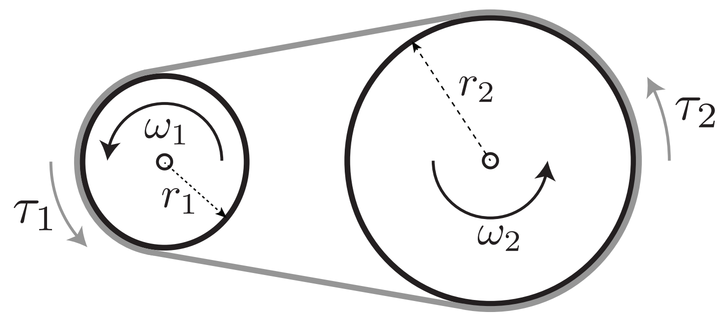
EE302 - Feedback Systems - Assignment 1
Department of Electrical and Electronics Engineering, Middle East Technical University

Due: 8-Mar-2019, @15:40 (There will be a box to drop the assignments in front of D-226. The box will be removed after 15:40.)

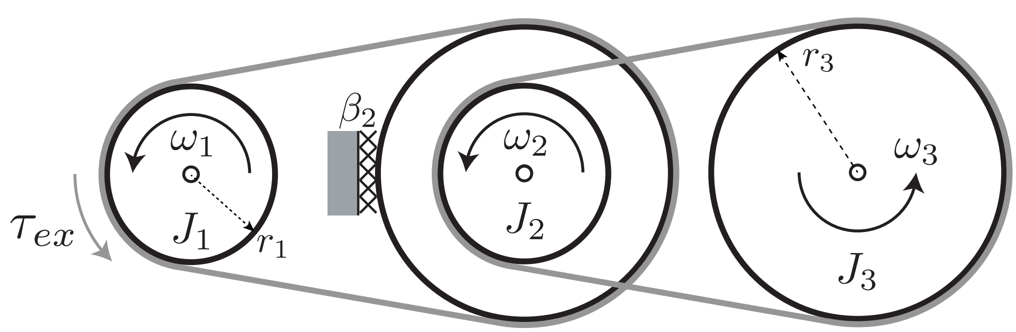
1. (60 Points) The mechanism illustration given below is an ideal belt-pulley mechanism. Fundamentally, it has the same kinematic relations with a gear pair system. The only difference is that, the direction of motion is preserved in a pulley system. r_1 and r_2 correspond to the radii of the first and second pulleys respectively.



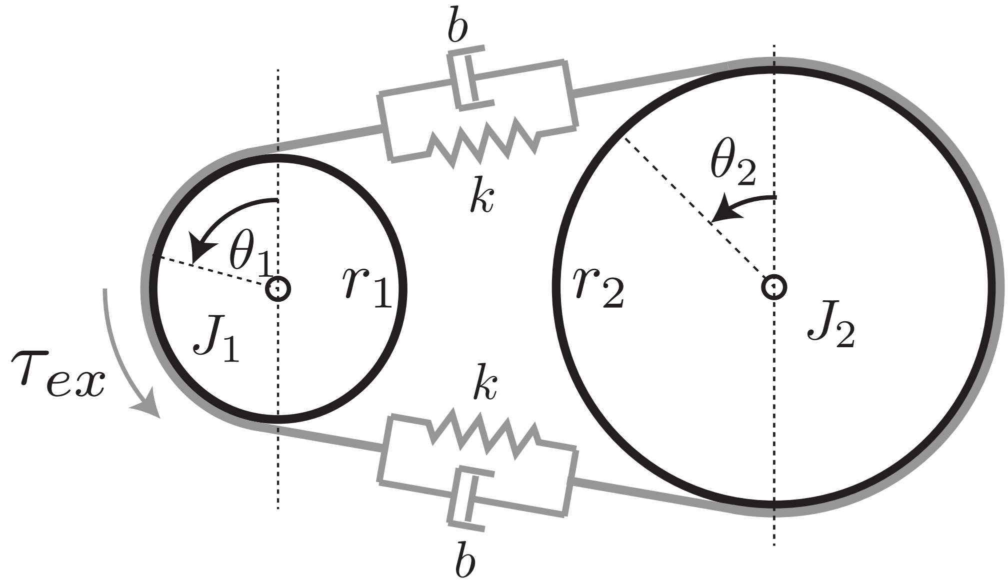
$$\frac{\omega_1}{r_2} = \frac{\omega_2}{r_1}$$
$$\frac{\tau_1}{r_1} = \frac{\tau_2}{r_2}$$

   (a) (15 Points) In this part, you will analyze the following belt-pulley system consisting of three pulleys and two belts. First pulley has a radius of r_1 and an inertia of J_1. Third pulley has a radius of r_3 and an inertia of J_3. Second pulley (in the middle) is connected to the first pulley through its outer disk, which has the same radius with the third pulley, i.e., $r_{2,o} = r_3$. Second pulley is also connected to the first pulley via its inner disk, which has the same radius with the first pulley, i.e., $r_{2,i} = r_1$. Second pulley has an inertia of J_2 (outer and inner disks move together). A linear rotational viscous damping with a damping constant β_2 also affects the motion of the second pulley.

