EE 301 Fall 2018-2019

HW 1

Group Number:66

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1)	a)	check $x(t) = x(t+T)$ $4\cos(3t+\frac{\pi}{5}) = 4\cos(3t + \underbrace{3T}_{2\pi k} + \frac{\pi}{5})$ $3T = 2\pi k$ $T = \frac{2\pi}{3}k$ for $k=1$, $T_0 = \frac{2\pi}{3}$
	b)	check $x(t) = x(t+T)$ $e^{j\frac{\pi}{2}t} e^{-j2} = e^{j\frac{\pi}{2}t} \cdot e^{j\frac{\pi}{2}T} \cdot e^{-j2}$ $e^{j\frac{\pi}{2}T} = 1 = e^{j2\pi k}$ $\frac{\pi}{2}T = 2\pi k \Rightarrow T = 4k$ for $k=1$, $T_0 = 4$

	c)	check $x(t)=x(t+T)$ $e^{jt^2} = e^{jt^2} \cdot e^{jtT} \cdot e^{jT2}$ $e^{jtT} = e^{j2\pi k}$, $e^{jT^2} = e^{j2\pi k}$ $t \cdot T = 2\pi k$, $T^2 = 2\pi k$ $tT = T^2$ $\cancel{T=0}$, $T=t$ for $k=1$, $T=\sqrt{2\pi}$
	d)	check $x[n] = x[n+N]$ $\cos\left(\frac{3\pi}{7}n+\frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{7}n+\frac{3\pi}{7}N+\frac{\pi}{4}\right)$ $\frac{3\pi}{7}N = 2\pi k$ $N = \frac{14}{3}k$ for $k=3$, $N_0 = 14$ N should be integer

e)

check $x[n] = x[n+N]$.

$$\left[\sin\left(\frac{\pi}{3}n+\frac{\pi}{2}\right)\right]^2 = \frac{1-\cos\left(\frac{2\pi}{3}n+\pi\right)}{2}$$

$$\frac{1-\cos\left(\frac{2\pi}{3}n+\pi\right)}{2} = \frac{1-\cos\left(\frac{2\pi}{3}n+\frac{2\pi}{3}N+\pi\right)}{2}$$

$$\frac{2\pi}{3}N = 2\pi k$$

$$N = 3k$$

for $k=1$, $N_0=3$

f)

check $x[n] = x[n+N]$

$$\cos\left(\frac{\pi}{7}n^2\right) = \cos\left(\frac{\pi}{7}n^2+\frac{2\pi}{7}nN+\frac{\pi N^2}{7}\right)$$

