Math 202 A01 **Instructor: W. Thompson**

Fall 2023 **Midterm 1 VA**

October 6th 2023 **Time Limit: 50 Minutes**

- **DO NOT open the exam booklet until you are told to begin. Write your name and student number below and read the instructions below. This exam is out of 100 points.**

Solutions

- **The only approved calculator is the Sharp EL-510 R models.** All other calculators will be confiscated.
- **There is a bubble sheet at the back of the exam.** Do not rip it out. Fill in your answers to the multiple choice on this sheet. You must show some relevant work for your multiple choice to be graded. No part marks are given for multiple choice.
- **Understand what each question is asking.** There is more than one way to solve some problems. If you are asked to use a specific method, you are being tested on using that method and not getting the answer. If no particular method is asked, you are free to use any method introduced in the course.
- **Organize your work** in a reasonably neat and coherent way in the space provided below each question. Work scattered over a page without a clear ordering will receive little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

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Multiple Choice

Read all directions carefully and write all work below each question. Circle your answers on the bubble sheet at the back of the exam. Do not rip out the bubble sheet.

1. (10 points) Determine the radius of the sphere given by

$$x^2 + y^2 + z^2 + 4x - 6y + 8z + 4 = 0$$

(A) $R = 2$; (B) $R = 4$; (C) $R = 5$; (D) $R = 6$; (E) $R = 25$

$\Rightarrow (x^2+4x)+(y^2-6y)+(z^2+8z) = -4$

$\Rightarrow (x+2)^2-4+(y-3)^2-9+(z+4)^2-16 = -4$

$\Rightarrow (x+2)^2+(y-3)^2+(z+4)^2 = 25$

so $R = \sqrt{25} = 5$

2. (10 points) Determine the following limit

$$\lim_{(x,y)\to(0,0)} \frac{\sin(2x^2+2y^2)}{x^2+y^2} + 3\cos(x^2+y^2)$$

(A) 2; (B) 3; (C) 4; (D) 5; (E) 6

Let $u = x^2+y^2$ then

$= \lim_{u\to 0^+}\left(\frac{\sin(2u)}{u} + 3\cos(u)\right)$

$= \left(\lim_{u\to 0^+} \frac{\sin(2u)}{u}\right) + 3\cos(0)$ $\sim \frac{0}{0}$

$\overset{l'H}{=} \left(\lim_{u\to 0^+} \frac{2\cos(2u)}{1}\right) + 3$

$= 2\cos(0)+3$

$= 2+3 = 5$

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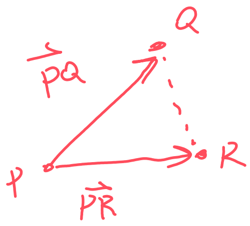
3. (10 points) Determine the area of a triangle formed by the vertices $P(1,2,0)$, $Q(1,2,1)$ and $R(5,-1,1)$.

(A) $\frac{1}{2}\sqrt{26}$; (B) $\frac{5}{2}$; (C) 3; (D) $\sqrt{26}$; (E) 6

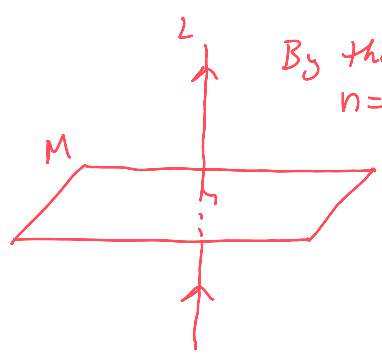


$$A = \frac{1}{2}\|\vec{PQ} \times \vec{PR}\|$$
$$= \frac{1}{2}\sqrt{\|\vec{PQ}\|^2\|\vec{PR}\|^2 - (\vec{PQ}\cdot\vec{PR})^2}$$
$$= \frac{1}{2}\sqrt{(0+0+1)(16+9+1) - 1^2}$$
$$= \frac{1}{2}\sqrt{25}$$
$$= 5/2$$

$\vec{PQ} = \langle 0,0,1\rangle$
$\vec{PR} = \langle 4,-3,1\rangle$

4. (10 points) Let L be the equation of the line containing point $P(2,0,1)$ with direction $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$. Let M be the equation of the plane containing point $P(2,1,2)$ and orthogonal to the line L. What is the z-value of M when $(x,y) = (4,-2)$?

