MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 462 Utilization of Electrical Energy

First Midterm Examination

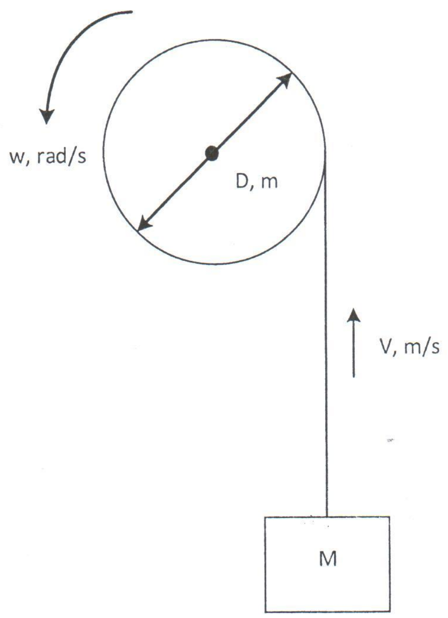
Duration: 100 minutes
Attempt all questions
Show all your calculations.

21 April 2016

NAME and SURNAME ___Solutions___

Q1	
Q2	
Q3	
Q4	
Total	

Q1 (15 pts). Show that $w = 2v / D$. Show all steps of the derivation.



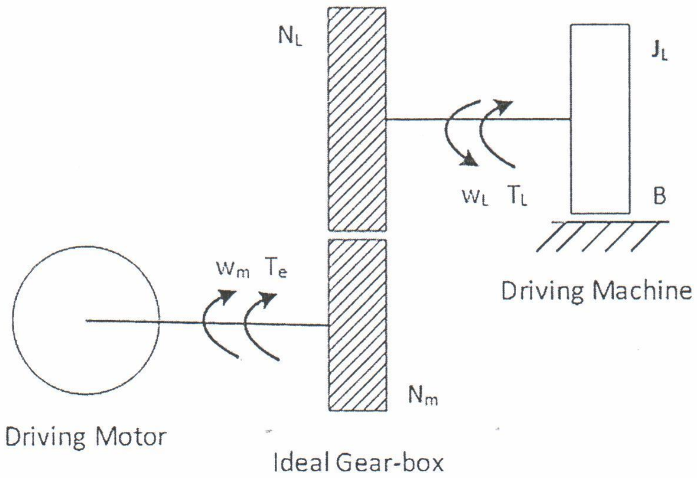
Circumference of the drum, πD

Drum speed in revolutions per second, $\frac{V}{\pi D}$

Drum speed in radians per second, $\omega = \frac{V}{\cancel{\pi} D} \cdot 2\cancel{\pi}$

Therefore, $\omega = 2V/D$ //

Q2 (20 pts). Consider the following gear drive which is operating **in the steady-state**.



Driven machine side quantities are given as:

$T_L = 3000$ Nm, $w_L = 50\pi/6$ rad/s, $J_L = 1.0$ Kg-m^2, $B = 100$ Nm-s/rad, and the number of teeth of the gear box, $N_m = 20$, $N_L = 60$.

Calculate w_m, T_e, and the referred values J_L' and B'.

$$\omega_m = \left(\frac{N_L}{N_m}\right)\omega_L = \left(\frac{6\cancel{0}}{2\cancel{0}}\right) \times \frac{50\pi}{\cancel{6}} = 25\pi \text{ rad/s} \quad //$$

$$J_L \frac{d\omega_L}{dt} + B\cdot\omega_L + T_L = \left(\frac{N_L}{N_m}\right)\cdot T_e$$

Since $J_L \frac{d\omega_L}{dt} = 0$ in the steady-state, then

$$\left(\frac{N_L}{N_m}\right) T_e = T_L + B\cdot\omega_L \Rightarrow T_e = \left(\frac{N_m}{N_L}\right)\left[T_L + B\cdot\omega_L\right]$$

