METU EE462 Utilization of Electric Energy

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Fundamentals of Electric Machines

- Lorentz Force
- Reluctance Force
- Torque Representation in Electric Machines
- Force Determination Using Energy Balance
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EE361 Energy Conversion
Chapter Review!

Forces in Electromechanical Systems

An electromechanical energy converter is a power converter. It converts electrical power in the form of current and voltage to mechanical power in the form of translational speed (or rotational) and force (or torque). This conversion process is reversible. It can convert mechanical power into electrical power as well.

All magnetic forces are the result of the fact that the magnetic field lines tend to shorten, minimum energy state.

Force production can be related to two different mechanisms:

1. **Lorentz force:** Force on a current-carrying conductor in a magnetic field
2. **Reluctance force:** Force caused by the change in magnetic resistance (reluctance)

1. Lorentz Force

In a purely magnetic system (no electric field):

- $\vec{F} = q \cdot (\vec{v} \times \vec{B})$ in Newtons (N)

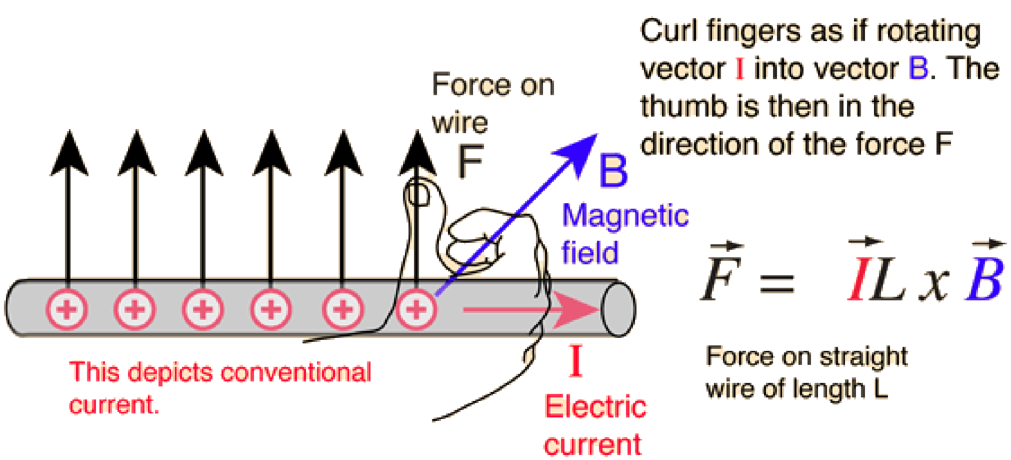
Alternatively force density:

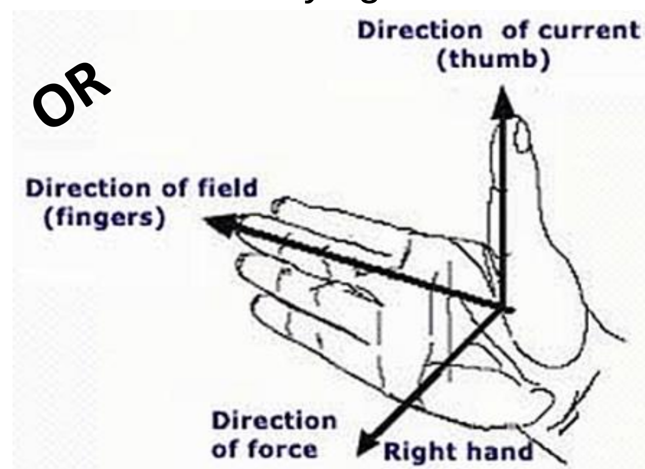
- $\vec{F} = \rho \cdot \left(\vec{v} \times \vec{B}\right) = \vec{J}\, x\, \vec{B}$ in N/m^3

Alternatively force as a function of current:

- $\vec{F} = \int \rho \cdot \left(\vec{v} \times \vec{B}\right) dA\, dl = \int \int \vec{J}\, dA\, dl\; x\, \vec{B} \;= I \cdot \left(\vec{l} \times \vec{B}\right)$ in N

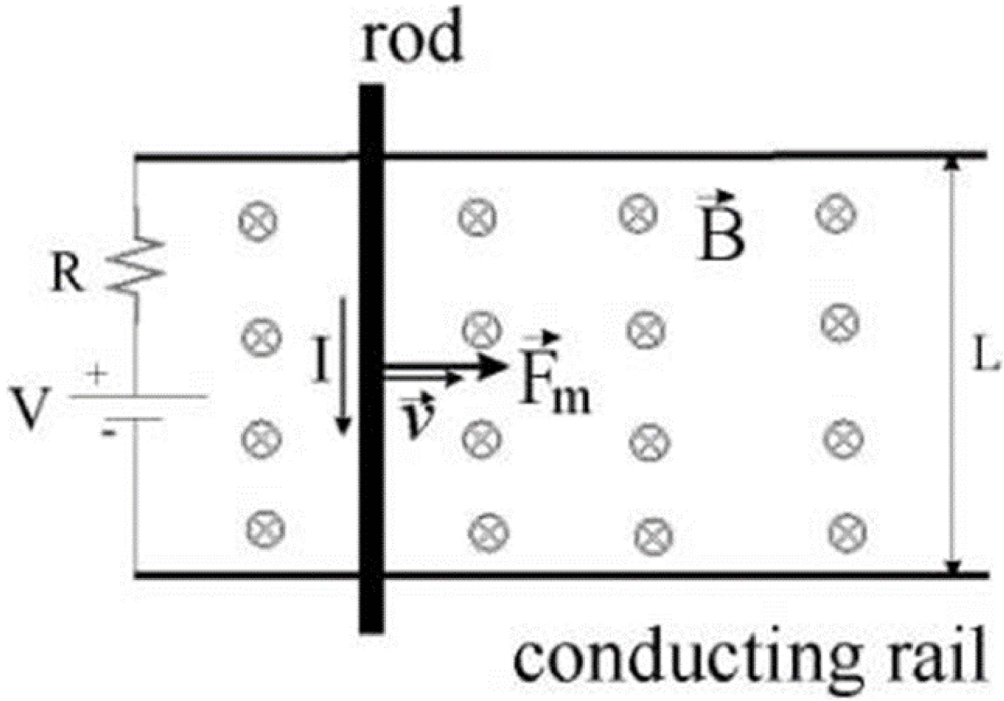
ρ : Charge density (C/m^3) , $\vec{v}$: velocity (m/s), $\vec{l}$: Length of the current carrying wire





