Assignment 3
Applied Stochastic Processes
Habib University – Fall 2023

Ali Asghar Yousuf - ay06993
Muhammad Murtaza - mm06369

December 10, 2023

1. Dave fails quizzes with probability $\frac{1}{4}$, independent of other quizzes.

   (a) What is the probability that Dave fails exactly two of the next six quizzes?

   Solution:

   $$\begin{aligned} P(\text{Dave fails exactly two of the next six quizzes}) &= \binom{6}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 \\ &= 15 \times \frac{1}{16} \times \frac{81}{256} \\ &= \frac{1215}{4096} \\ &= 0.296875 \end{aligned}$$

   (b) What is the expected number of quizzes that Dave will pass before he has failed three times?

   Solution:
   No. of times he failed $= 3$
   Total no. of quizzes taken to fail 3 times $= n$

   $$\begin{aligned} n * \frac{1}{4} &= 3 \\ n &= 12 \end{aligned}$$

   Dave takes 12 quizzes to fail 3 times. Therefore, he passes 9 quizzes.

(c) What is the probability that the second and third time Dave fails a quiz will occur when he takes his eighth and ninth quizzes, respectively?

Solution:
1st Fail $\rightarrow 1-7$ quizzes
2nd Fail $\rightarrow$ 8th quiz
3rd Fail $\rightarrow$ 9th quiz

$$\begin{aligned} P(X) &= P(\text{1 fail in 7 tests}) \cdot P(\text{2nd fail in 8th test}) \cdot P(\text{3rd fail in 9th test}) \\ &= \binom{7}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^6 \cdot \frac{1}{4} \cdot \frac{1}{4} \\ &= \frac{7 \cdot 3^6}{4^9} = \frac{5103}{262144} \\ &= 0.0194568634 \end{aligned}$$

(d) What is the probability that Dave fails two quizzes in a row before he passes two quizzes in a row?

Solution:
$F =$ Fail, $P =$ Pass

$$\begin{aligned} P(X) &= P(\text{Dave fails two quizzes in a row before he passes two quizzes in a row}) \\ &= P(FF \cup PFF \cup FPFF \cup PFPFF \cup FPFPFF \cup \dots) \\ &= \frac{[P(F)]^2}{1 - P(F) \cdot P(P)} + \frac{P(P) \cdot [P(F)]^2}{1 - P(F) \cdot P(P)} \\ &= \frac{\left(\frac{1}{4}\right)^2}{1 - \frac{1}{4} \cdot \frac{3}{4}} + \frac{\frac{3}{4} \cdot \left(\frac{1}{4}\right)^2}{1 - \frac{1}{4} \cdot \frac{3}{4}} \\ &= \frac{7}{52} \end{aligned}$$