In the Name of God

Communication Systems (25751-4)

Problem Set 03

Fall Semester 1401-02

Department of Electrical Engineering

Sharif University of Technology

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Due on Aban 17, 1400 at 18:00

(∗) starred problems are optional and have a bonus mark!

1 Autocorrelation of a Triplet Pulse

Determine the autocorrelation function of the triplet pulse shown in figure bellow:

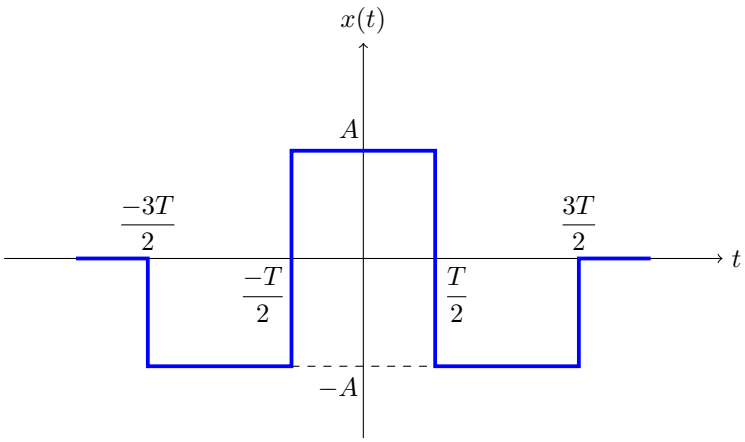

Figure 1: Triplet Pulse

2 Autocorrelation, Power Spectral Density and Periodic Signals

1. Let $x(t)$ be periodic with period T_0. Show that $R_x(\tau)$ has the same periodicity.
2. Show that if the periodic signal $x(t+T_0) = x(t)$ passes through an LTI system with the frequency response $H(f)$, then the power spectral density of the output signal equals to:

$$\mathcal{S}_y(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 |H(\frac{n}{T_0})|^2 \delta(f - \frac{n}{T_0}),$$

where c_ns are the coefficients of the Fourier series expansion of $x(t)$.

3 Power Spectral Density Estimation

Show that if $x_T(t)$ denotes the truncated signal corresponding to the power-type signal $x(t)$; that is:

$$x_T(t) = \begin{cases} x(t) & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases},$$

and if $\mathcal{S}_{x_T}(f)$ denotes the energy spectral density of $x_T(t)$, then $\mathcal{S}_x(f)$, the power-spectral density of $x(t)$, can be expressed as

$$\mathcal{S}_x(f) = \lim_{T\to\infty} \frac{\mathcal{S}_{x_T}(f)}{T}$$

4 Energy and Power Spectral Density Calculation

Determine whether these signals are energy-type or power-type. In each case, find the energy spectral density and the energy content or the power spectral density and the power content of the signal.

1. $x_1(t) = e^{-\alpha|t|}\sin(\beta t)$ $\qquad (\alpha, \beta > 0)$
2. $x_2(t) = \text{sinc}^2(\alpha t)$ $\qquad (\alpha > 0)$
3. $x_3(t) = \sum_{n=-\infty}^{\infty} \Lambda(t - kn)$ $\qquad (k \in \mathbb{N}, k > 2)$
4. $x_4(t) = Au(-t)$
5. $(*)$ $x_5(t) = |\cos(2\pi f_0 t)|$
6. $(*)$ $x_6(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-t^2}{2\sigma^2}}$

5 Double Side-Band Modulation

In a DSB system the carrier is $c(t) = A\cos(2\pi f_c t)$ and the message signal is given by $m(t) = 2\text{sinc}(t) + \text{sinc}^2(2t)$.

Find the frequency domain representation and the bandwidth of the modulated signal.

6 Amplitude Modulation

An AM signal has the form $x_c(t) = 4\cos(2800\pi t) + 20\cos(3000\pi t) + 4\cos(3200\pi t)$.

1. Determine the modulating signal $m(t)$ and the carrier $c(t)$.
2. Determine the modulation index.
3. Determine the ratio of the power in the sidebands to the power in the carrier.