1. Lorentz Force and Faraday's Law

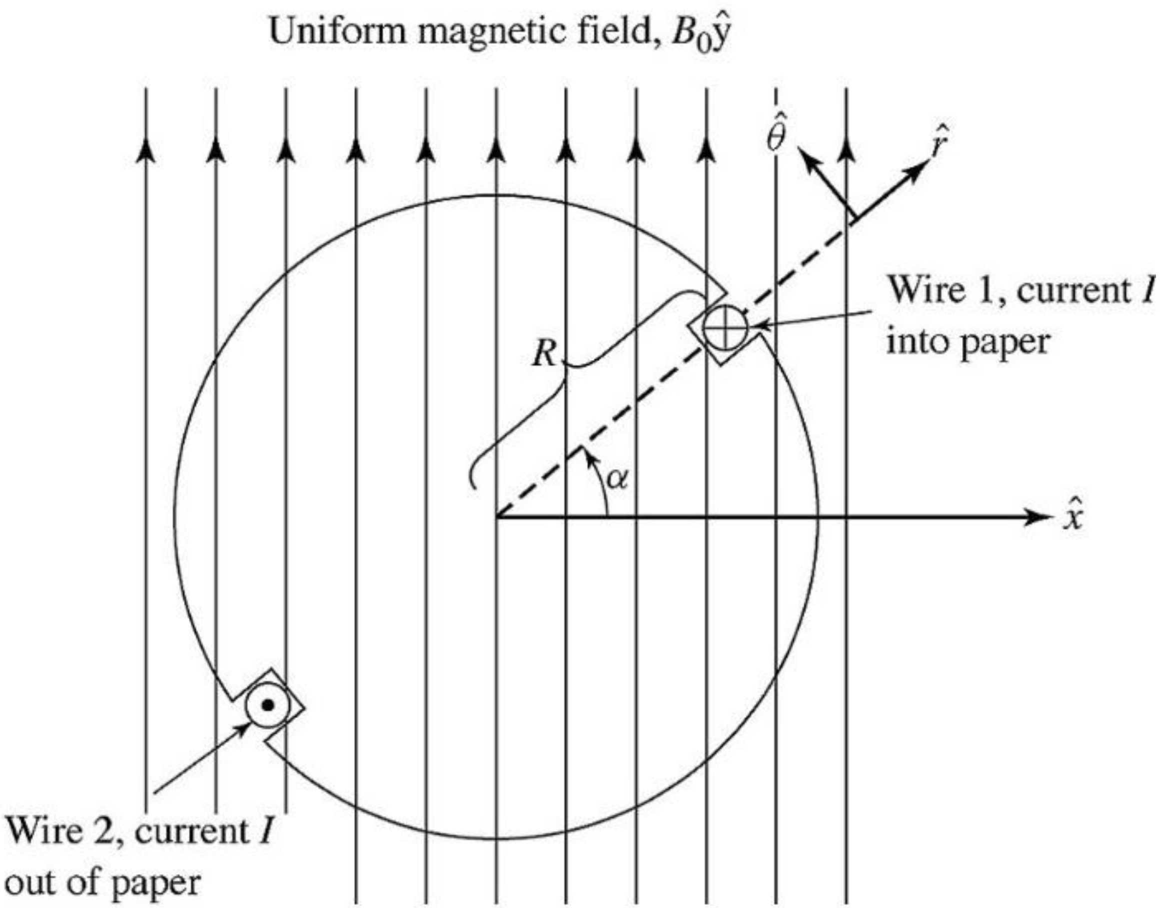
Motoring and Generator Actions



<u>Motoring:</u> Sliding rode experience a force due to Lorentz force

<u>Generator:</u> When no voltage is applied, and only rode moves, total flux linkage increase so that a voltage is induced

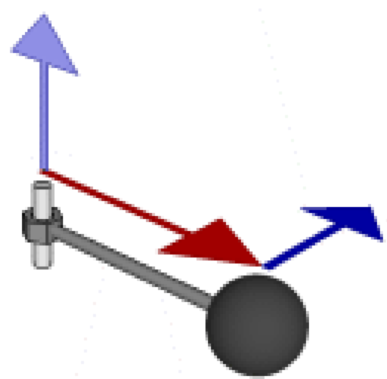
1. Lorentz Force



Mounted on the shaft and free to rotate.

Will there be a rotation?
- Yes!

