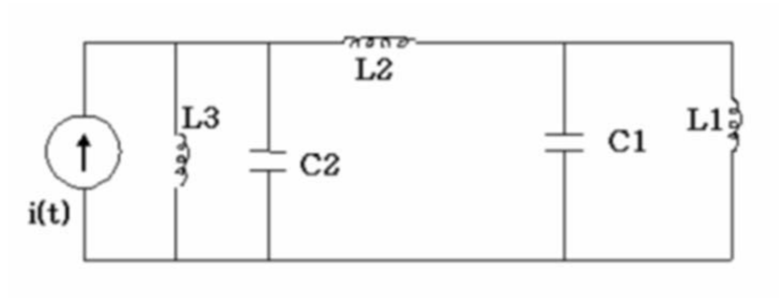
EE302 HW1 **Due Date: 23.03.2016 Thursday Time: 23:55**

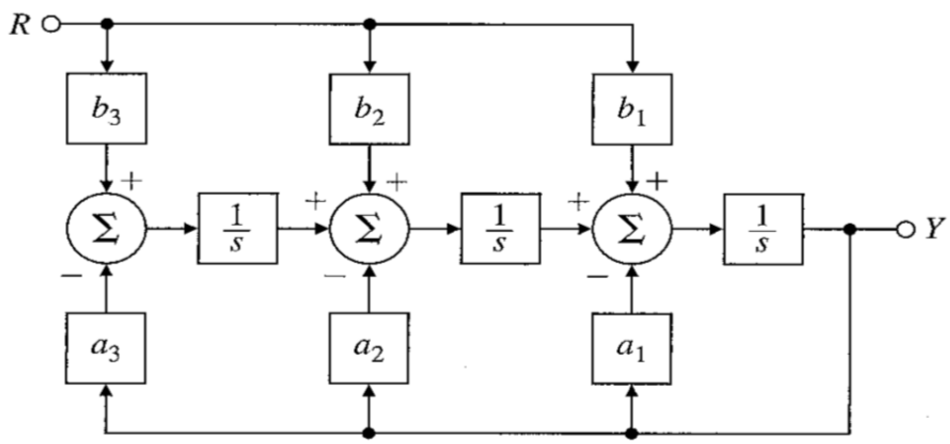
1. Problem 6 from Nise (7th edition, Chapter 3).

2. Consider the following circuit where the output is the voltage of C_1 and the input is the current source *i(t)*. Obtain the differential equation, transfer function and state space representations for the system.

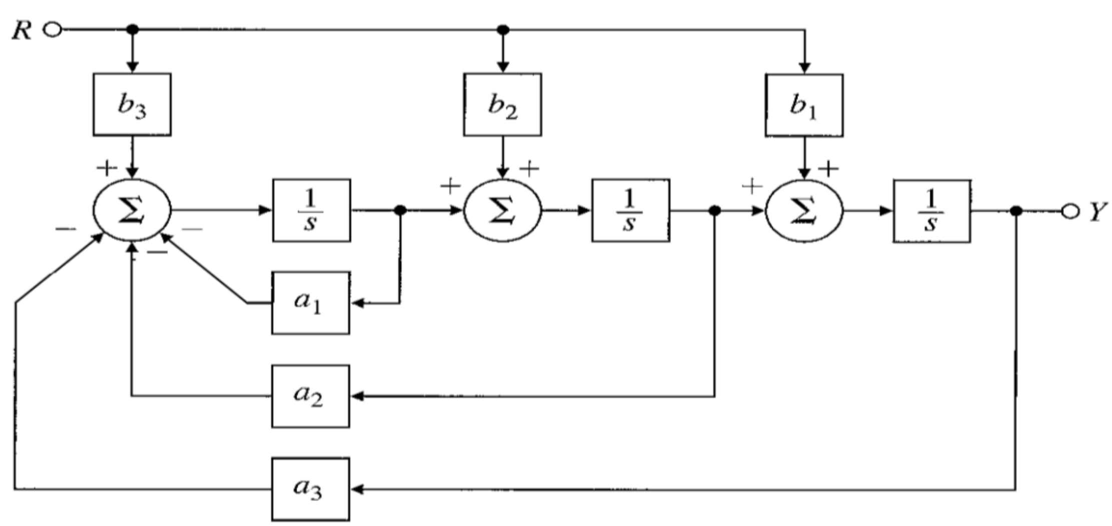


Electric Circuit

3. Find the transfer functions associated with the following block diagram representations:



(a)



