Middle East Technical University
Department of Electrical and Electronics Engineering
EE301: Signals and Systems
Fall 2018-2019

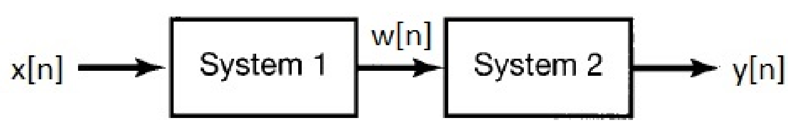
Homework 2
due: 23:55, Nov. 4

Reading assignment: Following sections from Oppenheim: 2.0-2.4, 3.0-3.3

1. In this question, we will practice our convolution skills.
   (a) Consider two DT signals: $x[n] = \left(\frac{1}{2}\right)^n u[n-1]$ and $w[n] = u[n+1]$. Compute $x[n] * w[n]$.
   (b) Now let $x[n] = \left(\frac{1}{2}\right)^n (u[n-1] - u[n-100]) + \delta[n]$, and $w[n] = u[2n]$. Compute again $x[n] * w[n]$.
   (c) Suppose the input to a CT LTI system is as follows:
   $$x(t) = \begin{cases} 2-t, & \text{if } 0 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$
   The impulse response of the system is given by $h(t) = u(t) - u(t-2)$. Find the response of the system for this input signal. (**Hint:** Plot $x(\tau)$, $h(-\tau)$, $h(t_0 - \tau)$ for different t_0's.)

2. In this question, we will consider DT systems described by difference equations. Note that a DT LTI system whose impulse response has a finite duration (i.e. finite number of nonzero values) is often called a *finite impulse response (FIR) system*. Conversely, if a DT LTI system has an impulse response of infinite duration, the system is referred as *infinite impulse response (IIR) system*.
   (a) Suppose the input-output relation is given by $y[n] = x[n] - \frac{1}{2}x[n-2]$. Is this system LTI? (Do you need to have initially at rest condition to prove your result? Explain.) If the system is LTI, find its impulse response and state whether it is FIR or IIR.
   (b) Suppose the input-output relation is given by $y[n] = x[n] - \frac{1}{2}y[n-2]$, and the system is initially at rest. Is this system LTI? If so, find its impulse response and state whether it is FIR or IIR.

3. Consider the following interconnected system:



   System 1 is an LTI system with impulse response $u[n+2]$. The input-output relationship for System 2 is
   $$y[n] = \sum_{m=-3}^{-1} w[n-m].$$
   (a) Is System 1 memoryless, causal, and/or stable? Justify your answers.
   (b) Is System 2 LTI? Justify your answer.
   (c) Find the impulse response of the overall system. Simplify your answer as much as possible.
   (d) Find the input-output relationship for the overall system as a difference equation. Simplify your answer as much as possible.
   (e) Is the overall system LTI, memoryless, causal, and/or stable? Justify your answers.

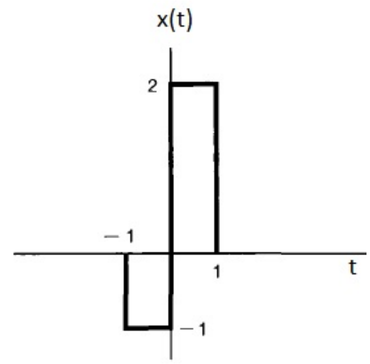
4. Consider a system whose input-output relationship is given by

$$y(t) = \int_{-\infty}^{t} e^{-2(t-\tau-1)} x(\tau) d\tau$$

(a) Verify that this is an LTI system.

(b) Find the impulse response of this LTI system.

(c) Find the output of the system for the following input signal:



(d) Determine whether the system is memoryless, causal, and/or stable. Justify your answers. (**Hint:** You can use your answer to part (b) together with the properties of LTI systems.)

(e) **(BONUS)** Repeat part (b) when the input-output relationship is

$$y(t) = \int_{t-4}^{t+4} e^{-2(\tau-t)} x(\tau - 1) d\tau + x(t-1)$$

5. Let

$$x(t) = \begin{cases} 2, & 0 < t < 1 \\ 0, & 1 < t < 3 \end{cases}$$

be a periodic signal with fundamental period $T_0 = 3$.

(a) Plot the signal $x(t)$ in the interval $[-5, 5]$.

(b) Find the CTFS coefficients of $x(t)$ analytically.

(c) Plot the CTFS coefficients a_k obtained in part (b) in MATLAB for $-8 \leq k \leq 8$.

(d) Let's define an approximation to $x(t)$ as follows:

$$x_M(t) = \sum_{k=-M}^{M} a_k e^{jk\omega_0 t}$$

which includes up to and including Mth harmonic components. In MATLAB, plot the approximation $x_M(t)$ using only

i. the DC component (i.e. $M = 0$)
ii. upto and including the fundamental components
iii. upto and including the 2nd harmonic components
iv. upto and including the 4th harmonic components
v. upto and including the 8th harmonic components

Is $x_M(t)$ equal to $x(t)$ for some finite M? For any finite M, are there always ripples on $x_M(t)$? Comment on your results using Gibbs phenomenon.

6. **(BONUS)** This question aims to illustrate that we should be cautious when we apply the convolution operator on modified signals (such as after scaling, reversal, or shifting).

(a) Suppose $y(t) = x(t) * h(t)$. Compute the following convolution: $x(2t) * h(2t)$. You can provide an integral that involves the signals x and h, but try to simplify it as much as possible. Is the result equal to $y(2t)$? (**Hint:** When computing $x(2t) * h(2t)$, you should first treat each signal as a new signal like $u(t) = x(2t)$ and $v(t) = h(2t)$, and write the convolution integral for $u(t) * v(t)$.)

(b) Now consider the DT case. Suppose $y[n] = x[n] * h[n]$. Compute the following convolution: $x[2n] * h[2n]$. You can provide a summation that involves the signals x and h, but try to simplify it as much as possible. Is the result equal to $y[2n]$? (Same hint above.)