Rotation direction?
- Clockwise



$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

τ: $Torque$

r: Radius

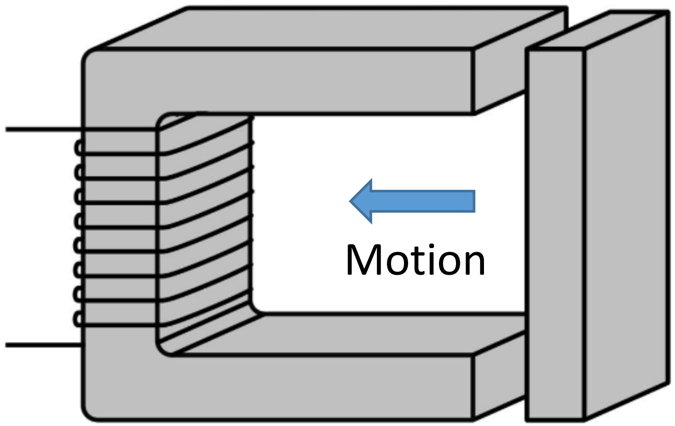
F: Force

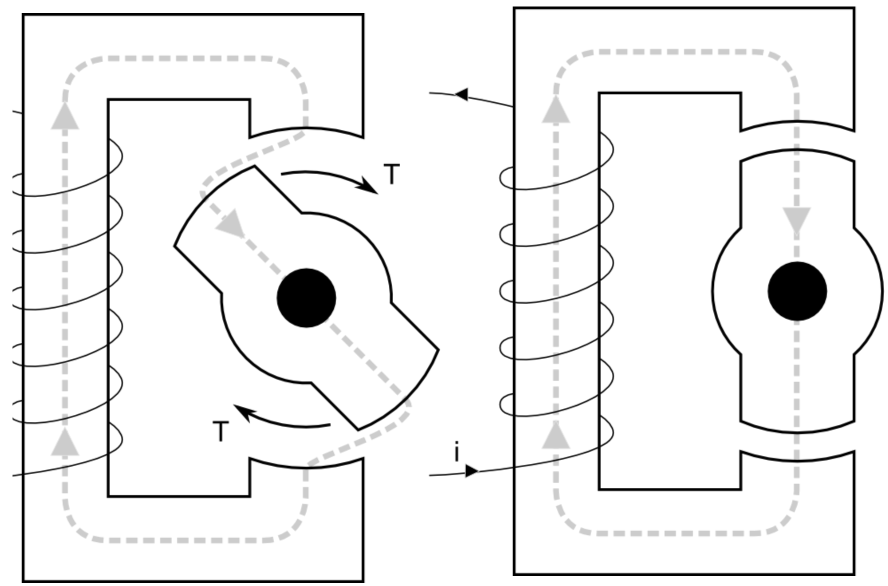
L: Angular momentum

r: Radius

p: Linear momentum (mv)

2. Reluctance Force





What will happen if you apply current to these magnetic systems?

Reluctance force is generated through a change of magnetic resistance. A non-magnetic field is induced in a ferromagnetic material due to saliency. System tries to reduce the reluctance (magnetic resistance).

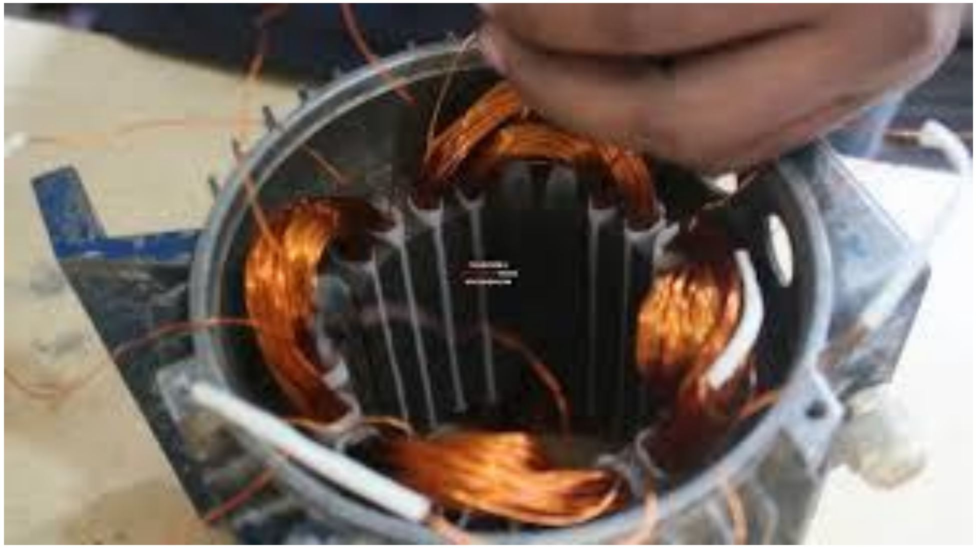
$Reluctance = MMF/Flux$ = $\mathcal{F} = \mathcal{R}\Phi$ in (1/H)

$$\mathcal{R} = \frac{Ni}{\emptyset} = \frac{H\ l}{B\ A} = \frac{H\ l}{\mu_0\ \mu_r\ H\ A} = \frac{l}{\mu_0\ \mu_r\ A}$$

Torque Representation in EMs

We have learned two types of magnetic forces, Lorenz and Reluctance.

- First of those, Lorenz force equation can be merely used for electromechanical actuators.
- As you can see below, windings (current carrying copper) is placed into stator slot made of electrical steel with a high relative permeability. So flux created from rotor (Winding or PM) will penetrate through the stator iron core, it won’t travel over the copper.
- So, we can <u>not</u> use Lorenz Force equation directly.



Torque Representation in EMs

1. Phase voltage equation

$$v = Ri + {}^{d\lambda}/_{dt}$$

where $\lambda = L(\theta)i + \lambda_{ext}$

2. Power equation

$$v\, i = R\, i^2 + L(\theta)\frac{\partial i}{\partial t}\, i + {}^{d\lambda_{ext}}/_{dt}\, i$$

3. Energy equation

$$\underbrace{\int v\, i\, dt}_{\text{Electrical energy}} = \underbrace{\int R\, i^2\, dt}_{\text{Resistive loss}} + \underbrace{\int L(\theta)\, i\, di}_{\text{Magnetically stored energy}} + \underbrace{\lambda_{ext} i}_{\text{Mechanical energy}}$$

