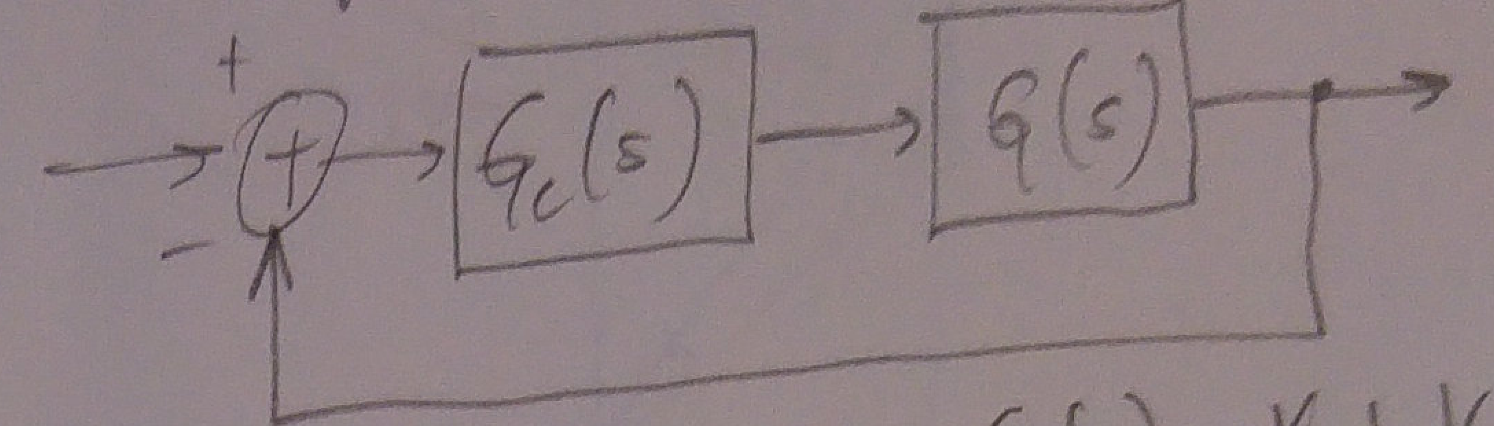
03/2019
re-write.

Proportional-Integral-Derivative Control (PID) and Transient/Steady-state Analysis.

