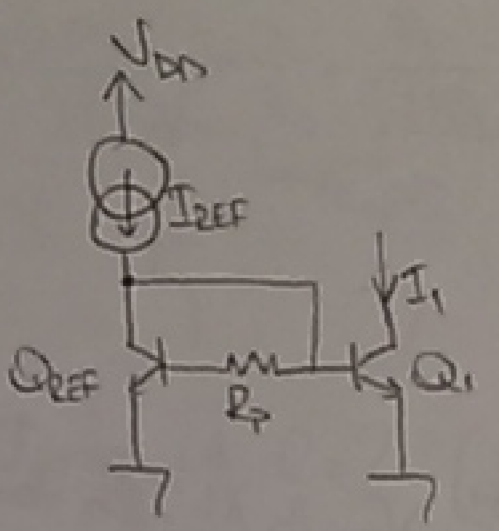
Arif Can GÜNGÖR – 1814219 March 2015

EE414 THE 2

1)




manufacturing error.

$I_1 \longrightarrow 1.1 \times I_1$

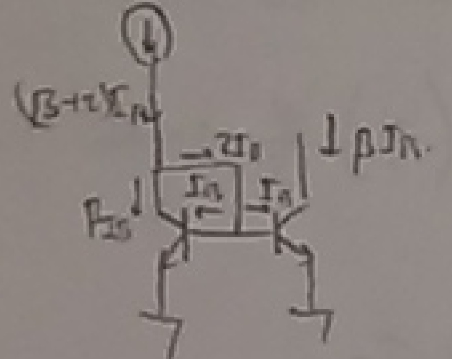
$R_P = 0 \longrightarrow R_P = R_P$

$Q_{REF}$ & $Q_1$ identical

$R_P = ?$

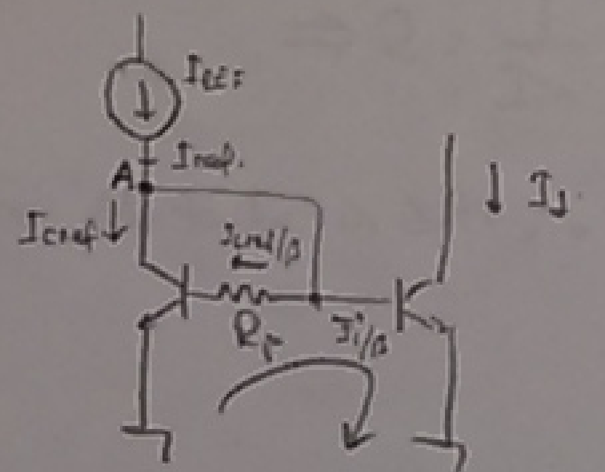
Let me first find $I_1 - I_{REF}$ relationship without $R_P$;

$Q_{REF} - Q_1$ matched:




$$\frac{I_1}{I_{REF}} = \frac{\beta I_B}{(\beta+2)I_B} \rightarrow I_1 = \frac{\beta}{\beta+2} I_{REF}$$

In our case, then; $\quad I_1 = 1.1 \cdot \frac{\beta}{\beta+2} \cdot I_{REF} \quad (1)$




at Loop; $V_{BEref} + V_{R_P} = V_{BE1}$

$$V_{RP} = \frac{I_{c,ref}}{\beta} \cdot R_P = V_{BE1} - V_{BEref}$$

$$= V_T \ln \frac{I_1}{I_{CREF}}$$

$$R_P = \frac{\beta}{I_{cref}} \cdot V_T \ln \frac{I_1}{I_{c,ref}} \quad (2)$$

at node A; $\quad I_{cref} + \frac{I_{cref}}{\beta} + \frac{I_1}{\beta} = I_{ref}$

$$I_{cref} = \frac{I_{ref} - \frac{I_1}{\beta}}{1 + \frac{1}{\beta}} \quad (3)$$

Now, pluging (1) into (3).;

$$I_{cref} = \frac{I_{ref}\left(1 - \frac{1.1}{\beta+2}\right)}{\left(1+\frac{1}{\beta}\right)} = I_{ref}\,\frac{(\beta+2-1.1)}{\beta+2}\,\frac{\beta}{\beta+1} = I_{ref}\cdot\frac{(\beta+0.9)\beta}{(\beta+2)(\beta+1)} \quad (4)$$