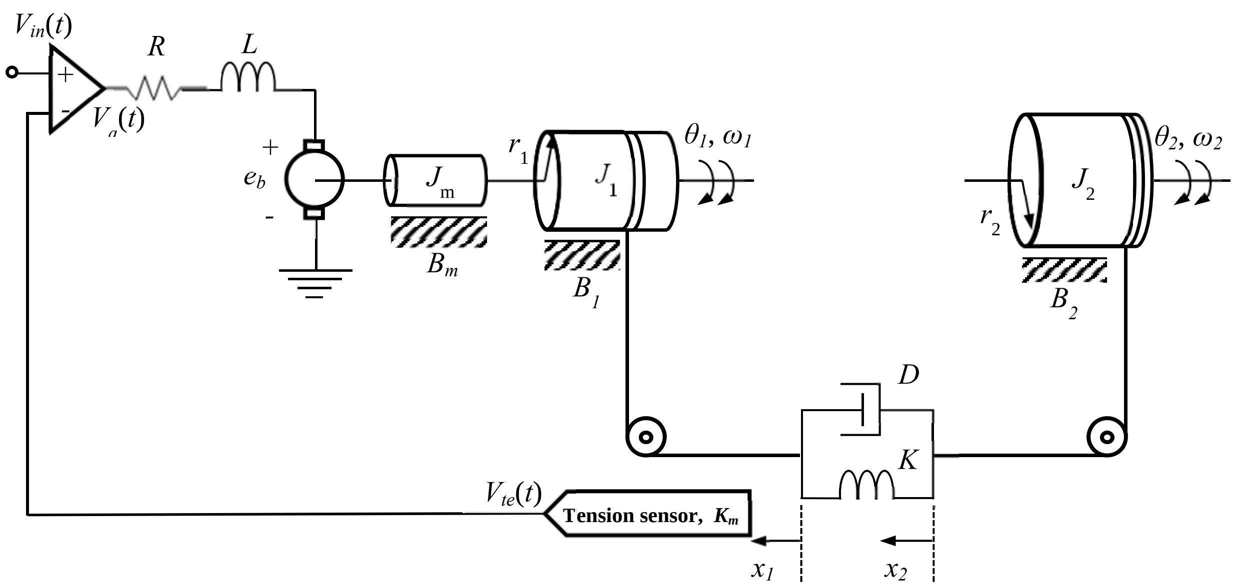
(b)

4. Consider the tape transport system (passing a long piece of tape from one reel to another) sketched below where the tape is modeled by a translational spring K and damper D attached to two inextensible strings that lead to the tape packs. An armature controlled DC motor drives the first reel which is denoted with inertia J_1 and has the radius r_1 and the friction coefficient B_1. Connected to the first reel via the tape is the second reel shown with inertia J_2 which has the radius r_2 and the friction coefficient B_2. The DC motor has the following parameters: armature resistance R, armature inductance L, back emf constant K_b, torque constant K_T, rotor inertia J_m, friction coefficient of the rotor B_m. The tension T_e of the tape is proportional to the difference of the displacements x_1 and x_2 as

$$T_e = K_{te}(x_1 - x_2).$$

The tension T_e is measured using a tension sensor which gives the voltage $V_{te}(t)$ which is proportional to the tension with the constant K_m, i.e., $V_{te}=K_m T_e$. This voltage is fed back to the input side and is subtracted from the input voltage $V_{in}(t)$ to obtain the armature voltage $V_a(t)$.

Find the transfer function from the input voltage, $V_{in}(t)$, to the tension T_e .



6. Represent the rotational mechanical system shown in Figure P3.6 in state space, where $\theta_1(t)$ is the output. [Section: 3.4]

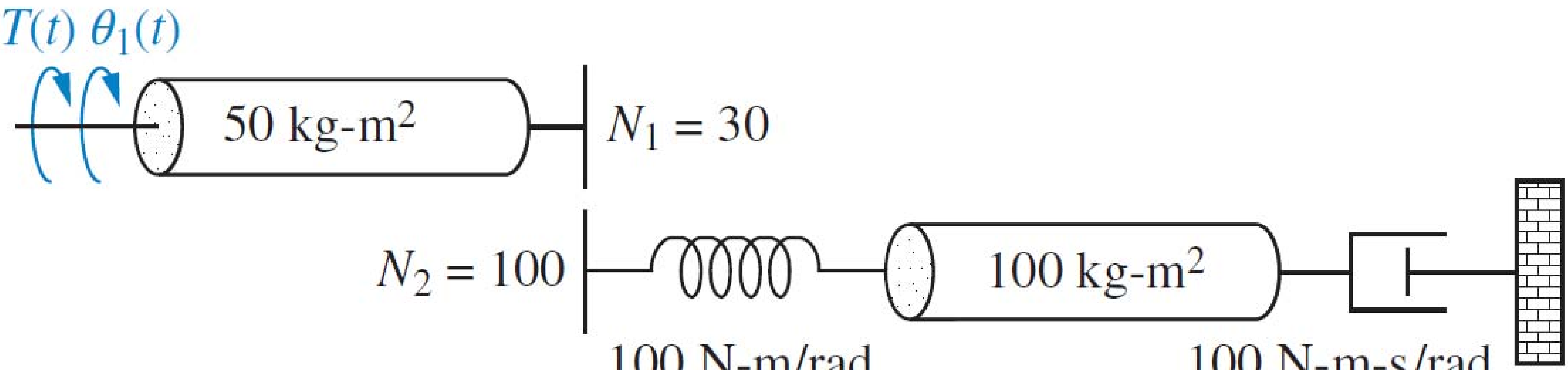


FIGURE P3.6

EE302 - Spring 2017 Homework #2 Due: April 07, 2017 Friday – 23:55

1. (Adapted from Nise – 6th Ed.) **Part A.** For each of the following second-order systems, find ξ, w_n, t_r, t_p, t_s and $M_p(\%)$. (Note that these are not in the standard form but you can focus on the denominator for the approximate analysis)

