Chapter 6

Sampling

We live in an analog world. Whether recording sounds, capturing images, or processing an electromagnetic wave, many sources of information are of analog or CT nature. We usually want to store/exchange/manipulate this information using digital computers, microprocessors, and so on.

Sampling: The process of converting a CT signal to a DT signal (i.e. discrete sequence of numbers)

$$x(t) \longrightarrow \ldots, x(-2T), x(-T), x(0), x(T), x(2T), \ldots$$

Ex:

Sampling theorem states that *under certain conditions* a CT signal can be completely represented by its values ("samples") at points equally spaced in time. That is, no information is lost in the sampling process.

Why is sampling theorem very important/useful?

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6.1 Sampling Theorem

Observation: *Only under some specific conditions*, equally-spaced samples uniquely specify a CT signal.
Ex:

Definition: A signal $x(t)$ is bandlimited if

Ex:

Ideal impulse-train sampling:
Periodic impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$
Sampling of a CT signal x(t):

Notice that we have one-to-one correspondence between time-domain and frequency domain. (Given $x(t)$ or $X(j\omega)$, we can find the other). For the sampled signal, we want to preserve this 1-1 correspondence.
When $\omega_s > 2\omega_M$ (sampling freq. greater than twice the largest frequency in the signal),

Sampling Theorem:
Let $x(t)$ be

Definition: The *Nyquist rate* $2\omega_M$ is the minimum sampling rate required for unique representation of the signal.
Ex: When $\omega_s < 2\omega_M$,

Ideal lowpass filtering is necessary for perfect reconstruction of a bandlimited signal. In practice, non-ideal filters are used, and this causes some discrepancy between $x(t)$ and $x_r(t)$. Hence there will always be a distortion on $x(t)$, but for each application there will be an acceptable distortion level (which determines the choice of the non-ideal filter).

6.1.1 Effect of undersampling: Aliasing

Question: What happens if $\omega_s < 2\omega_M$ (undersampling)?
Ex:

The problem of resulting overlap in the frequency domain is referred as **aliasing**.
Ex: Consider sampling $x(t) = \cos(4\pi t)$ at a rate of $\omega_s = 5\pi$.

Ex: [Challenge yourself!] For the above example, plot $x(t)$ and $x_r(t)$ in Matlab, and observe that they have the same values at the sampling points.

6.1.2 Practical sampling with a zero-order hold and reconstruction

In practice, a narrow large-amplitude pulse that approximates an impulse is difficult to generate (and transmit). A practical analog-to-digital converter (ADC) generates the sampled signal with a *zero-order hold*.

Zero-order hold sampling: Sample at $t = nT$ and hold that value until $t = (n+1)T$ (until next sampling instant). Here T is the sampling interval.

Mathematically, zero-order hold corresponds to

Question: What is the effect of zero-order hold sampling in the frequency domain?
$X_o(j\omega) =$

Question: How to reconstruct $x(t)$ from zero-order hold sampled signal $x_o(t)$, or equivalently $X(j\omega)$ from $X_o(j\omega)$?

6.1.3 Reconstruction of a signal from its samples as a process of interpolation

Interpolation: Fitting a CT signal to samples

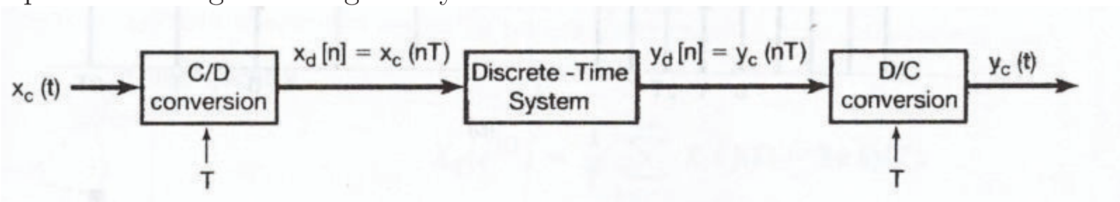
Ex: zero-order hold (convolution with a rect)
first-order hold/linear interpolation (convolution with a triangle)
higher-order holds (connection of sample points by higher order polynomials)

Sinc interpolation for exact reconstruction:
This is ideal lowpass filtering viewed in the time domain:

As discussed before, in practice, a less accurate but a simpler filter is preferable $\Leftrightarrow$ a simpler interpolating function than sinc is preferable.

6.2 DT processing of sampled CT signals

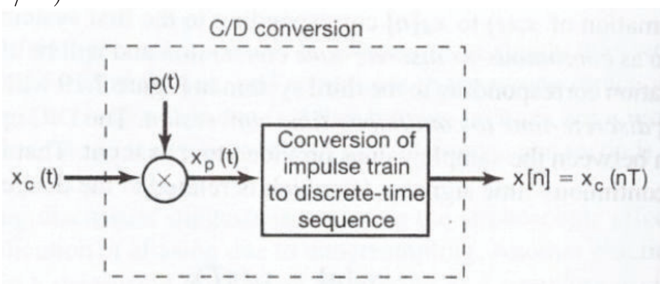
We often process CT signals using DT systems as follows:



instead of processing in CT:

We will discuss that it is possible to make these two approaches equivalent with proper choice of sampling frequency and filters.

C/D conversion:
Continuous-to-discrete (C/D) conversion can be modeled as follows:



What is the relationship between the spectrums of $x_c(t)$ and $x_d[n]$?

(Cont'ed)

Ex:

D/C conversion:
Remember that the <u>ideal</u> way of discrete-to-continuous (D/C) conversion is

Equivalent processing in CT or DT:

Both of the two processing result in the same output if

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Ex: