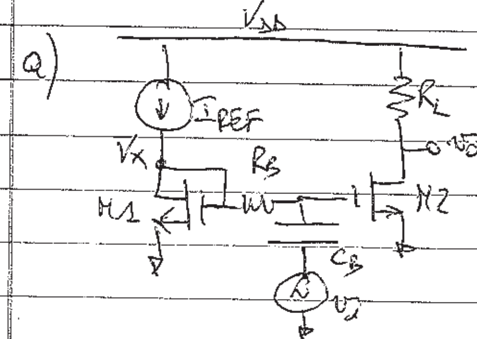
EE 140/EE 240A

HW5 Solutions

Problem 1

a)



$L = 120\,nm \quad V_{TH,0} = 0.5V \quad K_n' = 200\mu A/V^2 \quad \lambda = 0$

$V_{DD} = 1.5V \quad W_1 = 1.92\,\mu m \quad I_{REF} = 100\,\mu A$

$R_L = 1\,k\Omega$

? W_2 assuming $\lambda = 0$

$$A_v = \frac{v_o}{v_i} = g_m R_L = 5 \Rightarrow g_m = 5\,mS = \sqrt{2K_n'\left(\frac{W}{L}\right)_2 I_{D2}}$$

Call M the current mirror ratio $\left(\frac{W}{L}\right)_2 = M\left(\frac{W}{L}\right)_1$ & $I_{D2} = M I_{D1}$

$$\Rightarrow g_m = \sqrt{2K_n'\left(\frac{W}{L}\right)_1 I_{D1} M^2} = \sqrt{2\,200\mu A/V^2 \cdot \frac{1.92\mu m}{120\,nm} \cdot 100\mu A\, M^2}$$

$$\Rightarrow M = 6.25 \quad \left(\frac{W}{L}\right)_2 = \frac{12\mu m}{120\,nm} \quad I_{D2} = 625\,\mu A$$

b) Now $\lambda = 0.4V^{-1}$? I_{D2}

$$I_{D1} = \frac{1}{2}K_n'\left(\frac{W}{L}\right)_1 (V_X - V_{TH})^2(1 + \lambda V_X) = 100\mu A$$

$$= \frac{1}{2}K_n'\left(\frac{W}{L}\right)\left[\lambda V_X^3 + V_X^2(1 - 2\lambda V_{TH}) + V_X(\lambda V_{TH}^2 - 2V_{TH}) + V_{TH}^2\right] = 100\mu A$$

Can solve numerically to find $V_X = 0.72V$

$$I_{D2} = \frac{1}{2}K_n'\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH})^2(1 + \lambda V_{DS2}) \quad V_{GS2} = V_X \quad V_{DS2} = V_O$$

$$V_O = V_{DD} - I_{D2}R_L = V_{DD} - R_L\left[K_n'\left(\frac{W}{L}\right)_2 (V_X - V_{TH})^2(1 + \lambda V_O)\right]$$

Solving for $V_O \rightarrow V_O = 0.383V$

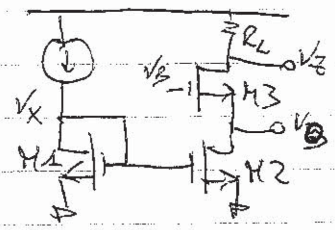
$\Rightarrow I_{D_2} = 670\,\mu A$

Also $r_{o2} = \dfrac{1}{\lambda I_{D2}} = 3.7\,K\Omega$ $\quad g_{m_2} = K_n'\left(\frac{W}{L}\right)_2 (V_X - V_{TH})(1+\lambda V_o) = 6.08\,mS$

$\Rightarrow \dfrac{v_o}{v_x} = g_{m_2}\left(r_{o2} \| R_L\right) = 4.8$

I_{D_2} and g_{m_2} are larger than from part a) because of the channel modulation effect. On the other hand, the finite output impedance of the output transistor degrades the overall output impedance, so the gain decreases.

c)



Set $I_{D_2} = 625\,\mu A$

$\Rightarrow V_X = V_O$ To have $V_{DS_1} = V_{DS_2}$

$\Rightarrow V_O = 0.72\,V$

$V_Z = V_{DD} - I_{D2} R_L = 0.875\,V$

Now compute V_B such that M3 can drive I_{D2}!

Assume M3 in saturation

$$I_{D_3} = \frac{1}{2} K_n'\left(\frac{W}{L}\right)_3 \left(V_{GS_3} - V_{TH}\right)^2 \left(1 + \lambda V_{DS_3}\right) = I_{D_2} = 625\,\mu A$$

$$\Rightarrow \frac{1}{2} K_n'\left(\frac{W}{L}\right)_3 \left(V_B - V_O - V_{TH}\right)^2 \left(1 + \lambda\left(V_Z - V_O\right)\right) = I_{D2}$$

$\Rightarrow V_B = 1.44\,V$

BUT $V_{GD_{M3}} = V_B - V_Z = 0.59 > 0.5\,V = V_{TH}$

$\Rightarrow$ assume triode operation

$$I_{D_3} = \frac{1}{2} K_n'\left(\frac{W}{L}\right)_3 \left[2\left(V_B - V_O - V_{TH}\right)\left(V_Z - V_O\right) - \left(V_Z - V_O\right)^2\right] = 625\,\mu A$$

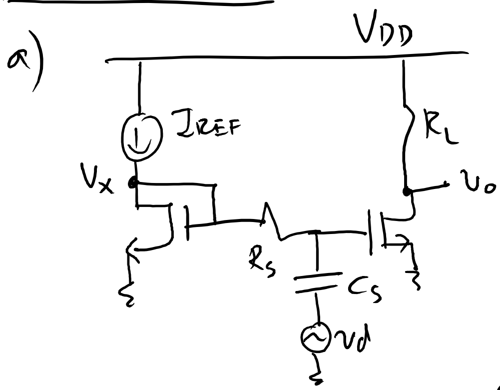
$\Rightarrow V_B = 1.465\,V$

$V_{GD_{M3}} = V_B - V_Z = 0.615\,V > V_{TH} \Rightarrow$ TRIODE!

HW5 Solutions - add-on

Problem 1

a)



$L = 120\,\text{nm}$, $V_{th_0} = 0.5\,\text{V}$, $k_n' = 200\,\mu\text{A}/\text{V}^2$, $\gamma = 0$

$V_{DD} = 1.5\,\text{V}$, $W_1 = 4 \times 1.92\,\mu\text{m}$, $I_{REF} = 100\,\mu\text{A}$

$R_L = 1\,\text{k}\Omega$ $\quad W_2 = ?$ assuming $\lambda = 0$

$$|A_v| = \left|\frac{v_o}{v_i}\right| = g_m R_L = 5 \Rightarrow g_m = 5\,\text{mS}$$

$$g_m = \sqrt{2 k_n' \left(\frac{W}{L}\right)_2 I_{D2}}$$

$$\left(\frac{W}{L}\right)_2 = M \left(\frac{W}{L}\right)_1 \Rightarrow I_{D2} = M I_{D1}$$

$$g_m = \sqrt{2 k_n' \cdot \left(\frac{W}{L}\right)_1 \cdot I_{D1} \cdot M^2} = M \cdot \sqrt{2 k_n' \left(\frac{W}{L}\right)_1 \cdot I_{D1}}$$

$$M = \frac{g_m}{\sqrt{2 k_n' \left(\frac{W}{L}\right)_1 \cdot I_{D1}}} = 3.125$$

$$\left(\frac{W}{L}\right)_2 = \frac{24\,\mu\text{m}}{120\,\text{nm}} \Rightarrow I_{D2} = 312.5\,\mu\text{A}$$

b) Now $\lambda = 0.4\,\text{V}^{-1}$

$$I_{D1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 (V_x - V_{th})^2 (1 + \lambda V_x) = 100\,\mu\text{A}$$

Can solve numerically for V_x. $\quad V_x = 0.612\,\text{V}$

$$I_{D2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_2 (V_x - V_{th})^2 (1 + \lambda V_o)$$

$$V_o = V_{DD} - I_{D2} R_L \qquad V_o = \frac{V_{DD} - I_{D2SAT} R_L}{(1 + I_{D2SAT} \cdot R_L \cdot \lambda)} = 1.13\,\text{V}$$

$$I_{D2} = 364.8\,\mu\text{A}$$

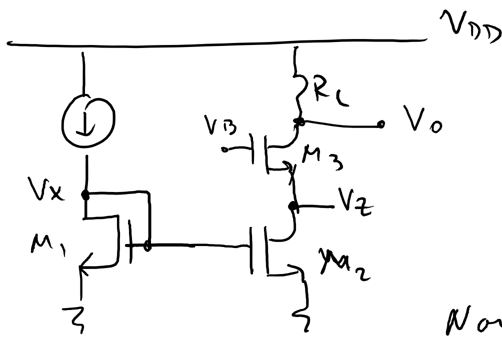
HW5 Solutions - add-on

$$r_{o2} = \frac{1}{\lambda I_{D2}} = 6.8\,k\Omega$$

$$g_{m2} = k_n'\left(\frac{W}{L}\right)_2 (V_X - V_{th})(1+\lambda V_O) = 6.5\,mS$$

$$\Rightarrow A_V = -g_{m2}\, R_L \| r_{o2} = -5.7$$

c)



$V_X = V_Z$ to have $V_{DS1} = V_{DS2}$

$V_Z = 0.612\,V$

$V_O = V_{DD} - I_{D2} R_L = 1.13\,V$

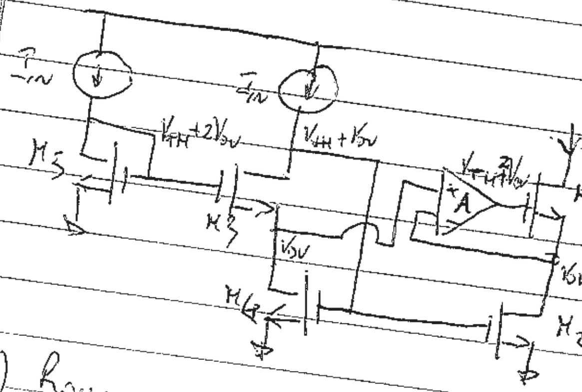
Now compute V_B such that M_3 can drive I_{D2}.

