EE 301 Fall 2018-2019

HW 2

Group Number:66

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1)	a)	$x[n] * w[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k-1) \cdot u[n-k+1]$ $u(k-1)$: $k \geq 1$ → 1; o.w → 0 $u(n-k+1)$: $k \leq n+1$ → 1; o.w → 0 $n>0$ $x[n] * w[n] = \sum_{1}^{n+1} \left(\frac{1}{2}\right)^k$ (for $n>0$ $= \frac{1-\left(\frac{1}{2}\right)^{k+1}}{1-\frac{1}{2}} - 1 = 1-\left(\frac{1}{2}\right)^{n+1}$ $\boxed{x[n] * w[n] = \left(1-\left(\frac{1}{2}\right)^{n+1}\right) \cdot u[n]}$
	b)	$x[n] * w[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k \left(u[k-1] - u[k-100] + \delta[k]\right) \cdot u[2n-2k]$ $= \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k-1] \cdot u[2n-2k] - \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k-100)$ $+ \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k \cdot u[2n-2k]$ (Distributive property of convolution) $\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k-1] \cdot u[2n-2k]$ $n>1$ $k \geq 1$ $k \leq n$ $\sum_{k=1}^{n} \left(\frac{1}{2}\right)^k = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-1/2} - 1 = \left(1-\left(\frac{1}{2}\right)^n\right) u[n-1]$ $\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k-100)\, u(2n-2k)$ $n>100$ $k \geq 100$ $k \leq n$ $\left[\left(2-\left(\frac{1}{2}\right)^{[illegible]}\right) - \left(2-\left(\frac{1}{2}\right)^{100}\right)\right] u[n-100]$

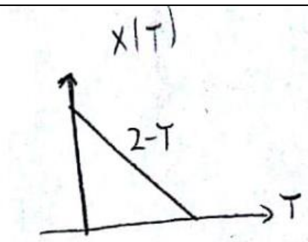
$$\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k \delta(k)\, u(2n-2k)$$

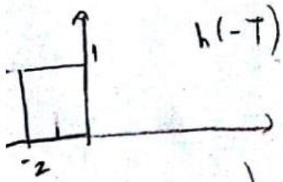
$n \geq 0 = u[2n)$

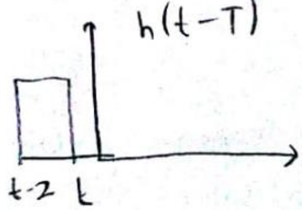
$$x[n] * y[n] = u[2n] + \left(1-\left(\frac{1}{2}\right)^n\right) u[n-1]$$

$$-\left(\left(\frac{1}{2}\right)^{100} - \left(\frac{1}{2}\right)^n\right) u[n-100]$$

c)







$t < 0 \quad \to 0$

$0 < t < 2 \qquad \int_0^t (2-T)\, dT$

$$2t - \frac{t^2}{2}$$

$2 < t < 4 \qquad \int_{t-2}^{2} (2-T)\, T$

$$8 - 4t + \frac{t^2}{2}$$

$$x(t) * h(t) \begin{cases} 0 & t < 0, t > 4 \\ 2t - \frac{t^2}{2} & 0 \leq t \leq 2 \\ 8 - 4t + \frac{t^2}{2} & 2 < t \leq 4 \end{cases}$$

2)	a)	for linearity $x_1[n] \rightarrow y_1[n]$ $x_2[n] \rightarrow y_2[n]$ $ax_1[n]+bx_2[n] \rightarrow ax_1[n]+\frac{1}{2}ax_1[n-2]$ $+bx_2[n]+\frac{1}{2}bx_2[n-2]$ $=ay_1[n]+by_2[n]$ System is linear for Time-Invariant $x_1[n] \rightarrow y[n]$ $x_1[n-n_0] \rightarrow x_1[n-n_0]-\frac{1}{2}x_1[n-n_0-2]$ $=y[n-n_0]$ System is Time invariant $x[n]=\delta[n]$, we can find impulse response $h[n]=\delta[n]-\frac{1}{2}\delta[n-2]$ FIR because only for $n=0$ and $n=2$ we can give non zero value and they are finite number.

