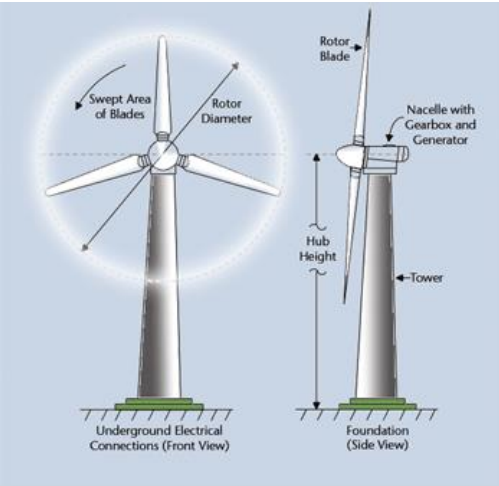
EE 462 Homework #1

Assume that we have a wind turbine with a variable speed synchronous generator connected to blades via a gear box. Its characteristics are given below:



Drawing of the rotor and blades of a wind turbine, courtesy of ESN

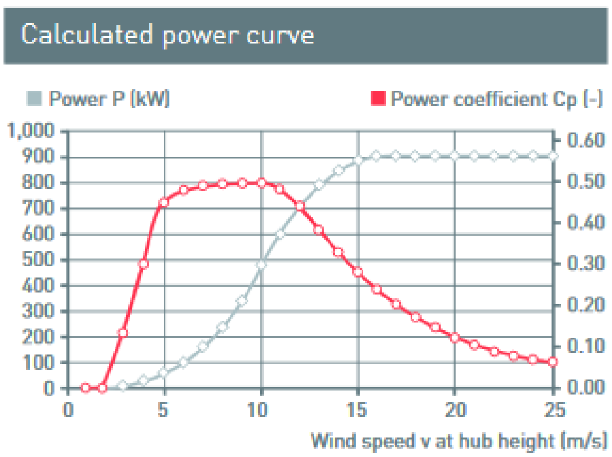
$$n\ (number\ of\ blades) = 3$$

$$r\ (radius\ of\ blades) = 44\ m$$

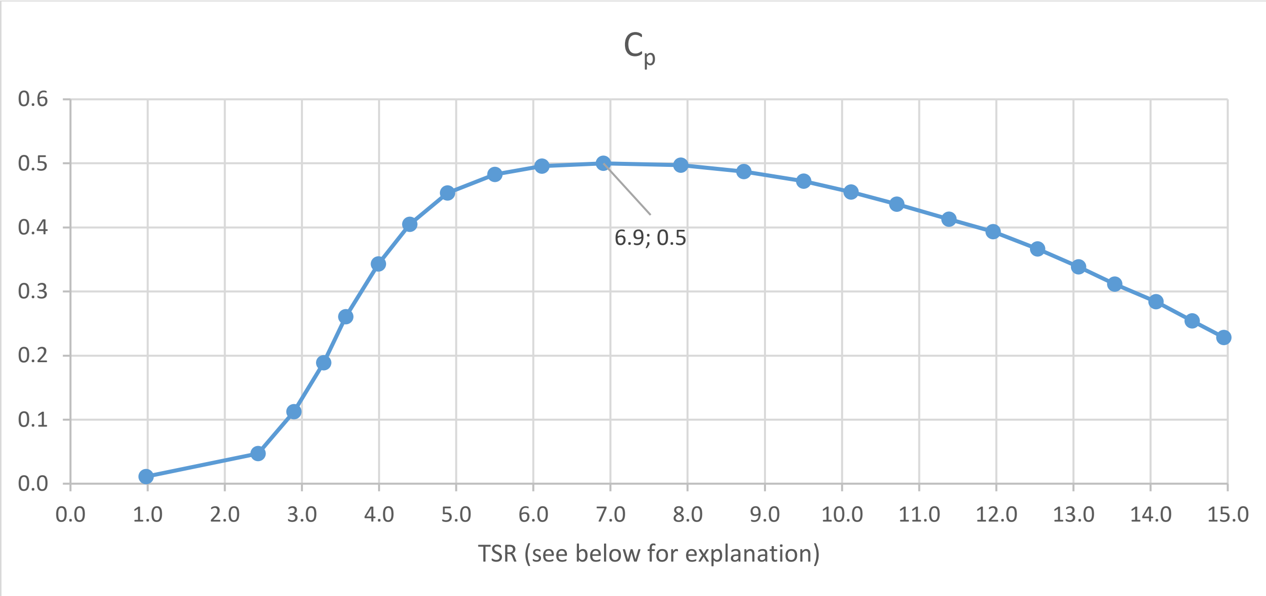
$$g(gear\ ratio) = \left(\frac{\omega_{blades}}{\omega_{rotor}}\right) = \frac{1}{90}$$