(a) $T(s) = \frac{s+50}{s^2+3s+16}$

(b) $T(s) = \frac{1}{s^2+0.02s+0.04}$

Part B. Use Matlab's `LTIViewer` (or a sequence of Matlab's `TF(.)` and `step(.)` commands) to obtain again for systems in Part A, the *rise time*, *peak time*, *settling time*, and *percent overshoot*. Also consider the *standard form* systems with the same denominator to obtain the same parameters and compare the results.

2. (Past Midterm Question) Consider the control system given below.

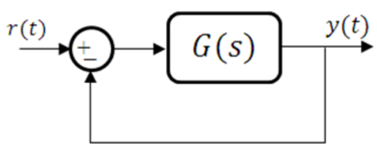


Figure 1

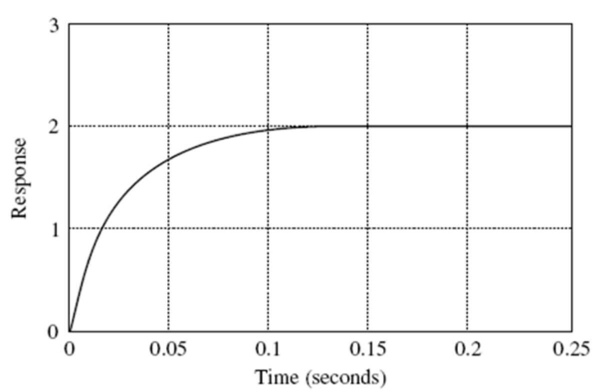
$$G(s) = \frac{K}{(s+\alpha)(s+\beta)^2}$$

where α and β are real scalars. The following facts are known about the system above.

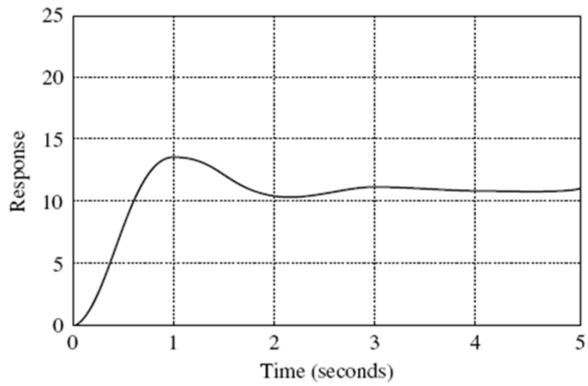
- $e_{ss} = 9/8$ to unit-ramp input.
- The closed-loop system is approximately a critically-damped second-order system with 5% settling time $t_s = 3/2$ seconds.

Based on the information given above, find α, β and K.

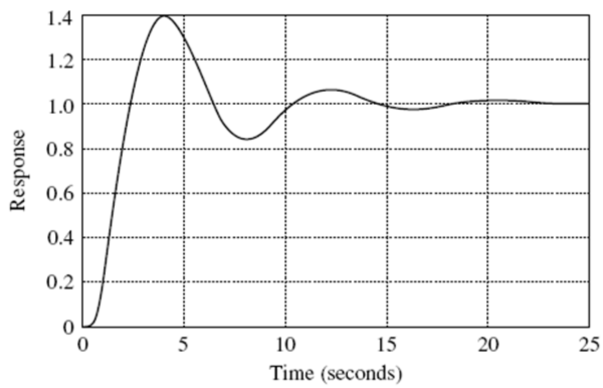
3. (Nise – 6th Ed) For each one of the following three unit-step responses, find the approximate transfer function of the system.



(*a*)



(*b*)



(*c*)

4. A system is represented in state-space by the equations

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 & 3 \\ 0 & 6 & 5 \\ 1 & 4 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t),$$

$$y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \mathbf{x}$$

Without solving the state equations, find

(a) *the characteristic polynomial* (the denominator of the transfer function),

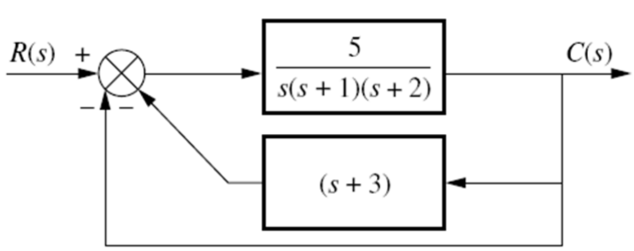
(b) the poles of the system.

5. (Nise – 6th Ed.) For the system shown below, find

(a) The positional, velocity and acceleration error constants K_p, K_v, K_a.