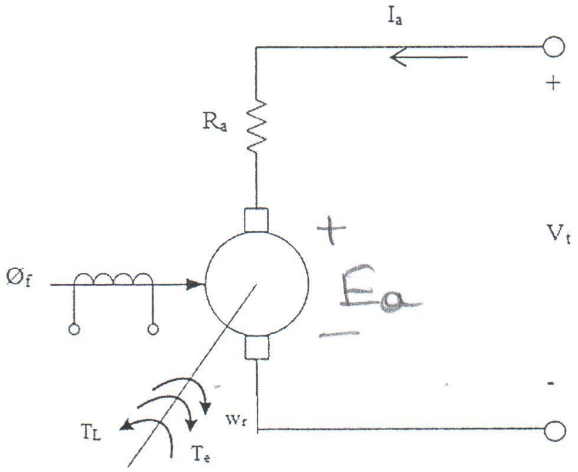
$$T_e = \left(\frac{20}{60}\right)\left[3000 + 100 \times \frac{50\pi}{6}\right]$$

$$T_e = 1873 \text{ Nm} \quad //$$

$$J_L' = \left(\frac{N_m}{N_L}\right)^2 \cdot J_L = \left(\frac{20}{60}\right)^2 \cdot 1.0 = 0.\overline{1} \text{ Kg-m}^2 \quad //$$

$$B' = \left(\frac{20}{60}\right)^2 \cdot 100 = 11.\overline{1} \text{ Nm-s/rad} \quad //$$

Q3 (20 pts). Consider the dc motor drive shown in the figure. Assume that it is operating **in the steady-state**. Friction and windage torque is included in the load torque. Show all steps of the derivation.



Show that

$$w_r = \frac{V_t}{K_a \emptyset_f} - \frac{R_a}{(K_a \emptyset_f)^2} . T$$

$$J\frac{d\omega_r}{dt} = T_e - (T_L + T_{f\&w})$$

Since $J\frac{d\omega_r}{dt} = 0$ in the steady-state, then

$$T_e = T_L + T_{f\&w} = T$$

$$V_t = E_a + R_a I_a \quad \text{---} (1)$$

$$E_a = K_a \Phi_f \omega_r \quad \text{---} (2)$$

$$T_e = K_a \Phi_f I_a \quad \text{---} (3) \Rightarrow I_a = T_e / K_a \Phi_f \quad \text{---} (4)$$

By substituting (2) and (4) in (1)

$$V_t = K_a \Phi_f \omega_r + \frac{R_a}{K_a \Phi_f} . T \quad (5)$$ where, $T = T_e = T_L + T_{f\&w}$

By rearranging (5)

$$\omega_r = \frac{V_t}{K_a \Phi_f} - \frac{R_a}{(K_a \Phi_f)^2} . T \quad //$$

Q4 (55 pts). A mechanical load is driven by a chopper-fed separately-excited dc motor as shown below. Throughout the problem, assume **continuous** armature current.