4. Energy balance

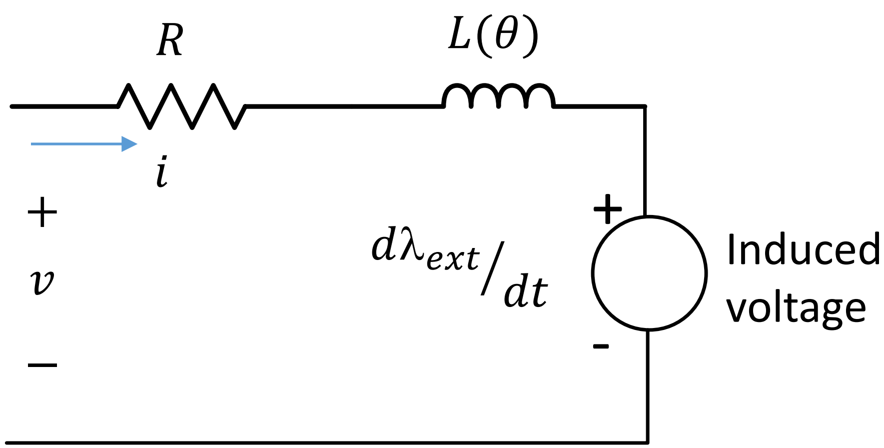
Torque Representation in EMs

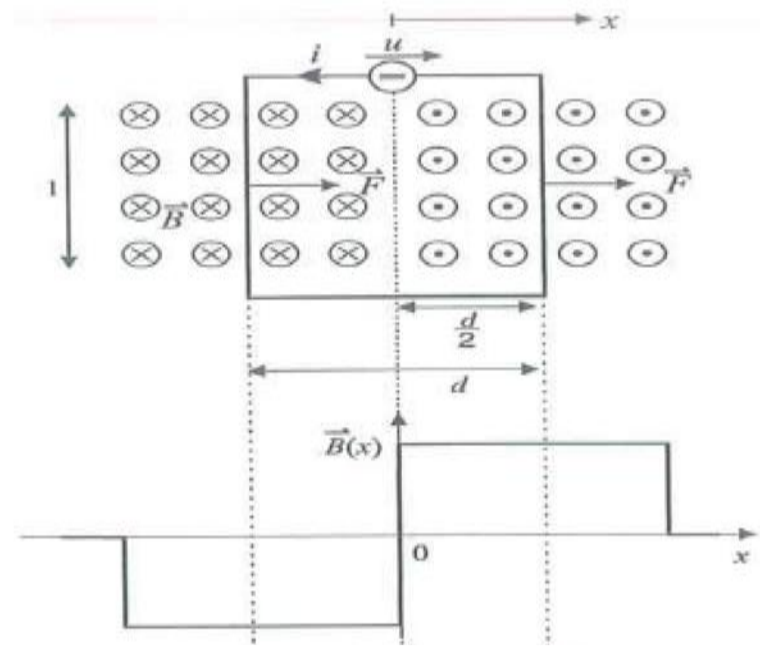
Phase voltage equation, $v = Ri + {}^{d\lambda}/_{dt}$

where $\lambda = L(\theta)i + \lambda_{ext}$

Power equation, $v\,i = R\,i^2 + L(\theta)\,\frac{\partial i}{\partial t}\,i + {}^{d\lambda_{ext}}/_{dt}\,i$

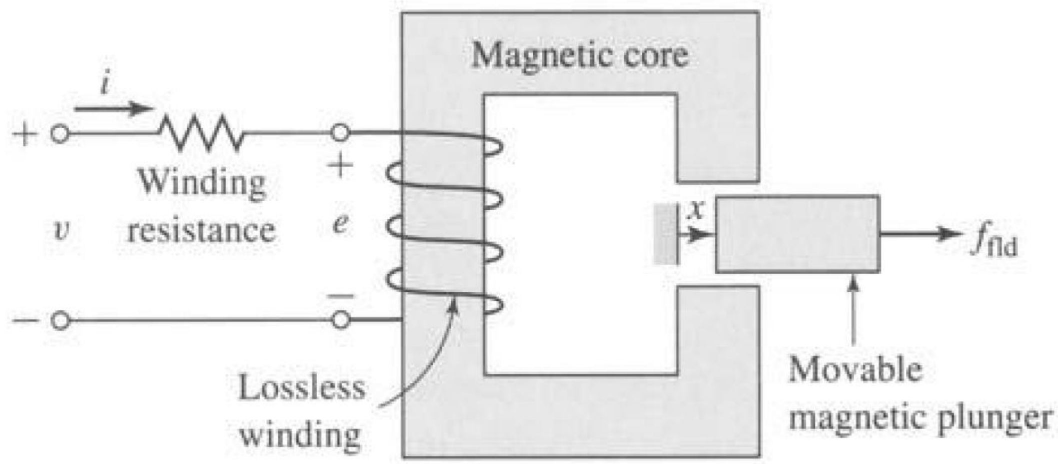
Mechanical power, $\mathrm{P}_{mech} = \mathrm{F}\,v = {}^{d\lambda_{ext}}/_{dt}\,i$



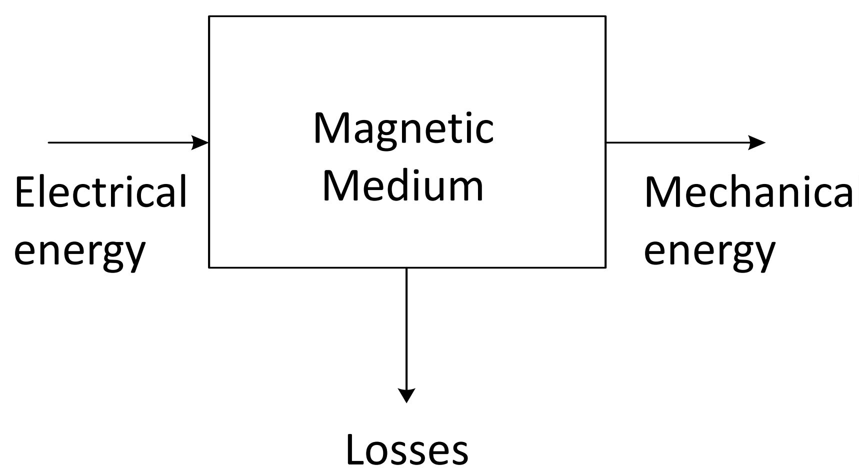


This induced voltage represent the mechanical motion!

