MIDDLE EAST TECHNICAL UNIVERSITY

MECHANICAL ENGINEERING DEPARTMENT

ME 205 STATICS – FALL 2018

SECTION 1

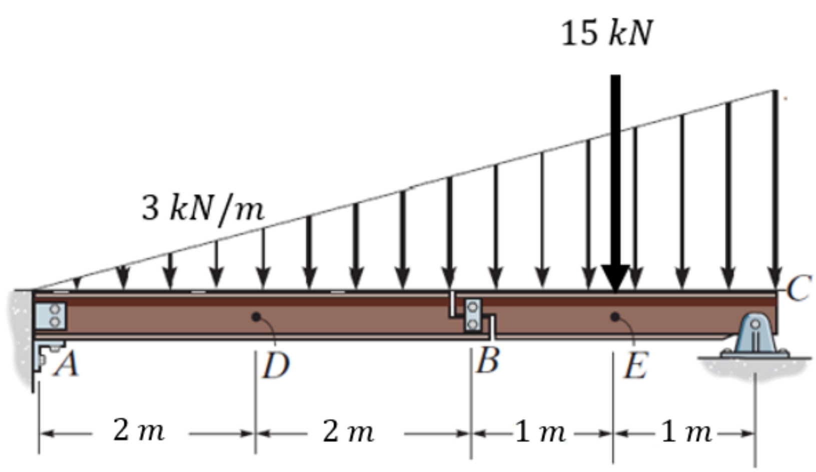
HOMEWORK #5

Prepared by: Koray Eskiduman
Room: D-106
E-mail: korayesk@metu.edu.tr

Assigned Date: 27.12.2018
Due Date: 03.01.2019
Due Time: 17.00
Grading Due Date: 17.01.2019

Please include your name, student ID, due date, a proper headline, page number with total page number, and units in your homework. Neatness will be graded.

1. For the given beam,
   a. Find the internal normal force, shear force, and moment at points *D* and *E.* Point *E* is located at just to the left of the 15 kN force. *(5 pts)*
   b. Draw the shear and bending moment diagrams. *(20 pts)*
   c. Determine the magnitude and location of the maximum absolute value of the shear force and the bending moment. *(5 pts)*



Solution:
a)

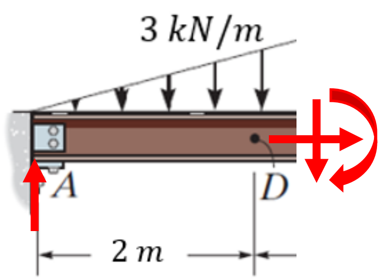
$$\sum F_x = 0;\ \ A_x = C_x = 0$$

$$\sum M_c = 0;\ \ A_y(6) - (15)(1) - \left[\frac{1}{2}(3)(6)\right](2) = 0$$

$$A_y = 5.5\ kN\ (\uparrow)$$

$$\sum F_y = 0;\ \ A_y + C_y - 15 - \frac{1}{2}(3)(6) = 0$$

$$C_y = 18.5\ kN\ (\uparrow)$$



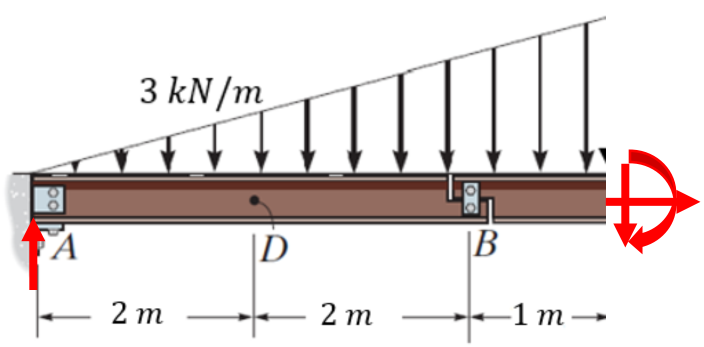
$$\sum F_x = 0;\ \ N_D = 0$$

$$\sum F_y = 0;\ \ A_y - V_D - \frac{1}{2}(1)(2) = 0$$

$$V_D = 4.5\ kN\ (\downarrow)$$

$$\sum M_D = 0;\ \ A_y(2) - \left[\frac{1}{2}(1)(2)\right]\left(\frac{2}{3}\right) - M_D = 0$$

