In the Name of God

Communication Systems (25751-4)

Problem Set 01

Fall Semester 1401-02

Department of Electrical Engineering

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($*$) starred problems are optional and have a bonus mark!

1 Fourier Transform

Determine the Fourier transform of each of the following signals:

1. $x_1(t) = e^{-\alpha|t|}\cos(\beta t). \qquad (\alpha > 0)$
2. $x_2(t) = \Lambda(t) = (1 - |t|)u(t+1)u(-t+1)$.
3. $x_3(t) = \frac{t}{(a^2+t^2)^2}$.
4. $x_4(t) = \displaystyle\sum_{n=-\infty}^{\infty} (-1)^n \Lambda(\tfrac{t}{T} - kn). \qquad (T > 0, k > 0)$

2 Parseval's Theorem

Let $x(t)$ and $y(t)$ be two energy-type signals, and let $X(f)$ and $Y(f)$ denote their Fourier transforms, respectively. Show that:

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

3 Fourier Transform and Real Integrals

Use the known properties of the Fourier transform to obtain the following:

1. $I_1 = \displaystyle\int_0^{+\infty} \frac{\text{sinc}(x)}{a^2+x^2}dx$
2. $I_2 = \displaystyle\int_0^{+\infty} e^{-\alpha t}\text{sinc}^2(\beta t)dt \qquad (\alpha > 0)$
3. $I_3 = \displaystyle\int_{-\infty}^{+\infty} \frac{\sin(t) - t\cos(t)}{t^3}dt$
4. $(*)I_4 = \displaystyle\int_0^{+\infty} \frac{\sin^4(t)}{t^4}dt$

4 Inverse Fourier Transform

1. Determine $x_1(t)$, whose Fourier transform $X_1(f)$ has the following magnitude and phase. Express $x_1(t)$ as a closed-form and sketch its function of time.

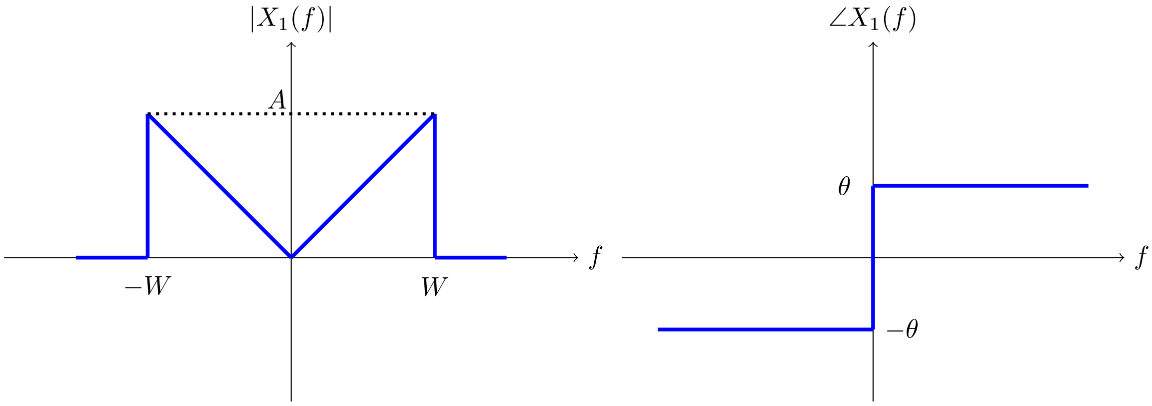

Figure 1: Problem 2 – part 1

2. Determine $x_2(t)$, whose Fourier transform $X_2(f)$ has the following magnitude and phase. Express $x_2(t)$ as a closed-form and sketch its function of time.

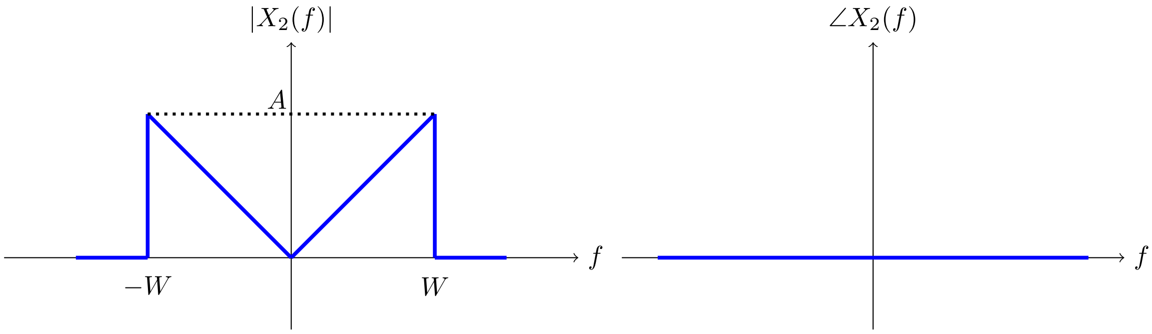

Figure 2: Problem 2 – part 2

3. What are important similarities and differences between $x_1(t)$ and $x_2(t)$? How do those similarities and differences manifest in their Fourier transforms?