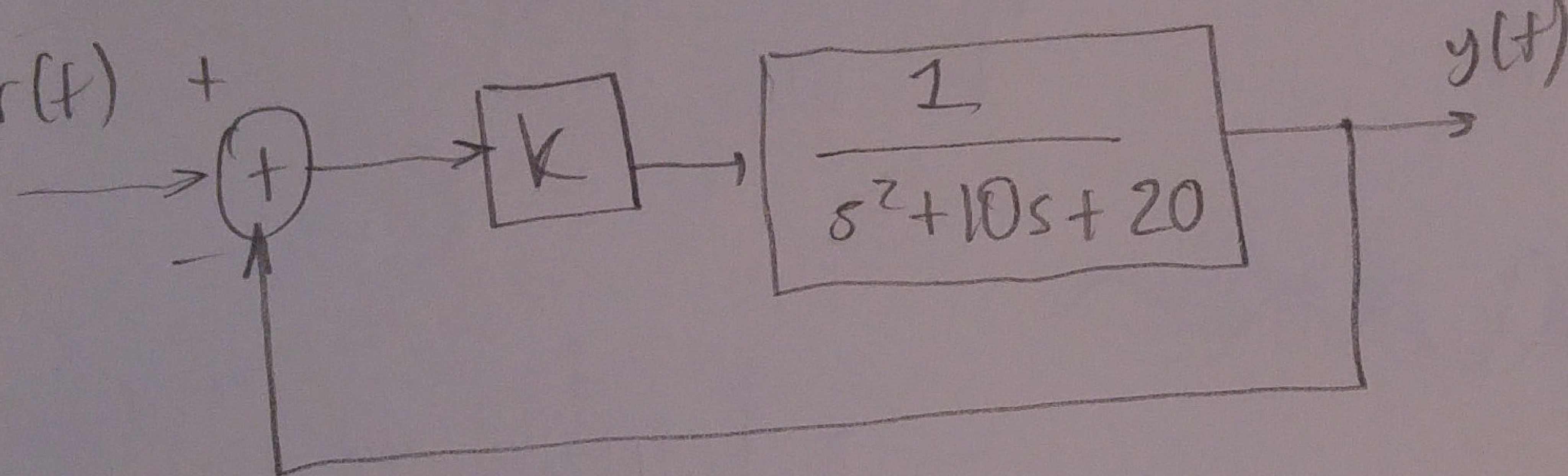


$G_c(s) = K + K_d s + \frac{K_I}{s}$

I. Proportional Control.

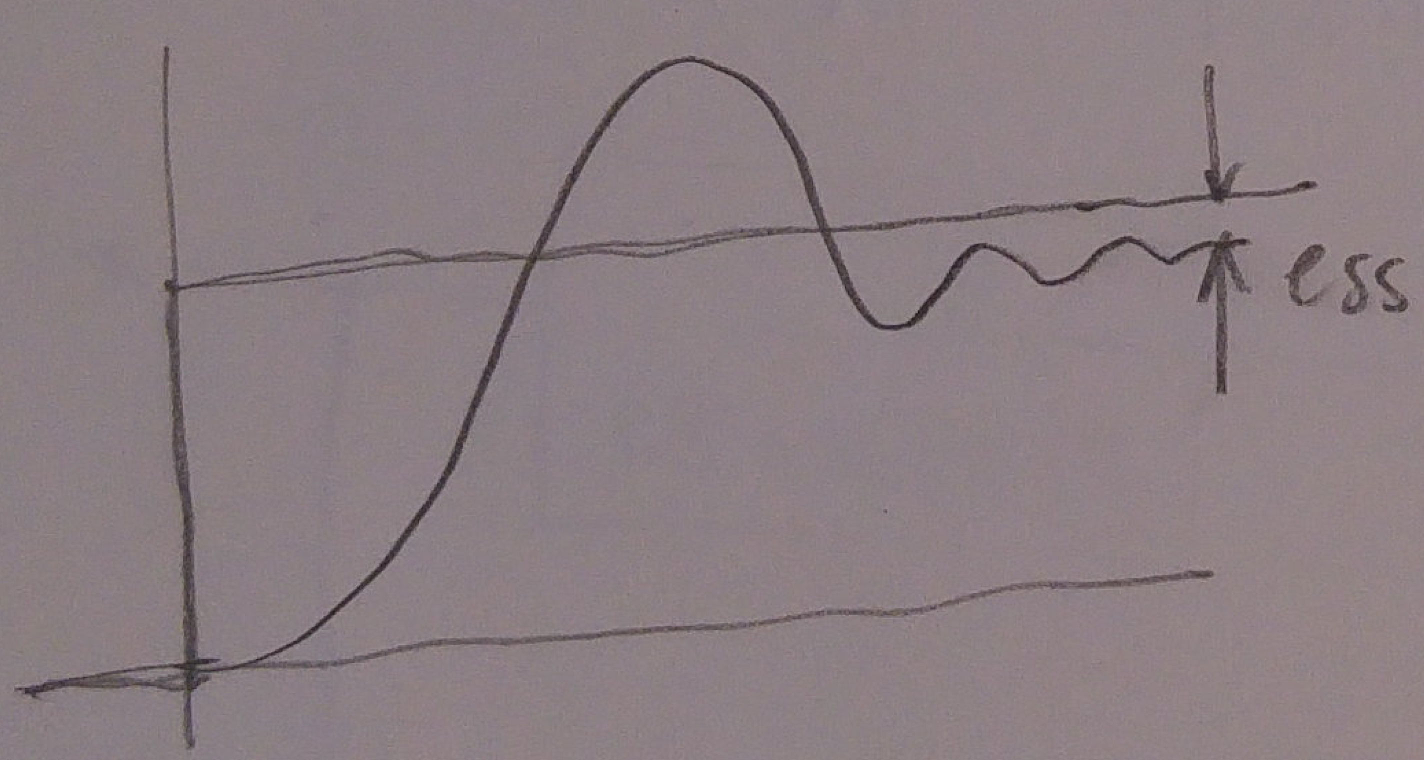
Given Plant Model $G(s) = \dfrac{1}{s^2+10s+20}$

P-control: Only a simple gain as controller.



