PA - Curs 1

6.10.2011

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e = element dintr-o algebră liberă
expresie

(S, Σ) signatura

$X = \{X_s\}_{s \in S}$

Alg. expr. = alg. liberă

$X \longrightarrow T_\Sigma(X) \ni e$. Se dă

$f \searrow \mathcal{A} \swarrow$ $\exists! f^\#$ cu $f^\#/_X = f$

A da valori variabilelor din X în alg $\mathcal{A}$ = a da o funcție $f: X \to A$

= a da un morfism $h: T_\Sigma(X) \to A$

Rezultatul evaluării lui e este $f^\#(e)$

$f: A \to B \overset{\text{bijectivă}}{\iff} (\forall b \in B)(\exists! a \in A)$ a.î. $f(a) = b$

$Alg_\Sigma(T_\Sigma(X), A) \longrightarrow \underset{\text{(mulțime)}}{Set(X, \mathcal{A})}$ $r(h) = h/_X$

$\forall f \in Set(X, \mathcal{A})$ $\exists! h \in ?_\Sigma(T_\Sigma(X), \mathcal{A})$

$r(h) = f$

Σ-alg. inițială T_Σ

$\underset{\text{(inițial)}}{I} \xrightarrow{\exists! \alpha} \forall O$

Cine e mulțimea inițială? $\emptyset$ (de la ea în $\forall$ altă mulțime e o funcție - funcția incluziunii

Monoid inițial
Grup inițial (∃ un morfism ✗ numai unul) {e}
{e}

semigrup inițial $\emptyset$

inel inițial $\mathbb{Z}$

semiinel inițial: IN $\quad (\mathcal{P}(S\times S), \cup, \emptyset, ;, \Delta\}$

$\quad (\mathcal{C}(A^*), \cup, \emptyset, \cdot, \{\lambda\})$

Ecuație condiționată $\quad (\forall X)\, l \doteq_s r \ \underline{\text{if } H} \quad l, r \in T_\Sigma(X)_s$

mulț. finită de egalități formale

$$(\forall X)\, \underbrace{H \Rightarrow}\, l \doteq_s r$$

$$\forall x\, y\, z\ (x\cdot y = x\cdot z \Rightarrow y = z)$$ Legea simplificării (ex. de ec. condiționată)

Rescrierea = semantică operațională pt PA.

Algebră satisface o ec condiționată

$$\mathcal{A} \models (\forall X)\, l \doteq_s r \text{ if } H \Leftrightarrow (\forall h: T_\Sigma(X) \to \mathcal{A}\ \Sigma\text{-morf}) \text{ dacă}$$
$$(\forall u \doteq_t v \in H\ \ h_t(u) = h_t(v)) \Rightarrow h_s(l) = h_s(r)$$

(lipsește dacă ec. e necondiționată)

Prog logică ediția a 3-a + articolul lui Țuțu

În ce condiții o specificație e program? - să se termine (∄ rescrieri infinite)
(rescrierea generată de specificație)
- confluența (rez. unicității f.n. unică)

cartea: Țiplea - Fundamentele algebrice ale informaticii
(ultimul capitol)

$Sen(\mathcal{A}) = \mathcal{A}\times\mathcal{A} = \{a \doteq_s b \mid a, b \in A_s\}_{s\in S}$

sentence $\quad = \{(a,b) \mid a, b \in A_s\}_{s\in S}$

Γ - mulț. ec cond

$A \models_\Sigma \Gamma \Leftrightarrow (\mathcal{A} \models_\Sigma \gamma,\ \forall \gamma \in \Gamma)$

- congruența semantică a lui $\mathcal{A}$

$$a \equiv_\Gamma^{\mathcal{A}} b \iff \forall h: \mathcal{A} \to \mathcal{M} \models \Gamma \quad h(a) = h(b)$$

$$\equiv_\Gamma^{\mathcal{A}} = \bigcap_{h: A \to \mathcal{M} \models \Gamma} Ker(h)$$

- conține toate consecințele lui Γ din algebra $\mathcal{A}$

	Programare logică ecuațională
$\Gamma \models (\forall X)\, G$	$\Gamma \models (\exists X)\, G$

EQLOG

Prog log. { ecuațională ; relațională }

(AI) Prolog

lb. pt Princ. prog. decl.

