MIDDLE EAST TECHNICAL UNIVERSITY
Department of Electrical and Electronics Engineering

EE301 SIGNALS and SYSTEMS 1

HOMEWORK 3

Due: 18/11/2018, 23:55

Q1) One period of rectangular pulse train $x(t)$ with period 4 is $g(t) = \begin{cases} 1, & -1 \le t < 1 \\ 0, & 1 \le t < 3 \end{cases}$, where the pulse train is expressed as $x(t) = \sum_{m=-\infty}^{\infty} g(t+4m)$. A practical parameter called the "duty ratio" of the pulse train is defined as $d = \frac{\text{Duration of the Nonzero Values of the Rectangular Pulse Train in a Period}}{\text{Period of the Rectangular Pulse Train}}$.

(Notice that $d = 1/2$ for the given $x(t)$; and $g(t)$ could be expressed as $g(t) = rect(t/2) = \begin{cases} 1, & |t| \le 1 \\ 0, & else \end{cases}$.)

- **a)** Calling the FS coefficients of $x(t)$, X_k, state X_k's in terms of the duty ratio d (& then substitute d).
- **b)** Another periodic signal is defined as $z(t) = \int_0^4 x(\tau)x(t-\tau)d\tau$. Sketch $z(t)$. Calling its FS coefficients Z_k, use the "convolution in time" property of the CTFS representation to express Z_k in terms of X_k.
- **c)** Define $y(t) = \frac{d}{dt}z(t)$ and express it in terms of $x(t)$.
- **d)** Find the FS coefficients Y_k of $y(t)$ using your answer to part (c).
- **e)** Compute Z_k from Y_k using the differentiation (or integration) property.
- **f)** Compare Z_k's found in part (b) with Z_k's of part (e). Are they equal? If so, show it.

Q2) a) Compute $f[k] = \sum_{n=0}^{N-1} e^{j(2\pi/N)kn}$ for all integer values of k.

b) Repeat part (a) if $f[k] = \sum_{n=M}^{M+N-1} e^{j(2\pi/N)kn}$, where M is an integer.

Q3) a) An LTI system has the impulse response $h(t) = e^{-t}u[t]$. Is this system causal? Is it stable? Find its output $y(t)$ corresponding to the input $x(t) = (j)^t$ by using the concept of eigenfunctions.

b) An LTI system has the impulse response $h[n] = 2^{-n}u[n+1]$. Is this system causal? Is it stable? Find its output $y[n]$ corresponding to the input $x[n] = (j)^n$ by using the concept of eigenfunctions.

Q4) Determine whether the following CT signals in part (a) and DT signals in part (b) are periodic. If they are, find their fundamental period and compute the corresponding FS coefficients.

a) $i.\ \sin(2t) + \cos(3t)$, $ii.\ \sin\left(\frac{\pi}{2}t\right) + \cos\left(\frac{\pi}{3}t\right)$, $iii.\ \sin(2t) + \cos(\frac{\pi}{3}t)$.

b) $i.\ \sin(2n) + \cos(3n)$, $ii.\ \sin\left(\frac{\pi}{2}n\right) + \cos\left(\frac{\pi}{3}n\right)$, $iii.\ \sin(2n) + \cos(\frac{\pi}{3}n)$.

Q5) A rectangular pulse train can be expressed as $x[n] = \sum_{m=-\infty}^{\infty} A\, rect[(n+mN)/(2N_1+1)]$, where the pulse duration is less than the period; i.e., $2N_1 + 1 < N$ and $rect[n/(2N_1+1)] = \begin{cases} 1, & |n| \leq N_1 \\ 0, & else \end{cases}$.

Notice that the "duty ratio" of the pulse train is $d = (2N_1+1)/N$.

- **a)** Compute the Fourier Series (FS) coefficients a_k of $x[n]$ in terms of A, N and d.
- **b)** Notice that $x[n]$ is a real-valued and even sequence. Derive the condition that these two (i.e.; being real-valued and even) imposes on the DTFS coefficients a_k.
- **c)** Letting the duty ratio $d = 1/2$, find c and n_0 such that the shifted signal, $y[n] = x[n-n_0] + c$, has purely imaginary FS coefficients b_k. Also determine b_k's from a_k's by making use of the DTFS properties.

Q6) A periodic sequence $x[n]$ is defined as $x[n] = \sum_{m=-\infty}^{\infty} g[n+Nm]$, where $g[n] = \begin{cases} 1, & |n| \leq 2 \\ 0, & else \end{cases}$, and $N > 5$. Write a MATLAB code that computes the DTFS coefficients a_k of $x[n]$,

i. directly from the DTFS analysis equation; *ii.* using the result found in part (a) of Q5.

- **a)** Display your MATLAB codes together with the plots of
  - *i.* $x[n]$ for $N = 10$ and $N = 20$ (versus n for $n \in [-20, 20]$).
  - *ii.* DTFS coefficients a_k of $x[n]$ (versus $k \in [-20, 20]$) for $N = 10$ and $N = 20$.
- **b)** Use the FFT (Fast Fourier Transform) command of MATLAB to find the FFT of the single period of $x[n]$, where $n \in [0,\ N-1]$, for $N = 10$ and $N = 20$. Then compare these FFT's with a_k's where $k \in [0,\ N-1]$; for $N = 10$ and $N = 20$.