METU EE462 Utilization of Electric Energy

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Content

Basic motion

Two Inertia System

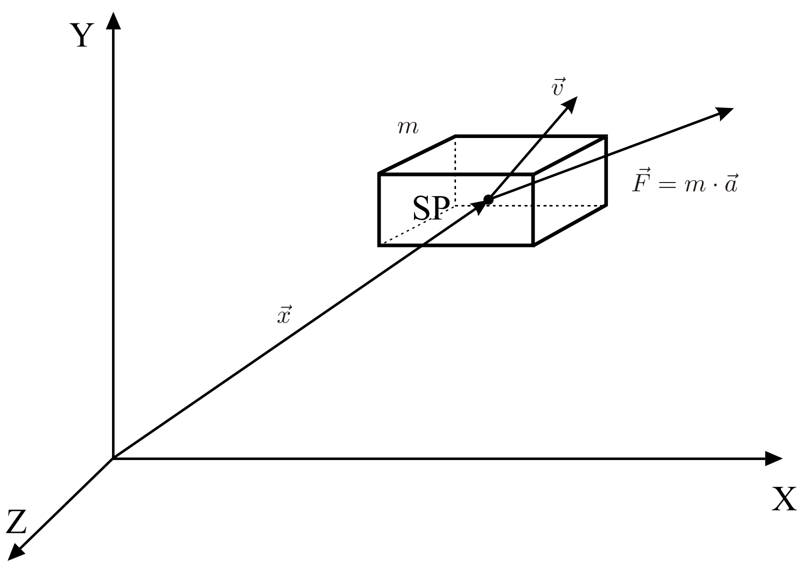
Sensors

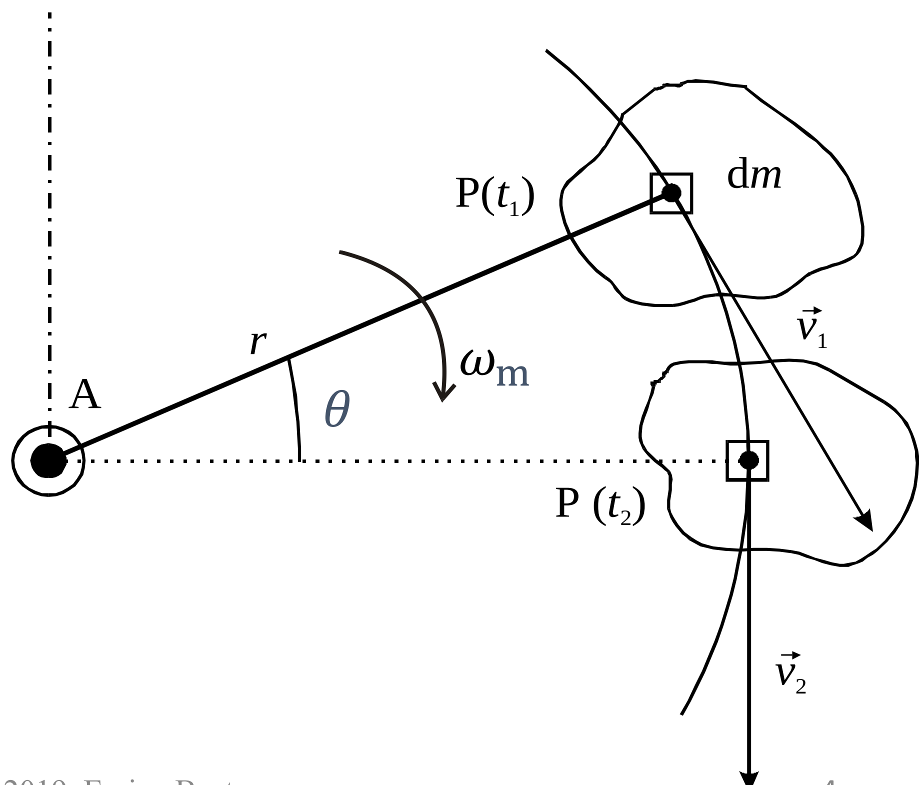
Linear vs. Rotational Motion

Translation			Rotation		
Quantity	**Equation**	**Unit**	**Quantity**	**Equation**	**Unit**
Distance, s	$s = r.\theta$	m	**Angle, θ**	$\theta = s/r$	rad
Velocity, v	$v = \frac{ds}{dt}$ $v = r.\omega$	m/s	**Angular velocity, ω**	$\omega = \frac{d\theta}{dt}$ $\omega = v/r$	1/s
Acceleration, a	$a = \frac{dv}{dt}$	m/s^2	**Angular acc., $\boldsymbol{\alpha_m}$**	$\alpha_m = \frac{d\omega}{dt}$	$1/s^2$
Mass, M		kg	**Inertia, J**	$J = \iiint r^2\, dm$	$kg\ m^2$
Force, F	$F = m\frac{dv}{dt}$	N	**Torque, T**	$T = J\frac{d\omega}{dt}$	Nm
Power, P	$P = F.v$	W	**Power, P**	$P = T.\omega$	W
Energy, E	$E = \int P\, dt$	J (Ws)	**Energy, E**	$E = \int P\, dt$	J (Ws)

Linear vs. Rotational Motion

Motion Equations	$F = m\dfrac{dv}{dt} + \mathrm{F_{load}}$	$T = J\dfrac{d\omega}{dt} + T_{load}$
Stored Kinetic Energy	$W = \dfrac{1}{2}mv^2$	$W = \dfrac{1}{2}J\omega^2$





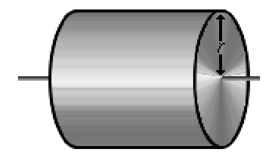
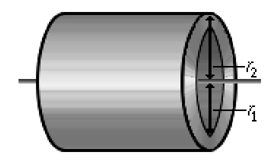
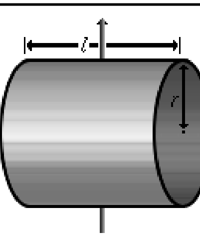
Moment of Inertia - Rotational Mass Equivalent

$$T_{\mathrm{tot}}(t) = \sum_i T_i\,(t) = J\,\frac{\mathrm{d}\Omega_{\mathrm{m}}(t)}{\mathrm{d}t}$$

General: $J = \iiint (\vec{r} - \vec{r}_0)^2\, \rho(\vec{r})\mathrm{d}V$

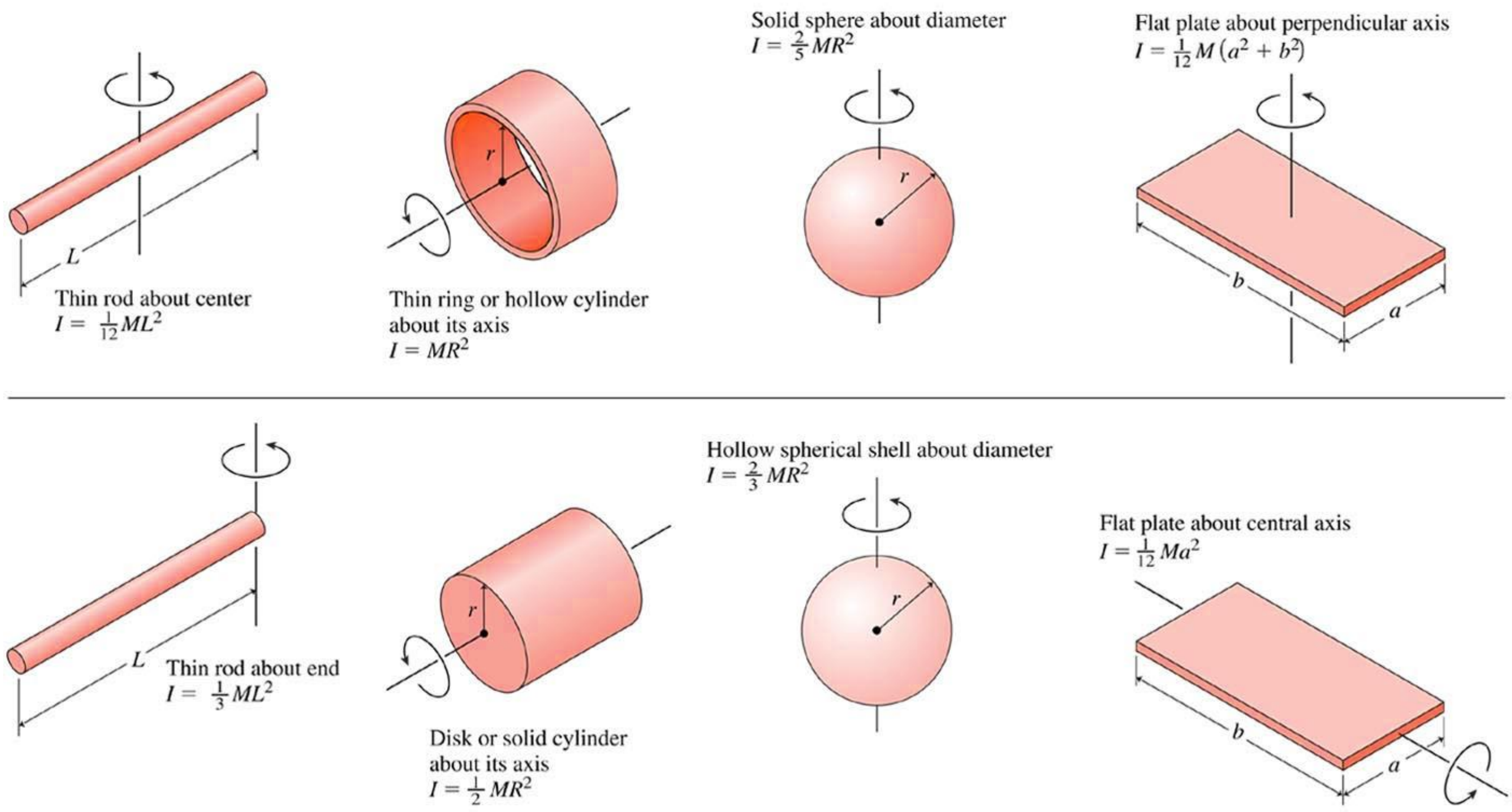
⇒ Inertia depends on spatial mass distribution respective to the rotational axis

Simple geometries:

Solid cylinder:	$J_{\mathrm{s}} = \frac{1}{2} m r^2$	e.g. machine rotor	
Hollow cylinder:	$J_{\mathrm{h}} = \frac{1}{2} m (r_1^2 - r_2^2)$	e.g. hollow shaft	
Solid cylinder: (transversal axis)	$J_{\mathrm{s,trans}} = \frac{1}{4} m r^2 + \frac{1}{12} m l^2$		