OBJ — J. Goguen

Maude — J. Meseguer

CAFEObj — japonez, R. Diaconescu (român)

CASL

$$\Gamma \models (\forall X)\, G \iff (\forall \mathcal{M} \models \Gamma)\ \mathcal{M} \models (\forall X) G \iff \forall h: T_\Sigma(X) \to \mathcal{M} \models \Gamma$$

orice gama-alg

$h_t(u) = h_t(v)$ pt orice $u \doteq_t v \in G$

$$\Gamma \models (\forall X)\, G \iff G \subseteq \equiv_\Gamma^{T_\Sigma(X)}$$

$\forall x \exists y \Leftarrow \exists y \forall x$

Dem.

$$\Gamma \models (\forall X)\, G \iff \forall h: T_\Sigma(X) \to \mathcal{M} \models \Gamma \ \forall u \doteq_t v \in G \ h_t(u) = h_t(v)$$

$$\Updownarrow$$

$$\forall u \doteq_t v \in G \ \forall h: T_\Sigma(X) \to \mathcal{M} \models \Gamma \quad h_t(u) = h_t(v)$$

$$\Updownarrow$$

$$\forall u \doteq_t v \in G \quad u \equiv_\Gamma^{T_\Sigma(X)} v$$

$$\Updownarrow$$

$$\forall u \doteq_t v \in G \quad (u, v) \in \equiv_\Gamma^{T_\Sigma(X)}$$

$$\Updownarrow$$

$$G \subseteq \equiv_\Gamma^{T_\Sigma(X)}$$

$h: A \to B \quad a \equiv_\Gamma^{\mathcal{A}} b \Rightarrow h(a) \equiv_\Gamma^{\mathcal{B}} h(b)$ $\qquad h(\equiv_\Gamma^{\mathcal{A}}) \subseteq \equiv_\Gamma^{\mathcal{B}}$

Dem.: Fie $a \equiv_\Gamma^{\mathcal{A}} b$ $\quad$ tb. ar. $h(a) \equiv_\Gamma^{B} h(b) \iff$

$\forall m: B \to \mathcal{M} \models \Gamma$

$m(h(a)) = m(h(b))$

Fie m un m.f

$m: B \to \mathcal{M} \models \Gamma$

$\mathcal{A} \xrightarrow{h;m} \mathcal{M} \models \Gamma$ $\quad (h;m)(a) = (h;m)(b)$

$\qquad \qquad \| $

$\qquad m(h(a))$

$A \xrightarrow{h} B$

$h: Sen(A) \to Sen(B)$

$h(a \doteq b) = h(a) \doteq h(b)$

$h: \mathcal{P}(Sen(A)) \to \mathcal{P}(Sen(B))$

$h(G) = \{h(g) \mid g \in G\}$

Consecință

$h: A \to B \quad h(\equiv_\Gamma^{\mathcal{A}}) \subseteq \equiv_\Gamma^{B}$

$h: T_\Sigma(X) \to T_\Sigma(Y)$

$\Gamma \models (\forall X) G \Rightarrow \Gamma \models (\forall Y) h(G)$

$\Downarrow$ dem

$G \subseteq \equiv_\Gamma^{T_\Sigma(X)}$

Aplic. $\Gamma \models (\forall X) G \iff G \subseteq \equiv_\Gamma^{T_\Sigma(X)}$ (mai sus)

$\Downarrow$ aplic h $\qquad h(G) \subseteq \equiv_\Gamma^{T_\Sigma(X)}$

$h(G) \subseteq h(\equiv_\Gamma^{T_\Sigma(X)}) \subseteq \equiv_\Gamma^{T_\Sigma(Y)}$

$\mathcal{A} \xrightarrow{\eta} \mathcal{A}/_{\equiv_\Gamma} = \mathcal{A}_\Gamma$ (factorizare) $\models \Gamma$ (Γ-alg)

$\forall h$ unif $\downarrow$ $\quad \exists h^\#$ $\quad B \models \Gamma \qquad \eta; h^\# = h$

$\eta(a) = \eta(b) \iff a \equiv b$

$\cdot$ ~~$A_\Gamma \models \Gamma$ (e Γ-alg)~~

$$\begin{tikzcd} A \arrow[r, "\forall h"] & B \\ T_\Sigma(X) \text{ projective} \arrow[u, "\exists f"] \arrow[ur, "\forall g"'] & \end{tikzcd}$$

$\forall s \in S$

$h_s : A_s \to B_s$ surj.

