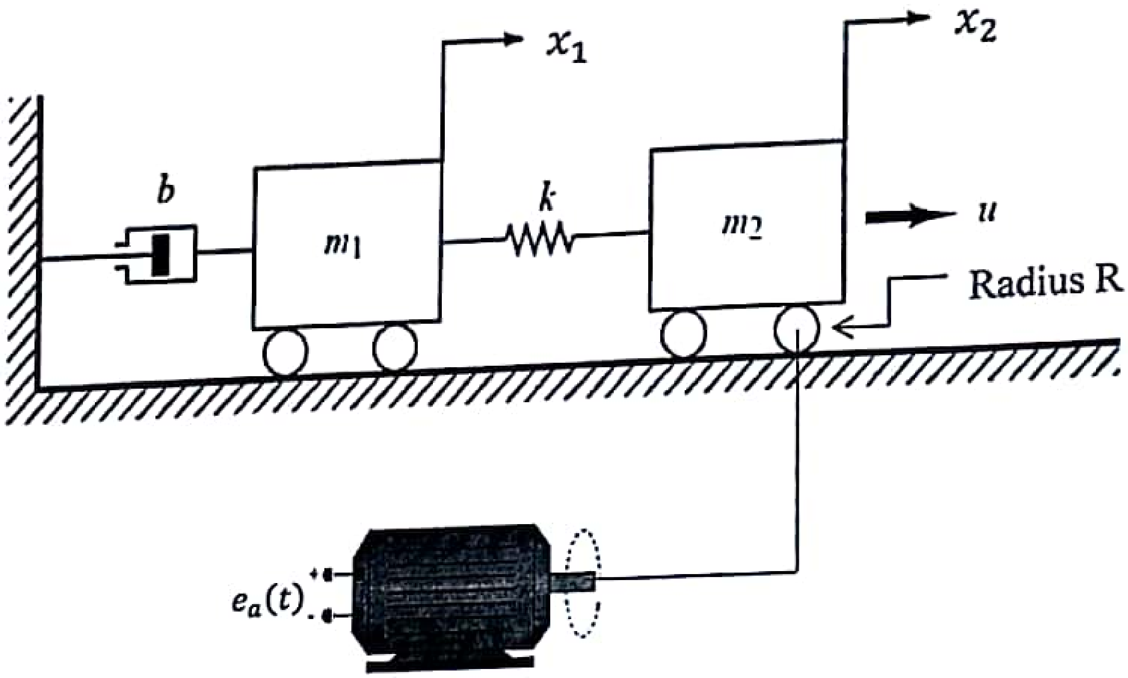
Q1. Consider the electromechanical system shown below. The mechanical part is driven by the force excitation $u(t)$. The force $u(t)$ is related to the torque T_m generated by an armature controlled DC motor whose rotor is connected to one of the wheels (of radius R) of the second mass m_2. The aim of the overall control system is to control the displacement x_2 of the second mass given a reference position denoted as v_{ref}. The input of the overall control system is the reference position v_{ref} and the output is the displacement x_2 of the second mass. The armature voltage of the DC motor is set to be proportional to the difference between the reference position v_{ref} and the system output x_2 (i.e., $e_a(t) = K_a(v_{ref}(t) - x_2(t))$. The other motor parameters are armature resistance R_a and inductance L_a, rotor friction coefficient B_m, motor back emf constant K_b, motor torque constant K_T, rotor moment of inertia J_m.

a. Obtain a block diagram representation of the system showing all the important variables, system input and output.

b. Obtain the transfer function of this system, $G(s) = \frac{X_2(s)}{V_{ref}(s)}$.