(b) Find the steady-state error e_{ss} for an input of $50u(t)$, $50tu(t)$ and $50t^2u(t)$,

(c) The system type.

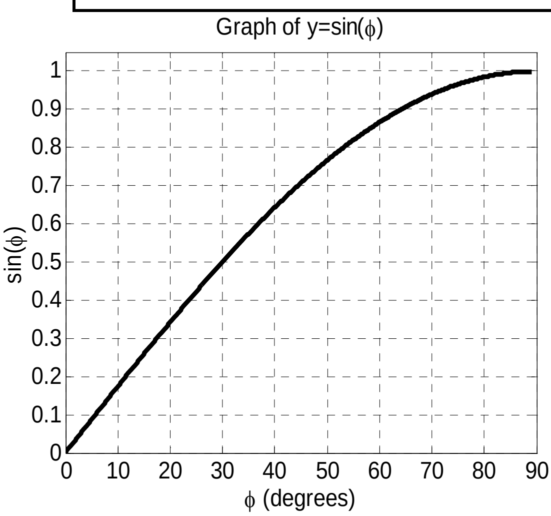


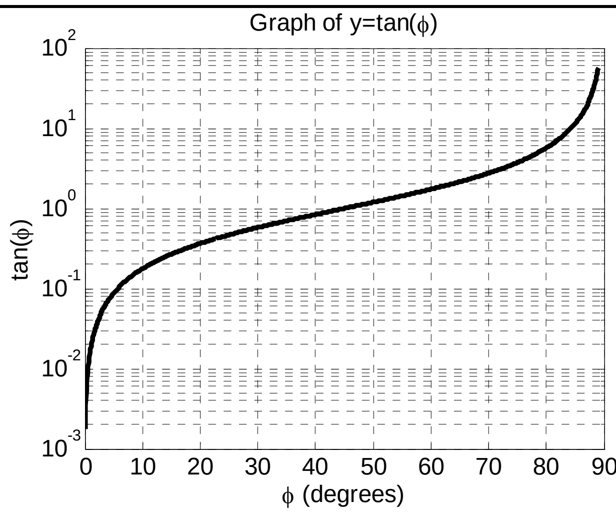
This is the formula and graph page you will get in the exam.

Table of logarithms and squares

x	log(x)	x^2	x	log(x)	x^2	x	log(x)	x^2	x	log(x)	x^2	x	log(x)	x^2
1,0	0	1	**3,0**	0,47712	9	**5,0**	0,69897	25	7,0	0,8451	49	**9,0**	0,95424	81
1,1	0,04139	1,2	3,1	0,49136	9,61	5,1	0,70757	26	7,1	0,85126	50,4	9,1	0,95904	82,8
1,2	0,07918	1,4	3,2	0,50515	10,2	5,2	0,716	27	7,2	0,85733	51,8	9,2	0,96379	84,6
1,3	0,11394	1,7	3,3	0,51851	10,9	5,3	0,72428	28,1	7,3	0,86332	53,3	9,3	0,96848	86,5
1,4	0,14613	2	3,4	0,53148	11,6	5,4	0,73239	29,2	7,4	0,86923	54,8	9,4	0,97313	88,4
1,5	0,17609	2,3	3,5	0,54407	12,3	5,5	0,74036	30,3	7,5	0,87506	56,3	9,5	0,97772	90,3
1,6	0,20412	2,6	3,6	0,5563	13	5,6	0,74819	31,4	7,6	0,88081	57,8	9,6	0,98227	92,2
1,7	0,23045	2,9	3,7	0,5682	13,7	5,7	0,75588	32,5	7,7	0,88649	59,3	9,7	0,98677	94,1
1,8	0,25527	3,2	3,8	0,57978	14,4	5,8	0,76343	33,6	7,8	0,8921	60,8	9,8	0,99123	96
1,9	0,27875	3,6	3,9	0,59107	15,2	5,9	0,77085	34,8	7,9	0,89763	62,4	9,9	0,99564	98
2,0	0,30103	4	**4,0**	0,60206	16	**6,0**	0,77815	36	**8,0**	0,90309	64	**10**	1	100
2,1	0,32222	4,4	4,1	0,61278	16,8	6,1	0,78533	37,2	8,1	0,90849	65,6			
2,2	0,34242	4,8	4,2	0,62325	17,6	6,2	0,79239	38,4	8,2	0,91381	67,2			
2,3	0,36173	5,3	4,3	0,63347	18,5	6,3	0,79934	39,7	8,3	0,91908	68,9			
2,4	0,38021	5,8	4,4	0,64345	19,4	6,4	0,80618	41	8,4	0,92428	70,6			
2,5	0,39794	6,3	4,5	0,65321	20,3	6,5	0,81291	42,3	8,5	0,92942	72,3			
2,6	0,41497	6,8	4,6	0,66276	21,2	6,6	0,81954	43,6	8,6	0,9345	74			
2,7	0,43136	7,3	4,7	0,6721	22,1	6,7	0,82608	44,9	8,7	0,93952	75,7			
2,8	0,44716	7,8	4,8	0,68124	23	6,8	0,83251	46,2	8,8	0,94448	77,4			
2,9	0,4624	8,4	4,9	0,6902	24	6,9	0,83885	47,6	8,9	0,94939	79,2			
3,0	0,47712	9	**5,0**	0,69897	25	**7,0**	0,8451	49	**9,0**	0,95424	81			

$$\phi = \arctan\left(\frac{\sqrt{1-\xi^2}}{\xi}\right); \quad t_r = \frac{\pi - \phi}{\omega_d}; \quad t_s = \frac{3}{\xi\omega_n}\ (5\%); \quad t_p = \frac{\pi}{\omega_d}; \quad M_p = e^{-\pi/\tan\phi}$$





