MIDDLE EAST TECHNICAL UNIVERSITY

MECHANICAL ENGINEERING DEPARTMENT

ME 205 STATICS – FALL 2018

SECTION 1

HOMEWORK #1

Submit the solution of the first problem to D-106.

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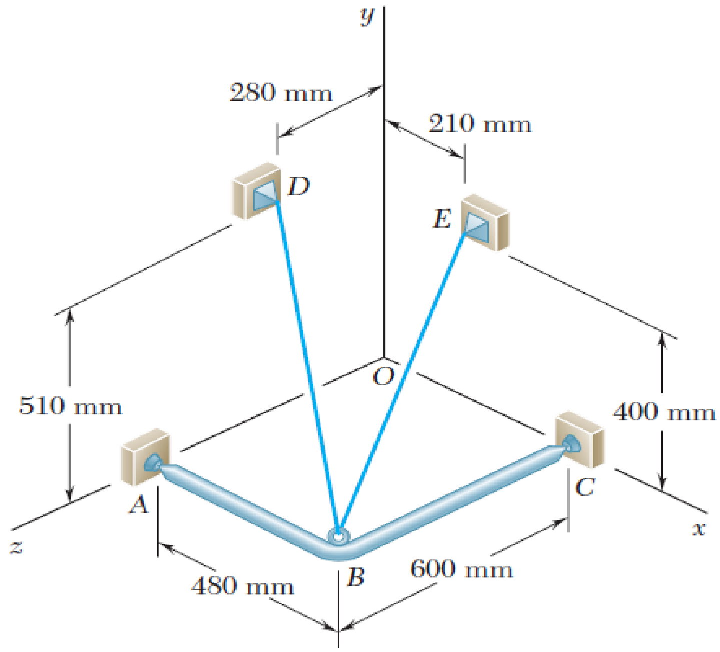
Assigned Date: 19.10.2018
Due Date: 26.10.2018
Due Time: 16.00
Grading Due Date: 09.11.2018

Please include your name, student ID, due date, a proper headline, page number with total page number, and units in your homework. Neatness will be graded.

1. The cable *DBE* that passes through a frictionless ring at *B* supports the frame *ABC.* If the tension in the cable is 300 N, determine
   - **a.** The unit vectors along the lines *BD* and *BE,*
   - **b.** The resultant force at *B* due to the tension forces,
   - **c.** The component of the tension forces along the line *OB*,
   - **d.** The angle between the tension forces.



Solution:

a. The position vector along *BD* can bex expressed as

$$\overrightarrow{BD} = -480\boldsymbol{i} + 510\boldsymbol{j} - (600 - 280)\boldsymbol{k}\ mm$$
$$\overrightarrow{BD} = -480\boldsymbol{i} + 510\boldsymbol{j} - 320\boldsymbol{k}\ mm$$

Likewise, the position vector along *BE* is

$$\overrightarrow{BE} = -(480 - 210)\boldsymbol{i} + 400\boldsymbol{j} - 600\boldsymbol{k}\ mm$$
$$\overrightarrow{BE} = -270\boldsymbol{i} + 400\boldsymbol{j} - 600\boldsymbol{k}\ mm$$

To obtain the unit vectors, we need to divide vector to its magnitude. Then,

$$\vec{u}_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{-480\boldsymbol{i} + 510\boldsymbol{j} - 320\boldsymbol{k}}{\sqrt{(-480)^2 + 510^2 + (-320)^2}} = \frac{-480\boldsymbol{i} + 510\boldsymbol{j} - 320\boldsymbol{k}}{770}$$
$$\vec{u}_{BD} = -0.623\boldsymbol{i} + 0.662\boldsymbol{j} - 0.416\boldsymbol{k}$$

$$\vec{u}_{BE} = \frac{\overrightarrow{BE}}{BE} = \frac{-270\boldsymbol{i} + 400\boldsymbol{j} - 600\boldsymbol{k}}{\sqrt{(-270)^2 + 400^2 + (-600)^2}} = \frac{-270\boldsymbol{i} + 400\boldsymbol{j} - 600\boldsymbol{k}}{770}$$
$$\vec{u}_{BE} = -0.351\boldsymbol{i} + 0.52\boldsymbol{j} - 0.78\boldsymbol{k}$$

b. Since we obtained the unit forces along the cables, we can express the tension forces as

$$\vec{F}_{BD} = F_{BD}\vec{u}_{BD} \quad \& \quad \vec{F}_{BE} = F_{BE}\vec{u}_{BE}$$
$$\vec{F}_{BD} = 300(-0.623\boldsymbol{i} + 0.662\boldsymbol{j} - 0.416\boldsymbol{k})\ N$$
$$\vec{F}_{BE} = 300(-0.351\boldsymbol{i} + 0.52\boldsymbol{j} - 0.78\boldsymbol{k})\ N$$

