METU EE462 Utilization of Electric Energy

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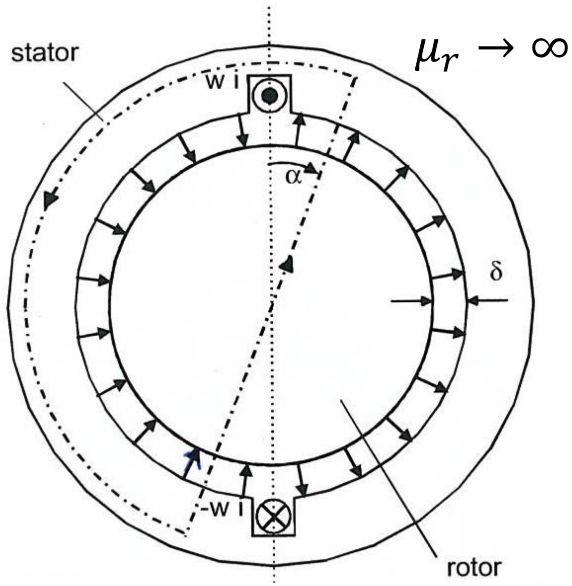
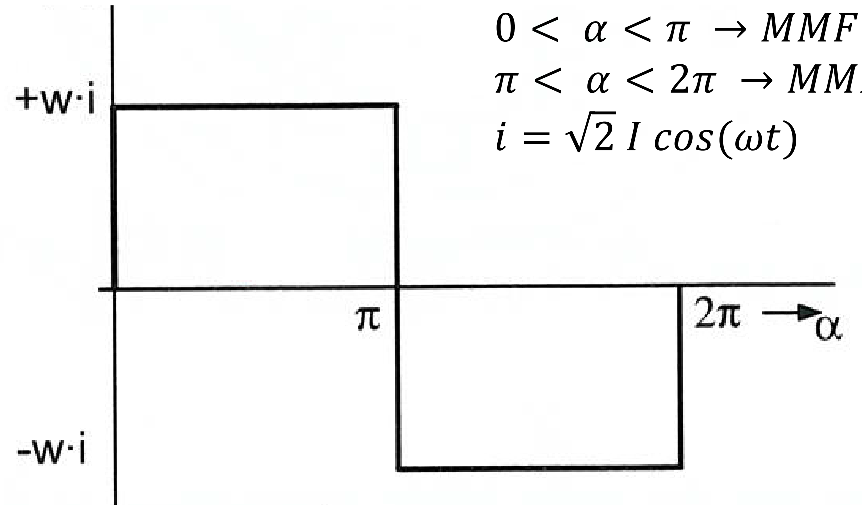
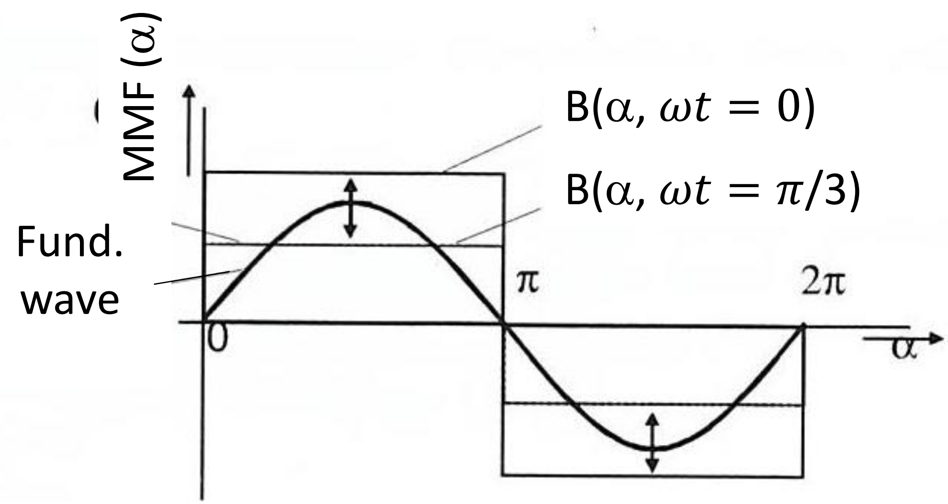
Content

