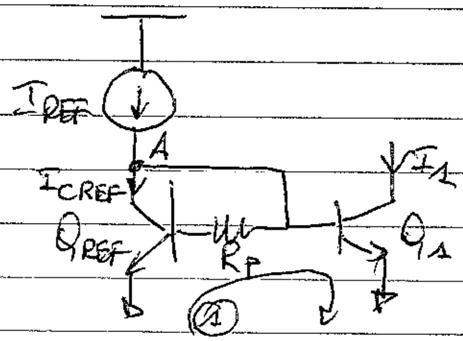
EE140/EE240A

HW4 Solutions

Problem 1



$I_1 = 1.1\, I_{1NOM}$? R_p

First, compute I_{1NOM}, i.e. I_1 with $R_p = 0$

$$I_{1NOM} = I_{REF} \cdot \frac{1}{1+\frac{2}{\beta}}$$

$$\Rightarrow I_1 = 1.1\, I_{REF} \cdot \frac{1}{1+\frac{2}{\beta}}$$

Now, let's consider R_p:

KVL @ LOOP 1: $V_{BEREF} + \frac{I_{CREF}}{\beta} R_p = V_{BE1}$

$$V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{CREF}}{I_S}\right) = \frac{I_{CREF}}{\beta} R_p$$

$$R_p = \frac{\beta}{I_{CREF}} V_T \ln\left(\frac{1.1\, I_{REF} \cdot \frac{1}{1+\frac{2}{\beta}}}{I_{CREF}}\right) \qquad (1)$$

KCL @ A: $I_{REF} = I_{CREF} + \frac{I_{CREF}}{\beta} + \frac{I_1}{\beta}$

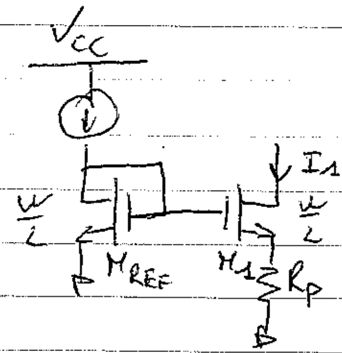
$$I_{CREF} = \frac{I_{REF}\left(1 - \frac{1.1}{\beta\left(1+\frac{2}{\beta}\right)}\right)}{1+\frac{1}{\beta}} = \frac{\beta(\beta+0.9)}{(\beta+1)(\beta+2)} I_{REF}$$

Now substitute back in (1)

$$R_p = \frac{(\beta+1)(\beta+2)}{\beta+0.9}\frac{V_T}{I_{REF}}\ln\left[\frac{1.1\cancel{\beta}(\beta+1)\cancel{(\beta+2)}}{\cancel{\beta}\cancel{(\beta+2)}(\beta+0.9)}\right]$$

$$\Rightarrow \boxed{R_p = \frac{(\beta+1)(\beta+2)}{\beta+0.9}\frac{V_T}{I_{REF}}\ln\left[\frac{1.1(\beta+1)}{\beta+0.9}\right]}$$

Problem 2



? R_p s.t. $I_1 = \frac{I_{REF}}{2}$

$$V_{GS,REF} = V_{TH} + \sqrt{\frac{2I_{REF}}{K_n \frac{W}{L}}}$$

KVL: $V_{GS_1} = V_{GS\,REF} - I_1 R_p$

$$\Rightarrow V_{GS_1} = V_{TH} + \sqrt{\frac{2I_{REF}}{K_n \frac{W}{L}}} - \frac{I_{REF}}{2} R_p$$

$$(*) \quad I_1 = \frac{1}{2} K_n \frac{W}{L}\left[\sqrt{\frac{2I_{REF}}{K_n \frac{W}{L}}} - \frac{I_{REF}}{2} R_p\right]^2 = \frac{I_{REF}}{2}$$

