Digital Communication

Mohammad Hadi

mohammad.hadi@sharif.edu

@MohammadHadiDastgerdi

Fall 2022

Overview

Digital Communication

Advantages and Disadvantages

- ✓ Hardware stability.
- ✓ Operational flexibility.
- ✓ Reliable reproduction.
- ✓ Noise immunity (low Bit Error Rate (BER)).
- ✓ Different multiplexing techniques.
- ✗ Complex implementation.

Advantages and Disadvantages

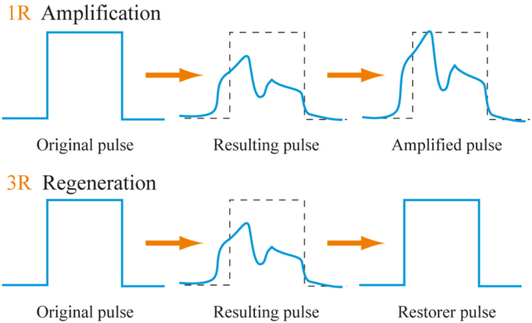


Figure: Regeneration versus amplification.

Advantages and Disadvantages

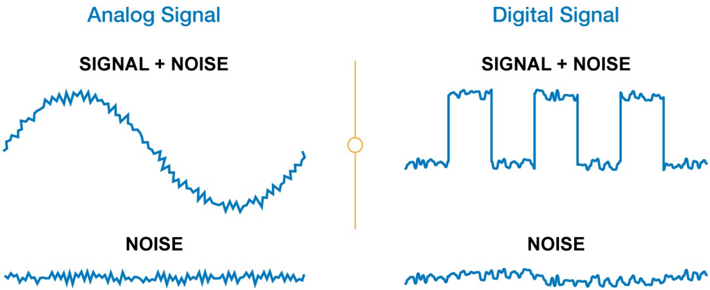


Figure: Noise immunity in digital communication.

Baseband and Passband Transmission

1. Baseband digital transmission
    1. Lowpass channel.
    2. Carrier-less.
    3. Usually short distance.
    4. Usually wired.
2. Passband digital transmission
    1. Bandpass channel.
    2. Carrier-oriented.
    3. Usually long distance.
    4. Usually wireless.

Baseband Digital Transmission

PAM

Statement (Digital Pulse Amplitude Modulation Signal)

A digital pulse amplitude modulation signal is expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$$

, where the kth symbol a_k belongs to a set of $M = 2^n$ levels and $p(t)$ is a pulse that satisfies the condition

$$p(KD) = \begin{cases} 1, & K = 0 \\ 0, & K = \pm 1, \pm 2, \cdots \end{cases}$$

- ✓ Clearly, $x(KD) = \sum_{k=-\infty}^{\infty} a_k p(KD - kD) = a_K$.
- ✓ The process of assigning pulses to the digital data is called line coding.

Baud Rate and Bit Rate

Definition (Baud Rate)

The baud or symbol rate of a PAM signal is defined as

$$r = \frac{1}{D}$$

Definition (Bit Rate)

The bit rate of a PAM signal is defined as

$$r_b = r \log_2(M) = \frac{\log_2(M)}{D} = \frac{n}{D} = rn$$

Line Codes

Binary PAM formats with rectangular pulses include

1. Unipolar return to zero with $p(t) = \sqcap(\frac{t}{D/2})$ and $a_k = b_k A$.
2. Unipolar nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and $a_k = b_k A$.
3. Polar return to zero with $p(t) = \sqcap(\frac{t}{D/2})$ and $a_k = (b_k - 0.5)A$.
4. Polar nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and $a_k = (b_k - 0.5)A$.
5. Twinned binary with $p(t) = \sqcap(\frac{t}{D/2}) - \sqcap(\frac{t-D/2}{D/2})$ and $a_k = (b_k - 0.5)A$.

✓ The formats differ in average DC value, power, power spectral density, bandwidth, and synchronization.

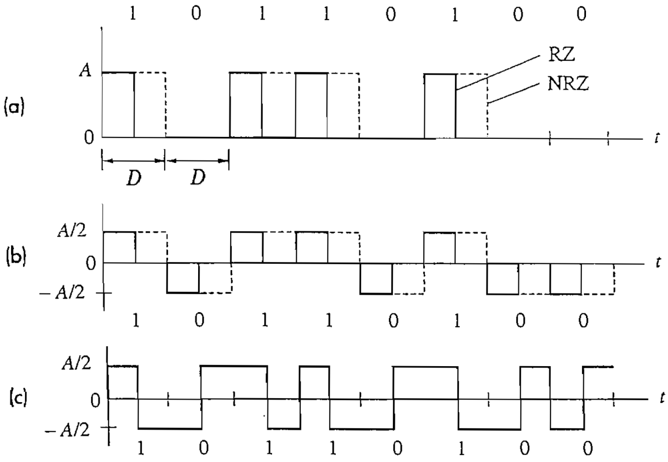


Figure: Binary PAM formats with rectangular pulses (line codes). (a) Unipolar RZ and NRZ (On-off RZ and NRZ) (b) Polar RZ and NRZ (c) Twined binary.