1-Phase Magnetic Air-gap B-Field

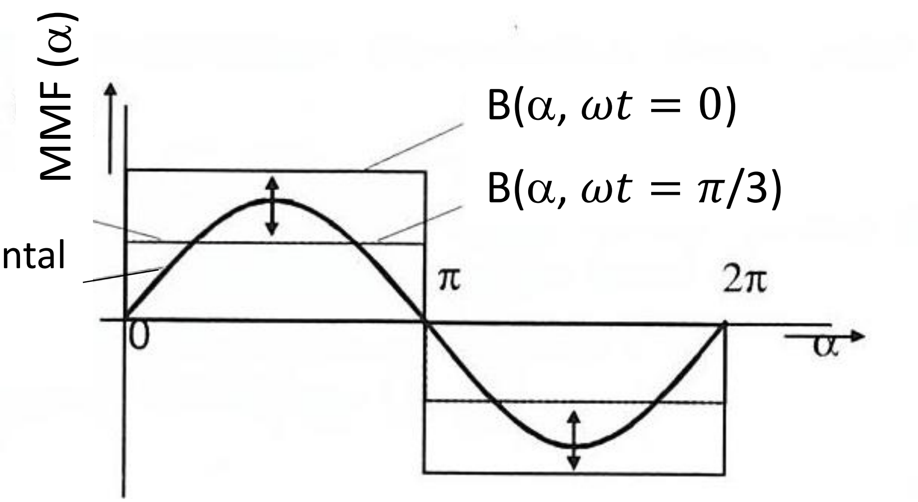


$0 < \alpha < \pi \rightarrow MMF = w\, i$
$\pi < \alpha < 2\pi \rightarrow MMF = -w\, i$
$i = \sqrt{2}\, I\, cos(\omega t)$





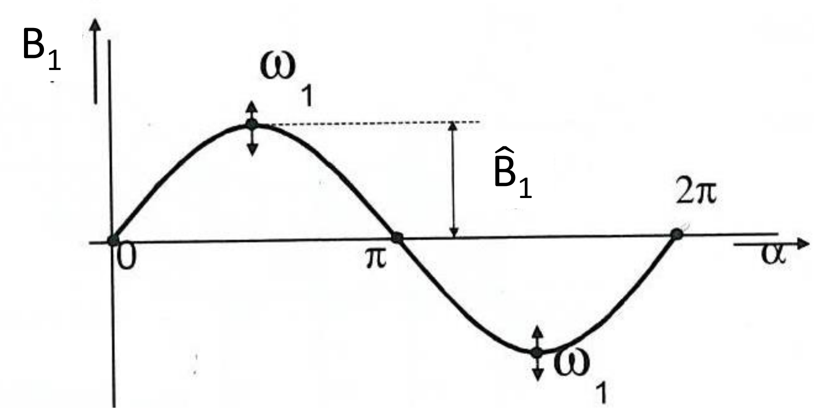
1-Phase Magnetic Field Distribution



$$\mathrm{MMF} = \mathrm{H}(\alpha)2\delta = \frac{\mathrm{B}(\alpha)}{\mu_0}2\delta$$

$$0 < \alpha < \pi \rightarrow \mathrm{B}(\alpha, t) = w\sqrt{2}\, I\, cos(\omega t)\frac{2\delta}{\mu_0}$$

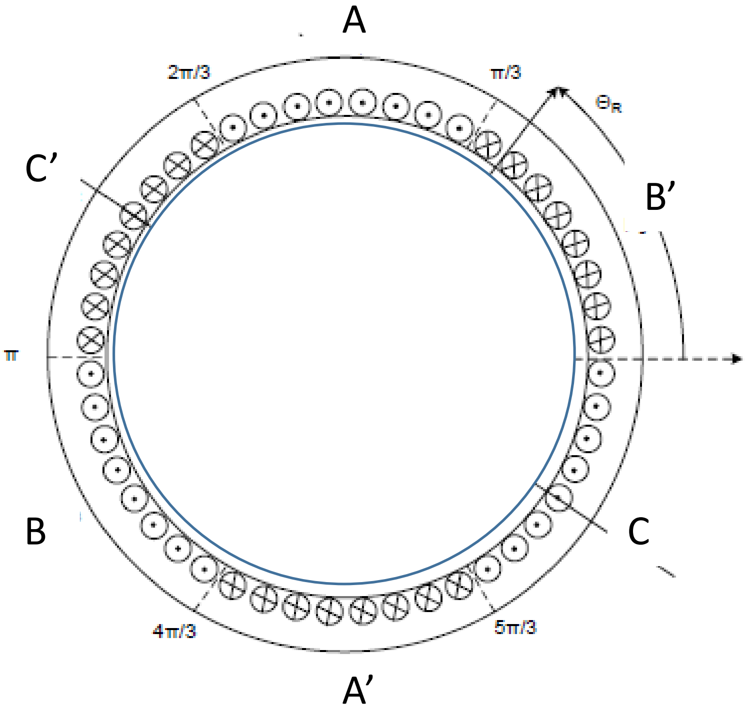
$$\pi < \alpha < 2\pi \rightarrow \mathrm{B}(\alpha, t) = -w\sqrt{2}\, I\, cos(\omega t)\frac{2\delta}{\mu_0}$$