$(\exists f) \cdot f ; h = g$

Δ Curs 2

14.10.2011

(A) $\Gamma \models (\forall X) G \iff \forall \mathcal{A} \models \Gamma\ \forall h: T_\Sigma(X) \to \mathcal{A}\ \ h_s(l) = h_s(r)\ \ \forall l \doteq_s r \in G$

(B) $\Gamma \models (\exists X) G \iff \forall \mathcal{A} \models \Gamma\ \exists h: T_\Sigma(X) \to \mathcal{A}$
$h_s(l) \doteq h_s(r)\ \forall l \doteq_s r \in G$

$c \equiv^{\mathcal{A}}_\Gamma b \iff$
$\iff \forall h: \mathcal{A} \to \mathcal{M} \models \Gamma$
$h(c) = h(b)$

(P1) $\Gamma \models (\forall X) G \iff G \subseteq \equiv^{T_\Sigma(X)}_\Gamma$

(P2) $h: \mathcal{A} \to \mathcal{B}\quad h(\equiv^{\mathcal{A}}_\Gamma) \subseteq \equiv^{\mathcal{B}}_\Gamma$

(P3) $h: T_\Sigma(X) \to T_\Sigma(Y)\quad \Gamma \models (\forall) G \Rightarrow \Gamma \models \forall Y\ h(G)$

TM. HERBRAND echivalente pt $G \subseteq T_\Sigma(X) \times T_\Sigma(X)$

1) $\Gamma \models (\forall X) G$

2) $T_{\Sigma,\Gamma} \models (\exists X) G$

3) $\exists \psi: T_\Sigma(X) \to T_\Sigma\quad \Gamma \models (\forall \emptyset)\, \psi(G)\ \ (\iff \psi(G) \subseteq \equiv^{T_\Sigma}_\Gamma)$

1 => 2) $T_{\Sigma,\Gamma} \models \Gamma$

2 => 3 $\quad h: T_\Sigma(X) \to T_{\Sigma,\Gamma}\ \ \forall l \doteq_s r \in G\ \ h_s(l) = h_s(r)$

$$\begin{tikzcd} T_\Sigma \arrow[r, "\eta"] & T_{\Sigma,\Gamma} = T_\Sigma/_{\equiv_\Gamma} \\ T_\Sigma(X) \arrow[u, "\psi"] \arrow[ur, "h"'] & \end{tikzcd}$$

$T_\Sigma(X)$ alg liberă => proiectiv

$T_\Sigma(X)$ proiectiv $\exists \psi: T_\Sigma(X) \to T_\Sigma\quad \psi;\eta = h$

Fie $l =_s r \in G\quad h_s(l) = h_s(r)$

$(\psi\eta)_s(l) = (\psi\eta)_s(r) \Rightarrow$

$\Rightarrow (\psi_s;\eta_s)(l) = (\psi_s;\eta_s)(r) \Rightarrow$

$\Rightarrow \eta_s(\psi_s(l)) = \eta_s(\psi_s(r)) \Rightarrow$

$\Rightarrow \psi_s(l) \equiv_\Gamma \psi_s(r)\qquad \boxed{\psi(G) \subseteq \equiv_\Gamma}$ (A) $\Rightarrow$

$\Rightarrow \Gamma \models (\forall \emptyset)\, \psi(G)$

3 => 1: Se dem pt Σ-alg $\forall \mathcal{M} \models \Gamma$

$\cancel{\mathcal{M} \models (\forall\emptyset)\psi(G)\ \forall f: T_\Sigma \to \mathcal{M}\ \forall l \doteq_s r \in G\ f(\psi(l)) = f(\psi(r))}$

