MIDDLE EAST TECHNICAL UNIVERSITY

MECHANICAL ENGINEERING DEPARTMENT

ME 205 STATICS – FALL 2018

SECTION 1

HOMEWORK #2

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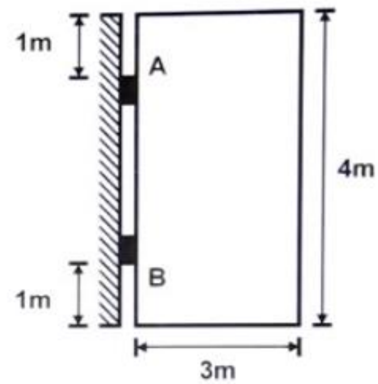
Assigned Date: 13.11.2018
Due Date: 20.11.2018
Due Time: 16.00
Grading Due Date: 04.12.2018

Please include your name, student ID, due date, a proper headline, page number with total page number, and units in your homework. Neatness will be graded.

1. A 80-kg door which is made of a uniform material is fixed by using two hinges at the points *A* and *B.* Assume that the reaction moments are zero at the hinges and it is required to find the reaction forces.

- **a.** Is the given information enough to find all of the reaction forces? Prove your answer mathematically without using the actual equations to find the numerical answers. If there are any, comment on why some of the forces cannot be found. You may use the free body diagram of the system. *(15 pts)*
- **b.** Find the reaction forces that can be obtained with the given information. *(25 pts)*
- **c.** Do the reaction forces depend on the material uniformity or hinge locations? Explain it verbally. *(10 pts)*



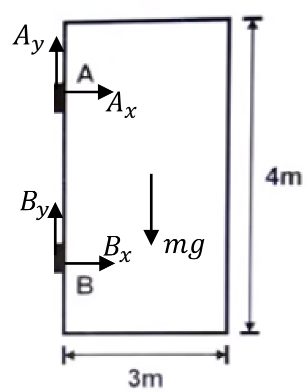
Solution:

a. In a 2D space, we can write 2 force and 1 moment equation, and that are

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_p = 0$$

However, there are 4 unknowns in the system, and they are the reaction forces at the hinges in the *x*-direction and *y*-direction, i.e. A_x, A_y, B_x, B_y. Since we have 4 unknowns but 3 equations, we cannot find all of the unknowns.

The free body diagram of the system is



Since the reaction forces along the *y*-axis are collinear, they cannot be used separately on the moment equation. Therefore, these forces cannot be found individually.

b. Now, let's write the force equations.

$$\sum F_x = 0\,; \quad A_x + B_x = 0$$
$$\sum F_y = 0\,; \quad A_y + B_y - mg = 0$$

We can find the forces A_x and B_x individually by writing a moment equation about the points *A* and *B* separately. Since the material is uniform,

$$\sum M_A = 0\,; \quad B_x(2\,m) - (80\,kg)(9.81\,m/s^2)(1.5\,m) = 0$$
$$\rightarrow B_x = 588.6\,N$$

Note that $A_x = -B_x$ from the force equation along the *x*-axis.

$$\rightarrow A_x = -588.6\,N \;\; (direction\ is\ to\ the\ left)$$

Also, we know that $A_y + B_y = (80\,kg)(9.81\,m/s^2) = 784.8\,N$

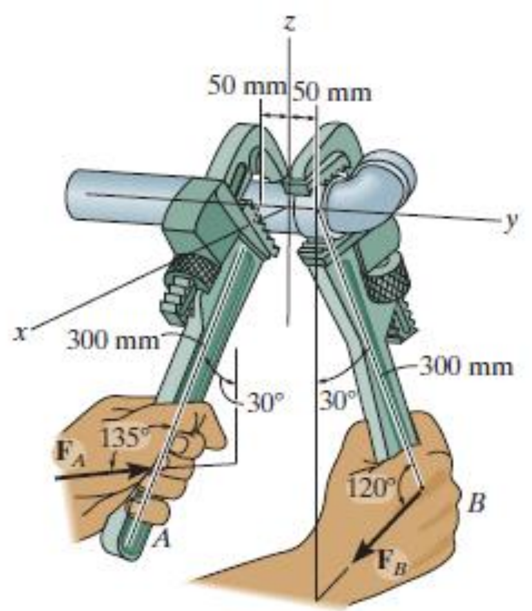
c. As we can see in the moment equations, the reaction forces along the *x*-direction depend on the length of the moment arms. Therefore, these forces depend on the material uniformity (since the location of the center of mass would change) and hinge locations.

On the other hand, since we cannot obtain the forces along the *y*-direction individually, we cannot say whether they depend on the material uniformity and hinge locations.

2. The fitting at the elbow of a pipe is tightened with the wrench *B.* The pipe is fixed in a way that it can rotate freely about *y*-axis; however, it cannot translate or rotate along any other axes. Therefore, the wrench *A* is used to provide a stationary position for the pipe.

a. What is the moment about the axis *y,* that is produced by the force F_B if $F_B = 90\,N$? Solve by using vector algebra. *(25 pts)*

b. What is the counter force applied by F_A so that the pipe remains stationary? *(25 pts)*



Solution:

a. The force F_B and its moment arm can be expressed as

$$\vec{F}_B = 90(\cos 60°\,\boldsymbol{i} - \sin 60°\,\boldsymbol{k})\,N$$
$$\vec{r}_B = 0.3(-\sin 30°\,\boldsymbol{i} - \cos 30°\,\boldsymbol{k})\,m$$

$$\left(\vec{M}_B\right)_y = \vec{r}_B \times \vec{F}_B = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -0.15 & 0 & -0.26 \\ 45 & 0 & -77.94 \end{vmatrix} = -[(-0.15)(-77.94) - (45)(-0.26)]$$
$$= -23.39\ Nm$$

Or,

$$\left(\vec{M}_B\right)_y = \vec{r}_B \times \vec{F}_B$$
$$= (90 \cdot 0.3)[(-\sin 30°\, \boldsymbol{i}) \times (\cos 60°\, \boldsymbol{i}) + (-\sin 30°\, \boldsymbol{i}) \times (-\sin 60°\, \boldsymbol{k})$$
$$+ (-\cos 30°\, \boldsymbol{k}) \times (\cos 60°\, \boldsymbol{i}) + (-\cos 30°\, \boldsymbol{k}) \times (-\sin 60°\, \boldsymbol{k})]$$
$$= 27(-0.433\boldsymbol{j} - 0.433\boldsymbol{j}) = -23.39\ Nm$$

b. The force F_A and its moment arm can be expressed as

$$\vec{F}_A = F_A(-\cos 15°\, \boldsymbol{i} + \sin 15°\, \boldsymbol{k})$$
$$\vec{r}_A = 0.3(\sin 30°\, \boldsymbol{i} - \cos 30°\, \boldsymbol{k})\ m$$
$$\left(\vec{M}_A\right)_y = \vec{r}_A \times \vec{F}_{\boldsymbol{A}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.15 & 0 & -0.26 \\ -F_A \cos 15° & 0 & F_A \sin 15° \end{vmatrix} = 0.21 F_A \boldsymbol{j} = 23.39 \boldsymbol{j}\ Nm$$
$$\rightarrow F_A = 110.2\ N$$