MIDDLE EAST TECHNICAL UNIVERSITY

MECHANICAL ENGINEERING DEPARTMENT

ME 205 STATICS – FALL 2018

SECTION 1

HOMEWORK #3

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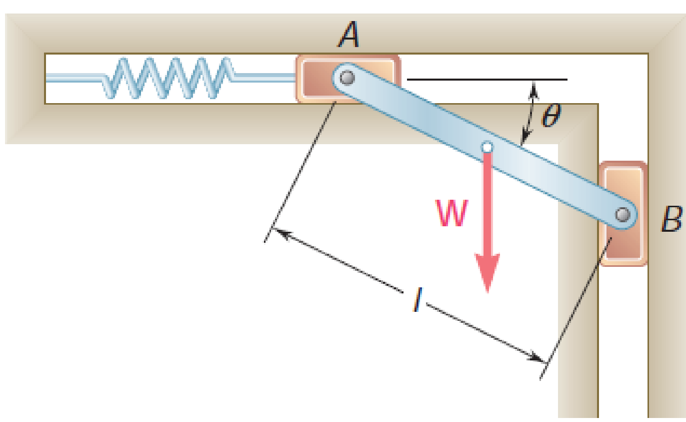
Assigned Date: 28.11.2018
Due Date: 05.12.2018
Due Time: 14.00
Grading Due Date: 12.12.2018

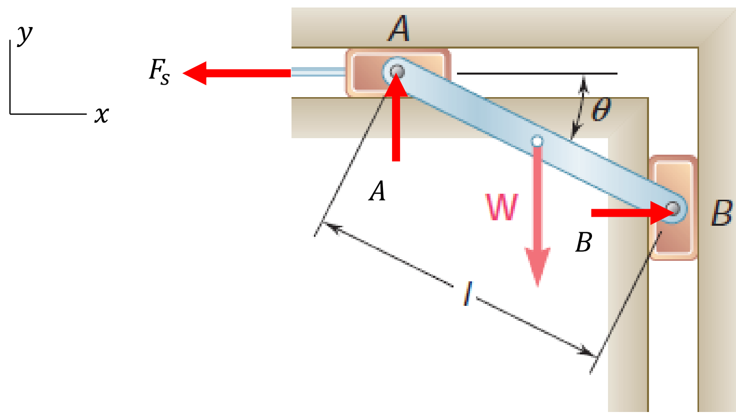
Please include your name, student ID, due date, a proper headline, page number with total page number, and units in your homework. Neatness will be graded.

1. The rod *AB* is attached to the block *A* which can translate freely in the $x -$ axis and the block *B* which can translate freely in the $y -$ axis. The weight of the rod is $W,$ the length of the rod is l, and the spring constant is k. The spring is unstretched when $\theta = 0$. The weights of the blocks are neglected. Determine,

a. The equilibrium equation in terms of W, k, l, and θ,
b. The value of θ when $W = 2kl$.



Solution:



$$F_s = k(l - l\cos\theta) = kl(1 - \cos\theta)$$

$$\sum F_y = 0; \quad A - W = 0$$

$$\rightarrow A = W$$

$$\sum M_B = 0; \quad W\frac{l\cos\theta}{2} - Al\cos\theta + F_s l\sin\theta = 0$$