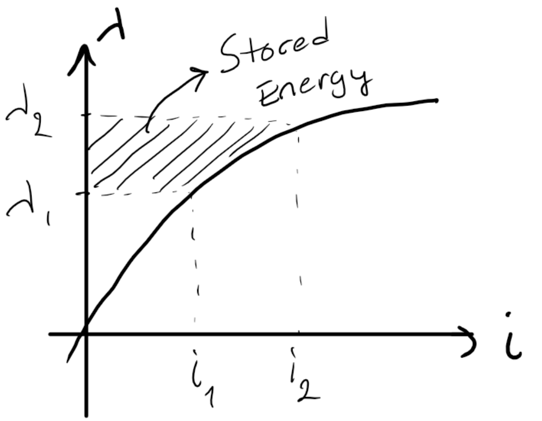
Energy Balance and The Energy Method

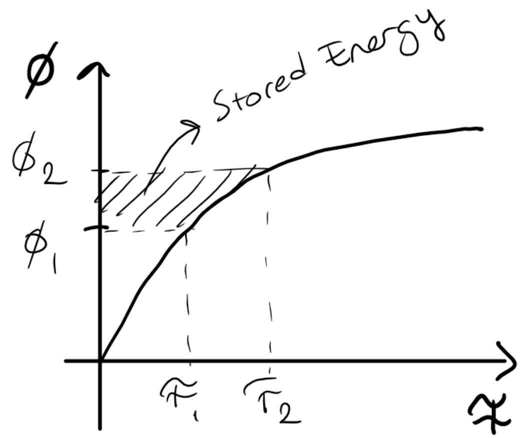


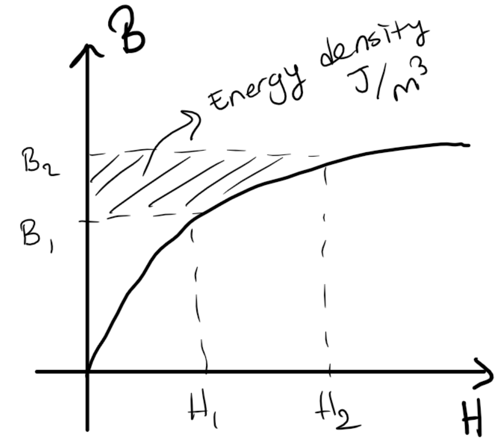
F_{fld}: Force
x: Position



Magnetic Energy







$$\Delta W = \int_{\lambda_1}^{\lambda_2} i \, d\lambda \quad or \quad \Delta W = \int_{N\Phi_1}^{N\Phi_2} i \, d(N\Phi) \; = \int_{\Phi_1}^{\Phi_2} \mathcal{F} \, d\Phi$$

$$\Delta W = \int_{\Phi_1}^{\Phi_2} \mathcal{F} d\Phi = \int_{B_1}^{B_2} H l_c \, A_c dB = l_c \, A_c \int_{B_1}^{B_2} H \, dB = V_c \int_{B_1}^{B_2} H \, dB$$

Energy Method

Electric energy input = Mechanical energy output + Increase in stored magnetic energy + Energy converted into heat

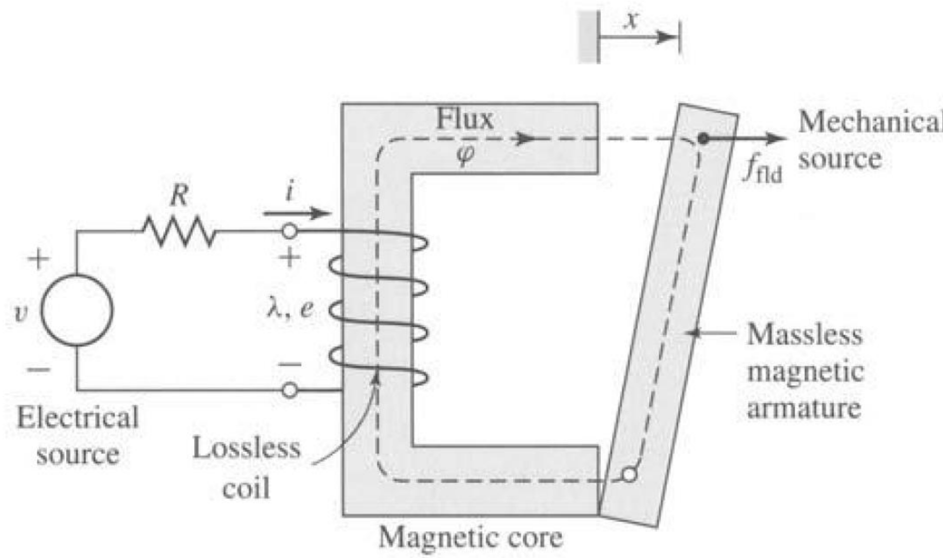
In a lossless system, power balance:

$$P_{elec} = P_{mech} + \frac{dW_{fld}}{dt}$$

ei $\quad f_{fld}\frac{dx}{dt}$

$$\frac{dW_{elec}}{dt} = \frac{dW_{mech}}{dt} + \frac{dW_{fld}}{dt}$$

$$\Rightarrow \frac{dW_{fld}}{dt} = ei - f_{fld}\frac{dx}{dt}$$



Which kind of a force do we have in this system?
- Reluctance → Use energy method

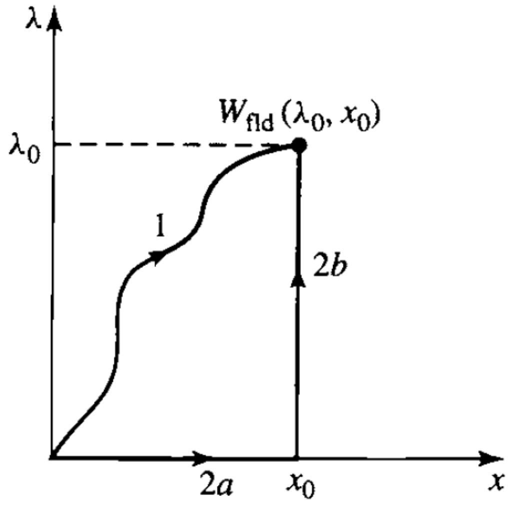
$$\boxed{For\ e = \frac{d\lambda}{dt} \Rightarrow dW_{fld} = i\, d\lambda - f_{fld}dx}$$

W_{fld}: Magnetically stored energy
f_{fld}: Force
x: Displacement

Energy in Singly Excited Systems

$dW_{fld} = i\,d\lambda - f_{fld}dx$

$W_{fld}(\lambda_0, x_0) = \int dW_{fld} = \int_{path\;2a} dW_{fld} + \int_{path\;2b} dW_{fld}$



Path 2a: $d\lambda = 0 \Rightarrow$ No magnetic force: $f_{fld} = 0$

Path 2b: $dx = 0$

$W_{fld}(\lambda_0, x_0) = \int dW_{fld} = \int_0^{\lambda_0} i\,(\lambda, x_0)\,d\lambda$

For $\lambda = L(x)i \Rightarrow \quad W_{fld}(\lambda_0, x_0) = \int_0^{\lambda_0} \frac{\lambda}{L(x_0)}\,d\lambda \; = \frac{1}{2}\frac{\lambda_0}{L(x_0)^2}$

$W_{fld}(\lambda, x) \; = \frac{1}{2}\frac{\lambda}{L(x)^2}$

If the system is lossless, it is a conservative system, so that total energy is path independent.