Moment of Inertia - Rotational Mass Equivalent

TABLE 10.2 Rotational Inertias



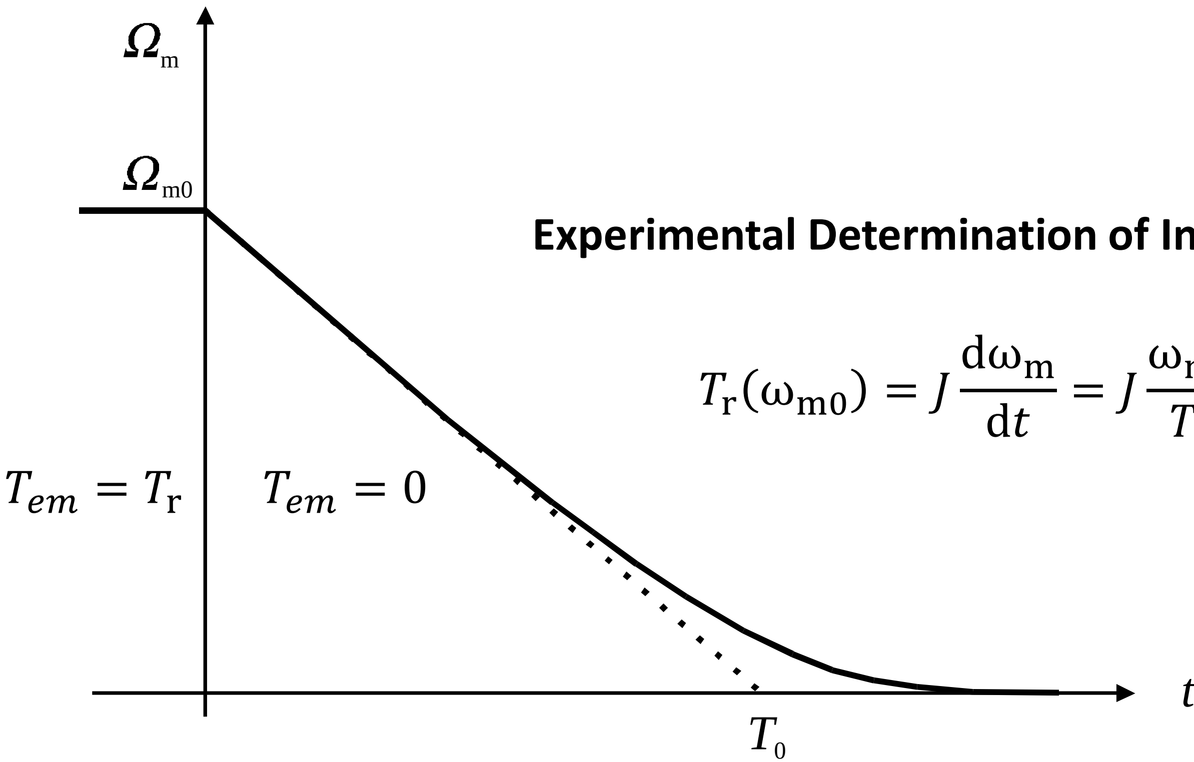
Moment of Inertia - Rotational Mass Equivalent

Importance of Inertia:

Flywheel store kinetic energy, reduce speed ripples

Reduces the dynamic performance of the drive

Large energy to dissipate in dynamic braking



Experimental Determination of Inertia (Retardation Test)

$$T_r(\omega_{m0}) = J\frac{d\omega_m}{dt} = J\frac{\omega_{m0}}{T_0} \Rightarrow J = \frac{T_0}{\omega_{m0}}T_r(\omega_{m0})$$

with T_r : friction torque
T_{em}: machine torque

Moment of Inertia - Rotational Mass Equivalent

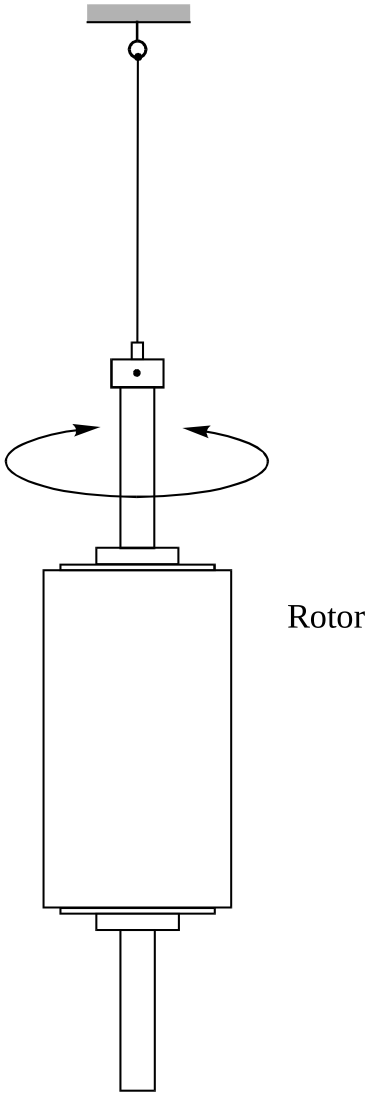
Experimental Determination of Inertia (Torsional Pendulum)

1. Hang up the rotor to a string connected to the rotational axis
2. Drill up the string & let the rotor perform free torsional oscillations
3. Determine the oscillation frequency f
4. Compare f to f_0 of a rotor with known inertia:

$$J = J_0 \left(\frac{f_0}{f}\right)^2$$

J_0: Inertia of the known motor

f_0: Oscillation frequency of the known motor



Net Tractive Torque

$$T_{\mathrm{tot}}(t) = \sum_i T_i\,(t) = J\,\frac{\mathrm{d}\omega_{\mathrm{m}}(t)}{\mathrm{d}t}$$

$$T_{\mathrm{tot}}(t) = T_{em} - T_r - T_{load}$$

T_{em}: Machine torque, it can be positive or negative.

T_r: Friction torque (torque losses) like aerodynamic windage and other mechanical friction. This component always directed against the direction of motion.

T_{load}: This is the sum of all load torques that produce the useable mechanical work and therefore represents function of the application. This can be the mass of the elevator or the aerodynamic resistance of a fan. In some applications friction can also be considered as a part of the load torque like Rolling and aerodynamic friction in vehicles.

$\sum_i T_i\,(t) > 0 \;\&\; \omega_{\mathrm{m}} > 0$: Acceleration

$\sum_i T_i\,(t) < 0 \;\&\; \omega_{\mathrm{m}} > 0$: Deceleration

$\sum_i T_i\,(t) = 0$: Constant speed

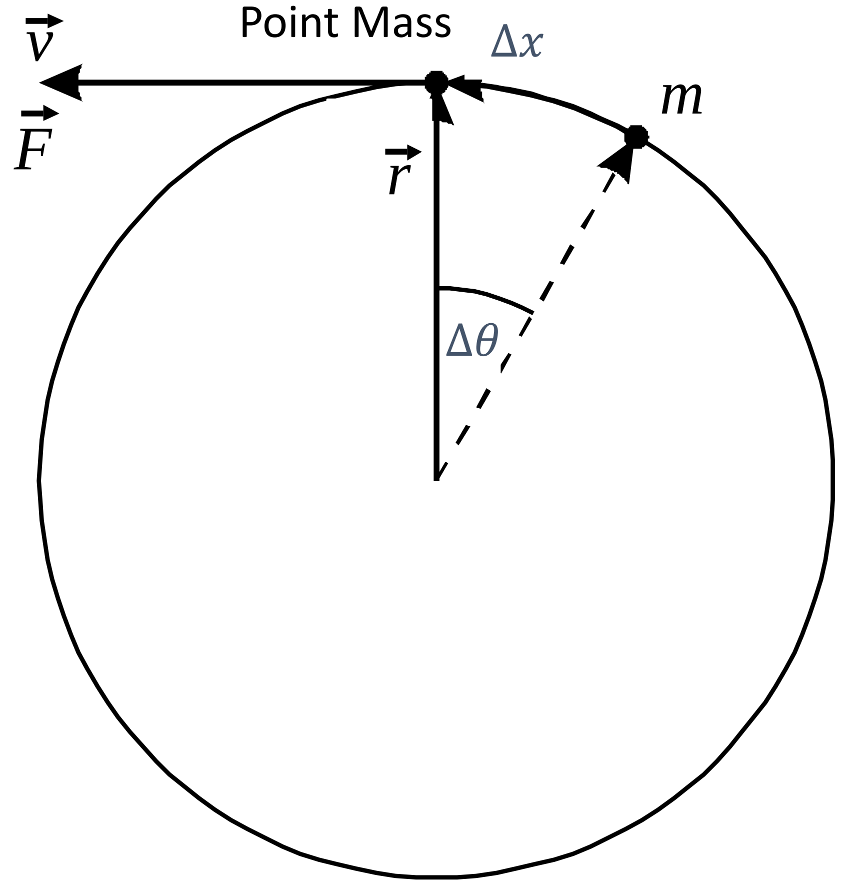
Circular Motion of a Point Mass

$$\|\vec{v}\| = \omega_{\mathrm{m}} \cdot r$$

$$T = r \cdot \left\|\vec{F}\right\|$$

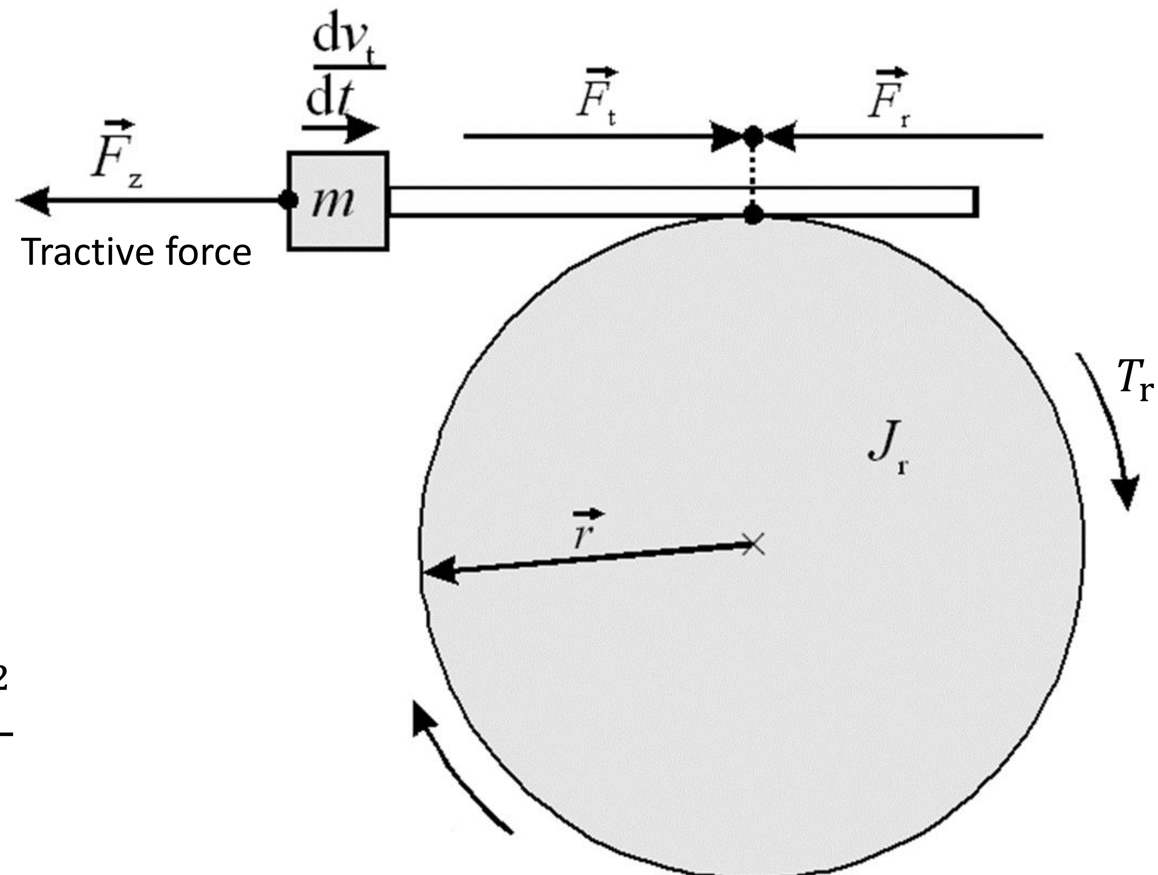
$$\Delta x = r \cdot \Delta\theta$$

$$J = m\, r^2$$



Conversion of Translation Motion to Rotation

Many drive system require the conversion from translatory to rotational motion and vice versa.



Angular Speed Ratio:

$$v = r\omega$$

Total Kinetic Energy:

$$\frac{mv^2}{2} + \frac{J_r\omega^2}{2} = \frac{J_{tot}\omega^2}{2}$$

Equivalent Total Inertia:

$$J_{tot} = J_r + mr^2$$

Equivalent inertia

t: translatory
r: rotary

Gear Transmission

Some drives cannot be connected to the load directly because they are assigned to run at different speeds (torques) or there is a limited space for mounting them directly to the load. Thus mechanical speed transducers are used.

