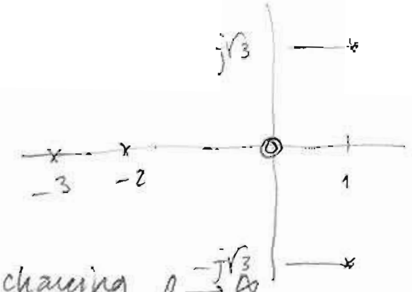
Example: Now consider again

$$q(s) = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$



with $a_1 = 3$, $a_3 = 8$, $a_4 = 24$ and a_2 is changing $0 \rightarrow \infty$

Sketch the Root Locus with a_2 changing.

$$q(s) = s^4 + 3s^3 + a_2 s^2 + 8s + 24$$

$$= 1 + a_2 \frac{s^2}{s^4+3s^3+8s+24} = 1 + \frac{a_2 s^2}{(s+3)(s+2)(s^2-2s+4)} = 1 + \frac{a_2 s^2}{(s+3(s+2)(s-1+j\sqrt{3})(s-1-j\sqrt{3})}$$

Rule 1: $n=4$ and $m=2$ $\max(m,n) = 4$ branches in the locus.

Rule 2: Starting and ending points: 2 branches start at poles and end at zeros. 2 branches start at poles and end at ∞.

Rule 3: Symmetry ✓

Rule 4: Asymptotes: $\phi = \frac{\pm 180}{n-m}(2l+1) = \pm 90(2l+1) = \pm 90^\circ$

Angle between asymptotes 180°.

Rule 5: $$\sigma_0 = \frac{\sum p_j - \sum z_i}{n-m} = \frac{-3-\cancel{2}+\cancel{1}+\cancel{1}-0-0}{2} = -\frac{3}{2} = -1.5.$$

Rule 6: Only region is between -3 and -2

Rule 7: Break away points:

$$\frac{d}{ds}\left[\frac{s^2}{s^4+3s^3+8s+24}\right] = 0 \rightarrow s(-2s^4 - 3s^3 + 8s + 48) = 0$$

One root at $s=0$ but it is difficult to determine the other roots.

However we know that it will be inside the region $-2, -3$.

(One can by numerical methods find $s = -2.45 \rightarrow a_2 \tilde{=} 0.61$.

Other roots are not in the valid region or do not lead to positive a_2.

Rule 8:

Rule 8: Intersection with $j\omega$ axis:

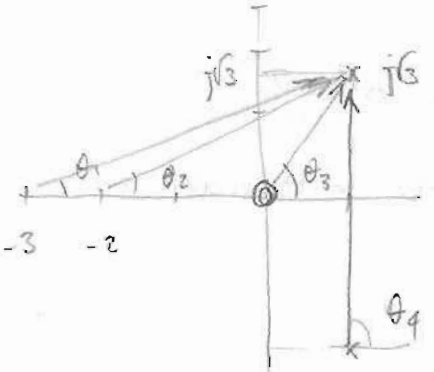
$$
\begin{array}{llll}
s^4 & 1 & a_2 & 24 \\
s^3 & 3 & 8 & 0 \\
s^2 & \frac{3a_2-8}{3} & 24 & \\
s^1 & \frac{24a_2-280}{3a_2-8} & & \\
s^0 & 24 & &
\end{array}
$$

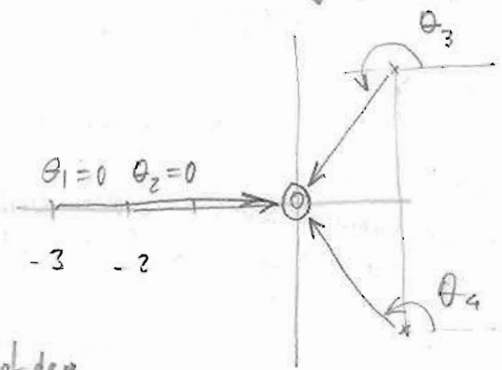
s^1 row → Equate to zero $a_2 = \frac{35}{3} \cong 11.7$

Substitute into auxiliary equation → $s_{1,2} = \pm j \frac{2\sqrt{6}}{3}$

Not that for $a_2 < \frac{35}{3}$ we have two roots in the RHP.

Rule 9: We seek the angle of arrival to the two zeros at $z=0$ and the angle of departure from the complex conjugate poles at $p_{3,4} = 1 \pm j\sqrt{3}$





(a)

$\theta_1 = 23.4°$
$\theta_2 = 30°$
$\theta_3 = 60°$
$\theta_4 = 90°$

angle of dep

$$2\theta_3 - \theta_1 - \theta_2 - \theta_4 - \theta_D = \pm 180(2\ell+1)$$ Angle of departure from poles.

$$\theta_D = 156.6°$$

(b) Angle of arrival to zeros.

$$2\theta_A - \underbrace{\cancel{\theta_1}}_{0} - \underbrace{\cancel{\theta_2}}_{0} - \underbrace{\theta_3 - \theta_4}_{360°} \rightarrow 2\theta_A = \pm 180(2\ell+1)$$

$$\boxed{\theta_A = \pm 90°}$$