$$M_D = 10.33\ kNm\ (\circlearrowleft)$$

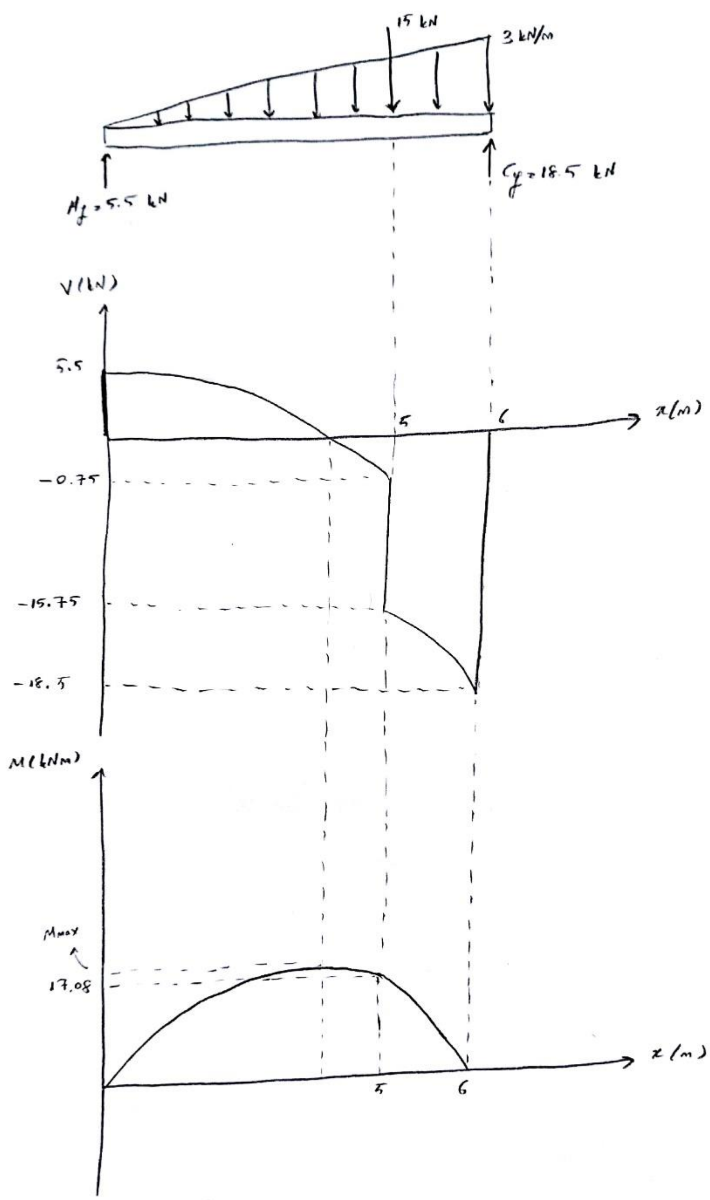


$$\sum F_x = 0;\ \ N_E = 0$$

$$\sum F_y = 0;\ A_y - V_E - \frac{1}{2}(2.5)(5) = 0$$
$$V_E = -0.75\ kN\ (\uparrow)$$

$$\sum M_E = 0;\ A_y(5) - \left[\frac{1}{2}(2.5)(5)\right]\left(\frac{5}{3}\right) - M_E = 0$$
$$M_E = 17.08\ kNm\ (\circlearrowleft)$$

b)



c) From the diagram $V_{max} = 18.5\ kN$. When moment is maximum or minimum, $\frac{dM}{dx} = 0$; therefore in the shear diagram, the zero points are the points where moment is maximum or minimum. In the shear diagram, the zero point is in between *A* and *E*. The shear equation between these points is

$$V(x) = 5.5 - \int w(x)dx$$

$$w(x) = \frac{1}{2}x$$

$$V(x) = 5.5 - \int \frac{1}{2}xdx = 5.5 - \frac{x^2}{4}$$

$$V(x) = 5.5 - \frac{x^2}{4} = 0 \rightarrow x = 4.69\ m$$

$$M(x) = \int V(x)\,dx = \int \left(5.5 - \frac{x^2}{4}\right)dx = 5.5x - \frac{x^3}{12}$$

