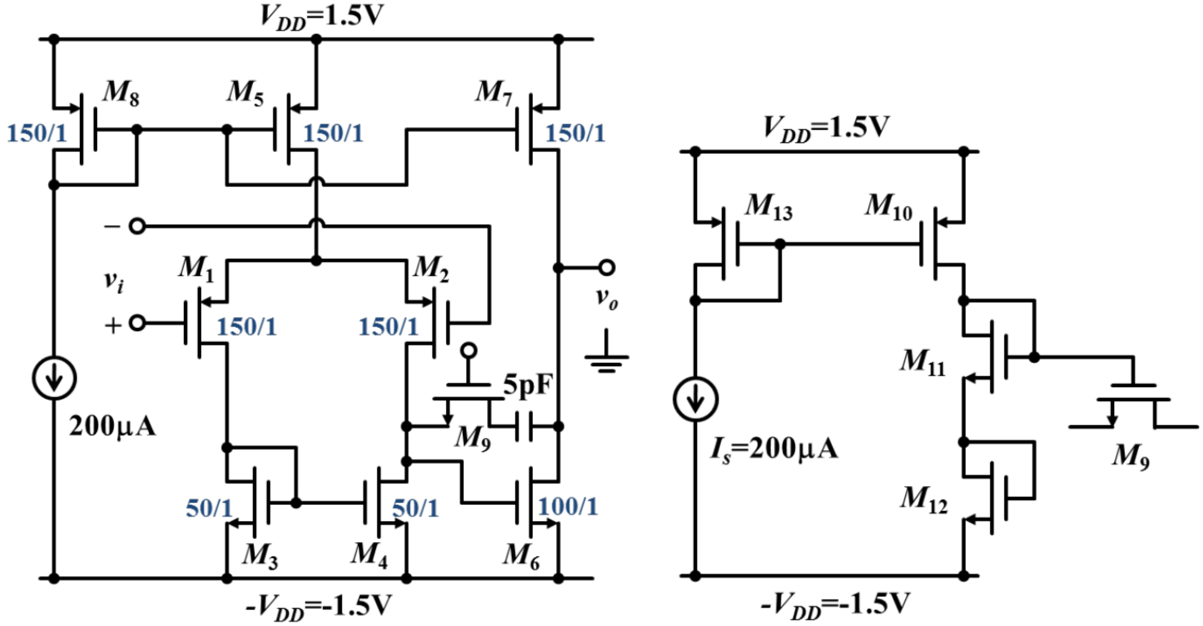
EE 414 Supplementary Problems

Problem 1: This problem concerns the CMOS op-amp shown in Fig. 1.



(a) (b)

Fig. 1.

Variable	NMOS	PMOS	Unit
X_d	0.1	0.1	µm
dX_d/dV_{ds}	0.02	0.04	µm/V
t_{ox}	80	80	A
µ	450	150	cm^2/Vs
V_t	0.7	0.7	V
γ	0	0	$V^{-1/2}$

Table 1

a. Calculate the open-loop voltage gain, unity-gain bandwidth, and slew rate, for the circuit in Fig. 1 (a). Use the parameters of Table 1. Assume that the gate of M_9 is connected to the positive power supply V_{DD} and that the W/L ratio of M_9 has been chosen to cancel the right half-plane zero.

b. If the circuit of Fig. 1 (b) is used to generate the voltage to be applied to the gate of M_9 in Fig. 1 (a), calculate the required W/L ratio of M_9 to move the right-half plane zero to infinity. Let V_{DD} = 1.5V and I_s = 200µA. Use L = 1µm for all transistors, $W_{13} = W_{10}$ = 150µm, and $W_{11} = W_{12}$ = 100µm. Use Table 1 for other parameters.

c. Assuming that the zero has been moved to infinity, determine the maximum load capacitance that can be attached directly to the output node of the circuit in Fig 1 (a) and still maintain a phase margin of 45°. Neglect all higher order poles except the one due to the load capacitance. Use the value of W/L ratio obtained in part (b) for M_9 with the bias circuit of Fig. 1 (b). Ignore junction capacitance for all transistors. Use Table 1 for other parameters.

