In the Name of God

Communication Systems (25751-4)

Problem Set 04

Fall Semester 1401-02

Department of Electrical Engineering

Sharif University of Technology

Instructor: Dr. M. Pakravan

Due on Aban 24, 1401 at 18:00

($*$) starred problems are optional and have a bonus mark!

1 DSB Modulation with Periodic Waveforms

A DSB signal is generated by multiplying the message signal $m(t)$ with the periodic rectangular waveform shown in Figure 2 and filtering the product with a bandpass filter tuned to the reciprocal of the period T_p, with bandwidth $2W$, where W is the bandwidth of the message signal.

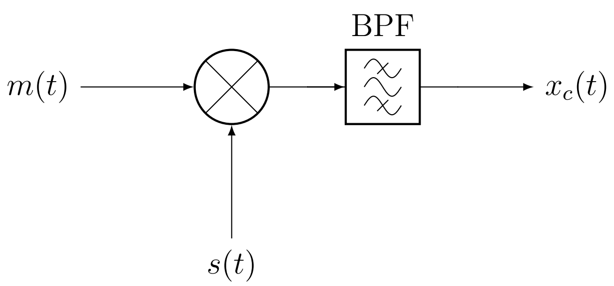

Figure 1: DSB Modulator

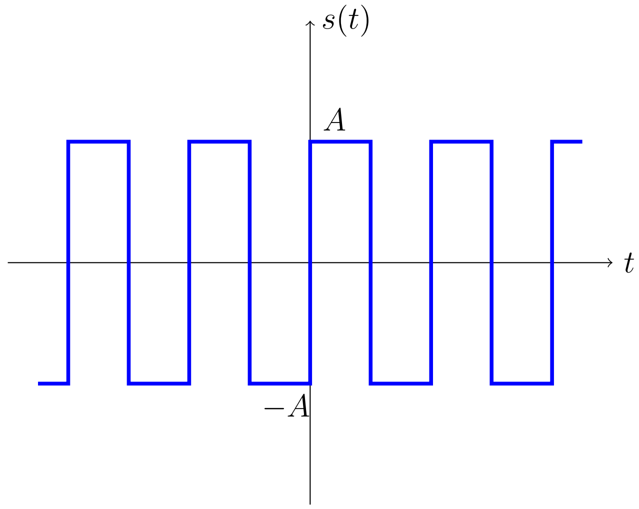

Figure 2: $s(t)$

1. Demonstrate that the output $x_c(t)$ of the BPF is the desired DSB signal

$$x_c(t) = A_c m(t) \sin(2\pi f_c t)$$

   where $f_c = \dfrac{1}{T_p}$, and find A_c.

2. Show that it is not necessary that the periodic signal be rectangular. This means that any periodic signal with period T_p can substitute for the rectangular signal in Figure 2.

2 Weaver's SSB Modulator

Weaver's SSB modulator is illustrated in Figure 3. By taking the input signal as $x(t) = \cos(2\pi f_m t)$, where $f_m < W$, demonstrate that by proper choice of f_1 and f_2 the output is a SSB signal.

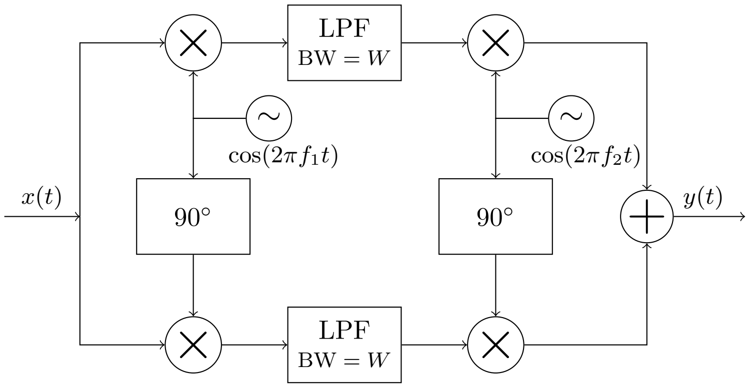

Figure 3: Weaver's SSB Modulator

3 VSB Signal

A VSB signal $y(t)$ is as below. α is a non negative constant less than one.

$$y(t) = \frac{\alpha}{2}\cos(2\pi(f_c + f_m)t) + \frac{1-\alpha}{2}\cos(2\pi(f_c - f_m)t) + \cos(2\pi f_c t)$$

1. Prove that the envelop of the signal can be calculated as below. $d(t)$ represents the distortion.

$$e(t) = \big[1 + \frac{1}{2}\cos(2\pi f_m t)\big] d(t)$$

$$d(t) = \sqrt{1 + [\frac{(1-2\alpha)\sin(2\pi f_m t)}{2 + \cos(2\pi f_m t)}]^2}$$

2. Find α such that it maximize $d(t)$.

4 A Simple Scrambler System

The message signal $m(t)$ has a Fourier transform shown in Figure 4. This signal is applied to the system shown in Figure 5 to generate the signal $y(t)$.

