Middle East Technical University
Department of Electrical and Electronics Engineering
EE301: Signals and Systems I

Solutions for MATLAB Question of Homework 4

December 30, 2018

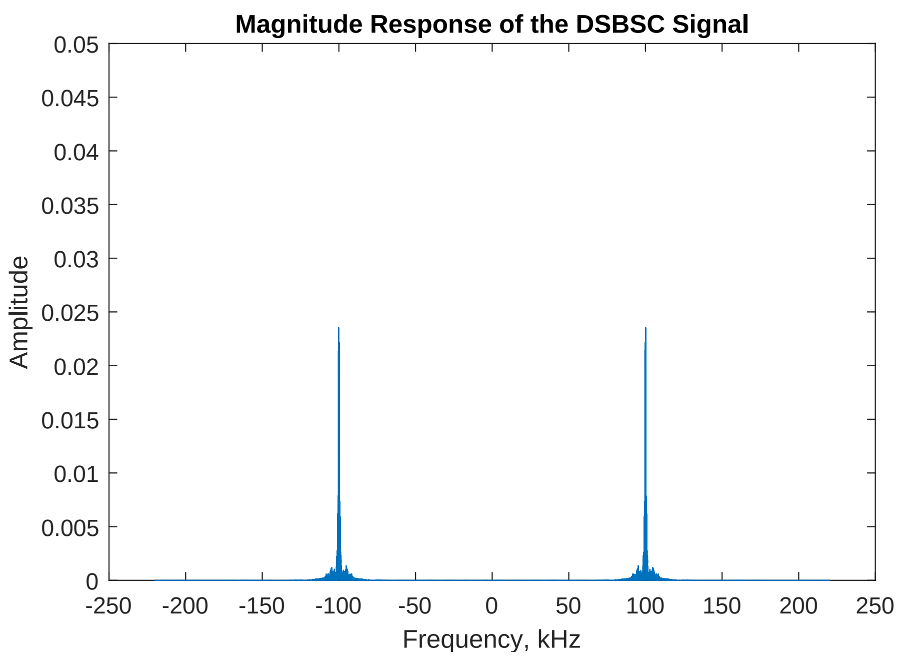
If you face any problem or mistake please contact Ömer Çayır, *ocayir@metu.edu.tr*, DZ-10.

1. The DSBSC signal is given as

$$s(t) = x_c(t)\,\cos(2\pi f_c t),$$

where $f_c = 100$ kHz.

The plot of the magnitude response of $s[n] = s(n/F_s)$, where $F_s = 441$ kHz, is given below.



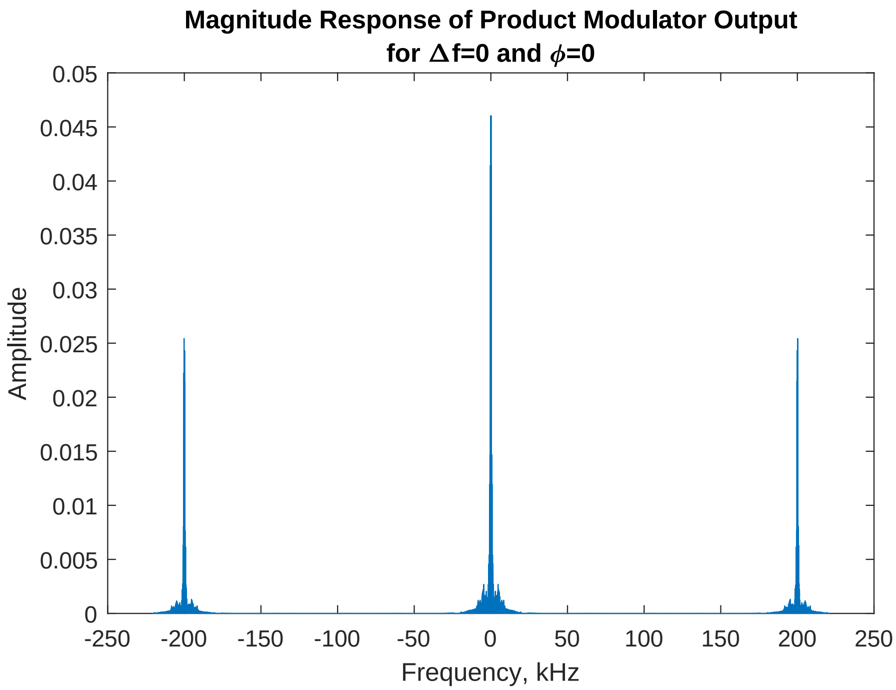
To plot the magnitude response of $s[n] = s(n/F_s)$, the MATLAB code is given below.

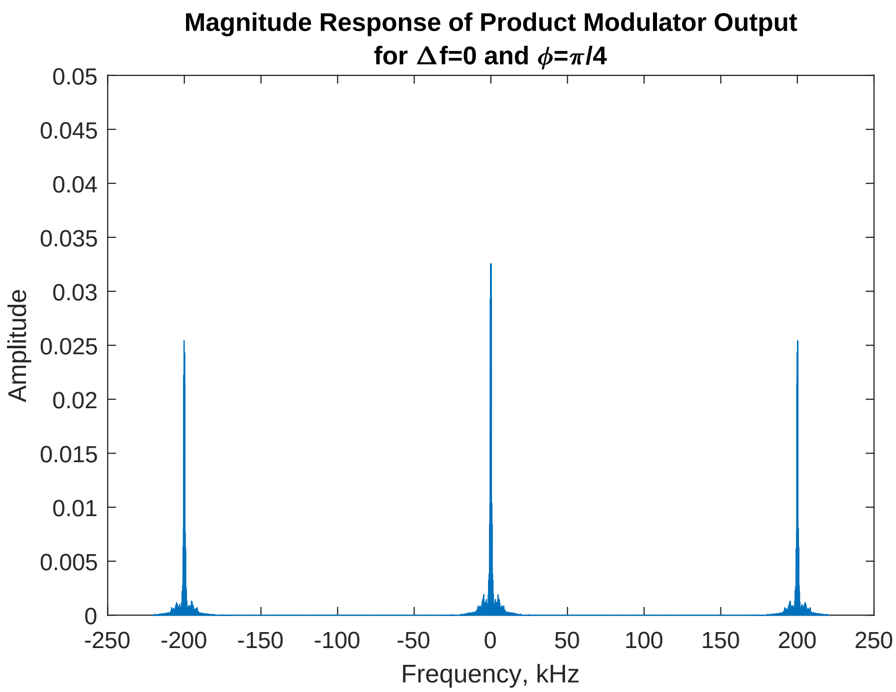
```
clc;close all;
fc = 100e3;
s = x.*cos(2*pi*fc*t');
w_axis = (-Fs/2:Fs/(upsmp_rate*voice_len-1):Fs/2)/1e3; % in kHz

% Observe magnitude response of the DSBSC signal
figure, plot(w_axis, fftshift(abs(fft(s)))/Fs);
title('Magnitude Response of the DSBSC Signal');
xlabel('Frequency, kHz'), ylabel('Amplitude');
set(gca,'ylim',[0, 0.05]);
```

2.	(a)	The product modulator output is

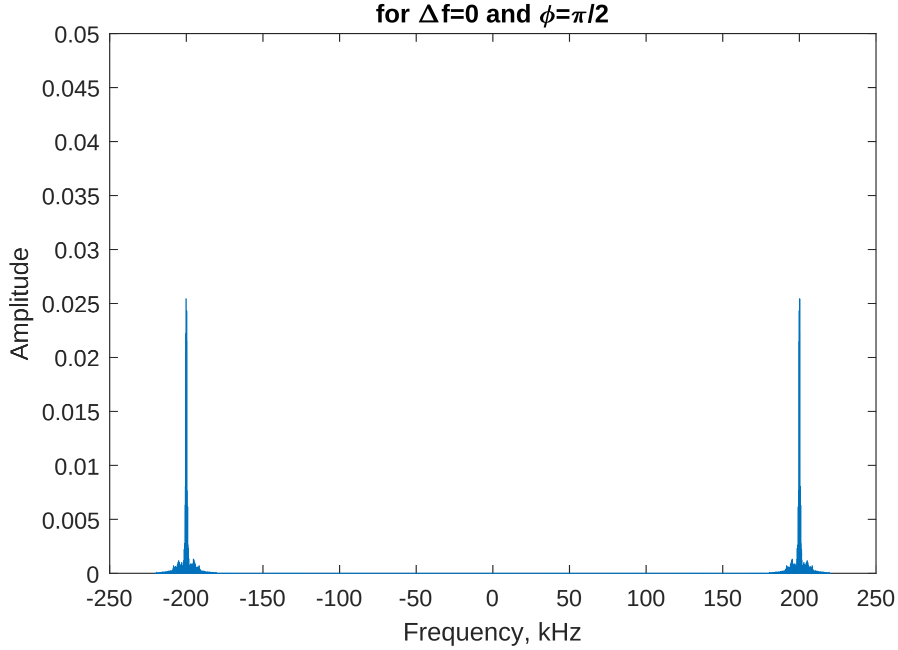
$$v(t) = 2\,s(t)\,\cos\left[2\pi(f_c + \Delta f)t + \phi\right].$$

For $\Delta f = 0$ and $\phi = 0, \pi/4, \pi/2, \pi$, plots of the magnitude response of $v[n] = v(n/F_s)$ are given below.