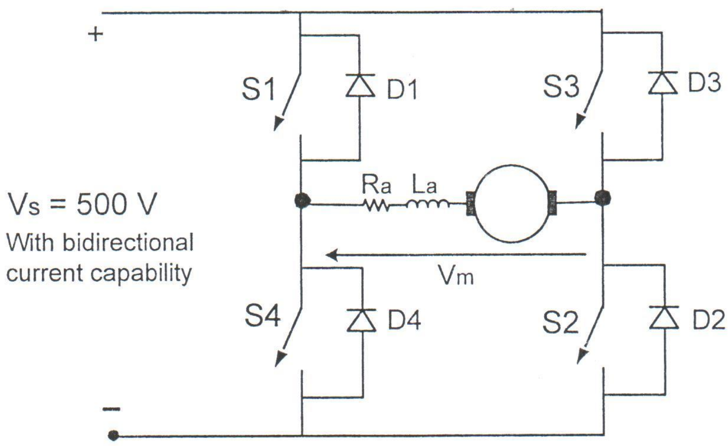
<u>DC Motor data</u>

$R_a = 0.1$ ohm

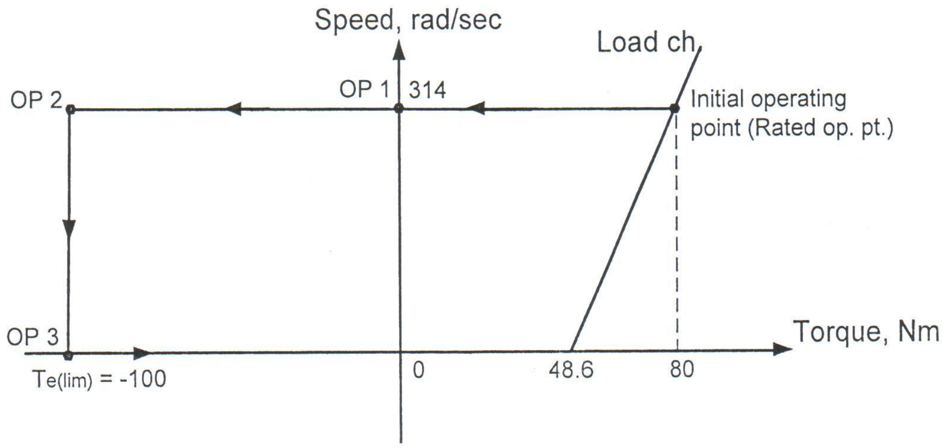
$L_a = 25$ mH

Magnetization characteristic : $E_a = 250$ V at $w_r = 314$ rad/s. Magnetic saturation is neglected and field flux ϕ_f is kept constant over the entire operation range.

Assume that combined inertia of the system is $J = 1.0$ kg-m^2.



DC Motor Drive



Loci of the operating point

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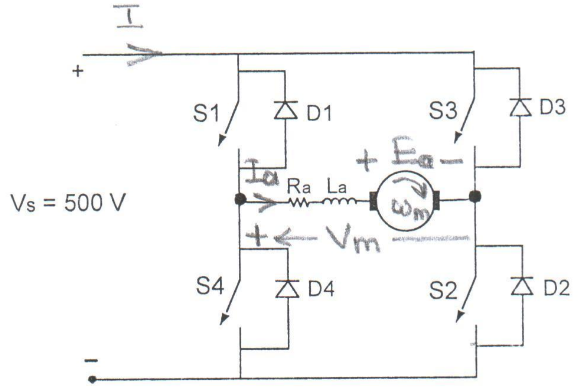
a) Calculate applied motor voltage, V_m, armature current, I_a, and induced armature emf, E_a, for the dc machine while it is operating in the steady-state at the initial operating point. Find the duty ratio, d, of the chopper by assuming **ideal chopper operation**.
Mark the polarities of voltages and direction of the armature current on the figure below.



$\omega_m = 314$ rad/s

$T_e = T_L = 80$ Nm in the steady-state

$E_a = 250$ V at $\omega_r = 314$ rad/s

$E_a = K_a\Phi_f \omega_r$

$250 = K_a\Phi_f \cdot 314 \Rightarrow$

$K_a\Phi_f = 0.7962$ //

$T_e = K\Phi_f I_a$

$80 = 0.7962 \times I_a \Rightarrow I_a = 100.5$ A //

$V_m = E_a + R_a I_a = 250 + 0.1 \times 100.5 = 260$ V //

$E_a = 250$ V // given

$V_m = D \cdot V_s$ where, $D = \frac{t_{on}}{t_{on} + t_{off}}$

$260 = D \cdot 500 \Rightarrow$

$D = 0.52$ //

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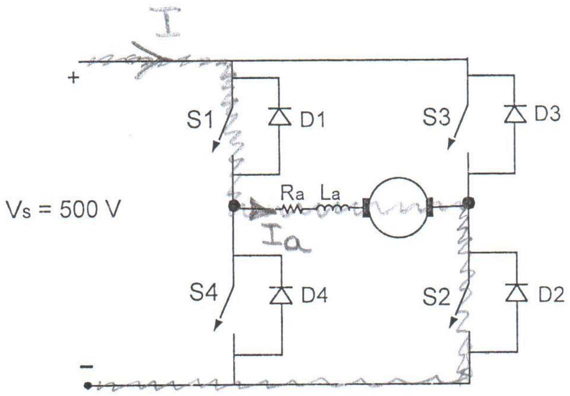
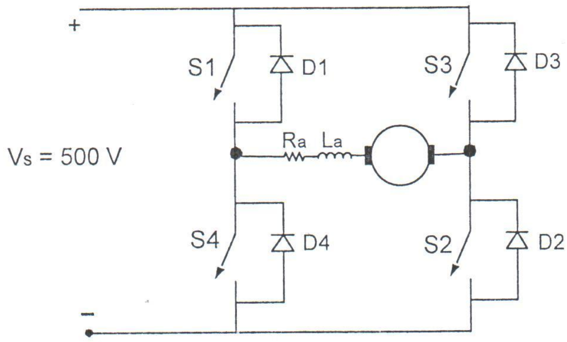
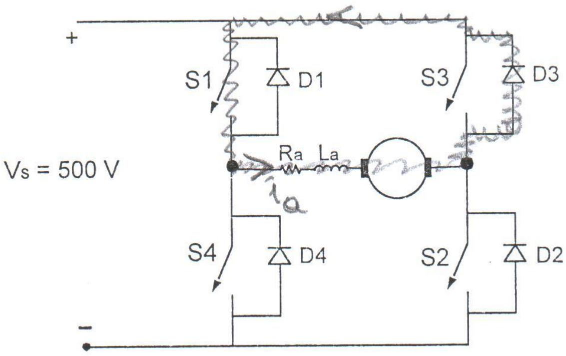
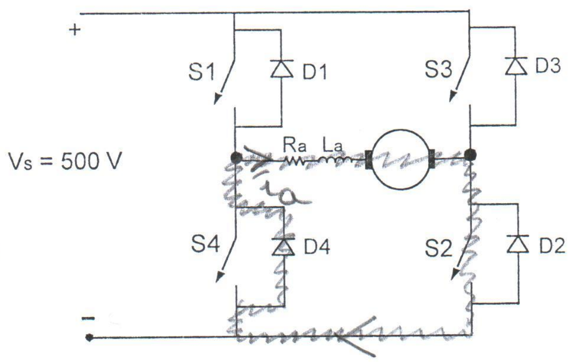
b) What are the recommended operation modes of the chopper for any two consecutive operation cycles while the system is operating at the **initial operating point**, Which power semiconductors are conducting in each operation mode?
Mark the corresponding current paths on the figures below by heavily drawn solid lines.