$$\frac{2\pi}{7}nN = 2\pi k \quad , \quad \frac{\pi}{7}N^2 = 2\pi k$$

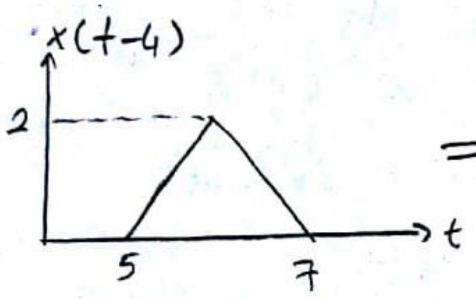
$$nN = 7k \qquad N^2 = 14k$$

for $k=14$, $\boxed{N_0=14}$

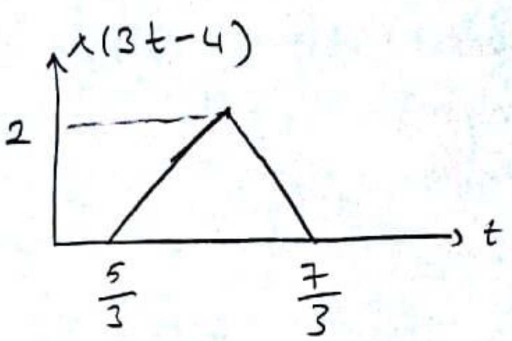
$n=7$

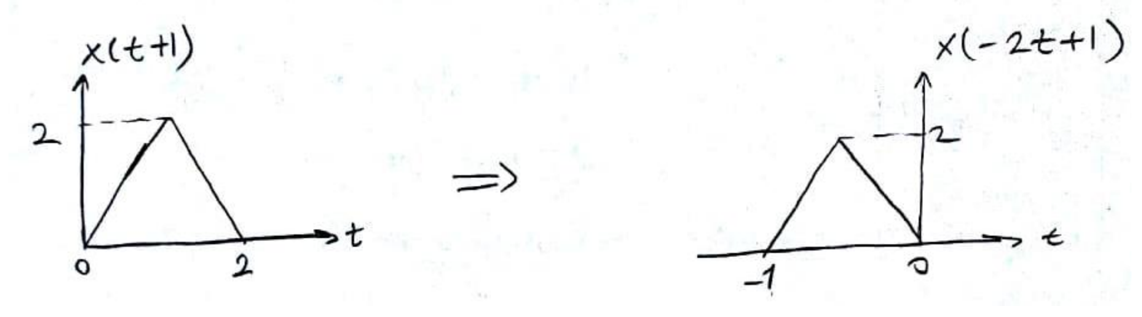
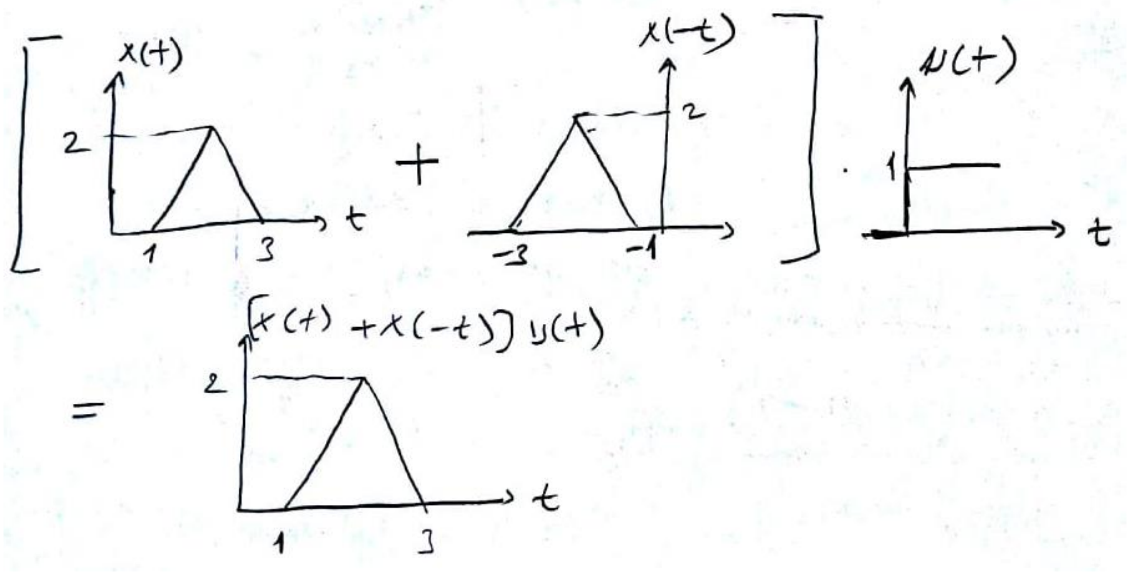
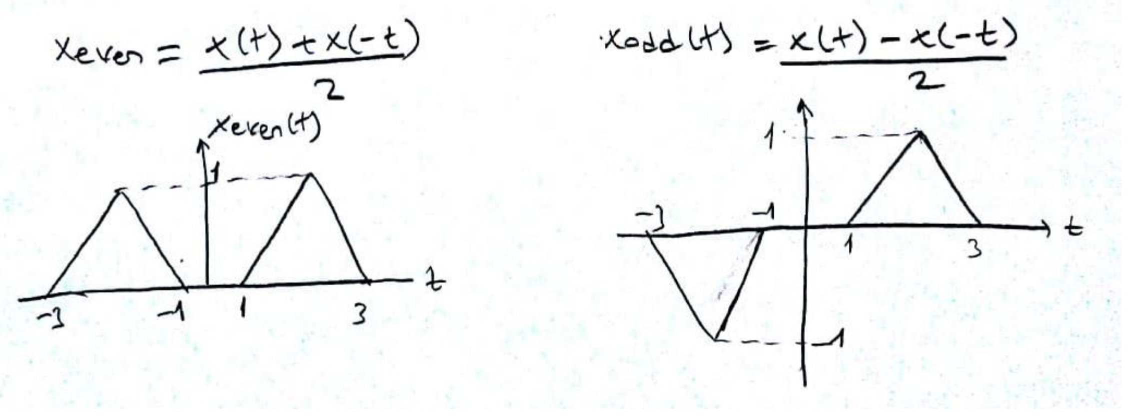
2) a)

