Analog to Digital Conversion

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Overview

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✓ In sampling, a discrete-time continuous-value signal from an analog signal is obtained.

✓ In quantization, a discrete-time discrete-amplitude signal from a discrete-time continuous-value signal is obtained.

✓ In encoding, a sequence of bits is assigned to different quantized values of a discrete-time discrete-amplitude signal.

Figure: Block diagram of analog to digital converter.

Sampling

Nyquist Sampling

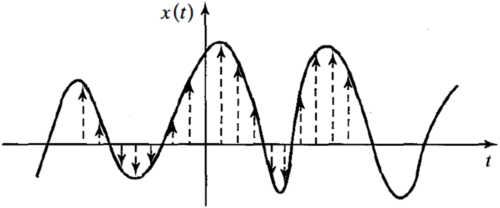


Figure: Nyquist sampling of a signal.

Nyquist Sampling

Theorem (Sampling Theorem)

Let the signal $x(t)$ have a bandwidth W, i.e., let $X(f) = 0$ for $|f| \geq W$. Let $x(t)$ be sampled at multiples of some basic sampling interval T_s, where $T_s \leq \frac{1}{2W}$, to yield the sequence $x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$. Then it is possible to reconstruct the original signal $x(t)$ from the samples values by the reconstruction formula

$$
\begin{aligned}
x(t) &= h(t) * x_\delta(t) = 2W' T_s sinc(2W' t) * x_\delta(t) \\
&= \sum_{n=-\infty}^{\infty} 2W' T_s x(nT_s) sinc[2W'(t - nT_s)]
\end{aligned}
$$

, where W' is any arbitrary number satisfying the condition $W \leq W' \leq \frac{1}{T_s} - W$.

Nyquist Sampling

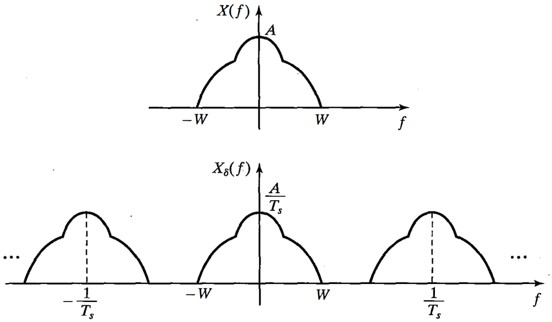


Figure: Frequency-domain representation of the nyquist sampled signal.

$$x_\delta(t) = x(t)\sum_{n=\infty}^{\infty}\delta(t-nT_s) = \sum_{n=\infty}^{\infty}x(nT_s)\delta(t-nT_s)$$

$$X_\delta(f) = X(f) * \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{T_s}) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(f-\frac{n}{T_s})$$

$$H(f) = T_s \sqcap (\frac{f}{2W'}) \Rightarrow X(f) = X_\delta(f)H(f) \Rightarrow x(t) = x_\delta(t) * h(t)$$

Zero-Order Hold Sampling

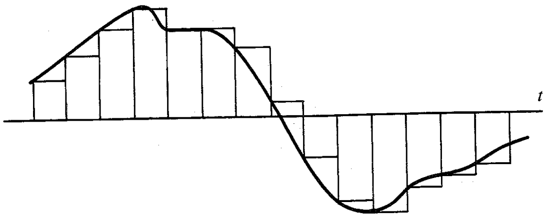


Figure: Flat-top sampling (zero-order hold sampling, sample and hold) of a signal.

$$x_p(t) = \sum_{n=\infty}^{\infty} x(nT_s)p(t - nT_s) = x_\delta(t) * p(t)$$

$$X_p(f) = X_\delta(f)P(f), \quad P_{eq}(f) = \frac{Ke^{-j2\pi f t_d}}{P(f)} \Rightarrow X_p(f)P_{eq}(f) = X_\delta(f)Ke^{-j2\pi f t_d}$$

$$X_p(f)P_{eq}(f)H(f) = X(f)Ke^{-j2\pi f t_d} \Rightarrow Kx(t - t_d) = x_p(t) * h(t) * p_{eq}(t) = x_p(t) * h_p(t)$$