$$p\ (machine\ pole\ number) = 10$$

C_p values for the selected blade geometry is given below as a function of wind speed:



Wind (m/s)	Power P (kW)	Power-coefficient Cp (-)
1	0.0	0.00
2	0.0	0.00
3	4.0	0.16
4	20.0	0.34
5	50.0	0.43
6	96.0	0.48
7	156.0	0.49
8	238.0	0.50
9	340.0	0.50
10	466.0	0.50
11	600.0	0.48
12	710.0	0.44



Wind and air characteristics:

$$V\ (wind\ speed) = 10\ m/s$$

$$\rho\ (\text{air density}) = 1.225\ \text{kg/m}^3$$

Calculate the followings by using the explanations attached to this assignment:

1. What is the swept area of blades?
2. What is the total power potential in a given area of wind?
3. What is the maximum power that can be converted to electrical power (use Betz constant)?
4. What is the optimum rotational speed of the blades in rpm (use optimal C_p vs. TSR graph)?
5. Find out the speed of the rotor in rpm at the optimum speed condition.
6. Calculate the torque on the shaft of the electric machine, assume a 97% efficiency for the mechanical transmission.
7. What is the frequency of the stator currents in Hz? Can we connect the output of the generator to the grid? Why?
8. What is the efficiency of the wind turbine, assume a 90% efficiency for the electric drive system including the generator and the power electronics (wind power to electrical power)?
9. What is purpose of using a gear box, comment on its effect on the size of the electric machine.

Some basic information regarding wind turbines are as follow:

Power of the wind:

$$P = \frac{1}{2}\rho A V^3$$

P= Power of wind

ρ= Air density (1.225 kg/m3)

A= Swept area of blades

V= Velocity of the wind

Betz limit: Albert Betz calculated that no wind turbine could convert more than 59.3% of the kinetic energy of the wind into the mechanical energy of the turning rotor. This is known as the Betz limit.

$$C_p = \frac{Electricity\ produced\ by\ wind\ turbine}{Total\ energy\ available\ in\ the\ wind}$$

For a wind turbine to be 100% efficient, it would need to stop 100% of the wind but then the rotor has to be solid disk and it would not turn, so that no kinetic energy would be converted. On the other extreme, if you have a wind turbine with one blade, most of the wind passing through the swept area will be missed. So, if a turbine is operating at Betz limit is taken as 100% efficiency. If a wind turbine converts 70% of the Betz limit into electricity, C_p of the turbine will be 0.59 0.7 = 41%. Good wind turbines generally operate with 35%-45% efficiency.

Tip speed ratio (TSR):

TSR is an important factor in wind turbine design, referring to the ratio between the wind speed and the speed of the tips of the wind turbine.

$$TSR = \frac{Tip\ speed\ of\ the\ blade}{Wind\ speed}$$

If the rotor of the wind turbine too slowly, most of the wind will pass through blades giving no power. However, if the turbine spins too fast, blades may act like a solid wall. At high speeds, blades may also cause turbulence, and next arriving blade may hit this turbulent air. Therefore, speed of the blades is very important. For a specific blade design and pitch angle, optimal rotor speed can be found from graph of power coefficient C_P versus the tip speed ratio TSR. The shape of this curve be made plausible with the following reasoning:

- At TSR = 0 the rotor does not rotate and hence cannot extract power from the wind
- At very high TSR, the rotor runs so fast that it seen by the wind as a completely blocked disc.