Problem 2:

What is the gain and -3 dB bandwidth (in Hz) of Fig. 2 if C_L=1 pF? Ignore reverse bias voltage effects on the pn junctions and assume the bulk-source and bulk-drain areas are given by Wx5μm. The W/L ratios for M1 and M2 are 10μm/1μm and for the remaining PMOS transistors the W/L ratios are all 2μm/1μm.

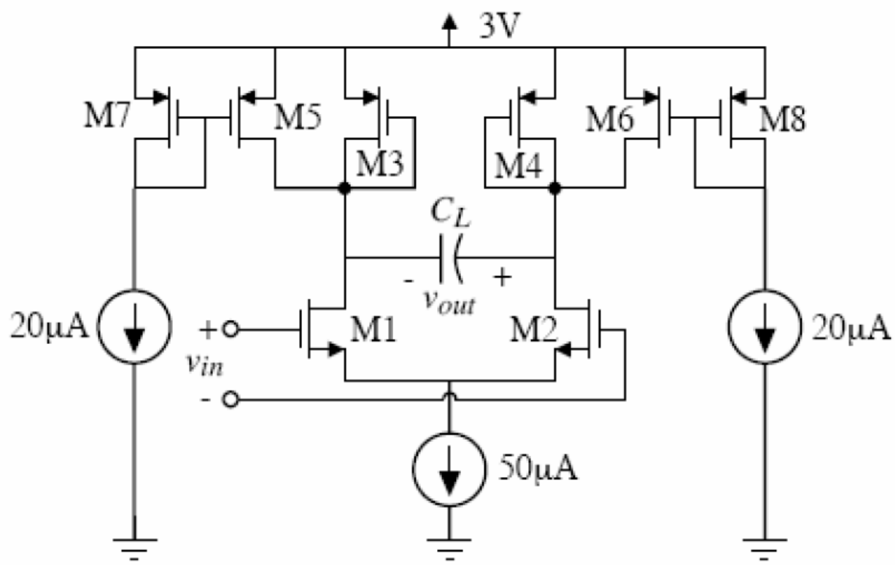


Fig. 2.

Problem 3:

A comparator consists of an amplifier cascaded with a latch as shown below. The amplifier has voltage gain of 10V/V and f_{-3dB} = 100MHz and the latch has a time constant of 10ns. The maximum and minimum voltage swings of the amplifier and latch are V_{OH} and V_{OL}. When should the latch be enabled after the application of a step input to the amplifier of $0.05(V_{OH}\text{-}V_{OL})$ to get minimum overall propagation time delay? What is the value of the minimum propagation time delay? It may useful to recall that the propagation time delay of the latch is given as $t_p = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{il}}\right)$ where v_{il} is the latch input ($v_{il} = x \cdot (V_{OH} - V_{OL})$).

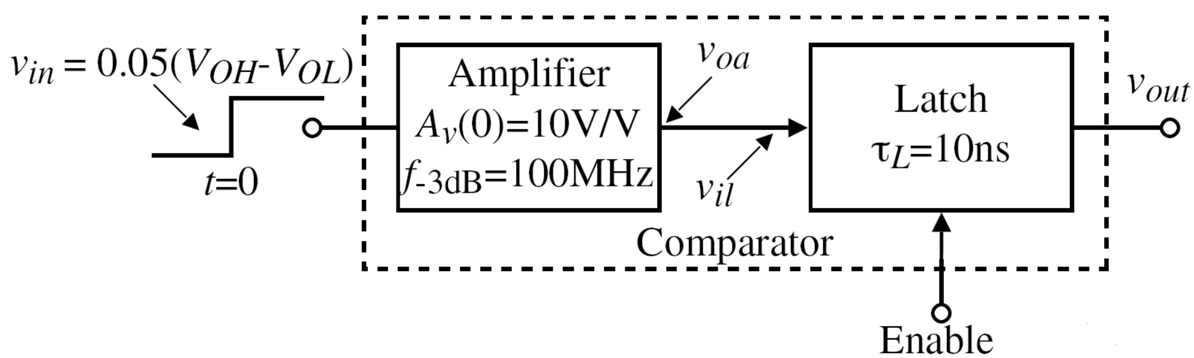


Fig. 3

EE414 Supplementary Problems Solutions

Problem 1:

For $L_d = 0$

(a) $L_{eff} = L - 2L_d - X_d = 0.9\,\mu m$

$|I_D| = 200\,\mu A$ for $M_5 - M_8$

$|I_D| = 100\,\mu A$ for $M_1 - M_4$

$$\frac{dX_d}{dV_{DS}} = 0.02\,\mu m/V \quad (NMOS)$$
$$= 0.04\,\mu m/V \quad (PMOS)$$

$$1/r_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{I_D}{L_{eff}} \frac{dX_d}{dV_{DS}}$$

$$1/r_{o2} = 100\,\mu A / 0.9\,\mu m \cdot 0.04\,\mu m/V = 4.44\,\mu A/V$$

$$1/r_{o4} = 100\,\mu A / 0.9\,\mu m \cdot 0.02\,\mu m/V = 2.22\,\mu A/V$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.9 \times 8.854 \times 10^{-12} / 80 \times 10^{-10} = 4.32 \times 10^{-3}\,F/m^2$$

$$K_p' = \mu_p C_{ox} = 150 \times 10^{-4} \times 4.32 \times 10^{-3} = 64.7\,\mu A/V^2$$

$$K_n' = \mu_n C_{ox} = 450 \times 10^{-4} \times 4.32 \times 10^{-3} = 194\,\mu A/V^2$$

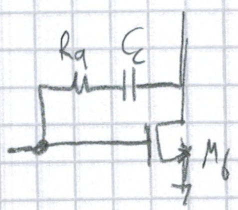
$$g_{m2} = \sqrt{2 \times 64.7 \times (150/0.9) \times 100} = 1.47\,mA/V$$

$$g_{m6} = \sqrt{2 \times 194 \times (100/0.9) \times 200} = 2.94\,mA/V$$

$$1/r_{o7} = \frac{200\,\mu A}{0.9\,\mu m} \times 0.04\,\mu m/V = 8.89\,\mu A/V$$

$$1/r_{o6} = \frac{200\,\mu A}{0.9\,\mu m} \times 0.02\,\mu m/V = 4.44\,\mu A/V$$

$$v_o/v_i = g_{m2}(r_{o2} \| r_{o4}) \cdot g_{m6}(r_{o6} \| r_{o7}) = \underline{\underline{48681}}$$



$$Z = \frac{1}{(1/g_{m6} - R_q)C_c}$$

Cancel this zero by moving it to infinity.

$$R_9 = \frac{1}{g_{m6}} = \frac{1}{2.94m} = 340.1\,\Omega$$

$$1/R_9 = K'\left(\frac{W}{L}\right)_9 \left(V_{GS_9} - V_{T9} - V_{DS_9}\right) \Rightarrow (V_{DS_9} \approx 0)$$

Assume $\gamma = 0$

$$1/R_9 = K'(W/L)_9 \left(\overset{V_{DD}}{V_{G9}} - V_{SS} - V_{GS6} - V_{T9}\right)$$

$$\therefore\ V_{OV_6} = \sqrt{\frac{2\times 200}{194(100/0.9)}} = 0.136\,V \Rightarrow V_{GS_6} = V_{OV_6} + \overset{0.6V}{V_{T6}} = 0.736V$$

$$\therefore\ 1/R_9 = \frac{1}{340} = 194\times10^{-6}\left(\frac{W}{L}\right)_9\left(3 - 0.736 - 0.7\right) \Rightarrow \left(\frac{W}{L}\right)_9 = 9.7$$

* M_9 is in triode region, so $X_d = 0 \Rightarrow L_9 = L_d - 2L_d - X_d = 1\,\mu m$

$\Rightarrow \underline{\underline{L_9 = 1\,\mu m}}$, $\underline{\underline{W_9 = 9.7\,\mu m}}$

At $f_{unity} \Rightarrow |A(j\omega)| = \left|\frac{g_{m_1}}{j\omega C_c}\right| = 1 \Rightarrow f_{unity} = \frac{g_{m1}}{C_c 2\pi} = \underline{\underline{46.8\,MHz}}$

$$SR = \frac{I_{max}}{C_c} = \frac{200\,\mu A}{5pF} = \underline{\underline{40\,V/\mu s}}$$