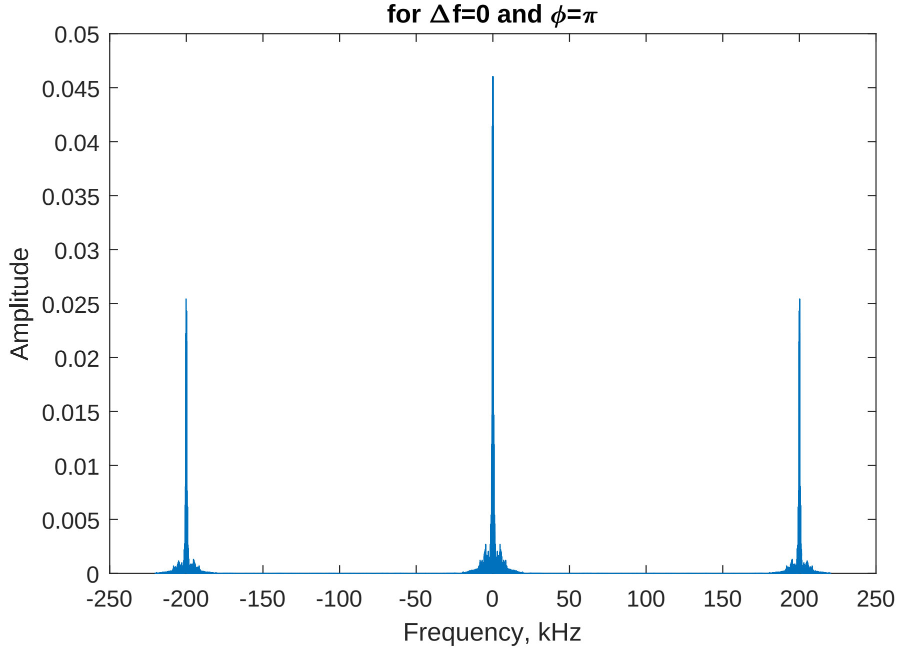




Magnitude Response of Product Modulator Output
for Δf=0 and $\phi=\pi/2$



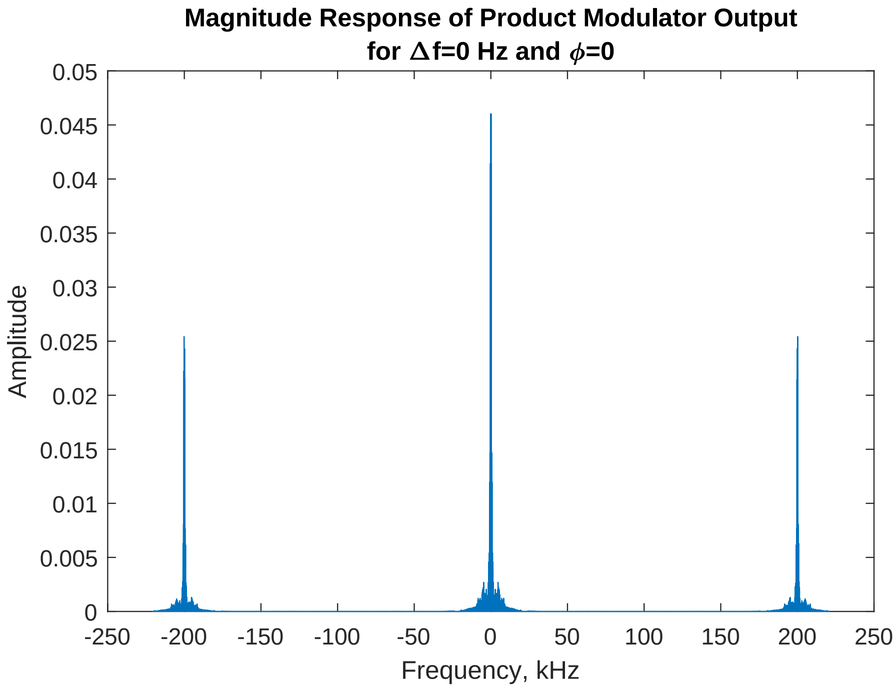
Magnitude Response of Product Modulator Output
for Δf=0 and $\phi=\pi$

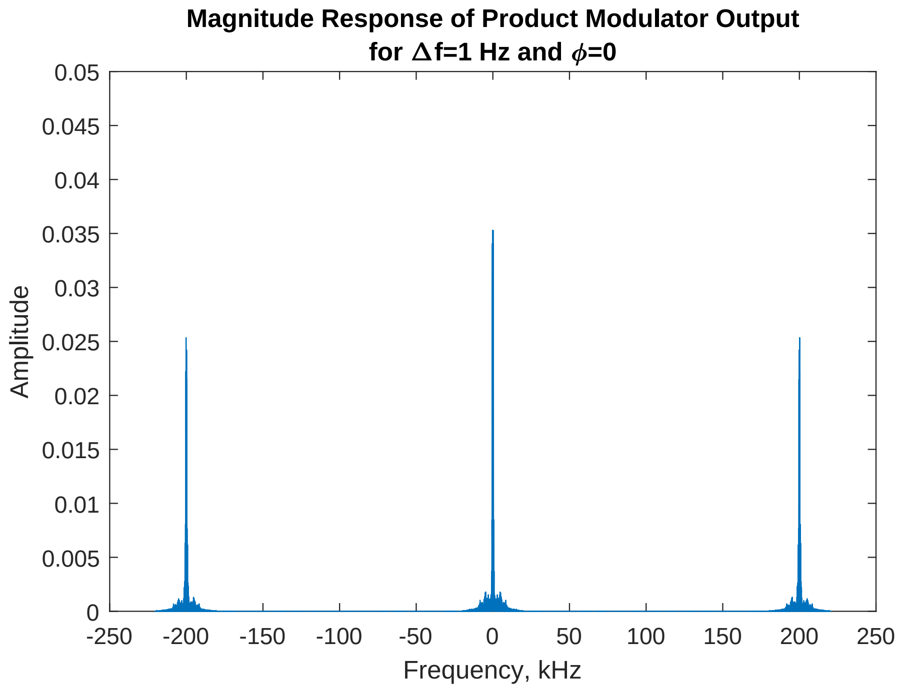


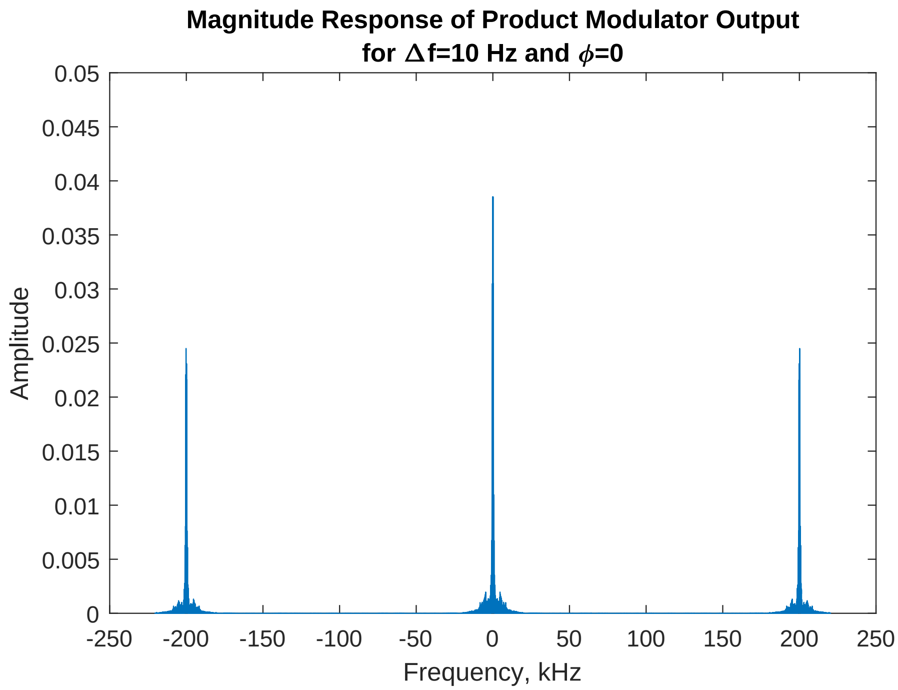
To obtain these plots, the MATLAB code is given below.