Fundamental wave:

$$\mathrm{B}(\alpha, t) = w\sqrt{2}\, I\, cos(\omega t)\frac{2\delta}{\mu_0}\frac{4}{\pi}\sin(\alpha)$$

3-Phase Magnetic Air-gap B-Field



$$B_{A1}(\alpha,t) = w\,\sqrt{2}\,I\,cos(\omega t)\,\frac{2\delta}{\mu_0}\frac{4}{\pi}\sin(\alpha)$$

$$B_{A1}(\alpha,t) = B_1 cos(\omega t)\,\sin(\alpha)$$

$$B_{B1}(\alpha,t) = B_1 cos\left(\omega t - \frac{2\pi}{3}\right)\sin(\alpha - \frac{2\pi}{3})$$

$$B_{C1}(\alpha,t) = B_1 cos\left(\omega t - \frac{4\pi}{3}\right)\sin(\alpha - \frac{4\pi}{3})$$

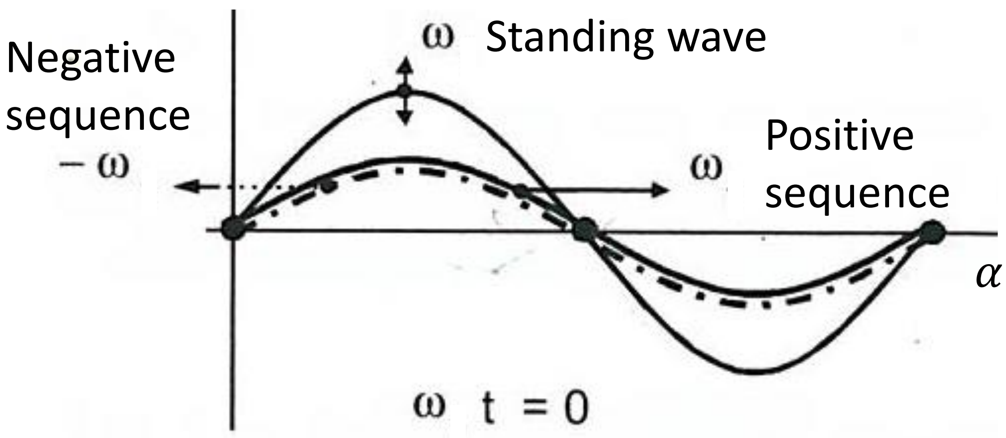
Fundamental component of the flux density distribution created by phase C.

$$i_{s1}(t) = \sqrt{2}\,I_{s1}\cos(\omega t)$$

$$i_{s2}(t) = \sqrt{2}\,I_{s1}\cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$i_{s3}(t) = \sqrt{2}\,I_{s1}\cos\left(\omega t - \frac{4\pi}{3}\right) = \sqrt{2}\,I_{s1}\cos\left(\omega t + \frac{2\pi}{3}\right)$$

3-Phase Magnetic Air-gap B-Field



$$\mathrm{B}_{\mathrm{A1}}(\alpha, t) = w\,\sqrt{2}\,I\,cos(\omega t)\,\frac{2\delta}{\mu_0}\frac{4}{\pi}\sin(\alpha)$$

$$\mathrm{B}_{\mathrm{A1}}(\alpha, t) = B_1 cos(\omega t)\,\sin(\alpha)$$

$$\mathrm{B}_{\mathrm{B1}}(\alpha, t) = B_1 cos\left(\omega t - \frac{2\pi}{3}\right)\sin(\alpha - \frac{2\pi}{3})$$

$$\mathrm{B}_{\mathrm{C1}}(\alpha, t) = B_1 cos\left(\omega t - \frac{4\pi}{3}\right)\sin(\alpha - \frac{4\pi}{3})$$