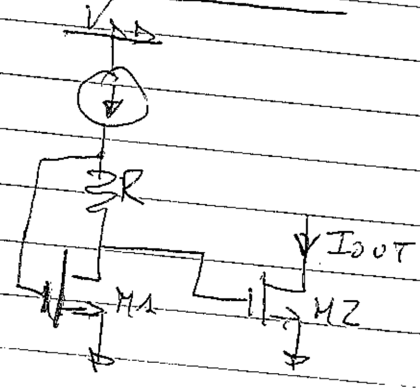
$$\sqrt{\frac{2I_{REF}}{K_n \frac{W}{L}}} - \frac{I_{REF}}{2} R_p = \sqrt{\frac{2\, I_{REF}/2}{K_n \frac{W}{L}}}$$

$$\frac{I_{REF}}{2} R_p = \sqrt{\frac{2I_{REF}}{K_n \frac{W}{L}}} - \sqrt{\frac{I_{REF}}{K_n \frac{W}{L}}} \qquad \frac{I_{REF}}{2} R_p = \left(\sqrt{2} - 1\right)\sqrt{\frac{I_{REF}}{K_n \frac{W}{L}}}$$

$$R_p = \frac{2\left(\sqrt{2} - 1\right)}{\sqrt{I_{REF} K_n \frac{W}{L}}}$$

I_1 does not change if the threshold voltage of the transistors change by the same amount ΔV. Looking at expression (*), we can see that there is no dependence on V_{TH} (because the V_{TH} of the two transistors cancel out).

Problem 3



$I_{OUT} = 0.1\mu A \quad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

part a) Given $I_{IN} = 1\mu A$ determine the range of $\frac{W}{L_1}$ s.t. M1 operates in saturation (strong inversion).

We need to determine the ~~vals~~ range of $\left(\frac{W}{L}\right)_1$ and for each value of $\left(\frac{W}{L}\right)_1$ compute the corresponding value of R that guarantees an output current of $I_{OUT} = 0.1\mu A$

From G&M 4.4.1.4

$$I_{OUT} = \frac{K'}{2}\left(\frac{W}{L}\right)_1 \left[\sqrt{\frac{2I_{IN}}{K'\frac{W}{L}}} - RI_{IN}\right]^2 \qquad (*)$$

solving for R

$$R = \frac{\sqrt{\frac{2I_{IN}}{K'\frac{W}{L}}} - \sqrt{\frac{2I_{OUT}}{K'\frac{W}{L}}}}{I_{IN}}$$

To operate in strong inversion:

$$V_{OV_1} = \sqrt{\frac{2I_{IN}}{K'\frac{W}{L}}} \geq 2nV_T = 78mV$$

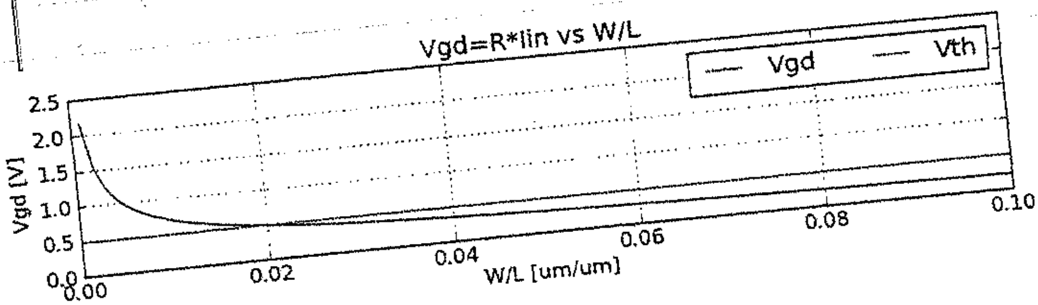
To operate in saturation:

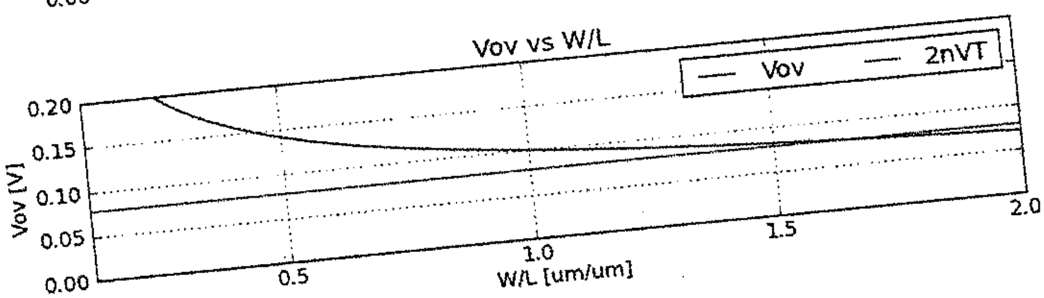
$$V_{GD_1} = RI_{IN} \leq V_{TH} = 0.5V$$

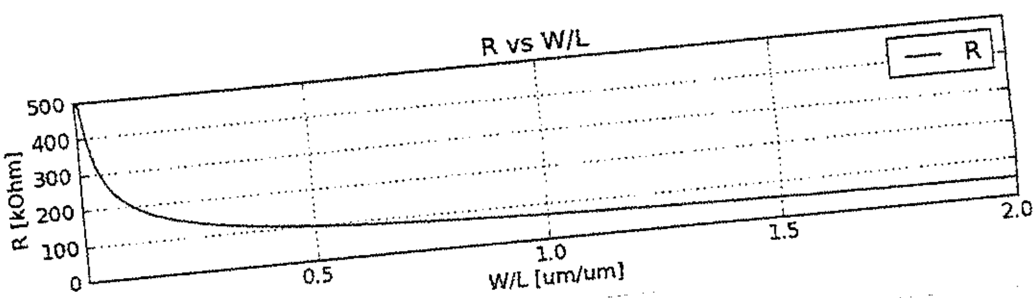
We can now sweep $\frac{W}{L}$ and determine the range of values that satisfy the above conditions. For low $\frac{W}{L}$ a bigger R will be required to obtain the desired I_{OUT}, so we can compute $\left(\frac{W}{L}\right)_{MIN}$ to maintain saturation. For high $\frac{W}{L}$ the V_{OV} of $M1$ will keep on decreasing until entering weak inversion. The plot below shows the trend of the quantities of interest. We get the limits

