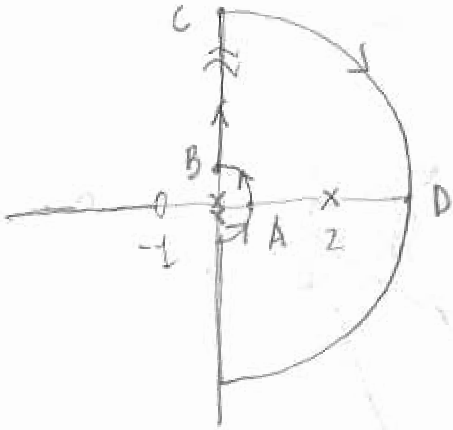
(Spring 2016 Extra)

Example: Double poles at the origin (slightly more tricky)

$$KG(s)H(s) = \frac{K(s+1)}{s^2(s-2)} \quad \text{Sketch the Nyquist Plot.}$$



$A \to B$ we have $s = \varepsilon e^{j\theta}$ $\theta : 0 \to \pi/2$

$$G(s)H(s)\Big|_{s=\varepsilon e^{j\theta}} = \frac{(\varepsilon e^{j\theta}+1)}{\varepsilon^2 e^{j2\theta}\underbrace{(\varepsilon e^{j\theta}-2)}_{\varepsilon \text{ very small.}}}$$

$$= \frac{1}{\varepsilon^2 e^{j2\theta}(-2)} = \frac{1}{2\varepsilon^2 e^{j2\theta} e^{j\pi}} = \frac{1}{2\varepsilon^2 e^{j(2\theta+\pi)}}$$

Careful: we have a phase contribution of π from this negative number

$$= \frac{1}{2\varepsilon^2} e^{-j(2\theta+\pi)} = Re^{j\phi} \text{ with } R \to \infty$$

and $\phi = -(\pi + 2\theta)$

as $\theta : 0 \to \pi/2$ we have $\phi : -\pi \to -2\pi$

Now $B \to C$ $s = j\omega$ $\omega : 0 \to \infty$

$$G(s)H(s)\Big|_{s=j\omega} = \frac{j\omega+1}{\underbrace{(j\omega)^2}_{-\omega^2}(j\omega-2)} = \frac{j\omega+1}{-\omega^2(-2+j\omega)} = \frac{\overbrace{(1+j\omega)(-2-j\omega)}^{-2-j\omega-2j\omega+\omega^2}}{-\omega^2(4+\omega^2)}$$

$$= \frac{-(\omega^2-2)+j3\omega}{\omega^2(\omega^2+4)} = \frac{2-\omega^2}{\omega^2(\omega^2+4)} + j\frac{3\omega}{\omega^2(\omega^2+4)}$$

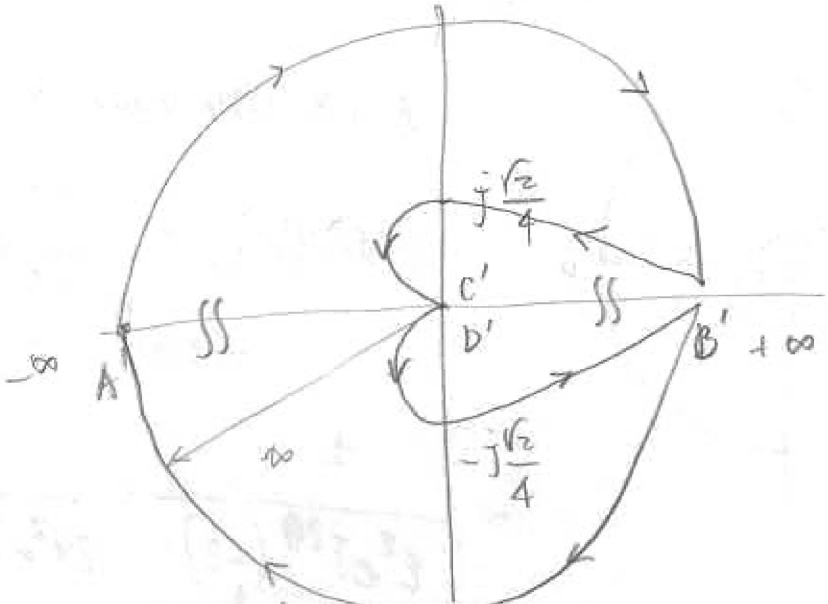
w	0	$\sqrt{2}$		$w \to \infty$
Re(GH)	$+\infty$	0	$-$	0
Im(GH)	$+\infty$	$+$	$+$	0

when $w = \sqrt{2}$ Re $= 0$

$$\text{Im} = j\frac{3\sqrt{2}}{2(6)} = j\frac{\sqrt{2}}{4}$$

$C \to D$ we have $s = Re^{j\theta}$ $R \to \infty$; $\theta : \pi/2 \to 0$

$$GH\Big|_{s=Re^{j\theta}} \longrightarrow 0$$



So consider $-\frac{1}{K}$ point around this sketch :

$-\infty < -\frac{1}{K} < 0 \longrightarrow 0 < K < \infty$ we have $N = 1$

$P = 1$

$$N = P_{cl} - P$$

$$1 = P_{cl} - 1 \longrightarrow \boxed{P_{cl} = 2} \text{ unstable poles.}$$

unstable.

If $0 < -\frac{1}{K} < +\infty \longrightarrow -\infty < K < 0 \longrightarrow \boxed{P_{CL} = 1}$

What about Root-Locus : System is always unstable.