Force Determination using Energy Balance

$$dW_{elec} = dW_{loss} + dW_{fld} + dW_{mech}$$

$$W_{elec} = \int v \cdot i \cdot dt$$

$$W_{loss\,(resistive)} = \int R i^2 dt$$

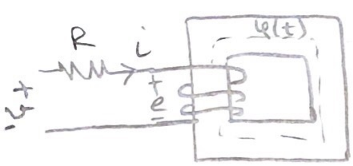
$$W_{elec} - W_{loss} = \int v \cdot i - R \cdot i^2 \, dt = \int i \underbrace{(v - Ri)}_{e} dt$$

1 Conversion of electrical energy to magnetic energy

* Assume that there are no movable parts in the system.

$$dW_{elec} = dW_{loss} + dW_{fld}$$

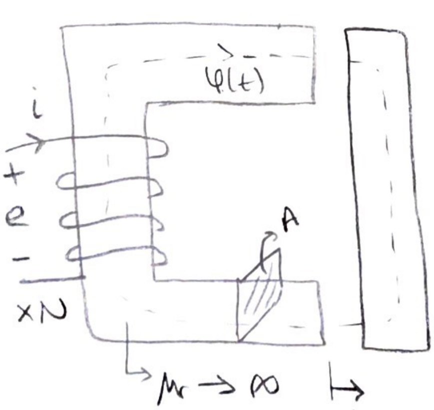
$$dW_{fld} = i \underbrace{(v - Ri)}_{e} dt = i \cdot \frac{d\lambda}{dt} dt = i\, d\lambda$$ → Stored energy in the magnetic field.



2 Conversion of electrical energy to mechanical energy

Electrical energy → Magnetic energy → Mechanical energy

* Assume a lossless system



$$Ni = \mathcal{R}(x)\phi$$

$$\mathcal{R}(x) = \frac{2x}{\mu_0 A}$$

$$L(x) = \frac{N^2}{\mathcal{R}(x)} = \frac{N^2 \mu_0 A}{2x}$$

$$\lambda = L(x) \cdot i$$

2a. Mechanical motion happens at constant flux

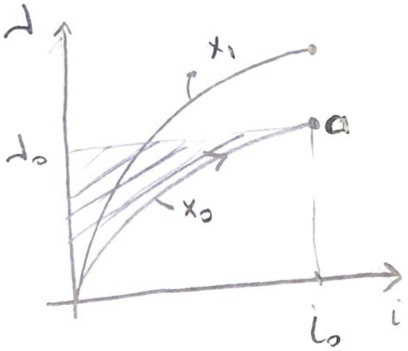
$$Ni = R(x)\,\phi$$

i: decreases; $R(x)$: decreases; ϕ: constant

This process can be divided into three steps:

1. $x = x_0$ & $0 \le \lambda \le \lambda_0$ (Flux linkage increases at constant x)
2. $x_0 \ge x \ge x_1$ & $\lambda = \lambda_0$ (Mechanical motion at constant flux)
3. $x = x_1$ & $\lambda_0 \ge \lambda \ge 0$ (Flux linkage is decreased to 0)