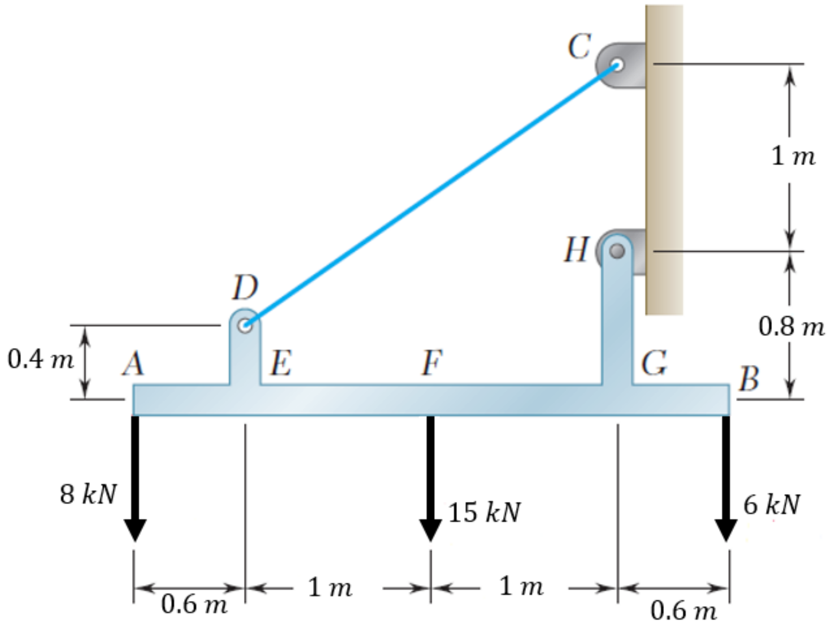
$$M(4.69) = 17.2\ kNm$$

Note: You may obtain the shear and moment equations by using method of sections as well.

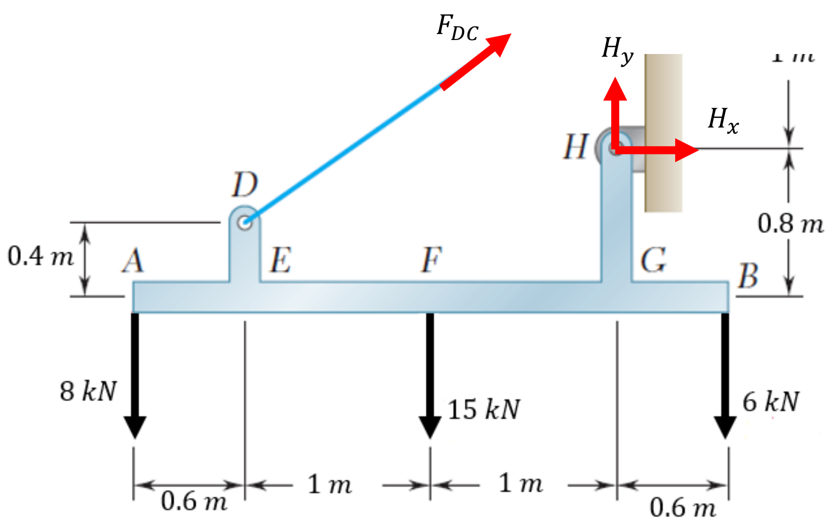
2. For the section *AB*,

a. Draw the shear and bending moment diagrams. *(20 pts)*

b. Determine the magnitude and location of the maximum absolute value of the shear force and the bending moment. *(5 pts)*



Solution:



The angle between F_{DC} and the horizontal is

$$\alpha = \tan^{-1}\frac{1.4}{2} = 35°$$

$$\sum M_H = 0;\ \ (8)(2.6) + (15)(1) - (6)(0.6) + (F_{DC}\cos 35°)(0.4) - (F_{CD}\sin 35°)(2) = 0$$