	b)	$y[n] + \frac{1}{2} y[n-2] = x[n]$ System is LTI because output depends only input $h[n] + \frac{1}{2} h[n-2] = \delta[n]$ for $n > 0$ $\quad h[n] + \frac{1}{2} h[n-2] = 0$ $h[n] = z^n$ $z^n + \frac{1}{2} z^{n-2} = 0$ $z^2 = -\frac{1}{2} \rightarrow z = \frac{i}{\sqrt{2}}$, $\rightarrow z = -\frac{i}{\sqrt{2}}$ $h[n] = A_1 \left(\frac{i}{\sqrt{2}}\right)^n + A_2 \left(-\frac{i}{\sqrt{2}}\right)^n \quad n > 0$ for $n=0$, $h[0] + \frac{1}{2} \underbrace{h[-2]}_{0} = 1$ $\boxed{h[0]=1}$ for $n=1$, $h[1] + \frac{1}{2} \underbrace{h[-1]}_{0} = 0$ $\boxed{h[1]=0}$ $h[0] = A_1 + A_2 = 1$ $h[1] = A_1 \frac{i}{\sqrt{2}} - A_2 \frac{i}{\sqrt{2}} = 0 \Rightarrow A_1 = A_2 = \frac{1}{2}$ $h[n] = \frac{1}{2} \left[\left(\frac{i}{\sqrt{2}}\right)^n + \left(-\frac{i}{\sqrt{2}}\right)^n \right], n \geq 0$ FIR because i^n can take only four value.

3)		Q3-) a-) $h_1[n] = u[n+2]$ System 1 is <u>not</u> memoryless since depends on future value. Also, <u>not</u> causal. $\sum_{k=-\infty}^{\infty} h[k]$ is not absolutely summable and System is <u>not</u> stable b) $h_2[n] = \sum_{m=-3}^{-1} \delta[n-m]$ $h_2[n] = \delta[n+3] + \delta[n+2] + \delta[n+1]$ System 2 is linear since it only depends on input. $\delta[n] \rightarrow h[n]$ $\delta[n-n_0] \rightarrow \delta[n-n_0+3] + \delta[n-n_0+2] + \delta[n-n_0+1]$ $= h_2[n-n_0]$ System 2 is Time invariant. c) we can take advantage of Associative property. $h[n] = h_1[n] * h_2[n]$ $\sum_{k=-\infty}^{\infty} (\delta[k+3] + \delta[k+2] + \delta[k+1])\, u[n-k+2]$ $= u[n+5] + u[n+4] + u[n+3]$ d) $y[n] = x[n] * h[n]$ $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\left(u[k+5] + u[k+4] + u[k+3]\right)$ $y[n] = \sum_{k=-\infty}^{n+5} x[n] + \sum_{k=-\infty}^{n+4} x[n] + \sum_{k=-\infty}^{n+3} x[n]$ $y[n] = y[n-1] + x[n+5] + x[n+4] + x[n+3]$ e) $h[n] = u[n+5] + u[n+4] + u[n+3]$ System is <u>not</u> memoryless and <u>not</u> causal since it depends on future value. $\sum_{k=-\infty}^{\infty} \lvert h[k] \rvert$ is not absolutely summable and System is <u>not</u> stable.

4)	a)	$x_1(t) \to y_1(t)$ $x_2(t) \to y_2(t)$ $ax_1+bx_2 \to \int e^{-2(t-\tau-1)}(ax_1+bx_2)\,d\tau$ $= a\underbrace{\int e^{-2(t-\tau-1)} x_1\,d\tau}_{y_1(t)} + b\underbrace{\int e^{-2(t-\tau-1)} x_2\,d\tau}_{y_2(t)}$ System is linear. $x(t) \to y(t)$ $x(t-t_0) \to \int_{-\infty}^{t-t_0} e^{-2(t-(\tau-1)^{+t_0})} x(\tau - t_0)^{+t_0}\,d\tau$ $= \int_{-\infty}^{t-t_0} e^{-2(t-(\tau+t_0)-1)} x(\tau)\,d\tau$ $= y(t-t_0)$ System is Time Invariant
	b)	$\int_{-\infty}^{\infty} x(\tau)\, \underline{h(t-\tau)}\,d\tau = \int_{-\infty}^{t} x(\tau) \cdot e^{-2(t-\tau-1)} \cdot d\tau$ $h(t-\tau) \cdot u(t-\tau)$ $\tau < t$ $\boxed{h(t) = e^{-2(t-1)} \cdot u(t)}$

c)

