MIDDLE EAST TECHNICAL UNIVERSITY
Department of Electrical and Electronics Engineering

EE301 SIGNALS and SYSTEMS 1

HOMEWORK 4

Due: 22/12/2018, 23:55

Q1) Let $X(j\omega)$ denote the continuous time Fourier transform (CTFT) of the signal $x(t)$ given below:

$$x(t) = \begin{cases} 2t+2 & -1 \le t < 0 \\ 2 & 0 \le t < 2 \\ -2t+6 & 2 \le t < 3 \\ 0 & elsewhere \end{cases}.$$

Please perform the following calculations without explicitly evaluating $X(j\omega)$

a) $X(j\omega)$ can be expressed as $A(j\omega)e^{j\theta(j\omega)}$, where functions $A(j\omega)$ and $\theta(j\omega)$ are both real-valued. Find $\theta(j\omega)$.
b) Find $X(j0)$.
c) Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$.
d) Evaluate $\int_{-\infty}^{\infty} X(j\omega)\frac{\sin\omega}{\omega}e^{j3\omega}d\omega$.
e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$.
f) Find and sketch the inverse Fourier transform of $Real\{X(j\omega)\}$.

Q2) Consider an LTI system S with impulse response given below:

$$h(t) = \frac{\sin(2t)}{\pi t}$$

Please determine the output of S for each of the following inputs:

a) $x_1(t) = \sin(3t)$
b) $x_2(t) = \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k \cos\left(\frac{k}{2}t\right)$
c) $x_3(t) = \frac{\sin(3(t-1))}{\pi(t-1)}$
d) $x_4(t) = \left[\frac{\sin(t/4)}{\pi t}\right]^2$

Q3) Let the following continuous time signal $y(t)$ be

$$y(t) = [x(t)\cos^2(4t)] * \frac{\sin(2(t-1))}{\pi(t-1)}$$

where $x(t)$ is real and its Fourier transform $X(j\omega) = 0$ for $|\omega| \ge 3$.

a) Show that there exists an LTI system with impulse response $h(t)$ such that $y(t) = h(t) * x(t)$. Find the corresponding impulse response $h(t)$.
b) For $X(j\omega) = u(\omega+3) - u(\omega-3)$, find $y(t)$ and compute the integral $\int_{-\infty}^{\infty}\frac{y(t)}{1-jt}e^{-jt}dt$ by using the following CTFT pair $e^{-t}u(t) \xrightarrow{F} \frac{1}{1+j\omega}$ and duality.

Q4) Let $X(e^{j\Omega})$ denote the discrete time Fourier transform (DTFT) of the signal $x[n]$ given below:

$$x[n] = \begin{cases} n+4 & -6 \le n < 0 \\ -2n+4 & 0 \le n < 2 \\ 2n-4 & 2 \le n < 4 \\ -n+8 & 4 \le n \le 10 \\ 0 & elsewhere \end{cases}$$

Please perform the following calculations without explicitly evaluating $X(e^{j\Omega})$

a) Evaluate $X(e^{j0})$.
b) Find the angle of $X(e^{j\Omega})$, i.e., $\measuredangle X(e^{j\Omega})$.
c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\Omega}) d\Omega$.
d) Find $X(e^{j\pi})$.
e) Determine and sketch the signal whose DTFT is $Real\{X(e^{j\Omega})\}$.
f) Evaluate $\int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega$.
g) Evaluate $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\Omega})}{d\Omega}\right|^2 d\Omega$.

Q5) Let $x[n]$ be a discrete-time (DT) signal with Fourier transform $X(e^{j\Omega})$ given below:

$$X(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} \left[u\left(\Omega + \frac{\pi}{4} - 2\pi k\right) - u\left(\Omega - \frac{\pi}{4} - 2\pi k\right)\right]$$

Please find and sketch the DTFT of $w[n] = x[n]p[n]$ for each of the following signals $p[n]$:

a) i. $p[n] = \cos(\pi n)$ ii. $p[n] = \cos(\pi n/2)$ iii. $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$.

b) Suppose that the signal $w[n]$ of part (a) is applied as the input to an LTI system with unit sample response $h[n] = \frac{\sin(\pi n/3)}{\pi n}$. Determine the output $y[n]$ for each of the choices of $p[n]$ in part (a).

Q6) The Discrete Fourier Transform (DFT) of an N-point signal $x[n]$ is given by

$$X[k] = \begin{cases} e^{j\theta} & k = 1 \\ e^{-j\theta} & k = N-1 \\ 0 & elsewhere \end{cases}$$

for $0 \le k \le N-1$. Find the N-point signal $x[n]$. Simplify your answer as much as possible. Is $x[n]$ real? If so, express your answer without using j.

Q7) MATLAB Question (related to DTFT and DFT analysis): This part will be uploaded to METU-Class on 10 December 2018.