Aliasing

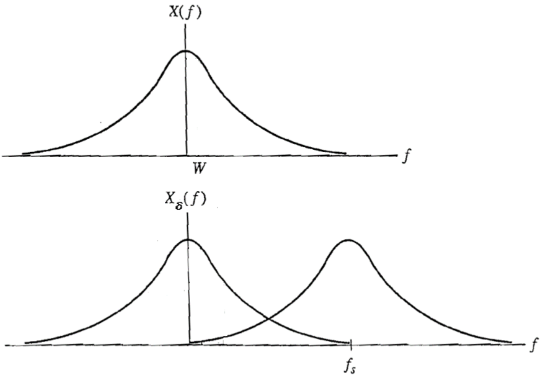


Figure: Aliasing in sampling.

✗ The unlimited bandwidth of messages creates aliasing.

Anti-aliasing

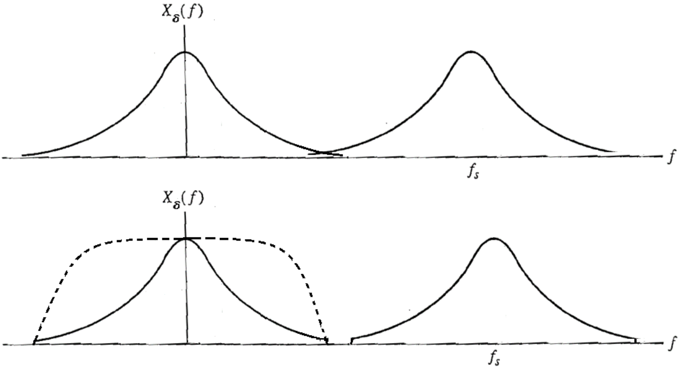


Figure: Anti-aliasing techniques in sampling.

✓ Increase sampling frequency and/or use anti-aliasing filter to mitigate aliasing effect.

Quantization

Quantization

Theorem (Quantization)

Quantization is a function defined as

$$Q(x) = \hat{x}_i : \quad x \in \mathbb{R}_i$$

where the sets $\mathbb{R}_i$ partition the set of real numbers $\mathbb{R}$.

Definition (Signal to Quantization Noise Ratio)

If the random variable X is quantized to $Q(X)$, the signal to quantization noise ratio is defined as

$$\text{SQNR} = \frac{E\{X^2\}}{E\{(X - Q(X))^2\}}$$

Uniform Quantization

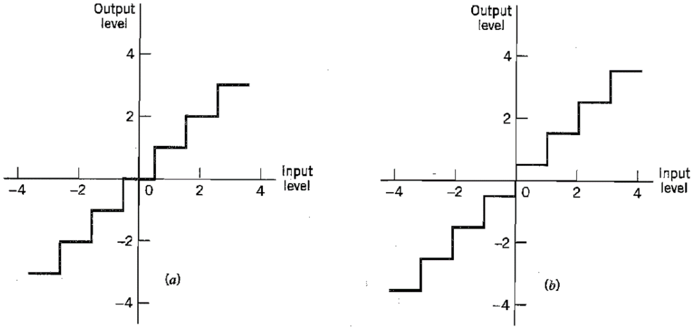


Figure: Two types of uniform quantization. (a) midtread and (b) midrise.

Nonuniform Quantization

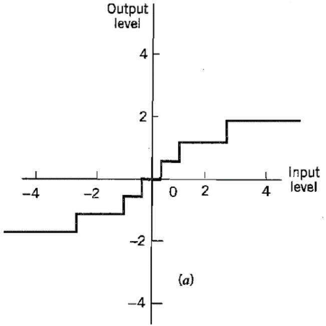


Figure: Two instances of nonuniform quantization.