Step 1



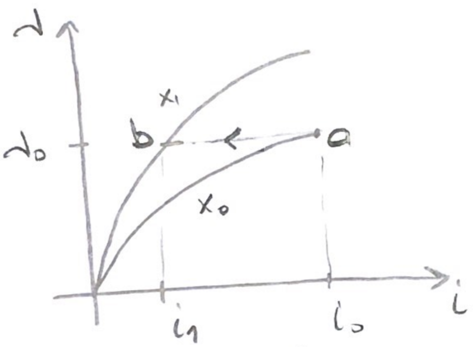
$\Delta W_{mech} = 0$

$\Delta W_{fld} = \Delta W_{elec}$

$\Delta W_{fld} = \int_0^{\lambda_0} i\, d\lambda \Big|_{x=x_0}$

Electrical energy supplied

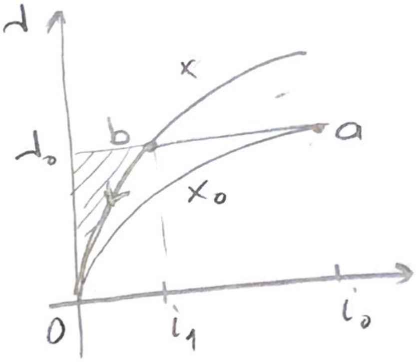
Step 2



$\Delta W_{fld} = i\cancel{d\lambda} - \Delta W_{mech}$, $d\lambda = 0$

$\Delta W_{fld} = -\Delta W_{mech}$

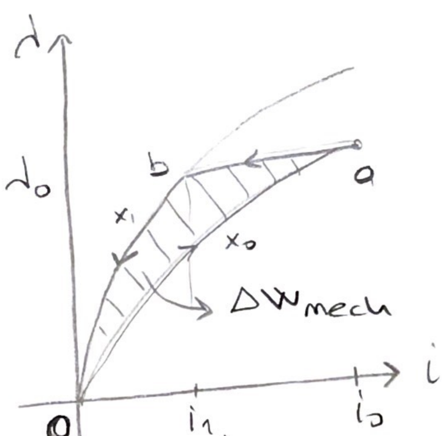
Step 3



$\Delta W_{mech} = 0$

$\Delta W_{fld} = \Delta W_{elec}$

$= \int_{\lambda_0}^{0} i\, d\lambda \Big|_{x=x_1}$



$*\ w_{fld}(\lambda_0, i_0) > w_{fld}(\lambda_0, i_1)$

$\Delta W_{fld} = -\Delta W_{mech}$

Minus sign indicates that mechanical force tries to reduce the energy stored in magnetic field

$$\Delta W_{mech} = \int_0^{\lambda_0} i\, d\lambda \Big|_{x=x_0} - \int_0^{\lambda_0} i\, d\lambda \Big|_{x=x_1} = f_{fld} \cdot dx$$

$$\boxed{f_{fld} = \frac{-\partial W_{fld}}{\partial x}\Big|_{\lambda \to const}}$$

2b. Mechanical motion happens at constant current

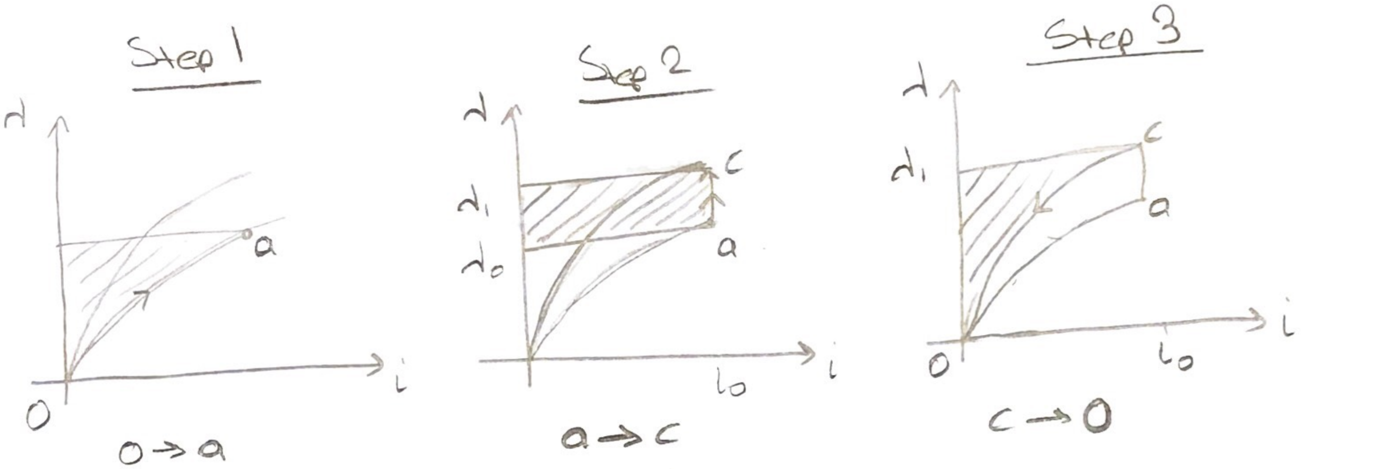
$$Ni = \mathcal{R}(x)\,\phi$$

$\rightarrow \quad \downarrow \quad \uparrow$

increases

This process can be also divided into three steps.