(A) -6; (B) -2; (C) 0; (D) 8; (E) 10



By the picture $n = \vec{v}$

$M \Rightarrow \vec{n}\cdot(\vec{x} - \vec{P}(2,1,2)) = 0$

$\Rightarrow 3(x-2) - 2(y-1) - 2(z-2) = 0$

when $(x,y) = (4,-2)$

$\Rightarrow 3(2) - 2(-3) - 2(z-2) = 0$

$\Rightarrow -2(z-2) = -12$

$\Rightarrow z = 6+2 = 8$

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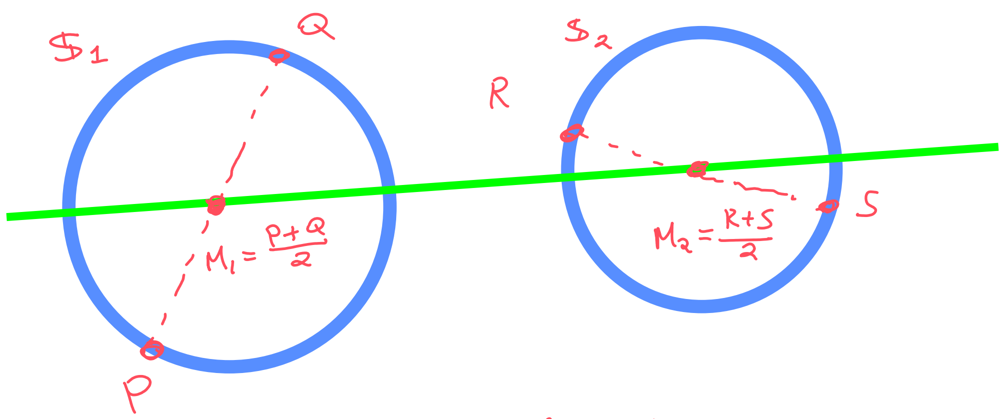
Written Questions

Read all directions carefully and write all work below each question. If you run out of space, you may continue your work on the back of the page but you must draw an arrow to indicate you have done so. Write your final answer in the box below each question.

5. (30 points) Let $\mathbb{S}_1$ be the sphere joining points $(0, 2, 3)$ and $(-4, -2, 3)$ as a diameter. Let $\mathbb{S}_2$ be the sphere joining points $(5, -2, 6)$ and $(1, 0, 4)$ as a diameter. Give equation of the line passing through the centers of both spheres.

P Q R S



The center of the spheres are

$$M_1 = \left(\frac{0+(-4)}{2}, \frac{2+(-2)}{2}, \frac{3+3}{2}\right) = (-2, 0, 3)$$

$$M_2 = \left(\frac{5+1}{2}, \frac{-2+0}{2}, \frac{6+4}{2}\right) = (3, -1, 5)$$

So $\vec{V} = \overrightarrow{M_1M_2} = \langle 5, -1, 2 \rangle$

and $\vec{L}(t) = M_1 + t\vec{V}$.

All possible expected answers:

1) $x = -2+5t$; $y = 0-t$; $z = 3+2t$ $\quad M_1 + t\overrightarrow{M_1M_2}$

2) $x = 3+5t$; $y = -1-t$; $z = 5+2t$ $\quad M_2 + t\overrightarrow{M_1M_2}$

3) $x = -2-5t$; $y = 0+t$; $z = 3-2t$ $\quad M_1 + t\overrightarrow{M_2M_1}$

4) $x = 3-5t$; $y = -1+t$; $z = 5-2t$ $\quad M_2 + t\overrightarrow{M_2M_1}$

x(t) = y(t) = z(t) =

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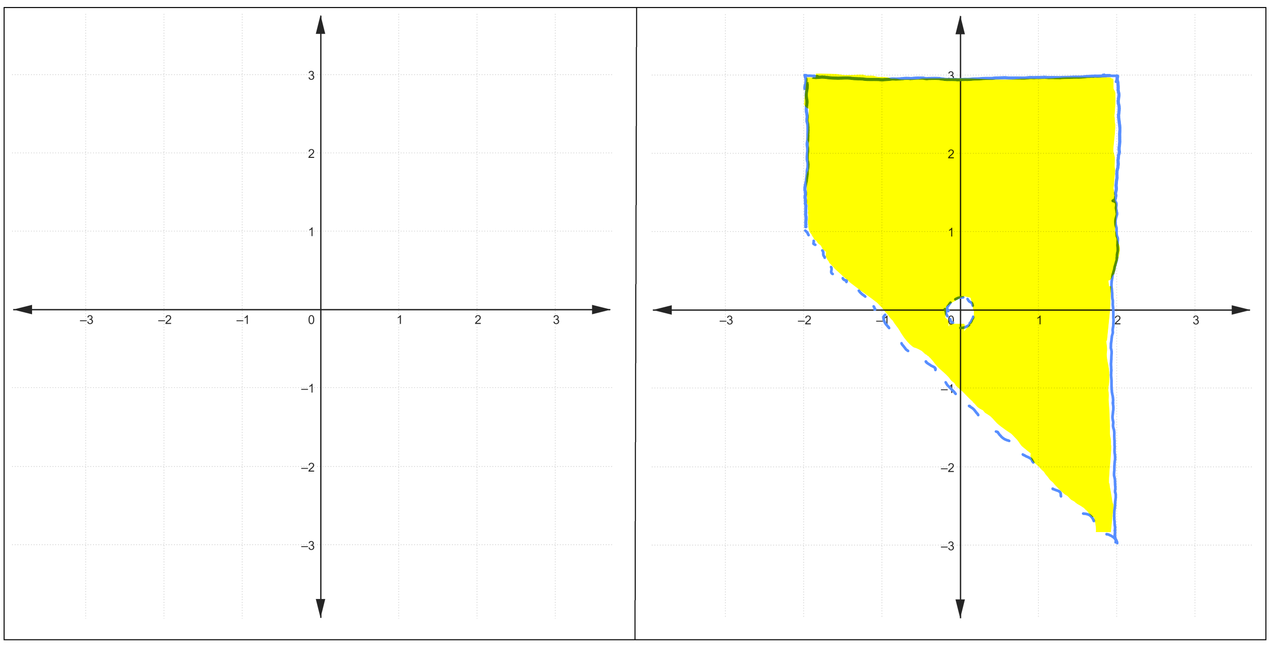
6. Consider the function below.

$$f(x,y) = \frac{\sqrt{4-x^2} + \ln(x^2+y^2) - \sqrt{9-y^2}}{\sqrt{x+y+1}}.$$



Graph for scratch work - *will not be graded* Final answer - *will be graded*

(a) (20 points) Sketch the domain for $f(x,y)$ in the graphs above. Shade the interior of the region and make sure to indicate whether a boundary is included (solid line) or not included (dashed line). The only thing being graded for this part is the accuracy of your sketch. Use the empty space below for any calculations.

Conditions

$$\Rightarrow \begin{cases} 4-x^2 \geq 0 & \Rightarrow & -2 \leq x \leq 2 \\ x^2+y^2 > 0 & \Rightarrow & (x,y) \neq (0,0) \\ 9-y^2 \geq 0 & \Rightarrow & -3 \leq y \leq 3 \\ x+y+1 > 0 & \Rightarrow & x+y > -1 \end{cases}$$

PARTS (b) AND (c) ARE CONTINUED ON THE FOLLOWING PAGE.

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(b) (5 points) Circle whether or not the domain is open, closed, or neither.

OPEN CLOSED NEITHER

Justify your claim below by referencing either appropriate definitions or results from lecture. No points will be awarded for an unjustified correct claim.

It is not open because it contains the boundary $x = \pm 2$ and $y = \pm 3$.

It is not closed because it does not contain $(0,0)$ or the boundary $x + y = -1$.

(c) (5 points) Circle whether or not the domain is bounded, unbounded, or neither.

BOUNDED UNBOUNDED NEITHER

Justify your claim below by referencing either appropriate definitions or results from lecture. No points will be awarded for an unjustified correct claim.

It is bounded because it fits in the ball $x^2 + y^2 < 100$ (say).

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[**Scrap Work**]

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