5 Types of Signals

Classify the following signals into energy-type, power-type, and neither energy-type nor power-type signals.
For energy-type or power-type signals find the energy or the power contents of the signal.

1. $x_1(t) = e^{-\alpha t}\cos(\beta t)u(t) \qquad (\alpha > 0)$
2. $x_2(t) = \text{sinc}(t)$
3. $x_3(t) = \sum_{n=-\infty}^{\infty} (-1)^n \Lambda(t - kn)$
4. $x_4(t) = \begin{cases} Kt^{-\frac{1}{4}} & t > 0 \\ 0 & t \leq 0 \end{cases}$

6 Poisson's Sum Formula

1. By computing the Fourier series coefficients for the periodic signal $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$, shows that:

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn\frac{2\pi t}{T_s}}$$

2. Using the result of part (1), prove that for any signal $x(t)$ and any T_s ,the following identity holds:

$$\sum_{n=-\infty}^{\infty} x(t - nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right) e^{jn\frac{2\pi t}{T_s}}$$

3. Conclude the following relation known as *Poisson's sum formula.*

$$\sum_{n=-\infty}^{\infty} x(nT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X\left(\frac{n}{T_s}\right)$$

7 Output Energy of an LTI System

Let $x(t)$ represent the input to an LTI system, where:

$$x(t) = \sum_{n=-\infty}^{\infty} \alpha^{|n|} e^{\frac{jn\pi}{4}t}$$

for $0 < \alpha < 1$. The frequency response of the system is:

$$H(f) = \begin{cases} 1 & |f| < W \\ 0 & o.w. \end{cases} .$$

Assume that $\alpha = 0.4$. What is the minimum value for W sush that the average energy in the output signal will be at least 90% of that in the input signal?

8 (∗) A Signal and Its Fourier Transform

Assume that $x(t)$ has the following Fourier transform:

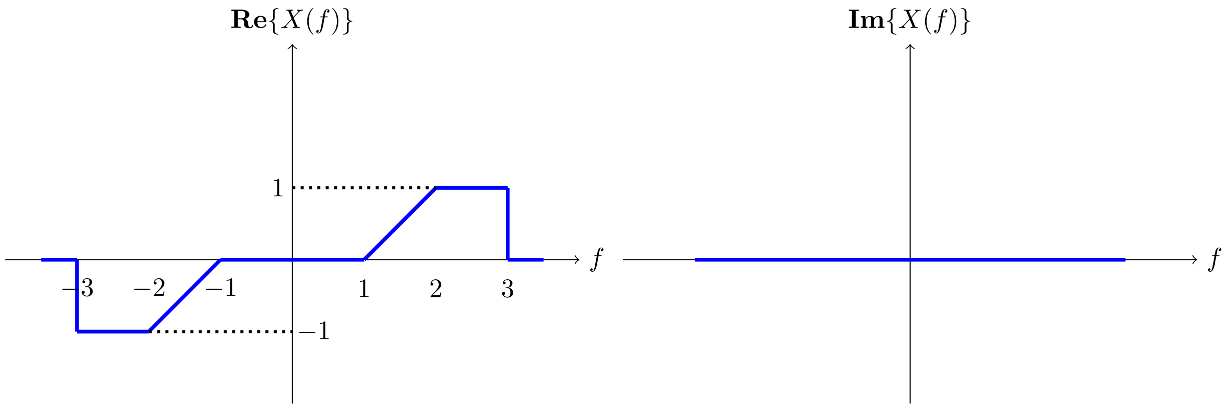

Figure 3: Problem 8

1. Find the following integral:

$$I = \int_0^{+\infty} 2tx(t)\cos(2\pi t)dt.$$

2. Is $x(t)$ an energy-type signal or a power-type signal?
3. find the energy or the power content of $x(t)$.