We have a "Type 0" system. $e_{ss} = \dfrac{1}{1+K_p}$

$K_p = \lim_{s\to 0} \dfrac{K}{s^2+10s+20} = \dfrac{K}{20} \longrightarrow e_{ss} = \dfrac{1}{1+\frac{K}{20}} = \dfrac{20}{20+K}$



To reduce e_{ss}, we need to increase K.

K ↗ e_{ss} ↘

But are there any consequences?

Calculate the closed-loop transfer function to see transient behavior.

$$T(s) = \frac{\frac{K}{s^2+10s+20}}{1 + \frac{K}{s^2+10s+20}} = \frac{K}{s^2+10s+20+K}$$

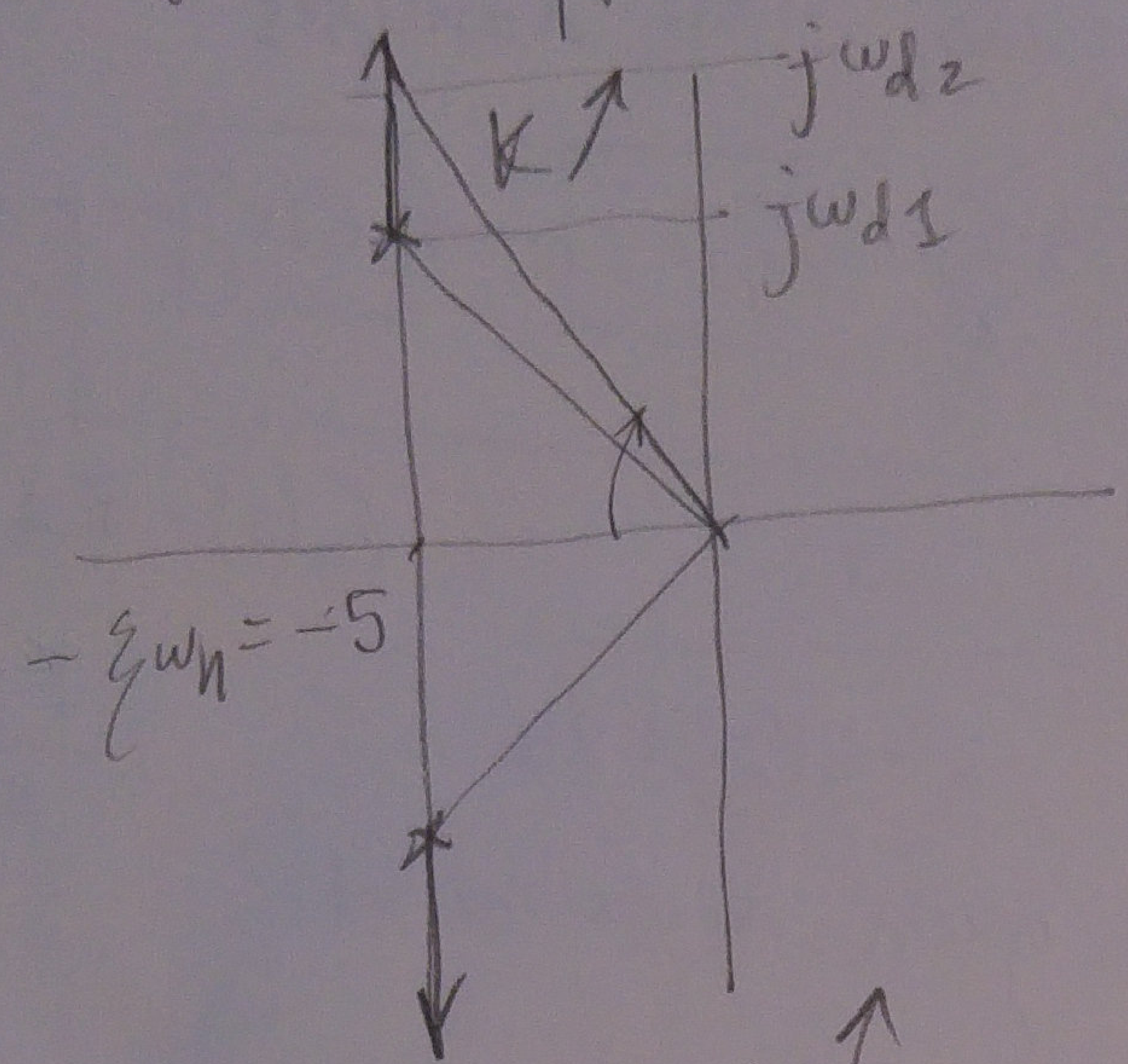
For $K \gg 20$ approximately standard form