$$x(t) = \left(u(t) - u(t-1)\right) - \left(u(-t) - u(-t-1)\right)$$

$$\int_{-\infty}^{\infty} e^{-2(t-T-1)}\, u(T).u(t-T).$$

$$t>0 \quad e^{2-2t}\int_{0}^{t} e^{2T}\, dT$$

$$t>1 \quad e^{2-2t}\int_{1}^{t} e^{2T}\, dT$$

$$t<0 \quad e^{2-2t}\int_{t}^{0} e^{2T}\, dT$$

$$t<-1 \quad e^{2-2t}\int_{t}^{-1} e^{2T}\, dT$$

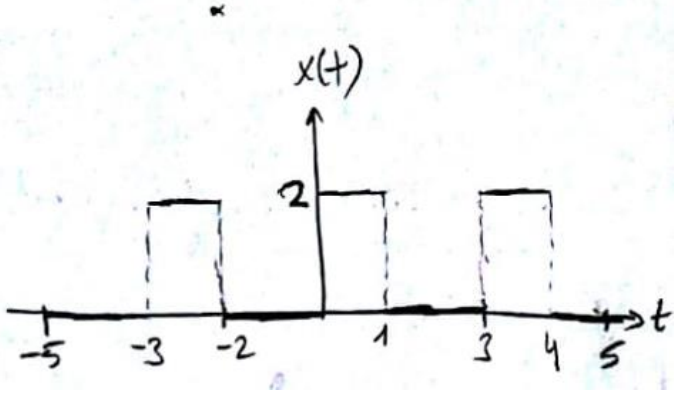
$$2\left(\frac{e^{2t}}{2} - \frac{1}{2}\right) e^{2-2t} = \underline{e^2 - e^{2-2t}} \;.\; u(t)$$

$$2\left(\frac{e^{2t} - e^2}{2}\right) e^{2-2t} = \underline{e^2} - e^{4-4t} \quad u(t-1)$$

$$\left(\frac{1-e^{2t}}{2}\right) e^{2-2t} = \frac{-e^2 + e^{2-2t}}{2} \quad u(-t)$$

$$\left(\frac{e^2 - e^{2t}}{2}\right) e^{2-2t} = \frac{-e^2 + e^{4-4t}}{2} \quad u(-t-1)$$

	d)	d) $h(t) = e^{-2(t-1)}$ System is not memoryless since it depends on past value. Because of this, system is causal. $\int_{-\infty}^{\infty} \lvert h(t) \rvert dt = \int_{-\infty}^{\infty} \lvert e^{-2(t-1)} \rvert dt$ $= \left\lvert \frac{e^{-2t} \cdot e^{2}}{-2} \right\rvert_{-\infty}^{\infty} = \infty$ System is not stable.
	e)	e) $x(t) = \delta(t)$ $h(t) = \int_{t-4}^{t+4} e^{-2(\tau - t)} \delta(\tau-1) d\tau + \delta(t-1)$ $h(t) = e^{2t-2} + \delta(t-1)$

5)	a)	

b)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$T_0 = 3 \quad , \quad a_0 = \frac{2}{T_0} = \frac{2}{3}$$

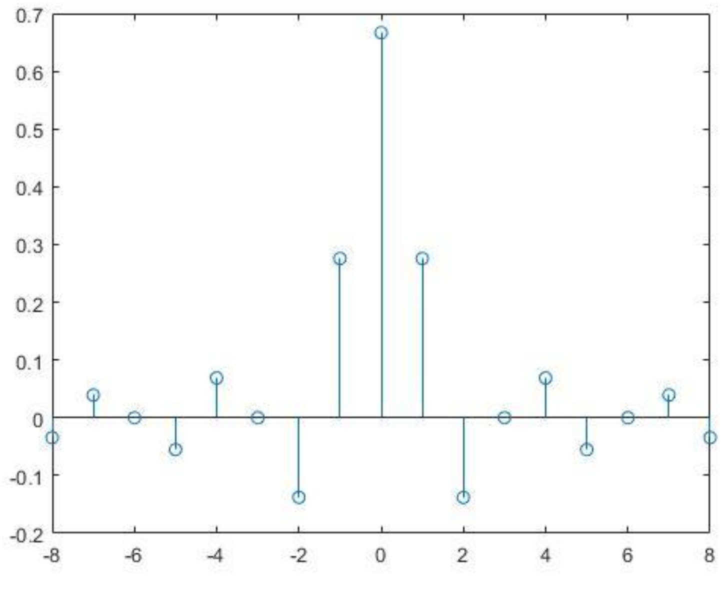
$$a_k = \frac{1}{3}\int_0^1 2 \cdot e^{-jk\omega_0 t}\, dt$$

