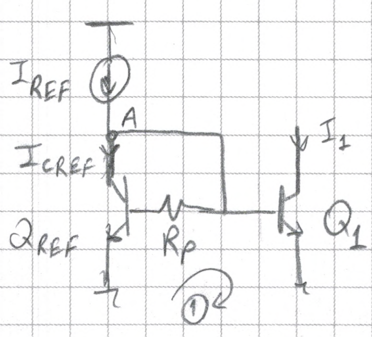
EE 414 - Introduction to Analog Integrated Circuits

Take Home Exam -2

Problem 1



$$I_1 = 1.1\ I_{1,NOM}$$

First compute $I_{1,NOM}$, i.e. I_1 with $R_P = 0$

$$I_{1,NOM} = I_{REF}\frac{\beta}{\beta+2} \Rightarrow I_1 = 1.1\ I_{REF}\frac{\beta}{\beta+2}$$

Now let's consider R_P:

KVL @ Loop 1 : $V_{BE} + \frac{I_{CREF}}{\beta}R_P = V_{BE_1}$

$$V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{CREF}}{I_S}\right) = \frac{I_{CREF}}{\beta}R_P$$

$$R_P = \frac{\beta}{I_{CREF}} V_T \ln\left(\frac{1.1\ I_{REF}\frac{\beta}{\beta+2}}{I_{CREF}}\right) \quad \text{--- (1)}$$

KCL @ A : $I_{REF} = I_{CREF} + \frac{I_{CREF}}{\beta} + \frac{I_1}{\beta}$

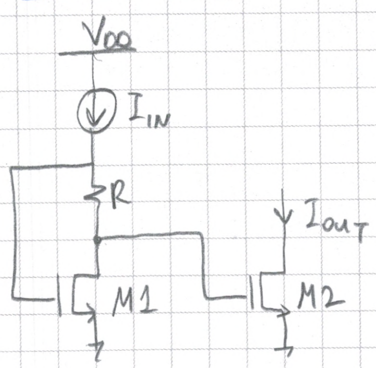
$$I_{CREF} = \frac{I_{REF}\left(1 - \frac{1.1}{\beta\left(\frac{\beta+2}{\beta}\right)}\right)}{1+\frac{1}{\beta}} = \frac{\beta(\beta+0.9)}{(\beta+1)(\beta+2)} I_{REF}$$

Now substitute back in (1)

$$R_p = \frac{(\beta+1)(\beta+2)}{\beta+0.9}\,\frac{V_T}{I_{REF}}\,\ln\left[\frac{1.1\,\cancel{\beta}(\beta+1)\cancel{(\beta+2)}}{\cancel{\beta}\,\cancel{(\beta+2)}\,(\beta+0.9)}\right]$$

$$\Rightarrow \boxed{R_p = \frac{(\beta+1)(\beta+2)}{\beta+0.9}\,\frac{V_T}{I_{REF}}\,\ln\left[\frac{1.1\,(\beta+1)}{\beta+0.9}\right]}$$

Problem 2:



$I_{OUT} = 0.1\,\mu A$ $\quad \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

a) $I_{in} = 1\,\mu A$

We need to determine the values range of $\left(\frac{W}{L}\right)_1$ and for each value of $\left(\frac{W}{L}\right)_1$ compute the corresponding value of R that guarantees an output current of $I_{out} = 0.1\,\mu A$

From Gray & Meyer:

$$I_{out} = \frac{K'}{2}\left(\frac{W}{L}\right)_1 \left(\sqrt{\frac{2 I_{IN}}{K'\left(\frac{W}{L}\right)}} - R I_{IN}\right)^2 \quad \text{----} \quad (1)$$

Solving for R;

$$R = \frac{\sqrt{\frac{2 I_{IN}}{K'\left(\frac{W}{L}\right)}} - \sqrt{\frac{2 I_{OUT}}{K'(W/L)}}}{I_{IN}} \quad \text{----} \quad (2)$$