$$I_{D2} = \frac{1}{2} k_n' \cancel{\left(\frac{W}{L}\right)_3}\left(\frac{W}{L}\right)_2 \cdot (V_B - V_Z - V_{th})^2 (1+\lambda\cdot(V_O - V_Z))$$

$$\Rightarrow V_B = \sqrt{\frac{2 I_{D2}}{k_n'\left(\frac{W}{L}\right)_2 \cdot (1+\lambda(V_O - V_Z))}} + V_{th} + V_Z = 1.235\,V$$

$V_B - V_O = 0{,}105\,V < V_{th} \Rightarrow M_3$ in SAT

Problem 2

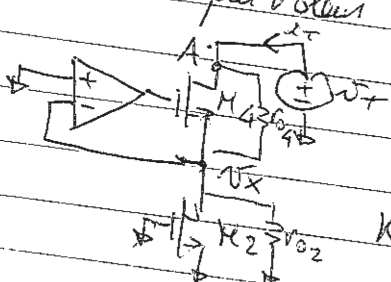


? R_{OUT}

? Size M_5 to minimize the systematic gain error at the output

a) R_{OUT}

To compute the output resistance, we disconnect all the input sources and apply a test source at the output. The simplified equivalent AC circuit becomes.



$$\text{KCL@A:}\ \begin{cases} v_X = i_T r_{o2} \\ i_T = \dfrac{v_T - v_X}{r_{o4}} + g_{m4}\left(-A v_X - v_X\right) \end{cases}$$

$$\Rightarrow r_{o4}\, i_T = v_T - i_T r_{o2} - r_{o4}\, g_{m4}(A+1)\, r_{o2}\, i_T$$

$$\Rightarrow R_{OUT} = \frac{v_T}{i_T} = r_{o4} + r_{o2} + g_{m4}(A+1)\, r_{o2} r_{o4} \approx g_{m4}(A+1)\, r_{o2} r_{o4}$$

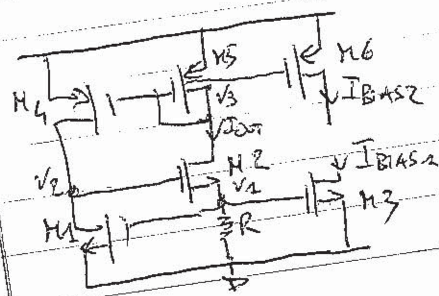
The super-cascode configuration boostes up the output impedance by $(A+1)$ with respect to the normal cascode.

b) If we size M5 with $\left(\frac{W}{L}\right)_{M5} = \frac{1}{4}\left(\frac{W}{L}\right)$ we get the DC voltages reported in the figure and NULL systematic error $\varepsilon = 0$.

The OPAMP will make sure that the voltages at the drain terminals of M4 & M2 are the same. Note that such a choice of $\left(\frac{W}{L}\right)_{M5}$ also minimizes the required $V_{OUT(MIN)}$

Problem 3

Ignore BODY EFFECT



Compute expressions for $I_{OUT}, I_{BIAS1}, I_{BIAS2}$ $(\lambda = 0)$

a) Compute expressions for

$$V_1 = V_{GS1} = V_{OV1} + V_{TH}$$
$$V_2 = V_{GS1} + V_{GS2} = V_{OV1} + V_{OV2} + V_{TH1} + V_{TH2}$$
$$V_3 = V_{DD} - V_{OV5} - |V_{TH5}|$$

By KCL: $I_{D1} = I_{D4}$, $I_{D2} = I_{D5} = I_{OUT}$

By KVL: $V_{OV4} = V_{OV5} = V_{OV6}$ $\quad V_{OV1} = V_{OV3}$

Let's define $K_i = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_i \Rightarrow I_{D_i} = K_i V_{OV_i}^2$

$$\left.\begin{aligned} \Rightarrow I_{D1} &= K_1 V_{OV1}^2 \\ I_{D4} &= K_4 V_{OV4}^2 = K_4 V_{OV5}^2 \\ I_{D5} &= I_{OUT} = K_5 V_{OV5}^2 \end{aligned}\right\} \Rightarrow I_{D4} = \frac{K_4}{K_5} I_{OUT}$$

$$\left.\begin{aligned} I_{D1} &= I_{D4} \\ I_{D4} &= \frac{K_4}{K_5} I_{OUT} \end{aligned}\right\} \Rightarrow \frac{K_4}{K_5} I_{OUT} = K_1 V_{OV1}^2$$

$$\Rightarrow V_{OV1} = \sqrt{\frac{K_4}{K_1 K_5} I_{OUT}}$$

$$I_{OUT} = \frac{V_{OV1} + V_{TH}}{R} = \frac{1}{R}\left(\sqrt{\frac{K_4}{K_1 K_5} I_{OUT}} + V_{TH}\right)$$

$$R I_{OUT} - \sqrt{\frac{K_4}{K_1 K_5}} \sqrt{I_{OUT}} - V_{TH} = 0 \Rightarrow \sqrt{I_{OUT}} = \frac{\sqrt{\frac{K_4}{K_1 K_5}} \pm \sqrt{\frac{K_4}{K_1 K_5} + 4 R V_{TH}}}{2R}$$