i)



$\Longrightarrow$



	ii)	x(t+1) 2 0 2 t x(-2t+1) 2 -1 0 t
	iii)	x(t) 2 1 3 t x(-t) 2 -3 -1 u(t) 1 t [x(t) + x(-t)] u(t) 2 1 3 t
	b)	Xeven = (x(t) + x(-t))/2 Xodd(t) = (x(t) - x(-t))/2 Xeven(t) 1 -3 -1 1 3 t 1 -1 -3 -1 1 3 t

3) a)

$$y(t) = \int_0^t x(\tau)\,d\tau$$

1-) Not Memoryless since it depends on past values.

2) check $x(t-t_0) \to y(t-t_0)$

$$y(t) = \int_0^t x(\tau)\,d\tau$$

$$= \int_{0-t_0}^{t-t_0} x(\tau - \overset{+t_0}{t_0})\,d\tau = \int_{-t_0}^{0} x(\tau)\,d\tau \qquad \text{NOT TI}$$

3-) $x_1(t) \to y_1(t)$

$x_2(t) \to y_2(t)$

$$a x_1(t) + b x_2(t) \to \int_0^t (a x_1(\tau) + b x_2(\tau))\,d\tau = a \underbrace{\int_0^t x_1(\tau)\,d\tau}_{y_1(t)} + b \underbrace{\int_0^t x_2(\tau)\,d\tau}_{y_2(t)}$$

Linear

4-) $y(t) = \int_0^t x(\tau)\,d\tau = x(t) - x(0)$

It is causal

5-) $-\beta_x \le x(t) \le \beta_x \Rightarrow -\beta_y \le y(t) \le \beta_y$

$$-\int_0^t \beta_x\,d\tau \le \int_0^t x(\tau)\,d\tau \le \int_0^t \beta_x\,d\tau$$

$-\beta_x t \le y(t) \le \beta_x t \to$ is not stable b)

$y(t) = (\sin 2t)\, x(t)$

1-) Memoryless, output depends on present value.

2-) check $x(t-t_0) \rightarrow y(t-t_0)$

$\sin(2t)\, x(t-t_0) \neq y(t-t_0)$ not TI

3-) $x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$$a x_1(t) + b x_2(t) \rightarrow \sin(2t)\left(a x_1(t) + b x_2(t)\right)$$

$$= a \underbrace{\sin(2t) x_1(t)}_{y_1(t)} + b \underbrace{\sin(2t) x_2(t)}_{y_2(t)}$$

Linear

4-) Causal since all memoryless system are causal

5-) $-\beta x \leq x(t) \leq \beta x \Rightarrow -\beta y \leq y(t) \leq \beta y$

$\Downarrow$

$-\beta x \sin(2t) \leq \sin 2t\, x(t) \leq \sin(2t)\, \beta x$

$-1 \leq \sin 2t \leq 1$, Then system is stable

c)

$$y(t) = \frac{dx(t)}{dt} = \lim_{\Delta \to 0} \frac{x(t) - x(t-\Delta)}{\Delta}$$

1-) NOT Memoryless, output depends on present and past values.

