Middle East Technical University
Department of Electrical and Electronics Engineering

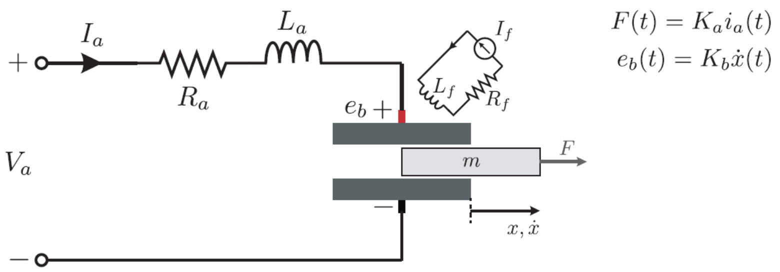
EE302 – Feedback Systems

Assignment 2

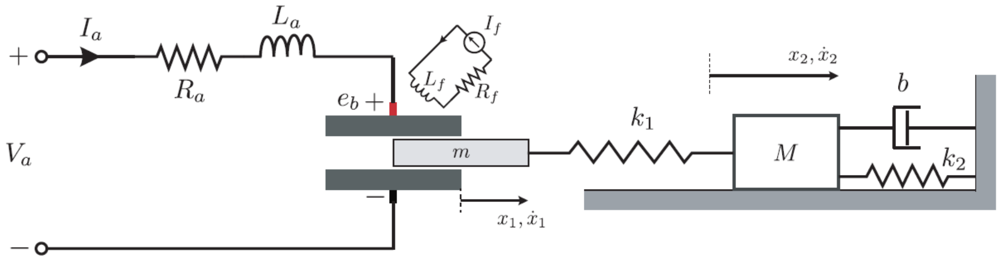
Deadline: 20-Mar-2019, @15:40

There will be a box to drop the assignments in front of D-226. The box will be removed after 15:40.

Q1. The figure below illustrates an armature controlled translational motor (or solenoid). Very similar to rotational motors, in armature controlled translational motor the force generated by the motor depends on armature current, i.e., $F(t) = K_a i_a(t)$, where Ka is a constant depending in motor properties. Again, depending on the translational velocity of the motor pin, a beck emf voltage is generated in the electrical circuit, i.e., $e_b(t) = K_b \dot{x}(t)$.

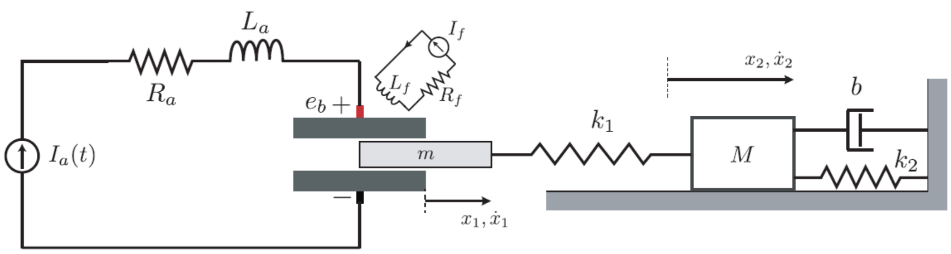


Now consider the following electromechanical system. In this system, the linear motor pin, which has a mass of m, is connected to a different body, with a mass of M, through a linear spring with a stiffness of k_1. This body also connected to the ground via a spring-mass pair ($[k_2, b]$).



