Math 202 A01
Spring 2019
Test 1 Version B
30/01/2019
Instructor: W. Thompson
Time Limit: 50 Minutes

Name (Print): Tom's Solutions

♪Hey I just met you,
and this is crazy, ♫
but here's my,

Student Number V00 ♪♫So call me maybe ♪♫

Tutorial Section T0 ______________________

This exam contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, calculator, or notes on this exam.
You are required to show your work on each problem on this exam. The following rules apply:

- **Understand what each question is asking.** There is sometimes more than one way to solve a problem. However, part of this course is making sure that you know how to solve a problem using the tools that are taught and not just on obtaining the correct answer. If a question asks you to solve something in a very specific way, you must do so.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- **Fully evaluate when possible.** If it is possible to evaluate an expression further, do so. For example, $\sin(\pi/4) = 1/\sqrt{2}$ is fully evaluated.
- **Don't simplify your answer if the question does not ask you to.** For the purposes of marking, it is easier to determine the accuracy of a method prior to simplification. Only simplify when told to do so.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	8	
3	7	
4	10	
Total:	35	

Do not write in the table above.

1. Answer the following short problems.

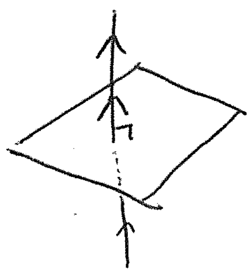
(a) (2 points) Form the equation of the plane containing the point $P(2,4,-1)$ with normal vector $\mathbf{n} = \langle 3,-4,1\rangle$.

$$\vec{n} \cdot (\langle x,y,z\rangle - \vec{P}) = 0$$

$$\Rightarrow \langle 3,-4,1\rangle \cdot \langle x-2, y-4, z+1\rangle = 0$$

$$\Rightarrow \quad 3(x-2) - 4(y-4) + (z+1) = 0 \; //$$

(b) (2 points) Construct the line perpendicular to the plane $x+y+z=1$ with starting point $Q(1,0,-2)$.



$\vec{v} = \vec{n}$ of plane $= \langle 1,1,1\rangle$

so

$$\vec{r}(t) = \vec{q} + t\vec{v} = \langle 1,0,-2\rangle + t\langle 1,1,1\rangle$$
$$= \langle 1+t, t, -2+t\rangle //$$

(c) (3 points) Find the area of the **paralellogram** formed by edges $\mathbf{u} = \langle -1,0,3\rangle$ and $\mathbf{v} = \langle 0,-2,5\rangle$.

$$A = \|\vec{u}\times\vec{v}\| = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 3 \\ 0 & -2 & 5 \end{vmatrix} \right\|$$

$$= \|(6)\hat{i} - (-5)\hat{j} + (2)\hat{k}\|$$

$$= \sqrt{36+25+4}$$

$$= \sqrt{65} \; //$$

(d) (3 points) Find the angle between the planes $x+y=5$ and $-2x-y+2z=4$. Leave your answer fully evaluated.

$$\vec{n}_1 = \langle 1,1,0\rangle \qquad \vec{n}_2 = \langle -2,-1,2\rangle$$

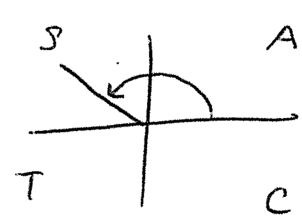
so $\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \cdot \|\vec{n}_2\| \cos\theta$

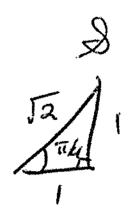
$$\Rightarrow -2-1 = \sqrt{1+1+0}\,\sqrt{4+1+4}\,\cos\theta.$$

$$\Rightarrow -3 = \sqrt{2}\times\sqrt{9}\cos\theta$$

$$\Rightarrow -\frac{1}{\sqrt{2}} = \cos\theta.$$

$$\Rightarrow \theta = \frac{3\pi}{4} \; //.$$





2. Determine whether or not the following limits exist. Justify your answer with an appropriate method and brief explanation.

(a) (4 points)

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y^2}{3x^4+y^2}$$

$$0 \leq \left|\frac{2x^2y^2}{3x^4+y^2}\right| = \frac{2x^2y^2}{3x^4+y^2} \leq \frac{2x^2(\cancel{3x^4}+y^2)}{3x^4\cancel{+}y^2}$$

$$= 2x^2 .$$

Since $\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} 2x^2 = 0$ then by Squeeze lemma.

$$\lim_{(x,y)\to(0,0)} \left|\frac{2x^2y^2}{3x^4+y^2}\right| = \lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{3x^4+y^2} = 0 //.$$

(b) (4 points)

$$\lim_{(x,y)\to(0,0)} \frac{3x^4\cos^2(y)}{(x+y)^4}$$

use the path $x=t, y=0$ then as $t\to 0$

$$\lim_{t\to 0} \frac{3t^4\cos^2(0)}{(t+0)^4} = \lim_{t\to 0} \frac{3t^4\times 1}{t^4} = 3 .$$

use the path $x=0, y=t$ then as $t\to 0$.