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Q2. Consider a unity (negative) feedback control system with open-loop transfer function $KG(s)$ where K is a non-negative scalar and $G(s)$ is given as

$$G(s) = \frac{s}{(s+2)(s+6)(s+7)} = \frac{s}{s^3 + 15s^2 + 68s + 84}$$

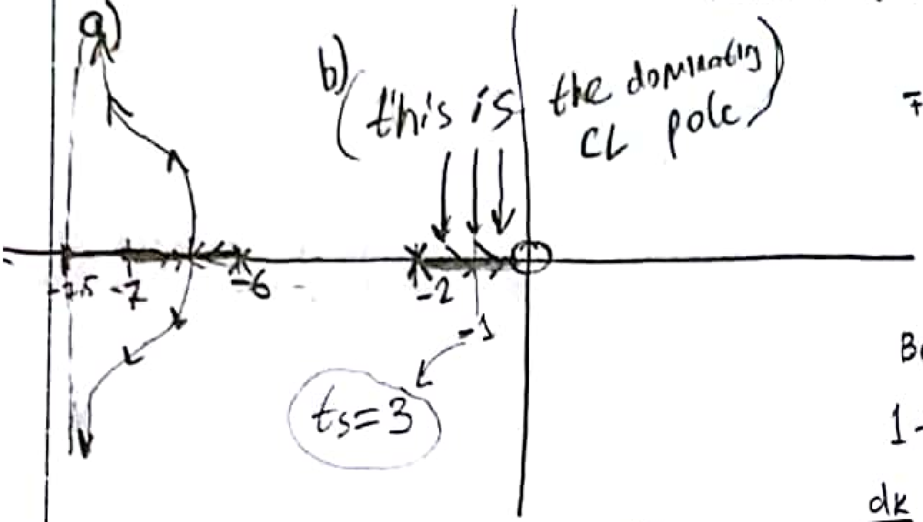
a. Draw the root-locus for the closed loop poles with all the details. (You need to show how to calculate the break-away/in points but you do not have to calculate them numerically).

b. Show on the root-locus the dominating closed-loop pole or poles.

c. Based on the root-locus, does the settling time (5%) of the closed loop system decrease or increase as K increases? Explain the reason clearly.

d. Find the value of $K \geq 0$ for which the closed loop system has settling-time (5%) $t_s = 3$ seconds.

e. Find all closed loop poles for the value of K you found in part-d?



$3-1 \rightarrow 2$ asymptotes

$\frac{\mp 180(2k+1)}{2} = +90, -90°$

$\sigma_0 = \frac{-7-6-2-0}{2} = -7.5$

Break away/in points

$1 + KG(s) = 0, \; K = -\frac{s^3+15s^2+68s+84}{s}$

$\frac{dK}{ds} = 0, \; (3s^2+30s+68).s - (s^3+15s^2+68s+84) = 0$

roots of this are breakaway-break in points.

$1 + K\frac{jw}{-jw^3 - 15w^2 + 68jw + 84} = 0$

$-jw^3 + 68jw + Kjw - 15w^2 + 84 = 0$

$w = \sqrt{\frac{84}{15}}, \; -\frac{84}{15}\sqrt{\frac{84}{15}} + 68\sqrt{\frac{84}{15}} + k = 0$

K is negative, so there is no jw crossing.

c) it increases. Because as K increases, dominant pole moves closer to origin, which causes the damping ratio to decrease.

d) $t_s = \frac{3}{\zeta w_n} \rightarrow t_s = 3$ when real part of dominant pole $= -1$

$1 + \frac{K.(-1)}{(-1+2)(-1+6)(-1+7)} = 0 \rightarrow K = 1.(5).(6) = 30$

e) $s_1 = -1$, $1 + \frac{30s}{(s+2)(s+6)(s+7)} = 0 \rightarrow s^3 + 15s^2 + 98s + 84 = 0$