Quantization Performance

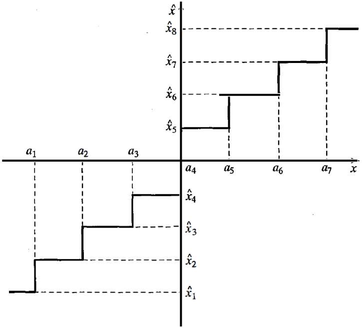
Example (SQNR)

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer with $a_1 = -60$, $a_2 = -40$, $a_3 = -20$, $a_4 = 0$, $a_5 = 20$, $a_6 = 40$, $a_7 = 60$, and $\hat{x}_1 = -70$, $\hat{x}_2 = -50$, $\hat{x}_3 = -30$, $\hat{x}_4 = -10$, $\hat{x}_5 = 10$, $\hat{x}_6 = 30$, $\hat{x}_7 = 50$, $\hat{x}_7 = 70$. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.

Quantization Performance

Example (SQNR (cont.))

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.



Example (SQNR (cont.))

The source $X(t)$ is a stationary Gaussian source with mean zero and power spectral density $S_x(f) = 2\sqcap(f/200)$. The source is sampled at the Nyquist rate and each sample is quantized using an eight-level quantizer. The SQNR for this quantization is $11.98 \equiv 10.78$ dB.

$$E\{X^2\} = \sigma^2 = R_X(0) = \int_{-\infty}^{\infty} S_X(f)df = 400$$

$$E\{(X - Q(X))^2\} = \int_{-\infty}^{a_1} (x - \hat{x}_1)^2 f_X(x)dx + \sum_{i=2}^{7} \int_{a_{i-1}}^{a_i} (x - \hat{x}_i)^2 f_X(x)dx$$

$$+ \int_{a_7}^{\infty} (x - \hat{x}_8)^2 f_X(x)dx = 33.38, f_X(x) = \frac{1}{\sqrt{800\pi}} \exp(-x^2/800)$$

$$\text{SQNR} = \frac{400}{33.38} = 11.98$$

Encoding

Encoding

Statement (Encoding)

In encoding, a unique sequence of ν bits is assigned to each $N = 2^{\nu}$ quantization level.

Natural Binary Coding and Gray Coding

Quantization Level	Level Order	NBC Code	Gray Code
$\hat{x}_1$	0	0000	0000
$\hat{x}_2$	1	0001	0010
$\hat{x}_3$	2	0010	0011
$\hat{x}_4$	3	0011	0001
$\hat{x}_5$	4	0100	0101
$\hat{x}_6$	5	0101	0100
$\hat{x}_7$	6	0110	0110
$\hat{x}_8$	7	0111	0111
$\hat{x}_9$	8	1000	1111
$\hat{x}_{10}$	9	1001	1110
$\hat{x}_{11}$	10	1010	1100
$\hat{x}_{12}$	11	1011	1101
$\hat{x}_{13}$	12	1100	1001
$\hat{x}_{14}$	13	1101	1000
$\hat{x}_{15}$	14	1110	1010
$\hat{x}_{16}$	15	1111	1011

Table: NBC and gray codes for 16-level quantization.

Pulse Code Modulation

PCM Transmitter

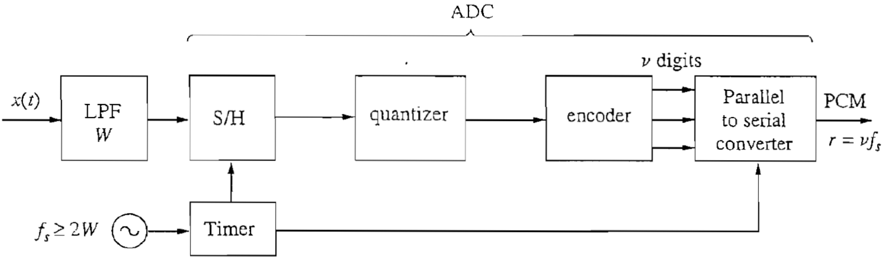


Figure: PCM transmitter.

✓ Output data rate is $r = \nu f_s$ bit/s.

