

Let $F_{\text{BIG}}: \mathbf{K} \times \mathbf{X} \rightarrow \mathbf{Y}$ be a PRF that has the extension property

$$F_{\text{BIG}}(k, x) = F_{\text{BIG}}(k, y) \quad \Rightarrow \quad F_{\text{BIG}}(k, \mathbf{x} \parallel \mathbf{w}) = F_{\text{BIG}}(k, \mathbf{y} \parallel \mathbf{w})$$

Generic attack on the derived MAC:

step 1: issue $|\mathbf{Y}|^{1/2}$ message queries for rand. messages in \mathbf{X} .

obtain (m_i, t_i) for $i = 1, \dots, |\mathbf{Y}|^{1/2}$

step 2: find a collision $t_u = t_v$ for $u \neq v$ (one exists w.h.p by b-day paradox)

step 3: choose some w and query for $t := F_{\text{BIG}}(k, \mathbf{m}_u \parallel \mathbf{w})$

step 4: output forgery $(\mathbf{m}_v \parallel \mathbf{w}, t)$. Indeed $t := F_{\text{BIG}}(k, \mathbf{m}_v \parallel \mathbf{w})$