Gear transmission:

$$\frac{r_a}{r_l} = \frac{T_a}{T_l} = \frac{\omega_l}{\omega_a}$$

Total Kinetic Energy:

$$\frac{J_a \omega_a{}^2}{2} + \frac{J_l \omega_l{}^2}{2} = \frac{J_{tot} \omega_a{}^2}{2}$$

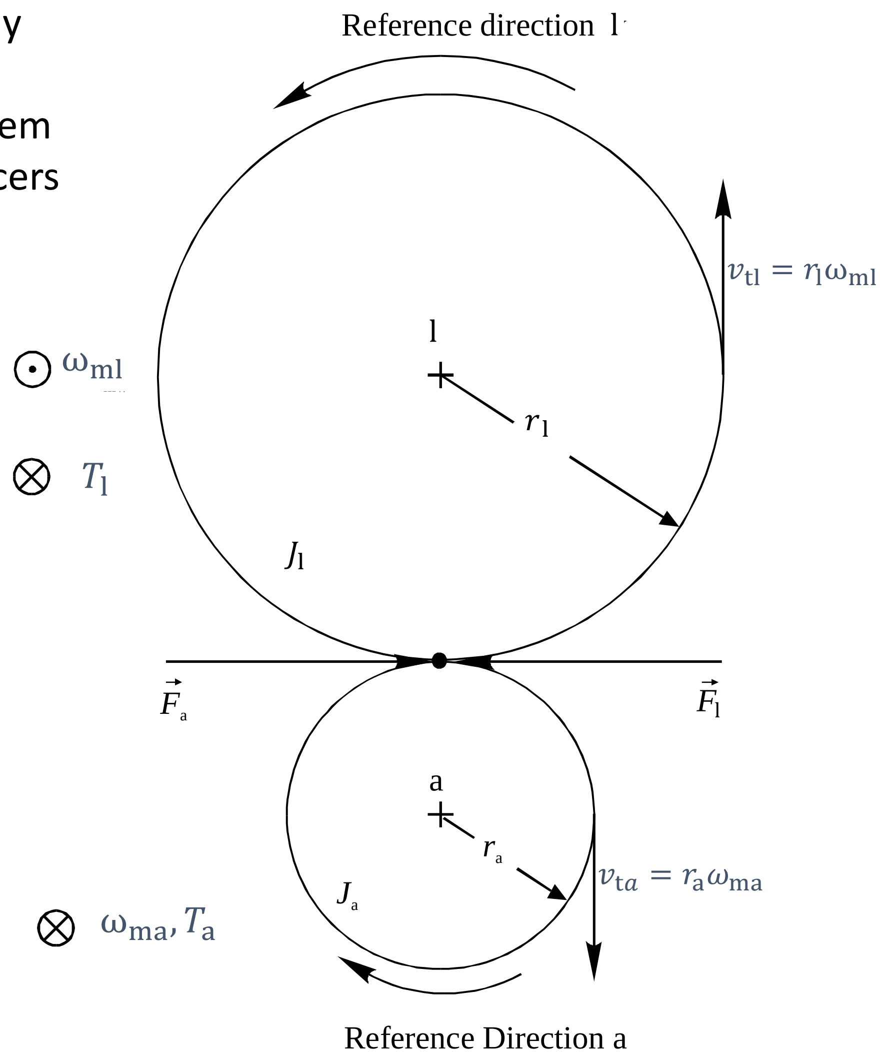
Angular Speed Ratio:

$$r_a \omega_a = r_l \omega_l$$

Equivalent Total Inertia:

$$J_a \omega_a{}^2 + J_l \left(\frac{r_a}{r_l} \omega_a\right)^2 = J_{tot} \omega_a{}^2$$

$$J_a + J_l \left(\frac{r_a}{r_l}\right)^2 = J_{tot}$$



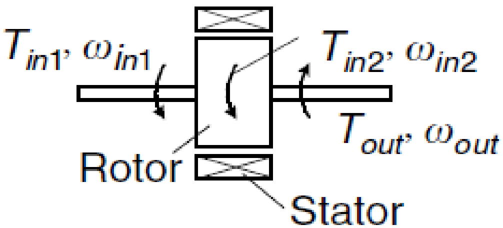
Coupling Mechanism - Rotating to Rotating

Torque coupler

$$T_{out} = k_1 T_{in1} + k_2 T_{in2}$$

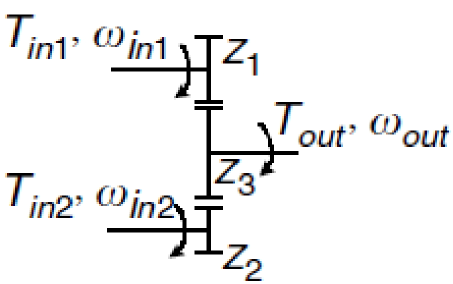
$$\omega_{out} = \frac{\omega_{in1}}{k_1} = \frac{\omega_{in2}}{k_2}$$

<u>1. Single shaft configuration:</u>



$$k_1 = 1$$
$$k_2 = 1$$

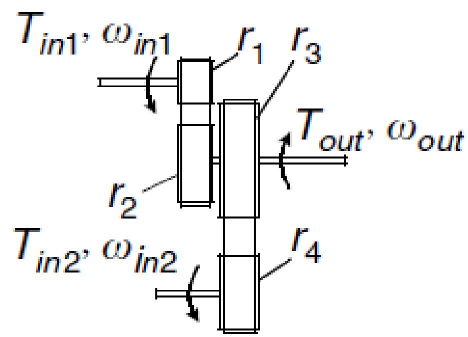
<u>3. Gear box:</u>



$$k_1 = \frac{Z_3}{Z_1},\; k_2 = \frac{Z_3}{Z_2}$$

z_1, z_2, z_3: Tooth number of the gears

<u>4. Pulley or chain assembly:</u>



$$k_1 = \frac{r_2}{r_1},\; k_2 = \frac{r_3}{r_4}$$

r_1, r_2, r_3, r_4: Radius of the pulleys

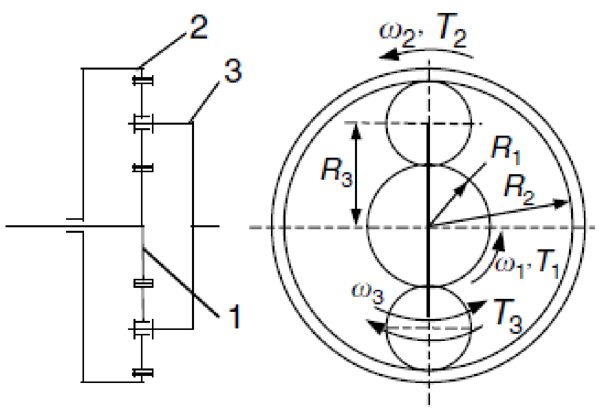
Coupling Mechanism - Rotating to Rotating

Speed coupler

$$\omega_{out} = k_1\omega_{in1} + k_2\omega_{in2}$$

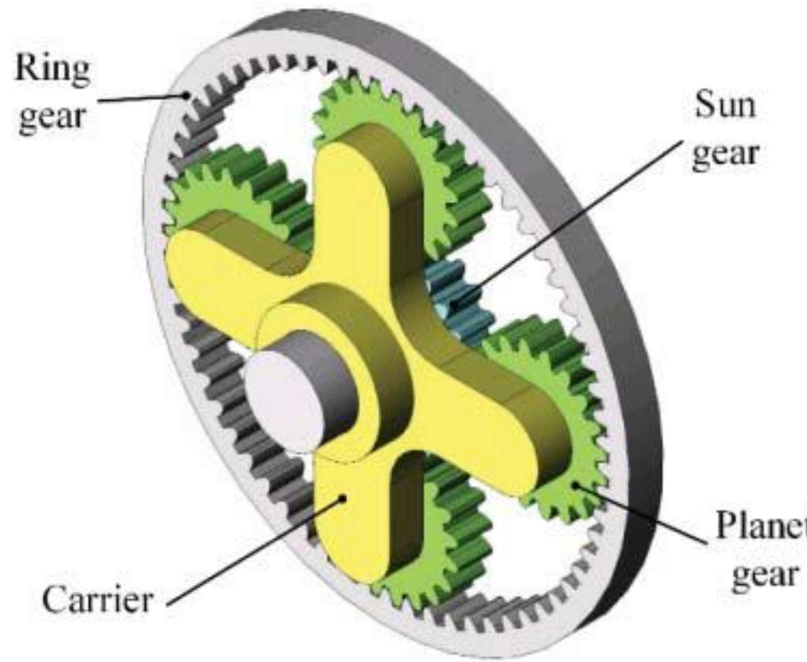
$$T_{out} = \frac{T_{in1}}{k_1} = \frac{T_{in2}}{k_2}$$

<u>1. Planetary gear unit:</u>



$$\omega_3 = \frac{R_1}{2R_3}\omega_1 + \frac{R_2}{2R_3}\omega_2 \qquad k_1 = \frac{R_1}{2R_3}$$

$$T_3 = \frac{2R_3}{R_1} \quad T_1 = \frac{2R_3}{R_2}T_2 \qquad k_2 = \frac{R_2}{2R_3}$$



Coupling Mechanism - Rotating to Rotating

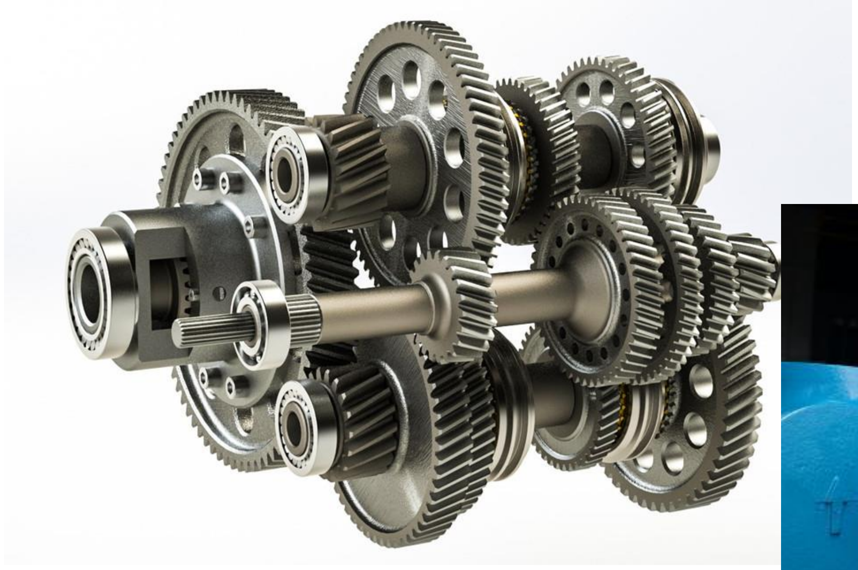
Direct Coupling
(Single Shaft)