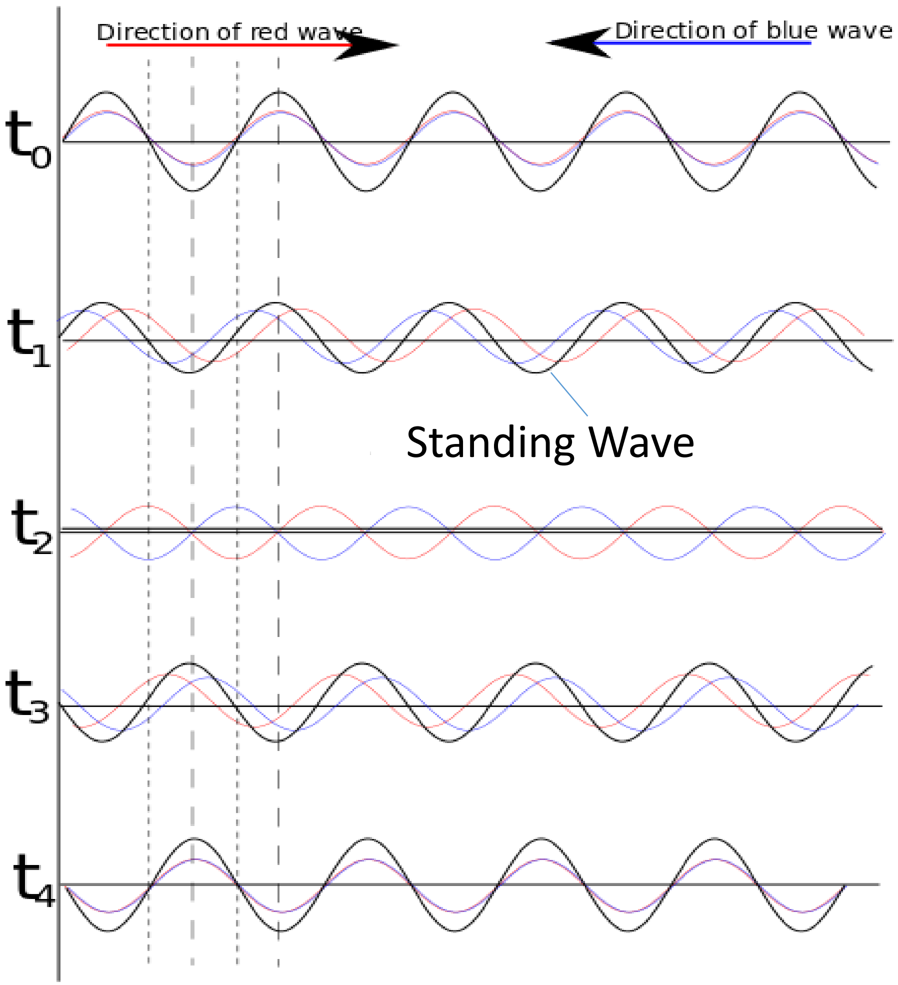
$$\underbrace{\frac{B_1}{2}\sin(\alpha - \omega t)}_{\text{Positive sequence}} + \underbrace{\frac{B_1}{2}\sin(\alpha + \omega t))}_{\text{Negative sequence}}$$

$\mathrm{B}_{\mathrm{A1}}(\alpha, t)$, $\mathrm{B}_{\mathrm{B1}}(\alpha, t)$ and $\mathrm{B}_{\mathrm{C1}}(\alpha, t)$ are the fundamental waves. The air-gap flux density distribution depends on

- position due to distribution of the winding conductors, $sin(\alpha)$
- time due to time-varying phase currents, $cos(\omega t)$.

Animation: https://upload.wikimedia.org/wikipedia/commons/7/7d/Standing_wave_2.gif

3-Phase Magnetic Air-gap B-Field



$$B_{A1}(\alpha,t)=w\sqrt{2}\,I\,cos(\omega t)\frac{2\delta}{\mu_0}\frac{4}{\pi}\sin(\alpha)$$

$$B_{A1}(\alpha,t)=B_1 cos(\omega t)\sin(\alpha)$$

$$B_{B1}(\alpha,t)=B_1 cos\left(\omega t-\frac{2\pi}{3}\right)\sin(\alpha-\frac{2\pi}{3})$$

$$B_{C1}(\alpha,t)=B_1 cos\left(\omega t-\frac{4\pi}{3}\right)\sin(\alpha-\frac{4\pi}{3})$$

$$B_{A1}(\alpha,t)=\frac{B_1}{2}\left(\sin(\alpha-\omega t)+\sin(\alpha+\omega t)\right)$$

$$B_{B1}(\alpha,t)=\frac{B_1}{2}\left(\sin(\alpha-\omega t)+\sin(\alpha+\omega t-4\pi/3)\right)$$

$$B_{C1}(\alpha,t)=\frac{B_1}{2}\left(\sin(\alpha-\omega t)+\sin(\alpha+\omega t-8\pi/3)\right)$$

$$\Rightarrow \mathbf{B_m}(\boldsymbol{\alpha},\boldsymbol{t})=\frac{\mathbf{3}\,\boldsymbol{B_1}}{\mathbf{2}}\,\mathbf{sin}(\boldsymbol{\alpha}-\boldsymbol{\omega t})$$

