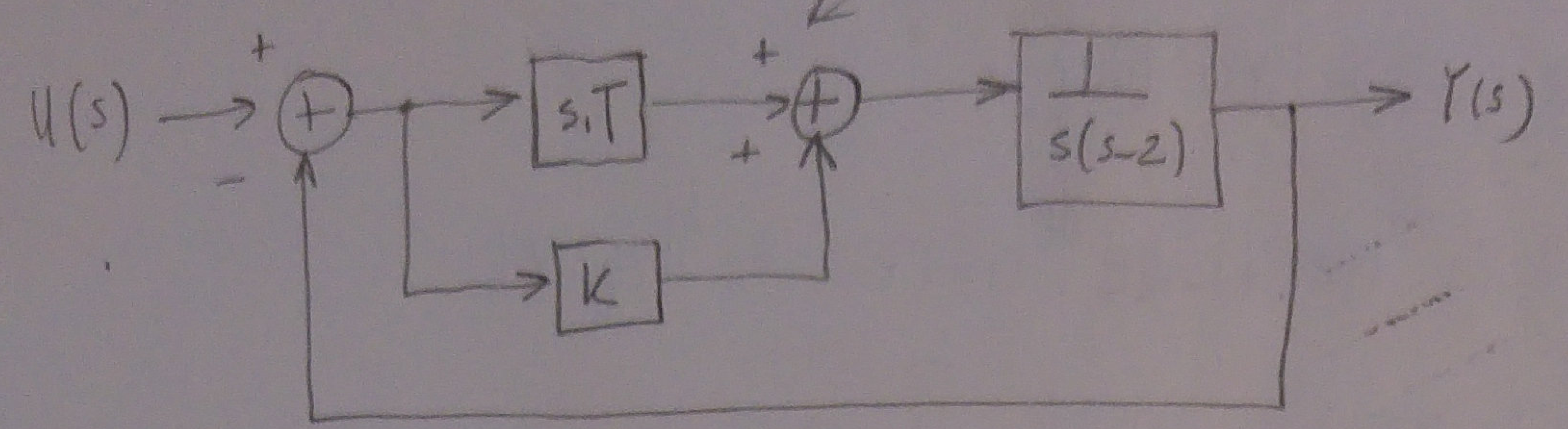
March 16, 2015
Handout ?

Transient Analysis & Design Example

Given the system Below:

This is a "PD" or proportional derivative controller.



(a) Select T and K such that $\xi = 0.6$ and $w_n = 10$ for the closed loop system.

(b) Calculate $y(t)$ when the input is a unit step function.

(c) Using the results of part b, Find M_p, t_p and t_s? (for the 2% tolerance)

Solution:

(a) First find the transfer function of the system and match with standard form

$$T(s) = \frac{\frac{sT+k}{s(s-2)}}{1+\frac{sT+k}{s(s-2)}} = \frac{sT+k}{\underbrace{s^2+(T-2)s+k}_{s^2+2\xi w_n s+w_n^2}}$$

$\xi = 0.6$

$W_n = 10 \longrightarrow w_n^2 = 100 \longrightarrow \boxed{K=100}$

$2\xi w_n = \underbrace{2(0.6)10}_{12} = T-2 \longrightarrow \boxed{T=14}$

(b) $Y(s) = \dfrac{14s+100}{s^2+12s+100}\, U(s)$ and $U(s) = \dfrac{1}{s}$

$$Y(s) = \frac{14s+100}{s(s^2+12s+100)} = \frac{A}{s} + \frac{Bs+C}{s^2+12s+100} = \frac{As^2+12As+100A+Bs^2+Cs}{s(s^2+12s+10)}$$

$$\left.\begin{aligned} A+B &= \emptyset \\ 12A+C &= 14 \\ 100A &= 100 \end{aligned}\right\} \quad \begin{aligned} A &= 1 \\ B &= -1 \\ C &= 14-12 = 2 \end{aligned}$$

$$= \frac{1}{s} - \frac{s+2}{s^2+12s+100}$$

$\Delta = 144 - 4 \cdot 100 < \emptyset$

Complex conjugate poles.

