EE414 Supplementary Problems Solutions

Problem 1:

For $L_d = 0$

(a) $L_{eff} = L - 2L_d - X_d = 0.9\,\mu m$

$|I_D| = 200\,\mu A$ for $M_5 - M_8$

$|I_D| = 100\,\mu A$ for $M_1 - M_4$

$$\frac{dX_d}{dV_{DS}} = 0.02\,\mu m/V \quad (NMOS)$$
$$= 0.04\,\mu m/V \quad (PMOS)$$

$$1/r_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{I_D}{L_{eff}}\frac{dX_d}{dV_{DS}}$$

$$1/r_{o2} = 100\,\mu A/0.9\,\mu m \cdot 0.04\,\mu m/V = 4.44\,\mu A/V$$

$$1/r_{o4} = 100\,\mu A/0.9\,\mu m \cdot 0.02\,\mu m/V = 2.22\,\mu A/V$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.9 \times 8.854 \times 10^{-12} / 80 \times 10^{-10} = 4.32 \times 10^{-3}\, F/m^2$$

$$K_p' = \mu_p C_{ox} = 150 \times 10^{-4} \times 4.32 \times 10^{-3} = 64.7\,\mu A/V^2$$

$$K_n' = \mu_n C_{ox} = 450 \times 10^{-4} \times 4.32 \times 10^{-3} = 194\,\mu A/V^2$$

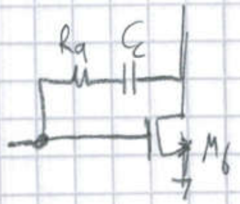
$$g_{m2} = \sqrt{2 \times 64.7 \times (150/0.9) \times 100} = 1.47\, mA/V$$

$$g_{m6} = \sqrt{2 \times 194 \times (100/0.9) \times 200} = 2.94\, mA/V$$

$$1/r_{o7} = \frac{200\,\mu A}{0.9\,\mu m} \times 0.04\,\mu m/V = 8.89\,\mu A/V$$

$$1/r_{o6} = \frac{200\,\mu A}{0.9\,\mu m} \times 0.02\,\mu m/V = 4.44\,\mu A/V$$

$$v_o/v_i = g_{m2}(r_{o2} \| r_{o4}) \cdot g_{m6}(r_{o6} \| r_{o7}) = \underline{\underline{48681}}$$



$$Z = \frac{1}{(1/g_{m6} - R_a)C_c}$$

Cancel this zero by moving it to infinity.

$$R_9 = \frac{1}{g_{m6}} = \frac{1}{2.94m} = 340.1\,\Omega$$

$$1/R_9 = K'\left(\frac{W}{L}\right)_9 \left(V_{GS_9} - V_{T9} - V_{DS9}\right) \quad \Rightarrow \left(V_{DS9} \approx 0\right)$$

Assume $\gamma = 0$

$$1/R_9 = K'(W/L)_9 \left(\overset{V_{DD}}{V_{G9}} - V_{SS} - V_{GS6} - V_{T9}\right)$$

$$\therefore\ V_{OV6} = \sqrt{\frac{2\times 200}{194(100/0.9)}} = 0.136\,V \Rightarrow V_{GS6} = V_{OV6} + \overset{0.6V}{V_{T6}} = 0.736V$$

$$\therefore\ 1/R_9 = \frac{1}{340} = 194\times10^{-6}\left(\frac{W}{L}\right)_9 \left(3 - 0.736 - 0.7\right) \Rightarrow \left(\frac{W}{L}\right)_9 = 9.7$$

* M_9 is in triode region, so $X_d = 0 \Rightarrow L_9 = L_d - 2L_d - X_d = 1\,\mu m$

$$\Rightarrow \underline{\underline{L_9 = 1\,\mu m}}\ ,\ \underline{\underline{W_9 = 9.7\,\mu m}}$$

At $f_{unity} \Rightarrow |A(j\omega)| = \left|\frac{g_{m1}}{j\omega C_c}\right| = 1 \Rightarrow f_{unity} = \frac{g_{m1}}{C_c 2\pi} = \underline{\underline{46.8\,MHz}}$

$$SR = \frac{I_{max}}{C_c} = \frac{200\,\mu A}{5pF} = \underline{\underline{40\,V/\mu s}}$$

(b) $1/R_9 = K'\left(\frac{W}{L}\right)_9 \left(V_{GS9} - V_{T9}\right)$

Since $I_{D12} = I_{D6}$ and $(W/L)_{12} = (W/L)_6 \quad \therefore V_{OV12} = V_{OV6} = 0.136\,V$

Therefore $V_{SG11} = V_{GS12} = V_{T12} + V_{OV12} = V_{SG9} = V_{GS6} = V_{T6} + V_{OV6}$

Because $V_{T12} = V_{T6}$ (no body effect)

Also: $V_{OV11} = V_{OV12}$ because $I_{D11} = I_{D12}$ and $(W/L)_{11} = (W/L)_{12}$

