Assignment 2
MATH 402 Applied Stochastic Processes
Habib University – Fall 2023

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1. **9.34** A server handles queries that arrive according to a Poisson process with a rate of 10 queries per minute.What is the probability that no queries go unanswered if the server is unavailable for 20 seconds?

Solution:

$$\lambda = 10 \text{ queries per minute}$$
$$\mu = 60 \text{ seconds per minute}$$
$$\lambda' = \frac{\lambda}{\mu} = \frac{10}{60} = \frac{1}{6} \text{ queries per second}$$
$$t = 20 \text{ seconds}$$
$$P_0(t) = e^{-\lambda' t} = e^{-\frac{1}{6} \times 20} = e^{-\frac{10}{3}} = 0.049787068367863944$$

2. **9.35** Customers deposit \$1 in a vending machine according to a Poisson process with rate λ. The machine issues an item with probability p. Find the PMF for the number of items dispensed in time t.

Solution:

$$\begin{aligned} \lambda &= \text{ rate of deposit} \\ p &= \text{ probability of issuing an item} \\ t &= \text{ time} \\ X &= \text{ number of items dispensed in time } t \end{aligned}$$

The PMF for X is given by:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \times p^k$$

For example, if $p = 0.5$ and $\lambda = 2$, then the probability of dispensing k items in time t is given by:

$$\begin{aligned} P(X = 0) &= \frac{2^0 e^{-2}}{0!} \times 0.5^0 = 0.1353352832366127 \\ P(X = 1) &= \frac{2^1 e^{-2}}{1!} \times 0.5^1 = 0.2706705664732254 \\ P(X = 2) &= \frac{2^2 e^{-2}}{2!} \times 0.5^2 = 0.2706705664732254 \\ &\vdots \end{aligned}$$

3. **6.8** During rush hour, from 8 AM to 9 AM, traffic accidents occur according to a Poisson process with a rate of 5 accidents per hour. Between 9 AM and 11 AM, they occur as an independent Poisson process with a rate of 3 accidents per hour. What is the PMF of the total number of accidents between 8 AM and 11 AM?

Solution:

$$\begin{aligned} \lambda_1 &= 5 \text{ accidents per hour} \\ \lambda_2 &= 3 \text{ accidents per hour} \end{aligned}$$

The rate of accidents between 8 AM and 11 AM is the sum of the rates of accidents between

8 AM and 9 AM and between 9 AM and 11 AM. Therefore,

$$\lambda = \lambda_1 + \lambda_2 = 5 + 3 = 8 \text{ accidents per hour}$$
$$t = 3 \text{ hours}$$
$$X = \text{ number of accidents in } t \text{ hours}$$

The PMF for X is given by:

$$P(X = k) = \begin{cases} \dfrac{e^{-8}8^k}{k!} & \text{if } k \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$