Then, the resultant force at *B* due to tension forces is

$$\vec{F}_R = \vec{F}_{BD} + \vec{F}_{BE} = -292.2\boldsymbol{i} + 354.54\boldsymbol{j} - 358.44\boldsymbol{k}\ N$$

c. We can obtain the component of the tension forces along the line *OB* by using the unit vector along *OB.*

$$\vec{u}_{OB} = \frac{\overrightarrow{OB}}{OB} = \frac{480\boldsymbol{i} + 600\boldsymbol{k}}{\sqrt{480^2 + 600^2}} = 0.625\boldsymbol{i} + 0.781\boldsymbol{k}$$

Then, the components of the tension forces can be found by using

$$\vec{F}_{BD} \cdot \vec{u}_{OB} \quad \& \quad \vec{F}_{BE} \cdot \vec{u}_{OB}$$

$$\vec{F}_{BD} \cdot \vec{u}_{OB} = 300(-0.623\boldsymbol{i} + 0.662\boldsymbol{j} - 0.416\boldsymbol{k}) \cdot (0.625\boldsymbol{i} + 0.781\boldsymbol{k})$$
$$\left(\vec{F}_{BD}\right)_{OB} = -214.2813\ N\ (in\ the\ opposite\ dir.of\ OB)$$

$$\vec{F}_{BE} \cdot \vec{u}_{OB} = 300(-0.351\boldsymbol{i} + 0.52\boldsymbol{j} - 0.78\boldsymbol{k}) \cdot (0.625\boldsymbol{i} + 0.781\boldsymbol{k})$$
$$\left(\vec{F}_{BE}\right)_{OB} = -248.5665\ N\ (in\ the\ opposite\ dir.of\ OB)$$

d. We can obtain the angle between the tension forces by using the properties of dot product.

$$\vec{F}_{BD} \cdot \vec{F}_{BE} = F_{BD} \cdot F_{BE} \cdot \cos\alpha$$

where α is the angle between the forces.

Then,

$$300(-0.623\boldsymbol{i} + 0.662\boldsymbol{j} - 0.416\boldsymbol{k}) \cdot 300(-0.351\boldsymbol{i} + 0.52\boldsymbol{j} - 0.78\boldsymbol{k}) = 300 \cdot 300 \cdot \cos\alpha$$
$$\cos\alpha = 0.8874$$
$$\alpha = \cos^{-1} 0.8874 = 27.45°$$

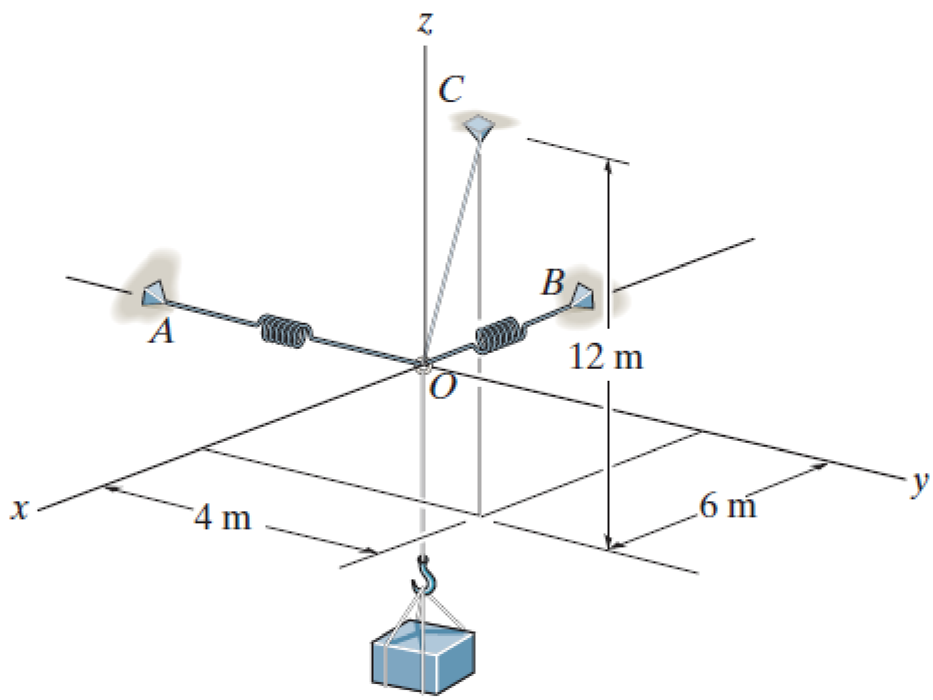
2. The 40-kg mass is held by two cables and two identical springs at the position given below. If the initial length of the springs is 1.5 m and the spring constant is 400 N/m, determine

