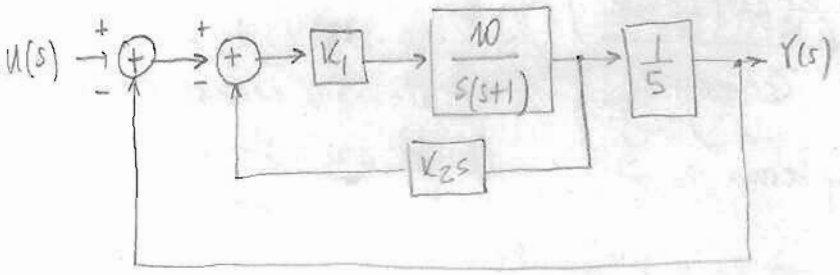
Problem:

Handout 7
2016

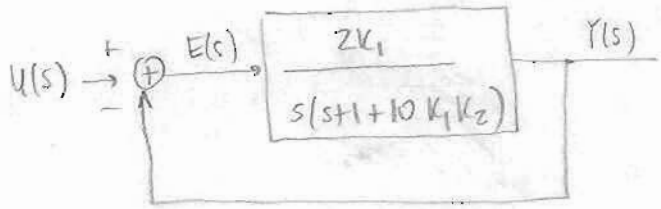
The block diagram of a control system is given below: For this system:



a) Find K_1 and K_2 so that e_{ss} due to a unit step function is zero, $t_s \cong 1$ sec; and $M_p \cong 5\%$ (5% band)

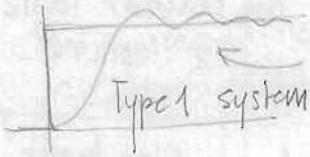
b) Find e_{ss} due to a unit ramp input.

Answer:



$$E(s) = \frac{1}{1+G(s)H(s)}U(s) = \frac{1}{1+G(s)}U(s) \longrightarrow sE(s) = \cancel{s}\cdot\frac{1}{1+\frac{2K_1}{s(s+1+10K_1K_2)}}\cdot\frac{1}{\cancel{s}} = \frac{s(s+1+10K_1K_2)}{s(s+1+10K_1K_2)+2K_1} = \frac{s(s+1+10K_1K_2)}{s^2+(1+10K_1K_2)s+2K_1}$$

we want $\lim_{s\to 0} sE(s) = \emptyset$



if K_1 and K_2 properly chosen (i.e., if the system is stable)

Consider the transient specs:

$$T(s) = \frac{2K_1}{s^2+(10K_1K_2+1)s+2K_1} = \frac{w_n^2}{s^2+2\xi w_n+w_n^2}$$

$$\left.\begin{aligned} w_n^2 &= 2K_1 \\ 2\xi w_n &= 1+10K_1K_2 \end{aligned}\right\} \quad \boxed{\xi = \frac{1+10K_1K_2}{2\sqrt{2K_1}}}$$

Can we get ξ value? (We know how!) Use M_p !!

$$M_p(\%) = 100e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 5 \longrightarrow e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 0.05 \longrightarrow \frac{-\pi\xi}{\sqrt{1-\xi^2}} \leq -3$$

Take the square of both sides: then $\boxed{\xi \geq 0.7}$ approx $\longrightarrow$ take $\boxed{\xi \cong 0.7}$

$$t_s \cong \frac{3}{\xi w_n} = 1 \longrightarrow \boxed{w_n \cong 4.29}$$

$$w_n^2 = 2K_1 \rightarrow K_1 = \frac{w_n^2}{2} = \frac{(4.29)^2}{2} \cong \boxed{9.18 = K_1}$$

$$2\xi w_n = 1+10K_1K_2 \longrightarrow 2(0.7)(4.29) = 1+10(9.18)K_2 \rightarrow \boxed{K_2 \cong 0.055}$$

b) For unit ramp $sE(s) = \frac{1}{s^2}\cdot\cancel{s}\frac{1}{1+\frac{2K_1}{s(s+1+10K_1K_2)}} = \frac{1}{\cancel{s}}\frac{\cancel{s}(s+1+10K_1K_2)}{s(s+1+10K_1K_2)+2K_1}$

$$\lim_{s\to 0} sE(s) = \frac{1+10K_1K_2}{2K_1} \cong$$

$$\frac{1+10(9.18)(0.055)}{2(9.18)} \cong \boxed{0.33}$$