$\div s+1$

$s^2 + 14s + 84 = 0$

$s^2 + 8s + 12$

$s^3 + 8s^2 + 12s + 7s^2 + 56s + 84$

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$$s^2 + 14s + 84 = 0$$

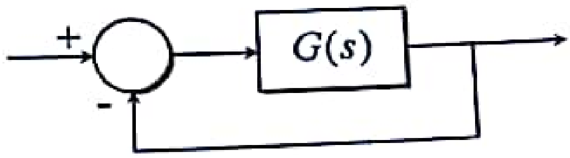
a b c

$$s_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} = \frac{-14 \mp \sqrt{196-336}}{2}$$

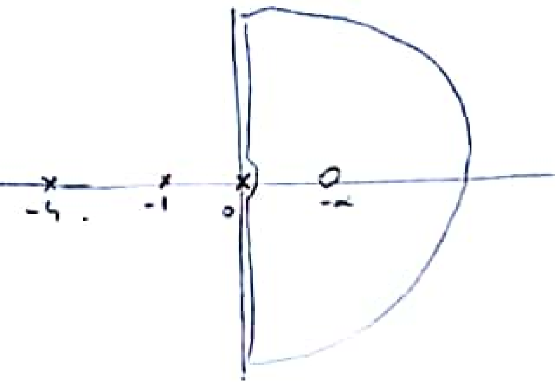
$$= -7 \mp \frac{\sqrt{-140}}{2} = \boxed{-7 \mp j\sqrt{35}}$$

Q3. The unity feedback system of the figure with the open-loop transfer function

$$G(s) = \frac{2(s+\alpha)}{s(s+4)(s+1)}$$



is given. Determine the range of "α" for stability, using <u>Nyquist</u> criterion. Show all the details of your analysis.



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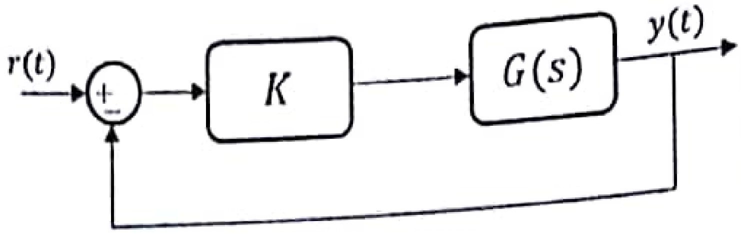
Q4. Consider the control system given on the right. The plant to be controlled has the transfer function given as