- Please discuss why system is not conservative if we take the losses into account.
- Please discuss why inductance is a function of mechanical position.

Determination of Magnetic Force and Torque from Energy

$$dW_{\text{fld}}(\lambda, x) = i\,d\lambda - f_{\text{fld}}\,dx$$

$$dW_{\text{fld}}(\lambda, x) = \left.\frac{\partial W_{\text{fld}}}{\partial \lambda}\right|_{x} d\lambda + \left.\frac{\partial W_{\text{fld}}}{dx}\right|_{\lambda} dx$$

$$i = \left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda}\right|_{x}$$

$$\boxed{f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\right|_{\lambda\ =\text{constant}}}$$

For a linear system: $f_{\text{fld}} = -\left.\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\lambda^2}{L(x)}\right)\right|_{\lambda} = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx}$

Info: For any function with two independent variables, total differential of F with respect to two state variables:

$$dF(x_1, x_2) = \left.\frac{\partial F}{\partial x_1}\right|_{x_2} dx_1 + \left.\frac{\partial F}{\partial x_2}\right|_{x_1} dx_2$$

For a linear system:

$$\boxed{f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx}}$$

Energy vs. Co-energy

$$f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\right|_{\lambda}$$

$$f_{\text{fld}} = -\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\lambda^2}{L(x)}\right)\bigg|_{\lambda} = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx}$$

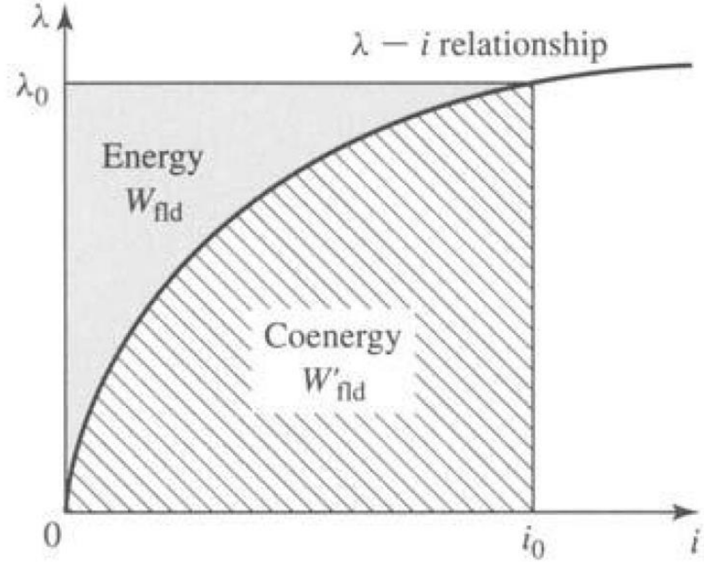


Figure 3.10 Graphical interpretation of energy and coenergy in a singly-excited system.

For linear magnetic systems for which $\lambda = L(x)i$

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx}$$

$$f_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}(i, x)}{\partial x}\right|_{i} \quad f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx}$$