$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

What happens as $K \nearrow$? We have $2\xi\omega_n = 10$

$\xi\omega_n = 5$ constant

But ω_n^2 getting large.



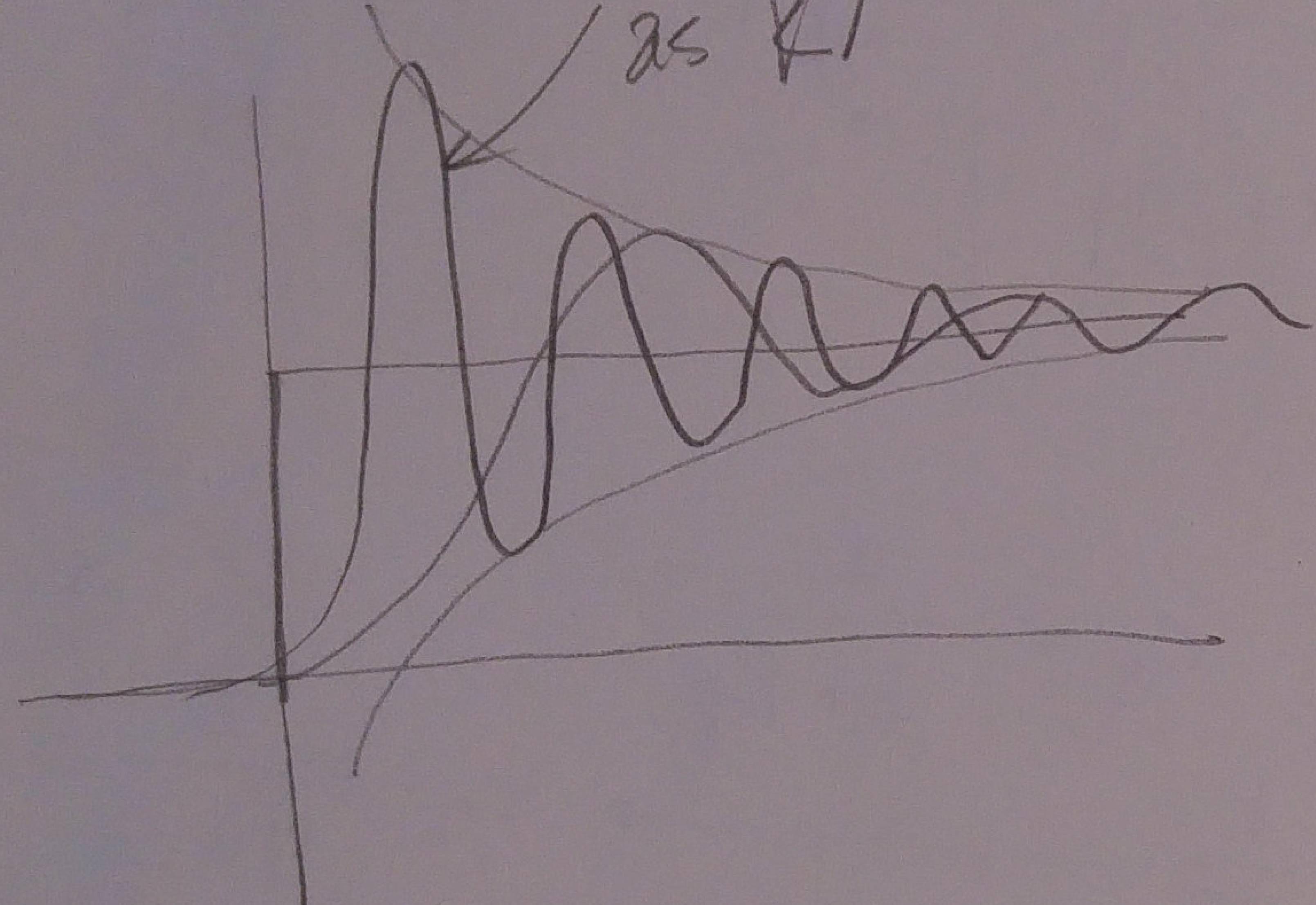
Exponential envelope constant

$\omega_d \nearrow$ Frequency of oscillations increase

$\theta \nearrow$ $\cos\theta \searrow$ $\xi \searrow$