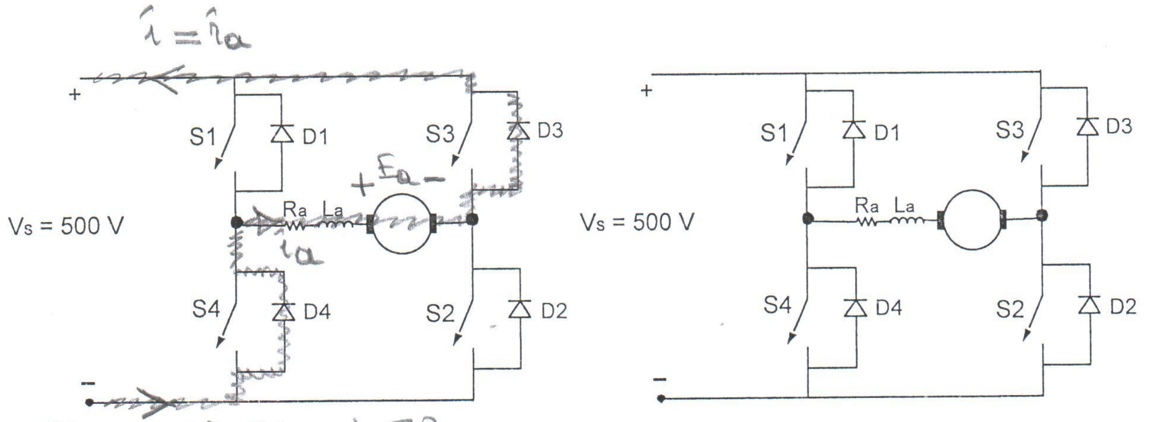
Power Transfer Mode



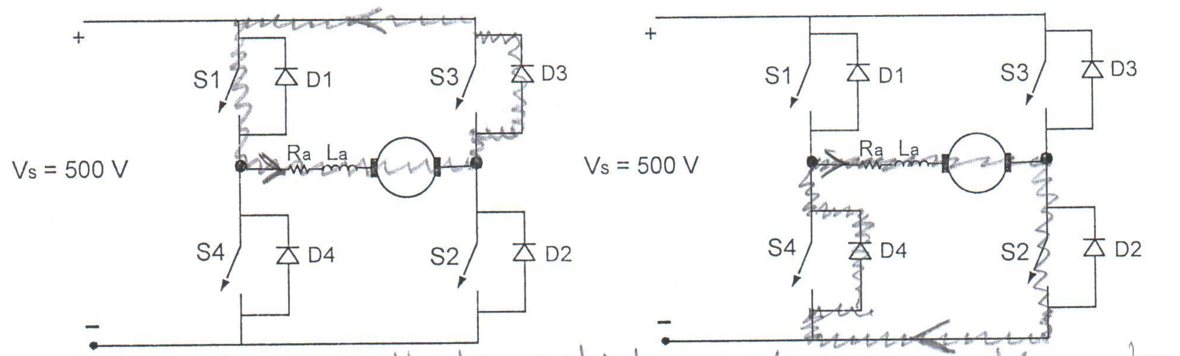


Periodic freewheling paths

c) Recommend a method to bring the operating point from initial operating point to OP1. Give your reasoning.
Draw the current path on the attached diagram by a heavily solid line. Estimate the time spent in this operation mode. Does the recommended method **minimize** the time spent to bring the current to zero. Assume that the shaft speed momentarily remains the same at its initial value.