$$G(s) = \frac{s+5}{s^2}$$

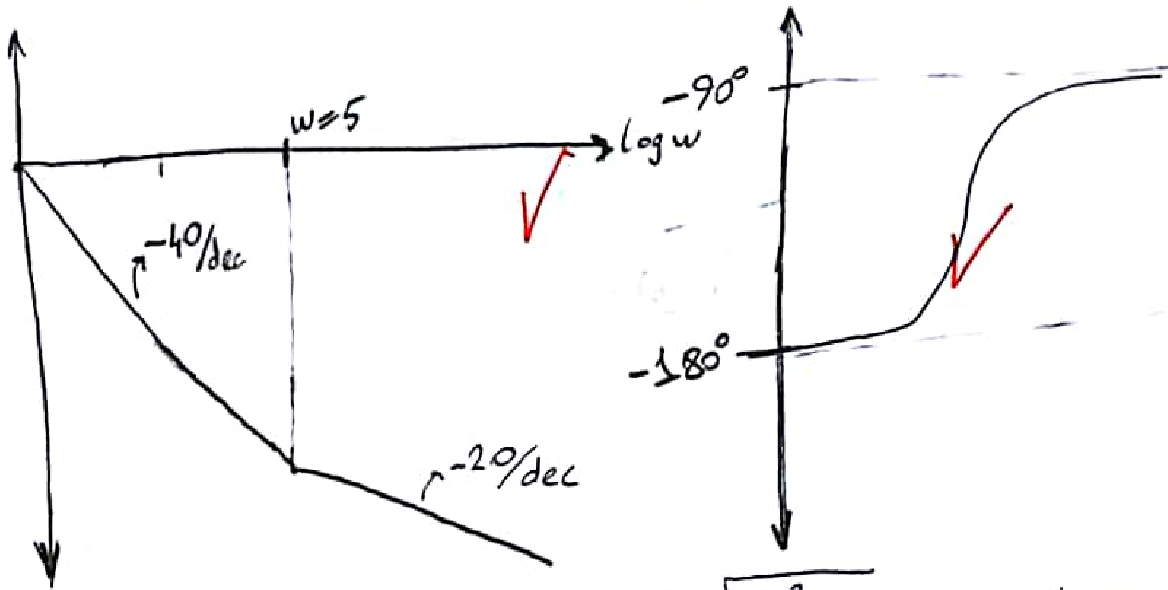


a. Draw an approximate Bode plot for $G(s)$.

b. Suppose that the system is controlled with a proportional controller as in the figure. Find K such that the system has PM=65 degrees. You need to calculate K analytically and you should not just give an approximate value based on your Bode plot in part-a.

c. Suppose that we would like to design a compensator for this system to increase the phase margin of the system above $\boxed{100}$ degrees. Which type of compensator would you use, phase-lead or phase-lag? (You do not have to design the compensator!) Explain your answer. An answer without a proper explanation will not be accepted.

a) $|G(j\omega)| = 20\log\sqrt{\omega^2+25} - 40\log\omega$, $\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{5}\right) - 180^\circ$



b) $|G(j\omega)| = \frac{K\sqrt{\omega^2+25}}{\omega^2} \rightarrow \frac{K\sqrt{5.5^2+25}}{5.5^2} = 1$, $K \approx 2.1$

$\angle G(j\omega_1) = 65 - 180 = \tan^{-1}\left(\frac{\omega_1}{5}\right) - 180$

$\tan^{-1}\left(\frac{\omega_1}{5}\right) = 65$, $\frac{\omega_1}{5} = 1.1 \rightarrow \omega_1 = 5.5$ rad/sec

c) if lead is used

$\phi_m = 100 + 5 - 65 = 40^\circ$

we will find good α, T values since ϕ_m is not too large.

if lag is used

$\angle KG(j\omega_g) = 180 - 105 = 75^\circ$

since $\angle KG(j\omega)$ never reach 75°, we would h[ave] to use lead compens[ator]

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b) (9 pts) Solution for the parameterization.

$$G_c(s) = \frac{1+aTs}{1+Ts} \qquad \frac{a>1}{T>0}$$

Calculate the internal gain.

$$G(s) \approx \frac{K_{original}}{s} \text{ as } s \to 0 \qquad |G(j\omega)| = \frac{K_{original}}{\omega} \text{ as } \omega \to 0$$

Choose $\omega \cong 5\,10^{-2} \Rightarrow G(j\omega) = 20\,dB = 10$

$$\frac{K_{original}}{5\,10^{-2}} = 10 \Rightarrow K_{original} = 0.5$$

$$K_v = \frac{1}{e_{ss}} = 10 = K\,K_{original} \Rightarrow K = \underbrace{20}_{2\times 10} \cong 26\,dB.$$

Uncomp. ω_g & PM.

$$\omega_g \cong \sqrt{2\times 3} = \sqrt{6} \qquad PM \cong -195^\circ + 180 = -15^\circ$$

$$\phi_m = PM_{req} - PM_{unc.} + 5^\circ = 30 - (-15) + 5 = 50^\circ$$

$$a = \frac{1+\sin 50^\circ}{1-\sin 50^\circ} = \frac{1+0.75}{1-0.75} = \frac{1.75}{0.25} = 7$$

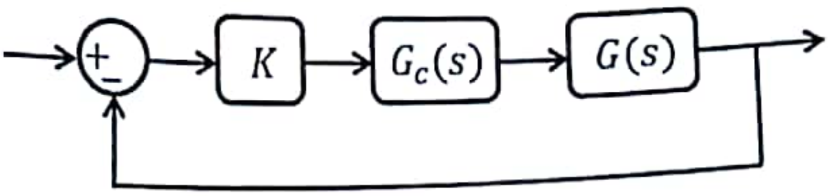
Find ω_{gc} (the new gain cross-over freq)

