In the Name of God

Communication Systems (25751-4)

Problem Set 06

Fall Semester 1401-02

Department of Electrical Engineering

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Due on Azar 24, 1401 at 23:55

($*$) starred problems are optional and have a bonus mark!

1 Sampling

The lowpass signal $x(t)$ with a bandwidth of W is sampled with a sampling interval of T_s, and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t - nT_s) \tag{1}$$

is reconstructed from the samples, where $p(t)$ is an arbitrary-shaped pulse (not necessarily time limited to the interval $[0, T_s]$).

1. Find the Fourier transform of $x_p(t)$.
2. Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.
3. Determine the required reconstruction filter.
4. Consider the signal $\tilde{x}(t)$ with the first-order interpolation as follows:

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} \left[\Big(x(nT_s) + \frac{t - nT_s}{T_s}\big(x((n+1)T_s) - x(nT_s)\big)\Big)\Big(u(t - nT_s) - u(t - (n+1)T_s)\Big)\right]$$

   Determine the spectral density of $\tilde{x}(t)$.

2 Nonideal Sampling

Some sampling devices extract from $x(t)$ its average value over the sampling duration, so $x(nT_s)$ in Eq. (1) is replaced by:

$$\overline{x}(nT_s) = \frac{1}{\tau}\int_{nT_s-\tau}^{nT_s} x(t)dt.$$

1. Devise a frequency-domain model of this process using an averaging filter, with input $x(t)$ and output $y(t)$, followed by instantaneous sampling. Then obtain the impulse response of the averaging filter and write the resulting expression for $X_p(f)$.
2. Find the equalizer needed when $p(t)$ is a rectangular pulse with duration T_s and unit amplitude.

3 Quantization Error

Consider the continues random variable X with pdf:

$$f_X(x) = \begin{cases} 2(1-x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let Y be a discrete version of X formed by uniformly quantizing to n level.

$$Y = \frac{1}{n}(\lfloor nX \rfloor + \frac{1}{2})$$

1. For $n = 3$, what values of Y are possible and what are their probabilities.
2. For $n = 3$, derive the quantization error $\mathbb{E}[(X-Y)^2]$.
3. Derive the value of quantization error $\mathbb{E}[(X-Y)^2]$ for arbitrary n.
4. Suppose a non-uniform quantizer is used to represent X. What values of a, b minimize $\mathbb{E}[(X-Z)^2]$ for fix t and

$$Z = \begin{cases} a & 0 \leq x \leq t \\ b & t < x \leq 1 \end{cases}$$

4 Quantization and Signal Transmission

A signal $m(t)$ bandlimited to 3 kHz is sampled at a rate $\frac{1}{3}$ higher than its Nyquist rate. The maximum acceptable error in the sample amplitude (the maximum quantization error) is 0.5% of the peak signal amplitude m_p (quantization steps are uniform). The quantized samples are binary coded. Find the minimum bandwidth of a channel required to transmit the encoded binary signal.
24 such signals are time-division-multiplexed. If 2% more bits are added to the multiplexed data for error protection and synchronization, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

5 μ-Law Compander

The message signal $m(t)$ with the power of 20 mWatts is applied to an analog-to-digital converter with the dynamic range of -1 volt to 1 volt.

1. To transmit this signal by PCM, uniform quantization is adopted. If the SQNR is required to be at least 43 dB, determine the minimum number of bits required to code the uniform quantizer. Determine the SQNR obtained with this quantizer.
2. Repeat part 1, if a μ-law compander is applied with $\mu = 100$.
Hint: You may have to use equation (5.35) of [1]

References

[1] B. P. Lathi and Z. Ding, *Modern digital and analog communication systems.* Oxford university press, 2019.