Only the solution with + has physical meaning:

$$I_{OUT} = \left(\frac{\sqrt{\frac{K_4}{K_1 K_5}} + \sqrt{\frac{K_4}{K_1 K_5} + 4RV_{TH}}}{2R} \right)^2$$

⇒ In first order, I_{OUT} doesn't depend on ~~Kco~~ V_{DD} but only on circuit parameters and process values (V_{TH})

$$\left.\begin{array}{l} I_{D1} = K_1 V_{OV_1}^2 \\ I_{D_3} = K_3 V_{OV_3}^2 \end{array}\right\} I_{D_3} = \frac{K_3}{K_1} I_{D_1} = \frac{K_3}{K_1} \frac{K_4}{K_5} I_{OUT}$$

$$I_{BIAS1} = \frac{K_3 K_4}{K_1 K_5} \left[\frac{\sqrt{\frac{K_4}{K_1 K_5}} + \sqrt{\frac{K_4}{K_1 K_5} + 4RV_{TH}}}{2R} \right]^2$$

Analogously:

$$I_{BIAS_2} = \frac{K_6}{K_5} \left[\frac{\sqrt{\frac{K_4}{K_1 K_5}} + \sqrt{\frac{K_4}{K_1 K_5} + 4RV_{TH}}}{2R} \right]^2$$

Now, plugging in numbers

$$K_1 = \frac{1}{2} \cdot 200 \frac{\mu A}{V^2} \cdot 50 = 5 \frac{mA}{V^2} \qquad K_2 = \frac{1}{2} \cdot 200 \frac{\mu A}{V^2} \cdot 25 = 2.5 \frac{mA}{V^2}$$

$$K_3 = \frac{1}{2} \cdot 200 \frac{\mu A}{V^2} \cdot 125 = 12.5 \frac{mA}{V^2} \qquad K_4 = \frac{1}{2} \cdot 100 \frac{\mu A}{V^2} \cdot 25 = 1.25 \frac{mA}{V^2}$$

$$K_5 = \frac{1}{2} \cdot 100 \frac{\mu A}{V^2} \cdot 50 = 2.5 \frac{mA}{V^2} \qquad K_6 = \frac{1}{2} \cdot 100 \frac{\mu A}{V^2} \cdot 62 = 3.1 \frac{mA}{V^2}$$

$$I_{OUT} = \frac{\sqrt{\frac{1.25}{5 \cdot 2.5 \cdot 10^{-3}}} + \sqrt{\frac{1.25}{5 \cdot 2.5 \cdot 10^{-3}} + 4 \cdot 1.75\,k\Omega \cdot 0.5\,V}}{2 \cdot 1.75\,k\Omega} = 0.4\,mA$$

$$I_{D_2} = I_{D5} = I_{OUT} = 0.4\,mA \Rightarrow V_{OV_2} = 0.4\,V \quad V_{OV5} = 0.4\,V$$

$$I_{D_1} = I_{D_4} = \frac{K_4}{K_5} I_{OUT} = 0.2\,mA \rightarrow V_{OV_1} = 0.2\,V \quad V_{OV_4} = 0.4\,V$$

$$I_{BIAS1} = \frac{K_3}{K_1} \frac{K_4}{K_5} I_{OUT} = \frac{12.5}{5} \frac{1.25}{2.5} \cdot 0.4\,mA = 0.5\,mA$$

$$I_{BIAS_2} = \frac{K_6}{K_5} I_{OUT} = 0.496\,mA$$

$V_1 = V_{ov_1} + V_T = 0.7V$ $\quad V_2 = V_{ov_1} + V_{ov_2} + 2V_T = 1.6\,V$

$V_3 = V_{DD} - V_{ov_5} - V_T = 2.1\,V$

$\Rightarrow$ all transistors are in saturation

$g_m = 2KV_{ov}$

$\Rightarrow g_{m_1} = 2mS \quad g_{m_2} = 2mS \quad g_{m_3} = 5mS \quad g_{m_4} = 1mS$

$g_{m_5} = 2mS \quad g_{m_6} = 2.48mS$

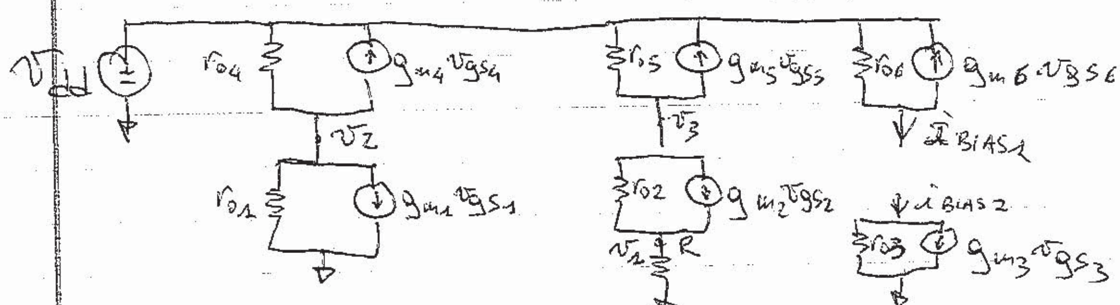
b) We assume the DC values don't change also when we set $\lambda = 0.05V^{-1}$, but now transistors have a small signal output impedance.