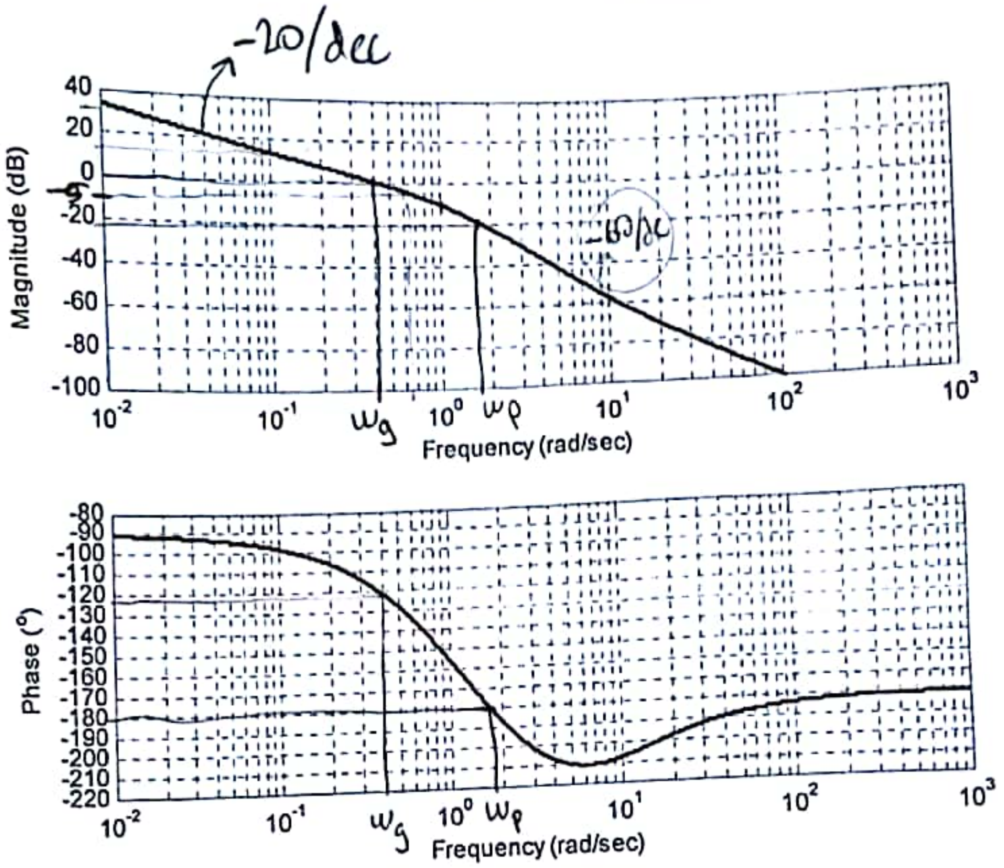
$$|G_{uncomp}(j\omega_{gc})| = \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{7}} = -10\log 7 \approx -8.5\,dB$$

$$\omega_{gc} \approx 4\text{ rad/sec} = \omega_m = \frac{1}{\sqrt{a}\,T}$$

$$\Rightarrow T = \frac{1}{\sqrt{7}\,4} = \frac{1}{2.7\times 4} \approx \underline{0.1} \Rightarrow \boxed{G_c(s) = \frac{1+0.7s}{1+0.1s}}$$

Q5. Consider the control system given on the right. The plant $G(s)$ to be controlled has the Bode plot given below.





a. What is the type of the system? Explain your answer.

b. If the plant $G(s)$ is known to have one zero, how many poles does $G(s)$ have? Explain your answer.

