F	F	**T**

Problem 2

Prove that the prepositional formulas

$$\mathbf{P} \vee \mathbf{Q} \vee \mathbf{R}$$

and

$$(\mathbf{P} \wedge \neg \mathbf{Q}) \vee (\mathbf{Q} \wedge \neg \mathbf{R}) \vee (\mathbf{R} \wedge \neg \mathbf{P}) \vee (\mathbf{P} \wedge \mathbf{Q} \wedge \mathbf{R})$$

are equivalent.

Response

P	Q	R	$P \vee Q \vee R$
T	T	T	**T**
T	T	F	**T**
T	F	T	**T**
T	F	F	**T**
F	T	T	**T**
F	T	F	**T**
F	F	T	**T**
F	F	F	**F**

P	Q	R	$P \wedge \neg Q$	$Q \wedge \neg R$	$R \wedge \neg P$	$P \wedge Q \wedge R$	(P $\wedge\neg$Q) $\vee$ (Q $\wedge\neg$R) $\vee$ (R $\wedge\neg$P) $\vee$ (P $\wedge$ Q $\wedge$ R)
T	T	T	F	F	F	T	**T**
T	T	F	F	T	F	F	**T**
T	F	T	T	F	F	F	**T**
T	F	F	T	F	F	F	**T**
F	T	T	F	F	T	F	**T**
F	T	F	F	T	F	F	**T**
F	F	T	F	F	T	F	**T**
F	F	F	F	F	F	F	**F**

Problem 3

(a) **Write the biconditional ($\iff$) using only implies ($\implies$) and and ($\wedge$). Prove that the new version is equivalent.**

(b) **Write it using only $\vee$ and $\neg$. Show your derivation.**

Response

(a) $(P \iff Q) \iff ((P \implies Q) \wedge (Q \implies P))$

P	Q	$P \iff Q$
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**T**

P	Q	$P \implies Q$	$Q \implies P$	$(P \implies Q) \wedge (Q \implies P)$
T	T	T	T	**T**
T	F	F	T	**F**
F	T	T	F	**F**
F	F	T	T	**T**

(b) $(P \iff Q) \iff \neg(\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$

Proven Equivalence in Class:

$$(P \implies Q) \iff (\neg P \vee Q) \qquad \& \qquad (Q \implies P) \iff (\neg Q \vee P)$$

From 3a:

$$(P \iff Q) \iff ((P \implies Q) \wedge (Q \implies P))$$

Using DeMorgan's Law:

$$((P \implies Q) \wedge (Q \implies P)) \iff \neg(\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P))$$

P	Q	$\neg P \vee Q$	$\neg Q \vee P$	$\neg(\neg P \vee Q)$	$\neg(\neg Q \vee P)$	$\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$
T	T	T	T	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

$\neg(\neg(\neg P \vee Q) \vee \neg(\neg Q \vee P))$
T
F
F
T