$r_{o_i} = \frac{1}{\lambda I_{D_i}}$

$r_{o_1} = 100K\Omega \quad r_{o_2} = 50K\Omega \quad r_{o_3} = 40K\Omega \quad r_{o_4} = 100K\Omega$

$r_{o_5} = 50K\Omega \quad r_{o_6} = 40.3K\Omega$

We use the following small signal model:



$v_{gs_1} = v_{gs_3} = v_1 \qquad v_{gs_2} = v_2 - v_1$

$v_{GS_4} = v_{GS_5} = v_{GS_6} = v_3 - v_{dd}$

$$\begin{aligned}
&\text{KCL @ 2} \quad \frac{v_2}{r_{o1}} + g_{m1} v_{gs1} + \frac{v_2 - v_{dd}}{r_{o4}} + g_{m4} v_{gs4} = 0 \\
&\text{KCL @ 3} \quad \frac{v_3 - v_1}{r_{o2}} + g_{m2} v_{gs2} + \frac{v_3 - v_{dd}}{r_{o5}} + g_{m5} v_{gs5} = 0 \\
&\text{KCL @ 1} \quad \frac{v_1}{R} + \frac{v_1 - v_3}{r_{o2}} - g_{m2} v_{gs2} = 0
\end{aligned}$$

$$\begin{pmatrix} g_{m1} & \frac{1}{r_{o1}} + \frac{1}{r_{o4}} & g_{m4} \\ -g_{m2} - \frac{1}{r_{o2}} & g_{m2} & g_{m5} + \frac{1}{r_{o2}} + \frac{1}{r_{o5}} \\ \frac{1}{R} + \frac{1}{r_{o2}} + g_{m2} & -g_{m2} & -\frac{1}{r_{o2}} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} g_{m4} + \frac{1}{r_{o4}} \\ g_{m5} + \frac{1}{r_{o5}} \\ 0 \end{pmatrix} v_{dd}$$

$$\begin{pmatrix} 2 & 0.02 & 1 \\ -2.02 & 2 & 2.04 \\ 2.591 & -2 & -0.02 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1.01 \\ 2.02 \\ 0 \end{pmatrix} v_{dd}$$

$$\Rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 5.851 \\ -2.4 \\ 998.3 \end{pmatrix} \cdot 10^{-3} v_{dd}$$

$$\Rightarrow i_{OUT} = \frac{v_1}{R} = \frac{5.851 \cdot 10^{-3}}{1.75\,k\Omega} v_{dd} \Rightarrow \frac{i_{OUT}}{v_{dd}} = 3.343 \cdot 10^{-3}\,mS$$

$\Rightarrow$ You see that for even 100 mV change in V_{DD}, I_{OUT} just changes for $\Delta i_{OUT} = 0.3\,\mu A$

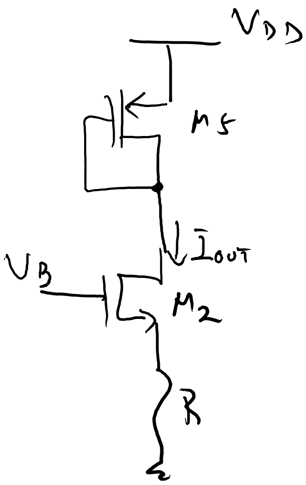
$$i_{BIAS1} = g_{m3} v_1 \Rightarrow \frac{i_{BIAS1}}{v_{dd}} = 5\,mS \cdot 5.851 \cdot 10^{-3} = 29.255\,10^{-3}\,mS$$

$$i_{BIAS2} = -g_{m6}(v_3 - v_{dd}) = 2.48\,mS \cdot 1.654\,10^{-3} v_{dd}$$

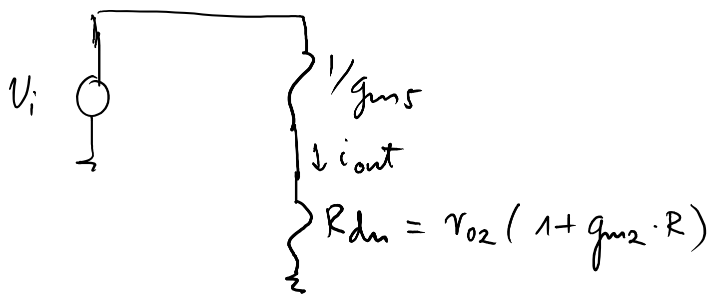
$$\Rightarrow \frac{i_{BIAS2}}{v_{dd}} = 4.1 \cdot 10^{-3}\,mS$$

HW5 Solutions - add-on

Problem 3 - simplification for (3b)



small-signal $\Longrightarrow$



$$\frac{i_{out}}{v_i} = \frac{1}{\frac{1}{g_{m5}} + r_{o2}(1 + g_{m2}R)} = 4.4\,\mu S$$