1. $x = x_0$ & $0 \le i \le i_0$ (Current increases at constant x)
2. $x_0 \ge x \ge x_1$ & $i = i_0$ (Mechanical motion at constant current)
3. $x = x_1$ & $i_0 \ge i \ge 0$ (Current decreases to 0)

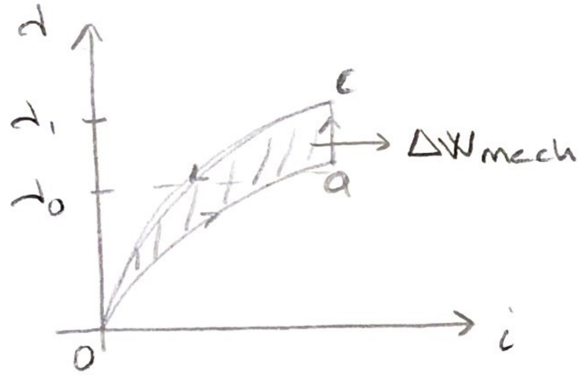


Supplied elec. energy

$$\Delta W_{fld} = \int_{\lambda_0}^{\lambda_1} i\,d\lambda - \Delta W_{mech}$$

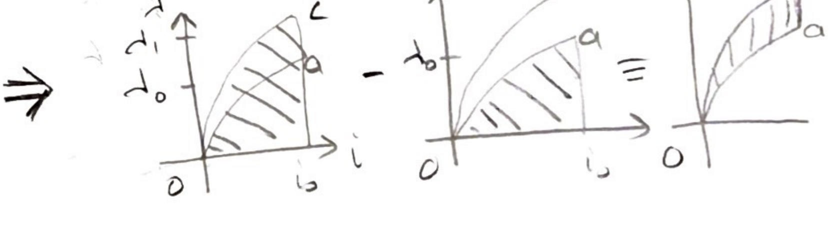
Returned electrical energy

$$\Delta W_{mech} = \text{Area}(0a\lambda_0) + \text{Area}(ac\lambda_1\lambda_0) - \text{Area}(c\lambda 0)$$
$$= \text{Area}(0ac)$$

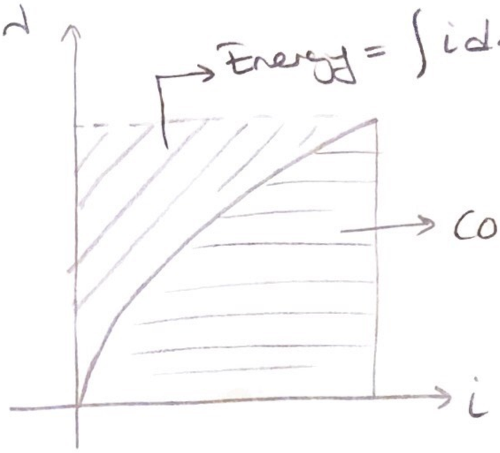


This area can not be represented as the change in the stored mag. energy.

However, it can be represented by the change in the area that is complementing magnetic energy.



Energy and Coenergy

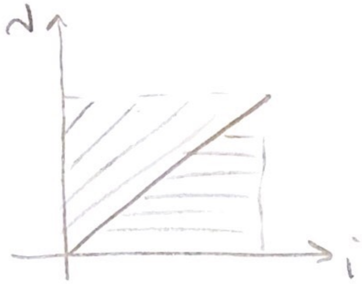


Energy = $\int i \, d\lambda = W_{fld}$

coenergy = $\int \lambda \, di = W'_{fld}$

* Here, we give only the definition of energy and co-energy. There is no change in mechanical position