$$\rightarrow kl^2(1 - \cos\theta)\sin\theta = W\frac{l\cos\theta}{2}$$

$$(1 - \cos\theta)\tan\theta = \frac{W}{2kl} \quad (\boldsymbol{a})$$

When $W = 2kl$,

$$(1 - \cos\theta)\tan\theta = 1$$

Solving numerically to obtain,

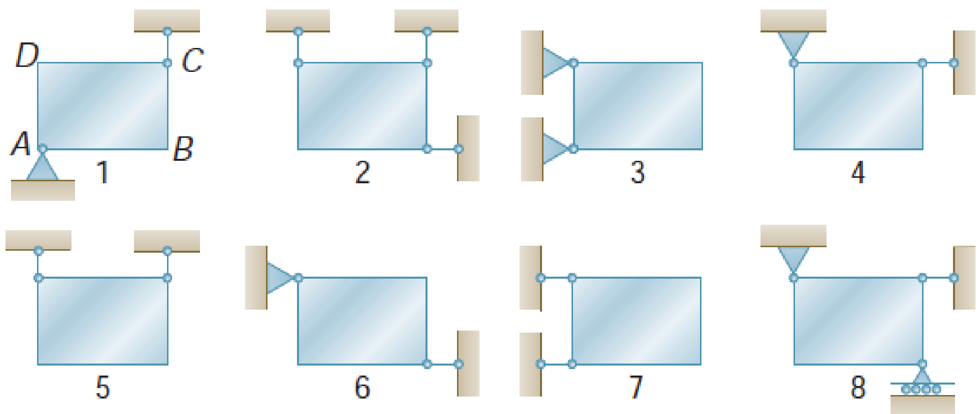
$$\theta = 62.03\,° \quad (\boldsymbol{b})$$

2. In the following figure, there are 8 plates, each of dimensions 400 x 250-mm and weight 200-N. The plates are held in vertical plane and take. In each case, determine

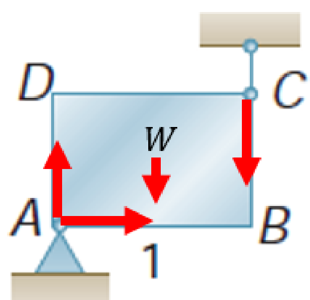
a. Whether the reactions are statically determinate or indeterminate,

b. Whether the equilibrium statically exist,

c. The reaction forces if possible.

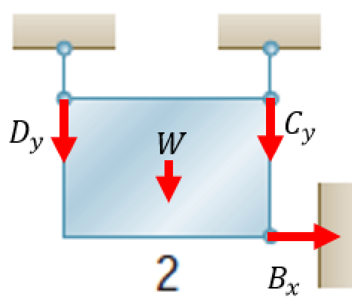


Solution:



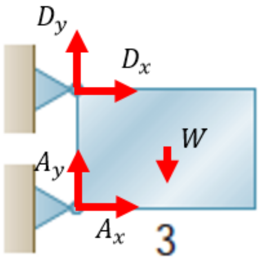
Reactions are determinate and equilibrium exists.

$$A_x = 0$$
$$A_y = -100\ N$$
$$C_y = 100\ N$$



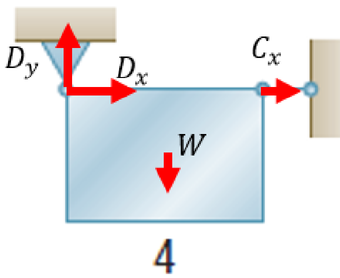
Reactions are determinate and equilibrium exists.

$$B_x = 0$$
$$C_y = D_y = -100\ N$$



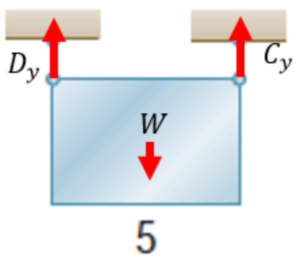
Reactions are indeterminate and equilibrium exists.

$$A_x = 160\ N$$
$$D_x = -160\ N$$
$$A_y + D_y = 200\ N$$



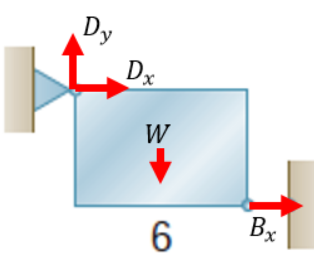
Reactions are indeterminate and equilibrium does not exist.

$$\sum M_D \neq 0$$



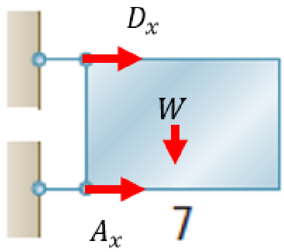
Reactions are determinate and equilibrium exists.

$$C_y = D_y = 100\ N$$



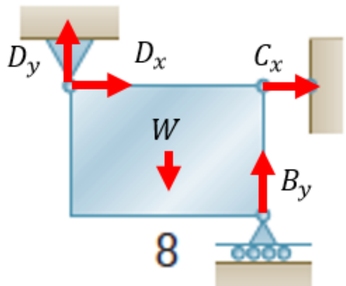
Reactions are determinate and equilibrium exists.

$$B_x = 160\ N$$
$$D_x = -160\ N$$
$$D_y = 200\ N$$



Reactions are determinate but equilibrium does not exist.