Polar quaternary nonreturn to zero with $p(t) = \sqcap(\frac{t}{D})$ and symbols a_k as

a_k	NBC Code	Gray Code
$3A/2$	11	10
$A/2$	10	11
$-A/2$	01	01
$-3A/2$	00	00

Table: Symbols in polar quaternary NRZ.

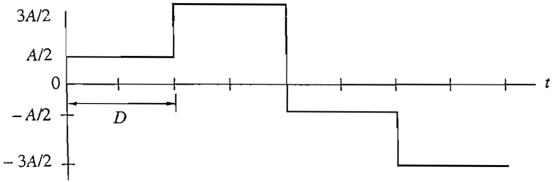


Figure: Polar quaternary NRZ.

Line Codes

A suitable line code should have

- Low bandwidth: Transmission bandwidth should be as small as possible.
- Power efficiency: For a given bandwidth and a specified detection error rate, the transmission power should be as low as possible.
- Favorable power spectral density: It is desirable to have zero power spectral density (PSD) at $f = 0$.
- Adequate timing content: It should be possible to extract timing or clock information from the signal.
- Transparency: It should be possible to correctly receive a digital signal regardless of the pattern of 1s and 0s.
- Error detection and correction capability: It is desirable to detect and preferably correct the detected errors.

PSD of PAM

Statement (Power Spectral Density of PAM)

Power spectral density of the pulse amplitude modulation signal $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$ *is*

$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n] e^{-j2\pi nfD}$$

, where $P(f)$ *is the Fourier transform of* $p(t)$ *and* $R_a[n] = E\{a_{n+k}a_k\}$ *is the autocorrelation of the stationary discrete random process* a_k*.*

✓ If a_k is a zero-mean uncorrelated discrete random process, $S_x(f) = \frac{R_a[0]}{D}|P(f)|^2$. ✓ Since $R_a[-n] = R_a[n]$ for real values, then $S_x(f) = \frac{|P(f)|^2}{D}[R_a[0] + 2\sum_{n=1}^{\infty} R_a[n]\cos(2\pi nfD)]$.

PSD of PAM

Statement (Power Spectral Density of PAM)

Power spectral density of the pulse amplitude modulation signal $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$ *is* $S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD}$*, where* $P(f)$ *is the Fourier transform of* $p(t)$ *and* $R_a[n] = E\{a_{n+k}a_k\}$ *is the auto-correlation of the stationary discrete random process* a_k*.*

$$S_x(f) = \lim_{T\to\infty} \frac{1}{T} E\{|X_T(f)|^2\}, \quad x_T(t) = x(t) \sqcap (\frac{t}{T})$$

$$x_T(t) = \sum_{k=-K}^{K} a_k p(t - kD) \Rightarrow X_T(f) = \sum_{k=-K}^{K} a_k P(f) e^{-j2\pi fkD}, \quad T = (2K+1)D, K \gg 1$$

$$|X_T(f)|^2 = X_T(f)X_T^*(f) = |P(f)|^2 \sum_{k=-K}^{K} a_k e^{-j2\pi fkD} \sum_{i=-K}^{K} a_i e^{-j2\pi fiD}$$

$$E\{|X_T(f)|^2\} = |P(f)|^2 \rho_K(f), \quad \rho_K(f) = \sum_{k=-K}^{K} \sum_{i=-K}^{K} E\{a_k a_i\} e^{-j2\pi f(k-i)D}$$

PSD of PAM

Statement (Power Spectral Density of PAM)

Power spectral density of the pulse amplitude modulation signal $x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$ *is* $S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD}$*, where* $P(f)$ *is the Fourier transform of* $p(t)$ *and* $R_a[n] = E\{a_{n+k}a_k\}$ *is the auto-correlation of the stationary discrete random process* a_k*.*

$$\rho_K(f) = \sum_{k=-K}^{K} \sum_{i=-K}^{K} E\{a_k a_i\} e^{-j2\pi f(k-i)D} = \sum_{k=-K}^{K} \sum_{i=-K}^{K} R_a[k-i] e^{-j2\pi f(k-i)D}, E\{a_k a_i\} = R_a[k-i]$$

$$\rho_K(f) = (2K+1) \sum_{n=-2K}^{2K} \left(1 - \frac{|n|}{2K+1}\right) R_a[n] e^{-j2\pi fnD}, \quad \lim_{n\to\infty} R_a[n] = 0$$

$$S_x(f) = \lim_{K\to\infty} \frac{1}{(2K+1)D} |P(f)|^2 \rho_K(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n] e^{-j2\pi nfD}$$

PSD of PAM

Example (PSD of rectangular NRZ polar line code)

Power spectral density of a rectangular NRZ polar code is $S_x(f) = \frac{DA^2}{4}\text{sinc}^2(Df)$ with the bandwidth r_b, average power $\frac{A^2}{4}$, nonzero PSD at $f = 0$, average DC value 0, and nontransparent synchronization.