PCM Receiver

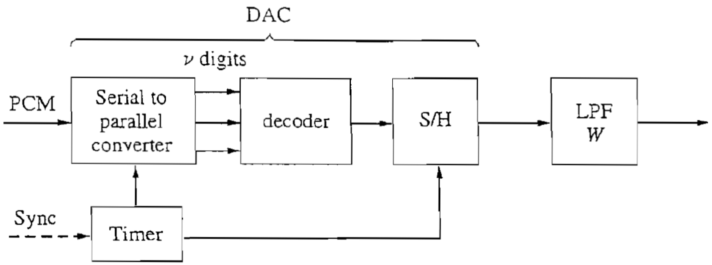


Figure: PCM receiver.

PCM Waveform

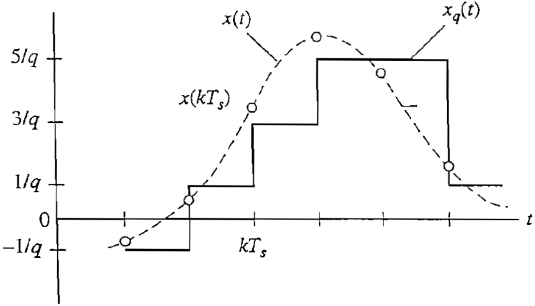


Figure: PCM waveform.

Companding

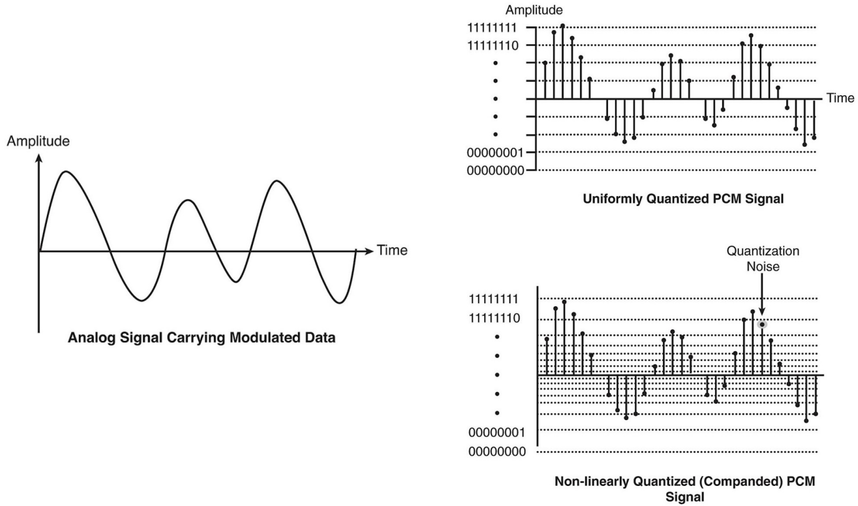


Figure: Uniformly-quantized and non-uniformly-quantized PCM signal.

Companding

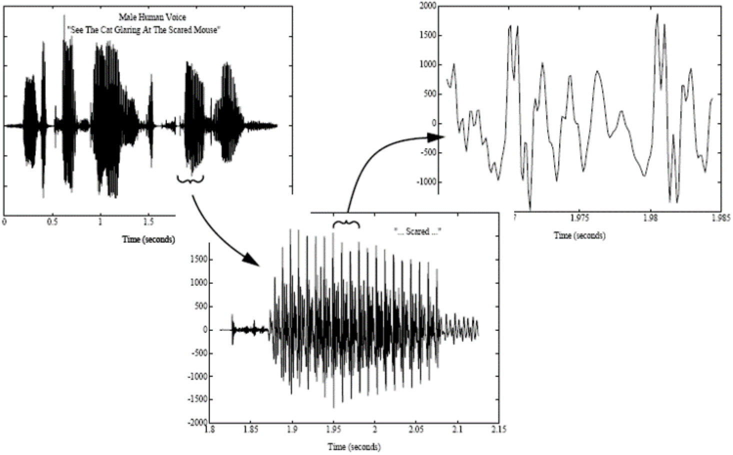


Figure: A sample of voice signal.