$g_{m5} = 2\,mS$

$r_{o2} = 50\,k\Omega$

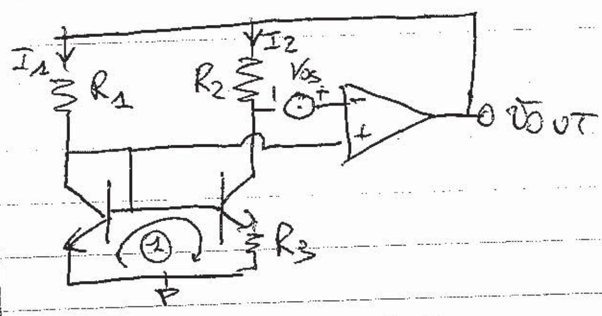
$R = 1.75\,k\Omega$

$g_{m3} = 5\,mS$

$$\frac{i_{bias2}}{v_i} = \frac{\left(\frac{W}{L}\right)_6}{\left(\frac{W}{L}\right)_5} \cdot \frac{i_{out}}{v_i} = 5.45\,\mu S$$

$$\frac{i_{bias1}}{v_i} = \frac{R}{\frac{1}{g_{m5}} + R_{dn}} \cdot g_{m3} = 38.8\,\mu S$$

Problem 4



$R_1 = R_3 = 1K\Omega$
$I_{S_1} = 10^{-15}A \quad I_{S_2} = 10^{-15} \cdot 8\,A$

$\beta_F \to \infty \quad V_A \to \infty$

a) First, we compute the expression for V_{OUT}

KVL @ ①: $V_{BE1} = V_{BE_2} + I_2 R_3 \quad (*)$

$$I_1 = \frac{V_{OUT} - V_{BE1}}{R_1} \qquad I_2 = \frac{V_{OUT} - V_{BE1} + V_{OS}}{R_2}$$

$\Rightarrow$ (*) becomes:

$$V_T \ln\left(\frac{I_1}{I_2} \cdot \frac{I_{S2}}{I_{S1}}\right) = \frac{R_3}{R_2}\left(V_{OUT} - V_{BE1} + V_{OS}\right)$$

$$V_T \ln\left[8\,\frac{R_2}{R_1} \cdot \frac{V_0 - V_{BE1}}{V_0 - V_{BE1} + V_{OS}}\right] = \left(\frac{R_3}{R_2}\right)\left(V_{OUT} - V_{BE} + V_{OS}\right)$$

$$\Rightarrow V_0 = V_{BE1} + \frac{R_2}{R_3} V_T \ln\left[8\,\frac{R_2}{R_1}\right] - V_{OS} + \underbrace{\frac{R_2}{R_3} V_T \ln\left[\frac{V_0 - V_{BE1}}{V_0 - V_{BE1} + V_{OS}}\right]}$$

Now we use the approximation $\ln(1+x) \approx x \quad x \to 0$

$$-\frac{R_2}{R_3} V_T \ln\left[\frac{V_0 - V_{BE1} + V_{OS}}{V_0 - V_{BE1}}\right] \approx -\frac{R_2}{R_3} V_T \frac{V_{OS}}{V_0 - V_{BE1}}$$

$$(**) \Rightarrow V_0 = V_{BE1} + \frac{R_2}{R_3} V_T \ln\left[8\,\frac{R_2}{R_1}\right] - \underbrace{\left(V_{OS} + \frac{R_2}{R_3} V_T \frac{V_{OS}}{V_0 - V_{BE1}}\right)}_{\varepsilon = \text{ERROR TERM}}$$

For $V_{OS}=0$, we set $\frac{\partial V_{OUT}}{\partial T}=0$

$$\frac{\partial V_{OUT}}{\partial T} = \underbrace{\frac{\partial V_{BE}}{\partial T}}_{-2\frac{mV}{K}} + \underbrace{\frac{R_2}{R_3}\ln\left(8\frac{R_2}{R_1}\right)}_{M=GAIN=23} \underbrace{\frac{\partial V_T}{\partial T}}_{+0.086\frac{mV}{K}} \qquad (***)$$

Plugging in the numerical values, we get:

$$R_2 \ln\left(8 R_2/R_1\right) = 23$$

$$\Rightarrow R_2 \simeq 6\,K\Omega \quad \text{(solving numerically)}$$

We can now compute the target voltage:

$$V_{OUT}^{TARGET} = V_{BE1} + \frac{R_2}{R_3} V_T \ln\left[8\frac{R_2}{R_1}\right] = 0.7V + 23\cdot 26mV = 1.298V$$

Unfortunately, at trimming time the offset will create an error in the calibration of R_2.

