EE 301 HOMEWORK 1 SOLUTIONS

Q1 A CT signal $x(t)$ is periodic with period T if $x(t)=x(t+T)$.
$x(t)=x(t+mT)$ $\forall m\in\mathbb{Z}$. The smallest value of T is T_0, fundamental period.

A DT signal $x[n]$ is periodic with period $N(\in\mathbb{Z})$ if $x[n]=x[n+N]$.
$x[n]=x[n+mN]$ $\forall m\in\mathbb{Z}$. The smallest value of N is N_0, fundamental period.

(a) $x(t)=4\cdot\cos\left(3t+\frac{\pi}{5}\right)$ is periodic with fundamental period $T_0=\frac{2\pi}{3}$,
since $3(t+T_0)+\cancel{\frac{\pi}{5}}=3t+\cancel{\frac{\pi}{5}}+2\pi \Rightarrow T_0=\frac{2\pi}{3}$.

(b) $x(t)=e^{j\left(\frac{\pi}{2}t-2\right)}$ is periodic with fundamental period of $T_0=4$,
since $\frac{\pi}{2}(t+T_0)-\cancel{2}=\frac{\pi}{2}t-\cancel{2}+2\pi \Rightarrow T_0=4$.

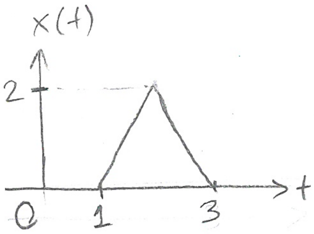
(c) $x(t)=e^{jt^2}$ is aperiodic (not periodic).
If $(t+T_0)^2=t^2+2\pi \Longrightarrow T_0^2+2t\cdot T_0-2\pi=0 \Longrightarrow T_0=-t\pm\sqrt{t^2+2\pi}$.
As T_0 depends on time t (T_0 is not a constant), we cannot find a fundamental period for $x(t)$. Hence, $x(t)$ is aperiodic.

(d) $x[n]=\cos\left(\frac{3\pi}{7}n+\frac{\pi}{4}\right)$ is periodic with fundamental period $N_0=14$,
since $\frac{3\pi}{7}(n+N_0)+\cancel{\frac{\pi}{4}}=\frac{3\pi}{7}n+\cancel{\frac{\pi}{4}}+k.2\pi$ where $k, N_0\in\mathbb{Z}^+$
$\frac{3\pi}{7}N_0=k.2\pi \Rightarrow N_0=k.\frac{14}{3} \Rightarrow$ Choose k as 3 (minimum for this case), $N_0=14$.

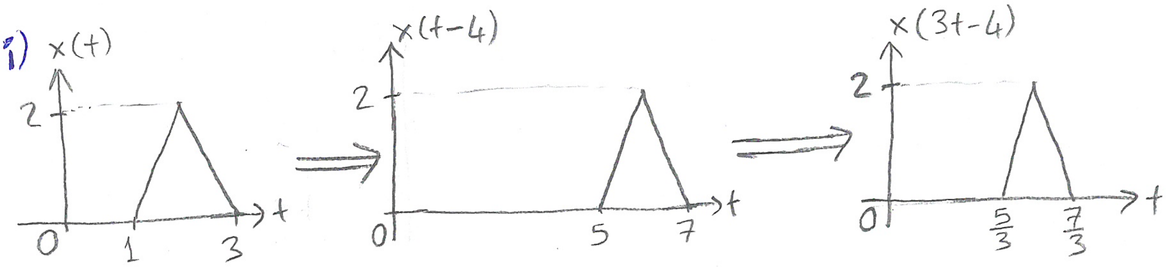
(e) $x[n]=\left[\sin\left(\frac{\pi}{3}n+\frac{\pi}{2}\right)\right]^2=\frac{1}{2}-\frac{1}{2}\cdot\cos\left(\frac{2\pi}{3}n+\pi\right)$ is periodic with $N_0=3$,
since $\frac{2\pi}{3}(n+N_0)+\cancel{\pi}=\frac{2\pi}{3}n+\cancel{\pi}+k.2\pi$ where $k, N_0\in\mathbb{Z}^+$
For $k=1$ (minimum for this case), $N_0=3$.