$$\lim_{t\to 0} \frac{3\times 0^4\cos^2(t)}{(0+t)^4} = \lim_{t\to 0} \frac{0}{t^4} = 0 .$$

By the two-path test, since two paths give different limits the limit DNE //

3. For the following consider the function $f(x, y, z) = x^2 + y^2 + z^2 + 8x - 6z$

(a) (3 points) Determine the radius and center of the sphere given by $f(x, y, z) = 0$.

$$x^2+8x+4^2-4^2+y^2+z^2-6z+3^2-3^2=0$$

$$\Rightarrow (x+4)^2+y^2+(z-3)^2-16-9=0$$

$$\Rightarrow (x+4)^2+y^2+(z-3)^2=25 \quad (=5^2)$$

Center $= (-4, 0, 3)$ and radius $= 5$.

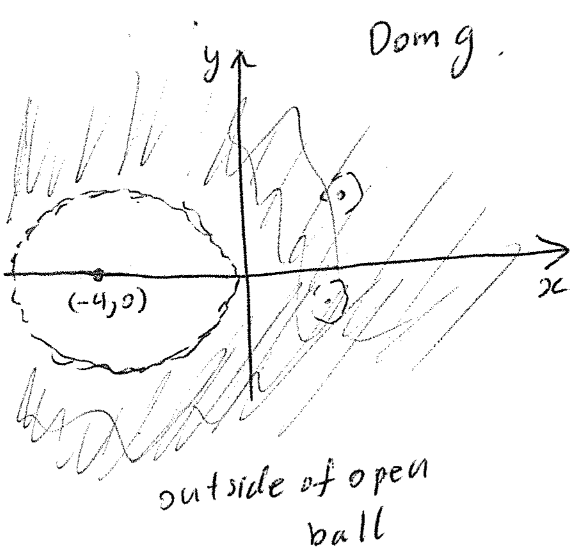
(b) (4 points) Graph the domain of the function $g(x, y) = \ln(f(x, y, 0))$. Is it open, closed, neither, or both? Is it bounded? Justify your answer by referring to their definition.
Hint: Use part (a).

$$g(x,y) = \ln((x+4)^2+y^2+(0-3)^2-25)$$

$$= \ln((x+4)^2+y^2+9-25)$$

$$= \ln((x+4)^2+y^2-16)$$

need $(x+4)^2+y^2-16>0 \Rightarrow (x+4)^2+y^2>16$



It is open because ~~this~~ every point is an interior point (or that it doesn't contain it's boundary $(x+4)^2+y^2=16$).

It is unbounded because you can't fit a ball around it.

4. Consider the function $f(x, y) = \arctan(x/y)$

(a) (1 point) What is the collection of points where $f(x, y)$ is continuous?

$$y \neq 0 \quad //$$

(b) (4 points) Compute the second partial derivative $\frac{\partial^2 f}{\partial x \partial y}$.

$$f_y = \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(-\frac{x}{y^2}\right) = \frac{1}{1+\frac{x^2}{y^2}} \times \left(-\frac{x}{y^2}\right) = \frac{-x}{y^2+x^2}.$$

$$f_{yx} = \frac{(y^2+x^2)\frac{\partial}{\partial x}[-x] - (-x)\frac{\partial}{\partial x}[x^2+y^2]}{(x^2+y^2)^2} = \frac{-(y^2+x^2)+x(2x+0)}{(x^2+y^2)^2}$$

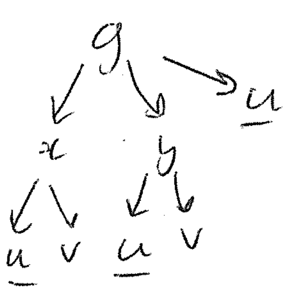
$$= \frac{-y^2+x^2+2x^2}{(x^2+y^2)^2}$$

$$= \frac{x^2-y^2}{(x^2+y^2)^2} \quad //$$

(c) (5 points) Consider the function $g(x, y, u) = f(x, y) + y^2u^2$ where $x = 2uv$ and $y = u^2 - v^2$. Construct a branch diagram and use the multivariable chain rule to compute $\frac{\partial g}{\partial u}$. Do not expand your final answer.

$$g(x,y,u) = \arctan\left(\frac{x}{y}\right) + y^2 u$$

Then by the branch



$$g_u = g_x x_u + g_y y_u + \frac{\delta g}{\delta u}.$$

$$= \left(\frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{1}{y}\right) + 0\right)(2v)$$

$$+ \left(\frac{1}{1+\left(\frac{x}{y}\right)^2}\left(-\frac{x}{y^2}\right) + 2yu^2\right)(2u) + 2y^2u$$

$$= \left(\frac{1}{1+\left(\frac{2uv}{u^2-v^2}\right)^2}\left(\frac{1}{u^2-v^2}\right)\right)(2v)$$

$$+ \left(\frac{1}{1+\left(\frac{2uv}{u^2-v^2}\right)}\left(-\frac{2uv}{u^2-v^2}\right)\right)(2u) + 2(u^2-v^2)^2 u \quad //$$

-This Page is for Scratch Work-