Put (4),(1) into (2);

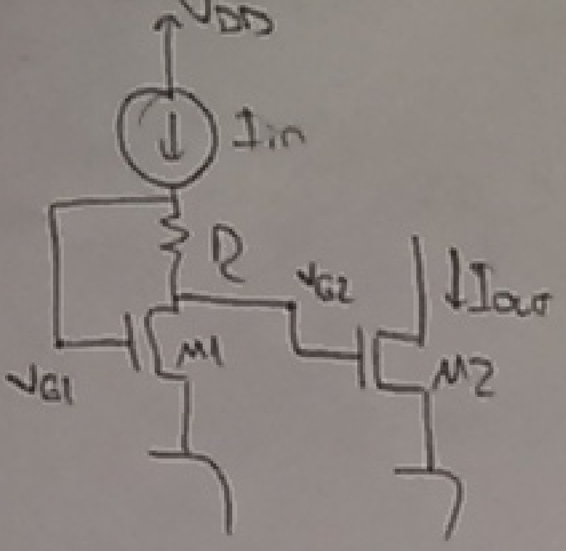
$$R_P = \frac{\beta}{I_{SREF}}\cdot V_T \ln\frac{I_1}{I_{C,ref}} = \frac{\cancel{\beta}(\beta+2)(\beta+1)}{I_{REF}\cdot(\beta+0.9)\cancel{\beta}}\cdot V_T\cdot \ln\left(\frac{(1.1)\frac{\cancel{\beta}}{\cancel{\beta+2}}I_{REF}}{\frac{\cancel{\beta}}{\cancel{\beta+2}}\cdot\frac{\beta+0.9}{\beta+1}I_{REF}}\right)$$

$$\boxed{R_P = \frac{(\beta+1)(\beta+2)}{(\beta+0.9)}\,\frac{V_T}{I_{REF}}\cdot \ln\left[\frac{1.1\,(\beta+1)}{(\beta+0.9)}\right]}$$

Let $\alpha = \dfrac{\beta+1}{\beta+0.9}$

$$R_P = \alpha(\beta+2)\,\frac{V_T}{I_{REF}}\cdot \ln\left[1.1\,\alpha\right]$$

2)




$I_{OUT} = 0.1\,\mu A$

$M_1$ & $M_2$ are identical $\rightarrow \left(\frac{W}{L}\right)_{M1} = \left(\frac{W}{L}\right)_{M2} = ($

$k' = 200\,\mu A/V^2$

$V_{th} = 0.5V$.

a) $I_{in} = 1\mu A$, $R = ?$, $\left(\frac{W}{L}\right) = ?$

$$I_{OUT} = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{G2} - V_T)^2 \quad , V_{G2} = V_{D1}$$

$$I_{in} = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{G1} - V_T)^2 \longrightarrow V_{G1} = \sqrt{\frac{2I_{in}}{k'\left(\frac{W}{L}\right)}} + V_T$$

$$V_{G1} = V_{G2} + RI_{in} \longrightarrow V_{G2} = V_{G1} - RI_{in}$$

$$I_{OUT} = \frac{k'}{2}\left(\frac{W}{L}\right)(V_{G1} - RI_{in} - V_T)^2 = \frac{k'}{2}\left(\frac{W}{L}\right)\left(\sqrt{\frac{2I_{in}}{k'\left(\frac{W}{L}\right)}} - RI_{in}\right)^2$$

$$-I_{in}R = \sqrt{\frac{2I_{OUT}}{k'\left(\frac{W}{L}\right)}} - \sqrt{\frac{2I_{in}}{k'\left(\frac{W}{L}\right)}}$$

$$\Rightarrow R = \frac{1}{I_{in}}\left(\sqrt{\frac{2I_{in}}{k'\left(\frac{W}{L}\right)}} - \sqrt{\frac{2I_{out}}{k'\left(\frac{W}{L}\right)}}\right) \quad (1)$$