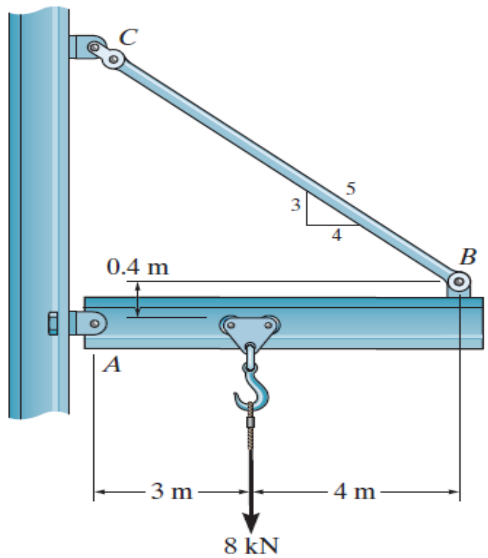
$$\sum F_y \neq 0$$



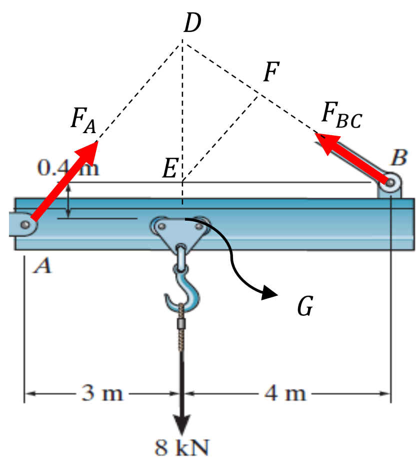
Reactions are indeterminate but equilibrium exists.

$$B_y = D_y = 100\ N$$
$$C_x + D_x = 0$$

3. In the following figure, find the reaction force at the pin *A* and the tension force at the link *CB* by using the properties of a three – force member.



Solution:



$$|EB| = 4\ m, |DE| = 3\ m, |EG| = 0.4\ m, |AG| = 3\ m, |AD| \mathbin{/\!/} |EF|$$

$$\angle DAG = \angle FEB = \theta = \tan^{-1}\frac{|DE| + |EG|}{|AG|} = \tan^{-1}\frac{3.4}{3} = 48.58\,°$$

$$\angle EDF = \alpha = \tan^{-1}\frac{|EB|}{|DE|} = \tan^{-1}\frac{4}{3} = 53.13\,°$$

$$\angle FED = \beta = 90 - \theta = 41.42\,°$$

$$\angle EFD = \gamma = 180 - \alpha - \beta = 85.45\,°$$

The force triangle is DEF. Apply the law of sines to get

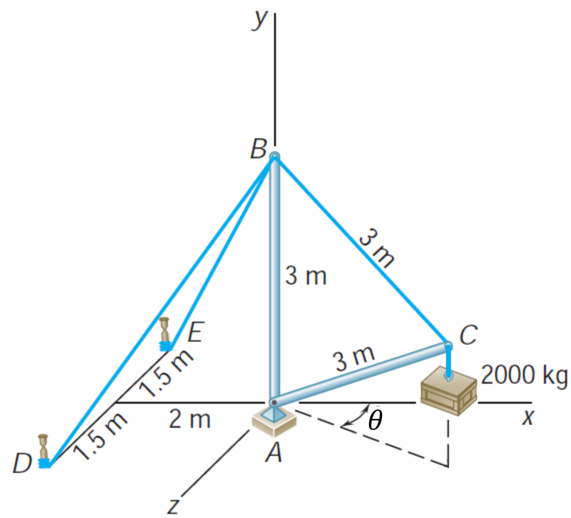
$$\frac{8\ kN}{\sin\gamma} = \frac{F_{BC}}{\sin\beta} = \frac{F_A}{\sin\alpha}$$