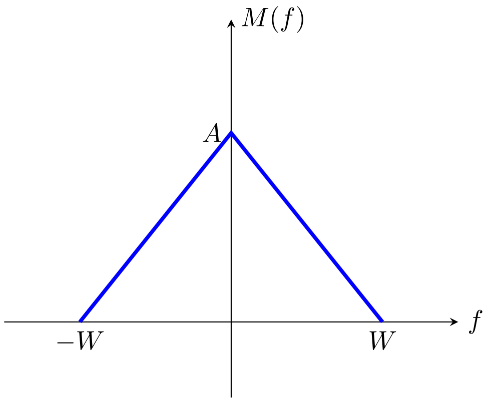

Figure 4: Fourier Transform of $m(t)$

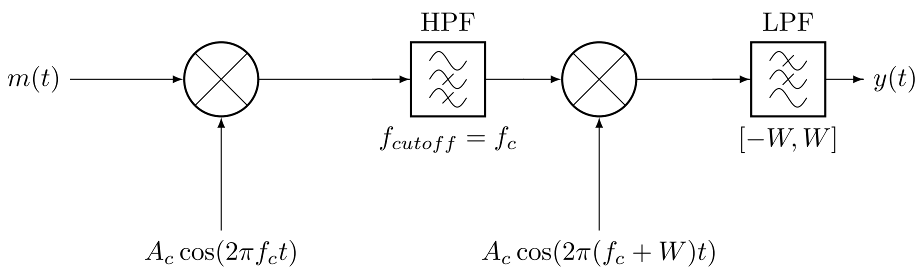

Figure 5: simple scrambler

1. Plot $Y(f)$, the Fourier transform of $y(t)$.
2. Show that if $y(t)$ is transmitted, the receiver can pass it through a replica of the system shown in Figure 5 to obtain $m(t)$ back. This means that this system can be used as a simple scrambler to enhance communication privacy.

5 VSB Modulation System

A vestigial sideband modulation system is shown in Figure 5. The bandwidth of the message signal $m(t)$ is W and the transfer function of the bandpass filter is shown in the figure 7.

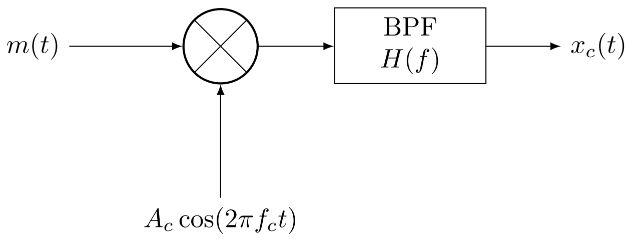

Figure 6: VSB Modulator

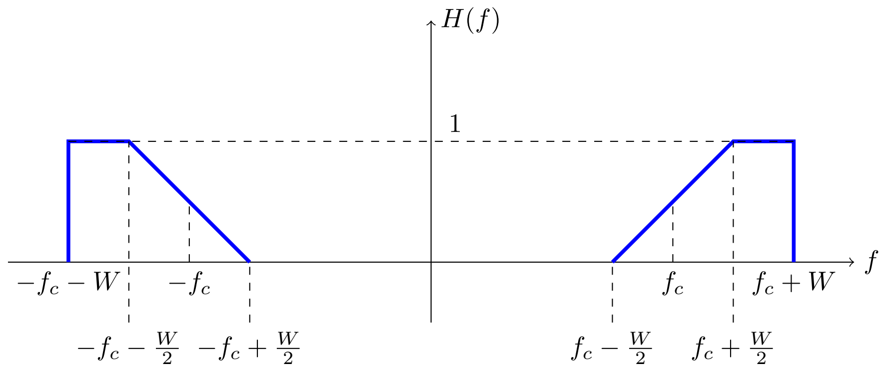

Figure 7: $H(f)$

1. Determine $h_{lp}(t)$, the lowpass equivalent of $h(t)$, where $h(t)$ represents the impulse response of the bandpass filter.
2. Derive an expression for the modulated signal $x_c(t)$.

6 (∗) Modulation and Chirp Signals

The message signal $m(t)$has a Fourier transform $M(f)$. This signal is applied to the system shown in Figure 8 to generate the signal $y(t)$. Assume that $h(t) = A_1 e^{j\pi\omega_0^2 t^2}$ and ω_0 is constant.

1. Find an expression for $y(t)$, in term of $m(t)$ and $M(f)$.
2. Design a system to reconstruct $m(t)$ from $y(t)$.

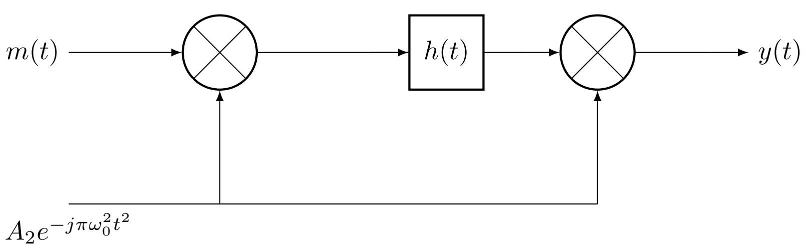

Figure 8: a system with chirp signals