For $M_1$ to be in SAT; • $V_{G1} - V_{th} > 2nV_T \rightarrow$ for strong inversion to be satisfied.

$$\sqrt{\frac{2I_{in}}{k'\left(\frac{W}{L}\right)}} > 78mV.$$

$$\boxed{\left(\frac{W}{L}\right) < 1643}$$

according to Gray & Meyer's book; $0 \leq V_{GS} - V_t < 2nV_t$ moderate inversion

$V_{GS} - V_{th} > 2nV_t$ strong inv.

from page 300 of Gray & Meyer

for strong inversion condition

$V_G - V_{th} > 78mV = \underbrace{\frac{3kT}{q}}_{\text{room temp}}$

• $V_{DS1} > V_{GS1} - V_{th}$

$V_{G2} - V_{G1} > -V_{th}$

$V_{G1} - RI_{in} - V_{G1} > -V_{th}$

$RI_{in} < V_{th} \quad (2)$.

put (1) into (2):

$$R I_{in} < V_{th} \rightarrow \sqrt{\frac{2 I_{in}}{k'(\frac{W}{L})}} - \sqrt{\frac{2 I_{out}}{k'(\frac{W}{L})}} < 0.5$$

$$\sqrt{\frac{2 \cdot 1}{200\,(\frac{W}{L})}} - \sqrt{\frac{2\,(0.1)}{200\,(\frac{W}{L})}} < 0.5$$

$$\sqrt{\left(\frac{W}{L}\right)^{-1}}\,(0.1 - 0.032) < 0.5$$

$$\frac{W}{L} > 0.0187$$

$$\boxed{1.6437 > \frac{W}{L} > 0.0187}$$

$$R = \frac{1}{I_{in}}\left(\sqrt{\frac{2 I_{in}}{k'(\frac{W}{L})}} - \sqrt{\frac{2 I_{out}}{k'(\frac{W}{L})}}\right)$$ putting limiting $\frac{W}{L}$ values to this expression

$$\boxed{53.33\,k\Omega < R < 500\,k\Omega}$$

b- $R = 10k\Omega$ , $I_{in} = ?$

2 SAT conditions;

- $V_{G1} - V_{th} > 78mV$

$$\sqrt{\frac{2I_{in}}{k'(\frac{W}{L})}} > 78mV \quad (b1)$$

- $R I_{in} < V_{th} \quad (b2)$

and from (1) • $R I_{in} = \left(\sqrt{\frac{2I_{in}}{k'(\frac{W}{L})}} - \sqrt{\frac{2I_{OUT}}{k'(\frac{W}{L})}}\right) \quad (b3)$

Solving (b1) & (b3) simultaneously gives $(\frac{W}{L})$'s upper limit:

From (b1) at boundary: $\sqrt{\frac{I_{in}}{(\frac{W}{L})}} = 7.8 \cdot 10^{-4}$ → put this into (b3)

$$10\,000(7.8 \cdot 10^{-4})^2\left(\frac{W}{L}\right) = 0.078 - 0.0316\sqrt{\frac{1}{(\frac{W}{L})}}$$

$\Rightarrow$ lower limit for $(\frac{W}{L}) = 0.1685$. (I utilized Wolfram Alpha for this solution.

Corresponding $I_{in} = 0.102\,\mu A$.

