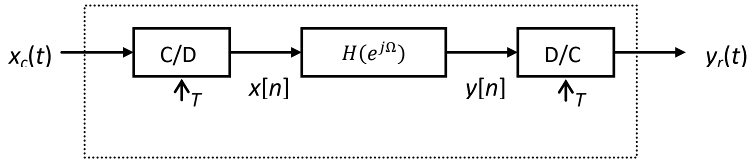
MIDDLE EAST TECHNICAL UNIVERSITY
Department of Electrical and Electronics Engineering

EE301 SIGNALS and SYSTEMS 1

HOMEWORK 5

Due: 05/01/2019, 23:55

Q1) For the given the system below, the frequency response of the digital filter is equal to $H(e^{j\Omega}) = 1 - e^{-j5\Omega}$, whereas its input is equal to $x_c(t) = 2 + 3\cos(\omega_1 t) + 4sin(\omega_2 t)$ for $-\infty < t < \infty$.



a) Given ω_1=200π and ω_2=440π, state and explain the Nyquist rate of this signal.
b) Let the sampling rate be 1000Hz. Given ω_1=500π and ω_2=1500π, sketch $|X(e^{j\Omega})|, |Y(e^{j\Omega})|$ and $|Y_r(j\omega)|$
c) For the same sampling rate 1000 Hz, now, let ω_1 and ω_2 be unknown and not equal to each other. Determine non-zero continuous-time frequencies ω_1 and ω_2 that cause the output signal $y_r(t)$ to be zero for the input $x_c(t)$. Pick ω_1 and ω_2, so that there is no aliasing.

Q2) The transfer function of an LTI system is given as $H(s) = \frac{1}{s(s+3)}$.

a) Find all possible expressions for the impulse response, $h(t)$. Comment on the causality of each system.
b) Is it possible to find an impulse response, $h(t)$, corresponding to a stable system? Explain your reasoning.
c) Suppose that $H(s)$ is modified so as to obtain the transfer function $G(s)$ of another LTI system,

$$G(s) = \frac{1}{(s-\alpha)(s+3)},$$

for which α is real. Without evaluating the inverse Laplace transform of $G(s)$, find the range of values for α, such that $G(s)$ corresponds to a causal and stable system. State the ROC and explain your reasoning.

Q3) The block diagram of a <u>causal</u> LTI system, with impulse response $h_1[n]$, input $x[n]$ and output $y[n]$ is given below in Fig.1. In this block diagram, the impulse response of the particular block is equal to $g[n] = \alpha\delta[n-1]$, where $\alpha>1$ and α is real. Another <u>causal</u> LTI system with impulse response $h_2[n]$ has the transfer function $H_2(z)$, whose pole-zero plot is given as in Fig.2.

a) Find $H_1(z)$, indicate its ROC and sketch its pole-zero plot. Determine $h_1[n]$. Check for the stability of the system that has the impulse response $h_1[n]$, while explaining its reason clearly.
b) Determine $H_2(z)$ (and its ROC) with all its multiplicative factors, if it is known *a priori* that $H_2(1)=1$.
c) Calculate the magnitude of DTFT, $|H_2(e^{j\Omega})|$, for $\alpha=2$, $\beta=3/2$ and for frequencies: $\Omega=\{0, \pi/2, \pi, 3\pi/2\}$. Comment on this result.
d) If the two LTI systems having the system functions $H_1(z)$ and $H_2(z)$ are cascaded, find the system function $H(z)$ (and its ROC) for the overall system. What should be the relation between any $\alpha>1$ and β such that the overall system is stable? Indicate the resulting ROC for $H(z)$, if this relation is satisfied.

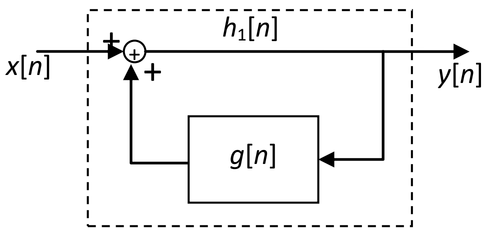


Fig.1. System 1

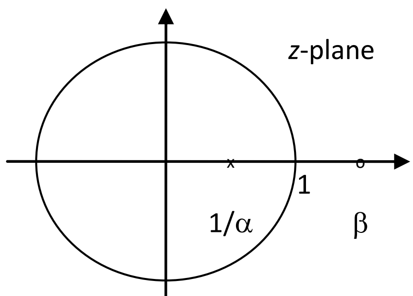


Fig.2. Pole-zero plot of $H_2(z)$

Q4) An LTI system with impulse response, $h[n]=\left(\frac{1}{3}\right)^{(n-n_0)}u[n-n_0]$, where n_0 is any integer, is excited by the input sequence $x[n]=\left(-\frac{1}{2}\right)^{n}u[n]$.

a) Is the system stable? Explain by making use of the system function $H(z)$. Is the system causal? If so, under which condition?

b) Find the *z*-transform $Y(z)$ of the output signal $y[n]$. Determine the output signal.

c) Does the Fourier transform of $y[n]$ exist? If so, find it. If not, explain the reason by making use of $Y(z)$ found in part (c).

MATLAB)

a) Read data from the audio file named `'hw5audio.wav'`, and obtain sampled data sequence, $x[n]$ in MATLAB:

```
[xn, Fs] = audioread('hw5audio.wav');
```

   i. You should observe that the length of the sequence "`xn`" is 500. After zero-padding, compute $X[k]$, the 512-point DFT of $x[n]$. Then, plot the magnitude of $X[k]$ versus k from 0 to 511.

   ii. You should observe that the magnitude of $X[k]$ contains peaks showing dominant frequencies. Find the indices of k corresponding to these peaks.

b) The sequence $x[n]$ is real-valued.

   i. Analytically show that $|X(e^{j\omega})| = |X(e^{-j\omega})|$, namely the magnitude of the DTFT is an even function, for a real signal. Then, plot the magnitude of $X[k]$ versus k from -256 to 255 by using "fftshift" command to rearrange $X[k]$. Are the indices of k, which correspond to dominant frequencies, symmetric with respect to $k = 0$?

   ii. How many dominant frequency components of this signal will be <u>heard</u>? [Hint: You may use the observation emerged from part (i).]

   iii. We know that the sampling frequency f_s, that is the "Fs" variable given in part (a), is related to the DFT length N, and we can find the frequency f_k for an analog signal by using $f_k = kf_s/N$, where k is the index of the DFT. What are the values (in Hz) of the frequencies in part (ii) for this analog audio signal?