EE-302 HW4 **Due Date: 20th May 2018 Sunday, Time: 23:55**

Q1. In this question we will use MATLAB to investigate certain topics we studied in the lectures.

Consider a unity (negative) feedback control system with open-loop transfer function:

G(s)= $\frac{10}{(s+1)(s+2)(s+3)}$.

We would like to sketch the **polar plot** of $G(s)$. Remember that **a polar plot** is the part of the Nyquist plot that corresponds to positive imaginary axis ($s = j\omega, 0 < \omega < \infty$) in the Nyquist path (It is indicated by the green line (segment-1) in Figure 1).

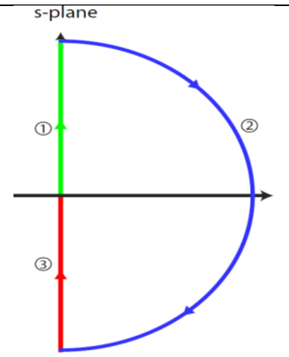


Figure 1: Nyquist path

In order to do that, we should evaluate $G(j\omega)$ for positive ω values. Let us create an ω-axis in MATLAB. The following line will create an array of ω values between 0.01 and 100 with increments of 0.01 in MATLAB.

```
w=0.01:0.01:100; %create omega axis
```

Now we can evaluate $G(j\omega)$ at these ω values.

```
Gw=10./((j*w+1).*(j*w+2).*(j*w+3)); % Evaluate G(jw)
```

Notice that we used ./ and .* instead of / and * to multiply and divide elementwise,
(see: https://www.mathworks.com/help/matlab/ref/times.html).

Now we are ready to sketch the polar plot of $G(s)$.

```
plot(real(Gw),imag(Gw)); %Plot G(jw) in the complex plane
grid on;
```

i) Given the polar plot, can you find the Gain Margin of the system?
ii) Can you find the phase margin?
Hint: You can plot the unit circle by first creating an array of theta values between 0 and 2π.
```
theta=0:0.01:2*pi; hold on;
plot(sin(theta), cos(theta),'r');
```
You can use the cursor tool (see: cursor) in the figure to find the relevant values.

So far we have plotted $G(j\omega)$ in the complex plane. Notice that, it is also possible to plot the magnitude and phase of $G(j\omega)$ separately which will correspond to the Bode plot. The following lines will sketch these plots.

```
figure(2),
plot(log10(w),20*log10(abs(Gw))); grid on;
figure(3),
plot(log10(w),angle(Gw)/pi*180); grid on;
```

iii) Given the magnitude and phase response of $G(j\omega)$, find the gain cross-over frequency, phase cross-over frequency, gain margin, phase margin of the closed loop system. Compare the phase and Gain margins with the ones you found in part (i) and (ii).

iv) One can also find the gain margin by inspecting the root locus. Remember that the gain margin is defined as the factor by which we can increase the open loop gain until the closed loop system becomes unstable, i.e., $GM = \frac{K_{unstable}}{K_{original}}$. Let us plot the root locus of the system using MATLAB. We can use the rlocus() function in MATLAB as follows. rlocus() function takes an LTI system representation as input. We can generate an LTI system representation from its transfer function as follows:

```
num=1;
den=poly([-3,-2,-1]);
s=tf(num, den);
rlocus(s);
```

(see:https://www.mathworks.com/help/matlab/ref/poly.html, https://www.mathworks.com/help/control/ref/tf.html)

From the root locus, find the K value, at which the closed loop system becomes unstable. You can use the cursor tool (see: cursor) in the figure to find the corresponding value of K. Find the gain margin from $GM = \frac{K_{unstable}}{K_{original}}$.

Notice that, to be able to use the root locus technique, one needs to know the exact open loop transfer function of the system, which is generally not possible. On the other hand, Bode-plot or the Nyquist-plot can be obtained from the frequency response of a system which can be found experimentally. In the next question, you will make a stability analysis, and design a controller for an unknown system whose frequency response is obtained experimentally.

Q2. In this question you are going to make a graphical design for an unknown plant whose frequency response information is given in a MATLAB data file. You can download the file from this link.
Load the .mat file in MATLAB. The file contains two variables: `w` and `Gunknownw` arrays, which correspond to ω axis, and $G_{unknown}(j\omega)$, evaluated at the ω values in `w` array.
Just as we did in Question-1, we can plot the magnitude and phase response of the plant:

```
figure(2),
plot(log10(w),20*log10(abs(Gunknownw))); grid on;
figure(3),
angleGW=angle(Gunknownw)/pi*180;
angleGW(angleGW>0)=angleGW(angleGW>0)-360;
plot(log10(w),angleGW);ylim([-270 0]); grid on;
```

a. What are the phase & gain cross-over frequencies, phase & gain margins of the system? Based on your results, what can you say about the stability of the closed loop system?

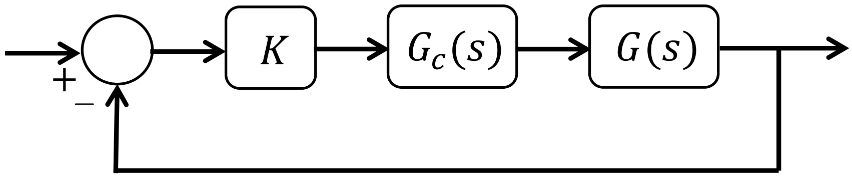
b. Design a phase-lead compensator to increase the phase margin of the closed loop system to 55^0. Please first write the transfer function $G_c(s)$ of the compensator you use and give the valid range of its parameters.

c. Obtain the Bode-plot of the compensated system. Find the new gain and phase margin of the compensated system.

d. Design a phase-lag compensator to increase the phase margin of the closed loop system to 70^0. Please first write the transfer function $G_c(s)$ of the compensator you use and give the valid range of its parameters.

e. Obtain the Bode-plot of the compensated system. Find the new gain and phase margin of the compensated system.

Q3. Consider the feedback system on the right where $G(s) = \frac{0.2}{s^2(s+100)}$.



a. Find the gain K so that the unit acceleration/parabola steady-state error is less than or equal to 0.25.

b. Calculate the gain cross-over frequency and phase margin of the uncompensated system (i.e., when $G_c(s) = 1$). Based on your results, what can you say about the stability of the closed loop system?

c. Do you prefer to use a phase-lead or a phase-lag compensator for this system? Explain your reasoning clearly.

d. Design the compensator $G_c(s)$ you choose in part-b to increase the phase margin of the closed loop system to 40^0. Please first write the transfer function $G_c(s)$ of the compensator you use and give the valid range of its parameters.