Maximum overshoot increase

$t_p = \frac{\pi}{\omega_d}$ getting smaller!



$\longrightarrow$ Terrible oscillations and overshoot as K gets very large

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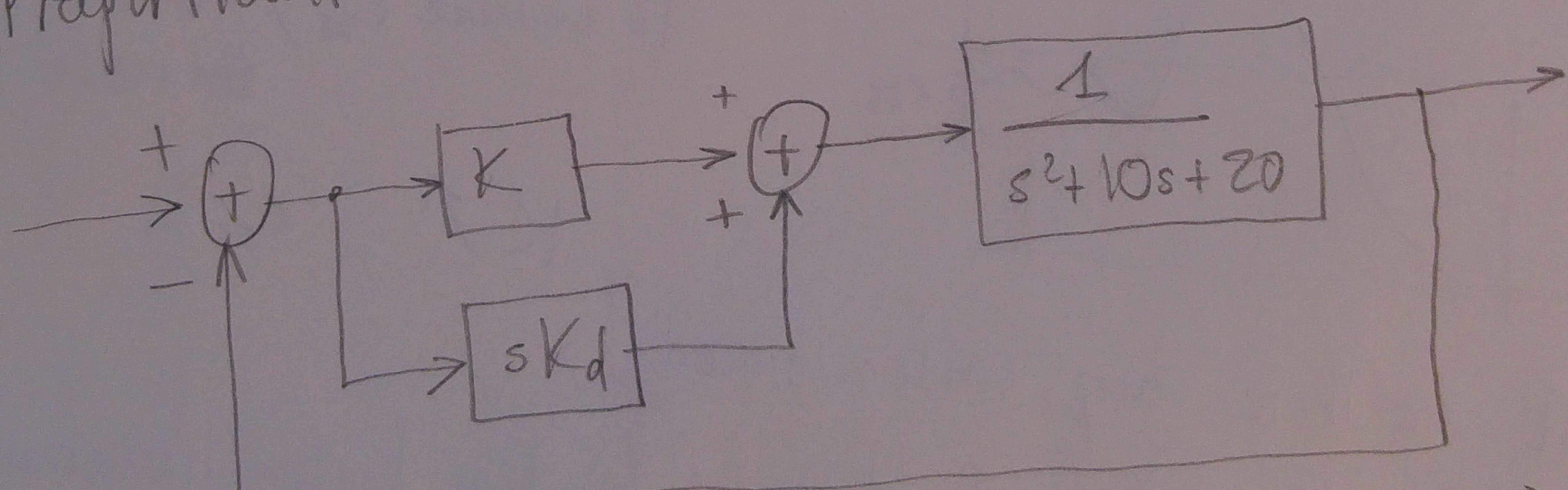
Proportional-Derivative Controller (PD)



Open-loop TF: $G(s) = \frac{(K + sK_d)}{s^2+10s+20}$ ← An open-loop zero introduced!

