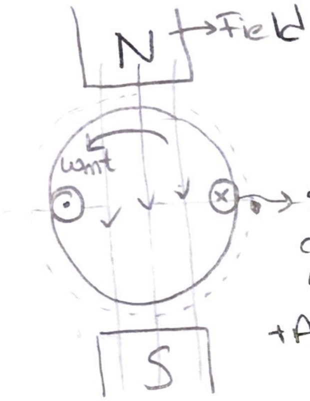
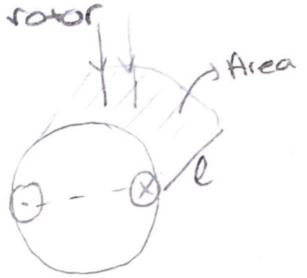
Induced Voltage in a Rotating Coil Placed in Constant Magnetic Field

(1) 2-pole system



→ Single coil placed on armature → zero position of the rotor

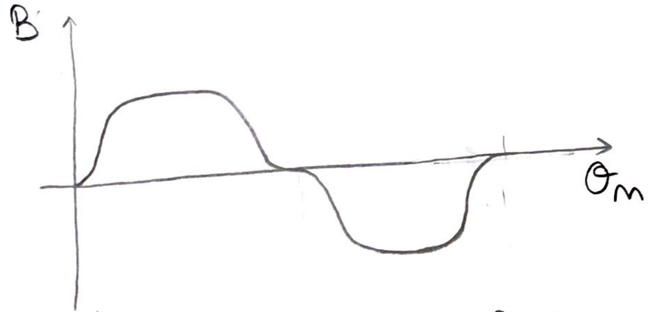


+ Armature rotates with $\omega_m t$.

ω_m: mechanical rotational speed

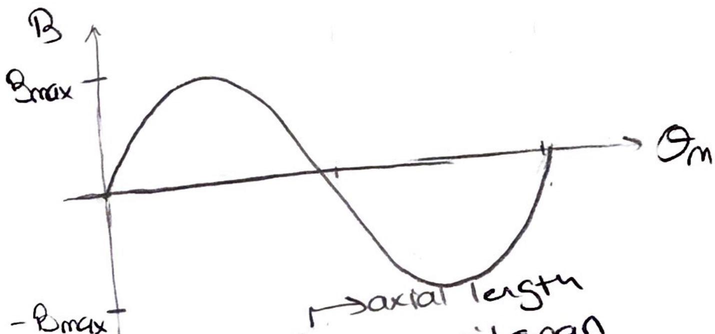
$\theta_m = \omega_m t + \theta_0$ ↳ initial position

Air-gap B-field Distribution



If we walk in the airgap in the ccw direction starting from initial (zero) position, we will observe this field distribution

Let's only consider the fundamental component



Question: Will there be an induced voltage if the armature rotates?

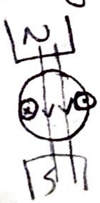
To get a voltage, we should have a time-dependent flux linkage. Let's calculate flux in the armature coil.

$$\phi_{max} = B \cdot A = \int_0^{\ell} \int_0^{\pi} B_{max} \sin\theta_m \, r \, d\theta_m \, d\ell$$

(ℓ → axial length, π → coil span)

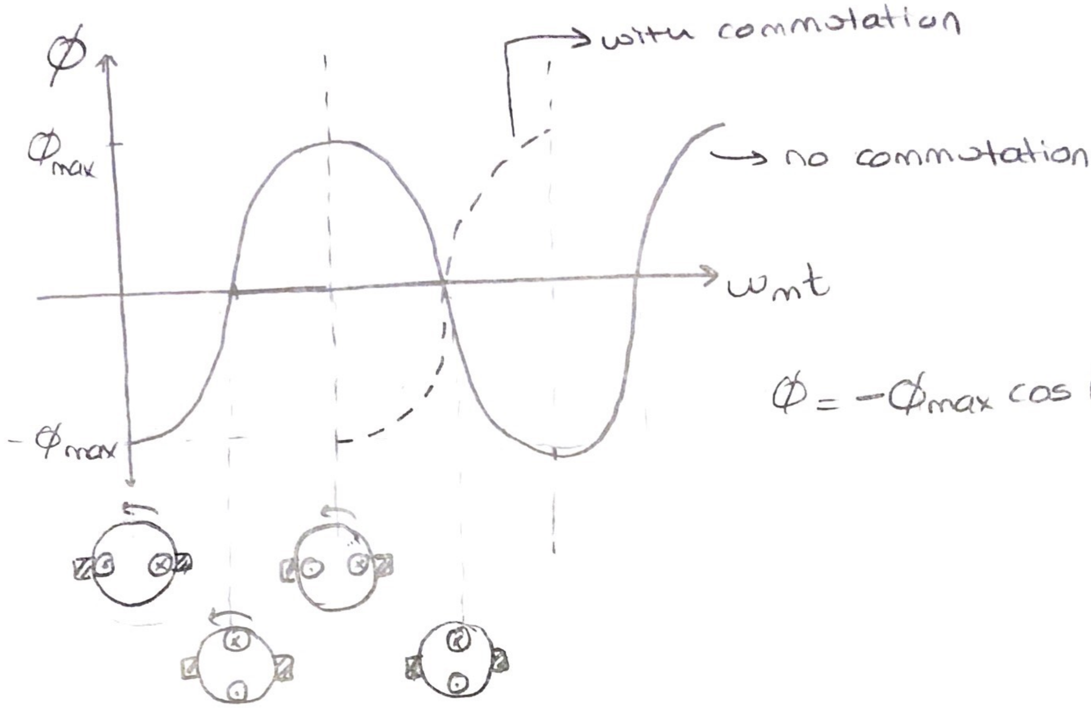
$$\underline{\underline{\phi_{max} = 2 r \ell B_{max}}}$$