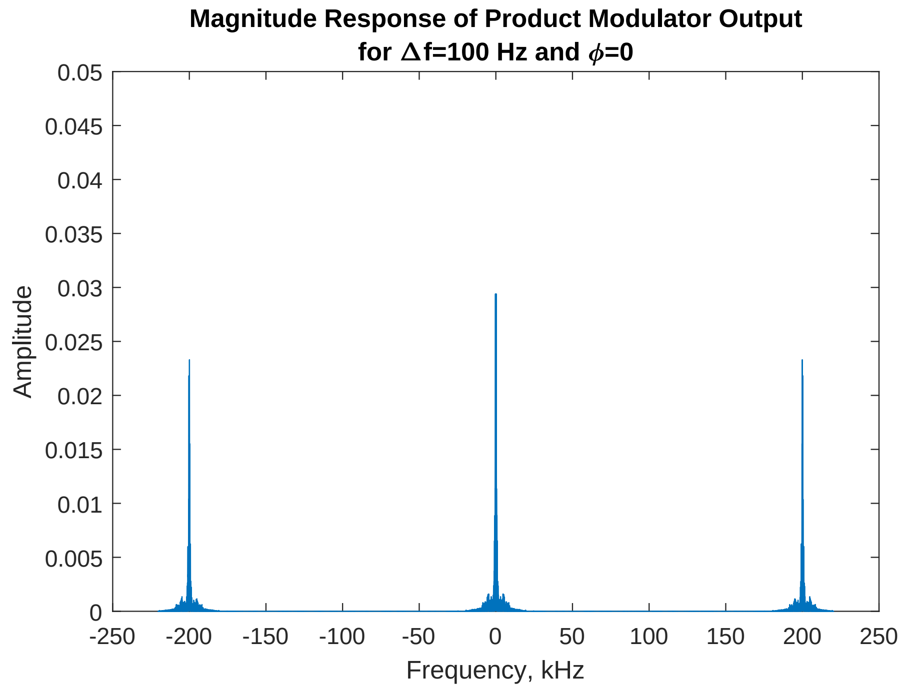
```
fdelta = 0;
PHI = [0, pi/4, pi/2, pi];
PHIstring = {'0','\pi/4','\pi/2','\pi'};
for i=1:numel(PHI)
    phi = PHI(i);
    v = 2*s.*cos(2*pi*(fc+fdelta)*t'+phi);
    w_axis = (-Fs/2:Fs/(upsmp_rate*voice_len-1):Fs/2)/1e3; % in kHz

    % Observe magnitude response of the product modulator output
    figure, plot(w_axis, fftshift(abs(fft(v)))/Fs);
    title(['Magnitude Response of Product Modulator Output' newline ...
        'for \Deltaf=' num2str(fdelta) ' and \phi=' PHIstring{i}]);
    xlabel('Frequency, kHz'), ylabel('Amplitude');
    set(gca,'ylim',[0, 0.05]);
end
```

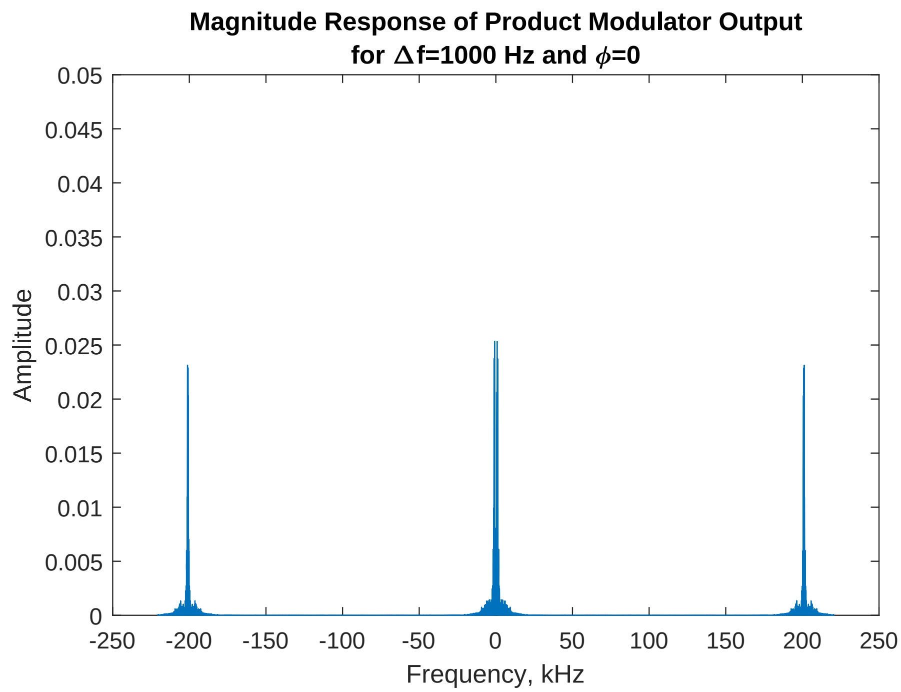
(b) For $\phi = 0$ and $\Delta f = 0, 1, 10, 100, 1000$ Hz, plots of the magnitude response of $v[n] = v(n/F_s)$ are given below.




Magnitude Response of Product Modulator Output
for Δf=1 Hz and ϕ=0
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-250
-200
-150
-100
-50
0
50
100
150
200
250
Frequency, kHz

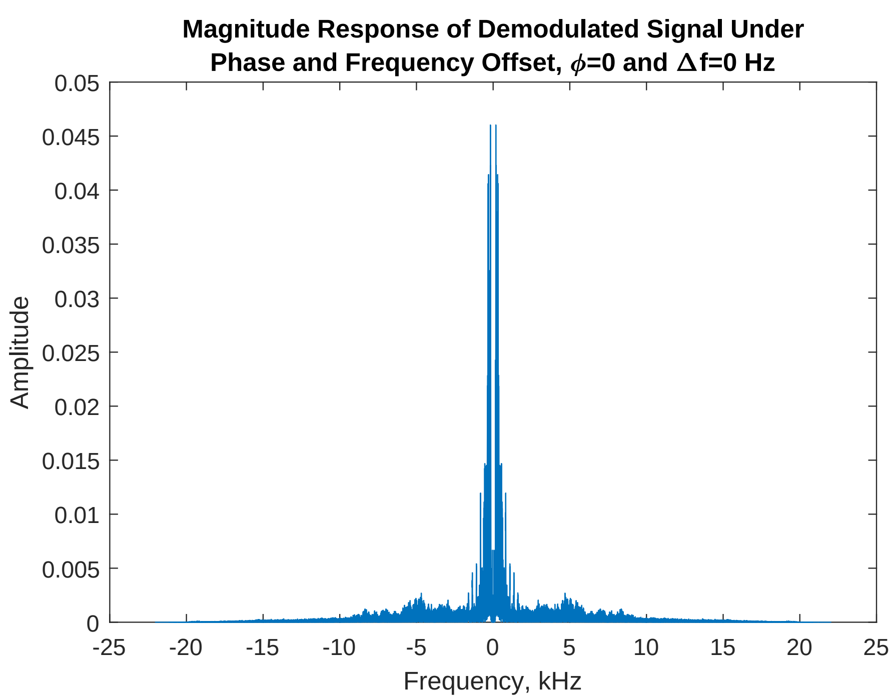

Magnitude Response of Product Modulator Output
for Δf=10 Hz and ϕ=0
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-250
-200
-150
-100
-50
0
50
100
150
200
250
Frequency, kHz