Sleeve Coupling

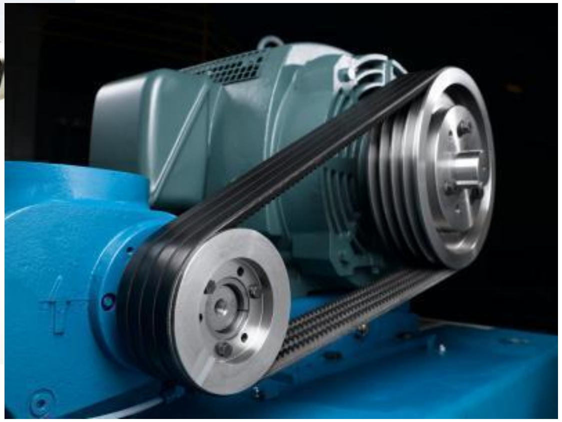
Jaw Type Coupling

Spiral Type Coupling

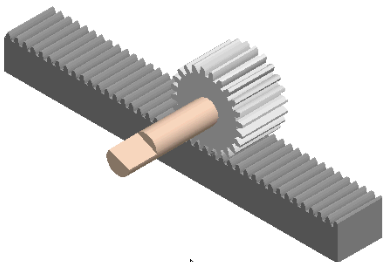
Gear Box



Belt Coupling – Pulley and Chain Assembly

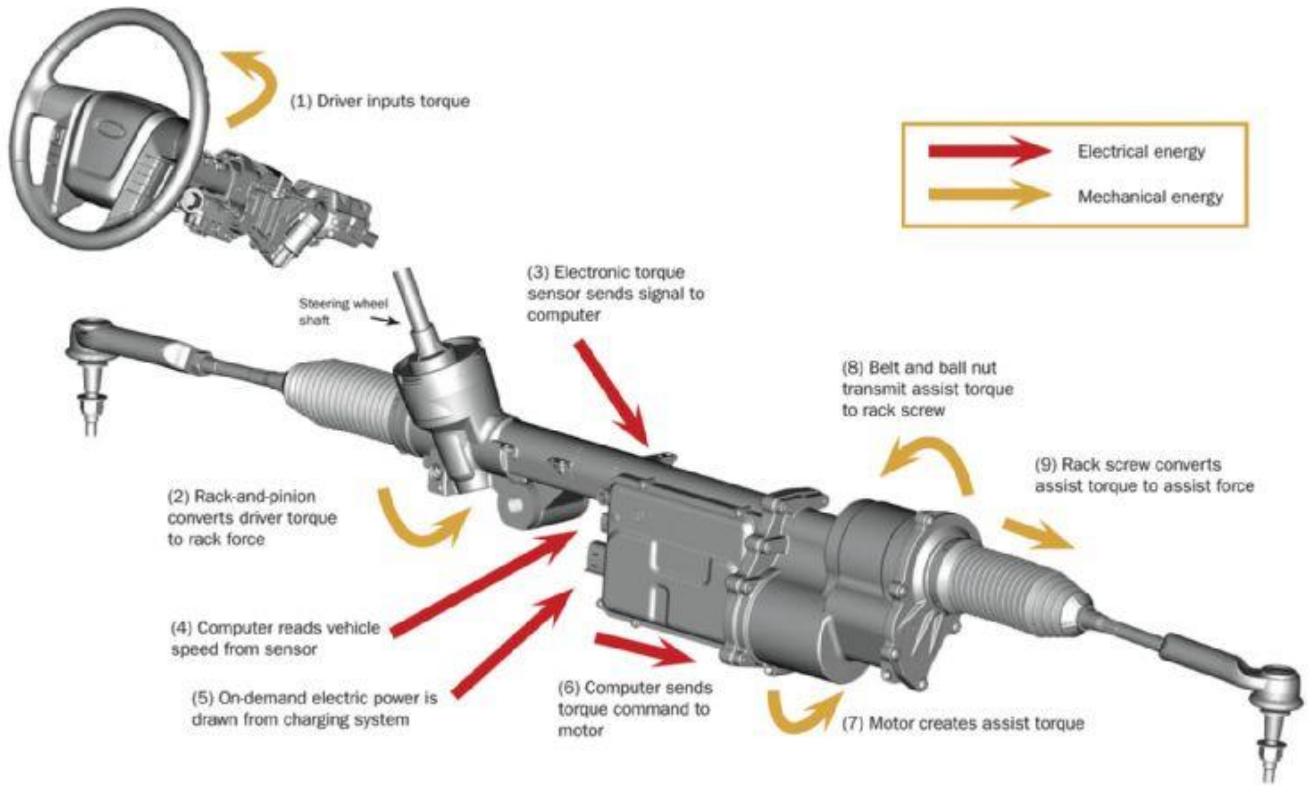


Coupling Mechanism - Rotating $\leftrightarrow$ Translatory



Rack and Pinion

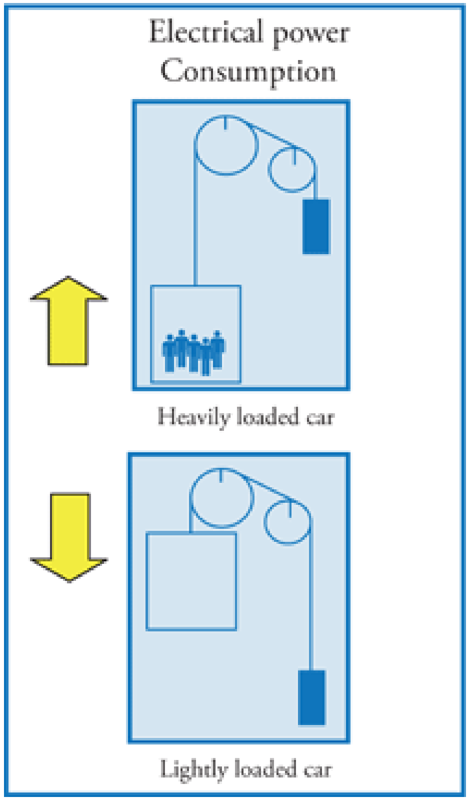
Application Example: Power Steering

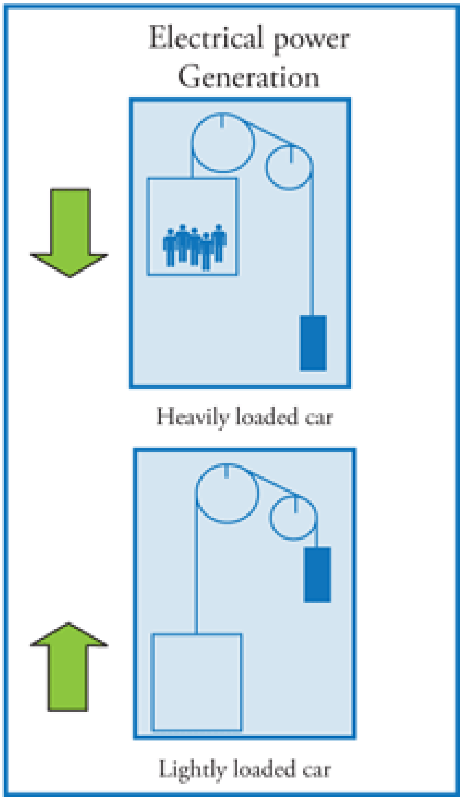


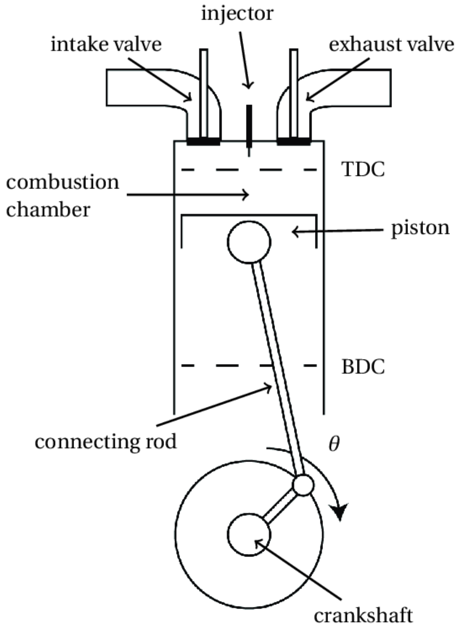
Why do not we liner actuators but rotating machines + converters?

Coupling Mechanism - Rotating $\leftrightarrow$ Translatory

Elevator





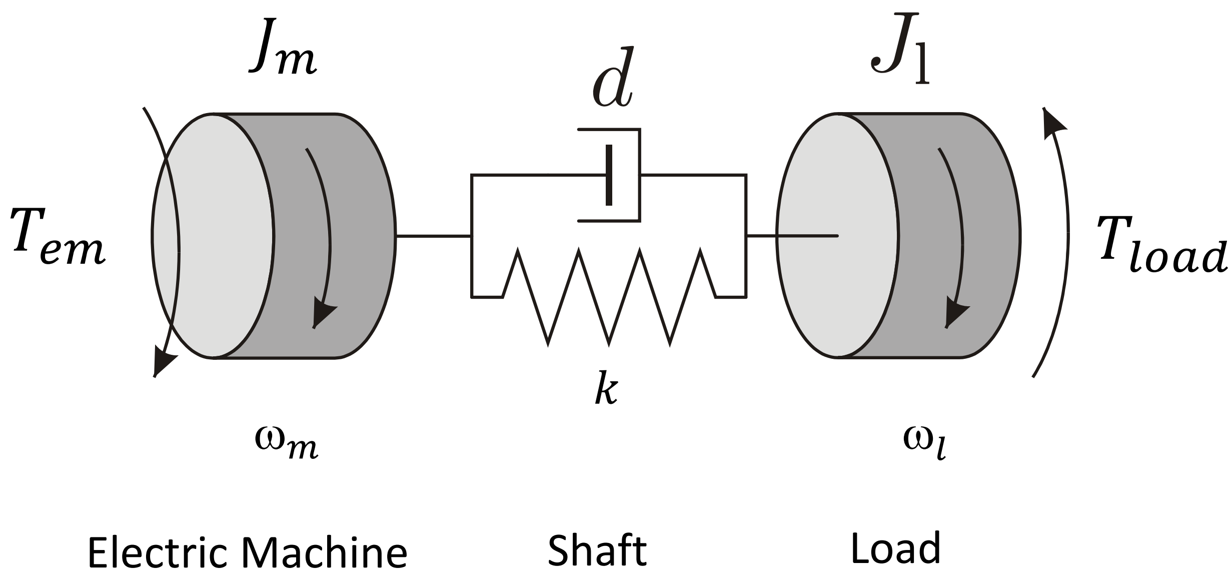


Internal combustion engine

Dynamic Model of a Drivetrain

Mass-spring-damper system with inertia *J*, stiffness *k* and damping *d*.

Example: Motor drives a load with significant mass via a shaft.



Stiffness (springiness) is the rigidity of an object, the extent to which it resists deformation in the response to applied force → Rotational stiffness: $T = k\theta$ where *k* in Nm/rad

Damping (viscous friction) is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations → $T = d\omega$ where *d* in Nm.s/rad

Dynamic Model of a Drivetrain

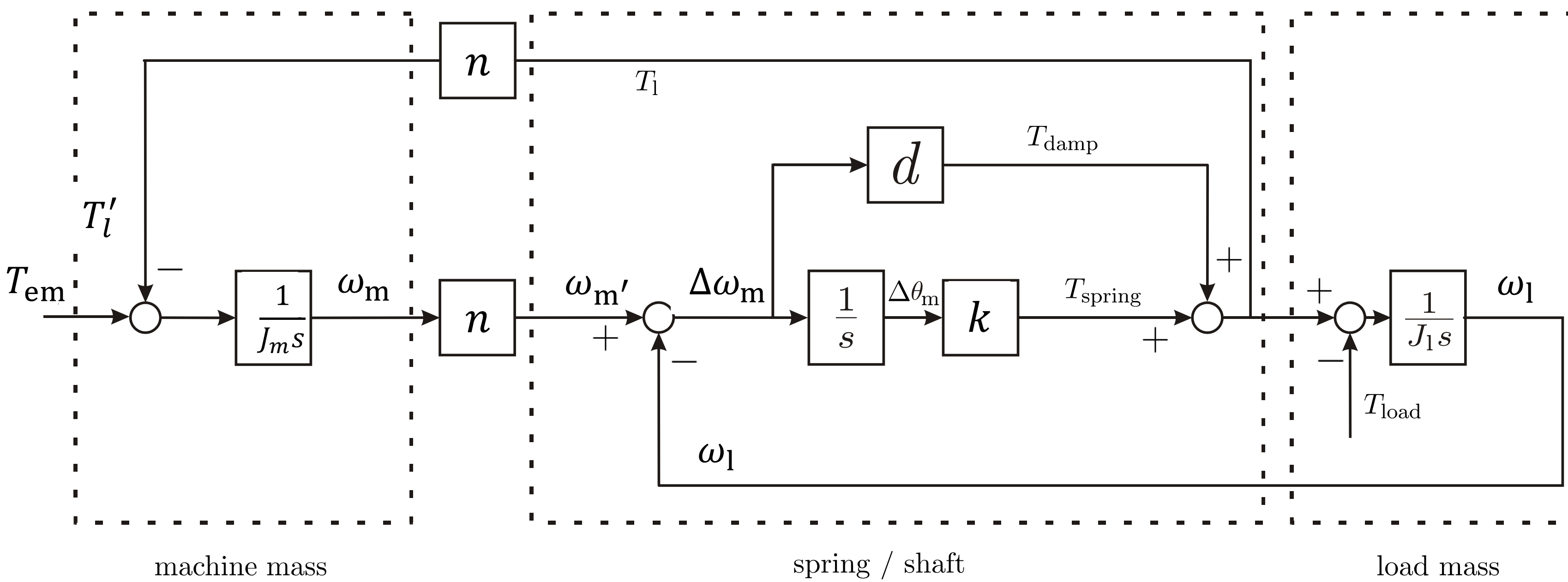
Shaft modeled as a linear damped torsion spring:

Spring equation: $T_{\mathrm{spring}} = k \cdot \int \Delta\omega_{\mathrm{m}} \mathrm{d}t \; = c \cdot \Delta\theta_{\mathrm{m}}$

Damping: $T_{\mathrm{damp}} \; = d \cdot (\omega_{\mathrm{l}} - \omega_{\mathrm{l}})$

$$J_{\mathrm{m}} \frac{\mathrm{d}\omega_{\mathrm{m}}(t)}{\mathrm{d}t} = T_{\mathrm{em}}(t) - k \underbrace{\big(\theta_{\mathrm{m}}(t) - \theta_{\mathrm{l}}(t)\big)}_{\Delta\theta} - d \underbrace{\big(\omega_{\mathrm{m}}(t) - \omega_{\mathrm{l}}(t)\big)}_{\Delta\omega_{\mathrm{m}}}$$

$$J_{\mathrm{l}} \frac{\mathrm{d}\omega_{\mathrm{l}}(t)}{\mathrm{d}t} = -T_{\mathrm{l}}(t) + k\big(\theta_{\mathrm{m}}(t) - \theta_{\mathrm{l}}(t)\big) + d\big(\omega_m(t) - \omega_{\mathrm{l}}(t)\big)$$

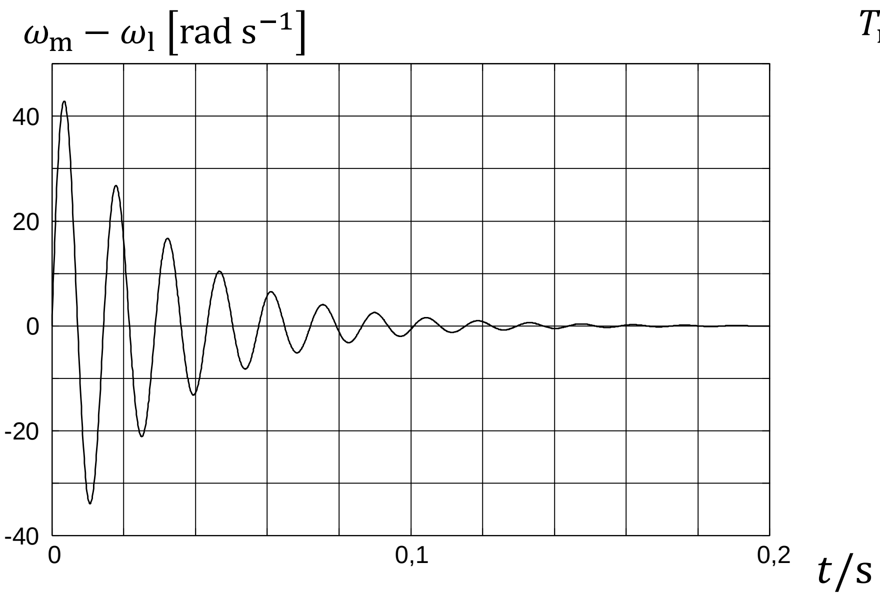


Dynamic Model of a Drivetrain

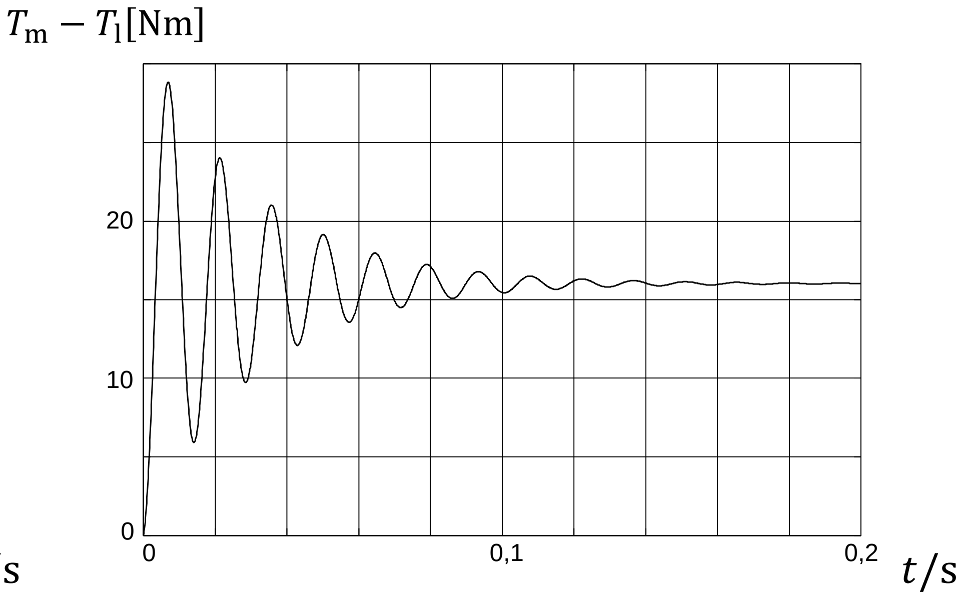
Step response of the load: $T_{\mathrm{em}} = 16\ \mathrm{Nm}; n = 1$

Time constant of the spring: $t_{\mathrm{spring}} = \frac{k}{d} \approx 30\ \mathrm{ms}$

$T_{\mathrm{l}} = 0\ \mathrm{Nm}$



Speed difference

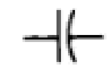


Effective load **acceleration** torque

d determines how fast the transient oscillations decay.
Frequency of oscillations depend on *c* and *J*.

Analogy between Mechanical & Electrical Systems

TABLE 1-3. TORSIONAL SYSTEM

Mechanical quantity	Electrical quantity	Mechanical energy stored or dissipated	Electrical energy stored or dissipated	Symbol used
Moment or torque $= M$	Electromotive force $= E$			
Angle of twist $= \theta$	Charge $= q$			
Angular velocity $= \omega$	Current $= i$			
Moment of inertia $= I$	Inductance $= L$	$\frac{1}{2} I\omega^2$	$\frac{1}{2} Li^2$	
Moment of resistance $= R_M$	Electrical resistance $= R$	$\int_0^t R_M\omega^2\, dt$	$\int_0^t Ri^2\, dt$	
Moment of compliance $= C_M$	Capacitance $= C$	$\frac{1}{2}\frac{\theta^2}{C_M} = \frac{1}{2} C_M M^2 = \frac{1}{2}(\text{torque})^2 C_M$	$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} CE^2$	

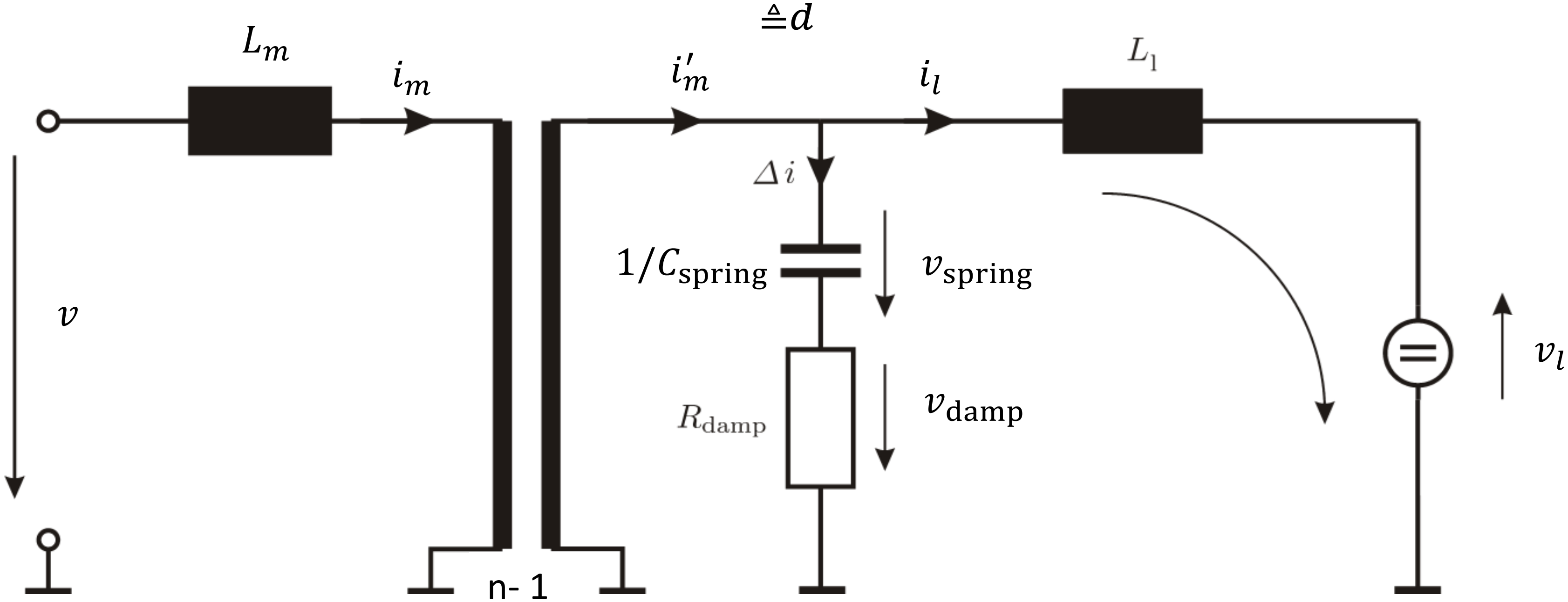
Electrical “Two-Inertia” System

$$v(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \Leftrightarrow T(t) = J\frac{\mathrm{d}\omega_\mathrm{m}(t)}{\mathrm{d}t}$$

$$\boxed{\begin{array}{c} i \Leftrightarrow \omega_\mathrm{m} \\ \mathrm{v} \Leftrightarrow T \end{array}}$$

„Spring equation“: $v_\mathrm{spring}(t) = \frac{1}{\underbrace{C_\mathrm{spring}}_{\triangleq k}} \int \Delta i(t)\mathrm{d}t$

Damping: $v_\mathrm{damp}(t) = \underbrace{R_\mathrm{damp}}_{\triangleq d} \cdot \Delta i(t)\mathrm{d}t$