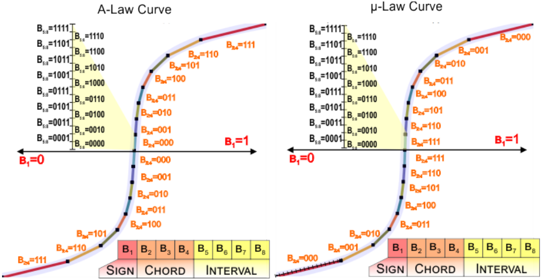
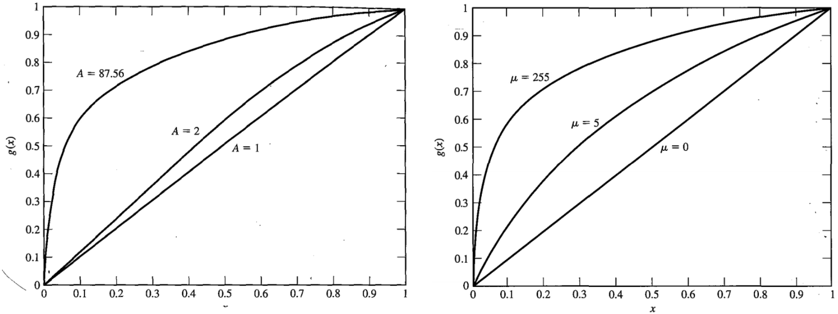


Figure: A-law and μ-law companding.

$$g(x) = \mathrm{sgn}(x) \begin{cases} \frac{A|x|}{1+\ln(A)}, & |x| \leqslant \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln(A)}, & |x| \geqslant \frac{1}{A} \end{cases}, \quad g(x) = \frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \mathrm{sgn}(x)$$

Companding



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E1 Digital Voice Multiplexing

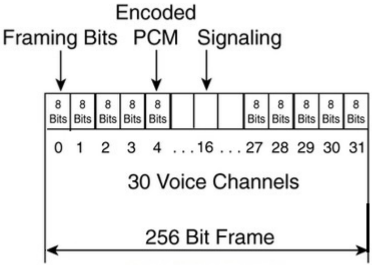

Figure: E1 frame.

✓ Each E1 frame carries 32 PCM channels with $f_c = 8000$ Hz and $\nu = 8$, which results in a net rate of $32 \times 8 \times 8000 \times 10^{-6} = 2.048$ Mb/s.

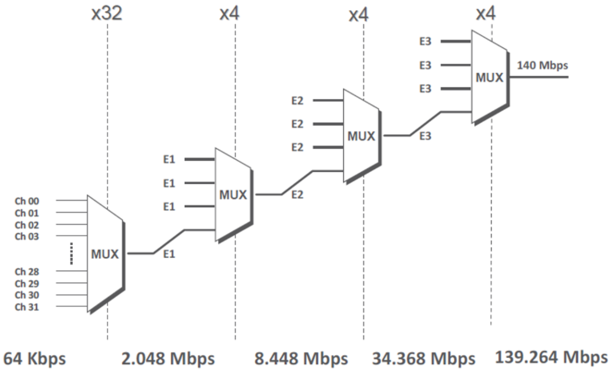

Figure: E-carrier digital multiplexing.

✓ Ex carrier carries $32 \times 4^{x-1}$ PCM channels with $f_c = 8000$ Hz and $\nu = 8$.
✓ Considering the overhead bits, Ex carriers provide rates of 2.048, 8.448, 34.368, 139.264 Mb/s.

Source Coding

Source Coding

Figure: Source coding reduces the number of bits required to represent the message signal.

✓ Source coder compresses the bits representing a message signal at the cost of quality reduction.

✓ Source coder creates computational complexity and may reduce the quality of the restored signal.

✓ In source coding, variable-length encoders are usually used.

Compression

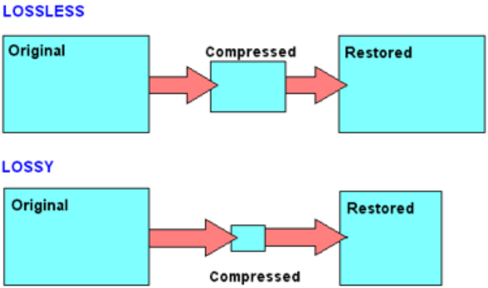

Figure: Lossless and lossy signal compression.