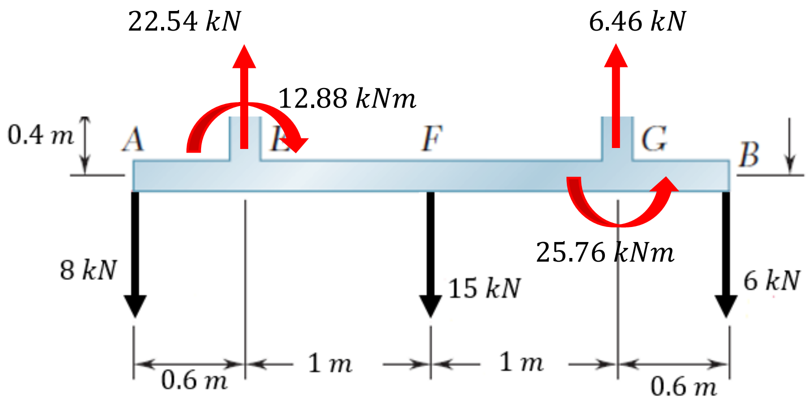
$$F_{CD} = 39.3\ kN$$

$$\sum F_x = 0;\ \ H_x + F_{CD}\cos 35° = 0$$

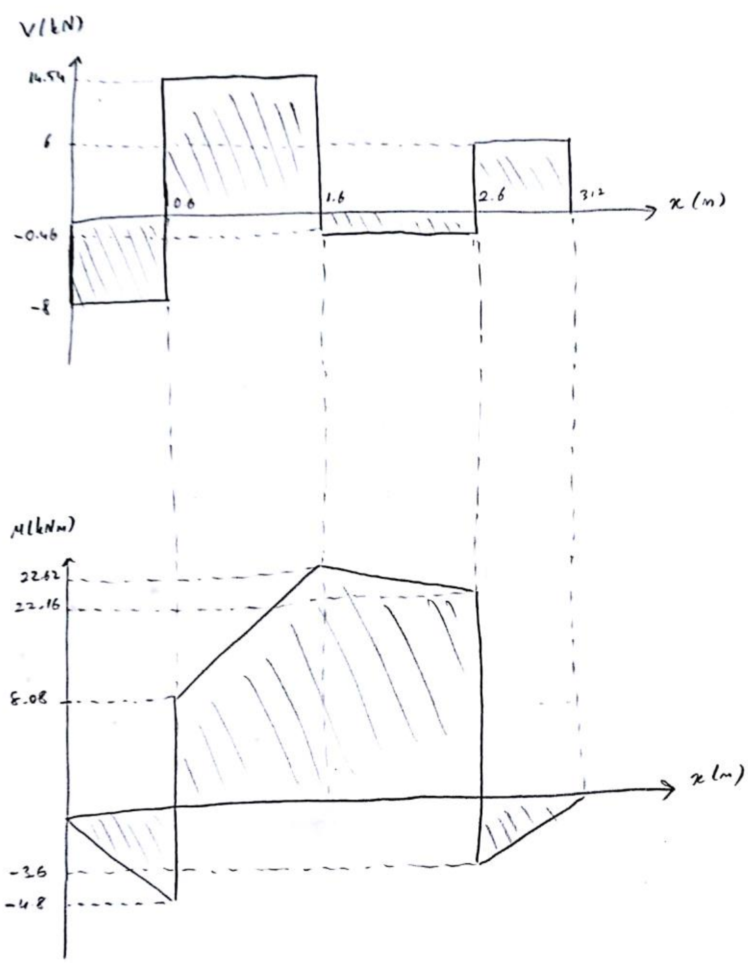
$$H_x = -32.2\ kN\ (\leftarrow)$$

$$\sum F_y = 0;\ \ H_y + F_{DC}\sin 35° - 8 - 15 - 6 = 0$$
$$H_y = 6.46\ kN$$

We need to obtai the equivalent system fot the section *AB.*



a)

