Q1. Consider the system given by

$$\dot{x} = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x$$

a. Find the transfer function of the system.

b. Check if the system is completely controllable or not.

c. Design an state feedback rule $u = r - Kx$ for the system by placing closed loop poles at $s = -1, -1 \mp j$.

Q2. Consider the system given by

$$\dot{x} = \begin{bmatrix} -4 & 2 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$$

a. Check if the system is controllable or not?

b. Can you find a state feedback law $u = r - Kx$ where $K = [k_1, k_2]$ to place the closed loop poles at $s = -4$ and $s = -4$?

c. Can you find a state feedback law $u = r - Kx$ where $K = [k_1, k_2]$ to place the closed loop poles at $s = -4$ and $s = -6$?

Q3. Consider the system given by

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 0 & -20 & -40 \end{bmatrix} x$$

a. Check if the system is completely controllable or not.

b. Check if the system is completely observable or not.

c. Noting that state feedback does not change zeros of the open loop system (i.e., the open loop and closed loop zeros are the same), determine a feedback law $u = r - Kx$ so that the resulting system is reduced to a second order system with a settling time of $t_s = 1$second (2%) and damped natural frequency of $\omega_d = 2$ rad/sec.

Hint: You might consider canceling one of the zeros of the system to reduce the order to 2.

d. Check if the closed loop system obtained in part-c is observable or not.