EE 301 Fall 2018-2019

HW 5

Group Number:66

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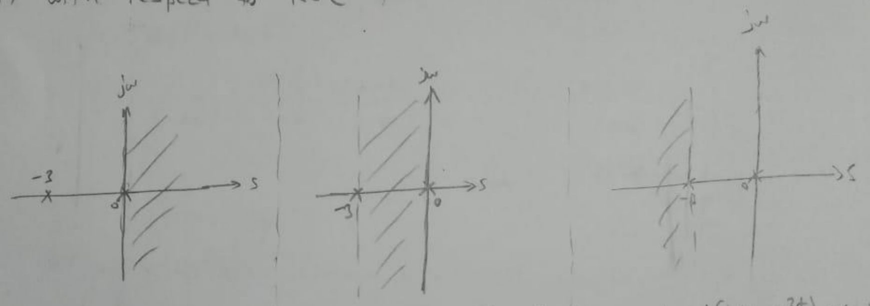
2)

Q2) a-)

$$H(s) = \frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3} \qquad A = \frac{1}{3} \quad B = -1/3$$

$$= \frac{1}{3}\frac{1}{s} - \frac{1}{3}\frac{1}{s+3}$$

poles of the system are $s_1 = 0$ and $s_2 = -3$. There are different $h(t)$ with respect to ROC



* $h(t) = \frac{1}{3}(1 - e^{3t})u(t)$
* system is causal since $h(t) = 0$, $t < 0$

* $h(t) = -\frac{1}{3}(u(-t) + e^{3t}u(t))$
* System is not causal since $h(t) \neq 0$, $t < 0$.

* $h(t) = \frac{1}{3}(-1 + e^{3t})u(-t)$
* System is not causal since $h(t) \neq 0$, $t < 0$

b) It is possible to corresponde to a stable sytem by its pole-zero plot. If pole-zero plot contains jw axes, it is a stable system. However, none of pole-zero plot has jw-axes. Due to that, systems are not stable.

c) To be causal system, pole-zero plot should be right sided. To be a stable system, pole-zero plot should contain jw axes. Therefore, α should be smaller than zero. When α is greater than −3, ROC contains right side of α. When α is smaller than −3, ROC contains right-side of −3. Then, both situation, ROC contains jw axes and right sided. Then range of α is $\underline{-\infty < \alpha < 0}$

3)

Q3) a-)

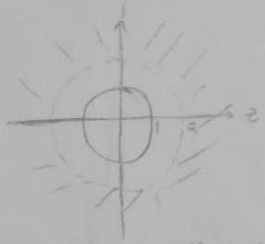
$y[n] = x[n] * h_1[n] \Rightarrow \frac{Y(z)}{X(z)} = H_1(z)$

$y[n] = y[n] * g[n] + x[n]$

$Y(z) = Y(z) \cdot G(z) + X(z) \Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1-G(z)} = H_1(z)$

$G(z) = \sum_{n=-\infty}^{\infty} a\delta[n-1] z^{-n} = a z^{-1}$

$\boxed{H_1(z) = \frac{1}{1-az^{-1}}}$ ROC: $|z| > a$

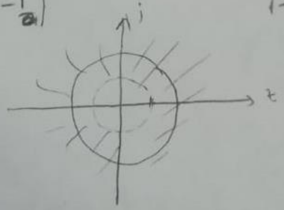


$\boxed{h_1[n] = a^n u[n]}$

System is not stable since ROC does not contains unit circle.

b) $H_2(z) = \frac{k(z-\beta)}{(z-\frac{1}{a})}$, $H_2(1) = k\left|\frac{1-\beta}{1-\frac{1}{a}}\right| = 1 \Rightarrow k = \frac{1-\frac{1}{a}}{1-\beta}$