EE302 HW3 **Due date: 25.04.2017 Tuesday Time: 23:55**

1. (Nise (6th edition)) Using the Routh-Hurwitz stability criterion determine the number of roots of the following polynomial in the right half plane, in the left half plane and on the jw axis?

$$P(s) = s^5 + 6s^3 + 5s^2 + 8s + 20$$

2. (Nise (6th edition)) Consider a unity feedback system with open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+2)(s+4)}.$$

a. Find the range of K for stability using Routh-Hurwitz stability criterion.

b. Find the value of K for which the system has closed loop poles on the imaginary axis.

c. For the value of K in part-b, find the location of all closed loop poles.

3. (Adapted from Nise (6th edition)) Consider a unity feedback system with open loop transfer function given as

$$G(s) = \frac{K(s+2)}{s(s+1)}.$$

a. Draw the root locus with respect to $K \geq 0$.

b. Mark the point on the root locus for which the closed loop system has minimum damping ratio.

c. Analytically express the damping ratio of the closed loop system in terms of K and find the value of K which achieves minimum damping ratio.

d. Find the value of K for which minimum settling time (5%) is achieved when the closed loop system has the damping ratio $\xi = \sqrt{3}/2$.

4. (Adapted from Nise (6th edition)) Consider a unity feedback system with open loop transfer function given as

$$G(s) = \frac{K(s+2)(s+3)}{(s^2+2s+2)(s+4)(s+5)(s+6)}.$$

a. Draw the root locus for the closed loop poles for $K \geq 0$ with all the details such as jw-crossings, break-away/in points etc. You might need to use Matlab (function roots(.)) to find e.g., the value of the break-away/in points.

b. What is the range of K for the stability of the closed loop system?

c. Draw a different root-locus than the one you have drawn in part-a by changing the poles which go to the asymptotes. Note that only one of the root loci you have drawn in part-a and part-c is the correct one. You will use Matlab to obtain the correct one below.

d. Using Matlab functions `tf(.)` and `rlocus(.)` (and possibly the function `poly(.)`), obtain the root locus for this system. Which root locus you have drawn in the previous parts is the correct one?

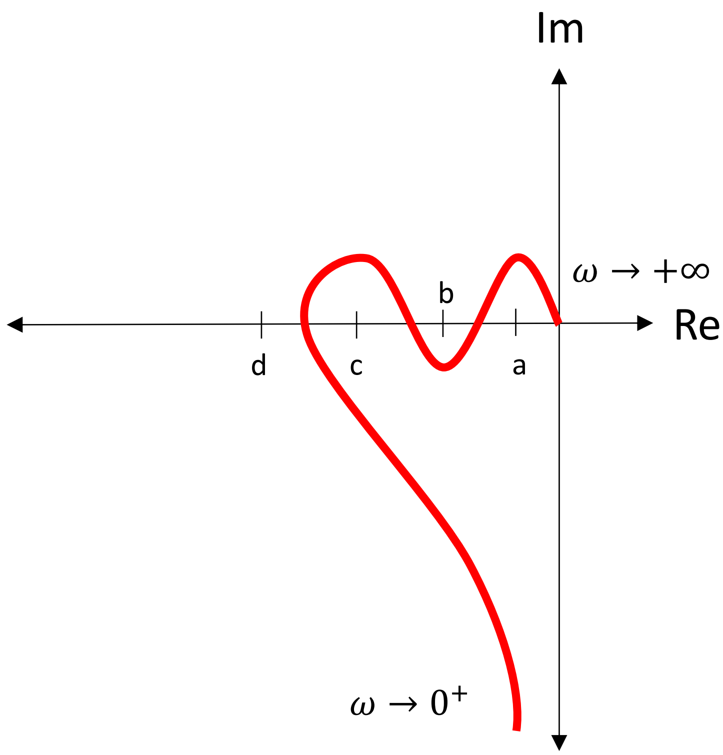
e. Repeat part-d by changing the open loop zeros into $-1, -2$ (instead of $-2, -3$).

PROBLEM SET

APRIL 2017

Problem 1) The figure below shows the polar plot of an open loop transfer function with a single pole at the origin, plus other poles and zeros in the left half plane.

a) Complete the Nyquist diagram and,
b) Determine the stability of the closed loop system for each of the four cases when the point –1 is at (a), (b), (c) and (d).
c) Now suppose that the open loop transfer function has 3 poles at the origin, instead of 1. What does the Nyquist plot look like now?