(b) $1/R_9 = K'\left(\frac{W}{L}\right)_9 (V_{GS_9} - V_{T9})$

Since $I_{D12} = I_{D6}$ and $(W/L)_{12} = (W/L)_6$ $\therefore V_{OV_{12}} = V_{OV_6} = 0.136\,V$

Therefore $V_{SB_{11}} = V_{GS_{12}} = V_{T12} + V_{OV12} = V_{SB9} = V_{GS_6} = V_{T6} + V_{OV6}$

Because $V_{T12} = V_{T6}$ (no body effect)

Also: $V_{OV_{11}} = V_{OV_{12}}$ because $I_{D11} = I_{D12}$ and $(W/L)_{11} = (W/L)_{12}$

Therefore: $V_{GS_9} = V_{GS11} + V_{GS12} - V_{GS6} = V_{G11} + V_{OV11} + \cancel{V_{G12}} + \cancel{V_{OV12}} - \cancel{V_{G6}} - \cancel{V_{OV6}}$

$= V_{G11} + V_{OV11}$

$\therefore V_{GS9} - V_{T9} = V_{T11} + V_{OV11} - V_{T9} = V_{OV11}$

Since $V_{GS9} - V_{T9} = V_{GS6} - V_{T6}$ and $1/R_9$ should be equal to g_{m6} to cancel the RHP zero.

$$1/R_9 = k'(W/L)_9 (V_{GS9} - V_{T9}) = g_{m6} = k'(W/L)_6 (V_{GS6} - V_{T6})$$

$$\Rightarrow (W/L)_9 = (W/L)_6 = 100/1$$

(c) To get $45°$ phase margin set the second pole = unity gain freq.

$$\left(|P_2| = \frac{g_{m6} C_c}{C_L C_1 + C_c C_L + C_1 C_c}\right) = \frac{g_{m2}}{C_c}$$ (C_1 is the parasitic cap. at first stage output)

$$\therefore C_1 = \cancel{C_{OL2}} + \cancel{C_{OL4}} + C_{GS6} + \cancel{C_{OL9}} \approx C_{GS6} = \left(\frac{2}{3} 100\mu m \cdot 0.9\mu m\right) \cdot 4.32 \frac{fF}{\mu m^2}$$

$$C_1 = 259.2 fF$$

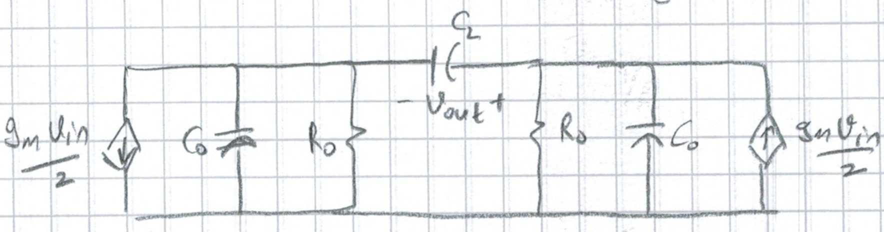
$$\frac{g_{m2}}{C_c} = \frac{1.47 mA/V}{5pF} = 294 \, Mrad/s$$

$$\therefore 294 \, Mrad/s = \frac{2.94 \, mA/V \cdot 5pF}{C_L \, 259.2f + C_L \, 5p + 259.2f \times 5p}$$

$$\Rightarrow C_L = 9.26 pF$$

Problem 2:

Let's first draw the small signal model of the circuit.



$$\frac{V_{out}}{V_{in}} = g_m R_o \quad \text{and} \quad W_{-3dB} = \frac{1}{(C_L + 0.5 C_o)\, 2R_o}$$

$$g_m = \sqrt{2 K_N' \left(\tfrac{W}{L}\right)_1 I_{D_1}} = \sqrt{2 \cdot 110 \cdot 10 \cdot 25}\ \mu A/V = 234.5\ \mu A/V$$

$$R_o \approx 1/g_{m_3} \,//\, r_{o1} \,//\, r_{o3} \,//\, r_{o5}\ , \quad g_{m_3} = \sqrt{2 K_P' (W/L)_3 I_{D_3}} = \sqrt{2 \cdot 50 \cdot 2.5}\ \mu A/V = 31.62\ \mu A/V$$

