Example:

A 4^{th} order system (will also illustrate how to analyze an arbitrary coefficient.)

$$q(s) = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

Example 1: let us have $a_1 = 3$, $a_2 = 3$, $a_3 = 1$ and $a_4 : 0 \to \infty$

$$q(s) = s^4 + 3s^3 + 3s^2 + s + a_4$$

$$= 1 + a_4 \frac{1}{s^4 + 3s^3 + 3s^2 + s} = 1 + a_4 \cdot \frac{1}{s(s+1)^3} = 1 + G(s)H(s)$$

with $G(s)H(s) = \dfrac{a_4}{s(s+1)^3}$

Now apply the Root Locus rules to observe the variation of the closed loop poles in the complex plane: as a_4 changes from $\emptyset$ to ∞

Rule 1: # of branches in the Locus:

# of branches $= \max(m,n) = \max(0,4) = 4$ branches.

Rule 2: Starts from open loop poles $\longrightarrow$ open loop zeros.

$a_4 = 0$; $p_1 = 0$, $p_{2,3,4} = -1$

$a_4 = \infty$: (No zero) $\longrightarrow$ All zeros are at infinity. $s = \infty$

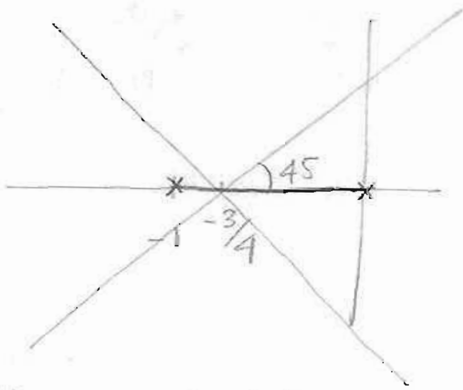
Rule 3: Symmetry

Rule 4: Asymptotes: $\phi = \dfrac{\pm 180}{n-m}(2l+1) = \dfrac{\pm 180}{4}(2l+1) = \pm 45(2l+1)$

Four asymptotes with

Rule 5: Asymptote intersection with real axis.

$$\sigma_0 = \frac{\sum p_j - \sum z_i}{n-m} = \frac{-3-\emptyset}{4} = -3/4$$



Rule 6: Locus on the real axis:

Part of Locus if # of poles & zeros at right is odd $\longrightarrow$ region between poles is part of Locus.

Rule 7: Break-away / break-in point.

$$\frac{d}{ds}\left(\frac{1}{s(s+1)^3}\right) = 0 \longrightarrow (s+1)^2(4s+1) = \emptyset \longrightarrow s_1 = -1,\ s_2 - \frac{1}{4}$$

$\dfrac{u'v - uv'}{v^2}$

Both of them are on the Locus

$a_4 = 0$ when $s_1 = -1$ and $a_4 = +0.105$ when $s_2 = -\frac{1}{4}$

Rule 8: Intersection with the $j\omega$ axis:

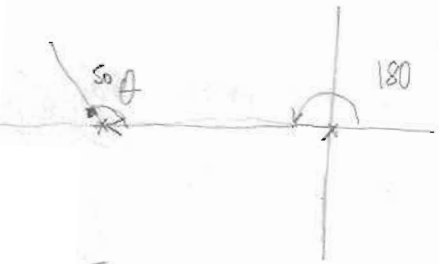
Routh Array

$$
\begin{array}{llll}
s^4 & 1 & 3 & a_4 \\
s^3 & 3 & 1 & 0 \\
\boxed{s^2} & \boxed{8/3} & \boxed{a_4} & \\
s^1 & 1-\dfrac{9a_4}{8} & & \\
s^0 & a_4 & &
\end{array}
\longrightarrow \quad 1-\frac{9a_4}{8}=0 \longrightarrow a_4=\frac{8}{9}\cong 0.89
$$

Auxiliary polynomial

$$\frac{8}{3}s^2+\frac{8}{9}=0 \longrightarrow s_{1,2}=\mp j0.577$$ As the intersection point with $j\omega$ axis at $a_4=0.89$

Rule 9: Angle of departure of branches at $s=-1$



$$-3\theta-180=\pm 180(2l+1)$$
$$3\theta+180=\pm 180(2l+1)$$
$$3\theta=\pm 180(2l)$$
$$\theta=\pm\frac{180}{3}(2l)=60(2l) \longrightarrow \begin{array}{ll} l=0 & \theta=0^\circ \\ l=1 & \theta=120^\circ \\ l=-1 & \theta=-120 \end{array}$$

Now, let us sketch the root Locus