(f) $x[n]=\cos\left(\frac{\pi}{7}n^2\right)$ is periodic with $N_0=14$.
$$e^{j\frac{\pi}{7}n^2}=e^{j\frac{\pi}{7}(n+N_0)^2}=e^{j\frac{\pi}{7}n^2}\cdot\underbrace{e^{j\frac{2\pi}{7}n\cdot N_0}}_{1}\cdot\underbrace{e^{j\frac{\pi}{7}N_0^2}}_{=e^{j\frac{\pi}{7}0^2}=1 \text{ if } N_0 \text{ exists.}}$$
$n=1 \Rightarrow \frac{2\pi}{7}N_0=k.2\pi \Rightarrow N_0=7, 14, \ldots$ for $k=1,2,\ldots$ Then $\frac{2\pi}{\cancel{7}}n.\cancel{7}=k.2\pi$, for all n
$$e^{j\frac{\pi}{7}N_0^2}=\begin{cases} e^{j7\pi} & \text{for } N_0=7 \ \times \\ e^{j28\pi} & \text{for } N_0=14 \end{cases}$$
Hence, $N_0=14$ is its fundamental period.

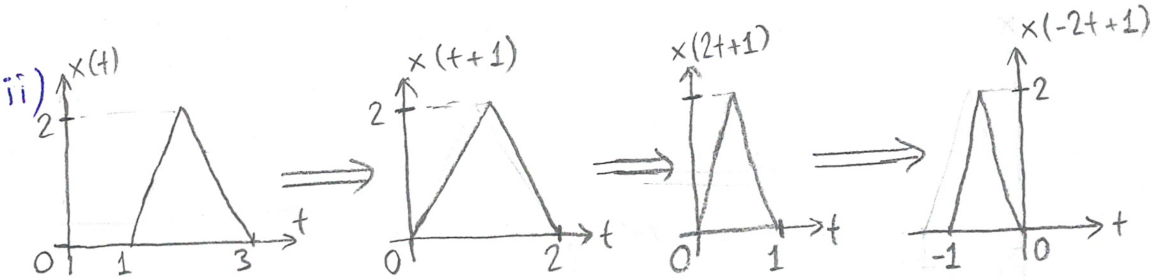
Q2



a) i)

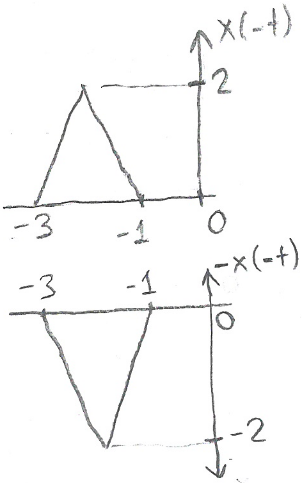


ii)

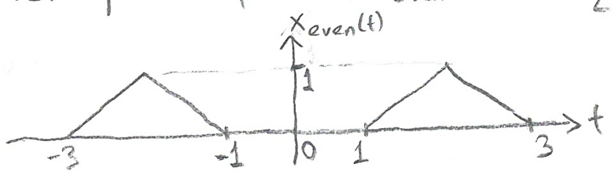


iii) $[x(t)+x(-t)]u(t)=x(t)$ due to following reasons:

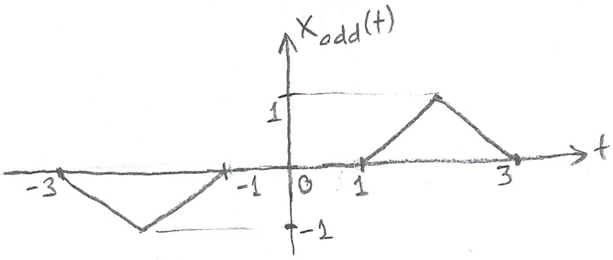
* $x(t)=0$ for $t\leq 0$ so $x(t).u(t)=x(t)$ for all t
* $x(-t)=0$ for $t>0$ so $x(-t).u(t)=0$ for all t



b) * Even part of $x(t)$, $x_{even}(t)=\frac{x(t)+x(-t)}{2}$



* Odd part of $x(t)$, $x_{odd}(t)=\frac{x(t)+(-x(-t))}{2}$



Note: If a function $z(t)$ is even, $z_{even}(t)=\frac{z(t)+z(-t)}{2}=z(t)=z(-t)$

$z_{odd}(t)=\frac{z(t)-z(-t)}{2}=0$

If a function $f(t)$ is odd, $f_{even}(t)=\frac{f(t)+f(-t)}{2}=0$

$f_{odd}(t)=\frac{f(t)-f(-t)}{2}=f(t)=-f(-t)$

Q3 a $y(t) = \int_0^t x(\tau).d\tau$

i) Not memoryless $y(t)$ depends on not only $x(t)$ but also $x(\tau)$ where $0 \leq \tau < t$.