$\rightarrow$ Again use similar approach to obtain transforms of sine and cosine terms: $s^2+12s+100 = (s^2+12s+36)+64$

$$Y(s) = \frac{1}{s} - \underbrace{\frac{s+6}{(s+6)^2+64}}_{e^{-6t}.\cos 8t} + \underbrace{\frac{8}{(s+6)^2+64}}_{e^{-6t}.\sin 8t} \longrightarrow y(t) = \left[1 - e^{-6t}\cos(8t) + e^{-6t}\sin(8t)\right].u(t)$$

(c) Find t_p, M_p and t_s:

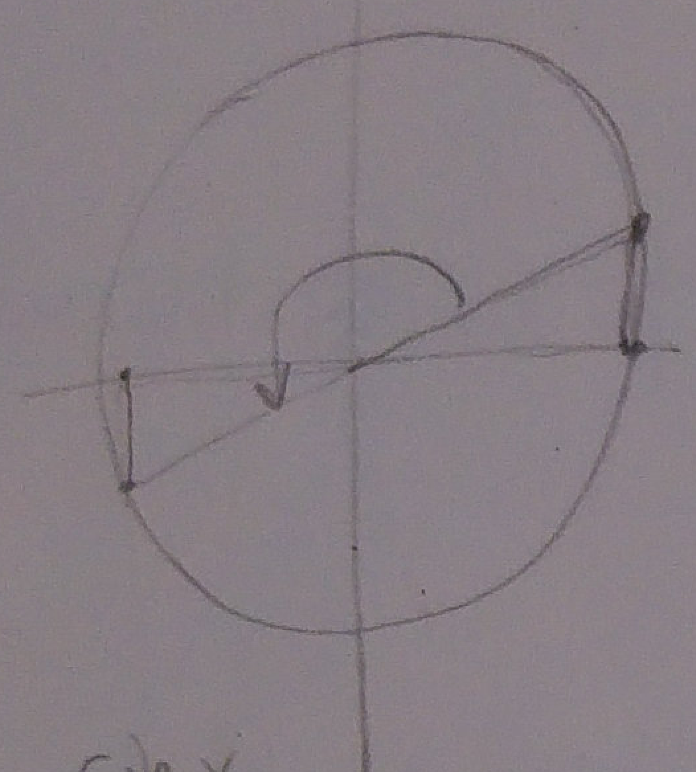
To find t_p, M_p we may use the formulas. But we are asked to use results of b:

At t_p; $\frac{dy(t)}{dt} = \emptyset$ for the first time:

$$\frac{dy(t)}{dt} = 6e^{-6t}\cos 8t + 8e^{-6t}\sin 8t - 6e^{-6t}\sin 8t + 8e^{-6t}\cos 8t$$

$$= \underbrace{2e^{-6t}}_{\neq 0}\left[7\cos(8t) + \sin 8t\right] = \emptyset$$

$$7.\cos 8t = -\sin 8t \longrightarrow$$



$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan(x+\pi) = \tan x$$

$$\underline{\tan 8t} = \frac{\sin 8t}{\cos 8t} = -7$$

We are seeking the <u>first positive t value</u>.

$$(8t_p + n\pi) = -1.43 \text{ rads.}$$

if $n=0 \longrightarrow t<0$ X

$n=-1 \longrightarrow t>0$ for the first time

$$8t - \pi = -1.43$$

$$8t = 1.71 \text{ rads.}$$

$$\boxed{t_p = 0.214 \text{ secs.}}$$

$$M_p = y(t)\Big|_{t=t_p} - 1 = \boxed{0.313} \quad \boxed{31.3\%}$$

$$\cos\left(8t + \frac{\pi}{4}\right) = \cos 8t \cos\frac{\pi}{4}$$

(c) $$y(t) = \left\{1 - \sqrt{2}e^{-6t}\cos\left[8t + \frac{\pi}{4}\right]\right\}u(t)$$

Use the envelope to get approximate settling time Envelope $= 1 \pm \sqrt{2}e^{-6t}$

2% band $\longrightarrow 0.98 = 1 - \sqrt{2}e^{-6t}$

$$\longrightarrow \boxed{t_s \cong 0.71 \text{ secs.}}$$