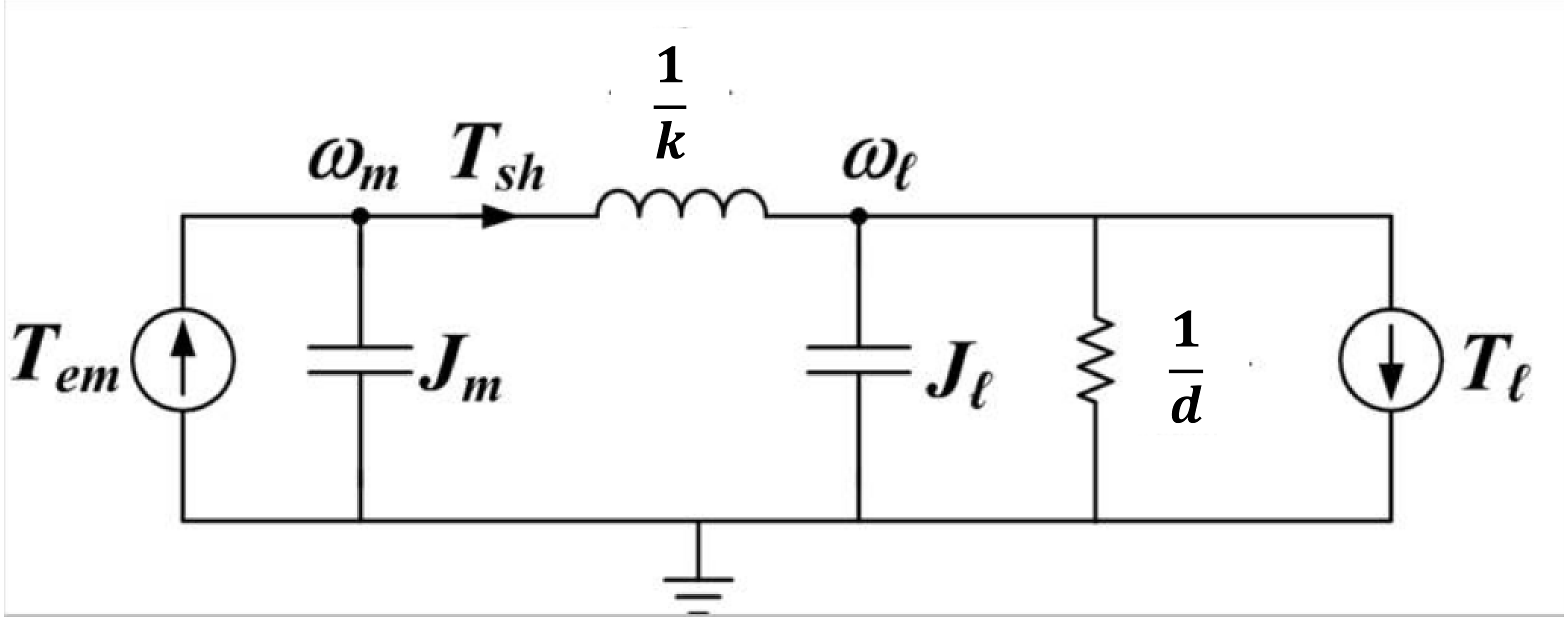


Electrical "Two-Inertia" System

Other electrical representation:

$$i \Leftrightarrow T$$
$$v \Leftrightarrow \omega_m$$

$$i(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t} \Leftrightarrow T(t) = J\frac{\mathrm{d}\omega_m(t)}{\mathrm{d}t}$$

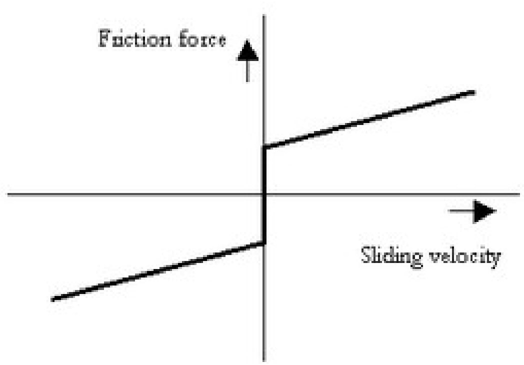


Frictional Forces

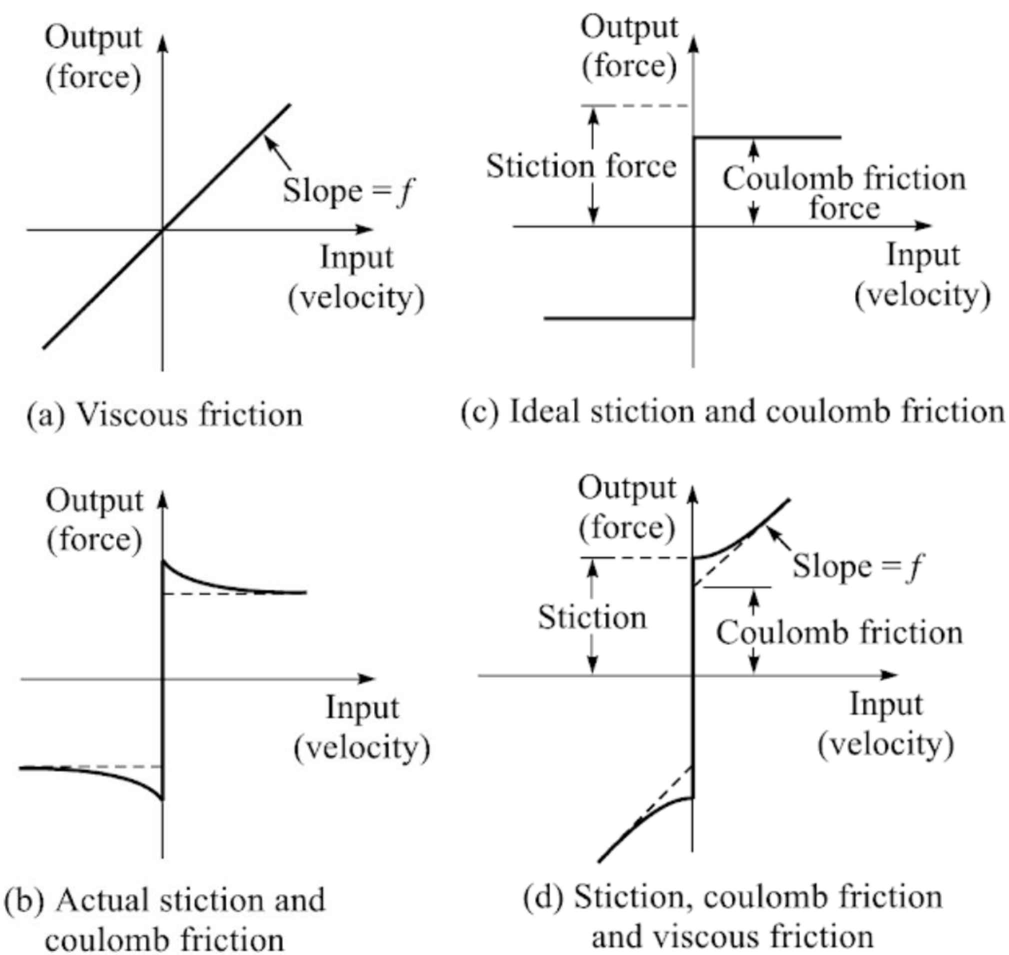
Viscous friction (damping): Proportional to speed ($T = d\omega$)

Coulomb friction: We assume it is a constant resisting force.

Extra friction (or stiction) at zero speed: Force required to initiate motion, usually small and ignored in linear models



[How to model in Simulink?]



(a) Viscous friction

(c) Ideal stiction and coulomb friction

(b) Actual stiction and coulomb friction

(d) Stiction, coulomb friction and viscous friction

Frictional Forces

Windage Torque

Proportional to ω^2

$T \propto \omega^2$

$P \propto \omega^3$

Example: Aerodynamic drag in vehicles

Turbulent air flow around vehicle body

Friction of air over vehicle body

Vehicle component resistance, from radiators, air vents and pipes under vehicle

$$F_w = \frac{1}{2}\rho A_f C_d (V + V_w)^2$$

Frontal area, m²

Vehicle speed, m/sec

Wind speed, m/sec

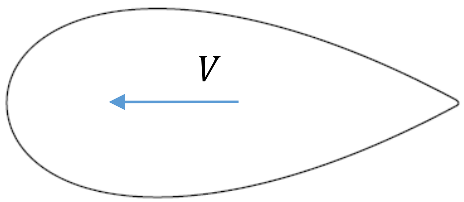
Coefficient of aerodynamic drag

Aerodynamic tractive effort in N

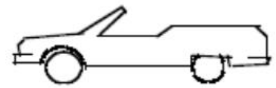
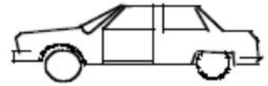
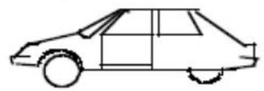
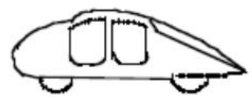
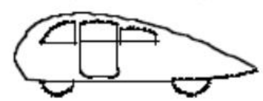
Density of air, typical value is 1.25kg/m³

Example: Aerodynamic Drag in Vehicles

Aerodynamic idea shape:



A teardrop of aspect ratio 2.4 with C_d=0.04

Vehicle Type	Coefficient of Aerodymanic Resistance
 Open convertible	0.5–0.7
 Van body	0.5–0.7
 Ponton body	0.4–0.55
 Wedge-shaped body; headlamps and bumpers are integrated into the body, covered underbody, optimized cooling air flow	0.3–0.4
 Headlamp and all wheels in body, covered underbody	0.2–0.25
 K-shaped (small breakway section)	0.23
 Optimum streamlined design	0.15–0.20
Trucks, road trains	0.8–1.5
Buses	0.6–0.7
Streamlined buses	0.3–0.4
Motorcycles	0.6–0.7

Questions:
Do we have windage friction electric machines?
Under what kind of losses do we consider it?

Answers:
Yes, mechanical losses.

Frictional Forces

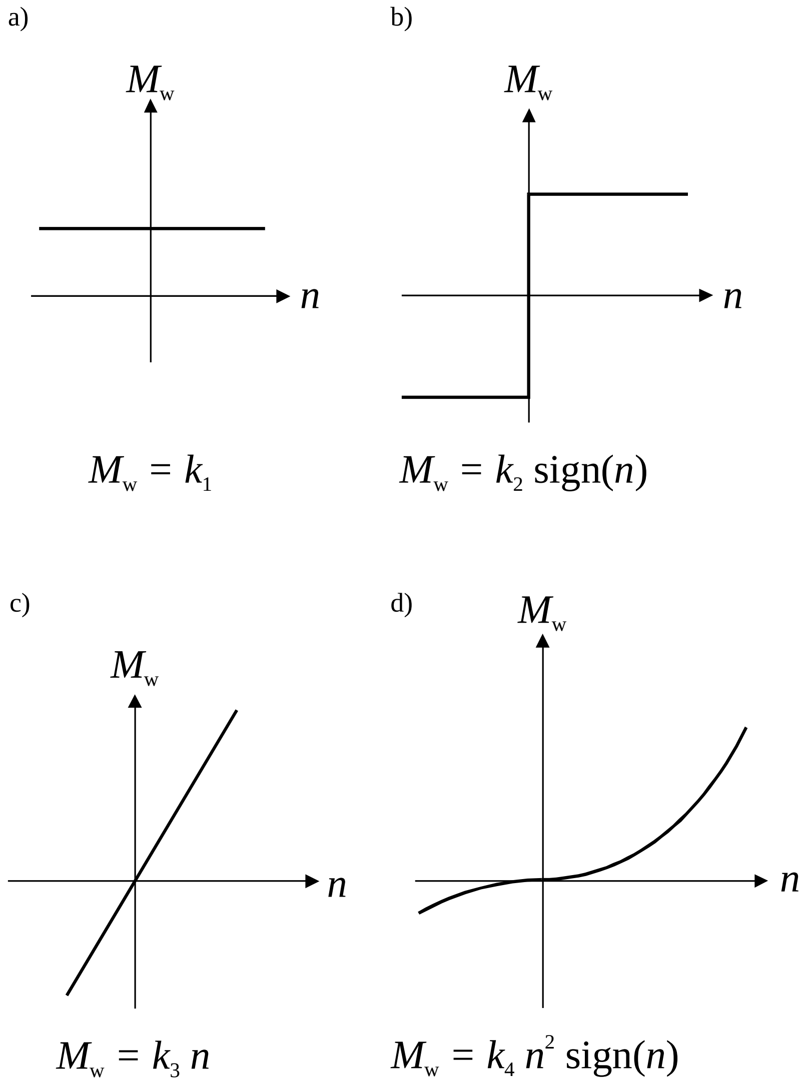
Examples:

a) gravity based load (elevator)