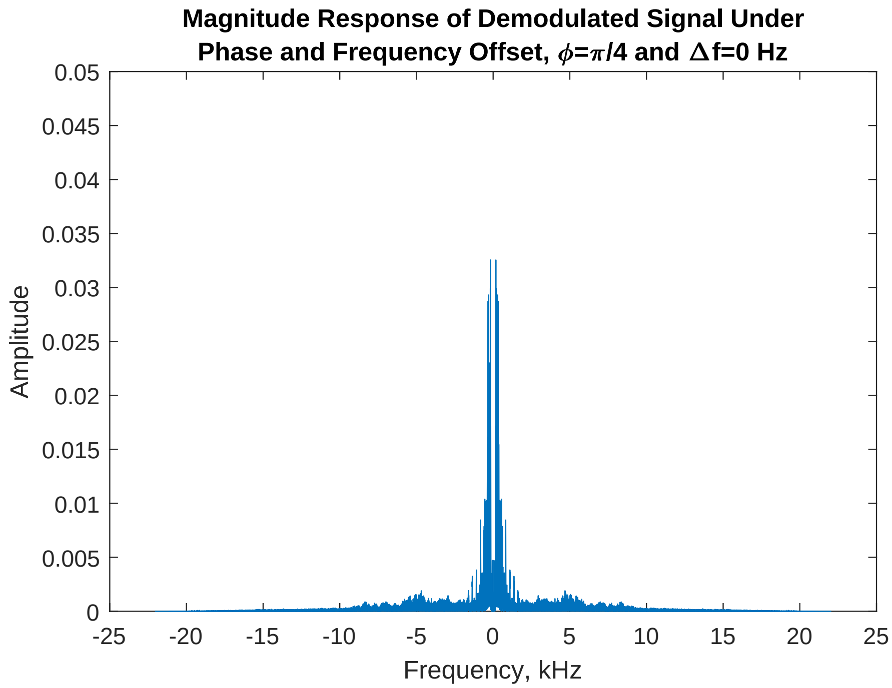




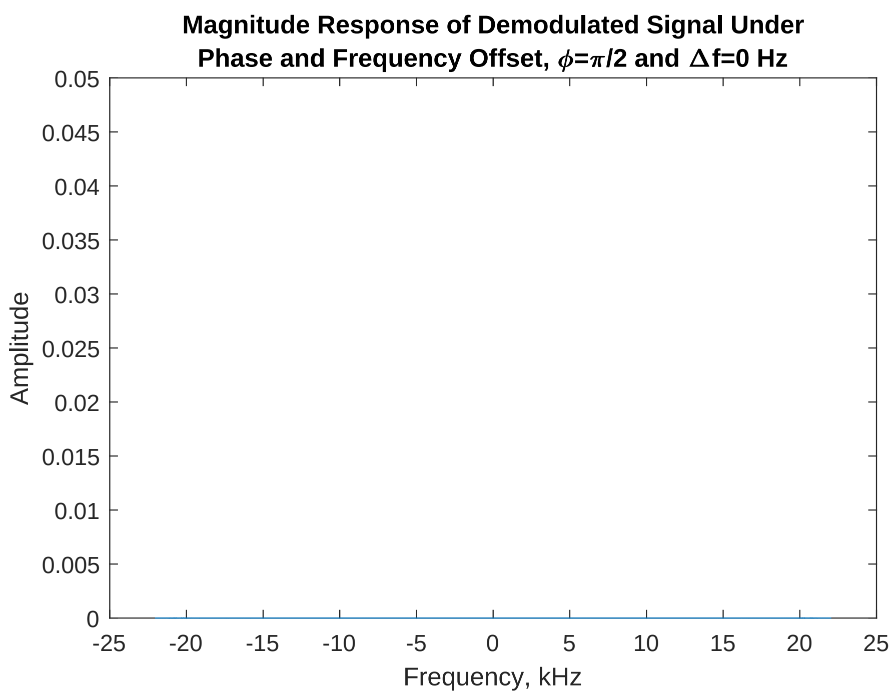
To obtain these plots, the MATLAB code is given below.

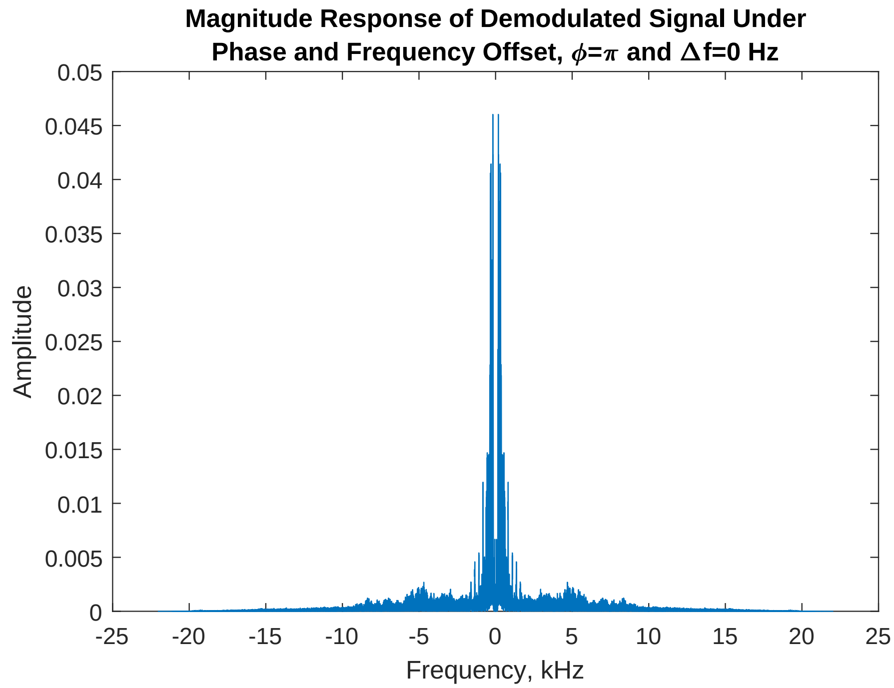
```
fDELTA = [0, 1, 10, 100, 1000];
phi = 0;
for i=1:numel(fDELTA)
    fdelta = fDELTA(i);
    v = 2*s.*cos(2*pi*(fc+fdelta)*t'+phi);
    w_axis = (-Fs/2:Fs/(upsmp_rate*voice_len-1):Fs/2)/1e3; % in kHz

    % Observe magnitude response of the product modulator output
    figure, plot(w_axis, fftshift(abs(fft(v)))/Fs);
    title(['Magnitude Response of Product Modulator Output' newline ...
        'for \Deltaf=' num2str(fdelta) ' Hz and \phi=0']);
    xlabel('Frequency, kHz'), ylabel('Amplitude');
    set(gca,'ylim',[0, 0.05]);
end
```

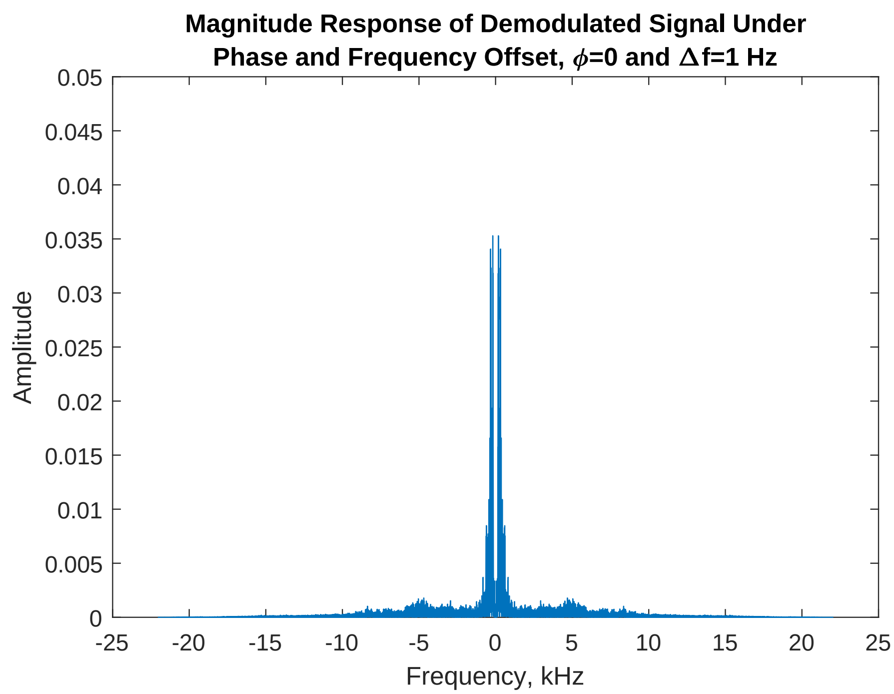
(c)	For the DSBSC signal $$s(t) = x_c(t)\,\cos(2\pi f_c t),$$ the output of product modulator can be expressed as follows: $$\begin{aligned} v(t) &= 2\,s(t)\,\cos\left[2\pi(f_c+\Delta f)t+\phi\right] \\ &= 2\,x_c(t)\,\cos(2\pi f_c t)\,\cos\left[2\pi(f_c+\Delta f)t+\phi\right] \\ &= 2\,x_c(t)\cdot\frac{e^{j2\pi f_c t}+e^{-j2\pi f_c t}}{2}\cdot\frac{e^{j(2\pi(f_c+\Delta f)t+\phi)}+e^{-j(2\pi(f_c+\Delta f)t+\phi)}}{2} \\ &= x_c(t)\cdot\frac{e^{j(2\pi(2f_c+\Delta f)t+\phi)}+e^{-j(2\pi(2f_c+\Delta f)t+\phi)}+e^{j(2\pi\Delta f t+\phi)}+e^{-j(2\pi\Delta f t+\phi)}}{2} \\ &= x_c(t)\Big(\cos\left[2\pi(2f_c+\Delta f)t+\phi\right]+\cos\left(2\pi\Delta f t+\phi\right)\Big) \end{aligned}$$ Assuming that $v(t)$ passes through an ideal low pass filter, whose cutoff frequency is below f_c, the demodulator output can be written as $$v_o(t) = x_c(t)\,\cos\left(2\pi\Delta f t+\phi\right)$$ If there is no offset in phase and the frequency of the local oscillator, i.e. $\phi = 0$ and $\Delta f = 0$, the demodulator output is $$v_o(t) = x_c(t)\,\cos(0) = x_c(t).$$
(d)	For different values of ϕ and Δf given in part (a) and (b), plots of the magnitude response of the demodulated signal are given below. 

