Dalhousie University Faculty of Computer Science
Design and Analysis of Algorithms
Assignment 5 CSCI 3110 Due: 2 Nov 2012

Problems are from the text (*Algorithms by Dasgupta, Papadimitriou, Vazirani p.120 - 126 and 148 - 155*).

1. **(6pts)** Let $G = (V, E)$ be a connected graph with n vertices and m edges with distinct edge costs. Let T be a spanning tree of G; we define the *bottleneck edge* of T to be the edge of T with the greatest cost. A spanning tree G is a *minimum bottleneck spanning tree* if there is no other spanning tree of G with a cheaper bottleneck edge.
   (a) Is every min. bottleneck tree of G a MST (min span. tree) of G? Prove or give a counter example.
   (b) Is every MST of G a min bottleneck tree of G? Prove or give a counter example.
2. **(8pts)** For every edge of a connected (communication) graph G, you have a bandwidth b_e. For every pair $u,\ v \in V$ you want to select a $u - -v$ path$P$ on which path the vertices will communicate. The bottleneck rate $b(P)$ of the path is the min bandwidth of any edge it contains ($b(P) = min_{e \in P} b_e$). The best achievable bottleneck rate for the pair $u, v \in G$ is the maximum, over all $U - -V$ paths P in G of the value $b(P)$. It is difficult to keep track of a path for each pair, so maybe a spanning tree T of G could be found such that for every pair of nodes, the unique path between them in T, actually attains the best achievable bottleneck rate for $u, v \in G$.

   Show that such a tree exists, and give an efficient algorithm to find one. That is, find an algorithm constructing a spanning tree T in which, for each pair $u,\ v \in V$, the bottleneck rate of the $u - -v$ path in T id the best achievable.
3. **(2 pts)** Ex. 4.4
4. **(2 pts)** Ex. 4.8
5. **(4 pts)** Ex. 4.14
6. **(4 pts)** Ex. 4.20
7. **(2 pts)** Ex. 5.7
8. **(10 pts)** Ex. 5.9