✓ In lossless compression, the source decoder recovers the exact original bit pattern.

✓ In lossy compression, the source decoder approximately replicate the original bit pattern.

✓ Type of message signal impacts the compression.

Vocoders

Voice signal can be compressed to reduce its bit rate using

1. Waveform source encoders
   1. Delta Modulation (DM)
   2. Sigma-Delta Modulation (SDM)
   3. Pulse Code Modulation (PCM)
   4. Differential PCM (DPCM)
   5. Adaptive DPCM (ADPCM)
2. Model-based source encoders
   1. Linear Predictive Coding (LPC)

Delta Modulation

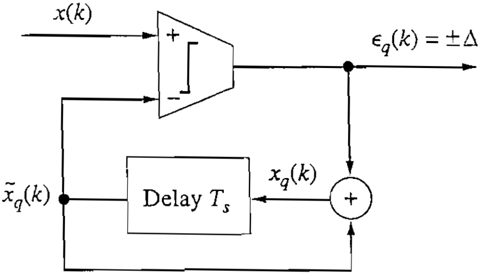

Figure: Delta transmitter sends the derivative of the input signal.

✓ Output data rate is $r = f_s$ bit/s.

Delta Modulation

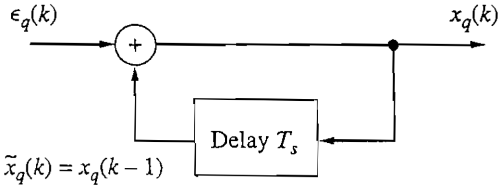

Figure: Delta receiver calculates the integral of the input signal.

Delta Modulation

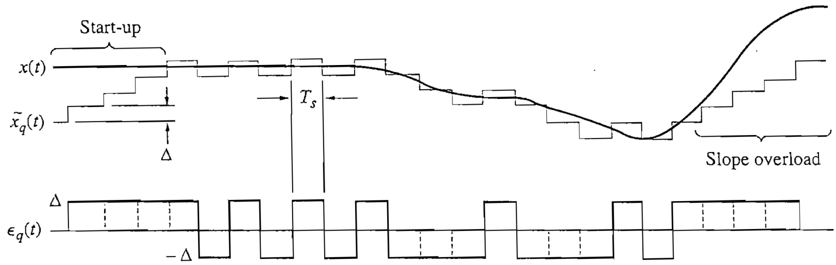

Figure: Delta waveform.

DPCM

✓ Voice signal samples are highly correlated and do not change drastically from one sample to the next.

✓ If $m[k]$ is the kth sample, instead of transmitting $m[k]$, the difference $d[k] = m[k] - m[k-1]$ can be transmitted.

✓ At the receiver, knowing $d[k]$ and the previous sample value $m[k-1]$, we can reconstruct $m[k]$.

✓ The signal $d[k]$ has a much smaller amplitude range. Therefore, the number of quantization levels required for its transmission reduces from 8 bits per sample to 4 bits per sample.

✓ Using this technique, voice signal rate can be reduced to 32 kbps.

LPC

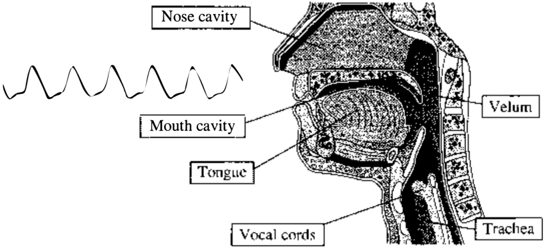

Figure: Human speech production mechanism. Human speech is produced by the joint interaction of lungs, vocal cords, mouth cavity, and the nose cavity.

✓ Voiced sounds are those made while the vocal cords are vibrating like the vowel "a" and the consonant "g".

✓ Unvoiced sounds are made while the vocal cords are not vibrating like the consonant "k".

✓ The vibrating vocal cords interrupt the air stream and produce the so-called pitch impulses.