c. What are the phase and gain cross-over frequencies of the system when $K = 1$ and $G_c(s) = 1$?

d. What are the phase and gain margins of the system when $K = 1$ and $G_c(s) = 1$?

e. Is the closed loop system stable when $K = 1$ and $G_c(s) = 1$? Explain your answer.

f. Design a phase-lead compensator (K and $G_c(s)$) such that

- $e_{ss} \leq 0.1$ to unit-ramp input
- $PM \geq 30$ degrees

Before designing the compensator, first write the transfer function $G_c(s)$ you use and the range of its parameters. In your design use the graphs of the sine/cosine and tangent functions given in the formula page if needed. Clearly denote the values you have read from the graph.

(Q5-) (SOLUTION) 13

a.-) System magnitude plot has slope $-20\,dB/dec$ as $w \to \infty$
(1pts) System phase plot $\to -90^\circ$ as $w \to 0$

$\Rightarrow$ System is Type-1

b-)
(2pts) $\underline{/G(jw)} \longrightarrow -180$ as $w \to \infty$

$\Rightarrow$ System has 2 more poles than the # of zeros.

$\Rightarrow G(s)$ has 3 poles.

c-) $w_p \cong 1.5\,rad/sec$ (phase cross-over freq.)
(2pts) $w_g \cong 0.4\,rad/sec$ (gain cross-over freq.)

d-)
(2pts) $PM = \underline{/G(jw_g)} + 180^\circ \cong -120 + 180^\circ = 60^\circ$

$GM = -20\log|G(jw_p)| = -(-20) = 20\,dB$

e-) Yes because both PM and GM (dB) are
2pts) positive.

(1 pts)

f-) Solution for the parameterization $G_c(s) = \dfrac{1+Ts}{1+\alpha Ts}$
(9pts)

$0 < \alpha < 1$

$T > 0$

Calculate the internal gain of the system.

$G(s) \approx \dfrac{K_{original}}{s}$ as $s \to 0 \Rightarrow |G(jw)| = \dfrac{K_o}{\ldots}$

as $w \to 0$

Choose $\omega \cong 5\ 10^{-2} \Rightarrow |G(j\omega)| = 20dB = 10$

$$\rightarrow \frac{K_{original}}{5\ 10^{-2}} = 10 \Rightarrow K_{original} = 0.5$$

$$K_v = \frac{1}{e_{ss}} = 10 = K\, K_{original}$$

$$\Rightarrow \boxed{K = 20} = 20 \log \underbrace{20}_{2\times 10} \cong 26\,dB$$

Uncompensated ω_g & PM

$$\Rightarrow \quad \omega_g \cong \sqrt{2\times 3} = \sqrt{6}$$

$$PM \cong -195^\circ + 180 = -15^\circ$$

$$\phi_m = PM_{required} - PM_{uncomp} + 5^\circ$$

$$= 30 - (-15) + 5 = 50^\circ$$

$$\Rightarrow \alpha = \frac{1 - \sin 50^\circ}{1 + \sin 50^\circ} = \frac{1 - 0.75}{1 + 0.75} = \frac{0.25}{1.75} = \frac{1}{7}$$

Find the new gain cross-over freq ω_{gc}.

$$|G_{uncomp}(j\omega_{gc})| = \sqrt{\alpha} =$$

$$\sqrt{\alpha} = \frac{1}{\sqrt{7}} = -10 \log 7$$

$$\cong -10 \log\sqrt{50} = -5 \log \frac{100}{2}$$

$$= -5(2-0.3) = -5 \times 1.7 = \underline{\underline{-8.5}} \text{ dB}.$$

$$\omega_{gc} \approx 4 \text{ rad/sec} = \omega_m = \frac{1}{\sqrt{\alpha}\,T}$$

$$\Rightarrow T = \frac{1}{4\sqrt{\frac{1}{7}}} \cong \frac{1}{\sqrt{2}} \approx 0.7$$