Turn-off S1 and S2
$i_a(t)$ ceases by regeneration in minimum time.



Alternative methods which need longer time to decay $i_a(t)$ to zero.

$$v_m(t) = e_a(t) + R_a i_a(t) + L_a \frac{di_a(t)}{dt}$$

$$\frac{di_a}{dt} + \frac{1}{\tau_a} i_a = v_m - e_a \quad \text{where, } \tau_a = L_a/R_a = 0.025/0.1 = 0.25\text{ s}$$

Solution of the above eqn is

$$i_a(t) = i_a(\infty)(1 - e^{-t/\tau_a}) + i_a(0)$$

In this mode, $i_a(0) = 100.5\text{ A}$, $v_m = -V_s = -500\text{ V}$, $e_a = 250\text{ V}$

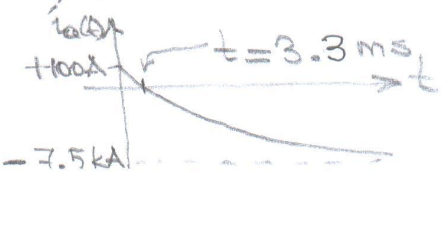
$$i_a(\infty) = \frac{-500-250}{0.1} = -7500\text{ A}$$

Therefore, $i_a(t) = -7600(1 - e^{-4t}) + 100.5$

Alternative expression
($i(t) = 7600e^{-4t} - 7500$

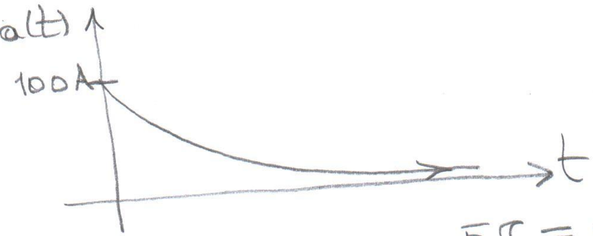
$$i_a(t) = -7600(1 - e^{-4t}) + 100.5 = 0$$

$\Rightarrow t = 3.3\text{ ms}$ //



pto ⟶

For freewheeling operation



$\tau_a = 0.25$ s $\rightarrow 5\tau = 1.25$ s

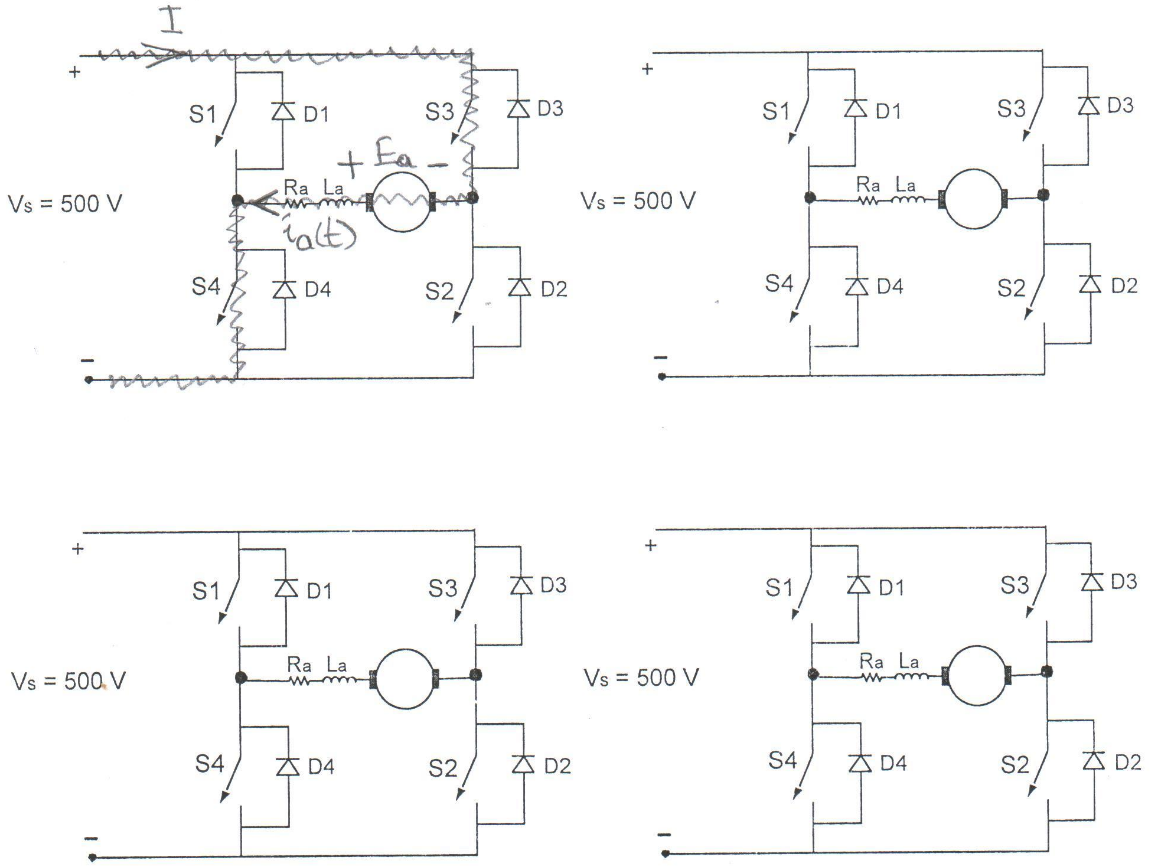
$i_a(\infty) = 0 \rightarrow t \rightarrow \infty$

