HW #4 Answers & Solutions

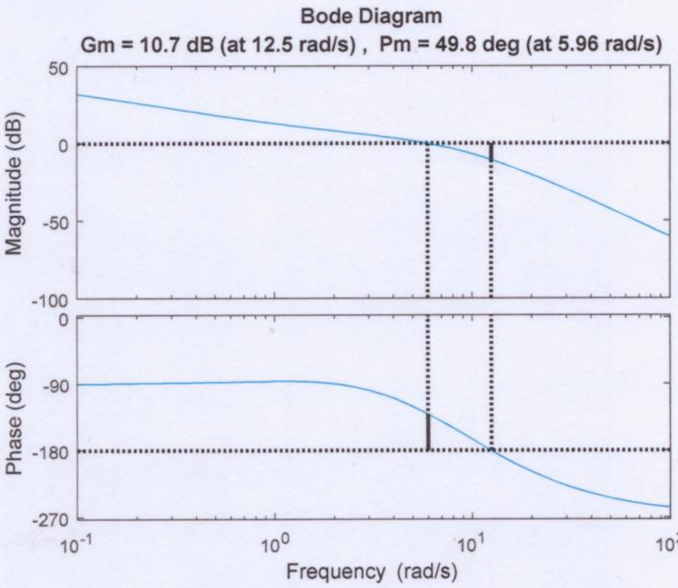
Questions 1-2: Computer Assignments:

Q1. You should be able to verify that the gain margin of the system is $\approx 15.5dB$ in three different ways (polar plot, Bode plot and root locus), by using the provided code in the question.

- Gain margin $\approx 15.5dB$
- Phase margin $\approx 90^0$.

Q2.

a.



- Gain cross-over frequency =5.96rad/s.
- Phase cross-over frequency =12. 5rad/s.
- Phase margin $\approx 50^0$
- Gain margin = $10.7dB$

b. Desired phase margin is 55^0. One can expect that a lead compensator with maximum phase (ϕ_{max}) of 10^0 (10 $\approx$ desired p.m. – actual p.m. $+ 5^0$) should be sufficient to increase the phase margin to 55^0. Unfortunately, since the phase of the system around the gain cross-over frequency changes too fast, it is not possible to achieve the desired phase margin with $\phi_{max} = 10^0$.

If you take the extra phase term added to be $> 20^0$ instead of $5^0 (i.e.,\ \phi_{max} > 25)$, you can achieve the desired phase margin.

c. The desired phase margin can be achieved by using a lag compensator $G_c(s) \approx \frac{1+2.5s}{1+4s}$.

Q3)

a) $G(s) = \dfrac{0.2}{s^2(s+100)}$

$e_{ss} = 1/K_a$, $K_a = \lim\limits_{s \to 0} s^2 K \cancel{G_c(s)}^{\,1} . G(s) = \lim\limits_{s \to 0} \dfrac{K/5}{s+100} = \dfrac{K}{500}$

$e_{ss} < 0.25 \Rightarrow \dfrac{500}{K} < 0.25 \Rightarrow K \geq 2000$, $K_{min} = 2000$

b) $G_{unc}(s) = K . G(s) = \dfrac{400}{s^2 . 100(1+\frac{s}{100})} = \dfrac{4}{s^2(1+\frac{s}{100})}$

$$|G_{unc}(j\omega)| = \frac{4}{\omega^2\Big(1+\underbrace{\big(\tfrac{\omega}{100}\big)^2}_{\approx 0 \text{ for small } \omega}\Big)}\Bigg|_{\omega=\omega_g} = 1 \Rightarrow \omega_g \approx 2 \text{ rad/s}$$

$\angle G_{unc}(j\omega) = -\angle -\omega^2 - \angle 100 + j\omega$

$p.m = \angle G_{unc}(j\omega_g) + 180 = -\angle -\omega_g^2 - \angle 100 + j\omega_g + 180$

$= -180 - \operatorname{atan}\frac{2}{100} + 180 \approx 0$

c) $\angle G_{unc}(j\omega)$ is always below $-180°$, therefore a phase-lag compensator cannot be used.

d) i)(Section: 1&2)

$0 + \phi_m = p.m. \text{desired} + 5$, $\boxed{G_c(s) = \dfrac{1+Ts}{1+\alpha Ts}, \quad T>0, \; 0<\alpha<1}$

$= 45 \Rightarrow \sin\phi_m = 0.71$, $\alpha = \dfrac{1-\sin\phi_m}{1+\sin\phi_m} \approx 0.16$, $\sqrt{\alpha} = 0.4$

$$|G_{unc}(j\omega)|\Big|_{\omega=\omega_{gnew}} = \sqrt{\alpha} = 0.4 = \frac{4}{\omega_{gnew}^2\sqrt{1+\frac{\omega_{gnew}^2}{100^2}}} \Rightarrow 100 = \omega_{gnew}^4 + \cancel{\frac{\omega_{gnew}^6}{100^2}}$$

(Because $1<\omega_g<10$)

$\omega_{gnew}^4 \approx 100 \Rightarrow \omega_{gnew} = \sqrt{10}$

$\omega_{gnew} = \omega_m = \dfrac{1}{\sqrt{\alpha}.T} \Rightarrow T = \dfrac{1}{\sqrt{10}.\sqrt{\alpha}} = 0.79$, $\alpha T = 0.16 \times 0.79 \approx 0.13$

$G_c(s) = \dfrac{1+Ts}{1+\alpha Ts} = \dfrac{1+0.79s}{1+0.13}$

d) ii) (Section: 3 & 4) $\Rightarrow G_c(s) = \dfrac{1+\alpha T s}{1+Ts}$, $\alpha > 1$, $T > 0$

$\phi_m = 45°$ (= p.m. desired + 5 − 0)

$$\frac{1}{\alpha} = \frac{1-\sin\phi_m}{1+\sin\phi_m} \Rightarrow \alpha \approx 6.25 //$$

$$\left| G_{unc}(j\omega) \right|_{\omega=\omega_{gnew}} = \sqrt{\frac{1}{\alpha}} = 0.4 = \frac{4}{\omega_{gnew}^2 \sqrt{1+\frac{\omega_{gnew}^2}{100^2}}} \Rightarrow 100 = \omega_{gnew}^4 + \frac{\omega_{gnew}^6}{100^2}$$

neglect (Because $1 < \omega_g < 10$)

$$\Rightarrow \omega_{gnew}^4 \approx 100 \Rightarrow \omega_{gnew} = \sqrt{10}$$

$$\omega_{gnew} = \omega_m = \frac{1}{\sqrt{\alpha}} \cdot \frac{1}{T} = 0.4 \times \frac{1}{T} = \omega_{gnew} = \sqrt{10} \Rightarrow T \approx 0.13 //$$

$$\alpha T \approx 0.81$$

$$G_c(s) = \frac{1+\alpha T s}{1+Ts} = \frac{1+0{,}81 s}{1+0{,}13 s} //$$