$(\mathcal{M} \models (\exists X)\, G)\ \exists s: T_\Sigma(X) \to \mathcal{M}\ \ \forall l \doteq_s r\ \ s(l) = s(r)$

Fie $\mathcal{M} \models \Gamma$ $\quad T_\Sigma(X) \xrightarrow{\psi} T_\Sigma \xrightarrow{\exists! \alpha} \mathcal{M}$

$\Delta ! = \psi ; \alpha$

Fie $l =_\Delta r \in G \xrightarrow{\text{din ip.}} \psi(l) \equiv_\Gamma^{T_\Sigma} \psi(r)$

$T_\Sigma \xrightarrow{\alpha} \mathcal{M} \models \Gamma$ $\quad \alpha(\psi(l)) = \alpha(\psi(r))$

$(\psi;\alpha)(l) = (\psi;\alpha)(r)$

$\Delta(l) = \Delta(r)$

Se aplică def congr. semantică

BULLSHIT

$x = f(y)$

$T_\Sigma(\{x,y\}) \longrightarrow T_\Sigma(\{y\})$

$\begin{cases} x \leftarrow f(y) \\ y \leftarrow y \end{cases}$

Def. Soluție pt $(\exists X) G$ în Σ-alg $\mathcal{A}$ $\quad s: T_\Sigma(X) \longrightarrow \mathcal{A}$

$s(G) \subseteq \equiv_\Gamma^{\mathcal{A}}$

Obs!

$$\begin{tikzcd} T_\Sigma(X) \arrow[r, "s"] \arrow[dr, "S"'] & \mathcal{A} \arrow[d, "\eta_{\mathcal{A}}"] \\ & \mathcal{A}_\Gamma = \mathcal{A}/\equiv_\Gamma \end{tikzcd}$$

a) $S := s ; \eta_{\mathcal{A}}$

b) Dat S, $T_\Sigma(X)$ proiectivă, η toate componentele surjective $\Big)$ $\exists s: T_\Sigma(X) \to \mathcal{A}$, $s ; \eta_{\mathcal{A}} = S$

1) s e soluție ~~clasica~~ $\Leftrightarrow$ S e soluție clasică: $\forall l \doteq r \in G$ $S(l) = S(r)$

$\Updownarrow$

Dem:

$s(G) \subseteq \equiv_\Gamma^{\mathcal{A}} \Leftrightarrow \forall l \doteq r \in G$ $\eta_{\mathcal{A}}(s(l)) = \eta_{\mathcal{A}}(s(l))$ $\Leftrightarrow$ $\forall l \doteq r \in G$ $(s;\eta_{\mathcal{A}})(l) = (s;\eta_{\mathcal{A}})$

$\Delta_{\mathcal{M}} = \{(m,m) \mid m \in \mathcal{M}\}$

$\Delta \subseteq T_\Sigma(X) \times T_\Sigma(X)$: $1_{T_\Sigma(X)}$ este soluție pt $(\exists X) \Delta$

$\Delta \subseteq \equiv$

$1_{T_\Sigma(X)}(\Delta) = \Delta \subseteq \equiv \subseteq \equiv_\Gamma^{T_\Sigma(X)}$

Prop

Compunerea unei soluții cu un mf e tot soluție

$s: T_\Sigma(X) \to \mathcal{A}$ soluție pt $(\exists X) G$ în $\mathcal{A}$, $\mathcal{A} \xrightarrow{h} \mathcal{B}$ morfism $\Big\} \Rightarrow s ; h$ este soluție pt $(\exists X) G$ în $\mathcal{B}$

Dem.

