

## Collaborated with Ivan Lin

Similar to part a, the professors start by agreeing on a number  $L$  that has a value much larger than one trillion. The first professor picks a random number  $R_1$  between 0 and  $L-1$  inclusive. However, instead of the first professor adding his salary to the random number, he splits it into  $n$  smaller random numbers that add up to the initial random number  $r_1$  to  $r_n$ . He then randomly picks a number between  $r_1$  to  $r_n$  inclusive before distributing the rest to the other professors. After the other  $n-1$  professors repeat the same process, each professor should have two random numbers: the random number they chose between 0 and  $L-1$  ( $R_n$ ) and the sum of all the random pieces they were given ( $\sum_{i=1}^n r_i$ ). The first professor then adds either  $R_1$  or  $\sum_{i=1}^n r_i$  to his salary (for the sake of this explanation, I'll say he uses  $R_1$ ) and mods it by  $L$  to maintain a uniform distribution.

$$\text{Current Total} = (S_1 + R_1) \bmod L$$

Rather than telling the professor next to him, the first professor says his number aloud for all professors to hear. The next professor volunteers and then adds his salary ( $S_2$ ) plus his random number ( $R_2$ ) modded by  $L$  to the current total.

$$\text{Current Total} = (S_1 + R_1 + S_2 + R_2) \bmod L$$

The remaining professors do the same resulting in:

$$\text{Total} = (\text{Total Professor Salaries } (\sum_{i=1}^n S_i) + \text{Total Random Numbers } (\sum_{i=1}^n R_i)) \bmod L$$

To figure out the total professor salaries, the professors all subtract the sum of their random pieces ( $\sum_{i=1}^n r_i$ ) then mod by  $L$

$$\sum_{i=1}^n R_i = \sum_{i=1}^n \sum_{i=1}^n r_i$$

$$\text{Total Professor Salaries } (\sum_{i=1}^n S_i) = (\text{Total} - \text{Total Random Pieces } (\sum_{i=1}^n \sum_{i=1}^n r_i)) \bmod L$$

To find the average of their salaries, the professors divide their total by  $n$ .

$$\text{Average Professor Salary} = \text{Total Professor Salaries} / n$$