Energy vs. Co-energy

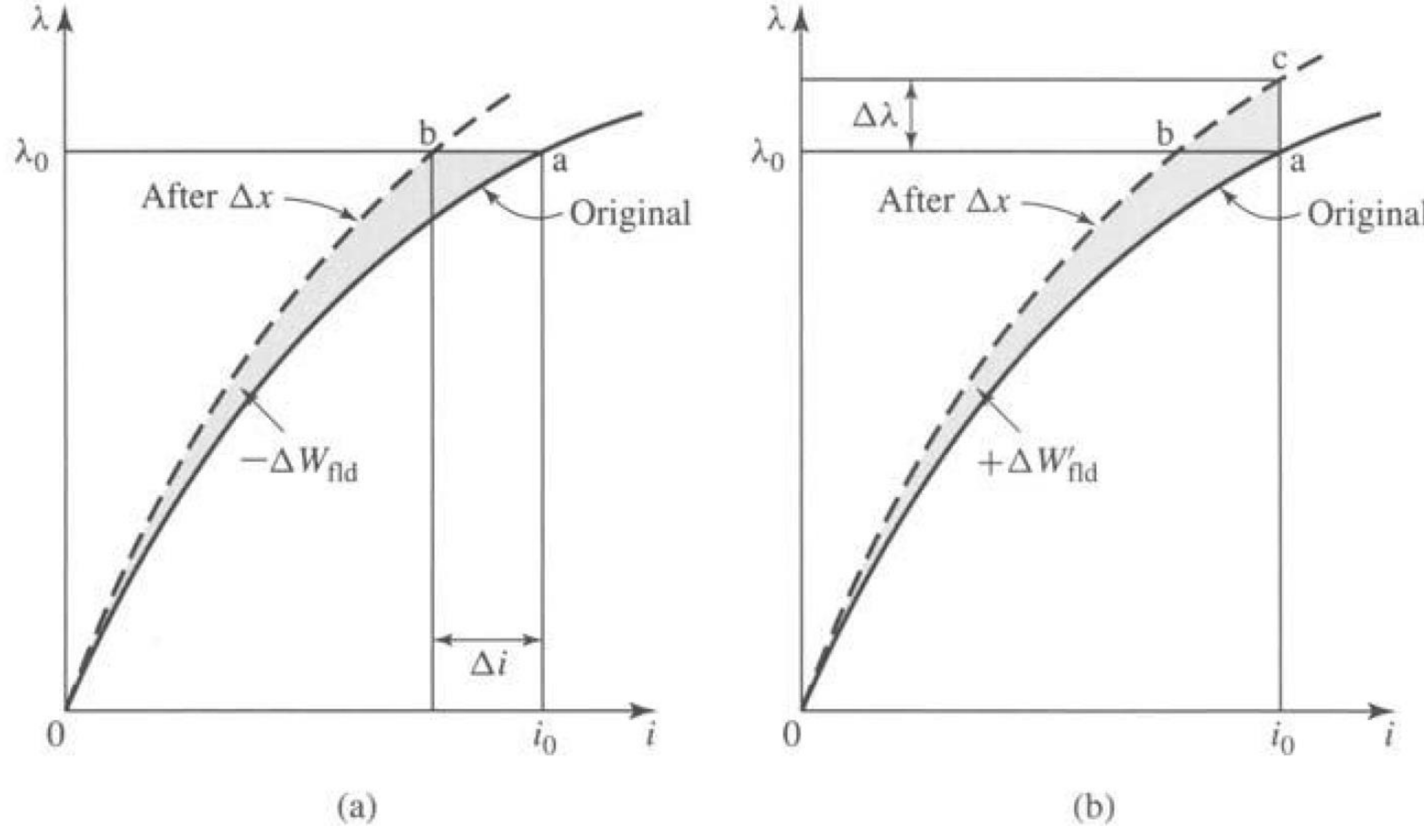


Figure 3.11 Effect of Δx on the energy and coenergy of a singly-excited device: (a) change of energy with λ held constant; (b) change of coenergy with i held constant.

Energy vs. Co-energy

- The force acts in the direction to decrease the magnetic field stored energy at constant flux or to increase the coenergy at constant current.

- In a singly excited system, the force acts to increase the inductance by pulling on members so as to reduce the reluctance of the magnetic path linking the winding.

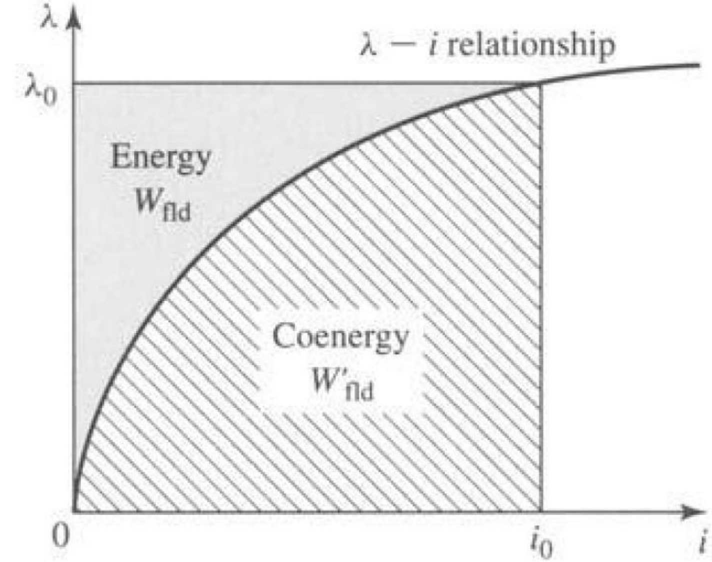


Figure 3.10 Graphical interpretation of energy and coenergy in a singly-excited system.

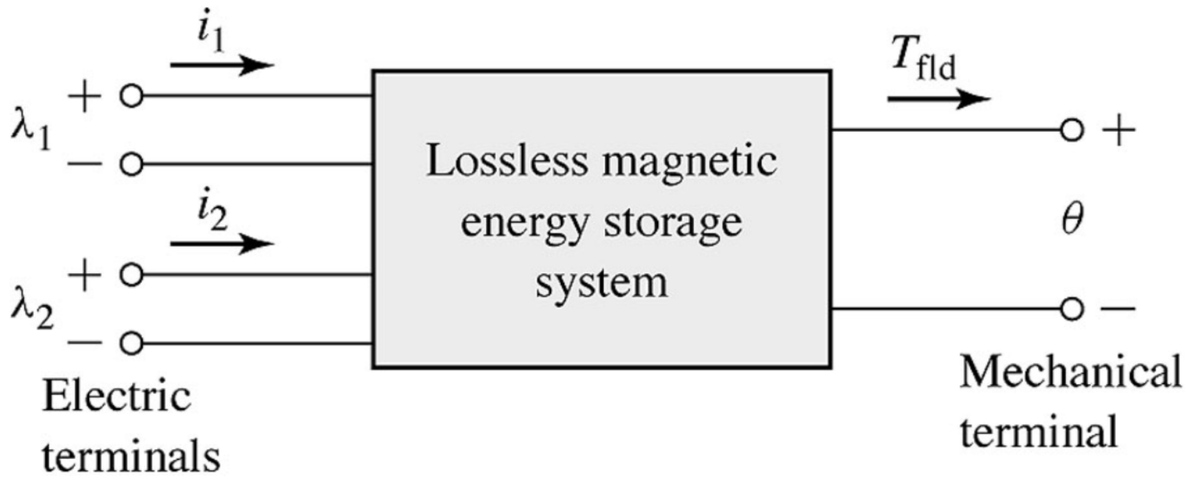
Force vs. Torque

For a linear system (inductance is not a function current), force and torque expressions are given blow:

$$f_{\mathrm{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx} \qquad\qquad T_{\mathrm{fld}} = \frac{i^2}{2}\frac{dL(\theta)}{d\theta}$$

Since $x = r\theta$ ➔ Just replace x with θ in the force equation to get torque equation.