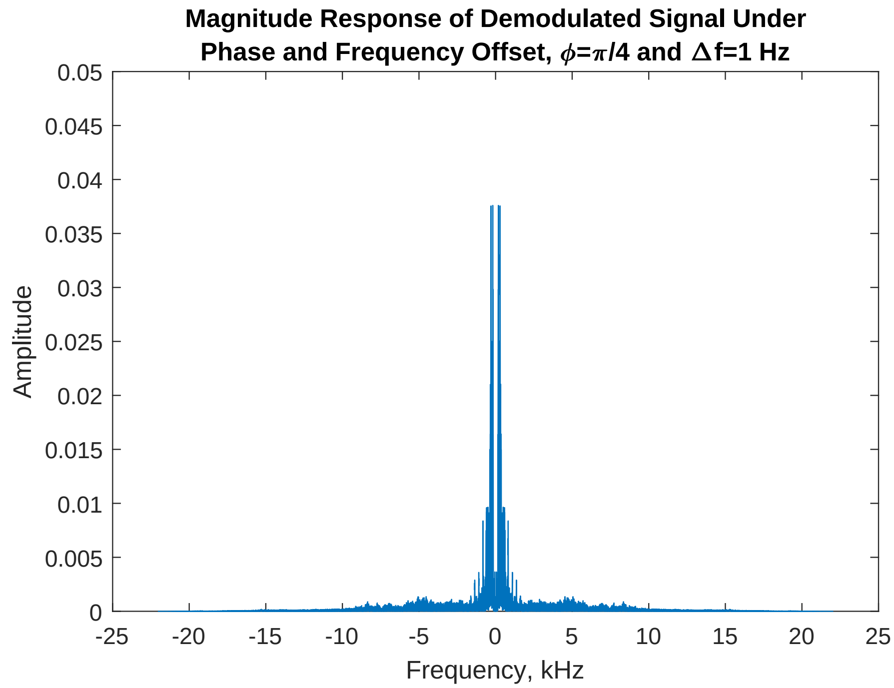


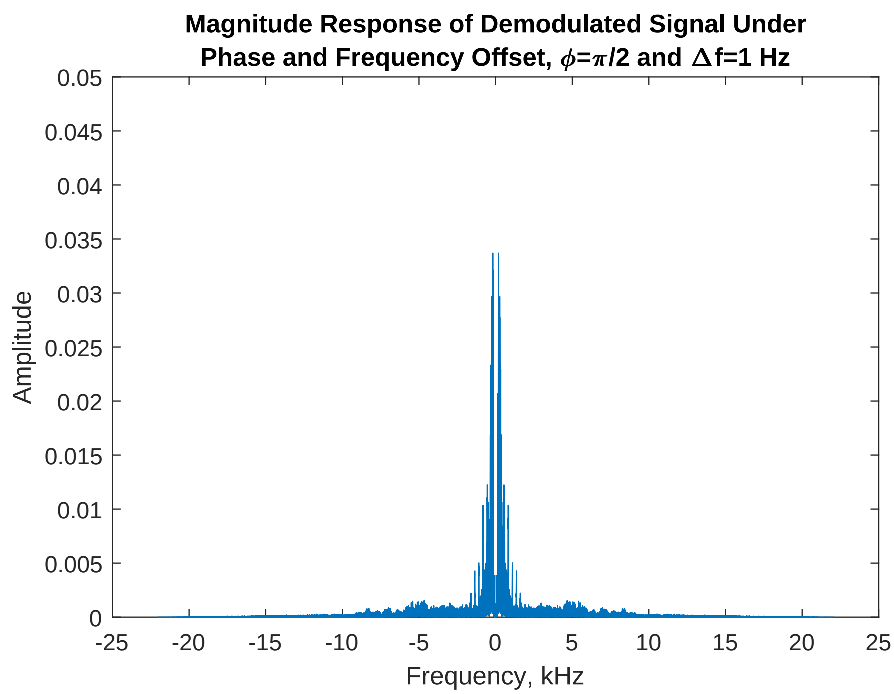
When $\phi = \pi/2$ and $\Delta f = 0$, $v_o[n] = v_o(n/F_s) = 0$.

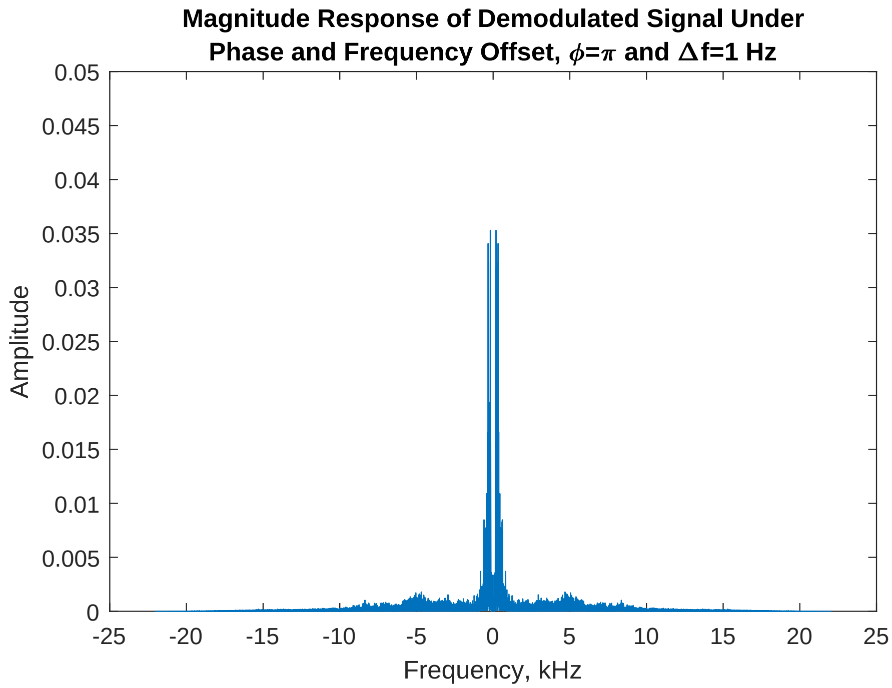


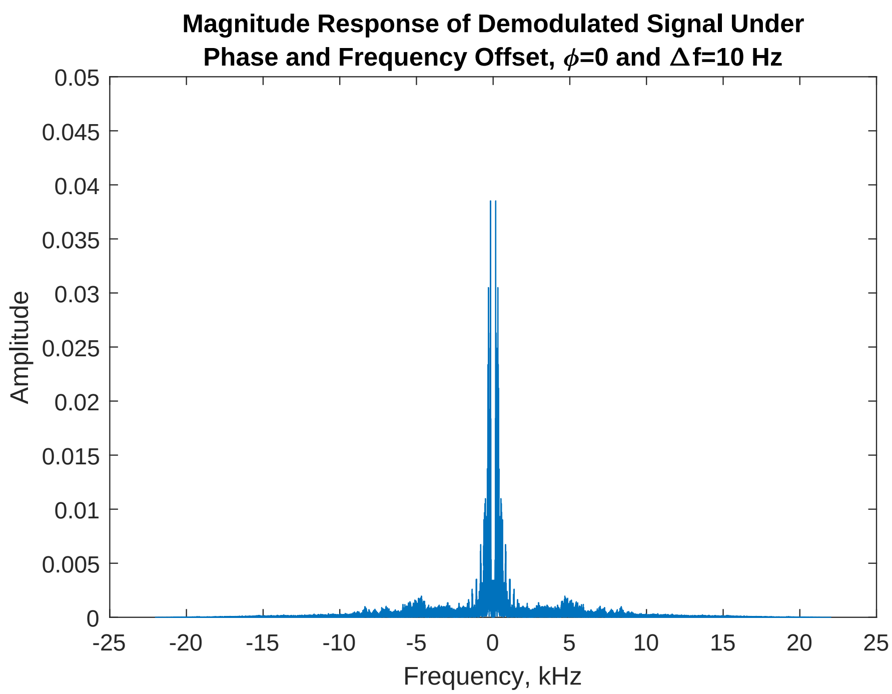

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi$ and Δf=0 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

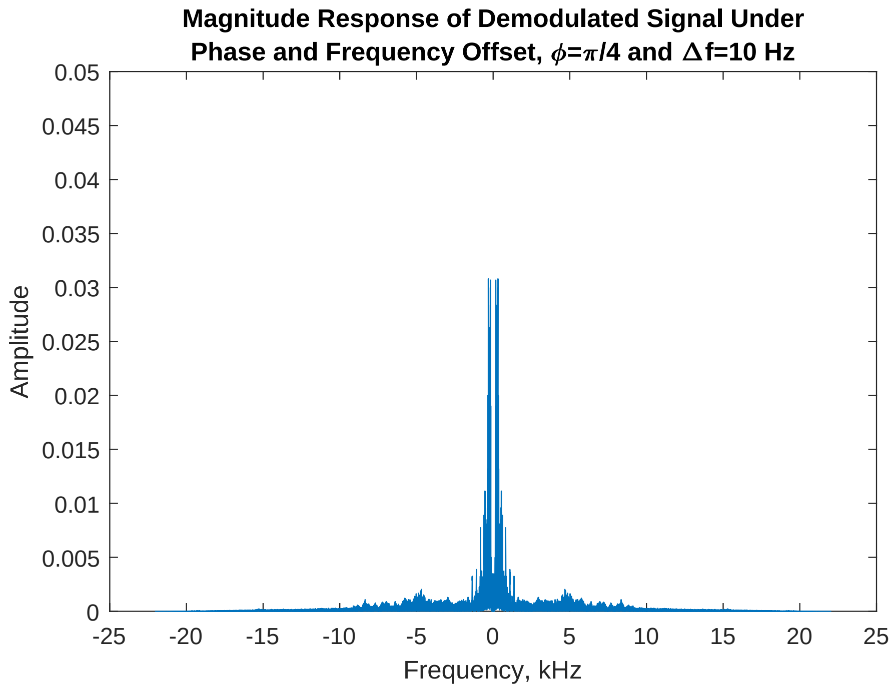

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=0 and Δf=1 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