ii) Time-variant $\int_0^t x(\tau - t_0).d\tau = \int_{-t_0}^{t-t_0} x(\tau).d\tau = \int_0^{t-t_0} x(\tau).d\tau + \int_{-t_0}^{0} x(\tau).d\tau$

$\int_0^t x(\tau - t_0).d\tau = y(t-t_0) + \int_{-t_0}^{0} x(\tau).d\tau \rightarrow$ Due to this term, it is time-variant.

iii) Linear If $x(t) = a x_1(t) + b x_2(t)$, $y_1(t) = \int_0^t x_1(\tau).d\tau$, $y_2(\tau) = \int_0^t x_2(\tau).d\tau$

$y(t) = \int_0^t [a x_1(\tau) + b x_2(\tau)]\, d\tau = a \int_0^t x_1(\tau).d\tau + b \int_0^t x_2(\tau).d\tau = a y_1(t) + b y_2(t)$

$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t)$ (Linearity for CT signals)

iv) Causal $y(t)$ depends only on the present and past values of $x(t)$.

v) Unstable Even if $x(t)$ is bounded, i.e, $|x(t)| < B < \infty$, $y(t)$ may go to infinity as times goes to infinity.

b $y(t) = [\sin(2t)]\, x(t)$

i) Memoryless $y(t)$ depends on only the present value of $x(\cdot)$ function, $x(t)$.

ii) Time variant $y(t-t_0) = [\sin(2t - 2t_0)]\, x(t-t_0) \Rightarrow$ The function $\sin 2t$ causes time variance. If $y(t-t_0)$ were $[\sin 2t]\, x(t-t_0)$, then it would be time invariant.

iii) Linear If $x(t) = a x_1(t) + b x_2(t)$, $y_1(t) = \sin 2t\, x_1(t)$, $y_2(t) = \sin 2t\, x_2(t)$,

$y(t) = \sin 2t [a x_1(t) + b x_2(t)] = a \sin 2t\, x_1(t) + b \sin 2t\, x_2(t) = a y_1(t) + b y_2(t)$

$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t)$ (Linearity for CT signals)

iv) Causal $y(t)$ depends on only the present value of $x(\cdot)$ function. Therefore, it is causal.

v) Stable If $x(t)$ is bounded, i.e., $|x(t)| < B < \infty$ for all t, $|y(t)| = |\sin 2t|.|x(t)| < B < \infty$. Hence, $y(t)$ is also bounded for all t.

(c) $y(t) = \frac{dx(t)}{dt}$

(i) Not memoryless It is sufficient to invalidate the system property by giving one example. Hence, let us assume that $x(t) = \cos wt$ Then $y(t) = -w.\sin(wt) = w.\cos(w(t-\frac{3\pi}{2w}))$. Therefore, $y(t)$ depends on the past values of the input and the system has memory.

(ii) Time invariant $y(t-t_0) = \frac{dx(t-t_0)}{dt}$ A time shift in input signal $x(t)$ results in identical time shift in output signal $y(t)$.

(iii) Linear If $x(t) = ax_1(t) + bx_2(t)$, $y_1(t) = \frac{dx_1(t)}{dt}$, $y_2(t) = \frac{dx_2(t)}{dt}$,

$$y(t) = \frac{d[ax_1(t)+bx_2(t)]}{dt} = a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt} = ay_1(t) + by_2(t)$$

$$ax_1(t) + bx_2(t) \longrightarrow ay_1(t) + by_2(t)$$

(iv) Causal This system can be implemented in causal manner due to the definition of derivation.

(v) Unstable Even if $x(t)$ is bounded, i.e, $|x(t)| < B < \infty$ for all t,

$|y(t)| = \left|\frac{dx(t)}{dt}\right|$ may be unbounded. If $x(t) = u(t)$ (step function),

$y(t) = \delta(t)$ (impulse function)

(d) $y[n] = x[2n]$

(i) Not memoryless $y[n]$ depends on the future values of $x[\cdot]$ function for $n>0$ and its past values for $n<0$. It has memory.

(ii) Not time-invariant If $x_2[n] = x[n-n_0]$, then $y_2[n] = x_2[2n]$.
$y_2[n] = x[2(n-n_0)] = x[2n-2n_0] = y[n-2n_0]$. n_0 shift in input signal causes $2n_0$ shift in output signal. Hence, it is time-variant.