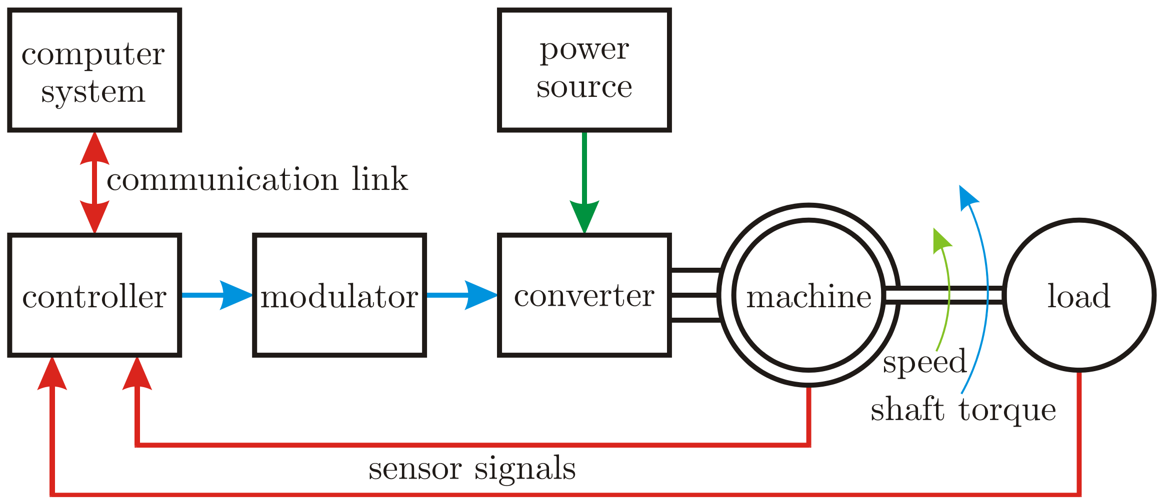
b) static friction (Coulomb friction)

c) linear kinetic friction (viscose friction)

d) air friction (car)



Sensors

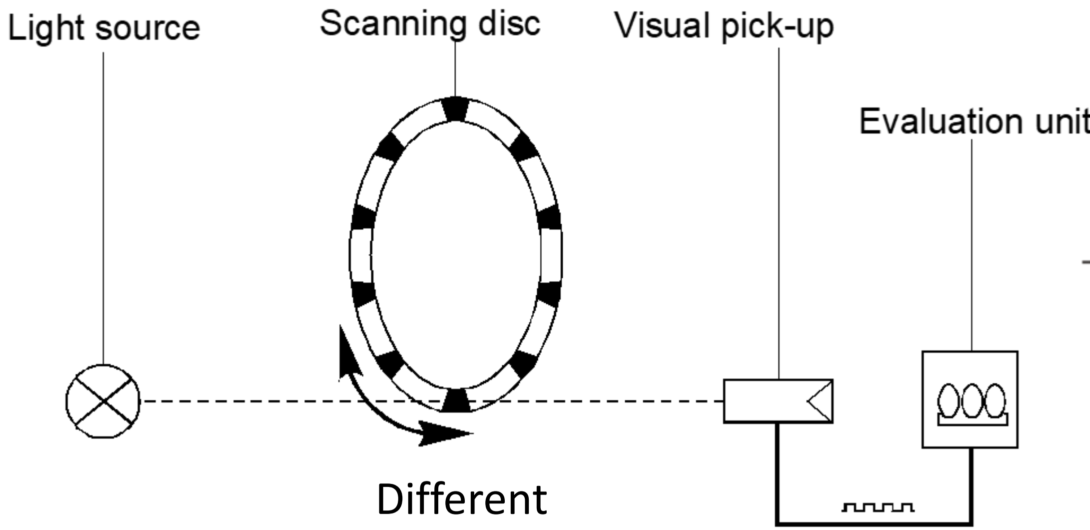


Typical sensors we use in an electrical drive:

- **Speed and position**
- **Torque (usually only used in test setups)**
- Current - Machine current(s) and DC-link current
- Voltage - DC-link
- Acceleration (not common, usually used for fault detection)
- Flux sensors (not common, usually used for fault detection)

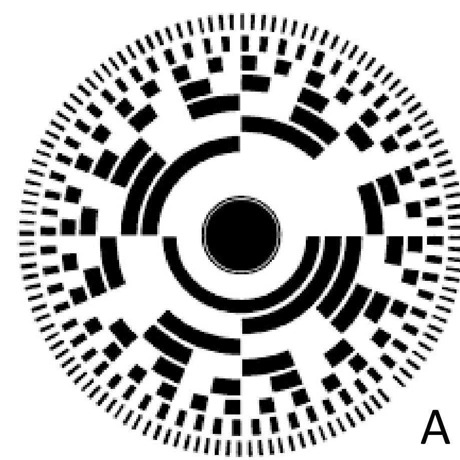
Incremental Position Sensors (1)

Optical Rotary Encoder



Different
transparency areas

→ rotational direction
<u>not</u> measurable



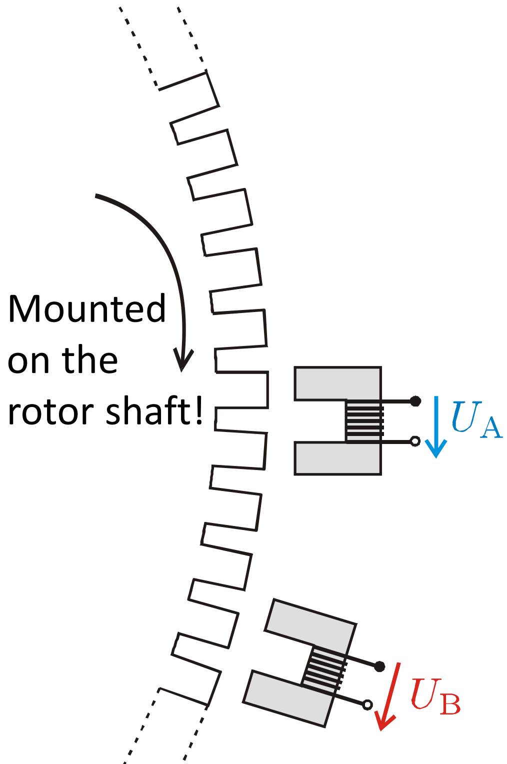
A finer
scanning disk

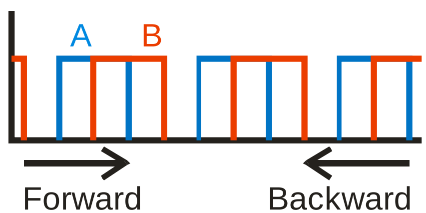
Incremental Position Sensors (2)

Magnetic Rotary Encoder

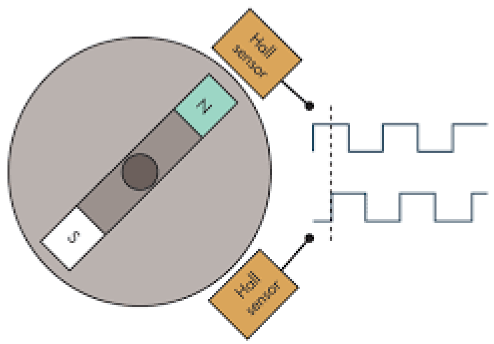
→ no absolute position knowledge

Inductive method: Like a rotating transformer





→ rotational direction is measurable



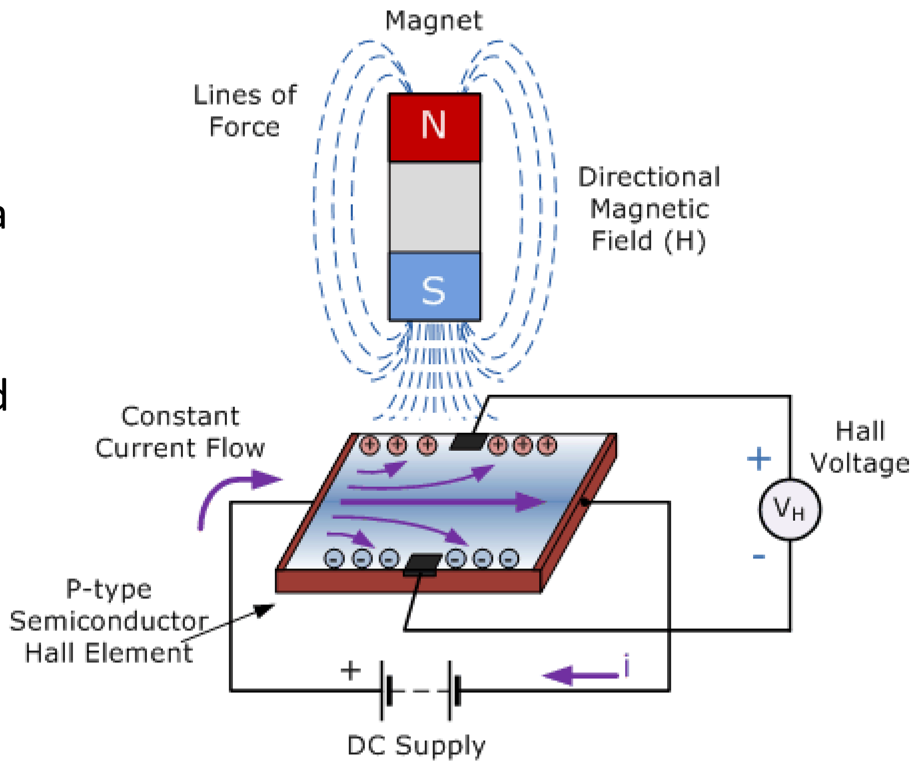
PM (coil) + Hall Effect combination also possible!

Hall Effect Sensor

A **Hall effect sensor** is used to measure the magnitude of a magnetic flux density. Its output voltage is directly proportional to the magnetic field strength through it.

There are also Hall effect switches, which output a constant voltage if flux exceeds a threshold value.

Hall effect sensors are used for proximity sensing, positioning, speed detection, and current sensing applications.



More information: https://www.electronics-tutorials.ws/electromagnetism/hall-effect.html

Questions:

Where do we place these sensors?