Generating Rotating Field

Standing wave animation

$$Standing\ wave\ (pulsating\ wave) = A\ cos(\omega_e t)\sin(\alpha)$$

$$A\ cos(\omega_e t)\sin(\theta_m) = \underbrace{\frac{A}{2}\sin(\alpha - \omega_e t)}_{\text{Positive sequence}} + \underbrace{\frac{A}{2}\sin(\alpha + \omega_e t)}_{\text{Negative sequence}}$$

3-phase system (positive sequence phase b is lagging phase a by 120 deg):

$$Phase\ 1 \Rightarrow \frac{A}{2}\sin(\alpha - \omega_e t) + \frac{A}{2}\sin(\alpha + \omega_e t)$$

$$Phase\ 2 \Rightarrow \frac{A}{2}\sin\left((\alpha - \frac{2\pi}{3}) - (\omega_e t - \frac{2\pi}{3})\right) + \frac{A}{2}\sin\left((\alpha - \frac{2\pi}{3}) + (\omega_e t - \frac{2\pi}{3})\right)$$

$$Phase\ 3 \Rightarrow \frac{A}{2}\sin\left((\alpha + \frac{2\pi}{3}) - (\omega_e t + \frac{2\pi}{3})\right) + \frac{A}{2}\sin\left((\alpha + \frac{2\pi}{3}) + (\omega_e t + \frac{2\pi}{3})\right)$$

$$= 3\frac{A}{2}\sin(\alpha - \omega_e t)$$

A constant wave with **3/2** times amplitude of the standing wave is generated.

Creating a Rotating Field – 5-Phase Case

Assignment!

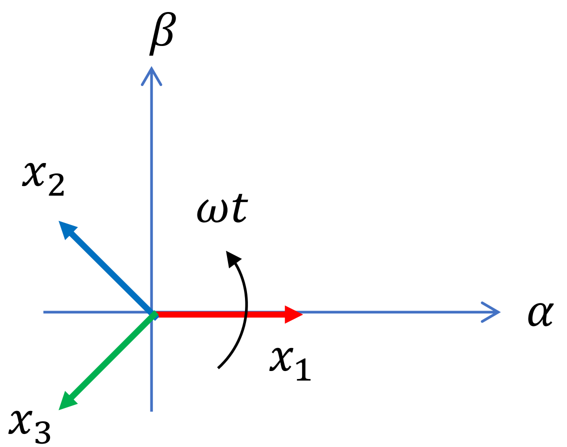
Space Vector Transformation

Since 3-phase systems are highly used, a transformation from 3-phase system into 2-phase quantities is beneficial.

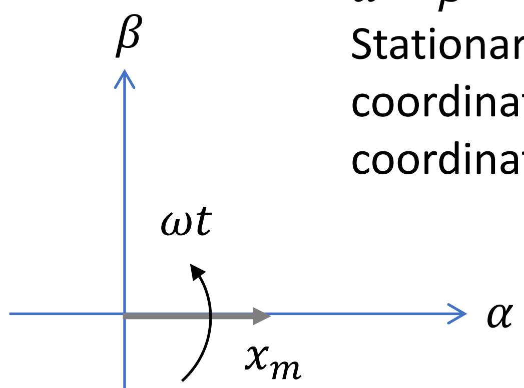
A general quantity x that may represent current, voltage and flux linkage.

Space vectors represent a physical interpretation for flux (linkages) but not for other quantities.

$\alpha - \beta$ Stationary coordinates (stator coordinates)



Phasor diagram



Space vector

$$x_m = \left\{ x_1 + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) x_2 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) x_3 \right\} = x_{m-\alpha} + j x_{m-\beta}$$

Space Vector Transformation (Clarke's Transformation)

$$x_m = \mathbf{\frac{2}{3}}\left\{x_1 + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)x_2 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)x_3\right\} = x_{m-\alpha} + jx_{m-\beta}$$

Amplitude invariant:

$$\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} = 2/3\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{bmatrix}\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} \qquad \rightarrow p_{3-phase} = 3/2\, p_{2-phase}$$

In electric machine analysis we use amplitude invariant transformation.