(iii) Linear If $x[n] = ax_1[n] + bx_2[n]$, $y_1[n] = x_1[2n]$, $y_2[n] = x_2[2n]$,

$y[n] = ax_1[2n] + bx_2[2n] = ay_1[n] + by_2[n]$ (Linearity for DT signals)

(iv) Not causal $y[n]$ depends on the future values of $x[\cdot]$ function.

(v) Stable If $x[n]$ is bounded, i.e., $|x[n]| < B < \infty$, for all n,

$|y[n]| = |x[2n]| < B < \infty$ is bounded, too. Hence, it is stable.

(e) $y[n]=x[-n]$

(i) Not memoryless For $n\neq 0$, $y[n]$ depends on either past or present values of $x[\cdot]$ function.

(ii) Time invariant $x[-(n-n_0)]=y[n-n_0]$ A time shift in input signal $x[\cdot]$ results in identical time shift in output signal $y[n]$.

(iii) Linear If $x[n]=ax_1[n]+bx_2[n]$, $y_1[n]=x_1[-n]$, $y_2[n]=x_2[-n]$, $y[n]=ax_1[-n]+bx_2[-n]=ay_1[n]+by_2[n]$ (Linearity for DT signals)

(iv) Not causal For $n<0$, $y[n]$ depends on the future values of $x[\cdot]$ function. Hence, it is not causal.

(v) Stable If $x[n]$ is bounded, i.e., $|x[n]|<B<\infty$ for all n, $|y[n]|=|x[-n]|<B<\infty$. is bounded, too. Hence, it is stable.

(f) $y[n]=\sum_{k=n-5}^{n+5} x[k]$

(i) Not memoryless $y[n]$ depends on not only the present values of $x[\cdot]$ function, $x[n]$, but also the past and future values of $x[\cdot]$ function. Hence, the system has memory.

(ii) Time invariant For $x_2[n]=x[n-n_0]$, $y_2[n]=\sum_{k=n-5}^{n+5} x[k-n_0]$.
$y_2[n]=\sum_{k=n-n_0-5}^{n-n_0+5} x[k]=y[n-n_0]$. Hence, the system is time-invariant.

(iii) Linear If $x[n]=ax_1[n]+bx_2[n]$, $y_1[n]=\sum_{k=n-5}^{n+5} x_1[k]$, $y_2[n]=\sum_{k=n-5}^{n+5} x_2[k]$,
$y[n]=\sum_{k=n-5}^{n+5}(ax_1[k]+bx_2[k])=a\sum_{k=n-5}^{n+5} x_1[k]+b\sum_{k=n-5}^{n+5} x_2[k]=ay_1[n]+by_2[n]$.
$ax_1[n]+bx_2[n]\longrightarrow ay_1[n]+by_2[n]$ (Linearity in DT signals)

(iv) Not causal $y[n]$ depends on not only present and past values of $x[\cdot]$ function but also future values of $x[\cdot]$ function. Hence, the system is not causal.

(v) Stable If $x[n]$ is bounded, i.e., $|x[n]|<B<\infty$ for all n,
$|y[n]|=\left|\sum_{k=n-5}^{n+5} x[k]\right| < [(n+5)-(n-5)+1]\,B=11.B<\infty$ is bounded, too.

Q4 **(a)** i) * If $a=1$, $\sum_{n=0}^{N-1} a^n = (N-1-0+1).1 = N$

* If $a\neq 1$, multiply $\sum_{n=0}^{N-1} a^n$ by $1-a\neq 0$ and then divide it by $1-a$.

$$\sum_{n=0}^{N-1} a^n = 1+\cancel{a}+\cancel{a^2}+\cancel{a^3}+\dots\dots+\cancel{a^{N-2}}+\cancel{a^{N-1}}$$

$$-a\sum_{n=0}^{N-1} a^n = -\cancel{a}-\cancel{a^2}-\cancel{a^3}-a^4 \qquad -\cancel{a^{N-2}}-\cancel{a^{N-1}}-a^N$$

$+$

$$(1-a)\sum_{n=0}^{N-1} a^n = 1+0+0+0+0+\dots+0+0-a^N = 1-a^N$$

Therefore, $\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$ if $a\neq 1$. $\boxed{\text{Hence, } \sum_{n=0}^{N-1} a^n = \begin{cases} N, & a=1 \\ \frac{1-a^N}{1-a}, & \text{otherwise} \end{cases}}$

ii) $\sum_{n=0}^{\infty} a^n = \lim_{N\to\infty} \frac{1-a^N}{1-a}$ if $a\neq 1$ from the formula in part i.

