Habib University
Course Code: MATH 402
Course Title: Applied Stochastic Processes
Instructor's name: Dr. M. Shahid Shaikh
Examination: Assignment #1   Given: 13 September 2023
Total Marks: 100   Due: 24 September 2023

Instructions:

1. Please type the answers or write neatly in hand.
2. Upload your PDF file (if you write in hand, scan and convert to PDF) Canvas.
4. No late submission will be accepted.

Q1. A number U is selected at random from the unit interval. Let the events A and B be: A = "U differs from $\frac{1}{2}$ by more than $\frac{1}{4}$" and B = "$1 - U$ is less than $\frac{1}{2}$." Find the events: (i) $A \cap B$, (ii) $A \cup B$, (iii) $A^c \cap B$.

Q2. Let A, B, and C be events. Find expressions for the following events:

(i) Exactly one of the three events occurs.

(ii) Exactly two of the events occur.

(iii) One or more of the events occur.

(iv) Two or more of the events occur.

(v) None of the events occur.

Q3. In a specified 8-AM-to-8-AM 24-hour period, a student wakes up at time t_1 and goes to sleep at some later time t_2.

(i) Find the sample space and sketch it on the x-y plane if the outcome of this experiment consists of the pair (t_1, t_2)

(ii) Specify the set A and sketch the region on the plane corresponding to the event "student is asleep at noon."

(iii) Specify the set B and sketch the region on the plane corresponding to the event "student sleeps through breakfast (9–10 AM)."

(iv) Sketch the region corresponding to $A \cap B$ and describe the corresponding event in words.

Q4. A dart is equally likely to land at any point inside a circular target of radius 2. Let R be the distance of the landing point from the origin.

(i) Find the sample space S and the range if R, S_R;

(ii) Show the mapping from S to S_R;

(iii) The "bull's eye" is the central disk in the target of radius 0.25. Find the event A in S_R corresponding to "dart hits the bull's eye." Find the equivalent event in S and $P(A)$.

(iv) Find and plot the cdf of R.

Q5. A voltage X is uniformly distributed in the set $\{-3, -2, \ldots, 3, 4\}$.

    (i) Find the mean and variance of X.

    (ii) Find the mean and variance of $Y = -2X^2 2 + 3$.

    (iii) Find the mean and variance of $Z = \cos(\pi X/8)$.

    (iv) Find the mean and variance of $W = \cos^2(\pi X/8)$.

Q6. A random variable X has pdf:

$$f_X(x) = \begin{cases} cx(1 - x^2), & 0 \le x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

    (i) Find c and plot the pdf and the cdf of X.

    (ii) Find $P(0 \le X \le 0.5)$, $P(X = 1)$, and $P(0.25 \le X \le 0.5)$.

Q7. Consider two RVs, X and Y, and an RV, Z, such that $P[Z = X] = p$ and $P[Z = Y] = 1 - p$.

    (i) Show that the pdf of Z is given by

$$f_Z(z) = pf_X(z) + (1 - p)f_Y(z).$$

    (ii) Calculate the cdf of two-sided exponential RV that has PDF given by

$$f_Z(z) = \begin{cases} p\lambda e^{\lambda z}, & z < 0, \\ (1 - p)\lambda e^{-\lambda z}, & z \ge 0. \end{cases}$$

    where $\lambda > 0$ and $0 < p < 1$.