LPC

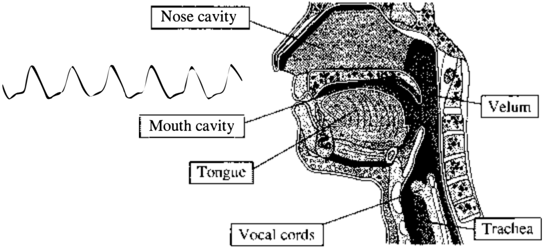

Figure: Human speech production mechanism. Human speech is produced by the joint interaction of lungs, vocal cords, mouth cavity, and the nose cavity.

✓ For voiced sound, the pitch impulses stimulate the air in the vocal tract (mouth and nasal cavities).

✓ For unvoiced sounds, the excitation comes directly from the air flow and can be considered as a broadband noise.

✓ Vocal cavities form resonators with characteristic resonance frequencies.

LPC

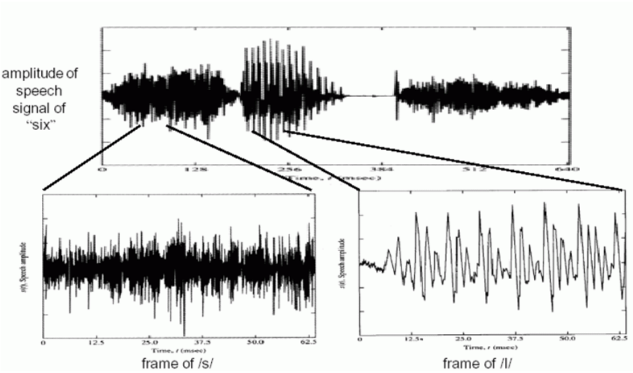

Figure: Speech signal of the word "six".

✓ Vocal tract can be approximately modeled by a simple linear digital filter with an all-pole transfer function of

$$H(z) = \frac{g}{A(z)} = \frac{g}{1 - \sum_{i=1}^{p} a_i z^{-i}}$$

LPC

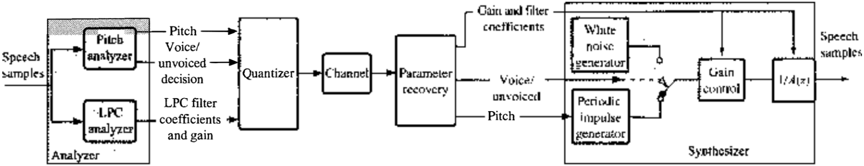

Figure: Analysis and synthesis of voice signals in an LPC encoder and decoder.

✓ The model-based vocoders analyze the voice signals segment by segment to determine the best-fitting speech model parameters.

✓ After speech analysis, the transmitter sends the necessary speech model parameters for each voice segment to the receiver.

✓ The receiver then uses the parameters for the speech model to set up a voice synthesizer to regenerate the respective voice segments.

LPC-10

Pitch Period	Voiced/Unvoiced	Gain g	10 LP Filter Parameters, bits/coefficient				
			$r_1 - r_4$	$r_5 - r_8$	r_9	r_{10}	
			5 bits	4 bits	3 bits	2 bits	Voiced
6 bits	1 bit	5 bits	5 bits	*Not used*			Unvoiced

Figure: Quantization bit allocation in LPC-10 vocoder.

✓ In the LPC-10 vocoder, the speech is sampled at 8 kHz.

✓ A total of 180 samples (22.5 ms) form an LPC frame for transmission.

✓ Each LPC frame requires between 32 (unvoiced) and 53 (voiced) bits.

✓ Adding frame control bits results in an average coded stream of 54 bits per speech frame, or an overall rate of 2.4 kbps.

Video Source Coding

✓ NTSC video signal has a bandwidth of around 4.2 MHz. Sampling and quantization of this signal results in a stream of 45-120 Mbps.
✓ Raw uncompressed HDTV signal requires 800 Mbps. Compression is very important for proper delivery of video over communication networks.
✓ Motion Picture Experts Group (MPEG) is an important standardization group for video compression.