$$R_a[0] = E\{a_k a_k\} = E\{(b_k - 0.5)^2 A^2\} = \frac{1}{2}\frac{A^2}{4} + \frac{1}{2}\frac{A^2}{4} = \frac{A^2}{4}$$

$$R_a[n] = E\{a_{n+k} a_k\} = E\{a_{n+k}\}E\{a_k\} = E\{(b_{n+k} - 0.5)A\}E\{(b_k - 0.5)A\} = 0, \quad n \neq 0$$

$$\mathcal{F}\{\text{rect}(\frac{t}{D})\} = D\text{sinc}(Df)$$

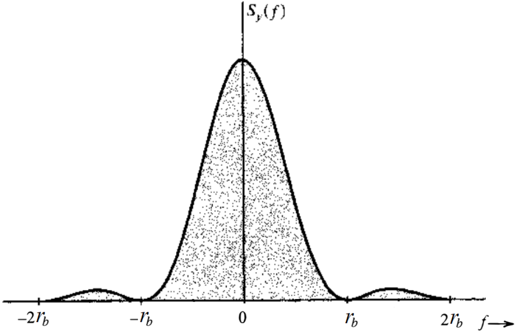
$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD} = \frac{1}{D}D^2\text{sinc}^2(Df)\frac{A^2}{4} = D\frac{A^2}{4}\text{sinc}^2(Df)$$

$$S_x(0) = D\frac{A^2}{4}, \quad \bar{P} = \frac{\frac{A^2}{4}D}{D} = \frac{A^2}{4}, \quad \text{DC} = E\{a_k\frac{D}{D}\} = 0$$

PSD of PAM

Example (PSD of rectangular NRZ polar line code)

Power spectral density of a rectangular NRZ polar code is $S_x(f) = \frac{DA^2}{4}\text{sinc}^2(Df)$ with the bandwidth r_b, average power $\frac{A^2}{4}$, nonzero PSD at $f = 0$, average DC value 0, and nontransparent synchronization.



PSD of PAM

Example (PSD of rectangular RZ polar line code)

Power spectral density of a rectangular RZ polar code is $S_x(f) = \frac{DA^2}{16}\text{sinc}^2(\frac{fD}{2})$ with the bandwidth $2r_b$, average power $\frac{A^2}{8}$, nonzero PSD at $f = 0$, average DC value 0, and transparent synchronization.

$$R_a[0] = E\{a_k a_k\} = E\{(b_k - 0.5)^2 A^2\} = \frac{1}{2}\frac{A^2}{4} + \frac{1}{2}\frac{A^2}{4} = \frac{A^2}{4}$$

$$R_a[n] = E\{a_{n+k} a_k\} = E\{a_{n+k}\}E\{a_k\} = E\{(b_{n+k} - 0.5)A\}E\{(b_k - 0.5)A\} = 0, \quad n \neq 0$$

$$\mathcal{F}\{\text{rect}(\frac{t}{D/2})\} = \frac{D}{2}\text{sinc}(\frac{D}{2}f)$$

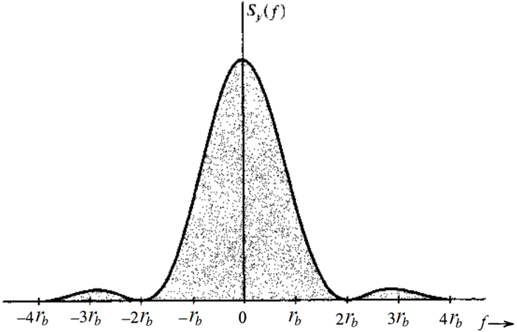
$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD} = \frac{1}{D}\frac{D^2}{4}\text{sinc}^2(\frac{D}{2}f)\frac{A^2}{4} = \frac{D}{4}\frac{A^2}{4}\text{sinc}^2(\frac{D}{2}f)$$

$$S_x(0) = \frac{D}{4}\frac{A^2}{4}, \quad \bar{P} = \frac{\frac{A^2}{4}\frac{D}{2}}{D} = \frac{A^2}{8}, \quad \text{DC} = E\{a_k \frac{D/2}{D}\} = 0$$

PSD of PAM

Example (PSD of rectangular RZ polar line code)

Power spectral density of a rectangular RZ polar code is $S_x(f) = \frac{DA^2}{16}\text{sinc}^2(\frac{fD}{2})$ with the bandwidth $2r_b$, average power $\frac{A^2}{8}$, nonzero PSD at $f = 0$, average DC value 0, and transparent synchronization.