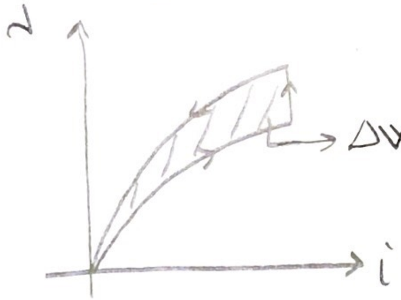
For a linear system



Energy = Coenergy

$\frac{1}{2}\lambda i = \frac{1}{2} L i$

Going back to previous results



$\Delta W_{mech} = \Delta W'_{fld}$

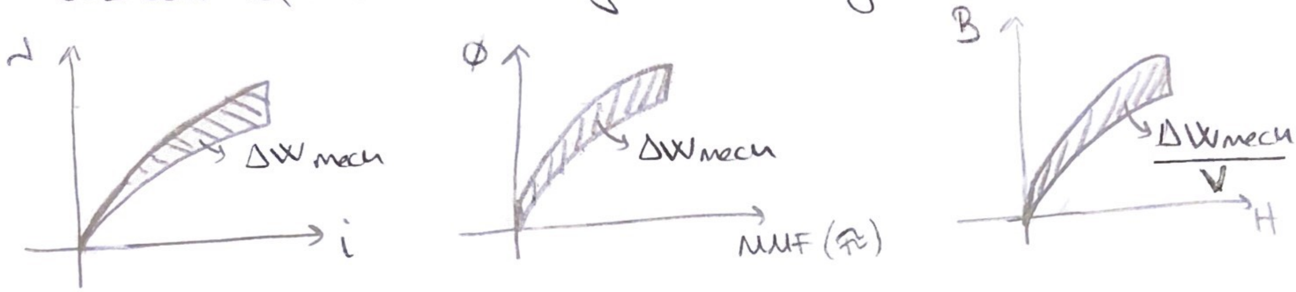
= Change in coenergy

$$f = \left. \frac{\partial W'_{fld}}{\partial x} \right|_{i \to constant}$$

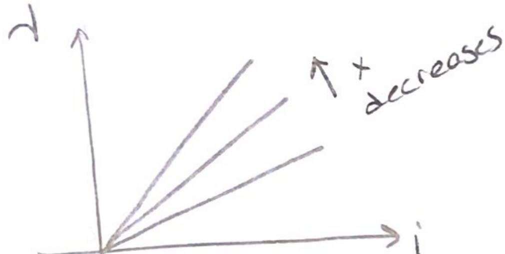
* There is no minus sign. System tries to increase coenergy.

In real life applications, transient flux linkage-current locus depends on the behaviour of the mechanical and electrical system. During mechanical motion both flux linkage and current may change.

We can repeat the analysis above for $\phi - \mathcal{F}$ and B-H curves



Special Case: Linear Analysis



Most of the electromechanical energy conversion devices are built with an air-gap.

$$R_g >> R_c$$

Energy Method

$$f = -\frac{\partial w_{fld}(\lambda, x)}{\partial x}\Big|_{\lambda \to \text{constant}}$$

$$w_{fld} = \int_0^{\lambda_0} i \, d\lambda$$

for $i = \frac{\lambda}{L(x)}$ $\quad w_{fld} = \frac{1}{2}\frac{\lambda^2}{L(x)}$

$$f = -\frac{1}{2}\lambda^2 \frac{\partial}{\partial x}\left(\frac{1}{L(x)}\right) = +\frac{1}{2}\frac{\lambda^2}{L(x)^2}\frac{dL(x)}{dx}$$

for $\lambda = L(x) \cdot i$

$$\boxed{f = \frac{1}{2} i^2 \frac{dL(x)}{dx}}$$

Be careful: We have to first derivate at constant λ, then replace it with current expression.

↳ Does not depend on current direction!

Coenergy Method

$$f = \frac{\partial w'_{fld}(i, x)}{\partial x}\Big|_{i \to \text{constant}}$$

$$W'_{fld} = \int_0^{i_0} \lambda \, di$$

for $\lambda = L(x) \cdot i$

$$W'_{fld} = \frac{1}{2} L(x) i^2$$

$$\boxed{f = \frac{1}{2} i^2 \frac{dL(x)}{dx}}$$

Energy and co-energy methods give the same result.

Virtual Work

The magnitude and direction of the electromechanical force f_{fld} were determined by considering the change in the energy balance if the moving part were allowed to move an incremental distance dx in the direction of f_{fld}.