Given that input of the system is the armature voltage, $u(t) = V_a(t)$, and output of the system is the position of the second body, $y(t) = x_2(t)$, answer/solve the following questions

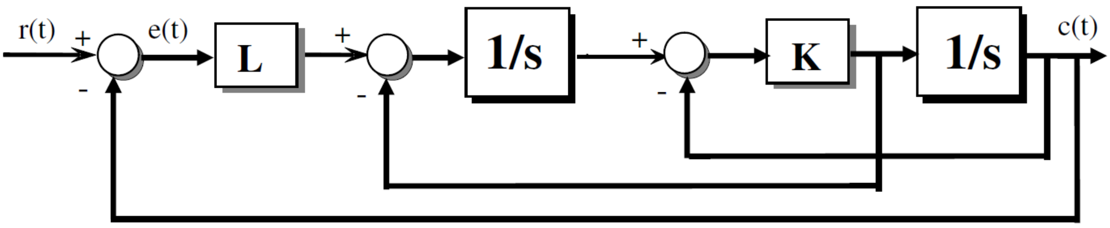
a) Find a state-space representation of the whole system
b) Draw a detailed block-diagram of the electro-mechanical system. (Do not use non-causal blocks in your block diagram representation).
c) Now we decide to adopt a current based motor control policy, and thus connect a variable high bandwidth current source to the input terminal of the electrical circuit.



In the above scenario, input of the system is the armature current, $u(t) = I_a(t)$, and output is unchanged, i.e., the position of the second body, $y(t) = x_2(t)$. Answer/solve following questions.

d) Update the state-space representation.
e) Update the block-diagram representation.

Q2. Given the following feedback system,



a) Determine the plant transfer function *G(s)*.
b) Find conditions on ***K*** and ***L*** that makes the system critically damped.
c) Assume that ***L*** > ***K***. Find the poles of this system comment on the change in the pole position as ***L*** increases further.
d) Comment on the change of the value of maximum overshoot, settling time and rise time as ***L*** increases.

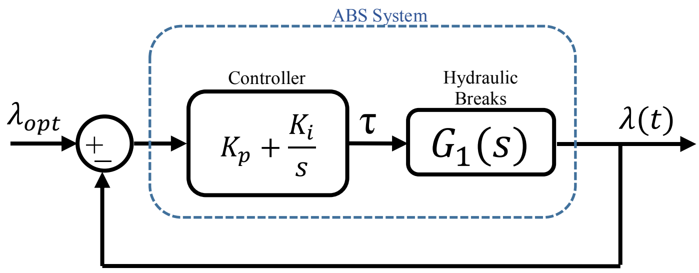
Q3. (Braking Bad) An Anti-lock Braking System (ABS) is used to prevent wheel lock and optimize breaking performance in various vehicles. For this purpose, the system monitors one quantity called the **longitudinal slip**, denoted by **λ(t)**, which is the relative motion between a tire and the road surface it is moving on. The equation which relates **λ(t)** with the angular velocity of the wheel $\boldsymbol{\omega(t)}$, and the longitudinal speed of the car $\boldsymbol{v}$ is given as $\boldsymbol{\lambda(t)} = \frac{\boldsymbol{v(t)-r\omega(t)}}{\boldsymbol{v(t)}}$, where **r** is the wheel radius. Notice that, if **λ(t) =0**, then there is no breaking; if **λ(t)= 1**, then the wheels are locked. In an ABS system, a dynamic torque **τ(t)** is applied to hydraulic breaking systems to reach the optimal slip value $\boldsymbol{\lambda_{opt}} \approx$ **0.15.** This is done by a controller in a feed-back loop. Assume the relationship between the torque **τ(t)** and the slip **λ(t)**, is given by the following differential equation.

$$\dot{\lambda}(t) = -\frac{r^2 C_0}{v_0 J}\lambda(t) + \frac{r}{v_0 J}\tau(t),$$

where r, C_0, v_0, m, J are constant parameters.

a) Find the transfer function $G_1(s) = \frac{\lambda(s)}{\tau(s)}$ for $r = 1$, $v_0 = 10$, $C_0 = 50$, $J = 0.1$.

b) Consider the configuration below.



By using the transfer function found in part (a) as $G_1(s)$, find the parameters of the controller (K_p and K_i) for the system to have a settling time (%5) of 0.06 seconds and percent overshoot of 10% to a unit step input *(assume that the effect of zeros of the transfer function on the step response can be neglected).*