2-) Check $x(t-t_0) \rightarrow y(t-t_0)$

$y(t-t_0) = \frac{d\, x(t-t_0)}{dt}$ TI

3-) $$a x_1(t) + b x_2(t) \rightarrow \frac{d}{dt}\left[a x_1(t) + b x_2(t)\right] = a \underbrace{\frac{d x_1(t)}{dt}}_{y_1(t)} + b \underbrace{\frac{d x_2(t)}{dt}}_{y_2(t)}$$

Linear

4-) Causal

5-) Unstable. If we give unit step as input, there is a discontinuity at 0.

	d)	$y[n] = x[2n]$ 1-) Not Memoryless, it depends on future value 2-) check $x[n-n_0] \rightarrow y[n-n_0]$ $x[n-n_0] \rightarrow y\left[\frac{n-n_0}{2}\right]$ Not TI 3-) $ax_1[2n] + bx_2[n] \rightarrow a\underbrace{x_1[2n]}_{y_1[n]} + b\underbrace{x_2[2n]}_{y_2[n]}$ Linear 4-) Not causal, it depends on future value 5-) $-Bx \leq x[2n] \leq Bx$ $\Downarrow$ $-Bx \leq y[n] \leq Bx \rightarrow$ Stable
	e)	$y[n] = x[-n]$ 1-) Not memoryless, for $n=-1$, $y[-1] = x[1]$ 2-) check $x[n-n_0] \rightarrow y[n-n_0]$ $x[-n+n_0] \rightarrow y[n-n_0]$ x not TI 3-) $ax_1[-n] + bx_2[-n] \rightarrow ax_1[-n] + bx_2[-n]$ <ins>Linear</ins> 4-) Not causal, for $n=-1$, $y[-1] = x[1]$ 5-) $-Bx \leq x[-n] \leq Bx$ $\Downarrow$ $-Bx \leq y[n] \leq Bx \rightarrow$ Stable

	f)	$y[n] = \sum_{k=n-5}^{n+5} x[k]$ 1) Not memoryless, it depends on past, present and future values 2-) check $x[n-n_0] \to y[n-n_0]$ $\sum_{n-5}^{n+5} x[k-n_0] = \sum_{n+n_0-5}^{n+n_0+5} x[k] = y[n-n_0]$ <u>TI</u> 3-) $ax_1[n] + bx_2[n] \to \sum_{k=n-5}^{n+5} (ax_1[k] + bx_2[k])$ $= a \underbrace{\sum_{k=n-5}^{n+5} x_1[k]}_{y_1[n]} + b \underbrace{\sum_{k=n-5}^{n+5} x_2[k]}_{y_2[n]}$ <u>Linear</u> 4-) Not causal, it depends on past, present and future values. 5-) $-\beta x \leq x[n] \leq \beta x$ $\Downarrow$ $-\sum_{k=n-5}^{n+5} \beta x \leq y[n] \leq \sum_{k=n-5}^{n+5} \beta x \Rightarrow -11\beta x \leq y[n] \leq 11\beta x$ <u>Stable</u>

4)	a)	i)	$a=1, \quad \sum_{n=0}^{N-1} 1 = \underbrace{1+1+1+\dots+1}_{N} = N$ $\sum_{n=0}^{N-1} a^n = 1 + a + a^2 \dots a^{N-1}$ $(1-a)(a^{N-1} + \dots a^2 + a + 1) = y(1-a)$ $1 - a^N = y(1-a) \Rightarrow y = \frac{1-a^N}{1-a}$
		ii)	$\lim_{N\to\infty} \sum_{n=0}^{N} a^n = \lim_{N\to\infty} \frac{1-a^{N+1}}{1-a}$ if $\|a\| < 1$, $= \frac{1}{1-a}$

	b)	i)	$\sum_{n=-2}^{7} e^{j\frac{\pi}{2}n} = e^{-j\pi} + e^{-\frac{j\pi}{2}} + \dots + e^{\frac{j\pi}{2}7}$ $= \underbrace{-1 - j + 1 + j}_{0} \underbrace{-1 - j + 1 + j}_{0} - 1 - j = -1 - j$
		ii)	$\int_0^8 e^{\frac{j\pi}{2}t}\,dt = \frac{2}{j\pi} e^{\frac{j\pi}{2}t}\Big\|_0^8 = \frac{2}{j\pi}\left[e^{j4\pi} - e^{0}\right] = 0$
		iii)	$\int_0^\infty e^{-t}\sin(t)\,dt = \int_0^\infty e^{-t}\left(\frac{e^{jt} - e^{-jt}}{2j}\right)dt = \frac{1}{2j}\int_0^\infty e^{t(j-1)} - e^{t(-j-1)}\,dt$ $= \frac{1}{2j}\left[\frac{e^{t(j-1)}}{j-1} + \frac{e^{t(-j-1)}}{j+1}\right]_0^\infty = \frac{1}{2j}\left(\frac{1}{j-1} + \frac{1}{j+1}\right) = -\frac{1}{2}$