To operate in saturation : $V_{GD_1} = R\, I_{IN} \leq V_{TH} = 0{,}5V$ ... ③

$$R \leq \frac{V_{TH}}{I_{IN}}$$

$$\frac{\sqrt{\frac{2 I_{IN}}{k'(\frac{W}{L})}} - \sqrt{\frac{2 I_{OUT}}{k'(\frac{W}{L})}}}{I_{IN}} \leq \frac{V_{TH}}{I_{IN}} \Rightarrow \left(\frac{W}{L}\right) \leq 1{,}643$$

=> From Gray & Meyer ; To operate in strong inversion;

$$V_{ov_1} = \sqrt{\frac{2 I_{IN}}{k'(W/L)}} \geq 3 V_T = 78mV \Rightarrow \left(\frac{W}{L}\right) \geq 0{,}018$$

=> Now we can compute corresponding R range by using eqn. ②

$$\boxed{\begin{array}{c} 0{,}018 \leq \left(\frac{W}{L}\right)_1 \leq 1{,}643 \\ 53\,k\Omega \leq R \leq 496\,k\Omega \end{array}}$$

b) Now size $R = 10\,k\Omega$ and compute the range of $\frac{W}{L}$ and I_{IN} to maintain $\frac{W}{L}$ in strong inversion and saturation. We use the same constrains as before,

From ③ $I_{IN} \leq 50\,\mu A$

When we put upper condition to the eqn ②;

$$\left(\frac{W}{L}\right) \leq 1{,}825$$

=> To operate in strong inversion

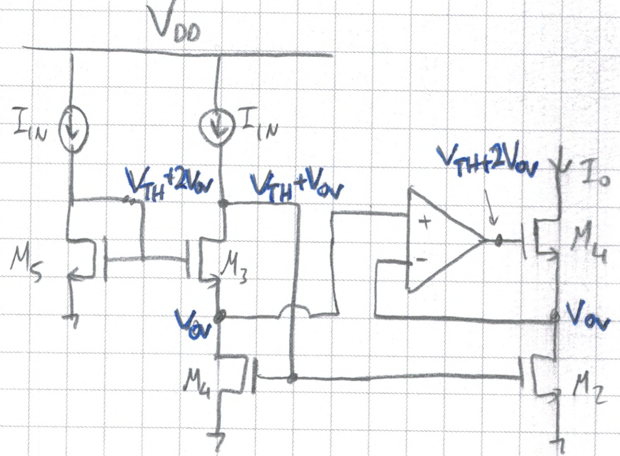
$$\sqrt{\frac{2 I_{IN}}{k'(W/L)}} > 78mV \Rightarrow \sqrt{I_{IN}} > 78\times10^{-5}\sqrt{\left(\frac{W}{L}\right)}$$

=> when we put it into eqn ②

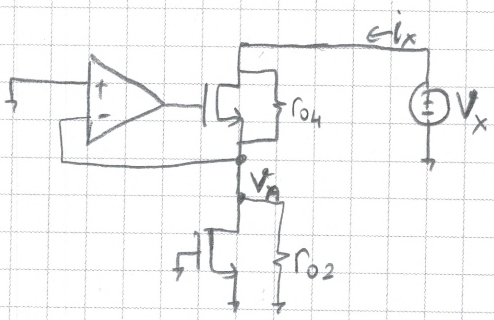
$$\frac{W}{L} \geq 0{,}34 \; , \; I_{IN} \geq 0{,}2\,\mu A$$

$$\boxed{\begin{array}{c} 0{,}34 \leq \frac{W}{L} \leq 1{,}825 \\ 0{,}2\mu A \leq I_{IN} \leq 50\,\mu A \end{array}}$$

Problem 3:



a) To compute the output resistance, we disconnected all the input sources and apply a test source at the output. The simplified equivalent circuit becomes.



$$V_A = i_x r_{o2} \quad \text{--- ①}$$

$$i_x = \frac{V_x - V_A}{r_{o4}} + g_{m4}\left(-AV_A - V_A\right) \quad \text{-- ②}$$