Problem 2) Consider the unity feedback system with

$$G(s) = K\frac{s+1}{s(s+4)(s-1)}$$

a) Sketch the root locus of the system

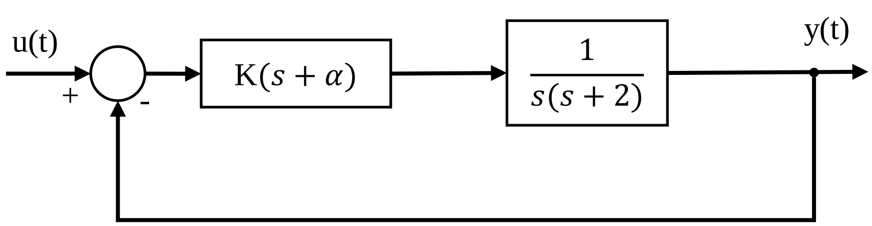
b) Determine the stability range for the parameter K, where $K \geq 0$

Problem 3) For the system given in **Problem 2**

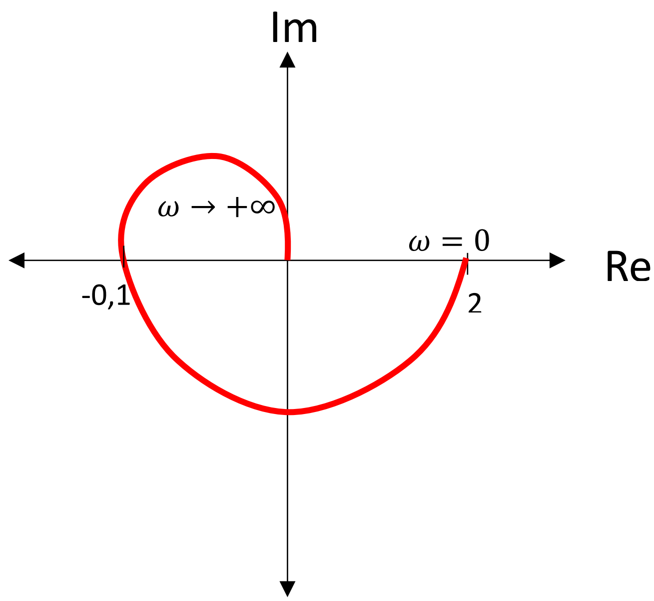
a) Sketch the Nyquist plot.
b) Determine the stability range for the parameter K based on the Nyquist criterion $-\infty \prec K \prec \infty$.

Problem 4)

a) Design a PD controller for the system below such that the closed loop system has a (%5) settling time of 1.5 sec and an overshoot of % 4.32.
b) Without changing the position of the zero that you found in part (a), find the minimum possible value for the settling time that can be achieved by varying the gain.



Problem 5) Consider a unity feed-back system with an open loop transfer function G(s). Polar plot of G(s) is given below.



a) What is the type of the system?
b) Determine the static position and velocity error constants K_p, and K_v.
c) Determine the number of asymptotes in the root locus.
d) Determine the gain margin of the system.

1. Consider the control system given in Figure 1. The plant to be controlled has the transfer function given as

$$G(s) = \frac{5}{(s+2)(s+3)}$$

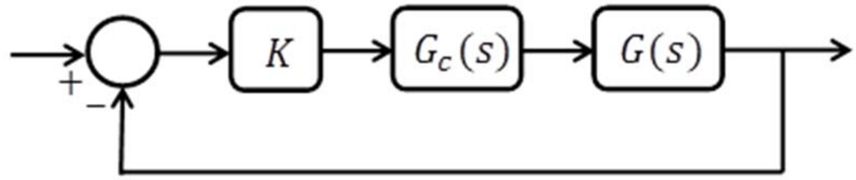


Figure 1. Control system considered in the homework.