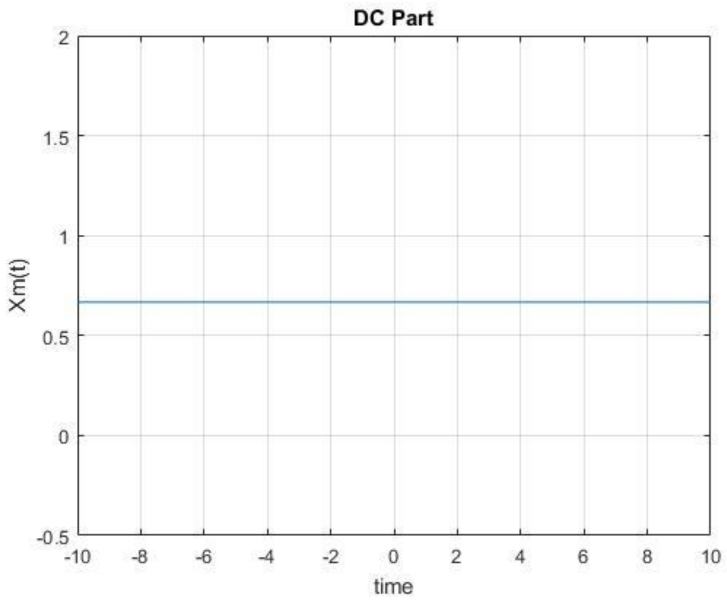
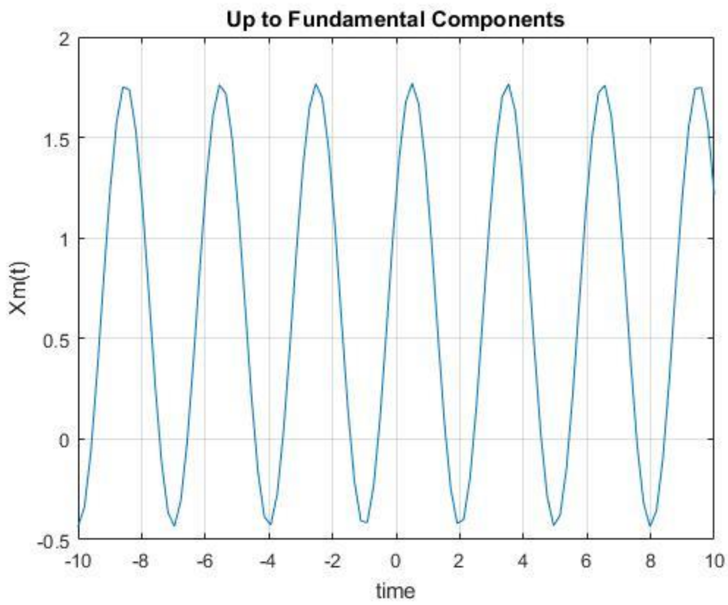
$$= \frac{2}{3(-j)k\omega_0} \cdot e^{-jk\omega_0 t}\Big|_0^1$$

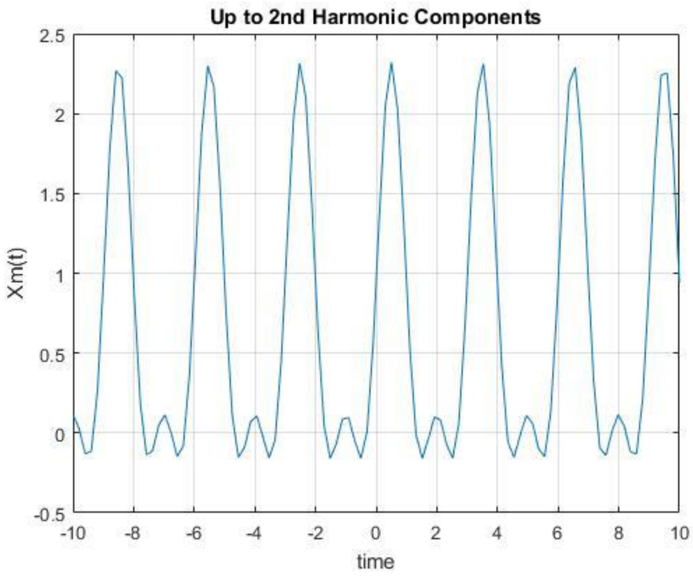
$$= \frac{2j}{3k\omega_0}\left(e^{-jk\omega_0} - 1\right), \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$$