$$\Rightarrow \boxed{G_c(s) = \frac{1+0.7s}{1+0.1s}}$$

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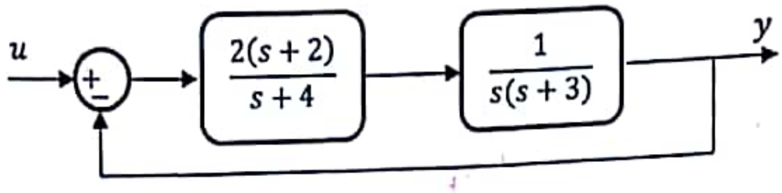
Q6. This question is composed of two independent parts.

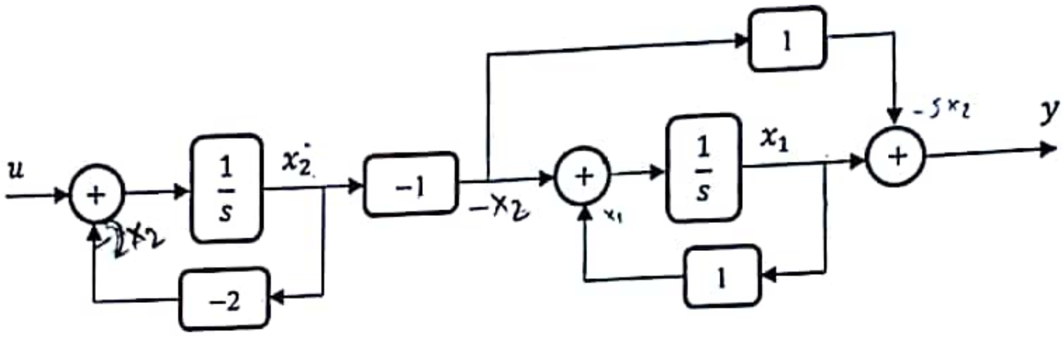
Part I:

a. Obtain the observable canonical state-space representation for the overall system given in block diagram form below:



b. Is the representation you have derived in (a) completely controllable? Clearly justify your result.

Part II: Consider the block diagram representation of a system given below where x_1 and x_2 are defined as system states:



a. Derive the state-space equations of the system and determine the A, B and C matrices,

b. Prove that the representation is completely controllable,

c. Draw the block diagram of a state-feedback control structure for the given system,

d. Using the pole-placement method and the structure you suggest in (c), design the state-feedback controller (determine feedback gains) to have both closed-loop poles of the overall system located at $s_{1,2} = -1$.

II a) $-2x_2 + u = sx_2$, $(-x_2 + x_1) = sx_1$

$-x_2 + x_1 = y$

$sx_1 = \dot{x}_1 = -x_2 + x_1$

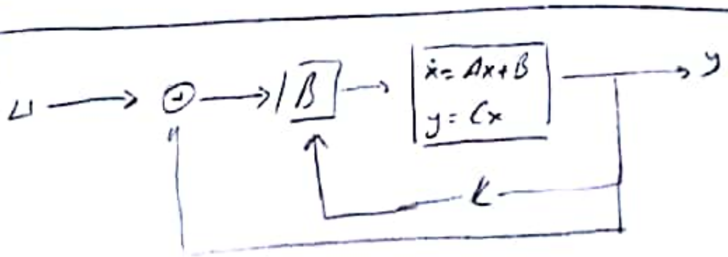
$sx_2 = \dot{x}_2 = -2x_2 + u$

2/2

$$\Rightarrow \dot{x} = \overbrace{\begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}}^{A} x + \overbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}^{B} u \qquad y = \overbrace{[1 \ -1]}^{C} x$$

IIb) $\underbrace{[B \mid AB]}_{Q} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$ $\det(Q) = 1 \neq 0$ $\Rightarrow$ Controllable.

2/2



0/2

d) 0/3