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<td>5)</td>
<td>a)</td>
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$y[-2] = \sum_{m=-\infty}^{\infty} x[m]h[-2-m] = x[-2]h[0] + x[-1]h[-1] = 1$

$y[-1] = \sum_{m=-\infty}^{\infty} x[m]h[-1-m] = x[-2]h[1] + x[-1]h[0] = 5+2 = 7$

$y[0] = \sum_{m=-\infty}^{\infty} x[m]h[-m] = x[-2]h[2] + x[-1]h[1] + x[0]h[0] = 10+10+3 = 23$

$y[1] = \sum_{m=-\infty}^{\infty} x[m]h[1-m] = x[-2]h[3] + x[-1]h[2] + x[0]h[1] + x[1]h[0] = 11+20+15+2 = 48$

$y[2] = \sum_{m=-\infty}^{\infty} x[m]h[2-m] = x[-2]h[4] + x[-1]h[3] + x[0]h[2] + x[1]h[1] + x[2]h[0] = 8+22+30+10+2 = 72$

$y[3] = \sum_{m=-\infty}^{\infty} x[m]h[3-m] = x[-2]h[5] + x[-1]h[4] + x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0] = 4+16+33+20+10+1 = 84$

$y[4] = \sum_{m=-\infty}^{\infty} x[m]h[4-m] = x[-2]h[6] + x[-1]h[5] + x[0]h[4] + x[1]h[3] + x[2]h[2] + x[3]h[1] = 1+8+24+22+20+5 = 80$

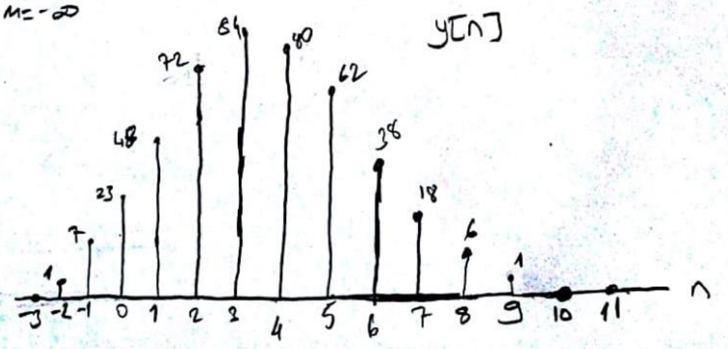
$y[5] = \sum_{m=-\infty}^{\infty} x[m]h[5-m] = x[-1]h[6] + x[0]h[5] + x[1]h[4] + x[2]h[3] + x[3]h[2] = 2+12+16+22+10 = 62$

$y[6] = \sum_{m=-\infty}^{\infty} x[m]h[6-m] = x[0]h[6] + x[1]h[5] + x[2]h[4] + x[3]h[3] = 3+8+16+11 = 38$

$y[7] = \sum_{m=-\infty}^{\infty} x[m]h[7-m] = x[1]h[6] + x[2]h[5] + x[3]h[4] = 2+8+8 = 18$

$y[8] = \sum_{m=-\infty}^{\infty} x[m]h[8-m] = x[2]h[6] + x[3]h[5] = 2+4 = 6$

$y[9] = \sum_{m=-\infty}^{\infty} x[m]h[9-m] = x[3]h[6] = 1$



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<tr>
<td></td>
<td>b)</td>
<td>

```
y= [1,2,3,2,2,1]; % input signal
indexy=[-2 -1 0 1 2 3 ]; % index of signal
h=[1,5,10,11,8,4,1]; % impulse response
figure(1);
stem(indexy,y);
title('input signal');
grid on;
xlim([-5,5]);
figure(2)
stem(h);
```

