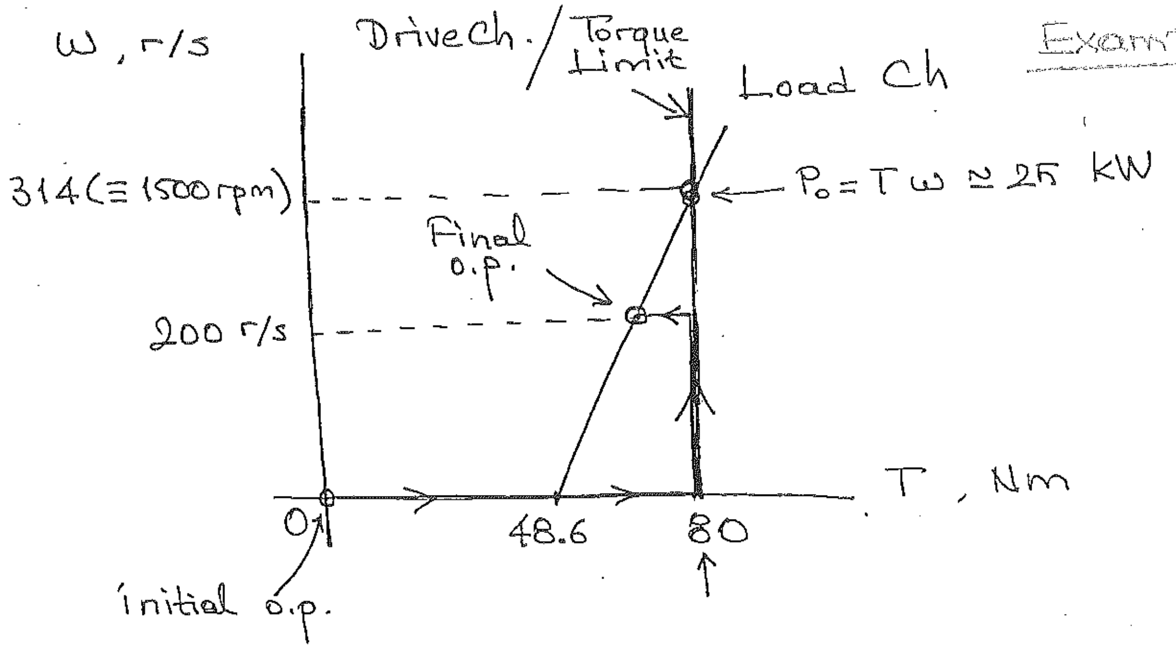
Example



$T_L = m\omega + a$ where, $a = 48.6$

$m = \frac{31.4}{314} = 0.1$

$\Delta T_L = 80 - 48.6 = 31.4$

$\Delta\omega = 314 - 0 = 314$

$T_L = 0.1\,\omega + 48.6$ //

Torque balance eqn:

$$J\frac{d\omega}{dt} = T_{lim} - (m\omega + a)$$

Let $J = 1.0$ $Kg\text{-}m^2$ (a high-inertia mechanical subsystem)

$$\frac{d\omega}{dt} + \frac{m}{J}\omega = \frac{T_{Lim} - a}{J}$$

$$\frac{d\omega}{dt} + \frac{1}{(J/m)}\omega = \frac{T_{Lim} - a}{J}$$

$$\frac{d\omega}{dt} + \frac{1}{\tau_m}\omega = \frac{T_{Lim} - a}{J}$$ where, $\tau_m = J/m$

Substitute numerical values

$\tau_m = 1.0/0.1 = 10$ s // $4\tau = 40$ s

$$\frac{T_{Lim} - a}{J} = \frac{80 - 48.6}{1.0} = 31.4 \quad //$$

Solution of this eqn is

$$\omega(t) = C_1\, e^{-t/\tau_m} + C_2$$

Since $\omega(0) = 0$ then $C_1 = -C_2$

$$\omega(t) = C_2\left(1 - e^{-t/\tau_m}\right)$$

as $t \to \infty$ $\qquad \omega(\infty) = C_2$

$$\left.\left(\frac{d\omega}{dt} + \frac{1}{\tau_m}\,\omega\right)\right|_{t\to\infty} = \frac{T_{lim} - a}{J}$$