PSD of PAM

Example (PSD of rectangular RZ unipolar line code)

Power spectral density of a rectangular RZ unipolar code is $S_x(f) = D\frac{A^2}{16}\text{sinc}^2(\frac{D}{2}f)[1 + \frac{1}{D}\sum_{n=-\infty}^{\infty}\delta(f - \frac{n}{D})]$ with the bandwidth $2r_b$, average power $\frac{A^2}{4}$, nonzero PSD at $f = 0$, average DC value $\frac{A}{4}$, and nontransparent synchronization.

$$R_a[0] = E\{a_k a_k\} = E\{b_k^2 A^2\} = \frac{1}{2}0 + \frac{1}{2}A^2 = \frac{A^2}{2}$$

$$R_a[n] = E\{a_{n+k} a_k\} = E\{a_{n+k}\}E\{a_k\} = E\{b_{n+k}A\}E\{b_k A\} = \frac{A^2}{4}, \quad n \neq 0$$

$$\mathcal{F}\{\text{rect}(\frac{t}{D/2})\} = \frac{D}{2}\text{sinc}(\frac{D}{2}f)$$

$$S_x(f) = \frac{1}{D}|P(f)|^2 \sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD} = \frac{1}{D}\frac{D^2}{4}\text{sinc}^2(\frac{D}{2}f)[\frac{A^2}{4} + \sum_{n=-\infty}^{\infty}\frac{A^2}{4}e^{-j2\pi nfD}]$$

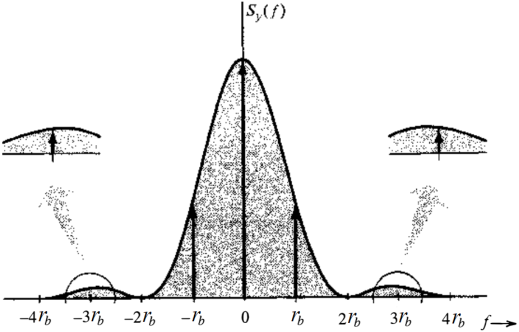
$$S_x(f) = D\frac{A^2}{16}\text{sinc}^2(\frac{D}{2}f)[1 + \frac{1}{D}\sum_{n=-\infty}^{\infty}\delta(f - \frac{n}{D})]$$

$$S_x(0) = D\frac{A^2}{16}[1 + \frac{1}{D}\delta(0)], \quad \bar{P} = \frac{\frac{1}{2}A^2\frac{D}{2} + \frac{1}{2}0}{D} = \frac{A^2}{4}, \quad \text{DC} = E\{a_k\frac{D/2}{D}\} = \frac{A}{4}$$

PSD of PAM

Example (PSD of rectangular RZ unipolar line code)

Power spectral density of a rectangular RZ unipolar code is $S_x(f) = D\frac{A^2}{16}\text{sinc}^2(\frac{D}{2}f)[1 + \frac{1}{D}\sum_{n=-\infty}^{\infty}\delta(f - \frac{n}{D})]$ with the bandwidth $2r_b$, average power $\frac{A^2}{4}$, nonzero PSD at $f = 0$, average DC value $\frac{A}{4}$, and nontransparent synchronization.

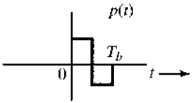


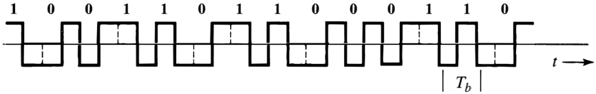
PSD of PAM

Example (DC null in PSD)

$\int_{-\infty}^{+\infty} p(t)dt = 0$ forces a DC null in the PSD of a line code.

$$S_x(0) = \frac{1}{D}|P(0)|^2 \sum_{n=-\infty}^{\infty} R_a[n] = 0 \Rightarrow P(0) = \int_{-\infty}^{+\infty} p(t)dt = 0$$

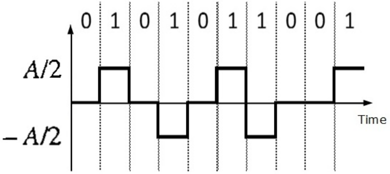




PSD of PAM

Example (Bipolar Line Code)

Bipolar line coding is used for PCM transmission in telephone networks. In bipolar line coding, a 0 is transmitted by no pulse, and a 1 is transmitted by alternating between $p(t)$ and $-p(t)$, depending on whether the previous 1 uses a $-p(t)$ or $p(t)$. Bipolar line coding is nontransparent and is capable of error detection or error correction. In Bipolar line coding, the consecutive symbols are dependent.



PSD of PAM

Example (Precoding in bipolar line code)

In bipolar line coding, the statistics of the amplitude sequences a_k are intelligently changed to create a DC null in the PSD $\frac{A^2D}{4}\text{sinc}^2(Df)\sin^2(\pi fD)$.

$$R_a[0] = E\{a_k a_k\} = \frac{1}{2}0 + \frac{1}{4}(\frac{A}{2})^2 + \frac{1}{4}(\frac{-A}{2})^2 = \frac{A^2}{8}$$

$$R_a[1] = E\{a_{k+1}a_k\} = \frac{1}{4}\times 0\times 0 + \frac{1}{4}\times 0\times\frac{\pm A}{2} + \frac{1}{4}\times\frac{\pm A}{2}\times 0 + \frac{1}{4}\times\frac{A}{2}\times\frac{-A}{2} = -\frac{A^2}{16}$$

$$R_a[2] = E\{a_{k+2}a_k\} = \frac{1}{8}\times\frac{A}{2}\times\frac{A}{2} + \frac{1}{8}\times\frac{A}{2}\times\frac{-A}{2} + \frac{6}{8}\times 0 = 0$$