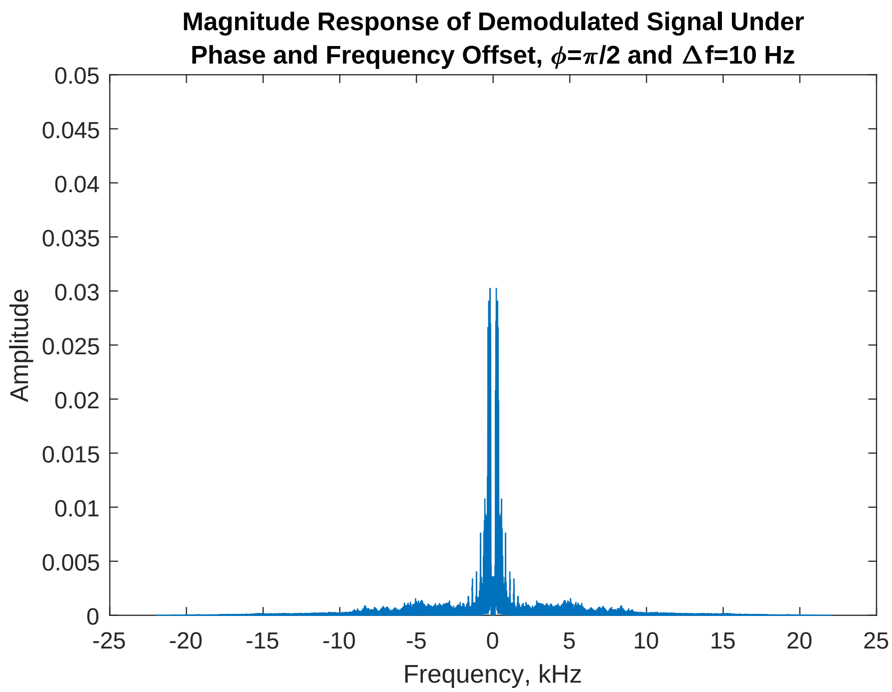

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi/4$ and Δf=1 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

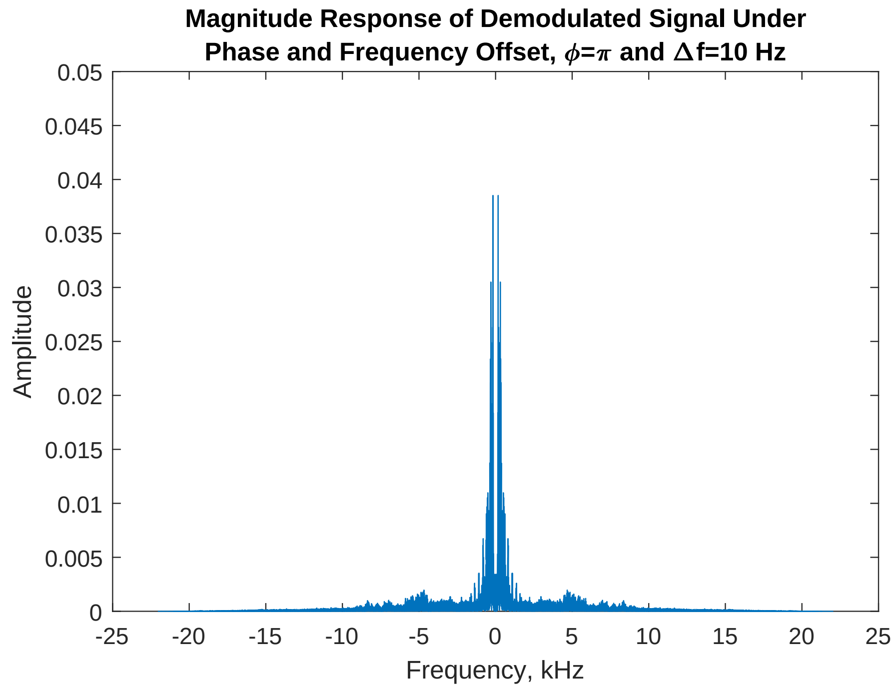

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi/2$ and Δf=1 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

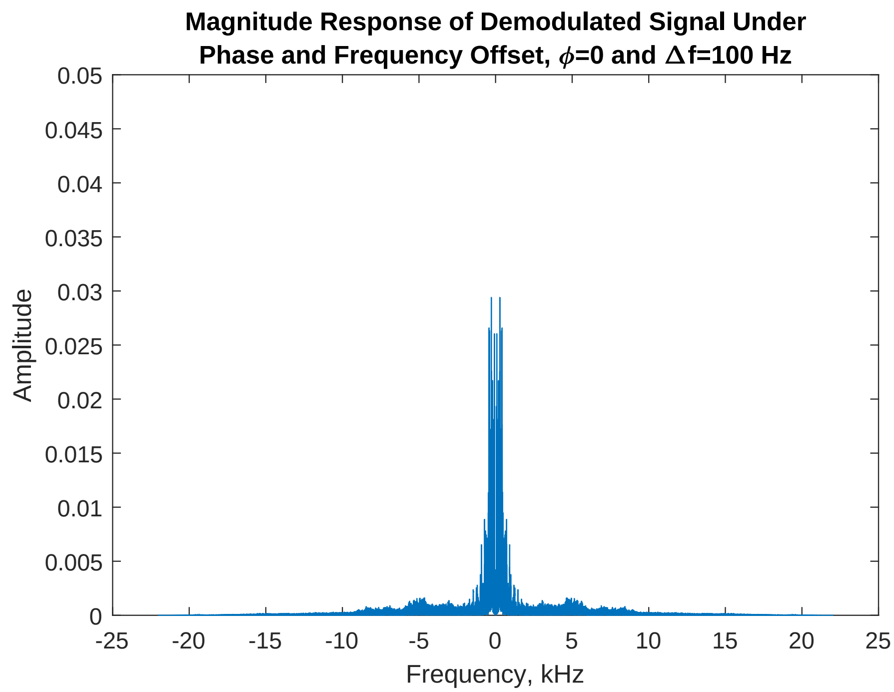

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi$ and $\Delta f=1$ Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

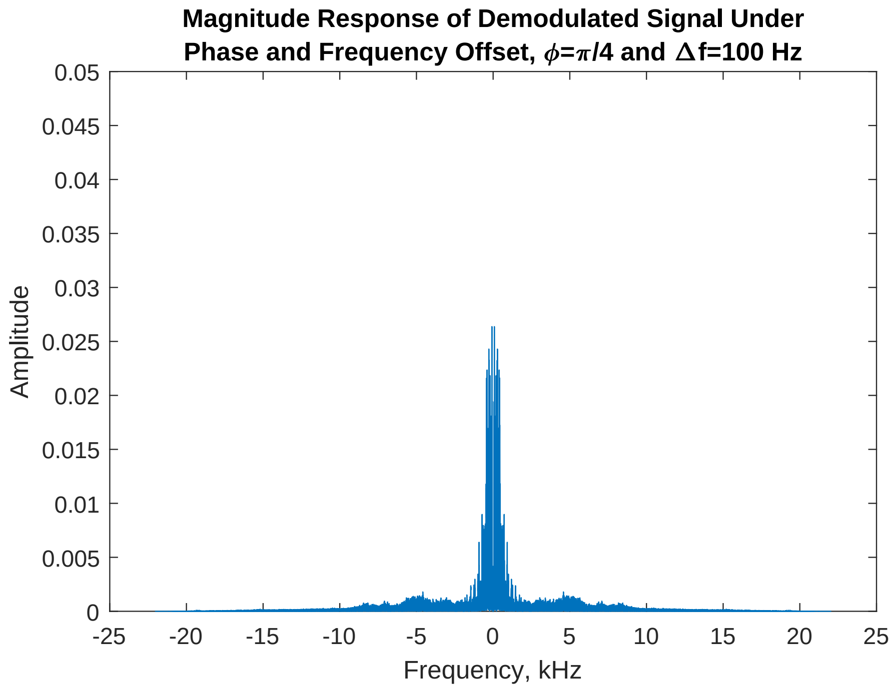

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=0$ and $\Delta f=10$ Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