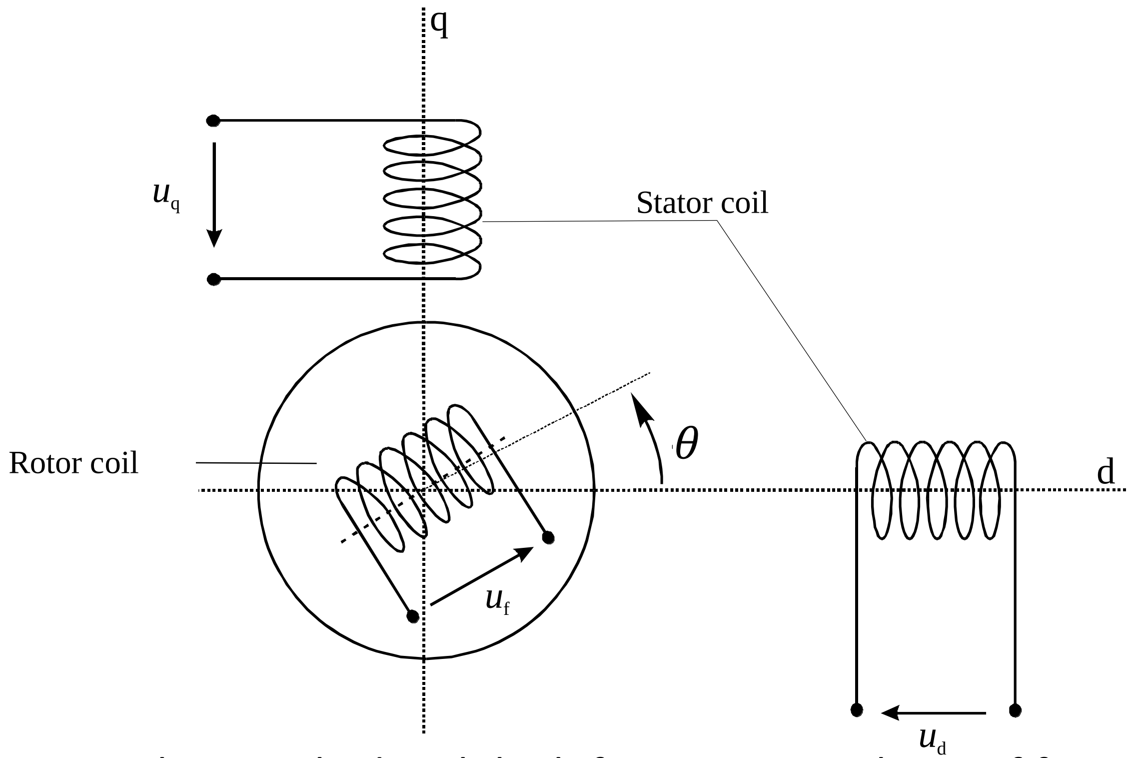
Is only one sensor enough?

Answers:

In the air-gap

No, we need multiples of them.

Absolute Position Encoder – The Resolver (1)



Rotor coil is supplied with high frequent ac-voltage of frequency ω_f

Flux-linkage of rotor coil: $\psi_f(t) = \hat{\psi}_f \cos(\omega_f t)$

Dependent on the rotor position θ the induced **flux-linkage in the air-gap** varies:

$$\vec{\psi}_g(t) = \psi_f(t)e^{j\theta} = \psi_{gd} + j\psi_{gq} = \hat{\psi}_f \cos(\omega_f t)\cos\theta + j\hat{\psi}_f \cos(\omega_f t)\sin\theta$$

Absolute Position Encoder – The Resolver (2)

For $\theta = \omega_m \, t$

Stator coil voltages

$$u_\mathrm{d} = \frac{\mathrm{d}\psi_\mathrm{gd}(t)}{\mathrm{d}t} = -\hat{\psi}_\mathrm{f}\omega_\mathrm{f}\sin(\omega_\mathrm{f}t)\cos\theta - \hat{\psi}_\mathrm{f}\omega_\mathrm{m}\cos(\omega_\mathrm{f}t)\sin\theta$$

$$u_\mathrm{q} = \frac{\mathrm{d}\psi_\mathrm{gq}(t)}{\mathrm{d}t} = -\hat{\psi}_\mathrm{f}\omega_\mathrm{f}\sin(\omega_\mathrm{f}t)\sin\theta + \hat{\psi}_\mathrm{f}\omega_\mathrm{m}\cos(\omega_\mathrm{f}t)\cos\theta$$

For $\omega_\mathrm{f} \gg \omega_\mathrm{m}$:

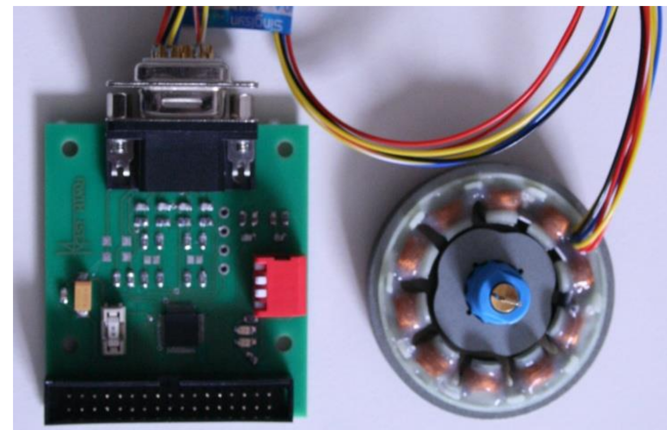
$$u_\mathrm{d} \approx -\hat{\psi}_\mathrm{f}\omega_\mathrm{f}\sin(\omega_\mathrm{f}t)\cos\theta$$

$$u_\mathrm{q} \approx -\hat{\psi}_\mathrm{f}\omega_\mathrm{f}\sin(\omega_\mathrm{f}t)\sin\theta$$

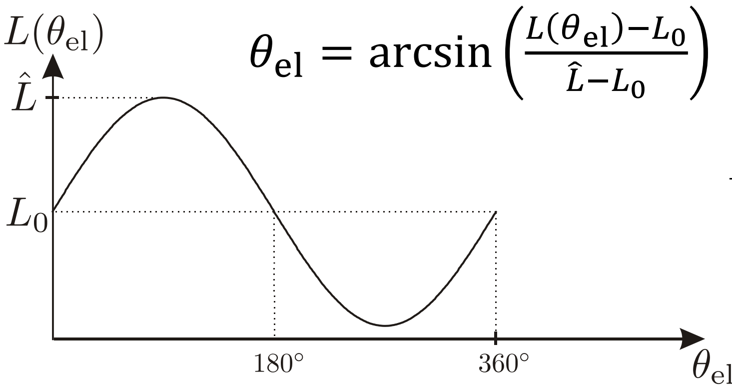
$$\Rightarrow \theta = \arctan 2\left(\frac{u_\mathrm{q}}{u_\mathrm{d}}\right)$$ (since image of arctan−function is $]-\pi;\pi[$)

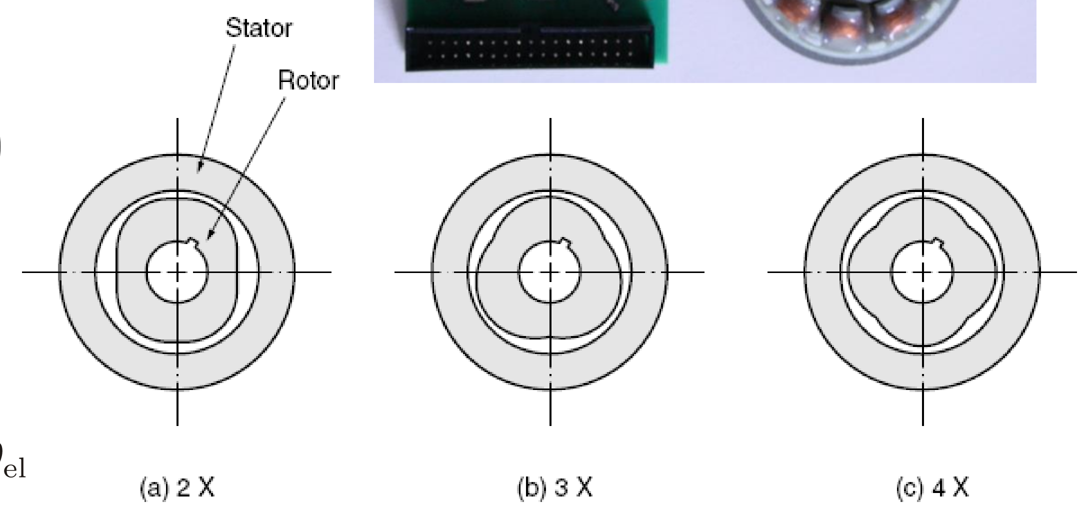
Absolute Position Encoders –VR-Type Resolvers

- State-of-the-Art: Variable Reluctance (VR) Type Resolvers
- Principle: Reluctance changes sinusoidally respective to rotor position
- Advantage: No field-winding attached to the rotor
- High freq. current injection; current slope is reversely proportional to inductance
- Inverse calculation of rotor position using the known inductance value



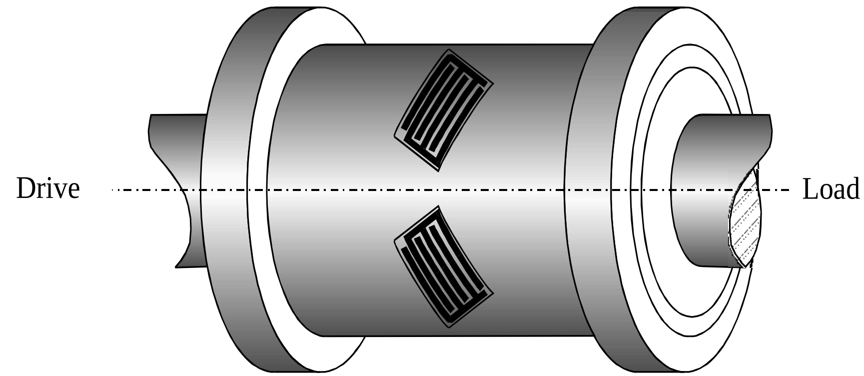
$$\theta_{\mathrm{el}} = \arcsin\left(\frac{L(\theta_{\mathrm{el}}) - L_0}{\hat{L} - L_0}\right)$$





Torque Sensors – Strain Gauges

Measurement of torque by evaluation of the strain gauge resistance (e.g. using a Wheatstone Bridge)

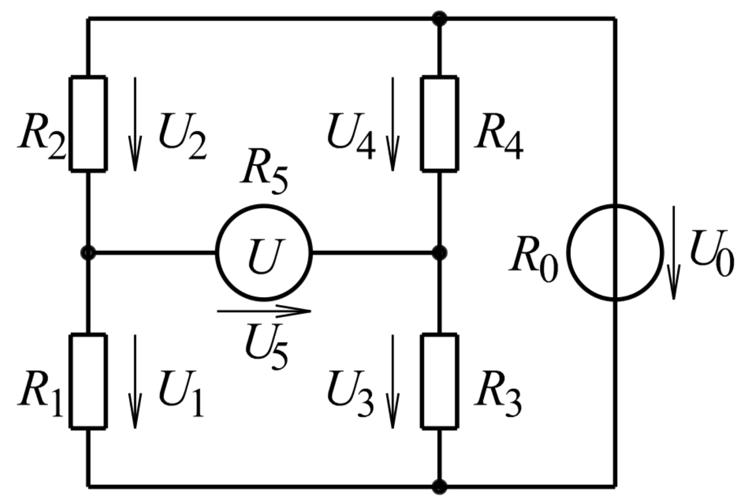


A **Strain gauge** (sometimes referred to as a Strain gage) is a sensor whose resistance varies with applied force; It converts force, pressure, tension, weight, etc., into a change in **electrical resistance** which can then be measured.

Weatstone Bridge $\frac{U_5}{U_0} = \frac{R_1 R_4 - R_2 R_3}{(R_1+R_2)(R_3+R_4)}$

Bridge in equilibrium:

$$\Longrightarrow U_5 \stackrel{!}{=} 0 \Leftrightarrow R_1 R_4 - R_3 R_2 \stackrel{!}{=} 0$$



Advantage of using 4 strain gauges:

- Temperature independency
- Higher voltage U_5, if strain gauges are mounted crossed for stretching and compressing

Disadvantage of using 4 strain gauges:

- Higher mounting effort
- Higher costs

Electrified Vehicle of the Day

Extra Material

Couplings

- Shaft Misalignment
- Unbalanced rotor
- Rotor Critical Speed

Understanding PLANETARY GEAR set !

Reducing Aerodynamic Drag