in 1.25 s time i_a ceases.

$1.25 \text{ s} \gg 0.0033$ s

d) Now bring the operating point from OP1 to OP2 **as quickly as possible** by applying a step voltage to the machine terminals. Assume that w_r remains momentarily constant. What is the required value of V_m ? Which power semiconductors are to be triggered into conduction ? Calculate d.

Mark the current path by a heavily drawn solid line on the figure below.



$v_m(t) - e_a(t) = L_a \frac{di_a(t)}{dt} + R_a i_a(t)$

$e_a(t) = E_a = 250$ V assumed to be momentarily constant.

$v_m(t) = -V_s$ by triggering S3 and S4 into conduction.

$-500 - 250 = L_a \frac{di_a}{dt} + R_a i_a(t)$ $\quad i_a(0) = 0$

at OP2 $\quad T_e = -100$ Nm $\quad T_e = K\Phi_f I_a \Rightarrow$

$I_a = T_e / K_a\Phi_f = -100/0.7962 = -125.6$ A

$i_a(\infty) = -7500$ A

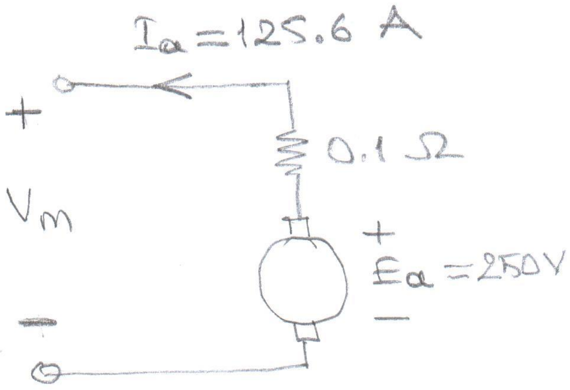
The soln $\quad i_a(t) = -7500(1 - e^{-4t})$