For the other condition (b2):

$$I_{in} < \frac{V_{th}}{R} = \frac{0.5}{10000} = 50\mu A \to \text{put into (b3)}$$

$$0.5 = \left(\sqrt{\frac{2 \cdot 50}{200(\frac{W}{L})}} - \sqrt{\frac{2 \cdot (0.1)}{200(\frac{W}{L})}}\right) = \sqrt{\frac{1}{\frac{W}{L}}}\left(\sqrt{0.5} - \sqrt{1/1000}\right)$$

$$\frac{W}{L} = 1.162$$

$$0.1685 < \left(\frac{W}{L}\right) < 1.162$$

$$0.102\mu A < (I_{in}) < 50\mu A$$

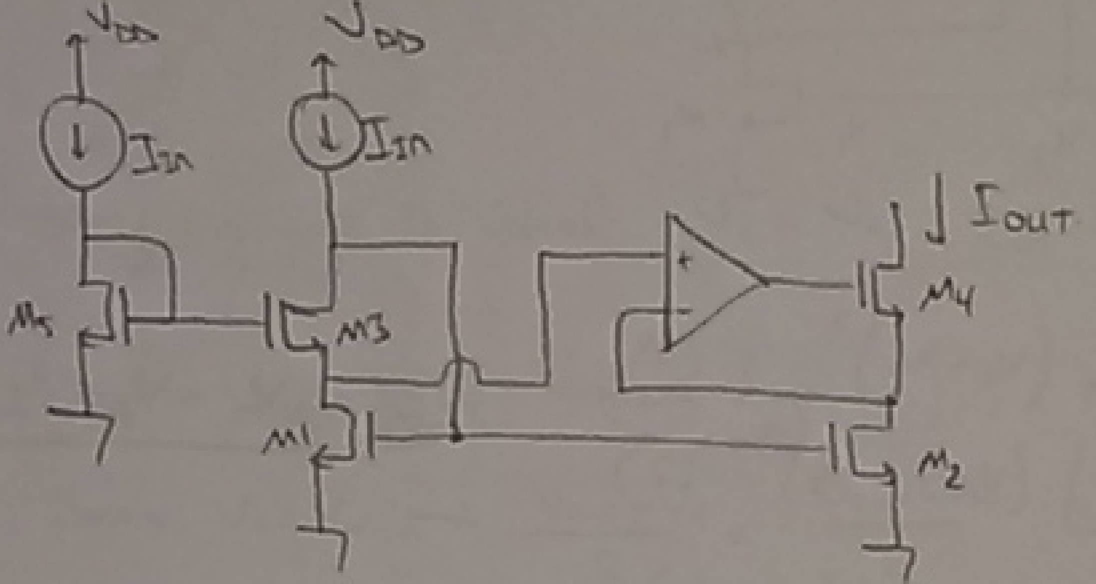
3) no body effect

$A = 100$, $R_{in} = \infty$, $V_o = A(V_+ - V_-) \rightarrow$ ac analysis

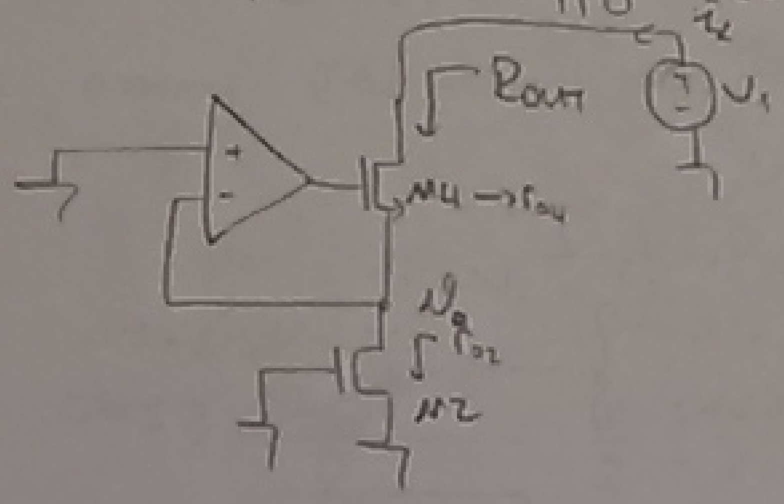
$V_+ = V_- \rightarrow$ DC analysis

$M_{1-4}: \frac{W}{L}$, $M_5 \rightarrow \left(\frac{W}{L}\right) = ?$ to min the gain error.