$H_2(z) = \frac{(1-\frac{1}{a})}{(1-\beta)} \frac{(z-\beta)}{(z-\frac{1}{a})}$



c) $H(z)\Big|_{z=e^{j\Omega}} = H(e^{j\Omega}) = \frac{1}{3} \cdot \frac{(e^{j\Omega} - 3/2)}{e^{j\Omega} - 1/2}$

for $\Omega = 0$, $H(e^{j0}) = \frac{1}{3} \frac{1-3/2}{1-1/2} = -\frac{1}{3}$, $|H(e^{j0})| = \frac{1}{3}$

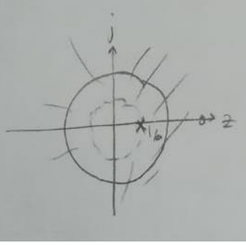
for $\Omega = \frac{\pi}{2}$, $H(e^{j\frac{\pi}{2}}) = \frac{1}{3} \frac{i-3/2}{i-1/2}$ $|H(e^{j\frac{\pi}{2}})| = \frac{1}{3}\sqrt{\frac{13}{5}}$

for $\Omega = \pi$, $H(e^{j\pi}) = \frac{1}{3} \frac{-1-3/2}{-1-1/2} = \frac{5}{9}$ $|H(e^{j\pi})| = \frac{5}{9}$

for $\Omega = \frac{3\pi}{2}$, $H(e^{j\frac{3\pi}{2}}) = \frac{1}{3} \frac{-i-3/2}{-i-1/2}$ $|H(e^{j\frac{3\pi}{2}})| = \frac{1}{3}\sqrt{\frac{13}{5}}$

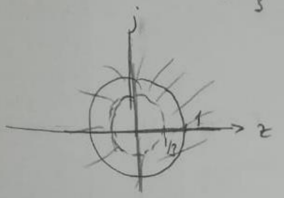
d) $H(z) = H_1(z) H_2(z)$

$= \frac{1}{1-az^{-1}} \cdot \frac{(1-\frac{1}{a})}{1-\beta} \frac{(z-\beta)}{(z-a)}$ ROC: $|z| > a$



4)

Q4) $H(z) = z^{-n_0} \dfrac{1}{1-\frac{1}{3}z^{-1}}$ ROC: $|z| > \frac{1}{3}$



a) This system is stable since ROC contains unit circle.

For causal system, $h[n] = 0$, $n < 0$. Then, $n_0 \geq 0$

b)

$Y(z) = H(z)X(z)$ where $X(z) = \dfrac{1}{1+\frac{1}{2}z^{-1}}$ ROC: $|z| > \frac{1}{2}$

$$Y(z) = z^{-n_0} \underbrace{\frac{1}{1-\frac{1}{3}z^{-1}} \cdot \frac{1}{1+\frac{1}{2}z^{-1}}}_{G(z)}$$

ROC: $\left(|z| > \frac{1}{2}\right) \cap \left(|z| > \frac{1}{3}\right)$

$= |z| > \frac{1}{2}$

$$G(z) = \frac{1}{1-\frac{1}{3}z^{-1}} \cdot \frac{1}{1+\frac{1}{2}z^{-1}} = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}}, \quad A = \frac{3}{5}, B = \frac{2}{5}$$

$$g[n] = \frac{3}{5} \cdot \left(-\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(\frac{1}{3}\right)^n u[n]$$

time shifting ↓

$$y[n] = \frac{3}{5}\left(-\frac{1}{2}\right)^{n-n_0} u[n-n_0] + \frac{2}{5}\left(\frac{1}{3}\right)^{n-n_0} u[n-n_0]$$

c) Yes, since the system is stable and ROC contains unit circle.