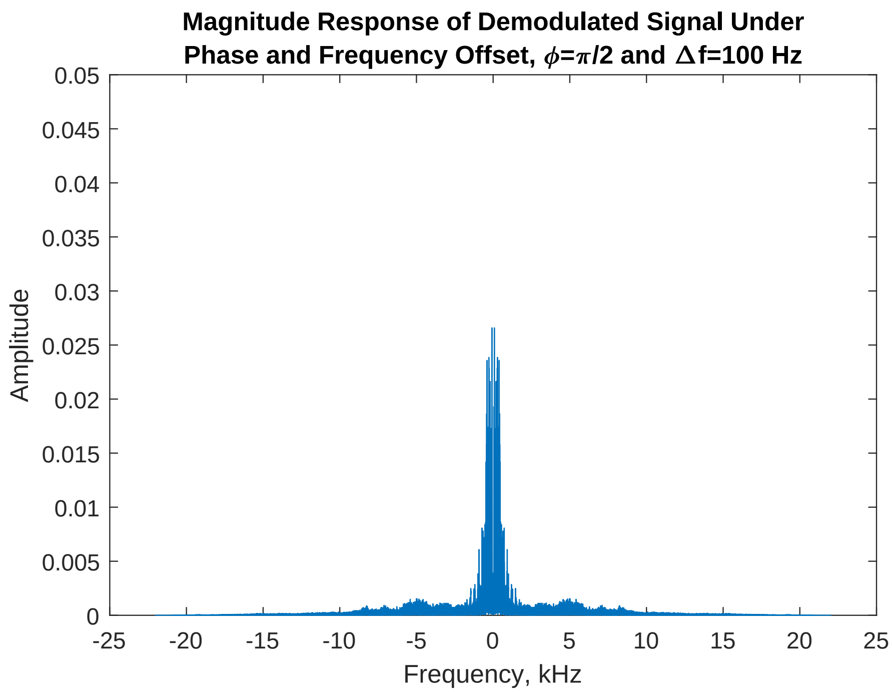

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi/4$ and Δf=10 Hz
Amplitude
Frequency, kHz
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25

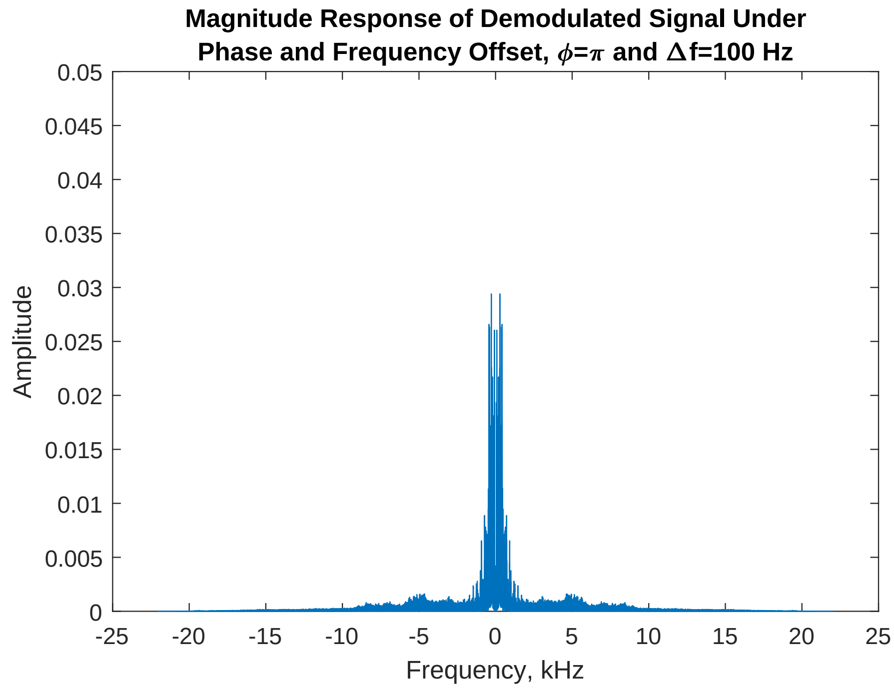

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi/2$ and Δf=10 Hz
Amplitude
Frequency, kHz
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25

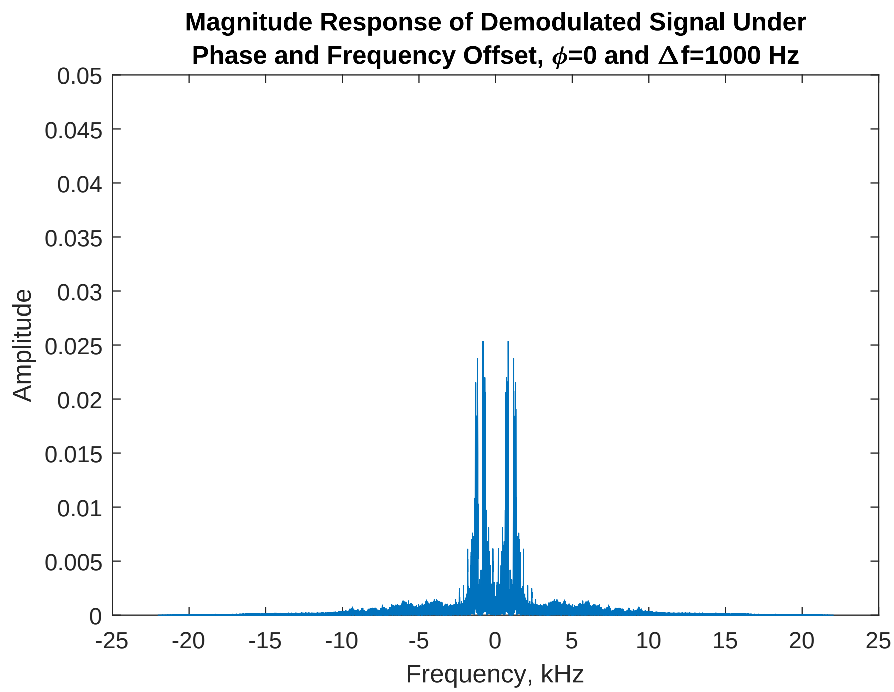

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi$ and Δf=10 Hz
Amplitude
Frequency, kHz

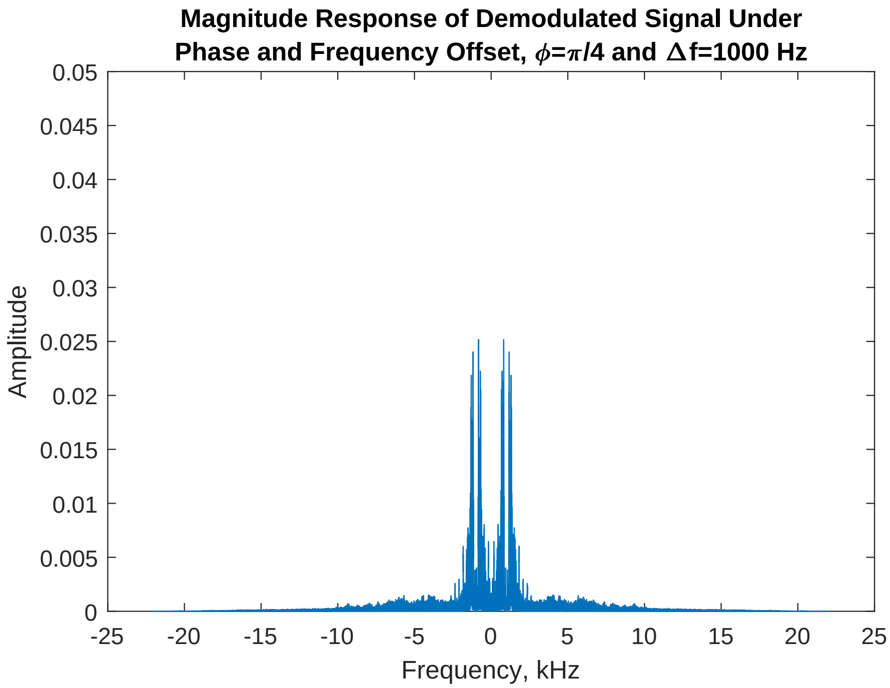

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=0 and Δf=100 Hz
Amplitude
Frequency, kHz

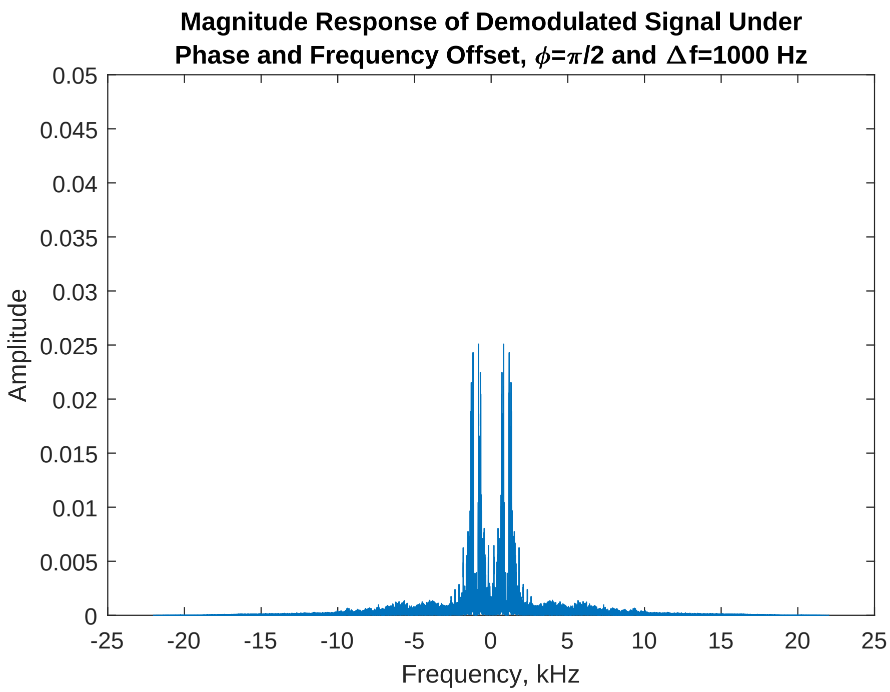

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=π/4 and Δf=100 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