Gain error? $R_{OUT} = ?$

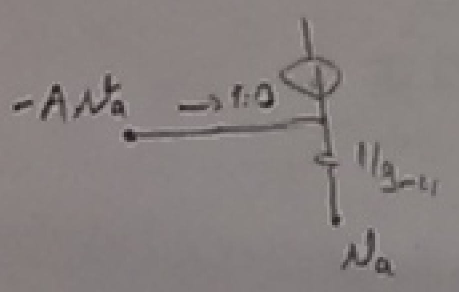



$R_{OUT} \rightarrow$ kill input sources; apply a test source to output;




$v_a = i_x r_{o2}$

$$i_x = \frac{v_x - v_a}{r_{o4}} + g_{m4}(-A v_a - v_a)$$



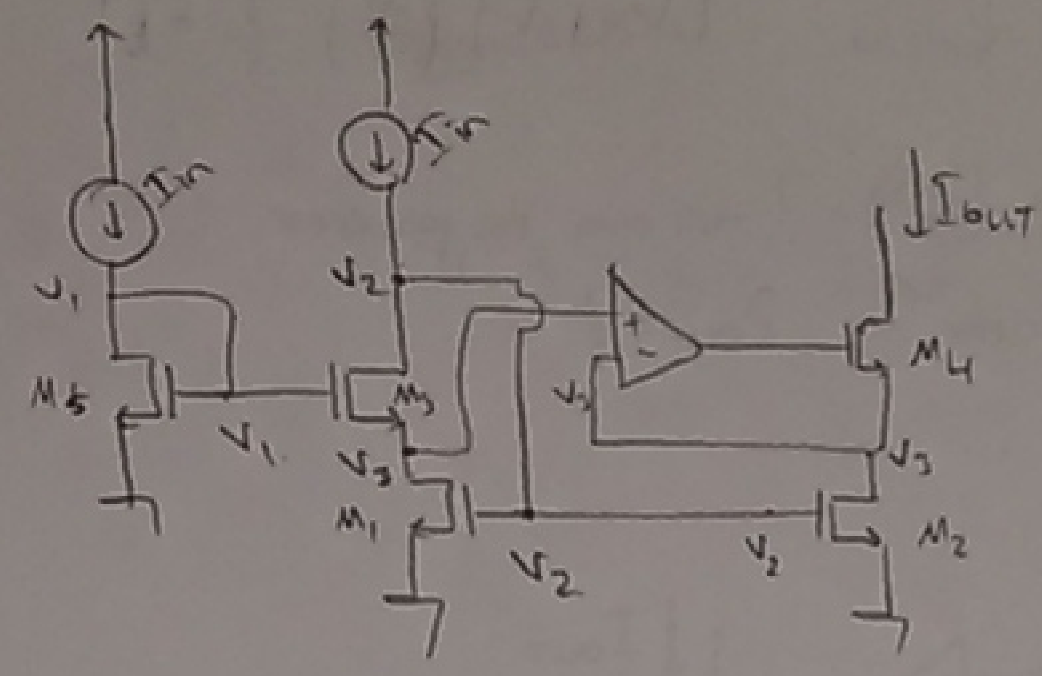

$$i_x = \frac{v_x - i_x r_{o2}}{r_{o4}} + g_{m4}(-A i_x r_{o2} - i_x r_{o2})$$

$$i_x(r_{o4} + r_{o2} + g_{m4} r_{o4}(A r_{o2} + r_{o2})) = v_x$$

$$\boxed{\frac{v_x}{i_x} = R_{OUT} = r_{o4} + r_{o2} + g_m r_{o4} r_{o2} (A+1)}$$

$\Rightarrow$ $R_{OUT}$ is further increased by the factor $(A+1)$

Consider DC voltages;




$V_+ = V_-$

$M_5 \rightarrow I_{in} = \frac{k}{2}\left(\frac{W}{L}\right)_5 (V_1 - V_T)^2$ (1)

$M_3 \rightarrow I_{in} = \frac{k}{2}\left(\frac{W}{L}\right) (V_1 - V_3 - V_T)^2$ (2)

$M_1 \rightarrow I_{in} = \frac{k}{2}\left(\frac{W}{L}\right) (V_2 - V_T)^2$ (3)

$\Rightarrow \underline{V_1 = V_2 + V_3}$

for both $M_1$ & $M_3$ to be open and in SAT.