$$r_{o1} = \frac{1}{\lambda I_{D1}} = \frac{1}{0{,}04 \times 25\ \mu A} = 1 M\Omega, \quad r_{o3} = 4 M\Omega,\ r_{o5} = 0{,}8\ M\Omega$$

$$\therefore R_o = 31{,}623 k\Omega \,//\, 1M\Omega \,//\, 4M\Omega \,//\, 0{,}8 M\Omega = 29{,}31\ k\Omega$$

$$C_o \approx C_{gs3} + C_{bd1} + C_{bd3} + C_{bd5} \qquad C_{gs3} = C_{GSO} W_5 + 0{,}67 C_{ox} W_5 L_5$$

$$= 220\ pF/m \cdot 2\times10^{-6} + 0{,}67 \times 24{,}7 \times 10^{-4}\ F/m^2 \times 2\ \mu m^2$$

$$C_{gs_3} = 3{,}73\ fF$$

$$C_{bd1} = C_J\, AS + C_{JSW} \cdot PS = 770\times10^{-6}\ F/m^2 \cdot 50\ \mu m^2 + 380\times10^{-12}\ F/m \times 30\ \mu m$$

$$C_{bd1} = 38{,}5\ fF + 11{,}4\ fF = 49{,}9\ fF$$

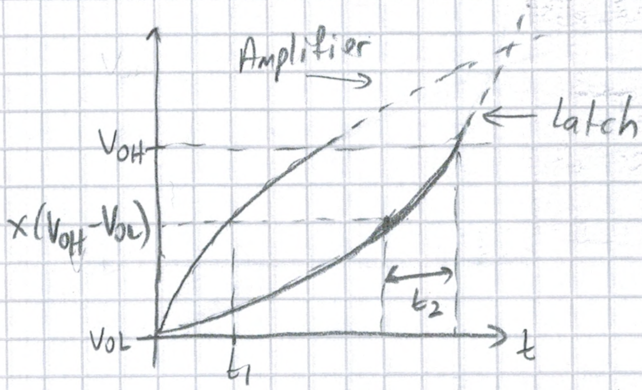
$$C_{bd3} = C_{bd5} = 560\times10^{-6}\ F/m^2 \cdot 10\ \mu m^2 + 350\times10^{-12}\ F/m \cdot 14\ \mu m = 10{,}5\ fF$$

$$\therefore C_o = 74.6\ fF \rightarrow W_{-3dB} = \frac{1}{(1{,}073\ pF)\, 58.62\ k\Omega} = 16.45\ Mrad/s$$

$$\underline{f_{-3dB} = 2{,}62\ MHz} \qquad \underline{A_v = 6{,}873\ V/V}$$

Problem 3 :

The solution is based on the figure shown.



Note that $\quad V_{oa}(t) = 10\left[1-e^{-\omega_{-3dB}t}\right]0{,}05(V_{OH}-V_{OL})$

If we define the input voltage to the latch as,

$v_{if} = x(V_{OH}-V_{OL})$ then we can solve for t_1 & t_2 as:

$$x(V_{OH}-V_{OL}) = 10\left[1-e^{-\omega_{-3dB}t}\right]0{,}05(V_{OH}-V_{OL})$$

$$\hookrightarrow x = 0{,}5\left[1-e^{-\omega_{-3dB}t}\right]$$

This gives,

$$t_1 = \frac{1}{\omega_{-3dB}}\ln\left(\frac{1}{1-2x}\right)$$

From the propagation time delay of the latch we get,

$$t_2 = \tau_L \ln\left(\frac{V_{OH}-V_{OL}}{2v_{i1}}\right) = \tau_L \ln\left(\frac{1}{2x}\right)$$

$$\therefore\ t_p = t_1 + t_2 = \frac{1}{\omega_{-3dB}}\ln\left(\frac{1}{1-2x}\right) + \tau_L \ln\left(\frac{1}{2x}\right)$$

$\frac{dt_p}{dx} = 0$ gives $x = \frac{\pi}{1+2\pi} = 0{,}4313$ $\qquad t_1 = \frac{10ns}{2\pi}\ln(1+2\pi) = \underline{\underline{3{,}16ns}}$

$$t_2 = 10ns\ln\left(\frac{1+2\pi}{2\pi}\right) = \underline{\underline{1{,}477ns}}$$