Combine ① & ②

$$r_{o4} i_x = v_x - i_x r_{o2} - g_{m4} r_{o4} (A+1) r_{o2} i_x$$

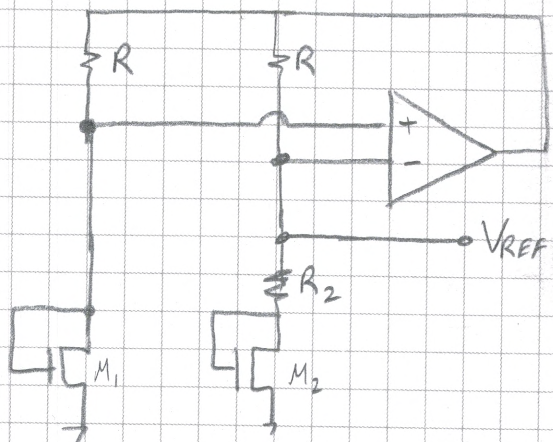
$$\boxed{R_{OUT} = \frac{v_x}{i_x} = r_{o2} + r_{o4} + g_{m4} r_{o4} r_{o2} (A+1) \simeq g_{m4} r_{o4} r_{o2} (A+1)}$$

$\Rightarrow$ The super-cascode configuration boostes-up the output impedance by $(A+1)$ with respect to the regular cascode.

b) If we size M5 with $\left(\frac{W}{L}\right)_5 = \frac{1}{4}\left(\frac{W}{L}\right)$ we get the DC voltages reported in the figure and systematic error $\varepsilon = 0$.

The OPAMP will make sure that the voltages at the drain terminals of M_4 & M_2 are the same. Note that such a choice of $\left(\frac{W}{L}\right)_5$ also minimizes the required $V_{OUT,MIN}$.

Problem 4:



$R_2 = 5{,}5\,K\Omega$

$W_1 = W$

$W_2 = K^* W_1 \Rightarrow K=4$

You can see from the left side of the circuit that V_{REF} is equal to the gate-to-source voltage of M_1. This results because the amplifier feedback holds its inputs at the same voltage. And due to same issue the current at each branch is same (I_D).

From KVL;

$$I_D = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R}\left(\sqrt{\frac{2I_D}{K_n'(\frac{W}{L})}} + V_{TH} - \sqrt{\frac{2I_D}{K_n' \cdot K(\frac{W}{L})}} - V_{TH}\right)$$

$$I_D = \frac{1}{R}\left(\sqrt{\frac{2I_D}{K_n'(W/L)}}\left(1 - \frac{1}{\sqrt{K}}\right)\right)$$

$$I_D = \frac{1}{R^2}\frac{2}{K_n'(\frac{W}{L})}\left(1 - \frac{1}{\sqrt{K}}\right)^2$$

$$V_{REF} = V_{GS1} = \sqrt{\frac{2I_D}{K_n'(W/L)}} + V_{TH} \Rightarrow \boxed{V_{REF} = \frac{2}{K_n'(\frac{W}{L})R}\left(1 - \frac{1}{\sqrt{K}}\right) + V_{TH}}$$

$\Rightarrow$ Let's drive the TCF of the circuit.

$$TCV_{REF} = \frac{1}{V_{REF}}\frac{\partial V_{REF}}{\partial T} \Rightarrow \frac{\partial V_{REF}}{\partial T} = \frac{\partial V_{TH}}{\partial T} + \frac{2}{(\frac{W}{L})(1-\frac{1}{\sqrt{K}})}\frac{\partial}{\partial T}\left(\frac{1}{K_n' R}\right)$$

$$\boxed{\frac{\partial V_{REF}}{\partial T} = \frac{\partial V_{TH}}{\partial T} - \frac{2}{(\frac{W}{L})(1-\frac{1}{\sqrt{K}})} \times \frac{1}{R K_n'}\left(\frac{1}{K_n'}\frac{\partial K_n'}{\partial T} + \frac{1}{R}\frac{\partial R}{\partial T}\right)}$$