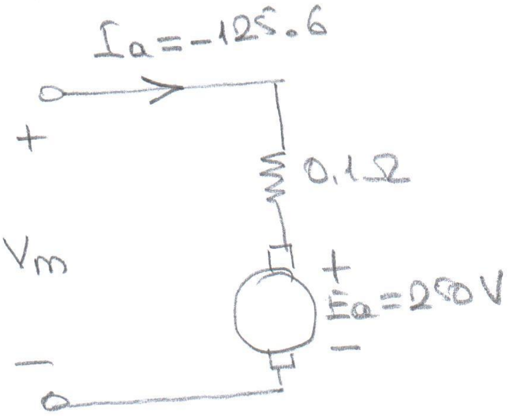
$-125.6 = -7500(1 - e^{-4t})$

$\Rightarrow t = 4.2$ ms //

pto →

at OP2





$E_a = R_a I_a + V_m$

$V_m = E_a - R_a I_a$

$= 250 - 0.1 \times 125.6$

$V_m = 237.4$ V

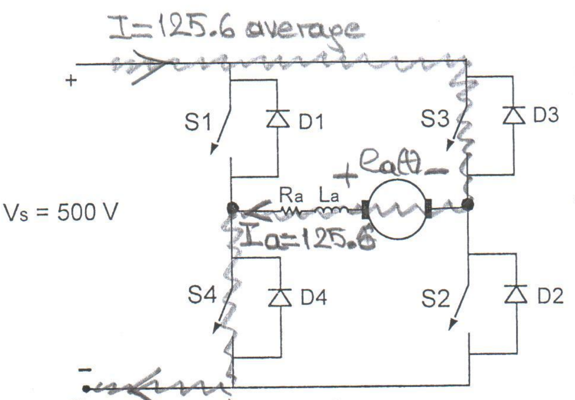
$V_m = E_a + R_a I_a$

$= 250 + 0.1(-125.6)$

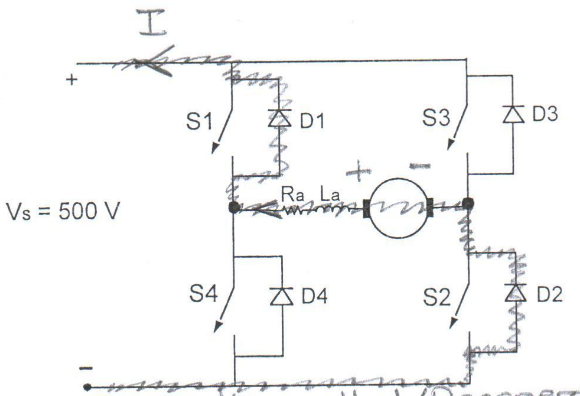
$V_m = 237.4$ V

$D = \frac{237.4}{500} = 0.475$ // at point OP2

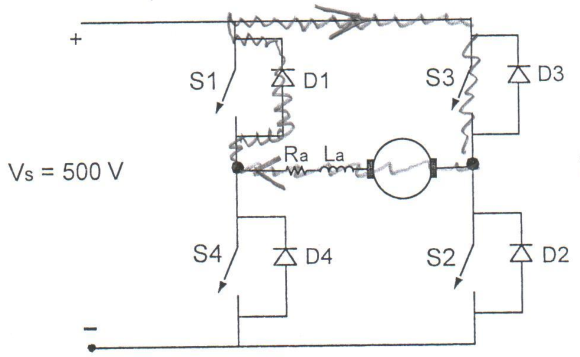
e) Repeat (a) and (b) for the operating point OP2. Operation modes, voltage polarities and current direction should represent the transition from OP2 to OP3.

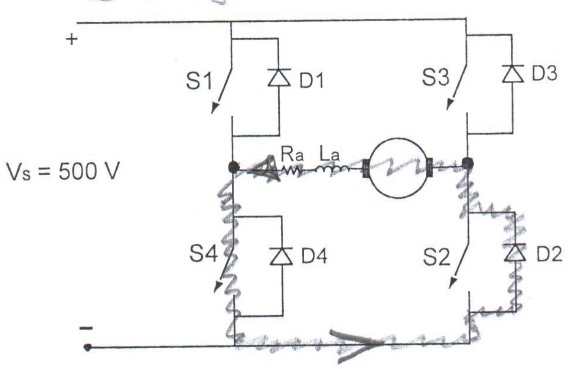


Power transfer mode, should be used together with periodic FW modes.



Alternative method / Regenerative braking which should be used together with periodic FW modes to charge the armature inductance.