$s(G) \subseteq \equiv_P^{\mathcal{A}}$

$h(s(G)) \subseteq h(\equiv_P^{\mathcal{A}}) \overset{\text{Prop 2}}{\subseteq} \equiv_P^{\mathcal{B}}$

$(s;h)(G) \subseteq \equiv_P^{\mathcal{B}}$

$s;h$ e sol pt $(\exists x) G$ în $\mathcal{B}$

sort nat < nlist < list

op 0 : -> nat

op s : nat -> nat

op nil : -> list

op __ : list list -> list [assoc]

op cap : nlist -> nat

op cdr : nlist -> list

var E : nat

var L : list

eq cap(E L) = E

eq cdr(E L) = L

op # : list -> nat

eq #(nil) = 0

eq #(E L) = s(#(L))

7b. Căutăm toate listele de lungime 2 care să înceapă cu 0.

$\#(L) = s(s(0))$

$cap(L) = 0$ $\qquad T_\Sigma(\{L\})$

?

Paramodulația

$c[a] \; s) \; G \longrightarrow_P \theta(c[r] \,) G)$

$(\forall Y)\ l = r \ c t$

Se calc. c.g.u. $T_\Sigma(X \cup Y) \longrightarrow T_\Sigma(Z)$

$c([a] \ ) \{l, a\}$

$\left\{ \begin{array}{l} (\exists x)\ x + 3 = 7 \\ (\exists y)\ y + 3 = 7 \end{array} \right.$ la fel

(numele unor var. legate pot fi schimbate)

$(\forall x)\ l = r$ -> ex de ex. second.

$\underline{\#(E\ L1)} = s(\#(L1))$

CGU $\{\#L, \#(E L1)\}$ $\quad L \leftarrow E\ L1$

$T_\Sigma(\{E, L1\})$

$s(\underline{\#(L1)}) = s(s(0))$, $cap(E\ L1) = 0$

$\underline{\#(E1\ L2)} = s(\#(L2))$ CGU $\{\#(L1), \#(E1\ L2)\}$

$C : s(Z) = s(s(0))$ $\quad L1 \leftarrow E1\ L2$

$s(s(\#(L2))) = s(s(0))$

$\underline{cap(E\ E1\ L2) = 0}$

$T_\Sigma(\{E, E1, L2\})$

$L3 \leftarrow E1\,L2$

$rap(E2\ L3) = E2$ $\qquad T_\Sigma(\{E, L1, L2\})$

$\big\downarrow L2 \leftarrow nil$

$T_\Sigma(\{E, L1\})\quad E = 0.$

$s(s(\#(L)) = s(s(0)) \qquad E = 0$

$\#(nil) = 0$

$s(s(0)) = s(s(0)) \qquad E = 0$

$L = E\,L1 =$
$= E\ E1\ L2$
$= E\ E1\ nil$
$= 0\ E1\ nil$

eliminarea egalităților adevărate $G \cup \{l = l\} \to G.$ m. calculat e identitatea

Reguli de deducție

- regula morfismului $h : T_\Sigma(X) \to T_\Sigma(Y)$

$G \xrightarrow[m]{} h(G)$ $\quad$ m.f. calculat e h

- Regula reflexiei extinse

$h : T_\Sigma(X) \to T_\Sigma(X) \quad h(l) = h(r)$

$G \cup \{l \doteq r\} \xrightarrow[re]{} h(G)$ $\quad$ m.f. calc. e h.

- R. reflexiei $h : T_\Sigma(X) \to T_\Sigma(Y)$ $\quad h = CGU\{l, r\}$

$G \cup \{l \doteq r\} \xrightarrow[r]{} h(G)$ m.f. calc. e h.

~~Regula rif, reflexie extinsă~~

Regula rif. $\longrightarrow$ Reflexiei extinse

$\big\downarrow$

Eliminarea egalităților adevărate $\longleftarrow$ Reflexie

$G \cup \{l \doteq r\} \xrightarrow[m]{} h(G) \cup \{h(l) \doteq h(r)\} \xrightarrow{\text{elimin. eg.}} h(G)$

$G \cup \{l = l\}$

$CGU\{l, l\} = 1$

$\mathcal{A} = (A_s, A_\sigma)$

$c \in T_\Sigma(A \cup \{\bullet\})$ e <u>context</u> $\iff nr_\bullet(c) = 1$

(bulina apare o sg dată)