$$0.018 \leq \frac{W}{L} \leq 1.643$$

$$496k\Omega \geq R \geq 53k\Omega$$





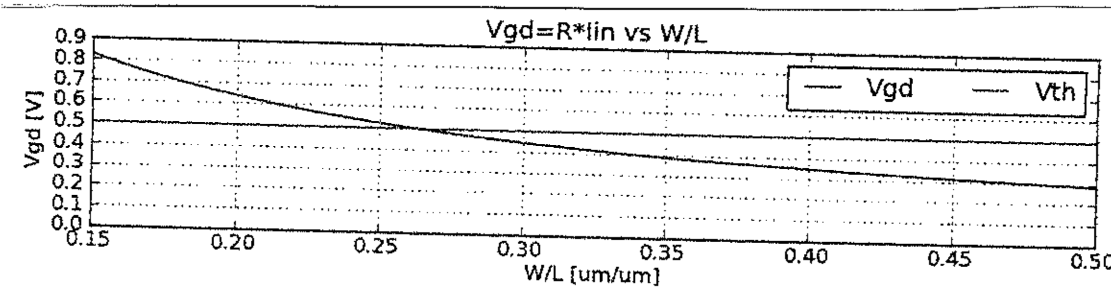


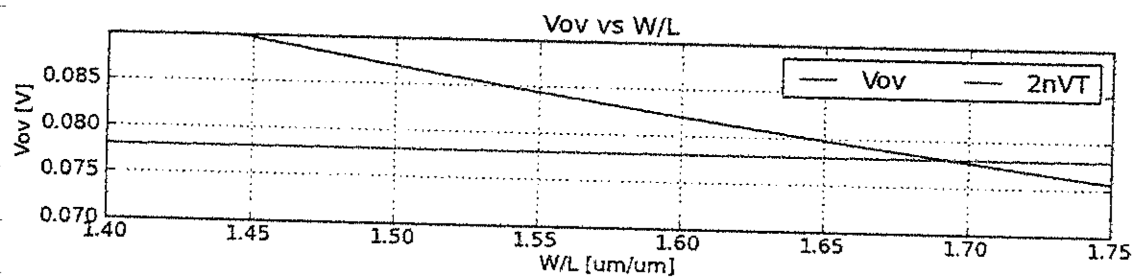
Part θ) Now fix $R = 100K\Omega$ and compute the range of $\frac{W}{L}$ and I_{in} to maintain $\frac{W}{L}$ in strong inversion and saturation.

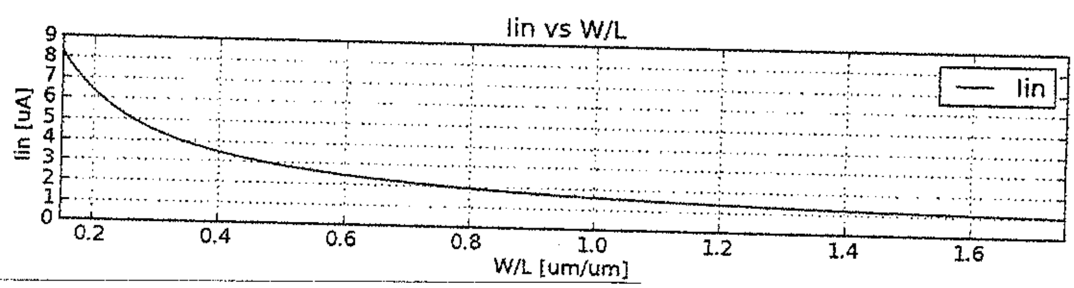
We use the same constraints as before but now solve Equation (*) numerically for simplicity. We get the following ranges:

$$0.262 \leq \frac{W}{L} \leq 1.691$$

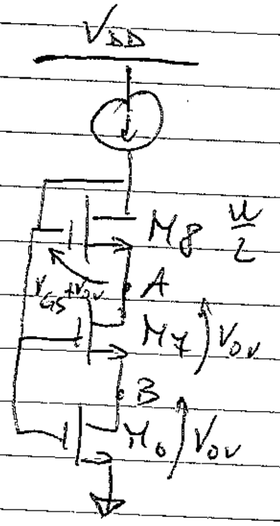
$$\cancel{400}\ 5\mu A \geq I_{in} \geq 1.028\,\mu A$$







Problem 4



Determine $\left(\frac{W}{L}\right)_{M_6}$, $\left(\frac{W}{L}\right)_{M_7}$ such that

$V_{DS_{M6}} = V_{DS_{M7}} = V_{OV_{M8}}$

M_8 is diode connected, so it operates in saturation. Further, the circuit connection forces M7 and M6 to operate in Triode because $V_{GD_{M6}} > V_{GD_{M7}} > V_{TH}$ when M8 is on

KCL @ A: $\frac{K'}{2}\left(\frac{W}{L}\right)_8 \left(V_{GS_8} - V_T\right)^2 = \frac{K'}{2}\left(\frac{W}{L}\right)_7 \left[2\left(V_{GS7} - V_T\right)V_{DS7} - V_{DS7}^2\right]$

we now enforce $V_{DS7} = V_{OV8}$ $\quad V_{GS8} = V_T + V_{OV}$

$\Rightarrow V_{GS7} = V_{GS8} + V_{DS7} = V_T + 2V_{OV}$

$\Rightarrow \frac{\cancel{K'}}{\cancel{2}}\left(\frac{W}{L}\right)_8 V_{OV}^2 = \frac{\cancel{K'}}{\cancel{2}}\left(\frac{W}{L}\right)_7 \left[2(2V_{OV})V_{OV} - V_{OV}^2\right]$