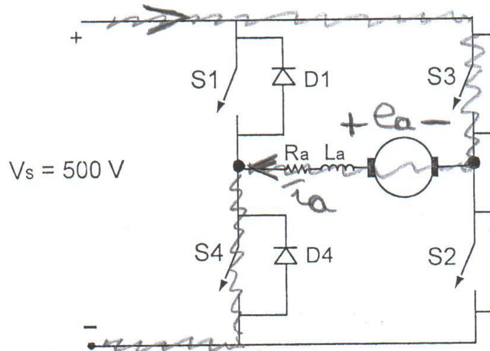
Periodic freewheeling modes

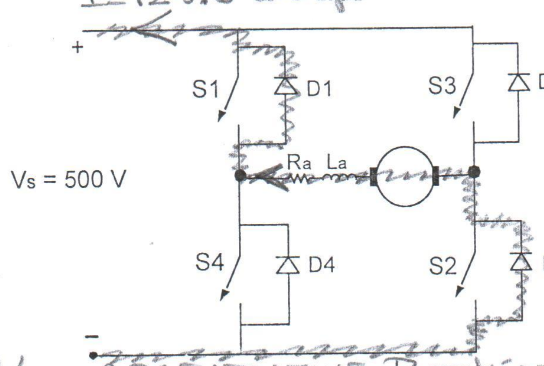
During deceleration, T_e should be kept constant at -100 Nm and hence I_a at $I_{a(limit)} = -125$ A

f) The dc motor is then brought to a stop by applying $T_{e(lim)} = -100$ Nm. Find an expression for $w_r(t)$. Calculate the deceleration time. Name the braking technique applied to the dc motor drive.

Mark the polarities of E_a and V_m, and the direction of I_a on the attached diagram. Define the operation modes of the chopper during acceleration. Which power semiconductors are in conduction in each operation mode? Mark the associated current current paths on the attached diagrams by heavily drawn solid lines.

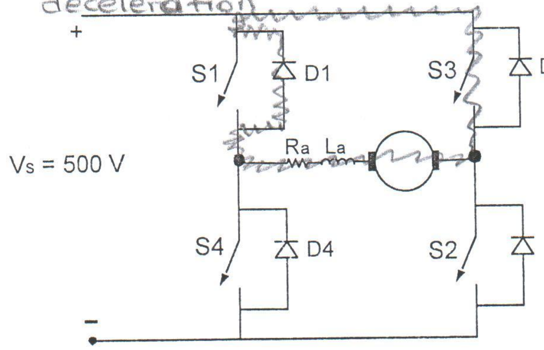
I=125.6 average

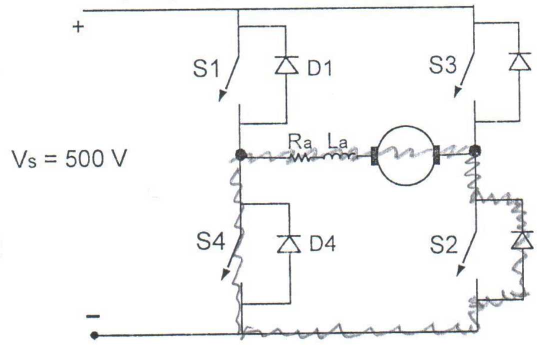




PLUGGING/not recommended/ it should be used together with periodic FW modes in order to keep I_a constant at $I_{a(lim)}$ during deceleration

REGENERATIVE Braking / recommended method/used together with periodic FW modes.





$$J\frac{dw}{dt} = T_{Lim} - T_L \quad \text{where, } T_L = mw + a = 0.1w + 48.6$$

$$\frac{dw}{dt} + \frac{1}{\tau_m}w = \frac{T_{lim} - a}{J} \quad \text{where, } \tau_m = \frac{J}{m} = 10\ s$$

$$\frac{dw}{dt} + 0.1w = \frac{-100 - 48.6}{1}$$

$$\frac{dw}{dt} + 0.1w = -148.6$$

Soln of this eqn

$$w(t) = C_1 e^{-t/\tau_m} + C_2$$

Since $w(0) = 314$ then $\quad 314 = C_1 + C_2$

pto →

Assume symmetrical load ch in QI and Q3

$T_L = 0.1\omega + 48.6 = T_e = 100$ Nm

Therefore, $\omega(\infty) = 514$ rad/s in the reverse direction

That is $\omega(\infty) = -514$

$\omega(\infty) = -514 = C_2$ and hence $C_1 = 828$

The soln is therefore

$$\omega(t) = 828\, e^{-0.1t} - 514$$

$$\omega(t) = 828\, e^{-0.1t} - 514 = 0$$

$$e^{-0.1t} = 514/828$$

$$\ln(e^{-0.1t}) = \ln(514/828)$$

$$-0.1t = -0.477$$

Therefore, $t = 4.77$ s //

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g) What will happen to the operation mode of the drive, if we do not reduce I_a to zero, after reaching point OP3?

I_a is negative, $T_e = -100\,Nm$ negative too
ω increases in the negative direction,
and hence the dc m/c starts to operate
as a reverse motor.

Dr. Ermiş