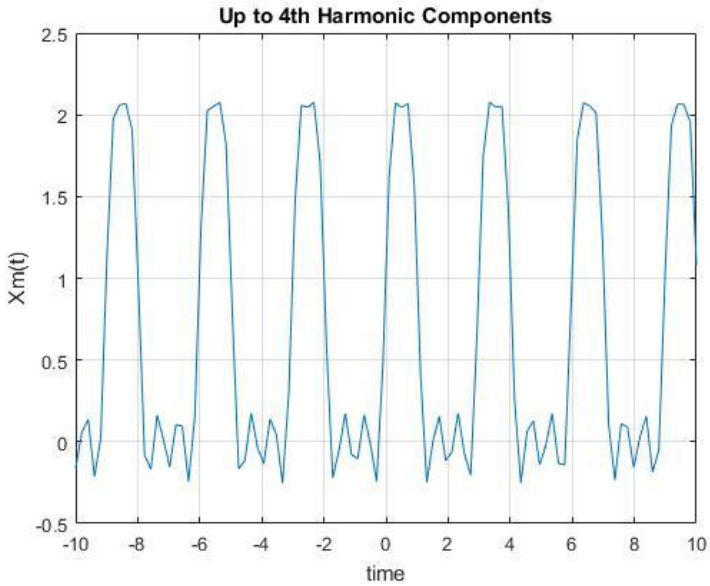
$$\boxed{a_k = \frac{j}{k\pi}\left(e^{-j\frac{2\pi k}{3}} - 1\right)}$$

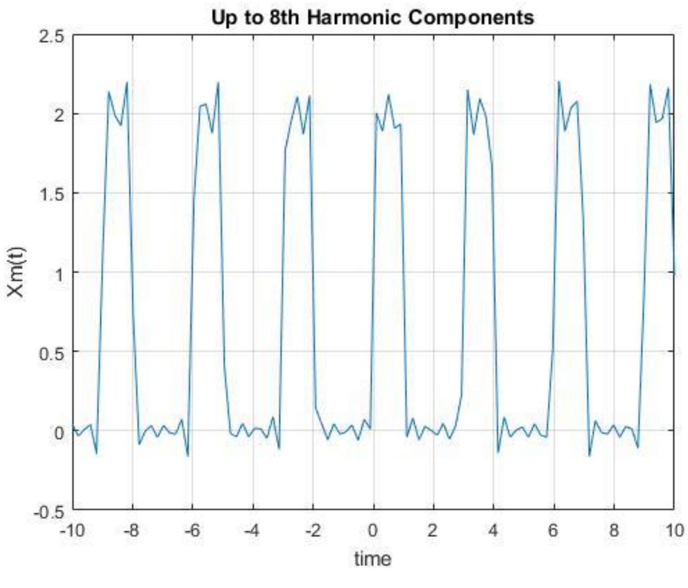
c)



	d)	DC Part Xm(t) time Up to Fundamental Components Xm(t) time


Up to 2nd Harmonic Components
Xm(t)
time


Up to 4th Harmonic Components
Xm(t)
time


Up to 8th Harmonic Components
Xm(t)
time

6	a)	$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$ $x(2t) * h(2t) = \int_{-\infty}^{\infty} x(2\tau)\, h(2t-2\tau)\, d\tau$ $2\tau = \lambda \Rightarrow 2d\tau = d\lambda$ $= \int_{-\infty}^{\infty} x(\lambda)\, h(2t-\lambda)\, \frac{d\lambda}{2}$ $= \frac{y(2t)}{2}$ //
	b)	b) $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$ $x[2n] * h[2n] = \sum_{k=-\infty}^{\infty} x[2k]\, h[2n-2k]$ $2k = \lambda \Rightarrow k = \frac{\lambda}{2}$ $= \sum_{\frac{\lambda}{2}=-\infty}^{\frac{\lambda}{2}=\infty} x[\lambda]\, h[2n-\lambda]$ $= \sum_{\lambda=-\infty}^{\infty} x[\lambda]\, h[2n-\lambda] = \underline{\underline{y[2n]}}$