We make the following approximation to simplify the expression for ε

$$\varepsilon = V_{OS}\left(1+\frac{R_2}{R_3}\frac{V_T}{V_0 - V_{BE1}}\right) \simeq V_{OS}\left(1+\frac{\cancel{R_2}}{\cancel{R_3}}\frac{\cancel{V_T}}{\frac{\cancel{R_2}}{\cancel{R_3}}\cancel{V_T}\ln\left(8\frac{R_2}{R_1}\right)}\right)$$

where we used $V_0 - V_{BE1} \simeq \frac{R_2}{R_3} V_T \ln\left(8\frac{R_2}{R_1}\right)$ from $(**)$

$$\Rightarrow \varepsilon = V_{OS}\left(1+\frac{1}{\ln(48)}\right)$$

At trimming time, the gain M in (***) will be set by:

$$V_{OUT} = V_{BE1} + M V_T - \varepsilon = V_{OUT}^{TARGET}$$

$$0.7V - \varepsilon + M \cdot 26mV = 1.298$$

$$\Rightarrow M = \frac{V_{OUT}^{TARGET} - V_{BE1} + \varepsilon}{V_T} \rightarrow \text{error Term} \qquad (****)$$

The gain error will translate in a residual Temperature coefficient

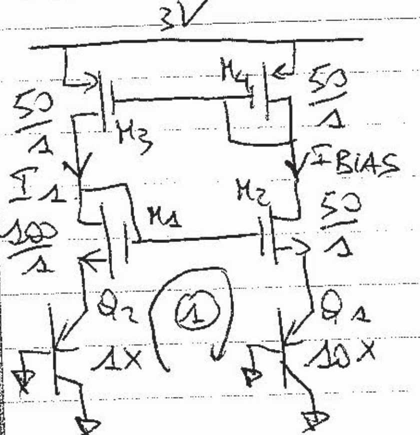
$$\frac{\partial V_{OUT}}{\partial T} = \frac{\varepsilon}{V_T} \cdot \frac{\partial V_T}{\partial T} = \frac{\varepsilon}{V_T} \cdot \frac{K}{q} \cdot \frac{T}{T} = \frac{\varepsilon}{T}$$

$$= \frac{30mV\left(1 + \frac{1}{\ln(48)}\right)}{300K} = 125 \frac{\mu V}{K}$$

$$TC_F = \frac{1}{V_{OUT}} \cdot \frac{\partial V_{OUT}}{\partial T} = 96 \frac{ppm}{K}$$

b) If $\varepsilon > 0$ (****) shows that the trimming will overcompensate the gain M. Since this gain is applied to the term in (***) with positive temperature coefficient ΔV_{BE}, the TC_F will be positive.
On the other hand, if $\varepsilon < 0$ the trimming will under compensate M and TC_F will be negative.

Problem 5



$|V_{TH}| = 0.5V$

$K_n' = 200\,\mu A/V^2$

$K_p' = 200\,\mu A/V^2$

a) ? I_{BIAS}

KVL @ (1): $V_{EB_2} + V_{GS_1} - V_{GS_2} - V_{EB_1} = 0$

$$\Rightarrow V_{GS_1} - V_{GS_2} = V_T \ln\left(\frac{I_{BIAS}}{10 I_{S2}} \cdot \frac{I_{S2}}{I_{BIAS}}\right) = -V_T \ln(10) \quad (*)$$

where we assumed $I_{BIAS} = I_1$ (enforced by current mirror)

$I_{BIAS} = I_1$

$$\Rightarrow \cancel{\tfrac{1}{2}}\cancel{K_n}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH})^2 = \cancel{\tfrac{1}{2}}\cancel{K_n}\left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{TH})^2$$

$$\sqrt{2}\,(V_{GS_1} - V_{TH}) = V_{GS_2} - V_{TH}$$

Combining with (*)

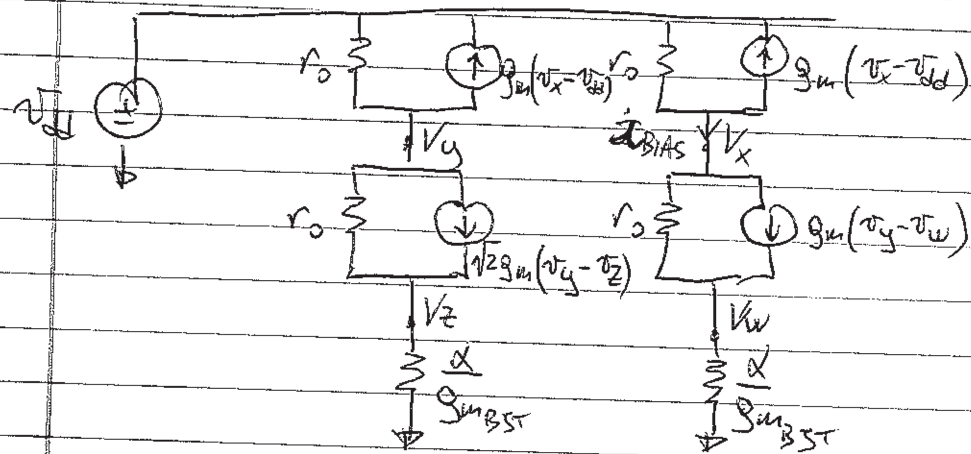
$$V_{GS_1} = \frac{V_T \ln(10)}{\sqrt{2} - 1} + V_{TH}$$

$$\Rightarrow I_{BIAS} = \frac{1}{2} K_n \left(\frac{W}{L}\right)_2 \left[\frac{\sqrt{2}\, V_T \ln(10)}{\sqrt{2} - 1} + \cancel{V_{TH}} - \cancel{V_{TH}}\right]^2 = 209\,\mu A$$

$$I_{BIAS} = K_n \left(\frac{W}{L}\right)_2 V_T^2 \frac{\ln^2(10)}{(\sqrt{2} - 1)^2}$$

b) $$\frac{\partial I_{BIAS}}{\partial T} = 2K_n\left(\frac{W}{L}\right)_2 \frac{K^2}{q^2}\,\frac{\ln^2(10)}{(\sqrt{2}-1)^2}\cdot T$$

c) We consider the following small signal circuit:



The r_o of all MOS is equal because they all draw the same current.