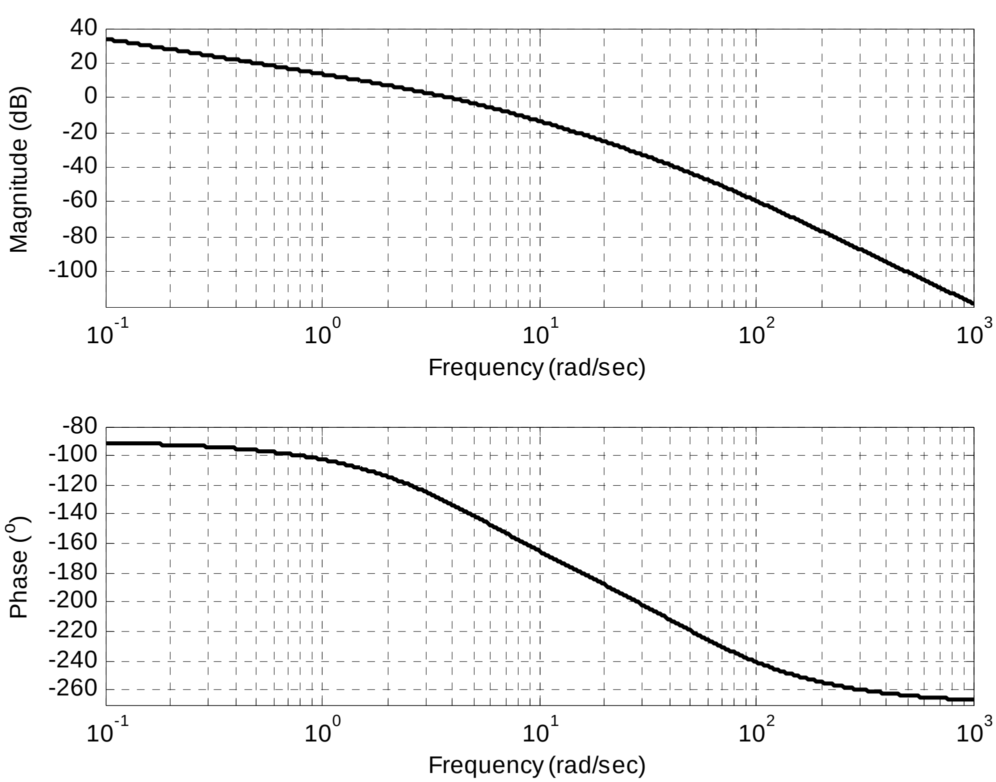
Design a phase-lead compensator for the closed loop system to have a phase margin of 45° and steady-state error to unit step input less than or equal to 0.05. Before designing the compensator, first write the transfer function $G_c(s)$ you use and the range of its parameters. In your design, use the graphs of the sine/cosine and tangent functions given at the end of the homework if needed.

2. Consider the control system given in Figure 1. The plant to be controlled has the transfer function given as

$$G(s) = \frac{1}{s(1+s)^2}.$$

Design a phase-lag compensator such that the phase margin 50° and the system has static velocity error constant $K_v = 10$. Before designing the compensator, first write the transfer function $G_c(s)$ you use and the range of its parameters. In your design, use the graphs of the sine/cosine and tangent functions given at the end of the homework if needed.

3. Consider the control system given in Figure 1. The plant $G(s)$ to be controlled has the Bode plot given below.



You are required to design a compensator such that

i. The steady-state error to unit ramp input is less than 0.001,

ii. PM is greater than or equal to 40°.

a. Find K to satisfy the steady-state error requirement.

b. What are the phase and gain cross-over frequencies of the uncompensated system (i.e., the closed loop system when $G_c(s) = 1$)?

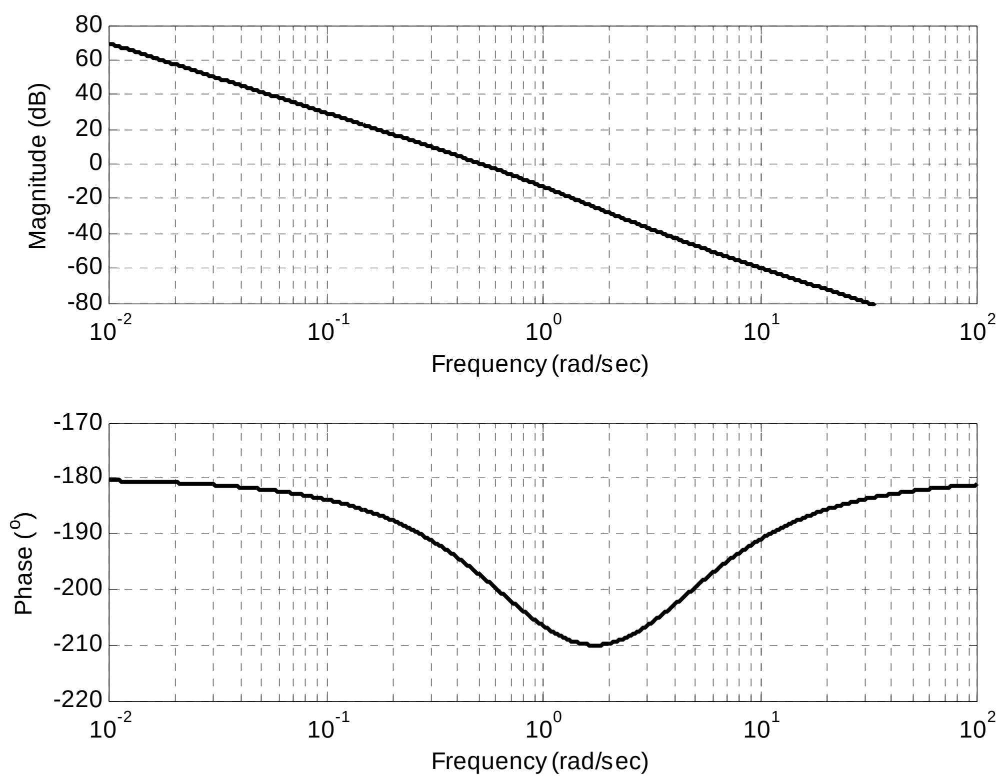
c. What are gain and phase margins of the uncompensated system?

d. Is the uncompensated system stable? Why?

e. What type of compensator do you need to use, phase-lead or phase-lag? Why? Write the transfer function of the compensator you use and the ranges of its parameters.

f. Design the compensator you choose in part-e such that the PM requirement is satisfied. In your design use the graphs of the sine/cosine and tangent functions given at the end of the homework if needed.