Amplitude invariant means that amplitude of 3-phase quantities and 2-phase quantities are going to be same.

Space Vector Transformation (Clarke's Transformation)

Assignment:

For amplitude invariant transformation, show that power of 3-phase system is 3-phase system is 3/2 times the power of the 2-phase system.

$$p_{3-phase} = v_1\, i_1 + v_2\, i_2 + v_3\, i_3 = \mathrm{Re}\{v\, i^*\}$$

$$p_{2-phase} = \mathrm{Re}\left\{ (v_\alpha + j\, v_\beta)\ (i_\alpha - j\, i_\beta) \right\} = v_\alpha\ i_\alpha + v_\beta\ i_\beta$$

$$\rightarrow p_{3-phase} = 3/2\ p_{2-phase}$$

Space Vector Transformation (Clarke's Transformation)

$$x_m = \sqrt{\frac{\mathbf{2}}{\mathbf{3}}}\left\{x_1 + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)x_2 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)x_3\right\} = x_{m-\alpha} + jx_{m-\beta}$$

Power invariant:

$$\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} = \sqrt{2/3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \sqrt{2/3}\begin{bmatrix} 1 & 0 & 1/\sqrt{2} \\ -1/2 & \sqrt{3}/2 & 1/\sqrt{2} \\ -1/2 & -\sqrt{3}/2 & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix}$$

Power

$\rightarrow p_{3-phase} = p_{2-phase}$

If we use power invariant transformation, the powers of 3-phase system and 2-phase system are going to be the same. But amplitues are going to be different.

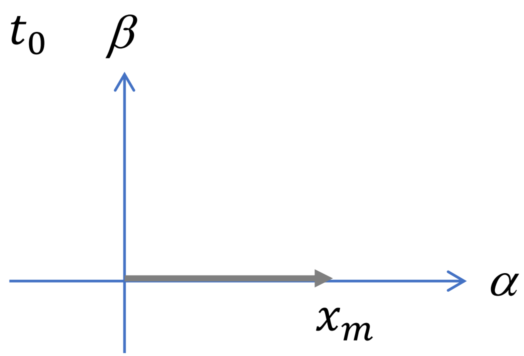
$$p_{3-phase} = v_1\, i_1 + v_2\, i_2 + v_3\, i_3 = \mathrm{Re}\{v\, i^*\}$$

$$p_{2-phase} = \mathrm{Re}\{\,(v_\alpha + j\, v_\beta)\ (i_\alpha - j\, i_\beta)\} = v_\alpha\ i_\alpha + v_\beta\, i_\beta$$

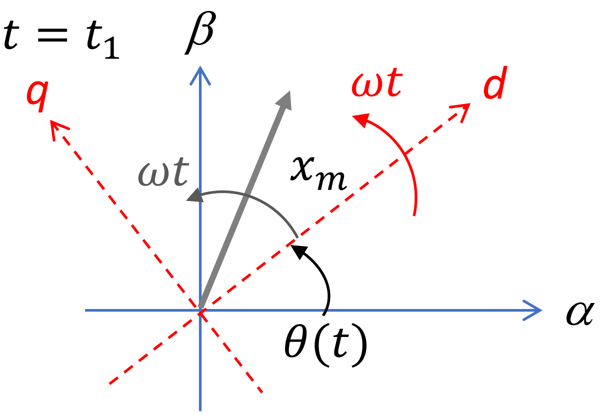
Coordinate Transformation (Park's Transformation)