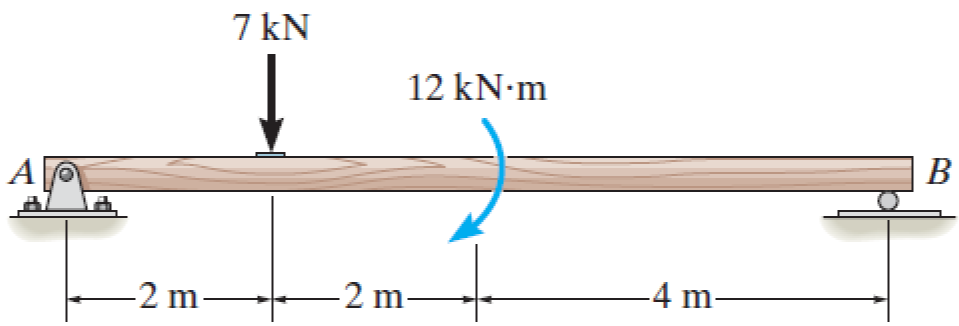


b) From the diagrams $V_{max} = 14.54\ kN$ and $M_{max} = 22.62\ kN$

3. The weight of the uniform beam is 6 kN.

a. Draw the shear and bending moment diagrams. (15 pts)

b. Determine the magnitude and location of the maximum absolute value of the shear force and the bending moment. (5 pts)



Solution:

The weight of the beam is distributed over the beam with $w(x) = \frac{6}{8} = 0.75\ kN/m$

$$\sum F_x = 0;\ \ A_x = 0$$

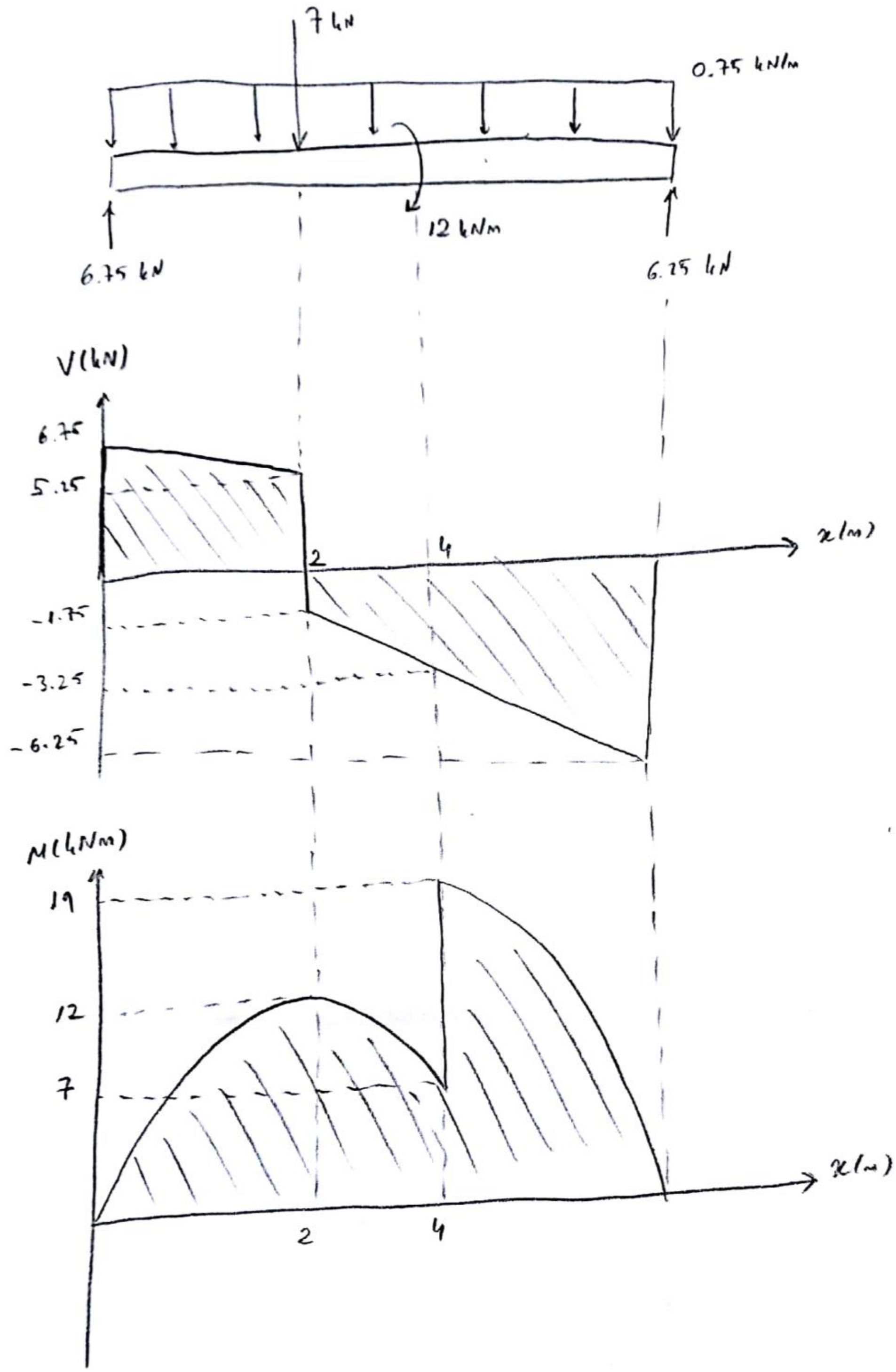
$$\sum M_A = 0;\ \ B_y(8) - (7)(2) - 12 - (6)(4) = 0$$