If $|a|<1$, $\lim_{N\to\infty} a^N = 0$. $\boxed{\text{Hence, } \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \text{ if } |a|<1}$

(b) i) $$\sum_{n=-2}^{7} e^{j\frac{\pi n}{2}} = \underbrace{e^{-j\pi}+e^{-j\frac{\pi}{2}}+e^{j0}+e^{j\frac{\pi}{2}}}_{A_1}+\underbrace{e^{j\pi}+e^{j\frac{3\pi}{2}}+e^{j2\pi}+e^{j\frac{5\pi}{2}}}_{A_2}+\underbrace{e^{j3\pi}}_{e^{j\pi}=-1}+\underbrace{e^{j\frac{7\pi}{2}}}_{e^{-j\frac{\pi}{2}}=-j}$$

Notice that $A_2 = \underbrace{e^{j2\pi}}_{1}.A_1 = A_1 = -1-j+1+j=0$. $\boxed{\text{Hence, } \sum_{n=-2}^{7} e^{j\frac{\pi n}{2}} = -1-j}$

Generalized version

$$\sum_{n=N_1}^{N_2} e^{j\frac{p}{q}\pi n} = \sum_{n=N_1}^{N_1+k-1} e^{j\frac{p}{q}\pi n} = \sum_{n=N_2-k+1}^{N_2} e^{j\frac{p}{q}\pi n}$$

where $\begin{cases} p \,\&\, q \text{ are coprime} \\ N_2 = N_1 + m.N_0 + k, \\ N_0 \text{ is fundamental period, } m,k\in\mathbb{Z} \end{cases}$

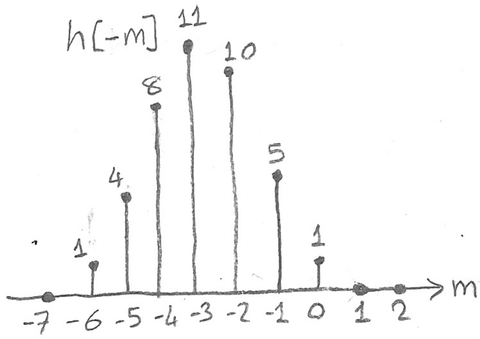
ii) $$\int_0^8 e^{j\frac{\pi t}{2}}.dt = \left[\frac{e^{j\frac{\pi}{2}t}}{j\frac{\pi}{2}}\right]_0^8 = \frac{e^{j4\pi}-e^{j0}}{j\frac{\pi}{2}} = 0$$

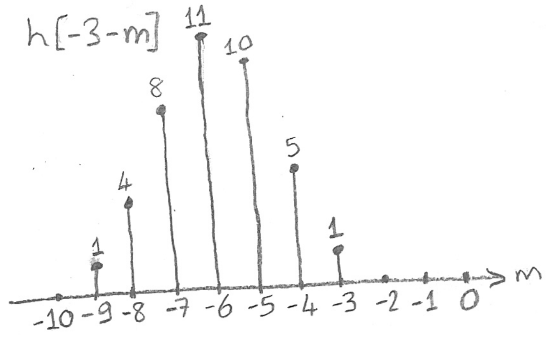
iii) $$\int_0^\infty e^{-t}.\sin(t).dt = \int_0^\infty e^{-t}\left(\frac{e^{jt}-e^{-jt}}{j2}\right)dt = \frac{1}{j2}\int_0^\infty \left[e^{(-1+j)t}-e^{-(1+j)t}\right]dt$$

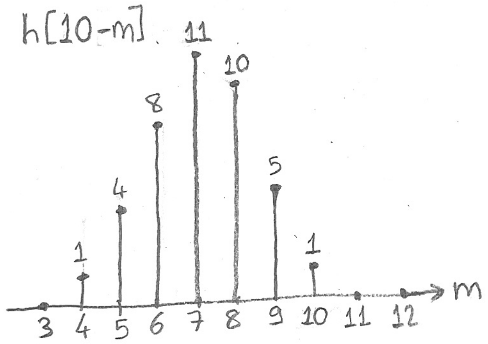
$$= \frac{1}{j2}\left[\frac{e^{(-1+j)t}}{-1+j}-\frac{e^{-(1+j)t}}{-1-j}\right]_0^\infty = \frac{1}{j2}\left[0-\left(\frac{1}{-1+j}+\frac{1}{1+j}\right)\right]$$

$$= \underset{(1-j)}{\frac{1}{2+j2}} + \underset{(1+j)}{\frac{1}{2-j2}} = \frac{(1-j)+(1+j)}{4} = \boxed{\frac{1}{2}}$$