$nr_\bullet(\bullet) = 1 \qquad nr_\bullet(a) = 0$

$nr_\bullet(\sigma(t_1, \dots, t_n)) = \sum_{i=1}^{n} nr_\bullet(t_i)$

$\bullet \leftarrow d$ (un el din A de ac. sort cu $\bullet$)

$\hat{A} : T_\Sigma(A \cup \{\bullet\}) \to \mathcal{A}$

$$\begin{cases} (\bullet \leftarrow d)(\bullet) = d \\ (\bullet \leftarrow d)(a) = a \end{cases}$$

$\hat{A}$

<u>Not</u>: $(\bullet \leftarrow d)(t) = t[d]$

$$\begin{array}{ccc} \mathcal{A} & \xrightarrow{h} & \mathcal{B} \\ {\scriptstyle \bullet \to d} \uparrow & & \updownarrow \\ T_\Sigma(\mathcal{A} \cup \{\bullet\}) & \xrightarrow{h^\bullet} & T_\Sigma(\mathcal{B} \cup \{\bullet\}) \end{array} \qquad \bullet \leftarrow h(d)$$

$h^\bullet(\bullet) = \bullet$

$h^\bullet(a) = h(a)$

$\hat{\mathcal{A}}$

$h(t[d]) = h^\bullet(t)[h(d)]$

$h : \mathcal{A} \to \mathcal{B}$

$h(a \doteq a) = h(a) \doteq h(a')$

$\mathcal{P}(\mathcal{A} \times \mathcal{A}) \xrightarrow{h} \mathcal{P}(\mathcal{B} \times \mathcal{B})$

$c \overset{\bullet}{=} a \quad h(c \doteq a) = h(c) \doteq h(a)$

$(c \doteq a)[d] = c[d] \doteq a$

$h(c[d]) = h^\bullet(c)[h(a)]$

Reguli de deductie, corectitudinea lor

$G \longrightarrow_R G'$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{S} A$

Paramodulatie

Pararescriere

Paramod. extinsă

R. morfismul → R. reflexie extinsă

R. reflexie

eliminare egalități adevărate

$R_m \perp EEA = R_m$

$G \cup \{l = r\} \to_{m}$ ⊛

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

$\theta(l) =_\Delta \theta(r)$

⊛ $\to_m \theta(G) \cup \{\theta(l) \doteq \theta(r')\} \to \theta(G)$

$R_m \quad G \longrightarrow \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

EEA $\quad G \cup \{l \doteq l\} \to G$

Reg. refl. extinsă

$G \cup \{l \doteq r\} \to_{RE} \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \quad \theta(l) =_\Delta \theta(r)$

Reg. reflexivă $\quad G \cup \{l \doteq_\Delta r\} \to_r \theta(G)$

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y)$

$\theta = c\, G \cup \{l, r\}$

Rr

$\Downarrow$

EEA

$cG \cup \{l, l\} = 1$

Regula e corectă dc

$\forall\, S : T_\Sigma(Y) \to A$ sol pt $(\exists Y)\, G'$

$\theta ; S$ este soluție pt $(\exists X)\, G$

Dem că regulile sunt corecte

Def $\quad S : T_\Sigma(X) \to A$ este sol pt $(\exists X)\, G$ dacă $S(G) \subseteq \equiv_\Gamma^A$

$G \xrightarrow[R]{} G' \xrightarrow[R']{} G''$ (aplic 2 reg. consec.)

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{\theta'} T_\Sigma(Z)$

$G \xrightarrow[R+R']{} G''$. $\quad$ R și R' corecte at. și aplicarea lor succesivă e corectă

$T_\Sigma(X) \xrightarrow[\theta;\theta']{} T_\Sigma(Z)$

Dem

$\forall s: T_\Sigma(Z) \to \mathcal{A}$ sol pt $(\exists Z) G''$ ——→ (ip.)

$(\theta, \theta'); s$ sol pt $(\exists X) G$ ——→ (de dem)

Fie $S: T_\Sigma(Z) \to \mathcal{A}$ sol pt $(\exists Z) G''$

R' corect: $\theta'; S$ sol pt $(\exists Y) G'$

R corect: $\theta; (\theta'; S)$ sol pt $(\exists X) G$

$\| \text{ asoc.}$

$(\theta; \theta'); S$

Rinfirmului e corectă.