Multiply Excited Systems



Electrical Energy = Magnetic Energy + Mechanical Energy

$$dW_{fld}(\lambda_1, \lambda_2, \theta) = i_1\, d\lambda_1 + i_2\, d\lambda_2 - T_{fld}\, d\theta$$

Multiply Excited Systems

$$W'_{\text{fld}}(i_1, i_2, \theta) = \frac{1}{2} L_{11}(\theta) i_1^2 + \frac{1}{2} L_{22}(\theta) i_2^2 + L_{12}(\theta) i_1 i_2$$

Stored Energy in Matrix Form

$$W'_{fld} = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix} \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

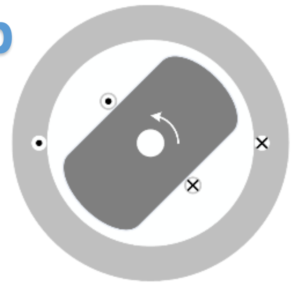
$$T_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial \theta} \right|_{i_1, i_2}$$

$$= \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}$$

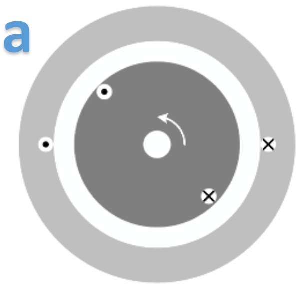
Basic Stator and Rotor Configurations

No magnetic saliency	Single mag. saliency	Double mag. saliency

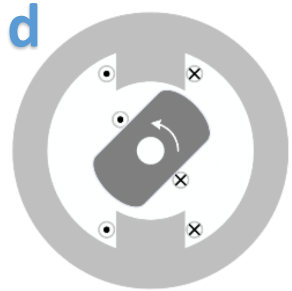
b



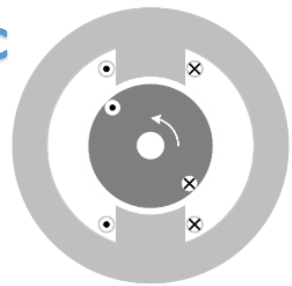
a



d



c



Double magnetic saliency
- Switched reluctance machine (SRM)

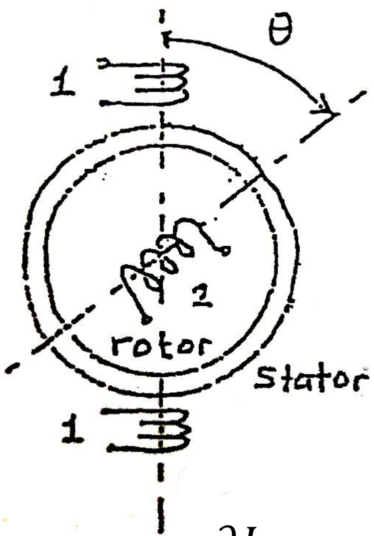
No magnetic saliency
- Induction machine (IM)
- Round rotor synchronous machine (SM)
- Surface mount permanent magnet synchronous machine (SM-PMSM)

Single magnetic saliency
- Interior permanent magnet synchronous machine (IPMSM)
- Synchronous reluctance machine (SyncRel)
- Salient pole synchronous machines (SP-SM)

➢ The production of a constant average torque is that the stator and rotor fields are standing still to each other, independent of the type of the machine. If both fields (stator & rotor) are rotating at different speeds, a pulsating torque is produced.

Basic Stator and Rotor Configurations

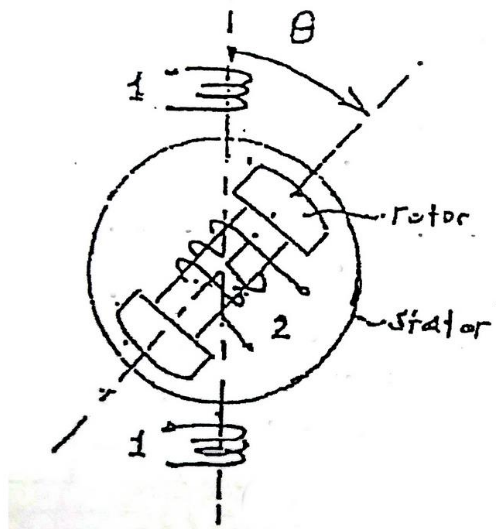
a **No magnetic saliency**



$$T = i_1 i_2 \frac{\partial L_{12}}{\partial \theta}$$

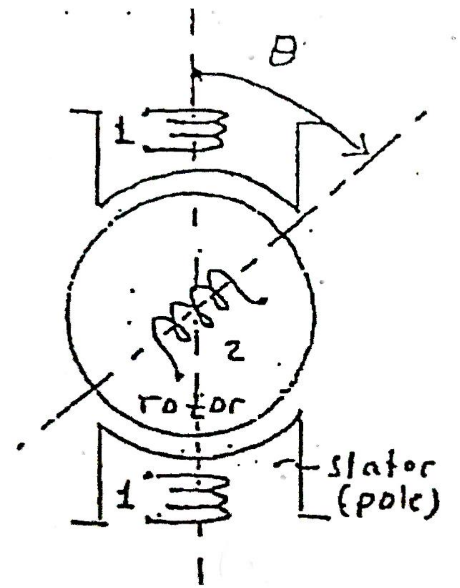
(L_{11} and L_{22} constant)

b **Single magnetic saliency**



$$T = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} \text{ (L}_{22}\text{ constant)}$$

c **Single magnetic saliency**



$$T = \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta}$$

(L_{11} constant)

Basic Stator and Rotor Configurations

Electric Machine Type		Rotor & Stator Configuration	i_1	i_2
Synchronous Machines	Cylindrical rotor	a	AC	DC or PM
	Salient pole	b	AC	DC
	Permanent magnet (surface mounted)	a	AC	PM
	Permanent magnet (interior)	b	AC	PM
	Synchronous reluctance	b	AC	=0
	Switched reluctance	d	AC	=0
Induction machine		a	AC	AC
DC machine		c	DC	AC
Universal machine		c	AC	AC

Losses in Electromechanical Systems

Mechanical friction: Mechanical losses – bearing friction & windage losses

Copper losses:

- Resistivity increases with temperature and also with frequency due to skin and proximity effects (in copper or aluminum conductors)

Core losses:

- Hysteresis losses: Losses due to domain movements in ferromagnetic materials
- Eddy current losses: Time-varying magnetic field produces eddy currents in conductive materials (in copper, stator and rotor cores, housing, PMs, etc.)