Q5 a) $y[n] = \sum_{m=-\infty}^{\infty} x[n-m] \cdot h[m] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m]$





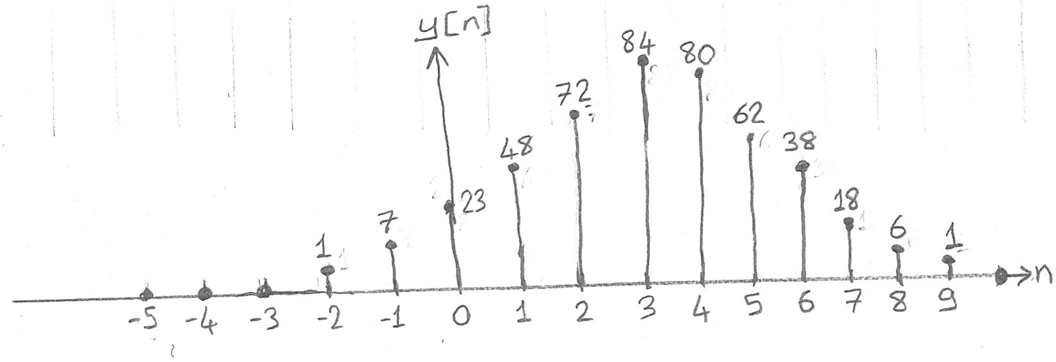


$$y[-3] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[-3-m] = 0$$

$$y[10] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[10-m] = 0$$

By using the following table, we will obtain $y[n] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m]$ for each n values.

m	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9		
x[m]									1	2	3	2	2	1								
h[-2-m]			1	4	8	11	10	5	1												1	y[-2]
h[-1-m]				1	4	8	11	10	5	1											7	y[-1]
h[m]					1	4	8	11	10	5	1										23	y[0]
h[1-m]						1	4	8	11	10	5	1									48	y[1]
h[2-m]							1	4	8	11	10	5	1								72	y[2]
h[3-m]								1	4	8	11	10	5	1							84	y[3]
h[4-m]									1	4	8	11	10	5	1						80	y[4]
h[5-m]										1	4	8	11	10	5	1					62	y[5]
h[6-m]											1	4	8	11	10	5	1				38	y[6]
h[7-m]												1	4	8	11	10	5	1			18	y[7]
h[8-m]													1	4	8	11	10	5	1		6	y[8]
h[9-m]														1	4	8	11	10	5	1	1	y[9]



Q5) PART B
MATLAB CODE

```
clc;
close all;
%first sequence
x=input('Enter x\n');
l1=input('Enter the lower limit:\n');
u1=input('Enter the upper limit:\n');
x1=l1:1:u1;%limit of sequence x(n)
%second sequence
h=input('Enter h:\n');
l2=input('Enter the lower limit:\n');
u2=input('Enter the upper limit:\n');
h1=l2:1:u2;%limit of sequence h(n)
l=l1+l2;
u=u1+u2;
a=l:1:u;%limit of output sequence y(n)
m=length(x);%length of sequence x(n)
n=length(h);%length of sequence h(n)
X=[x,zeros(1,n)];
subplot(311)
disp('x(n) is:')
disp(x)
stem(x1,x)
xlabel('n')
ylabel('x(n)')
title('First Sequence')
grid on;
H=[h,zeros(1,m)];
subplot(312)
disp('h(n) is:')
disp(h)
stem(h1,h)
xlabel('n')
ylabel('h(n)')
title('Second Sequence')
grid on;
%CONVULATION
for i=1:n+m-1
Y(i)=0;
for j=1:m
if((i-j+1)>0)
Y(i)=Y(i)+(X(j)*H(i-j+1));
else
end
end
end
subplot(313)
disp('y(n) is:')
disp(Y)
stem(a,Y)
xlabel('n')
ylabel('y(n)')
title('Output Sequence')
grid on;
```

COMMAND WINDOW

Enter x
[1 2 3 2 2 1]
Enter the lower limit:
-2
Enter the upper limit:
3
Enter h:
[1 5 10 11 8 4 1]
Enter the lower limit:
0
Enter the upper limit:
6
x(n) is:
    1   2   3   2   2   1

h(n) is:
    1   5   10   11   8   4   1

y(n) is:
    1   7   23   48   72   84   80   62   38   18   6   1

OUTPUT PLOT