Therefore: $V_{GS9} = V_{GS11} + V_{GS12} - V_{GS6} = V_{G11} + V_{OV11} + \cancel{V_{G12}} + \cancel{V_{OV12}} - \cancel{V_{G6}} - \cancel{V_{OV6}}$

$$= V_{G11} + V_{OV11}$$

$\therefore\ V_{GS9} - V_{T9} = V_{T11} + V_{OV11} - V_{T9} = V_{OV11}$

Since $V_{GS9} - V_{T9} = V_{GS6} - V_{T6}$ and $1/R_9$ should be equal to g_{m6} to cancel the RHP zero.

$$1/R_9 = k'(W/L)_9 (V_{GS9} - V_{T9}) = g_{m6} = k'(\tfrac{W}{L})_6 (V_{GS6} - V_{T6})$$

$$\Rightarrow (\tfrac{W}{L})_9 = (W/L)_6 = \underline{\underline{100/1}}$$

(c) To get 45° phase margin set the second pole = unity gain freq.

$$\left(|P_2| = \frac{g_{m6} C_C}{C_L C_1 + C_C C_L + C_1 C_C} \right) = \frac{g_{m2}}{C_C}$$ (C_1 is the parasitic cap. at first stage output)

$$\therefore\ C_1 = C_{OL2} + C_{OL4} + C_{GS6} + C_{OL9} \simeq C_{GS6} = \left(\tfrac{2}{3}\ 100\mu m \cdot 0.9\mu m\right) \cdot 4.32 \tfrac{fF}{\mu m^2}$$

$C_1 = 259.2 fF$

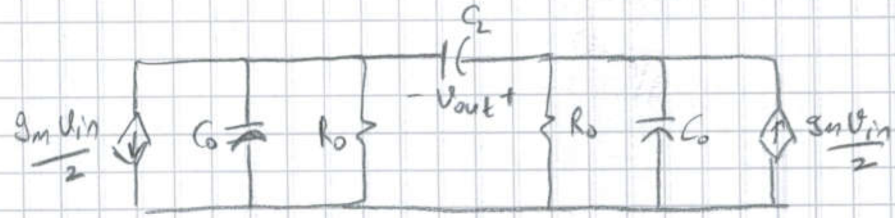
$$\frac{g_{m2}}{C_C} = \frac{1.47\ mA/V}{5pF} = 294\ Mrad/s$$

$$\therefore\ 294\ Mrad/s = \frac{2.94\ mA/V \cdot 5pF}{C_L\, 259{,}2f + C_L\, 5p + 259{,}2f \times 5p}$$

$$\Rightarrow C_L = \underline{\underline{9{,}26 pF}}$$

Problem 2:

Let's first draw the small signal model of the circuit.



$\frac{v_{out}}{v_{in}} = g_m R_o$ and $\omega_{-3dB} = \frac{1}{(C_L + 0.5C_o)\,2R_o}$

$g_m = \sqrt{2K_N' \left(\frac{W}{L}\right)_1 I_{D1}} = \sqrt{2 \cdot 110 \cdot 10 \cdot 25}\ \mu A/V = 234.5\ \mu A/V$

$R_o \approx 1/g_{m3} \| r_{o1} \| r_{o3} \| r_{o5}$, $g_{m3} = \sqrt{2K_P' (W/L)_3 I_{D3}} = \sqrt{2 \cdot 50 \cdot 2 \cdot 5}\ \mu A/V = 31.62\ \mu A/V$

$r_{o1} = \frac{1}{\lambda I_{D1}} = \frac{1}{0.04 \times 25\ \mu A} = 1M\Omega$, $r_{o3} = 4M\Omega$, $r_{o5} = 0.8\ M\Omega$

$\therefore R_o = 31.623k\Omega \| 1M\Omega \| 4M\Omega \| 0.8M\Omega = 29.31\ k\Omega$

$C_o \approx C_{gs3} + C_{bd1} + C_{bd3} + C_{bd5}$ $\quad C_{gs3} = C_{GSO} W_5 + 0.67 C_{ox} W_5 L_5$

$= 220\ pF/m \cdot 2\times10^{-6} + 0.67 \times 24.7\times10^{-4}\ F/m^2 \times 2\ \mu m^2$

$C_{gs3} = 3.73\ fF$

$C_{bd1} = C_J\ AS + C_{JSW} \cdot PS = 770\times10^{-6}\ F/m^2 \cdot 50\ \mu m^2 + 380\times10^{-12}\ F/m \times 30\ \mu m$

$C_{bd1} = 38.5\ fF + 11.4\ fF = 49.9\ fF$

$C_{bd3} = C_{bd5} = 560\times10^{-6}\ F/m^2 \cdot 10\ \mu m^2 + 350\times10^{-12}\ F/m \cdot 14\ \mu m = 10.5\ fF$

$\therefore C_o = 74.6\ fF \rightarrow \omega_{-3dB} = \frac{1}{(1.073\ pF)\ 58.62\ k\Omega} = 16.45\ Mrad/s$

$f_{-3dB} = 2.62\ MHz$ $\qquad A_v = 6.873\ V/V$