Homework 3

Assigned on November 3, 2023
Due on November 17, 2023

Maximum Points: 100

Learning Outcomes

After this homework, you should be able to:

(a) appreciate propagation of uncertainty of random variables through transformations;
(b) analyze and apply the KF and EKF for state estimation;
(c) appreciate the need for localizing a mobile robot and apply EKF for localization;
(d) analyze the assumptions of Bayes Filter and use it for estimation;
(e) refine their strategies for developing and debugging algorithms for robotics.

Instructions

- The homework assignment can be attempted in groups of two.
- Each group will register themselves as a group on Canvas under People/Groups/Homework 2.
- The homework submission on Canvas will be set up for group submission, so each group needs to make only one submission.
- **If it appears that a group member has not contributed to a homework assignment, then each member will be graded individually.**

Tasks

Problem 1
CLO-2/C-4
20 points

Consider a robot that lives in a 1-D coordinate system. Its location will be denoted by x, its velocity by $\dot{x}$, and its acceleration by $\ddot{x}$. Suppose we can only control the acceleration, $\ddot{x}$. Making use of equations of motion from school physics, write the discrete-time motion model for this system. Assume that the acceleration, $\ddot{x}$, is a sum of a commanded acceleration and a zero-mean noise term with variance σ^2, and assume that the actual acceleration remains constant in an interval Δt.

(a) Find the uncertainty/covariance in the pose $(x, \dot{x})$ after one time step. Are the two correlated?

(b) Suppose we control this robot with commanded acceleration sequence $a_1, a_2, a_3, \cdots$ for T time intervals. Will the final location, x, and the final velocity, $\dot{x}$, be correlated for some large value of T?

Problem 2
CLO-2/C-4

20 points

Suppose we have a mobile robot operating in a planar environment. Its state is its x-y location and its global heading direction θ. Suppose we know x and y with high certainty, but the orientation θ is unknown. This is reflected in our initial estimate:

$$\hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \Sigma_0 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{bmatrix}.$$

(a) Assume that the robot moves flawlessly without any noise. We'll consider the simple case when the robot's heading is not being controlled, i.e. $\omega = 0$. Observations of the robot are made at discrete points in time and consist of the robot's distance from the origin, d, and the bearing, θ, measured from the origin. Assume that the noise associated with these two measurements are independent. Develop a Kalman Filter that maintains an estimate of the robot's state.

(b) The location of the robot is a random vector. Draw 1000 samples of the initial state from a Gaussian distribution of the stated mean and covariance, propagate each initial state sample according to the motion equation, and plot the samples of the $x - y$ state only at time 1 in MATLAB. Assume that distance covered in one time step is 1.

(c) Use the prediction step of the EKF to make a prediction about the state at time 1 and its corresponding covariance. Plot the uncertainty ellipse of a Gaussian with mean equal to $\bar{x}_1$ and covariance $\bar{\Sigma}_1$ on the same plot as (a), and compare and comment on the two.
Resource: How to plot covariance error ellipse?

(d) Now incorporate a noisy measurement, i.e. $z = d + \epsilon$, where ϵ is zero-mean with covariance 0.01. Again, draw the uncertainty ellipse on the same plot after incorporating the measurement.

(e) What would have been your estimate for the $x - y$ at time 1 considering (a)? What would be your comments about the estimate provided by the EKF? What would have happened if the initial orientation were known, but we were uncertain about the y coordinate?

Problem 3
CLO-2/C-4

20 points

Suppose we live at a place where days are either sunny, cloudy, or rainy. The weather tomorrow is determined solely by the weather today (it's a Markov Chain) and is captured by the following state transition probabilities:

		Tomorrow's weather		
		Sunny	**Cloudy**	**Rainy**
	Sunny	0.8	0.2	0
Today's weather	**Cloudy**	0.4	0.4	0.2
	Rainy	0.2	0.6	0.2

Suppose that we cannot observe the weather directly, but instead rely on a sensor. Our sensor is noisy. The measurements are governed by the following measurement model:

		Sensor's Reading		
		Sunny	**Cloudy**	**Rainy**
	Sunny	0.6	0.4	0
Actual Weather	**Cloudy**	0.3	0.7	0
	Rainy	0	0	1

(a) Suppose Day 1 is sunny (this is known for a fact). At days 2 through 4, the sensor measures sunny, sunny, rainy. For each of the days 2 through 4, what is the most likely weather on that day. Answer the question in two ways: one in which only the data available to the day in question is used, and one in hindsight, where data from future days is also available.

(b) Consider the same situation. What is the most likely sequence of weather for Days 2 through 4? What is the probability of the most likely sequence?

Problem 4
CLO-2/C-3

20 points

In this problem, you'll implement EKF-based landmark localization, based on the case study in [1, 5.6.8.5]. Contrary to the example discussed in class, this problem does not have any physical landmarks. Instead, a map, M, is provided to the robot in the form of parameters for lines in the environment; one line corresponding to each wall in the environment. The robot is equipped with a Lidar that generates a 360° scan, lines are extracted from the lidar scan at each time step[1], and the parameters of these detected lines will be passed to the EKF algorithm as measurements.

- You'll have to launch `Gazebo Office` from your VMWare desktop.
- The main script file that will interact with Gazebo is `turtlebotEKFLocalization.m`. Add your IP.
- The wheel separation and wheel diameter are defined at the beginning of this file and can be accesses from the `params` structure.
- You'll have to complete the `incrementalLocalization` and all of its subsidiary functions.
- Run the simulation for a longer time and comment on the performance of the EKF-based localization.
- The measurement uncertainty covariance matrix R is being computed from the uncertainty of the lidar. Study Section 4.7.1.2 in [1] and provide an explanation in your own words with reference to code.
- **(Bonus)** Study and apply the Unscented Kalman Filter ([2]) to this problem

Problem 5
CLO-2/C-2

20 points

Answer the following questions individually:

(a) How many hours did each of you spend on this homework? Answer as accurately as you can, as this will be used to structure next year's class.

(b) Each group member is to specifically state their contribution in this homework assignment.

(c) Do you have any specific advice for students attempting this homework next year?

(d) Each group member is to provide a self-reflection in the form of a note or a concept map. This requires you to reflect on your learning in relation to each of the outcomes stated at the beginning of this document.

Some questions that may help in this regard are: Have I achieved this outcome? What do I currently understand about content related to this outcome? How does it help me

[1] Details of algorithms for line extraction can be found in [1, 4], but are not required for this problem.

understand or build any robot? Do I have unanswered questions? What went wrong? How can I enable myself to achieve this outcome? What could I do to know more or enhance my skills in this context?

Don't forget to indicate your name with your respective paragraph.

References

[1] R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, "Autonomous mobile robots," *A Bradford Book*, vol. 15, 2011.

[2] S. Thrun, W. Burgard, and D. Fox, "Probabilistic robotics," *Intelligent robotics and autonomous agents, The MIT Press*, 2006.

Grading:

To obtain maximal score for each question, make sure to elaborate and include all the steps.