- Transformation between stationary and rotatory coordinates

@ $t = t_0$



@ $t = t_1$



Space vector

Rotatory coordinates → Stationary coordinates

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

Stationary coordinates → Rotatory coordinates

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

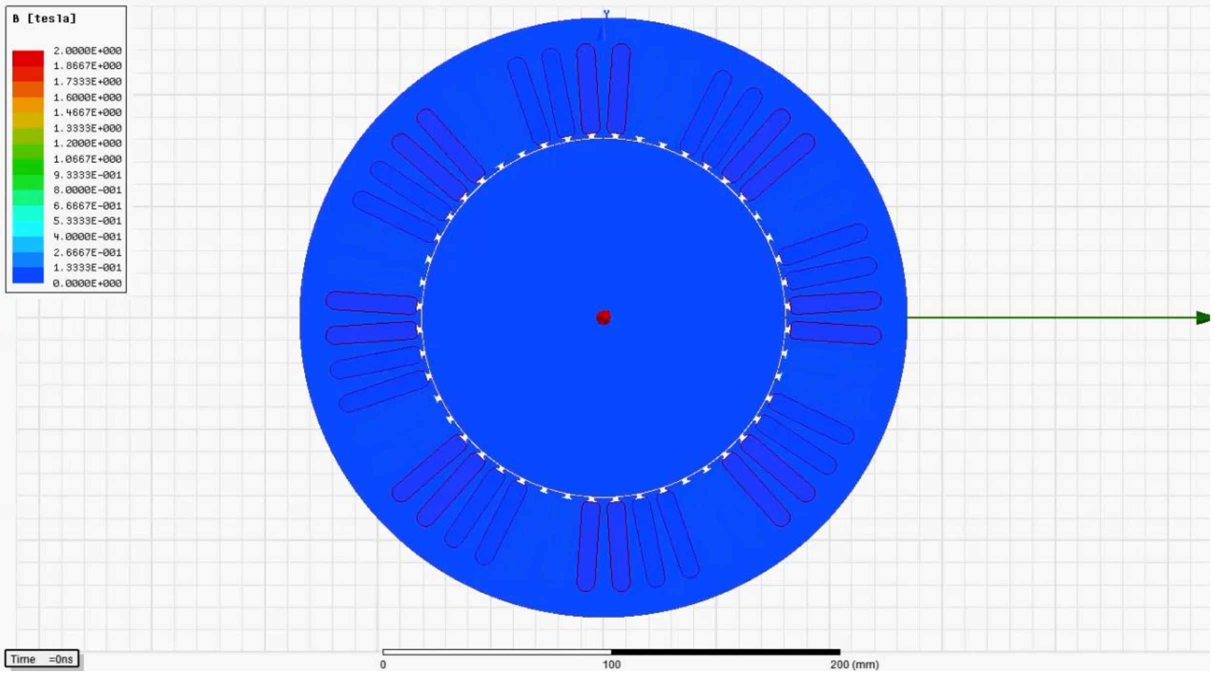
$\alpha\beta$: Stationary coordinate system
i.e. Stator reference frame

dq: Rotatory coordinate system,
i.e. Rotor reference frame

$\theta(t)$: angle between coordinate systems

Simulation of Standing Wave and Rotating Field with Ansys/Maxwell

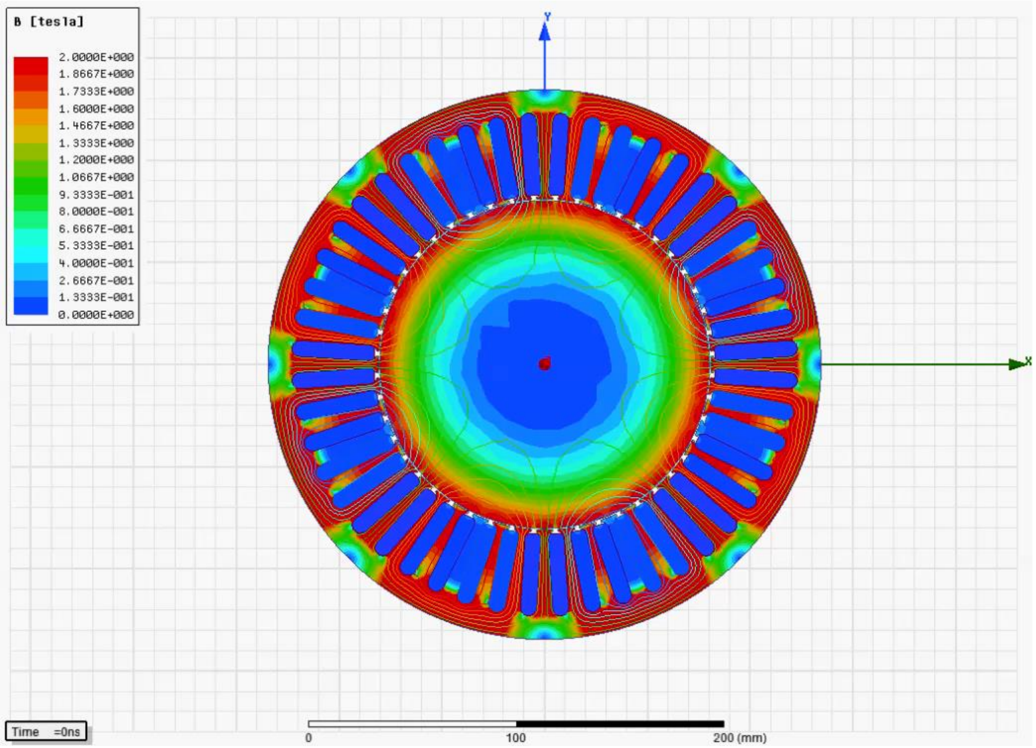
Standing Wave (Linear material)