$$B_y = 6.25\ kN$$

$$\sum F_y = 0; A_y + B_y - 7 - 6 = 0$$

$$A_y = 6.75\ kN$$

a)

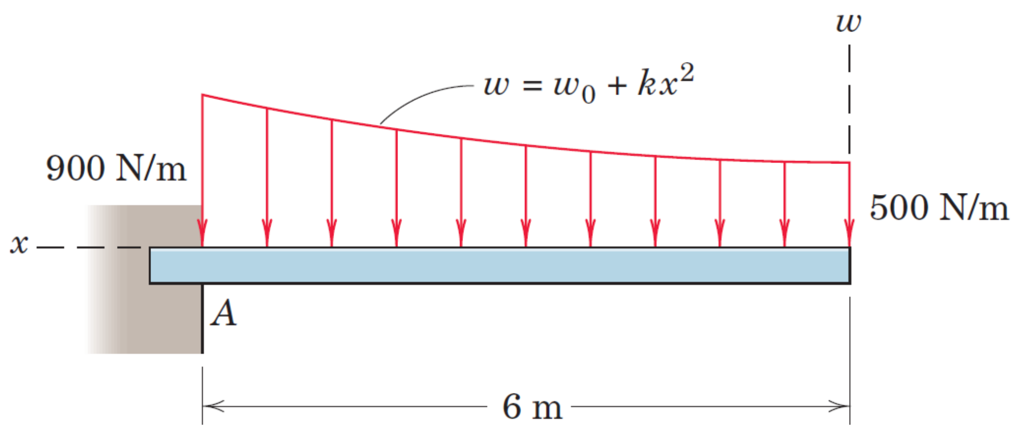


b) From the diagrams $V_{max} = 6.75\ kN$ and $M_{max} = 19\ kNm$

4. For the given beam,

a. Draw the shear and bending moment diagrams. *(20 pts)*

b. Determine the magnitude of the shear force and the bending moment at the middle of the beam. *(5 pts)*



Solution:

When $x = 0$, $w(0) = w_0 = 500\ N/m$

When $x = 6$, $w(6) = w_0 + 36k = 900\ N/m$

$$k = 11.11$$

Let W be the total distributed load, then

$$W = \int_0^6 w(x)dx = \int_0^6 (500 + 11.11x^2)dx = (500x + 3.7x^3)|_{x=0}^{x=6} = 3800\ N$$

The centroid of the load curve can be found from

$$\bar{x} = \frac{\int_0^6 xw(x)dx}{\int_0^6 w(x)dx} = \frac{\int_0^6 (500x + 11.11x^3)dx}{3800} = \frac{(250x^2 + 2.78x^4)|_{x=0}^{x=6}}{3800} = 3.32\ m$$