$G \xrightarrow[\;]{} \theta(G)$ (orice sol pt $\theta(G)$ e sol pt ?)

$T_\Sigma(X) \xrightarrow{\theta} T_\Sigma(Y) \xrightarrow{S} \mathcal{A}$

Fie S soluție pt $(\exists Y)\, \theta(G)$

$S(\theta(G)) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

$(\theta; S)$ e sol pt $(\exists X) G$

Eliminarea eg. adev. e corectă. Dem.

$G \cup \{ l \doteq l \} \to G$

$T_\Sigma(X) \xrightarrow{1} T_\Sigma(X) \xrightarrow{S} \mathcal{A}$

S sol pt $(\exists X) G \Rightarrow$

$S(G) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

$\overset{\equiv^{\mathcal{A}}_{\Gamma}}{\Rightarrow} S(G \cup \{ l \doteq l \}) =$

$= S(G) \cup \{ S(l) = S(l) \} \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

Deci S sol pt $(\exists X) G \cup \{ l \doteq l \}$

Corolar

1) Reg. refl. extinse e corectă

2) Reg. refl. e corectă (caz particular de refl. extinsă)

$\boxed{\text{Pararescrierea}}$ (egalități, perechi de termeni → para)

$G \cup \{c[h_s(l)]\}$ (context extras) $\Rightarrow_{Pr} G \cup \{c[h_s(l)]\} \cup h(H)$

$(\forall Y)\ l \doteq_\Delta r$ if $H \in \Gamma$, $h : T_\Sigma(X) \to T_\Sigma(Y)$

morfismul calculat e $1_{T_\Sigma(X)}$

$H \Rightarrow l \doteq r$

$\forall\ u \doteq v \in H$

$u = v$ (congr. semantic)

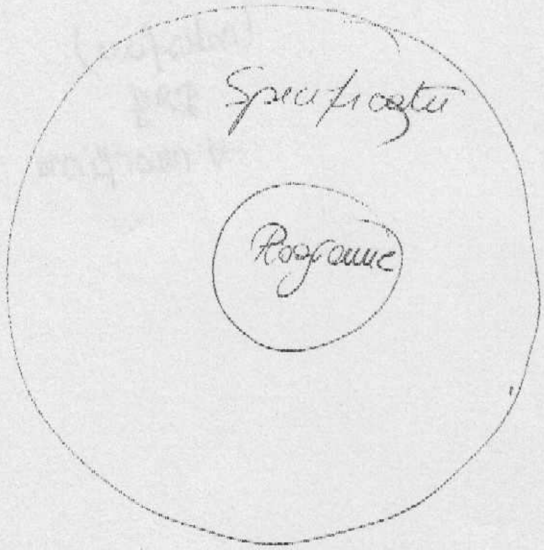
$\Updownarrow$

(~~retine~~ $u == v$

$fn(u) = fn(v)$

$u \doteq> fn(u)$

$v \doteq> fn(v)$



Nu ∀ specif. e program

specif. e program ⟺ sistemul de rescriere e confluent și are propr. de terminare

Thm. Pararescrierea e o regulă corectă.

Fie $S : T_\Sigma(Y) \to \mathcal{A}$ soluție pt $(\exists Y)\, G \cup c[h_s(r)] \cup h(H)$

$$S(G \cup c[h_s(r)] \cup h(H)) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$$

1) $S(G) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

2) $S(c[h_s(r)]) \in \equiv^{\mathcal{A}}_{\Gamma}$

3) $S(h(H)) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

Tb dem că $1_{T_\Sigma(X)}$ (ip); $S : T_\Sigma(X) \to \mathcal{A}$ este sol pt $(\exists X)\, G \cup \{c[h_s(l)]\}$

$$S(G \cup \{c[h_s(l)]\}) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$$

a) $S(G) \subseteq \equiv^{\mathcal{A}}_{\Gamma}$

b) $S(\{c[h_s(l)]\}) \in \equiv^{\mathcal{A}}_{\Gamma}$

Dem b)