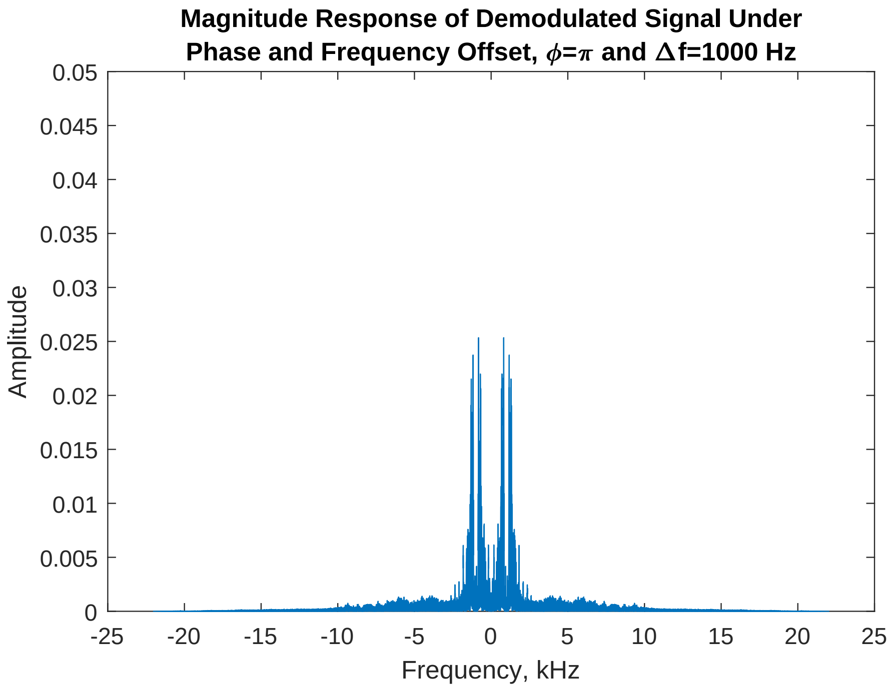

Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=π/2 and Δf=100 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz


Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=\pi$ and $\Delta f=100$ Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz


Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, $\phi=0$ and $\Delta f=1000$ Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz


Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=π/4 and Δf=1000 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz


Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=π/2 and Δf=1000 Hz
Amplitude
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25
Frequency, kHz

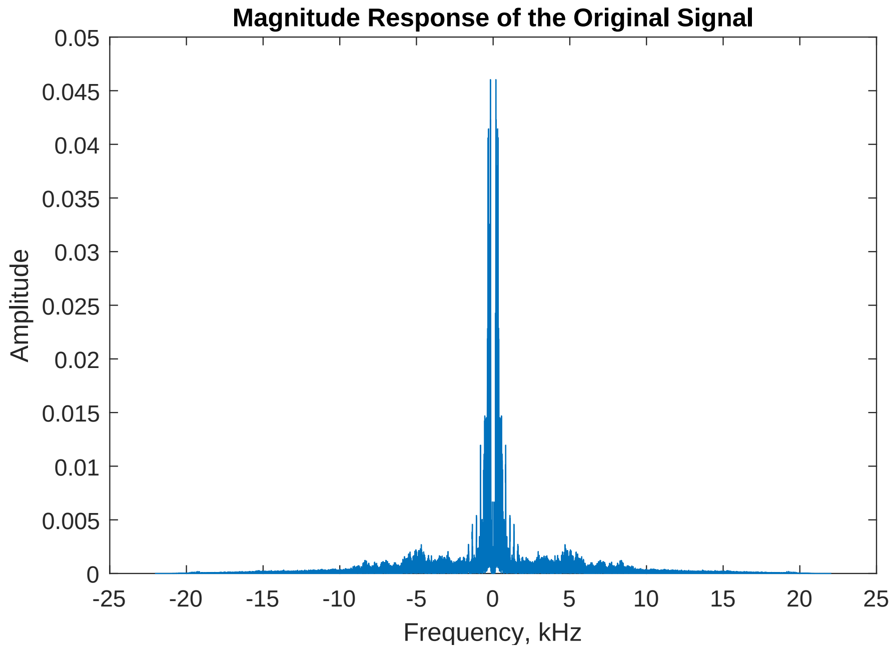


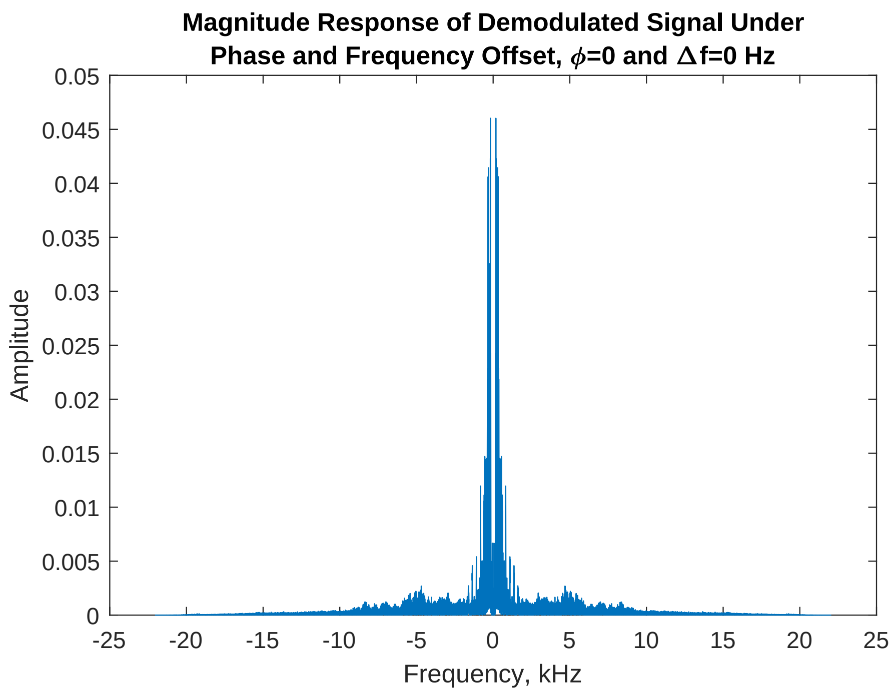
To obtain these plots, the MATLAB code is given below.

```
Fs = 441e3;
fir_order = 128; % The low-pass filter for demodulation will be an FIR
                 % filter of order 128
downsmp_rate = 10;
Fs = Fs / downsmp_rate;

fDELTA = [0, 1, 10, 100, 1000];
PHI = [0, pi/4, pi/2, pi];
PHIstring = {'0','\pi/4','\pi/2','\pi'};
for i=1:numel(fDELTA)
    for j=1:numel(PHI)
        fdelta = fDELTA(i);
        phi = PHI(j);
        v = 2*s.*cos(2*pi*(fc+fdelta)*t'+phi);
        demodulated_signal = decimate(v, downsmp_rate, fir_order, 'fir');
        w_axis = (-Fs/2:Fs/(voice_len-1):Fs/2)/1e3;

        % Observe magnitude response of the demodulated signal under phase
        % and frequency offset
        figure, plot(w_axis, fftshift(abs(fft(demodulated_signal)))/Fs);

        title(['Magnitude Response of Demodulated Signal Under' newline ...
            'Phase and Frequency Offset, \phi=' PHIstring{j} ...
            ' and \Deltaf=' num2str(fdelta) ' Hz']);
        xlabel('Frequency, kHz'), ylabel('Amplitude');
        set(gca,'ylim',[0, 0.05]);
    end
end
```

When $\phi \neq 0$ and $\Delta f \neq 0$, the demodulated signal has perceptional distortion.

As verified in part (c), the demodulated signal has not any perceptional distortion, if there is no offset in phase and the frequency of the local oscillator, i.e. $\phi = 0$ and $\Delta f = 0$. The magnitude response of the original signal is given below with the magnitude response of the demodulated signal for $\phi = 0$ and $\Delta f = 0$.


Magnitude Response of the Original Signal
Amplitude
Frequency, kHz
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25


Magnitude Response of Demodulated Signal Under
Phase and Frequency Offset, ϕ=0 and Δf=0 Hz
Amplitude
Frequency, kHz
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-25
-20
-15
-10
-5
0
5
10
15
20
25