$$\frac{1}{\tau_m}\,\omega(\infty) = \frac{T_{lim} - a}{J}$$

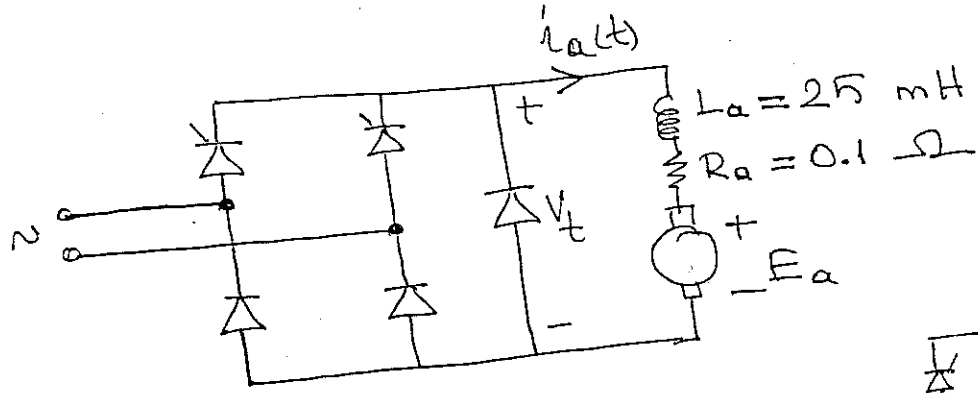
$$\omega(\infty) = 10\,(31.4) = 314 \ r/s \ //$$

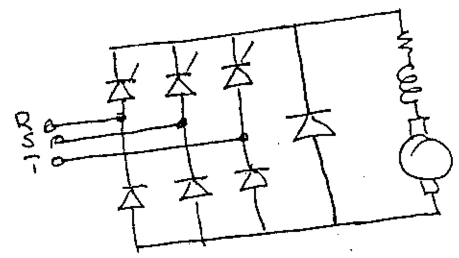
$\therefore \quad C_2 = 314 \ r/s \ //$

which results in

$$\omega(t) = 314\left(1 - e^{-t/10}\right) \quad //$$

On the other hand





$$V_t = e_a(t) + R\,i_a + L\frac{di_a}{dt}$$

$$\frac{di_a}{dt} + \frac{1}{(L_a/R_a)}\,i_a = V_t - e_a(t)$$

$$\frac{di_a}{dt} + \frac{1}{\tau_a}\,i_a = V_t - e_a(t) \qquad \text{when } \tau_a = L_a/R_a$$

Suppose now that $i_a(0) = 0$, $\omega(0) = 0$, a step voltage of $V_t = 200$ V is suddenly applied and the shaft speed momentarily remains constant owing to high inertia.

Since $\omega(0) = 0$ then $e_a(0) = 0$.

Solution of the eqn:

$$i(t) = i(\infty)\left(1 - \epsilon^{-t/\tau_a}\right)$$

$\omega = 0$ (locked rotor)

$$i(\infty) = V_t/R_a = 200/0.1 = 2\,000 \text{ A}$$

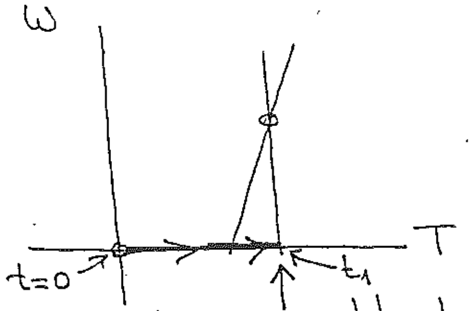
$$\tau_a = L_a/R_a = 25\times10^{-3}\text{ H}/0.1\,\Omega = 0.25 \text{ s} \;//$$

$$4\tau_a = 1.0 \text{ s} \;//$$

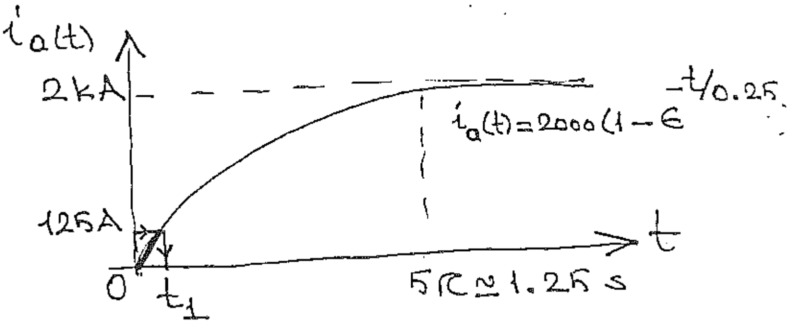
It is obvious that $\tau_a \ll \tau_m$ //

In constructing four-quadrant diagrams to illustrate dynamic operation of electric motor drives (how reversing takes place; how torque limit circuit acts etc) we may take $\tau_a \to 0$ and $\tau_m \to \infty$.

Let us plot the variations in i_a and ω.