</td>
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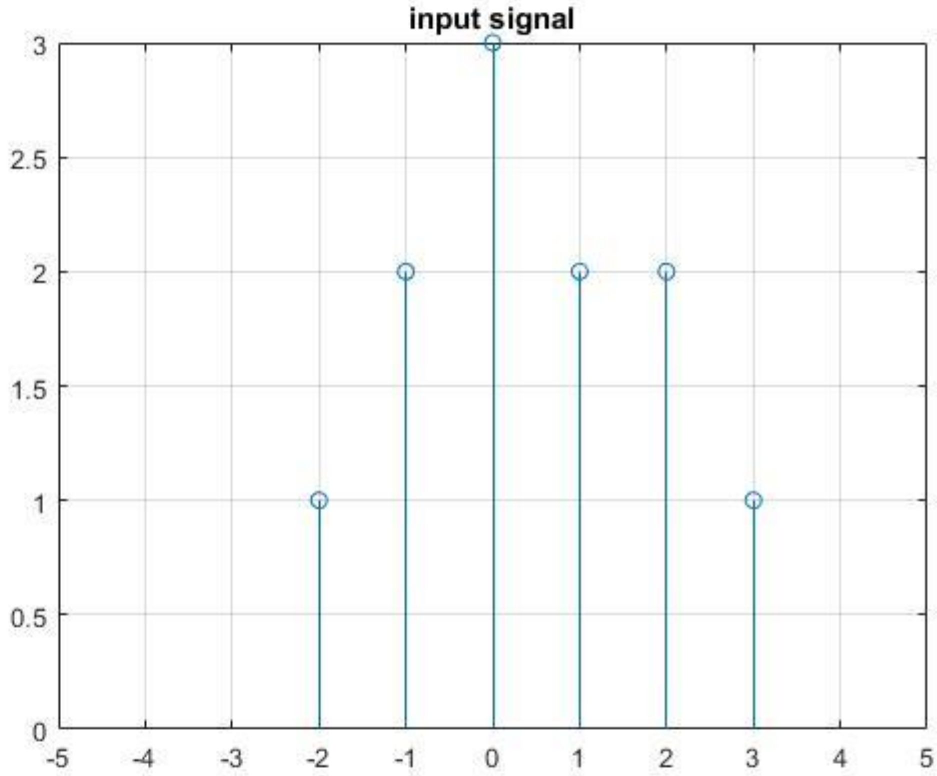
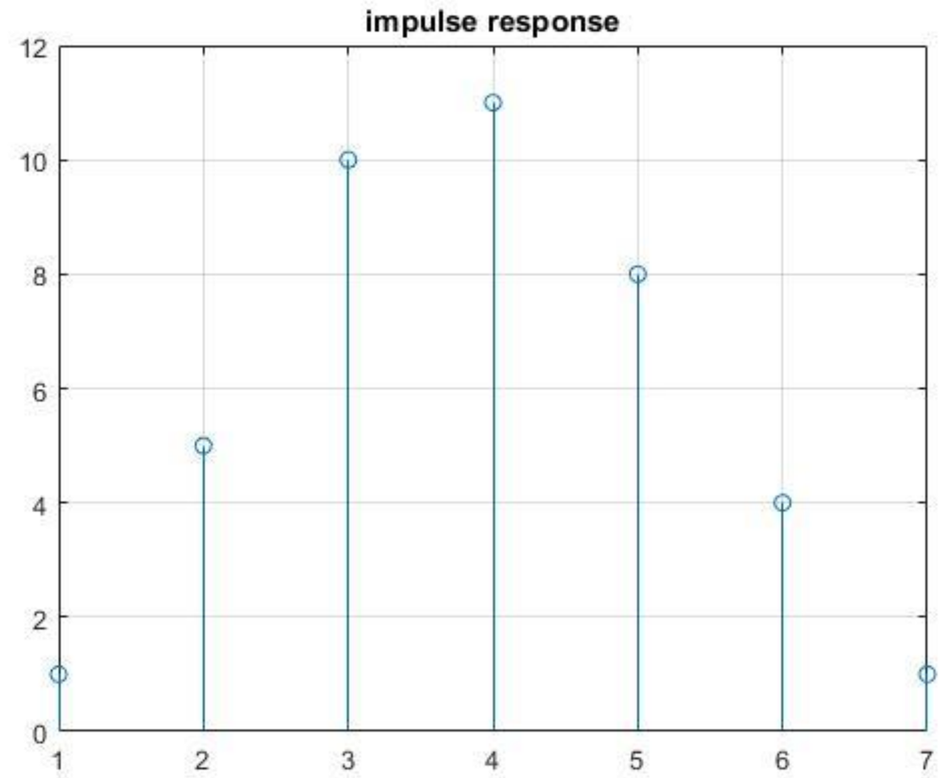
</table>