Did anything change for e_{ss}? $K_p = \lim_{s\to 0} \frac{K+sK_d}{s^2+10s+20} = \frac{K}{20}$

still "Type 0" system $\boxed{e_{ss} = \frac{20}{20+K}}$ same!

What about transient behavior?

$$T(s) = \frac{\frac{K + sK_d}{s^2+10s+20}}{1 + \frac{K+sK_d}{s^2+10s+20}} = \frac{K + sK_d}{s^2 + (10+K_d)s + K + 20}$$

$T(s)$ is no longer in standard form. But $K \gg K_d$ and $K \gg 20$ we can assume a standard form.

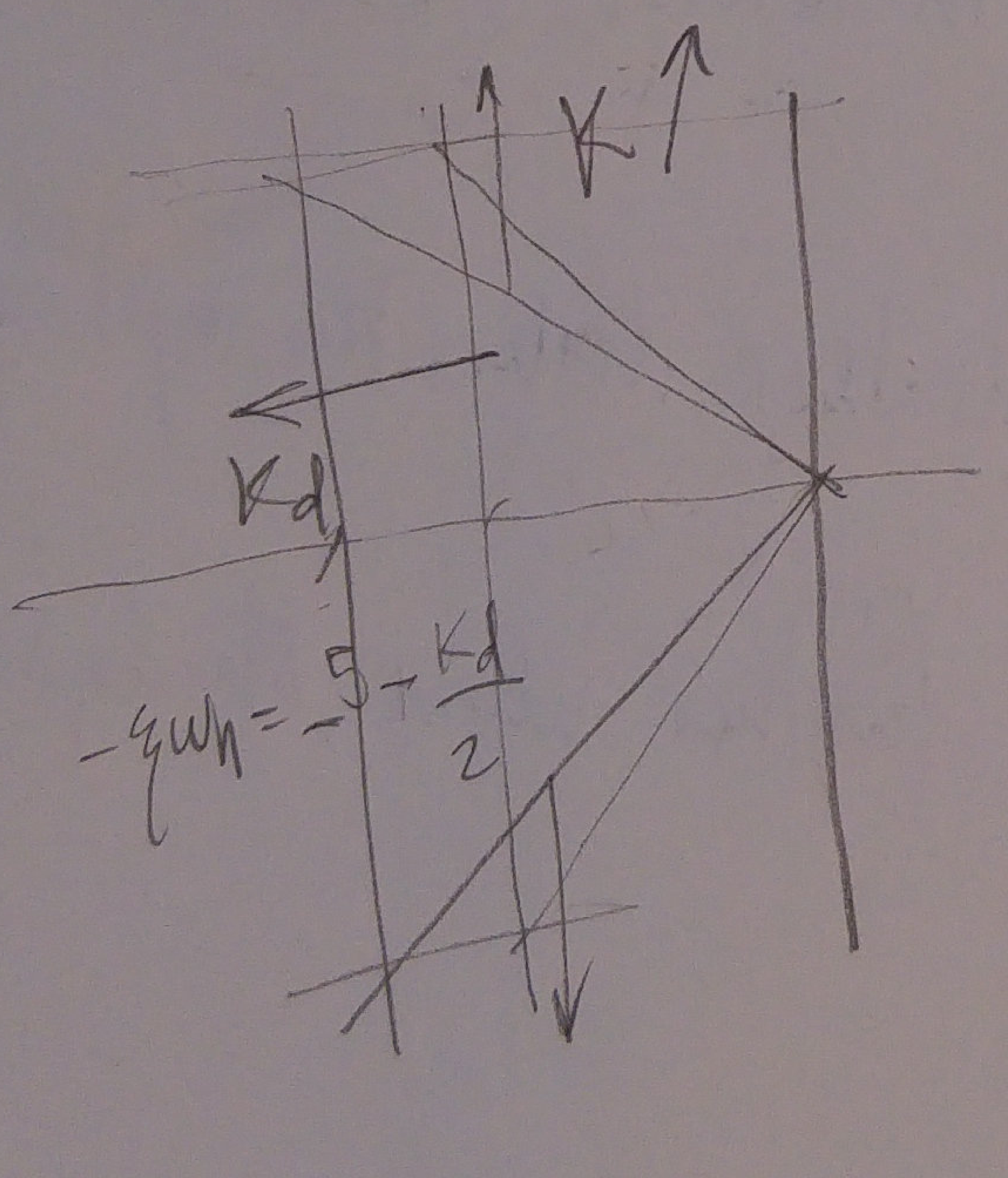
$$\omega_n^2 = K + 20 \longrightarrow \omega_n = \sqrt{K+20}$$