} This occurs when field and armature coil alligns



→ at 180 deg

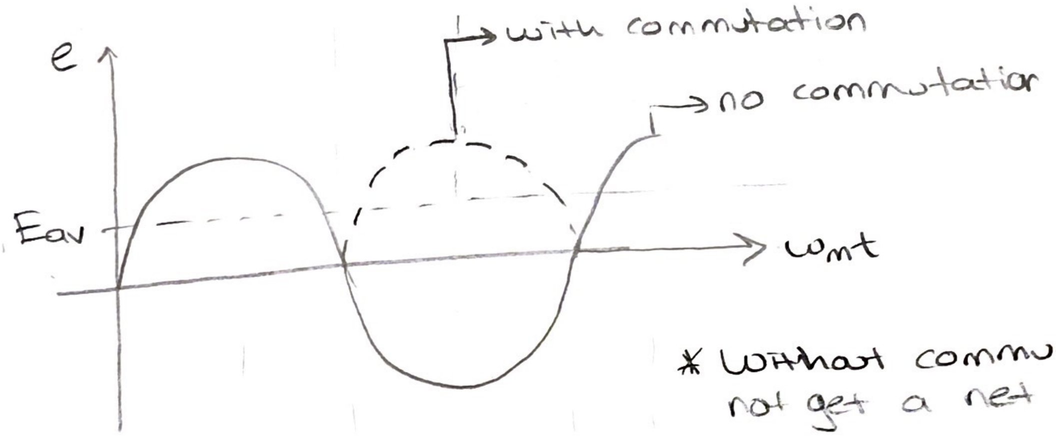
$\omega_m t = \pi$



$\phi = -\phi_{max} \cos(\omega_m t) \; [0 - \pi]$

$e = - N \dfrac{d\phi}{dt}$

We will use this sign later to decide the sign of the voltage



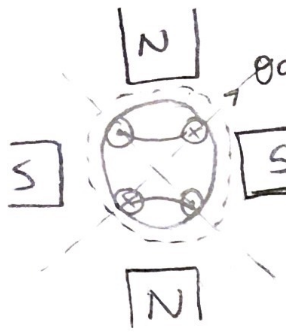
* Without commutation, we would not get a net DC voltage.

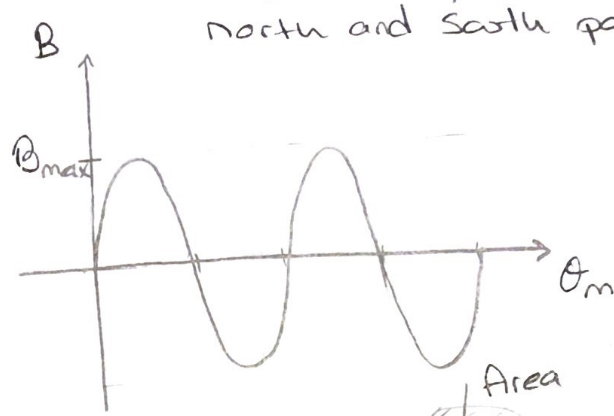
$e = N \phi_{max} \omega_m \sin(\omega_m t)$

$E_{max} = N \phi_{max} \omega_m$

$E_{av} = E_{max} \cdot \dfrac{2}{\pi} = \underbrace{\dfrac{2}{\pi} N}_{\text{Constant}} \omega_m \phi_{max} \rightarrow \underbrace{K}_{\text{Constant}} \omega_m \phi_{max}$

② 4-pole System → In one rotation, there will be two north and South poles.





