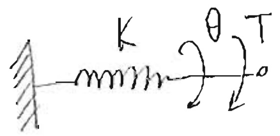
Rotational Mechanics Components & Example

March 01, 2015
Handout

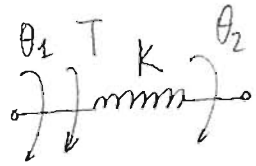
Components

T : Torque
θ : Angular displacement
$\dot{\theta}$: Angular velocity

1.) Rotational Spring

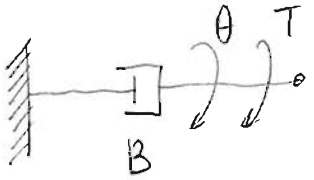


$$T = -K\theta$$

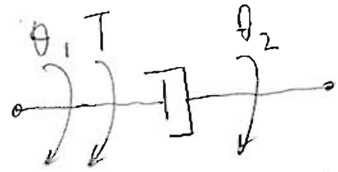


$$T = -K(\theta_1 - \theta_2)$$

2) Rotational Damper (Viscous Damper)

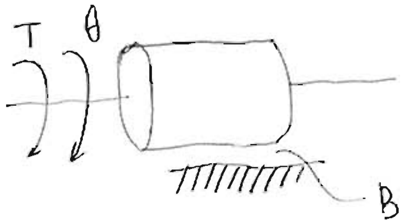


$$T = -B\dot{\theta}$$



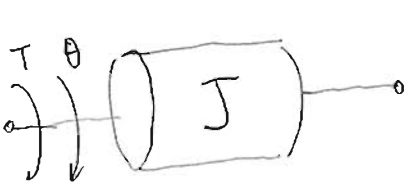
$$T = -B(\dot{\theta}_1 - \dot{\theta}_2)$$

3.) Rotational Viscous Friction



$$T = -B\dot{\theta}$$

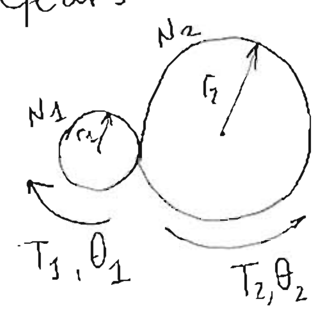
4.) Newton's 2nd Law of Motion



J : Rotational inertia
T_{net} : Net torque acting on the inertia
$\ddot{\theta}$: Rotational acceleration.

$$J\ddot{\theta} = T_{net}$$

5.) Gears



$$\frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{T_1}{T_2}$$

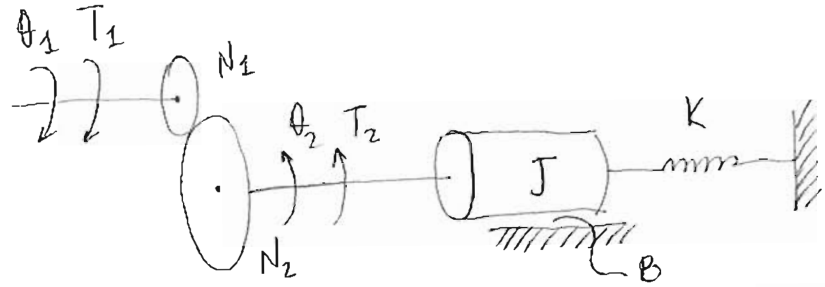
r_1, r_2 : Gear radius
N_1, N_2 : Number of gear teeth.

Example:

N_1, N_2 : Number of teeth of gears

T_1 : torque (input)

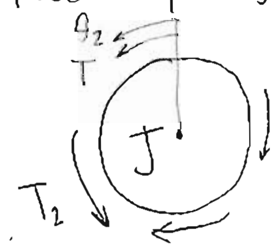
θ_1 : angular displacement (output)



Can be solved by "projecting" variables to either side of the gears

Approach 1. Project everything (all variables) to the θ_1, T_1 side.

Free body diagram: (on the right hand side)



$T_f = B\dot{\theta}_2$

$T_s = K\theta_2$

Newton's Law: $T_{net} = J \cdot \ddot{\theta}_2$

$$J\ddot{\theta}_2 = T_2 - B\dot{\theta}_2 - K\theta_2$$

$$\boxed{J\ddot{\theta}_2 + B\dot{\theta}_2 + K\theta_2 = T_2}$$

But we need an equation in terms of T_1 and θ_1 (the input and output)

Use $\left.\dfrac{N_1}{N_2} = \dfrac{\theta_2}{\theta_1} = \dfrac{T_1}{T_2}\right\} \rightarrow$ "project θ_2 and T_2 to the input" (write them in terms of θ_1 and θ_2)

$$\theta_2 = \frac{N_1}{N_2}\theta_1 \qquad T_2 = \frac{N_2}{N_1}T_1 \quad \rightarrow \quad J\left(\frac{N_1}{N_2}\right)\ddot{\theta}_1 + B\left(\frac{N_1}{N_2}\right)\dot{\theta}_1 + K\left(\frac{N_1}{N_2}\right)\theta_1 = \left(\frac{N_2}{N_1}\right)T_1$$

$$J\ddot{\theta}_1 + B\dot{\theta}_1 + K\theta_1 = \left(\frac{N_2}{N_1}\right)^2 T_1$$

$$(LT) \rightarrow (Js^2 + Bs + K)\,\theta_1 = \left(\frac{N_2}{N_1}\right)^2 T_1$$

$$T(s) = \frac{\theta_1(s)}{T_1(s)} = \frac{(N_2/N_1)^2}{Js^2 + Bs + K}$$

· State Space? Let $x_1 = \theta_1$; $x_2 = \dot{\theta}_1$

$$\dot{x}_1 = \dot{\theta}_1 = x_2$$

$$\dot{x}_2 = \ddot{\theta}_1 = -\frac{K}{J}\theta_1 - \frac{B}{J}\dot{\theta}_1 + \frac{1}{J}\left(\frac{N_2}{N_1}\right)^2 T_1$$

$$= -\frac{K}{J}x_1 - \frac{B}{J}x_2 + \frac{1}{J}\left(\frac{N_2}{N_1}\right)^2 T_1$$

$\Rightarrow$

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ \frac{1}{J}\left(\frac{N_2}{N_1}\right)^2 \end{bmatrix} T_1$$

$$\theta_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}$$