$i_{s1}(t) = 100\cos(\omega t - \pi/2)$ and $i_{s2}(t) = i_{s3}(t)$= 0

Simulation of Standing Wave and Rotating Field with Ansys/Maxwell

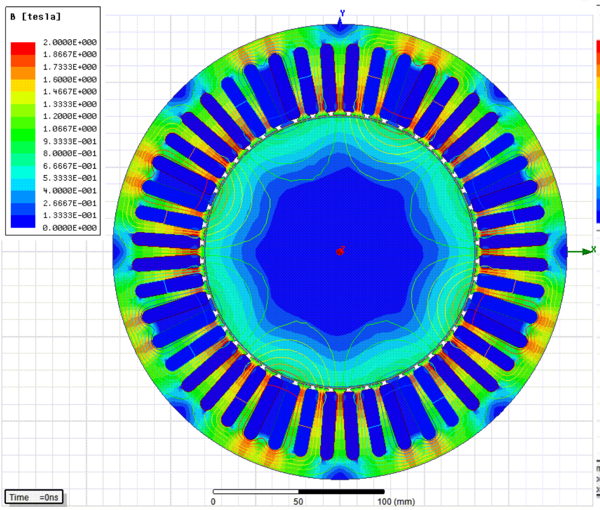
Rotating Field (Linear material)



$i_{s1}(t) = 100\sin(\omega t)$, $i_{s2}(t) = 100\sin\left(\omega t - \frac{2\pi}{3}\right)$, and $i_{s3}(t) = 100\sin\left(\omega t - \frac{4\pi}{3}\right)$

Simulation of an IPMSM

Stator rotating field

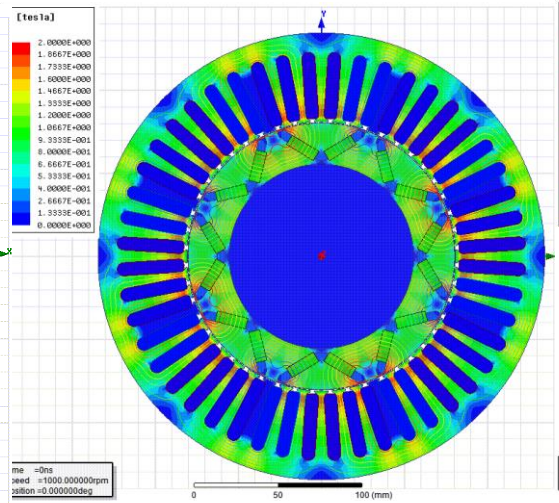


$$i_{s1}(t) = 100\sin(\omega t),$$

$$i_{s2}(t) = 100\sin\left(\omega t - \frac{2\pi}{3}\right), \text{ and}$$

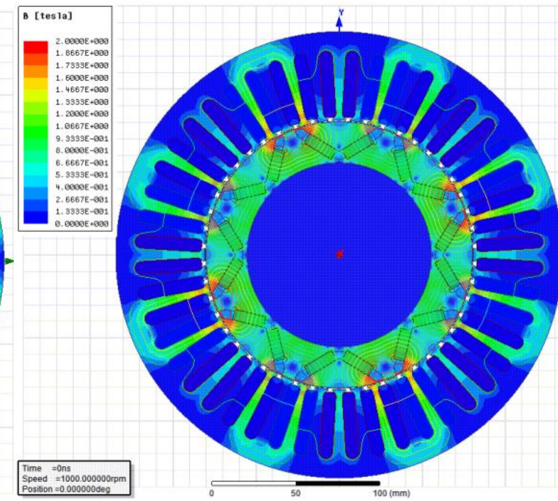
$$i_{s3}(t) = 100\sin\left(\omega t - \frac{4\pi}{3}\right)$$

Rotating rotor with PMs



$$i_{s1}(t) = i_{s2}(t) = i_{s3}(t) = 0$$

Stator rotating field & rotating rotor with PMs



$$i_{s1}(t) = 100\sin(\omega t),$$

$$i_{s2}(t) = 100\sin\left(\omega t - \frac{2\pi}{3}\right), \text{ and}$$

$$i_{s3}(t) = 100\sin\left(\omega t - \frac{4\pi}{3}\right)$$

Matlab/Simulink

Assignment:

Implement space vector and coordinate transformations in Matlab/Simulink.

https://www.mathworks.com/discovery/clarke-and-park-transforms.html