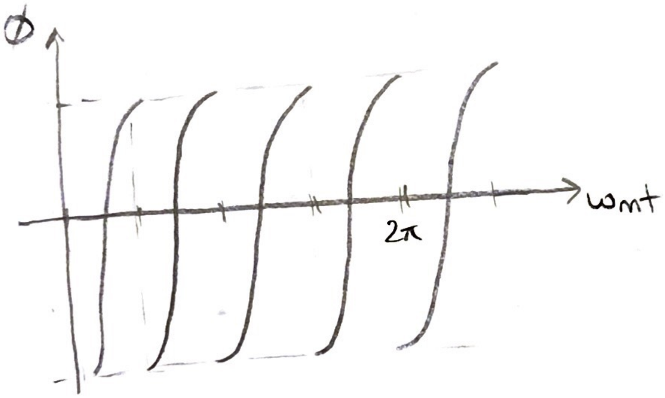
$$B = B_{max} \sin(2\theta_m)$$

$$\phi = \int_0^{l} \int_0^{\pi/2} B_{max} \sin 2\theta_m \, r d\theta_m dl$$

$$\phi_{max} = \frac{4 r l B_{max}}{4} = \frac{4 r l B_{max}}{p}$$

4 → is the pole number

$$\boxed{\text{For a 2-pole system } \phi_{max} = \frac{4 r l B_{max}}{2} = 2 r l B_{max}}$$



$$\phi = -\phi_{max} \cos(2\omega_m t)$$

$$[0 - \pi/2]$$

$$E_{max} = N \phi_{max} \underset{\downarrow p/2}{2} \omega_m$$

$$E_{max} = N \phi_{max} \frac{p}{2} \omega_m$$

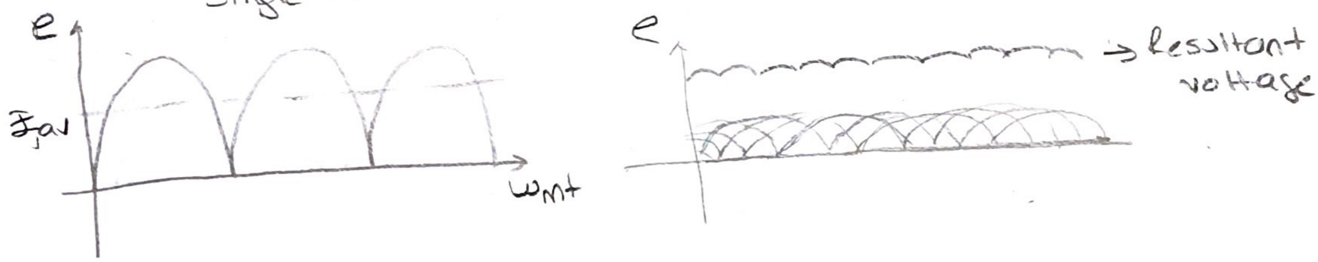
$$E_{av} = \frac{2}{\pi} N \phi_{max} \frac{p}{2} \omega_m \Rightarrow \boxed{\frac{p}{\pi} N \omega_m \phi_{max}}$$

General case, p: pole number.

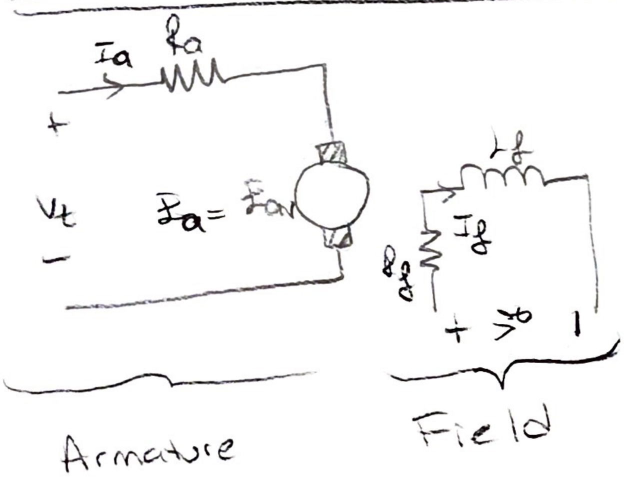
So, there will be an induced voltage due to mechanical rotation. This voltage has a non-zero DC component due to mechanical commutation.

By using distributed coils in armature, we can greatly reduce voltage ripple.

Single coil Distributed coils



Equivalent Circuit of a DC Machine



$* \; E_{av} = K \cdot \omega_m \, \phi$ → field flux

ω_m → speed

K → constant

$V_t = I_a R_a + E_a$

$\underbrace{V_t I_a}_{\text{Input power}} = \underbrace{I_a^2 R_a}_{\text{Loss}} + \underbrace{E_a I_a}_{\text{Output Power (before mechanical friction}}$

$\Rightarrow E_a I_a = T \omega_m$

$** \; T = \dfrac{E_a I_a}{\omega_m} = K I_a \phi$

K → constant

I_a → armature current

ϕ → field flux