$$R_a[n] = 0, \quad n \geq 2$$

$$\mathcal{F}\{\text{rect}(\frac{t}{D})\} = D\text{sinc}(Df)$$

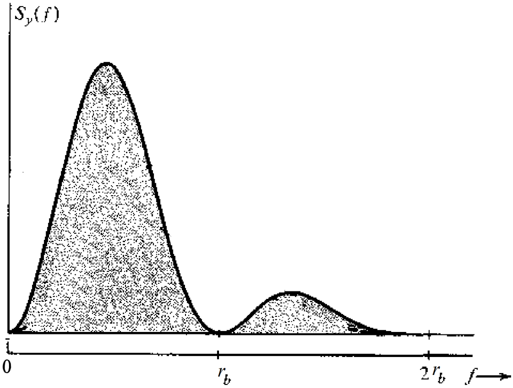
$$S_x(f) = \frac{|P(f)|^2}{D}[R_a[0] + 2\sum_{n=1}^{\infty} R_a[n]\cos(2\pi nfD)] = \frac{1}{D}D^2\text{sinc}^2(Df)[\frac{A^2}{8} - \frac{A^2}{8}\cos(2\pi fD)]$$

$$S_x(f) = \frac{A^2D}{4}\text{sinc}^2(Df)\sin^2(\pi fD) \Rightarrow S_x(0) = 0$$

PSD of PAM

Example (Precoding in bipolar line code)

In bipolar line coding, the statistics of the amplitude sequences a_k are intelligently changed to create a DC null in the PSD $\frac{A^2 D}{4}\mathrm{sinc}^2(Df)\sin^2(\pi f D)$.



Transmission Limitations

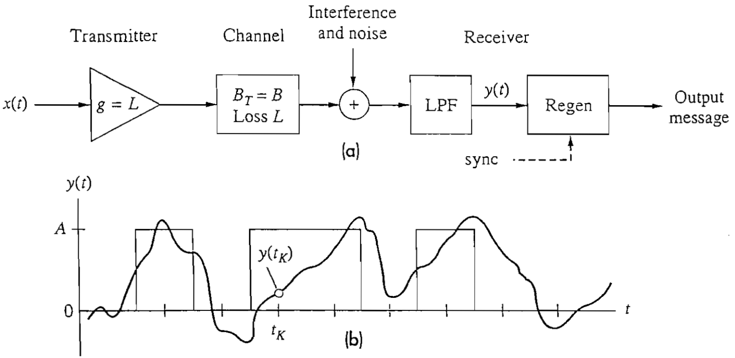


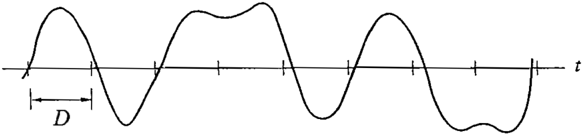
Figure: (a) Baseband transmission system (b) Signal-pluse-noise waveform.

$$y(t) = \sum_{k=-\infty}^{\infty} a_k \tilde{p}(t - t_d - kD) + n(t), \quad \tilde{p}(t) = p(t) * h_c(t) * h_{LPF}(t)$$

$$y(t_K) = y(KD + t_d + t_s) = a_K \tilde{p}(t_s) + \sum_{k \neq K}^{\infty} a_k \tilde{p}(KD + t_s - kD) + n(t_K)$$

Eye Diagram

$$y(t_K) = a_K \underbrace{\tilde{p}(t_s)}_{\text{Synchronization Mismatch}} + \underbrace{\sum_{k \neq K}^{\infty} a_k \tilde{p}(KD + t_s - kD)}_{\text{Inter-Symbol Interference (ISI)}} + \underbrace{n(t_K)}_{\text{Noise}}$$



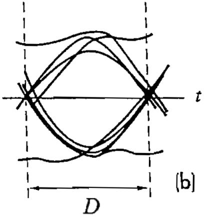


Figure: (a) Distorted polar binary signal (b) Eye diagram. To reduce the noise, the bandwidth of the LPF should be decreased, which in tern broadens the signal and increases ISI.

Eye Diagram

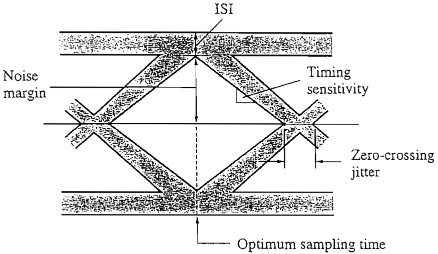


Figure: Binary eye pattern. The eye diagram is used for experimental adjustment of the receiver.

1. Inter-symbol interference
2. Optimum sampling time
3. Zero-crossing jitter
4. Noise margin
5. Timing sensitivity
6. Nonlinear distortion

Statement (Nyquist's Criterion for ISI Cancellation)

Given an ideal lowpass channel of bandwidth B, it is possible to transmit independent symbols at a rate $r \leq 2B$ baud without ISI. It is not possible to transmit independent symbols at $r > 2B$.