a. The unit vector along the lines *OA, OB,* and *OC,*

b. The resultant force at *O* due to the spring forces,

c. The final length of each spring,

d. The components of the spring forces along the line *OC.*



Solution:

a. The unit vectors along *OA* and *OB are*

$$\vec{u}_{OA} = -1\,\boldsymbol{j} \quad \& \quad \vec{u}_{OB} = -1\,\boldsymbol{i}$$

The unit vector along *OC* can be found from

$$\vec{u}_{OC} = \frac{\overrightarrow{OC}}{OC} = \frac{6\boldsymbol{i} + 4\boldsymbol{j} + 12\boldsymbol{k}}{\sqrt{6^2 + 4^2 + 12^2}} = 0.43\boldsymbol{i} + 0.286\boldsymbol{j} + 0.857\boldsymbol{k}$$

b. The weight of the crate is $W = mg = 40 \cdot 9.81 = 392.4\,N$. Since the resultant force due to all forces at point *O* is zero,

$$\vec{F}_{OC} - F_{OB}\boldsymbol{i} - F_{OA}\boldsymbol{j} - W\boldsymbol{k} = 0$$

Then,

$$(F_{OC})_z = W$$

But,

$$\vec{F}_{OC} = F_{OC}\vec{u}_{OC} = F_{OC}(0.43\boldsymbol{i} + 0.286\boldsymbol{j} + 0.857\boldsymbol{k}) = (F_{OC})_x\boldsymbol{i} + (F_{OC})_y\boldsymbol{j} + (F_{OC})_z\boldsymbol{k}$$

$$(F_{OC})_z = F_{OC} \cdot 0.857 = 392.4$$

$$F_{OC} = 457.876\,N$$

Then,

$$(F_{OC})_x = F_{OC} \cdot 0.43 = 193.887\,N \;\; \& \;\; (F_{OC})_y = F_{OC} \cdot 0.286 = 130.95\,N$$

Note that

$$F_{OB} = (F_{OC})_x = 193.887\ N \quad \& \quad F_{OA} = (F_{OC})_y = 130.95\ N$$

Then, the resultant force at *O* due to spring forces is

$$\left(\vec{F}_R\right)_{spring} = -F_{OB}\boldsymbol{i} - F_{OA}\boldsymbol{j} = -193.887\boldsymbol{i} - 130.95\boldsymbol{j}$$

$$(F_R)_{spring} = \sqrt{(-193.887)^2 + (-130.95)^2} = 233.97\ N$$

c. The forces along the springs are

$$F_{OB} = 193.887\ N \quad \& \quad F_{OA} = 130.95\ N$$

Then, we can write

$$F_{OB} = k \cdot (l_{OB} - l_0) = 400 \cdot (l_{OB} - 1.5) = 193.887$$
$$\rightarrow l_{OB} = 1.985\ m$$

Similarly,

$$F_{OA} = 400 \cdot (l_{OA} - 1.5) = 130.95$$
$$\rightarrow l_{OA} = 1.83\ m$$

d. The components of the spring forces along *OC* can be obtained from

$$(F_{OB})_{OC} = \vec{F}_{OB} \cdot \vec{u}_{OC} \quad \& \quad (F_{OA})_{OC} = \vec{F}_{OA} \cdot \vec{u}_{OC}$$
$$(F_{OB})_{OC} = (-193.887\boldsymbol{i}) \cdot (0.43\boldsymbol{i} + 0.286\boldsymbol{j} + 0.857\boldsymbol{k}) = -83.37\ N$$
$$(F_{OA})_{OC} = (-130.95\boldsymbol{j}) \cdot (0.43\boldsymbol{i} + 0.286\boldsymbol{j} + 0.857\boldsymbol{k}) = -37.45\ N$$