$V_1 > V_T$ $\left\{ \begin{array}{l} V_3 \geq V_2 - V_T \\ V_2 - \cancel{V_3} \geq V_1 - \cancel{V_3} - V_T \end{array} \right.$

$V_2 > V_T$

$\left\{ \begin{array}{l} V_3 - V_2 \geq -V_T \\ V_2 - V_1 \geq -V_T \end{array} \right\} \rightarrow \begin{array}{l} V_3 - V_1 \geq -2V_T \\ V_1 - V_3 \leq 2V_T \end{array}$

So the stable solution with systematic error 0 $\Rightarrow$ $\boxed{\begin{array}{l} V_3 = V_T \\ V_2 = 2V_T \\ V_1 = 3V_T \end{array}}$

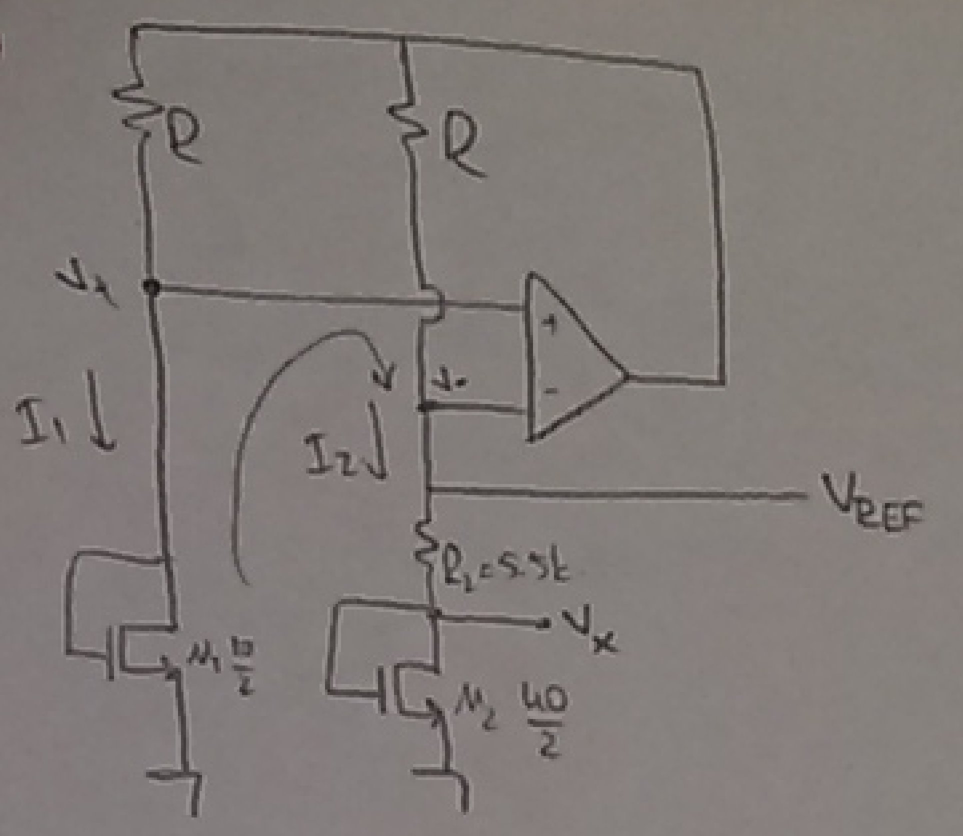
(all $V_{DS}$ s are equal)

Thus, $\boxed{\left(\frac{W}{L}\right)_5 = \frac{1}{4}\left(\frac{W}{L}\right)}$ from current equations (1) & (2).