4. Consider the control system given in Figure 1. The plant $G(s)$ to be controlled has the Bode plot given below.

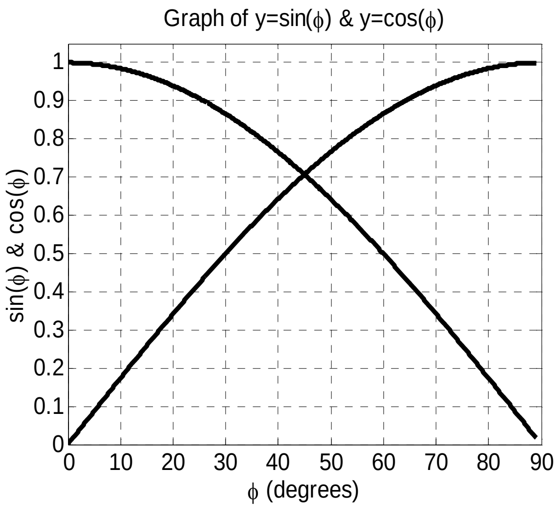


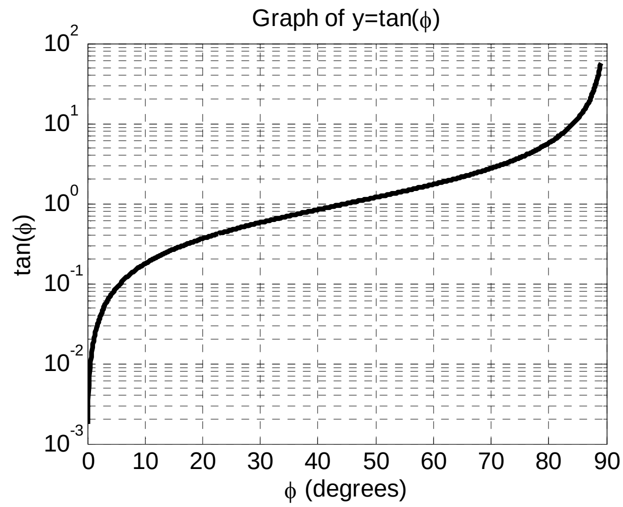
You are required to design a compensator such that

i. The steady-state error to unit acceleration input is less than 0.01,

ii. PM is greater than or equal to 30°.

What type of compensator do you need to use, phase-lead or phase-lag? Why? Design the compensator you choose such that requirements are satisfied. Before designing the compensator, first write the transfer function $G_c(s)$ you use and the range of its parameters. In your design use the graphs of the sine/cosine and tangent functions given at the end of the homework if needed.


Graph of y=sin(ϕ) & y=cos(ϕ)
sin(ϕ) & cos(ϕ)
ϕ (degrees)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0 10 20 30 40 50 60 70 80 90


Graph of y=tan(ϕ)
tan(ϕ)
ϕ (degrees)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
0 10 20 30 40 50 60 70 80 90

An Important Note about Phase Lead/Lag Compensator Parameterizations and Design

In EE302, different parameterizations of the phase lead/lag compensators are used in different sections. This situation is the same in the literature where you can find different parameterizations in different books about the subject.

For example, a parameterization of a lead compensator is

$$G_c^1(s) = \frac{1 + aT_1 s}{1 + T_1 s}, \qquad a > 1, \qquad T_1 > 0 \quad \text{(Section 2\&3)}$$

Another equivalent parameterization for the same compensator is

$$G_c^2(s) = \frac{1 + T_2 s}{1 + \alpha T_2 s}, \qquad 0 < \alpha < 1, \qquad T_2 > 0 \quad \text{(Section 1\&4)}$$

The design procedures for these two forms of the lead compensator are similar but the formulas for finding their parameters are different as expected. Note that although the parameters a & T_1 for the first form are different from α & T_2 in the second form, after the design the transfer functions $G_c^1(s)$ and $G_c^2(s)$ turn out the same (or similar due to numerical approximations). This is because of the facts that $T_1 = \alpha T_2$ and $T_2 = aT_1$ (as a result we have $a = 1/\alpha$).

Since HW4 involves phase-lead and lag compensator design, we provide two solution files for HW4 using different parameterizations so that you can check the solution containing the parameterization you learned.

In the final exam you can use the form which you have learned in your sections. In the case that you need to design a compensator in the final exam, first write the transfer function of the compensator you use ($G_c(s)$) and also denote the range of its parameters as is done above.