✓ Signaling at the maximum rate $r = 2B$ requires sinc pulse shaping $p(t) = \text{sinc}(rt)$.

✓ For the sinc pulse shaping $p(t) = \text{sinc}(rt)$, $P(f) = \frac{1}{r}\sqcap(\frac{f}{r}) = \frac{1}{2B}\sqcap(\frac{f}{2B})$, which suffers no distortion from ideal LPF with bandwidth B.

✓ Although the sinc pulse is not time-limited, it satisfies the zero-ISI conditions $p(0) = 1$ and $p(\pm kD) = 0, k = 1, 2, \cdots$.

✗ The sinc pulse falls off no faster than $\frac{1}{|t|}$ as $|t| \to \infty$. So, a small synchronization mismatch ϵ results in large ISI as $y(t_K) = a_K\text{sinc}(r\epsilon) + \sum_{k\neq K} a_k\text{sinc}(KD - kD + r\epsilon)$.

ISI

Statement (Nyquist's Pulse Shaping)

Nyquist pulse $p(t) = p_\beta(t)sinc(rt)$, where $P_\beta(f) = 0, |f| > \beta$, $0 \leq \beta \leq \frac{r}{2}$, and $p_\beta(0) = 1$, occupies the bandwidth $B = \frac{r}{2} + \beta$, satisfies the zeros ISI conditions $p(0) = 1$ and $p(\pm kD) = 0, k = 1, 2, \cdots$, and can be manipulated to reduce synchronization issues.

✓ The Nyquist pulse provides the baud rate $r = 2(B - \beta) \Rightarrow B \leq r \leq 2B$.

✓ For the Nyquist pulse, $P(f) = P_\beta(f) * [\frac{1}{r} \sqcap (\frac{f}{r})]$ requires the bandwidth $B = \frac{r}{2} + \beta$.

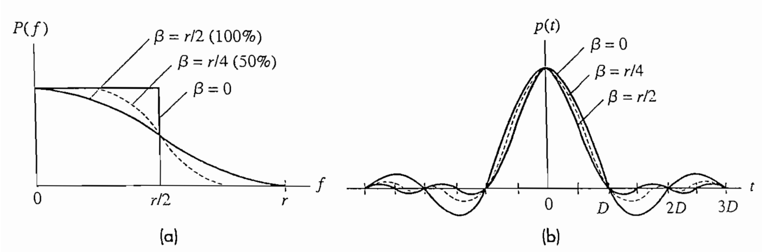


Figure: Nyquist pulse shaping (a) spectra (b) waveform. When the rolloff factor $\beta > 0$, the spectrum has a smooth rolloff and the leading and trailing oscillations decay more rapidly.

$$p_\beta(t) = \frac{\cos(2\pi\beta t)}{1-(4\beta t)^2} \Rightarrow P_\beta(f) = \frac{\pi}{4\beta}\cos(\frac{\pi f}{2\beta})\sqcap(\frac{f}{2\beta})$$

$$P(f) = \begin{cases} \frac{1}{r}, & |f| < \frac{r}{2}-\beta \\ \frac{1}{r}\cos^2\left(\frac{\pi}{4\beta}(|f|-\frac{r}{2}+\beta)\right), & \frac{r}{2}-\beta < |f| < \frac{r}{2}+\beta \\ 0, & |f| > \frac{r}{2}+\beta \end{cases}$$

$$p(t) = \frac{\cos(2\pi\beta t)}{1-(4\beta t)^2}\operatorname{sinc}(rt)$$

ISI

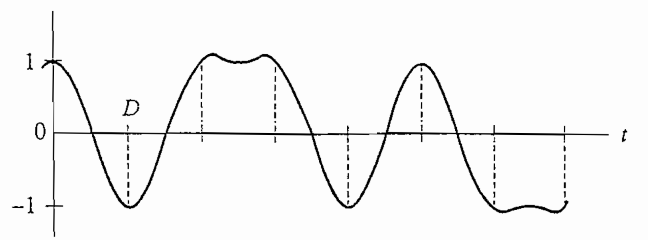


Figure: Binary polar raised cosine (100 percent rolloff) signaling for the bit sequence 10110100. The half-amplitude width of this pulse is exactly the symbol interval D, i.e., $p(\pm 0.5D) = 0.5$ and there are additional zero-crossings at $t = \pm 1.5D, \pm 2.5D, \cdots$ that facilitate synchronization.

$$p(t) = \frac{\mathrm{sinc}(2rt)}{1-(2rt)^2}, \quad P(f) = \frac{1}{r}\cos^2\left(\frac{\pi f}{2r}\right), \quad |f| < r$$

Synchronization

Statement (Suitable Synchronization)

A suitable synchronization can mitigate or eliminate synchronization mismatch. Zero-crossing in PAM signal has a key role in synchronization.