MPEG

- MPEG-1: Used for VCR-quality video and storage on video CD at a data rate of 1.5 Mbps. MPEG-1 decoders are available on most computers.
- MPEG-2: Supports diverse video coding applications for transmissions ranging in quality from VCR to HDTV. It offers 50:1 compression of raw video. MPEG-2 is a highly popular format used in DVD, HDTV, terrestrial digital video broadcasting (DVB-T), and digital video broadcasting by satellite (DVB-S).
- MPEG-4: Provides multimedia (audio, visual, or audiovisual) content streaming over different bandwidths including Internet. MPEG-4 is supported by Microsoft Windows Media Player, Real Networks, and Apple's Quicktime and iPod. MPEG-4 is converged with an ITU-T standard known as H.264.

MPEG

MPEG video compression is based on two main mechanisms of

- Temporal or interframe compression by predicting interframe motion and removing interframe redundancy.
- Spatial or intraframe compression, which forms a block identifier for a group of pixels having the same characteristics (color, intensity, etc.). For each frame, only the block identifier is transmitted.

MPEG

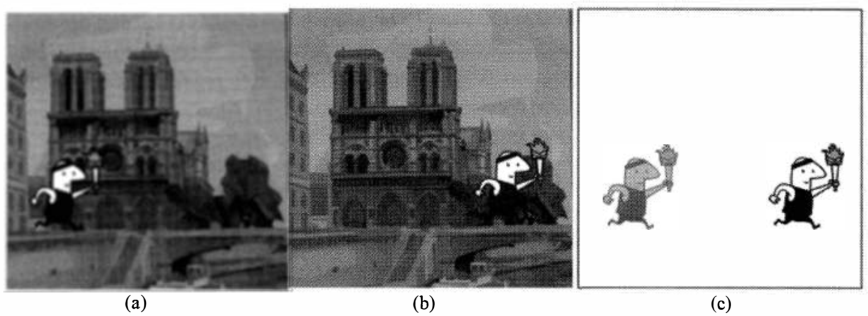

Figure: Motion estimation and compensation. (a) Frame 1. (b) Frame 2. (c) The residual image between frames 1 and 2.

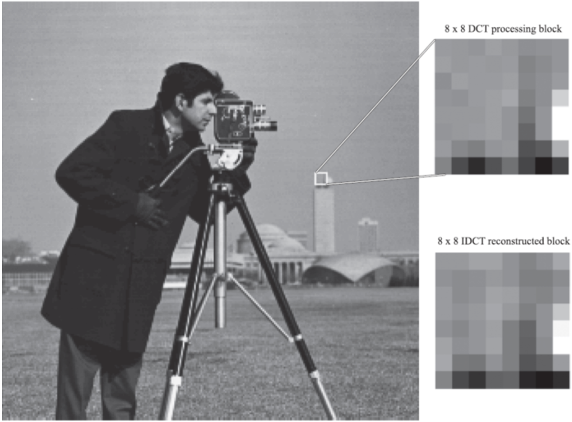

Figure: The famous cameraman image, a highlighted 8×8 block for compression and the correspondingly reconstructed block at the decoder after DCT compression.

MPEG

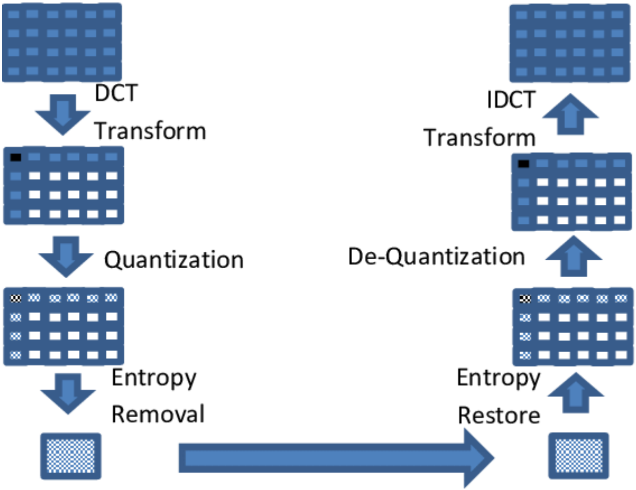

Figure: DCT compression.

The End