   Given that the external torque acting on the first pulley is the input, $u(t) = \tau_{ex}(t)$, and the angular velocity of the third pulley is the output, $y(t) = \omega_3(t)$, compute the transfer function of the system.



(b) (45 Points) In some belt-pulley applications, ignoring the elasticity of the belt can be very crude and can lead to substantial modeling errors. In order to overcome this problem, a very common method is modeling the belt with a linear (translational) spring-damper as shown in the belt-pulley mechanism below. In this mechanism, first pulley has a radius of r_1 and inertia of J_1, where as the second pulley has a radius of r_2 and inertia of J_2. The spring-mass dampers (above and below) that model the elasticity of the belt have spring stiffnesses of k and damping constants of b.



Given that the external torque acting on the first pulley is the input, $u(t) = \tau_{ex}(t)$, and the angular displacement of the second pulley is the output, $y(t) = \theta_2(t)$,

i. Find a state-space representation of the dynamics. (*Hint:* You can choose your state variables as $\mathbf{x} = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T$).

ii. Compute the transfer function, $G(s) = Y(s)/U(s)$

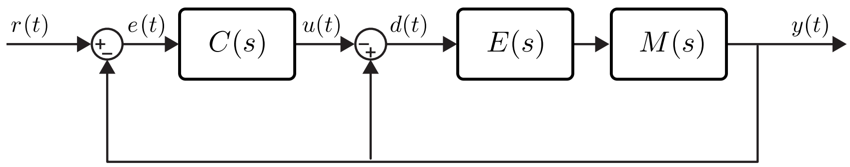
iii. Now, let the parameters of the system be equal to following numerical values

$$r_1 = 0.05\,m\ ,\ r_2 = 0.1\,m\ ,\ J_1 = 0.01\,kg \cdot m^2\ ,\ J_2 = 0.1\,kg \cdot m^2$$
$$k = 100\,N/m\ ,\ b = 10\,N/(m \cdot s)\ .$$

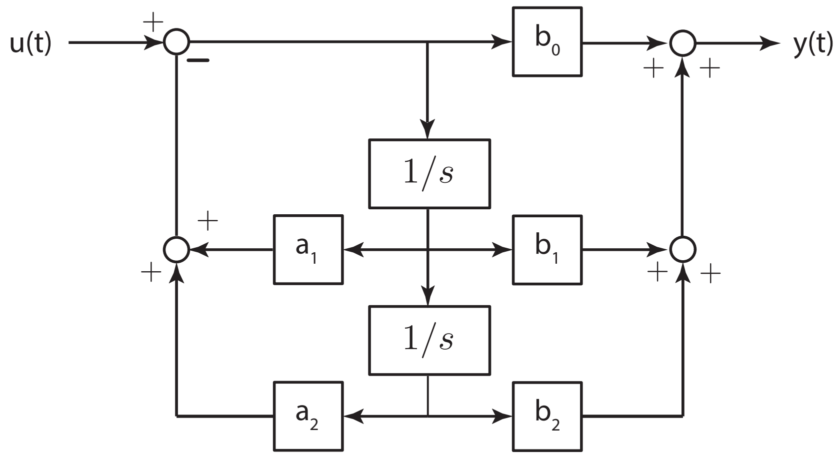
Simplify both the state-space and transfer function representations using these numerical values. Finally convert the state-space form to the transfer function form and verify that converted transfer function is equal to the previously computed one. (*Hint:* You can use MATLAB's *ss2tf* command for conversion).

2. (40 Points) In this problem, your goal is to simplify three different block-diagram topologies using block diagram simplification methods and compute the associated transfer functions.

   (a) (10 Points) Simplify the following block-diagram topology (using block-diagram simplification methods) and derive $Y(s)/R(s)$.



   (b) (15 Points) Simplify the following block-diagram topology (using block-diagram simplification methods) and derive $Y(s)/U(s)$.



3. (15 Points) Simplify the following block-diagram topology (using block-diagram simplification methods) and derive $Y(s)/U(s)$.