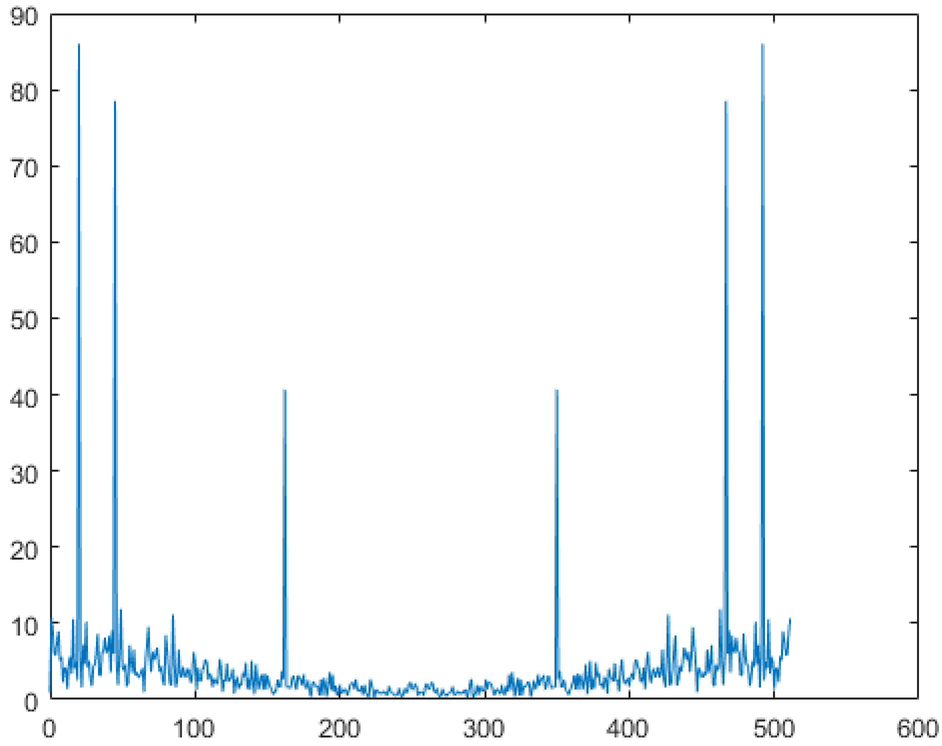
$$Y(z)\Big|_{z=e^{j\Omega}} = Y(e^{j\Omega}) = e^{-j\Omega n_0} \frac{1}{1-\frac{1}{3}e^{-j\Omega}} \cdot \frac{1}{1+\frac{1}{2}e^{-j\Omega}}$$

5)

Part a

Part i

```
k = 0:511;
xn_mag = abs(fft(xn, 512));
figure();
title("Magnitude of X[k]");
plot(k, xn_mag);
```



Part ii

a = find(xn_mag > 30)

a =

21

46

163

351

468

493

Part b

Part i

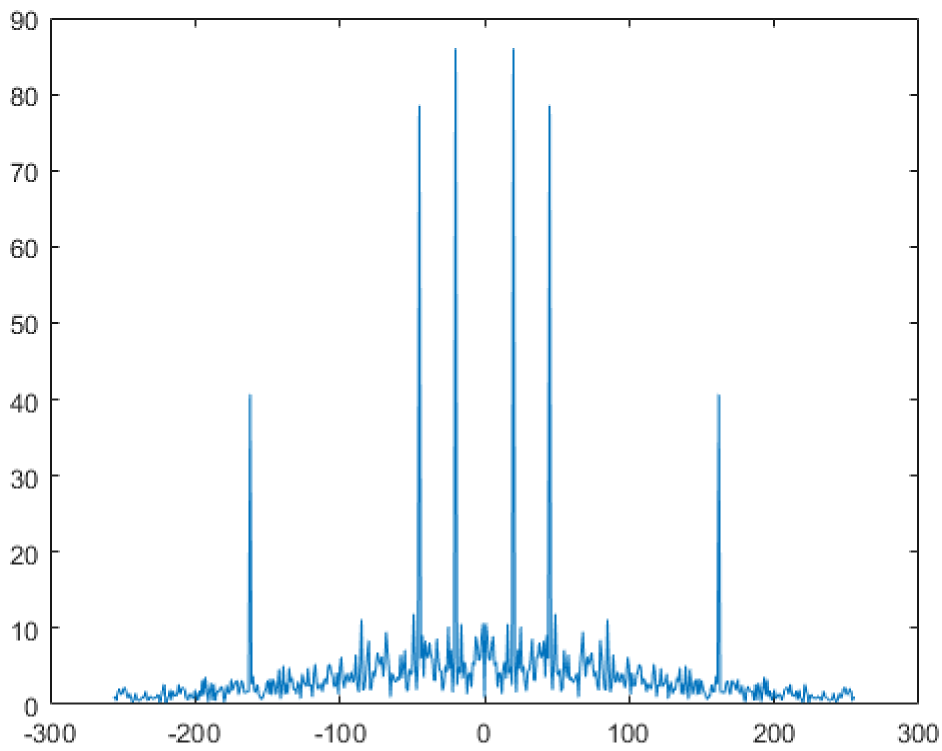
k = -256:255;