$a = b$ în $\mathcal{A} \Leftrightarrow \forall h : \mathcal{A} \to \mathcal{M} \models \Gamma \quad h(a) = h(b)$

Γ-algebră

Fie $\theta : \mathcal{A} \to \mathcal{M} \models$

$\theta(S(C[h_s(r)]))$ este o egalitate adevărată

$\theta(S(h(H)))$ (o mulț. de egalități adevărate) $\to h; S; \theta : T_\Sigma(Y) \to \mathcal{M} \models \Gamma$

$\mathcal{M} \models (\forall Y)\, l =_s r$ if H și $(h; S; \theta)(H)$ o mulț. de egalități adevărate

(satisface)

Def

$\forall$ morfism $m : T_\Sigma(Y) \to \mathcal{M}$ dacă $m(H)$ sunt egalități $\Rightarrow$

$\Rightarrow m(l) = m(r)$

$(h; S; \theta)_s(l) = (h; S; \theta)_s(r)$

$\theta(S(C[h_s(l)])) = (S; \theta)(C[h_s(l)]) =$

(regulă $m(C[a]) = m^*(c)[m(a)]$)

$= (S;\theta)^*(C)[(S;\theta)_s(h_s(l))] = (S;\theta)^*(c)[(h;S;\theta)_s(l)] =$

$= (S;\theta)^*(c)[(h;S;\theta)_s(r)] = (S;\theta)^*(c)[(S;\theta)(h_s(r))] =$

$= (S;\theta)(C[h_s(r)]) = \theta(S(C[h_s(r)]))$

$\Downarrow$

$\theta(S(C[h_s(l)]))$ este o egalitate $\quad S(C[h_s(l)]) \in \, =_s^{\mathcal{A}}$

Paramodulație

$G \cup \{C[a]\} \xrightarrow[P]{}$

$(\forall Y)\, l =_\Delta r \text{ if } H \in P$

$Y \cap X = \emptyset$

egale

$\theta = CGU\{a, l\}: T_\Sigma(X \cup Y) \to T_\Sigma(Z)$

Paramodulație extinsă

$\theta(a) = \theta(b)$ (doar unificator, nu cel mai general) $T_\Sigma(X) \xrightarrow{\theta/_{T_\Sigma(X)}} T_\Sigma(Z)$

Dem. R unif + Param rescriere = Paramodulație extinsă

$(\forall Y)\, l \doteq_\Delta r \text{ if } H \in P \quad Y \cap X = \emptyset$

$\theta: T_\Sigma(X \cup Y) \to T_\Sigma(Z)$

$\theta(a) = \theta(l)$

$G \cup C[a] \xrightarrow[u]{\theta/_{T_\Sigma(X)}} \theta(G) \cup \{\theta(C[a])\} = \theta(G) \cup \{\theta^*(c)\,[\theta(a)]\} =$

$= \theta(G) \cup \{\theta^*(c)\,[\theta_\Delta(l)]\} \xrightarrow[R_1]{} \theta(G) \cup \{\theta^*(c)[\theta_\Delta(r)]\} \cup \theta(H)$

$\| $

$\theta(G) \cup \{\theta(C[r])\} \cup \theta(H)$

$\|$

$\theta(G \cup \{C[r]\} \cup H)$

Corolar

1) Reg param extinse e corectă. (reg unif + reg param rescriere corecte)

2) Paramodulația e corectă (caz particular) ↑

$G \xrightarrow[R_1]{} G_1 \xrightarrow[R_2]{} G_2 \to \dots \xrightarrow[R_n]{} G_n \leftarrow$ mulț de ec adev. (identitatea e soluție)

$T_\Sigma(X) \xrightarrow{\theta_1} \quad \xrightarrow{\theta_2} \dots \xrightarrow{\theta_n}$

θ_n; id sol pt G_{n-1} etc.

$\theta_1; \theta_2; \dots \theta_n$ sol pt $(\exists X) G$

(compunere unif. calculate)