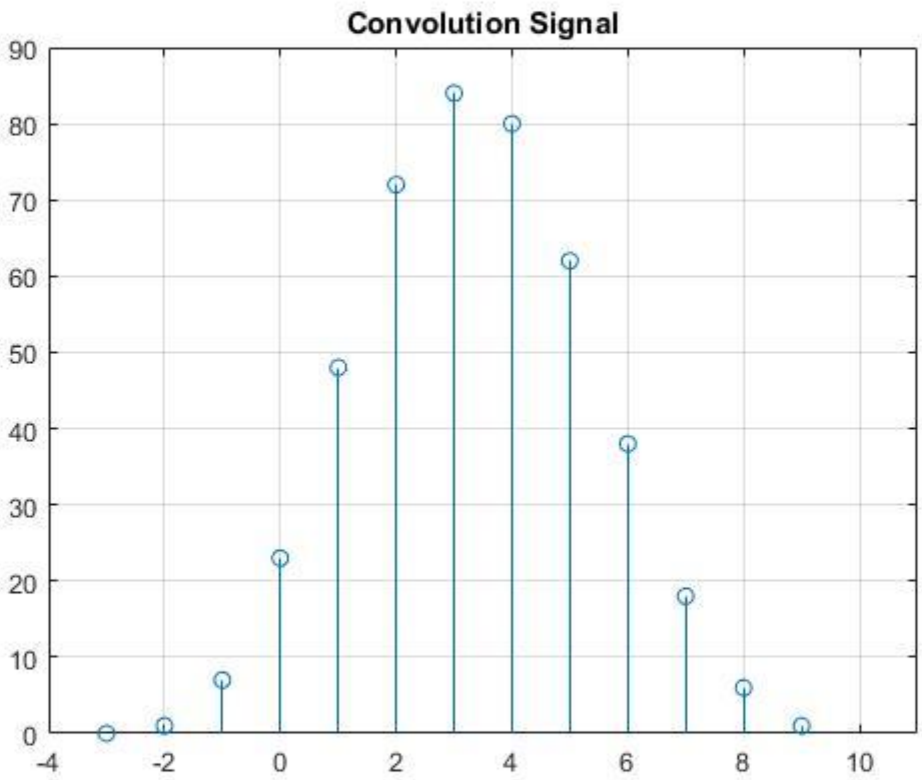
```
title('impulse response');
grid on;


lenConv= length(h)+length(y); % length of convolution sum
Conv=[]; % output of signal

for i=1:lenConv;
    sum=0;
        for j=1:length(y);

            if i-j>0 && i-j<(length(h)+1)
            sum= sum+ h(i-j)*y(j);
            end

        end
    Conv=[ Conv sum];

end


indexConv=[];

for i=1:lenConv
    indexConv= [indexConv ,(indexy(1)+i-2)];

end

figure(3);
stem(indexConv, Conv);
title('Convolution Signal');
xlim([-3 11]);
grid on;
```



input signal
3
2.5
2
1.5
1
0.5
0
-5
-4
-3
-2
-1
0
1
2
3
4
5



impulse response
12
10
8
6
4
2
0
1
2
3
4
5
6
7


Convolution Signal
90
80
70
60
50
40
30
20
10
0
-4
-2
0
2
4
6
8
10