Forces or torques in any direction can be calculated using this method.

$$f_x = \frac{\partial W'_{fld}}{\partial x}$$

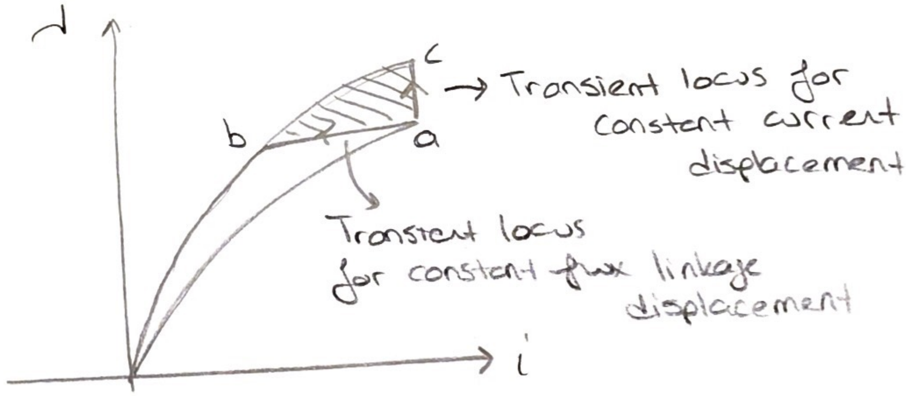
$$f_y = \frac{\partial W'_{fld}}{\partial y}$$

$$f_z = \frac{\partial W'_{fld}}{\partial z}$$

This method of determining forces by considering an <u>imaginary</u> or <u>virtual</u> displacement is called the principle of virtual work.

$\Rightarrow$ we assume a virtual displacement to calculate the force.

For $dx \to 0$, force at mechanical motion happening at constant flux and constant current has to be the same!



The difference between $|W_{fld}|_{\lambda \to constant}$ and $|W'_{fld}|_{i \to constant}$ is the area (abc).

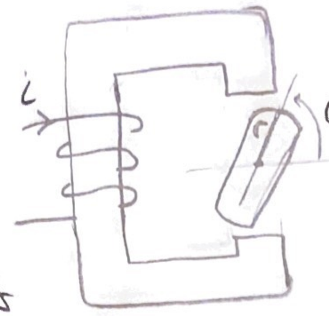
In the limit process as $dx \to 0$ Area $(abc) \to 0$

$\Rightarrow$ This shows that it make no difference how flux linkage and current is assumed to change.

Linear vs. Rotational Motion

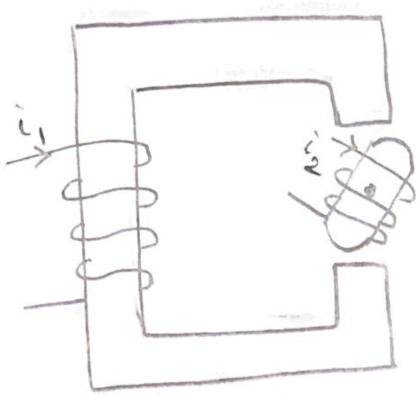
$$f = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

$$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$$



$r\theta = x \Rightarrow$ we donot need to multiply f with r once we replace x with θ.

Multiply Excited Systems



$$dW_{fld} = i_1 d\lambda_1 + i_2 d\lambda_2 - dW_{mech}$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = L_{22} i_2 + L_{21} i_1$$

L_{11} and L_{22} : self inductances

$L_{12} = L_{21} = M$: Mutual inductance

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{22} & L_{22} \end{bmatrix}$$

Most of the electromech. conversion systems have multiple excitations.

For example:
Multiple stator phases
+
Rotor excitation

Lets look at co-energy.

$$W'_{fld} = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2 + \frac{1}{2} L_{21} i_2 i_1$$

$$= \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2$$

$$T = \frac{\partial W'_{fld}}{\partial x} = \underbrace{\frac{1}{2} i_1^2 \frac{\partial L_{11}(\theta)}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}(\theta)}{\partial \theta}}_{\text{Reluctance torque}} + \underbrace{i_1 i_2 \frac{\partial L_{12}(\theta)}{\partial \theta}}_{\text{Mutual torque}}$$