$\Rightarrow \boxed{\left(\frac{W}{L}\right)_7 = \frac{1}{3}\left(\frac{W}{L}\right)_8}$

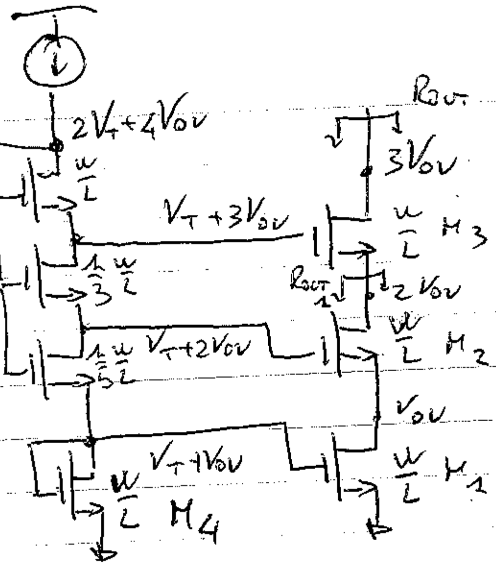
KCL @ B: $\frac{K'}{2}\left(\frac{W}{L}\right)_8 \left(V_{GS_8} - V_T\right)^2 = \frac{K'}{2}\left(\frac{W}{L}\right)_6 \left[2\left(V_{GS_6} - V_T\right)V_{DS_6} - V_{DS6}^2\right]$

we have: $V_{DS8} = V_{OV8}$ $\quad V_{GS8} = V_T + V_{OV}$

$V_{GS_6} = V_{GS8} + V_{DS7} + V_{DS6} = V_T + 3V_{OV}$

$\Rightarrow \frac{K'}{2}\left(\frac{W}{L}\right)_8 V_{OV}^2 = \frac{K'}{2}\left(\frac{W}{L}\right)_6 \left[2(3V_{OV})V_{OV} - V_{OV}^2\right]$

$\Rightarrow \boxed{\left(\frac{W}{L}\right)_6 = \frac{1}{5}\left(\frac{W}{L}\right)_8}$



The transistor sizes are the ones reported on the schematic.

$$R_{OUT_1} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$$

$$R_{OUT} = r_{o3} + R_{OUT_1} + g_{m3} r_{o3} R_{OUT_1} \simeq g_m^2 r_o^3$$

The minimum output voltage to guarantee all transistors in saturation is $3V_{OV}$.

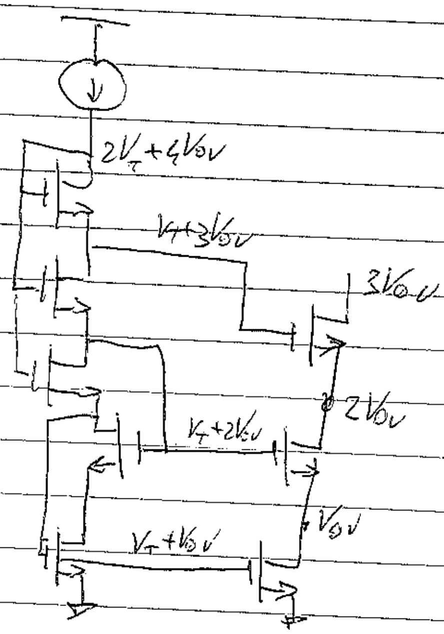
The voltage of all nodes in the input branch have been reported on the schematic

The systematic error is due to the difference in V_{DS} between transistors M1 and M4
Assuming a linear dependence of the output current of an MOS in saturation with V_{DS}, we get

$$\varepsilon = \frac{V_{DS1}-V_{DS4}}{V_A} \simeq \frac{V_{OV1}-(V_T+V_{OV3})}{V_A} \simeq -\frac{V_T}{V_A}$$

The error is negative because there is less current flowing in the output branch.

Using Sooch cascode Topology, we get:



$\boxed{\varepsilon = 0}$