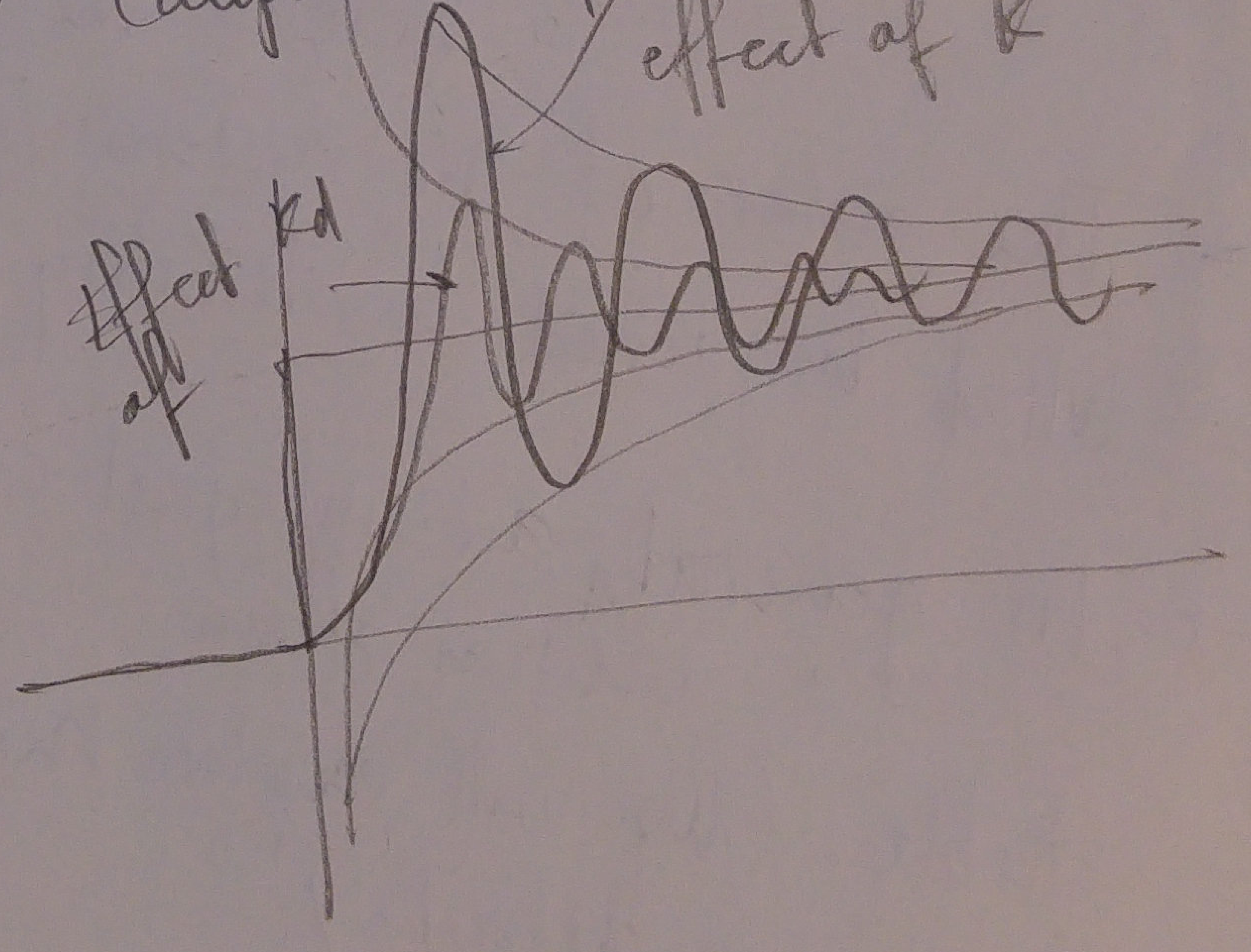
$$2\xi\omega_n = 10 + K_d \longrightarrow \xi = \frac{10+K_d}{2\sqrt{K+20}}$$