xn_shift = fftshift(abs(fft(xn, 512)));

figure();

title("Magnitude of X[k]");

plot(k, xn_shift);

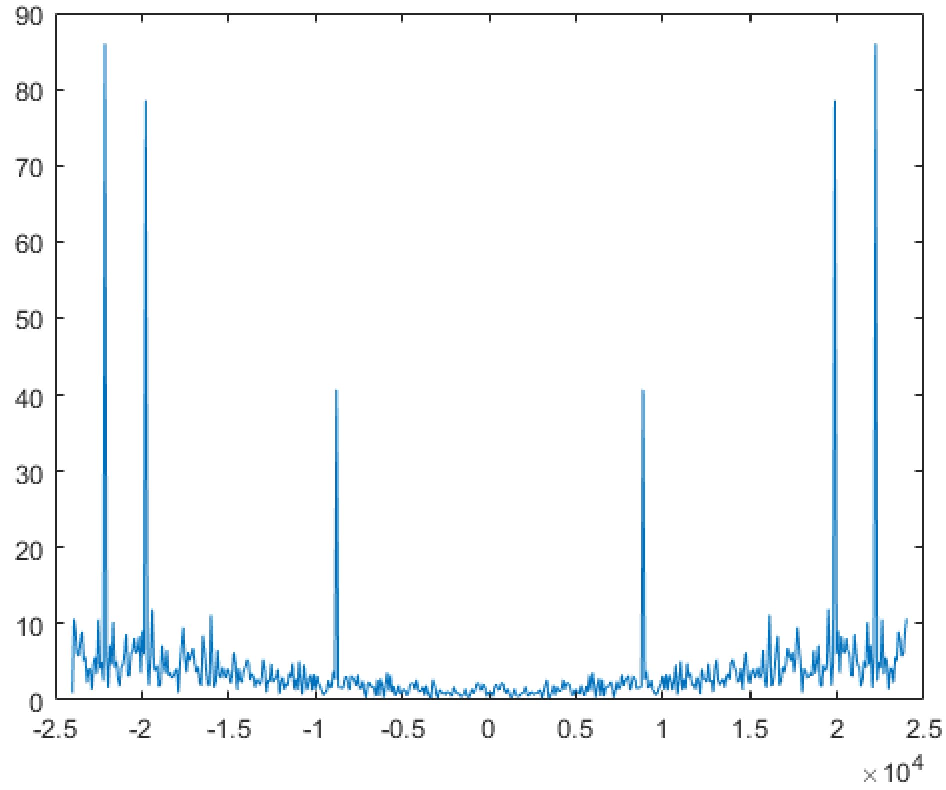


Part ii

There are six dominant frequency.

Part iii

```
w_axes = linspace(-Fs/2, Fs/2, 512);
figure();
title("Magnitude of X[k]");
plot(w_axes, xn_mag);
```


90
80
70
60
50
40
30
20
10
0
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
×10^4