$g_{m_1} = \sqrt{2}\, g_{m_{2-4}} = \sqrt{2} g_m$

Also $g_{m_{Q_1}} = g_{m_{Q_2}}$
$= g_{m_{BJT}}$

We write KCL at the four nodes marked in the ~~...~~ circuit. Our goal is to compute $i_{BIAS} = f(v_{dd})$

KCL @ y: $$\frac{v_{dd}-v_y}{r_o} - g_m(v_x - v_{dd}) - \frac{v_y - v_z}{r_o} - \sqrt{2}g_m(v_y - v_z) = 0$$

KCL @ z: $$\frac{v_y - v_z}{r_o} + \sqrt{2}g_m(v_y - v_z) - \frac{g_{m_{BJT}}}{\alpha}v_z = 0$$

KCL @ x: $$\frac{v_{dd}-v_x}{r_o} - g_m(v_x - v_{dd}) - \frac{v_x - v_w}{r_o} - g_m(v_y - v_w) = 0$$

KCL @ w: $$\frac{v_x - v_w}{r_o} + g_m(v_y - v_w) - \frac{g_{m_{BJT}}}{\alpha}v_w = 0$$

Now put in matrix format:

$$\begin{pmatrix} g_m & \frac{2}{r_o}+\sqrt{2}g_m & -\frac{1}{r_o}-\sqrt{2}g_m & 0 \\ 0 & -\frac{1}{r_o}-\sqrt{2}g_m & \sqrt{2}g_m+\frac{1}{r_o}+\frac{g_{mBJT}}{\alpha} & 0 \\ \frac{2}{r_o}+g_m & g_m & 0 & -\frac{1}{r_o}-g_m \\ -\frac{1}{r_o} & -g_m & 0 & \frac{1}{r_o}+\frac{g_{mBJT}}{\alpha}+g_m \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ v_w \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{r_o}+g_m \\ 0 \\ \frac{1}{r_o}+g_m \\ 0 \end{pmatrix} v_{dd}$$

$$r_o = \frac{1}{\lambda I_D} = 95\,K\Omega \qquad g_m = \sqrt{2K_n'\frac{W}{L}I_D} = 2\,mS$$

$$g_{mBJT} = \frac{I_D}{V_T} = 8\,mS$$

$$\begin{pmatrix} 2m & 2.84m & -2.83m & 0 \\ 0 & -2.83m & 10.83m & 0 \\ 2.02m & 2m & 0 & -2.01m \\ 0.01m & -2m & 0 & 10.01m \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ v_w \end{pmatrix} = \begin{pmatrix} 2.01m \\ 0 \\ 2.01m \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \\ v_W \end{pmatrix} = \begin{pmatrix} 0.96 \\ 0.042 \\ 0.01 \\ 0.0074 \end{pmatrix} v_{dd}$$

$$i_{BIAS} = \frac{g_{mBJT} v_w}{\alpha} = 59.2\cdot 10^{-6}\, v_{dd} \qquad \frac{i_{BIAS}}{v_{dd}} = 59.2\cdot 10^{-6}\, S$$

As expected the ~~gain~~ sensitivity is POSITIVE.

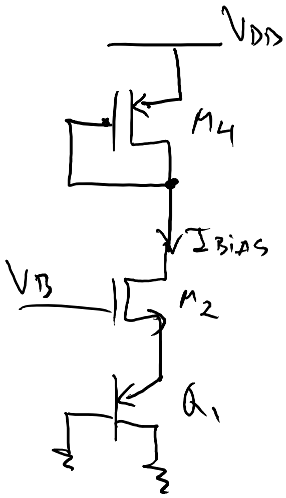
HW5 Solutions - add-on

$\boxed{\text{Problem 5}}$ - simplification for (5c)

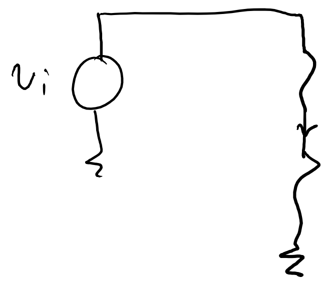
$r_{o2} = 95\,k\Omega$

$g_{m2} = 2\,mS$

$g_{m\,BJT} = 8\,mS$



$\Rightarrow$



$$R_{dn} = r_{o2}\left(1 + g_{m2} \cdot \frac{\alpha_F}{g_{m\,BJT}}\right)$$

$$\frac{i_{bias}}{v_i} = \frac{1}{\frac{1}{g_{m4}} + R_{dn}} = 6.4\,\mu S$$