$$\rightarrow F_{BC} = 5.31\ kN \quad \& \quad F_A = 6.42\ kN$$

4. The 2000-kg crate is supported by a link-and-cable system which is held by a ball-and-socket joint at *A* and by two cables attached at *D* and *E* as shown in the figure. The link *AC* forms an angle θ with the xy – plane and $0 < \theta < 90°$. Determine

a. The tension forces $\vec{F}_{BD}$ and $\vec{F}_{BE}$ in terms of θ,

b. The magnitude of the reaction force at *A* when $\theta = 30$ °,



Solution:

$$\vec{F}_{BD} = F_{BD}\vec{u}_{BD} \quad \& \quad \vec{F}_{BE} = F_{BE}\vec{u}_{BE}$$

$$\vec{u}_{BD} = \frac{-2\boldsymbol{i} - 3\boldsymbol{j} + 1.5\boldsymbol{k}}{\sqrt{(-2)^2 + (-3)^2 + 1.5^2}} = -0.51\boldsymbol{i} - 0.77\boldsymbol{j} + 0.384\boldsymbol{k}$$

$$\vec{u}_{BE} = \frac{-2\boldsymbol{i} - 3\boldsymbol{j} - 1.5\boldsymbol{k}}{\sqrt{(-2)^2 + (-3)^2 + 1.5^2}} = -0.51\boldsymbol{i} - 0.77\boldsymbol{j} - 0.384\boldsymbol{k}$$

$$W = (2000)(9.81) = 19.62\ kN$$

$$\sum F_x = 0; \quad A_x - 0.51F_{BD} - 0.51F_{BE} = 0 \quad (1)$$

$$\sum F_y = 0; \quad A_y - 0.77F_{BD} - 0.77F_{BE} - W = 0 \quad (2)$$

$$\sum F_z = 0; \quad A_z + 0.384F_{BD} - 0.384F_{BE} = 0 \quad (3)$$

$$\sum (M_A)_x = 0; \quad (0.384F_{BD} - 0.384F_{BE})(3) + (W)(3\cos 30° \sin\theta) = 0 \quad (4)$$

$$\sum (M_A)_z = 0; \quad (0.51F_{BD} + 0.51F_{BE})(3) - (W)(3\cos 30° \cos\theta) = 0 \quad (5)$$

From (4)

$$F_{BD} - F_{BE} = -2.255W\sin\theta \quad (6)$$

From (5)

$$F_{BD} + F_{BE} = 1.7W\cos\theta \quad (7)$$

By solving (6) & (7)

$$F_{BD} = 16.677\cos\theta - 22.12\sin\theta \quad kN$$
$$F_{BE} = 16.677\cos\theta + 22.12\sin\theta \quad kN$$

Then,

$$\vec{F}_{BD} = (-8.51\cos\theta + 11.28\sin\theta)\boldsymbol{i} + (-12.84\cos\theta + 17\sin\theta)\boldsymbol{j} + (6.4\cos\theta - 8.5\sin\theta)\boldsymbol{k} \ \ kN$$
$$\vec{F}_{BE} = (-8.51\cos\theta + 11.28\sin\theta)\boldsymbol{i} + (-12.84\cos\theta + 17\sin\theta)\boldsymbol{j} + (-6.4\cos\theta + 8.5\sin\theta)\boldsymbol{k} \ \ kN$$

At $\theta = 30°$

$$A_x = 0.51F_{BD} + 0.51F_{BE} = 14.73\ kN$$
$$A_y = 0.77F_{BD} + 0.77F_{BE} + W = 41.864\ kN$$
$$A_z = -0.384F_{BD} + 0.384F_{BE} = 8.5\ kN$$
$$F_A = \sqrt{A_x^2 + A_y^2 + A_z^2} = 45.1857\ kN$$