

Homework 2B

Problem 5

Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$ has the listed properties:

1. $f(x) = 2x$ (one-to-one but not onto)

Response

2. $f(x) = x + 1$ (one-to-one but not onto)

Response

3. $f(x) = \text{if } x \text{ is odd then } x - 1, \text{ lse } x + 1$ (bijective)

Response

Problem 6

Show that the product $(a + bi)(c + di)$ of two complex numbers can be evaluated using just three real-number multiplications. You may use a few extra additions

Response

Problem 7

Given a fixed function $f : A \rightarrow A$. An element $a \in A$ is called a *fixedpoint* of f if $f(a) = a$. Find the set of fixed points for each of the following functions.

1. $f : A \rightarrow A$ where $f(x) = x$

Response

2. $f : \mathbb{N} \rightarrow \mathbb{N}$ where $f(x) = x + 1$

Response

3. $f : \mathbb{N}_6 \rightarrow \mathbb{N}_6$ where $f(x) = 2x \bmod 6$

Response

4. $f : \mathbb{N}_6 \rightarrow \mathbb{N}_6$ where $f(x) = 3x \bmod 6$

Response

Problem 8

Let $f(x) = x^2$ and $g(x, y) = x + y$. Find compositions that use the functions f and g for each of the following expressions.

1. $(x + y)^2$
Response

2. $x^2 + y^2$
Response

3. $(x + y + z)^2$
Response

4. $x^2 + y^2 + z^2$
Response