Having chosen $\left(\frac{W}{L}\right)_5 = \frac{1}{4}\left(\frac{W}{L}\right)$ the systematic error is zero, '0'.

$V_{DS1} = V_{DS3} = V_{DS5}$

4)




$V_{REF} = ?$

Assuming op-amp is ideal ; $V_+ = V_-$ , $R_{in} = \infty$

$V_{GS1} = V_{GS2} + I_2 R_2$ (from loop eqn.)

Since $V_+ = V_- \longrightarrow \frac{V_{OUT} - V_+}{R} = \frac{V_{OUT} - V_-}{R} \Rightarrow I_1 = I_2$.

$$\frac{k_1}{2}\left(\frac{10}{2}\right)(V_+ - V_{T1})^2 = \frac{k_2}{2}\left(\frac{40}{2}\right)(V_x - V_{T2})^2$$

assuming $k_1 = k_2 = k$ and $V_{T1} = V_{T2} = V_T$

$$V_+ - V_T = 2(V_x - V_T) \qquad (1)$$

$$V_+ - V_x = V_x - V_T \longrightarrow V_- = V_+ = 2V_x - V_T$$

also $$\frac{\cancel{V_+ - V_x}}{R_2} = \frac{k}{2}\left(\frac{40}{2}\right)(V_x - V_T)^{\cancel{2}}$$

$$\frac{2}{k R_2 \left(\frac{W}{L}\right)_2} = V_x - V_T$$

$$V_x = \frac{2}{k R_2 \left(\frac{W}{L}\right)_2} + V_T$$

$R_2 = 5500\Omega$
$\left(\frac{W}{L}\right)_2 = \frac{40}{2}$

$$V_- = V_+ = 2V_x - V_T = \frac{4}{k R_2 \left(\frac{W}{L}\right)_2} + V_T = V_{REF} = \boxed{\frac{1}{27500\,k'} + V_T}$$

$$\boxed{V_{REF} = \frac{4}{kR_2\left(\frac{W}{L}\right)_2} + V_T}$$

Here, $V_T = \dfrac{\sqrt{2qN_A\epsilon(2\phi_F)}}{C_{ox}} + 2\phi_F + \phi_{ms} - \dfrac{Q_{ss}}{C_{ox}}$.

$$\phi_F = \frac{kT}{q}\ln\left[\frac{N_A\exp\left(E_g/2kT\right)}{\sqrt{N_cN_v}}\right]$$

$$\frac{dV_T}{dT} = -\frac{1}{T}\left[\frac{E_g}{2q} - \phi_f\right]\left[2 + \frac{\gamma}{\sqrt{2\phi_f}}\right]$$ from Grey & Meyer's equation 1.172

Thus $\dfrac{dV_{REF}}{dT} = \dfrac{4}{k\left(\frac{W}{L}\right)_2}\dfrac{1}{R_2^2}\cdot\dfrac{dR_2}{dT} - \dfrac{1}{T}\left[\dfrac{E_g}{2q} - \phi_f\right]\left[2 + \dfrac{\gamma}{\sqrt{2\phi_f}}\right]$

$$\boxed{\frac{dV_{REF}}{dT} = \frac{4}{k\left(\frac{W}{L}\right)_2}\frac{1}{R_2^2}\frac{dR_2}{dT} + \frac{dV_T}{dT}}$$

$$TCF = \frac{1}{V_{REF}}\frac{\partial V_{REF}}{\partial T} = \frac{1}{\left[\frac{4}{kR_2\left(\frac{W}{L}\right)_2} + V_T\right]}\cdot\left[\frac{4}{k\left(\frac{W}{L}\right)_2R_2^2}\frac{\partial R_2}{\partial T} + \frac{\partial V_T}{\partial T}\right]$$