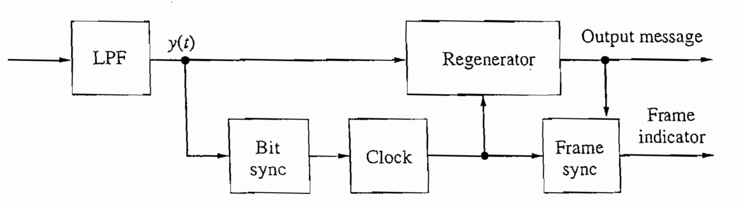


Figure: Synchronization in a binary receiver. A digital receiver requires different types of synchronization including carrier synchronization, bit synchronization, and frame synchronization.

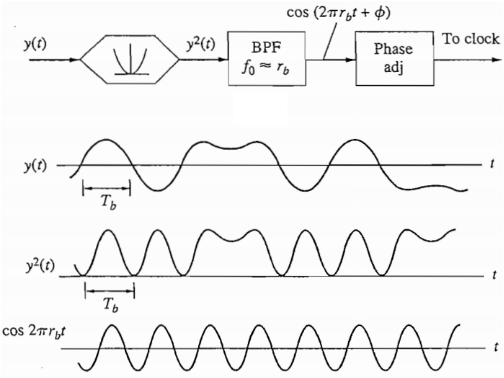


Figure: Open-loop bit synchronization. Open-loop bit synchronization is almost trivial for unipolar whose spectrum includes $\delta(f \pm r_b)$. This approach can be also used for polar line codes if the signal is first processed by a square-law device. Non-transparency of the line code can degrade the performance of the synchronization.

Synchronization

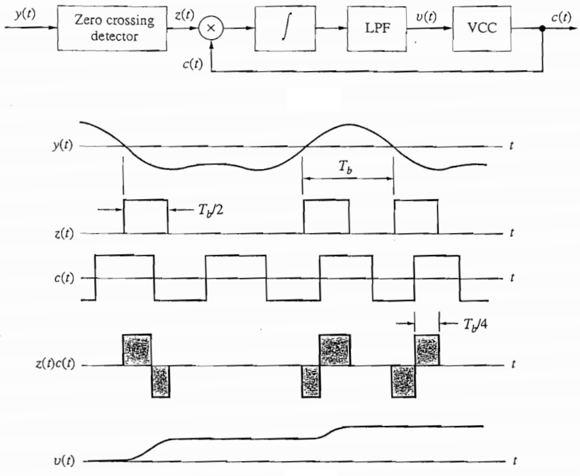


Figure: Closed-loop bit synchronization. Here, the zero-crossings of the line code are used for synchronization. Non-transparency of the line code can degrade the performance of the synchronization.

Synchronization

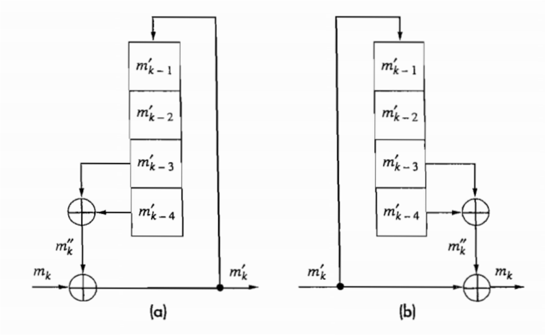


Figure: (a) Binary scrambler (b) binary unscrambler. Scrambling is a coding operation applied to the message that randomizes the bit stream to eliminate long strings of like bits. Scramblers are implemented using Linear Feedback Shift Registers (LFSRs).

Bit Error Probability

Statement (Bit Error Probability of Unipolar NRZ)

Assuming perfect ISI cancellation and synchronization, the bit error probability for unipolar NRZ binary signaling in zero-mean Gaussian noise with variance σ^2 *is* $Pe = Q(A/(2\sigma))$ *, where* $Q(x)$ *is the tail distribution function of the standard normal distribution, and* 0 *and* A *are the symbols corresponding to the equally-probable binary digits* 0 *and* 1*.*

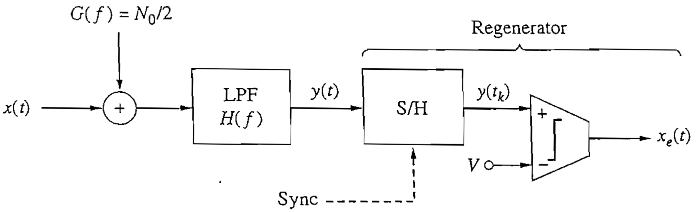
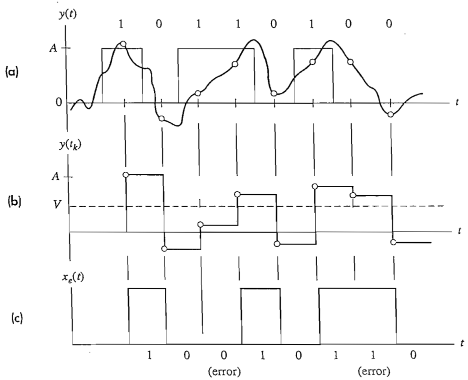


Figure: Baseband binary receiver.