$\longrightarrow$ $K \nearrow \longrightarrow e_{ss} \searrow$ Main action to reduce error

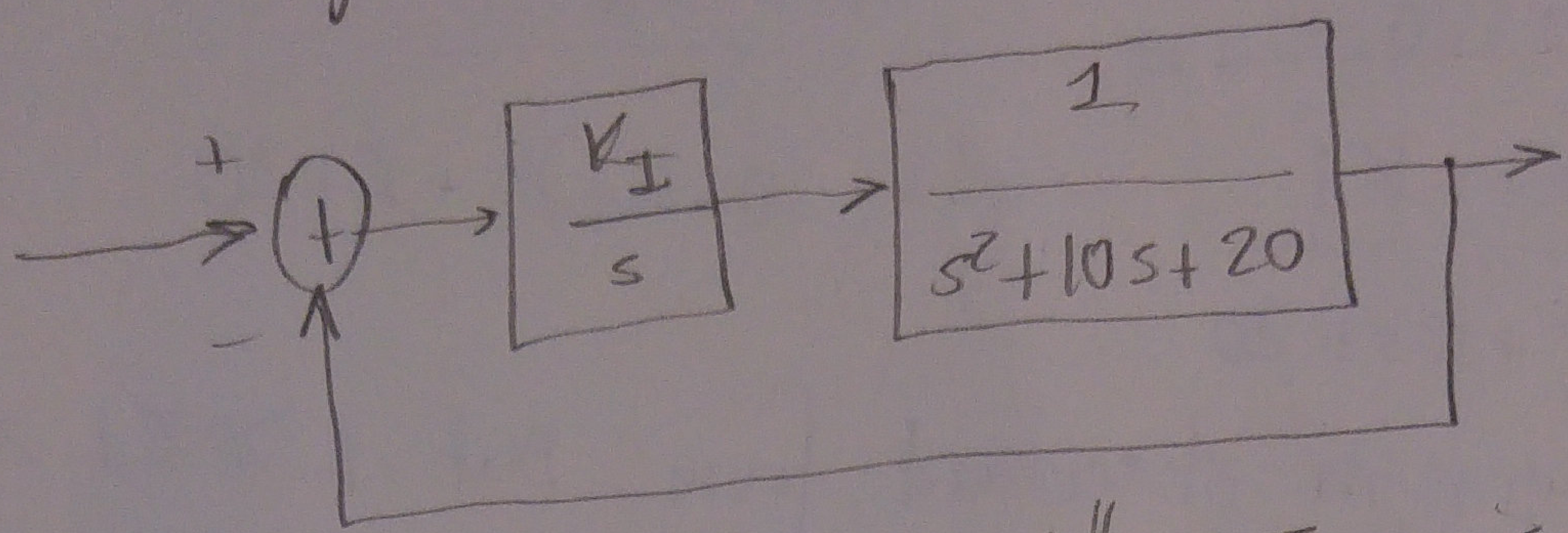
But $K \nearrow \longrightarrow \xi \searrow$ Effect is to increase oscillation.

$K_d \nearrow \longrightarrow \xi \nearrow$ Compensate for the oscillation effect of K





Integral Controller



Now → System is "Type 1" Type is increased;

$$K_p = \lim_{s \to 0} \frac{K_I}{s(s^2+10s+20)} \longrightarrow \infty$$

$$e_{ss} = \frac{1}{1+K_p} \longrightarrow 0$$

→ Integral control is usually applied to cope with steady-state error (in this case make it zero) increase the "precision" of the system.

→ The order of the system is increased. (Now 3)
What happened to stability?

→ In general, an addition of an integral controller makes the system less stable. (We will explore this further when studying stability in detail.)