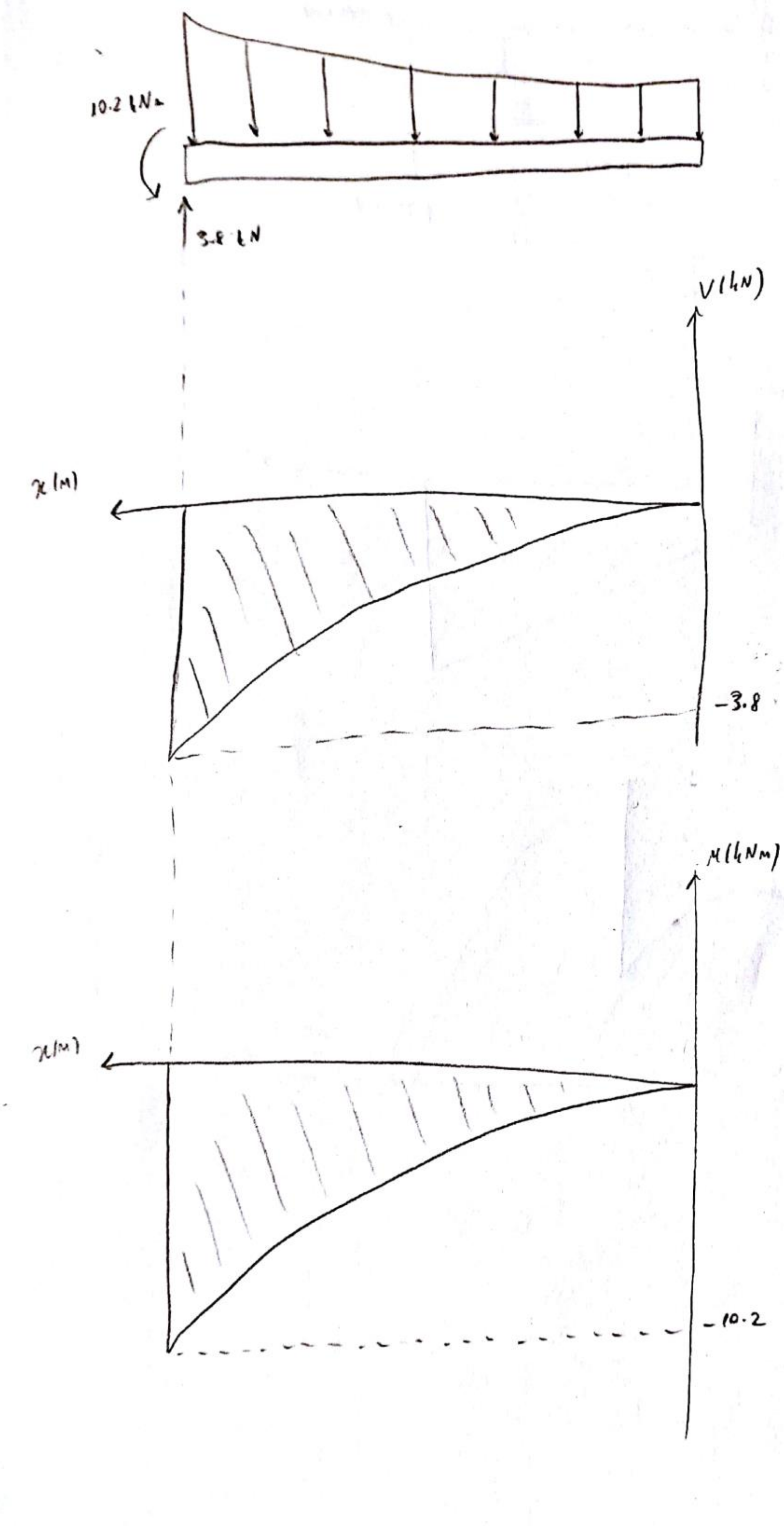
There are reaction forces in x and y directions and reaction moment at *A* since it is fixed.

$$\sum F_x = 0;\ A_x = 0$$

$$\sum F_y = 0;\ A_y = W = 3800\ N = 3.8\ kN$$

$$\sum M_A = 0;\ M_A = W(6 - \bar{x}) = 10200\ Nm = 10.2\ kNm\ (\circlearrowleft)$$

a)



b) At the middle of the beam $x = 3\ m$

$$V(x) = -\int w(x)\,dx = -\int (500 + 11.11x^2)dx = -500x - 3.7x^3$$

$$V(3) = -1600\ N$$

$$M(x) = \int V(x)dx = \int (-500x - 3.7x^3)dx = -250x^2 - 0.93x^4$$

$$M(3) = -2325\ Nm$$