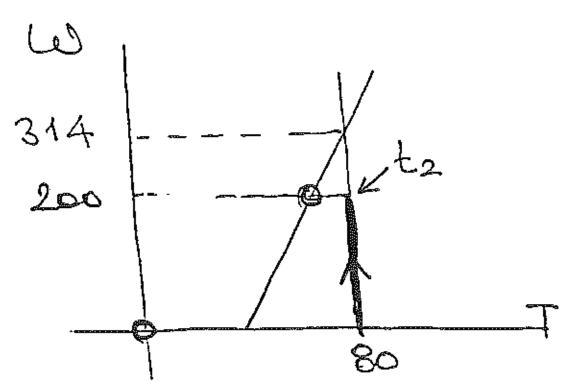


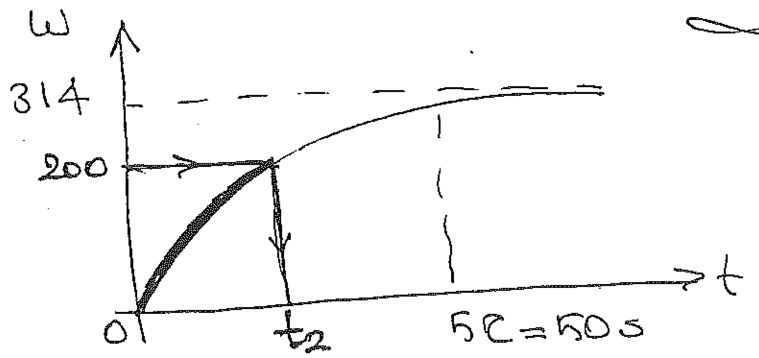
Assume that $I_{a(lim)}$ which corresponds to $T_{lim} = 80$ Nm is 125 A.



Compute t_1

$t_1 \cong 16$ ms

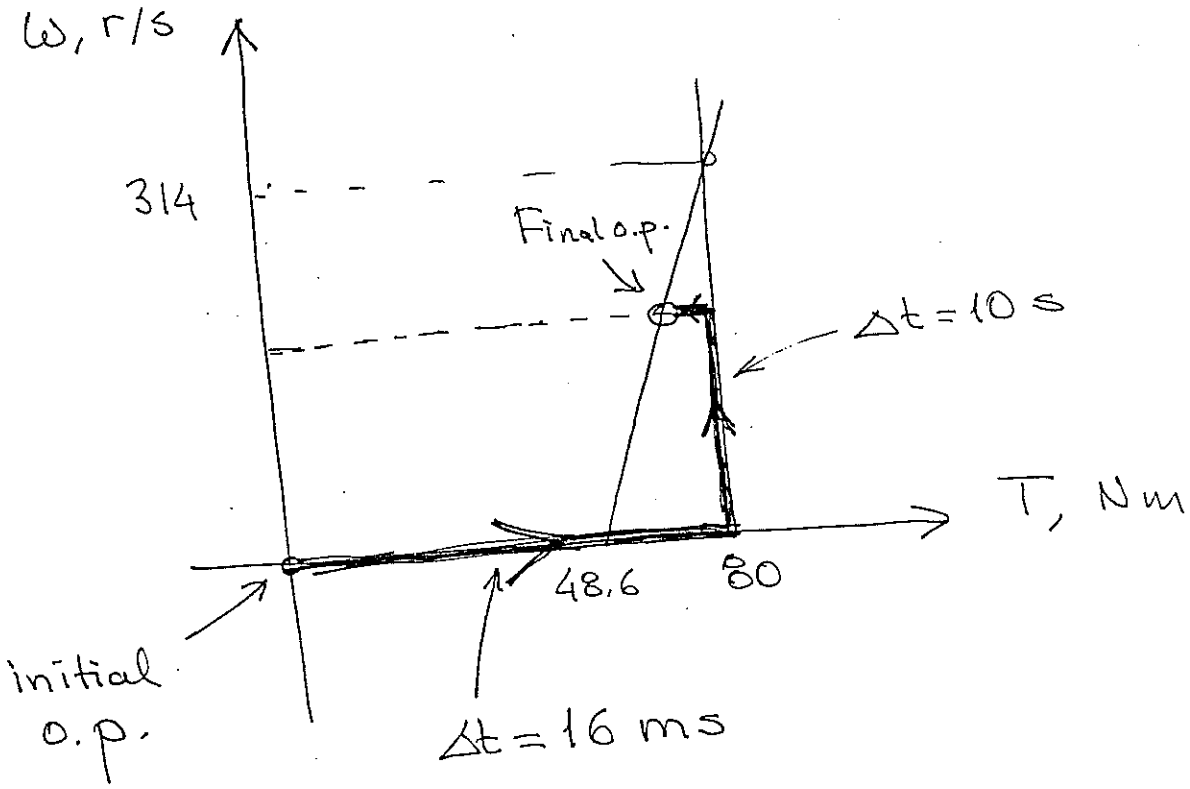




Compute t_2

$t_2 \cong 10$ s

This proves our proposition.



$$\omega(t) = 314\left(1 - e^{-t/10}\right)$$

at $t = 0.016$ s

$$\omega(0.016) = 0.5$$
$$\equiv 4.8 \text{ RPM}$$

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