Statement (Bit Error Probability of Unipolar NRZ)

Assuming perfect ISI cancellation and synchronization, the bit error probability for unipolar NRZ binary signaling is $Pe = Q(A/(2\sigma))$.



Bit Error Probability

Statement (Bit Error Probability of Unipolar NRZ)

Assuming perfect ISI cancellation and synchronization, the bit error probability for unipolar NRZ binary signaling is $Pe = Q(A/(2\sigma))$.

$$
\begin{aligned}
P_e &= P_0 P_{e|0} + P_1 P_{e|1} = \frac{1}{2}(P_{e|0} + P_{e|1}) \\
&= \frac{1}{2}(P[y(t_K) > V | a_K = 0] + P[y(t_K) \leq V | a_K = A]) \\
&= \frac{1}{2}(P[n(t_K) > V | a_K = 0] + P[n(t_K) + A \leq V | a_K = A]) \\
&= \frac{1}{2}(Q(\frac{V}{\sigma}) + Q(\frac{A-V}{\sigma})), \quad Q(x) = P\{X > x\} = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt
\end{aligned}
$$

$$
\frac{dP_e}{dV} = 0 \Rightarrow V = \frac{A}{2} \Rightarrow P_{e_{\min}} = Q(\frac{A}{2\sigma})
$$

Passband Digital Transmission

Transmission Techniques

Common passband digital transmission techniques

1. Amplitude Shift Keying (ASK)
2. Phase Shift Keying (PSK)
3. Frequency Shift Keying (FSK)

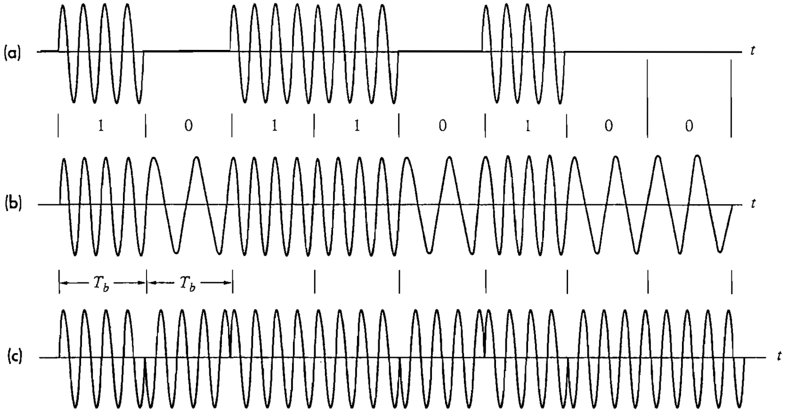


Figure: Binary modulated waveforms (a) ASK (b) FSK (c) PSK.

Passband Digital Signal

Statement (Passband Digital Signal)

Any modulated passband signal may be expressed in the quadrature-carrier form

$$x(t) = A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)]$$

. The carrier frequency f_c, amplitude A_c, and phase θ are constant, while the time-varying in-phase $x_i(t)$ and quadrature $x_q(t)$ components contain the message.

PSD of Passband Digital Signal

Statement (PSD of Passband Digital Signal)

If the in-phase $x_i(t)$ *and quadrature* $x_q(t)$ *components are statistically independent signals and at least one has zero mean, the power spectral density of the passband digital signal* $x(t) = A_c[x_i(t)\cos(2\pi f_c t+\theta) - x_q(t)\sin(2\pi f_c t+\theta)]$ *is*

$$S_x(f) = \frac{A_c^2}{4}[S_{x_i}(f-f_c) + S_{x_i}(f+f_c) + S_{x_q}(f-f_c) + S_{x_q}(f+f_c)]$$

, where $S_{x_i}(f)$ *and* $S_{x_q}(f)$ *are the power spectral density of the in-phase and quadrature components, respectively.*

✓ Defining the equivalent lowpass spectrum as $S_{lp}(f) = S_{x_i}(f) + S_{x_q}(f)$, then $S_x(f) = \frac{A_c^2}{4}[S_{lp}(f-f_c) + S_{lp}(f+f_c)]$.

ASK

Statement (ASK)

In ASK,

$$x_i(t) = \sum_k a_k p(t - kD), \quad x_q(t) = 0$$

$$a_k = 0, 1, \cdots, M - 1$$

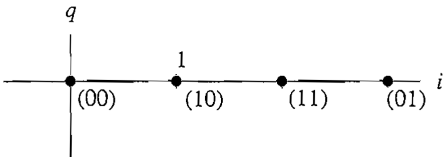


Figure: ASK constellation.

✓ The binary ASK is called on-off keying (OOK).

Example (PSD of ASK)

For ASK, the equivalent lowpass spectrum is $S_{lp}(f) = \frac{M^2-1}{12r}\text{sinc}^2(\frac{f}{r}) + \frac{(M-1)^2}{4}\delta(f)$ with effective bandwidth of $B_T \approx r$ and spectral efficiency $r_b/B_T \approx \log_2(M)$.

$$R_a[0] = E\{a_k a_k\} = \sum_{i=0}^{M-1} \frac{1}{M} i^2 = \frac{1}{M}\frac{(M-1)M(2M-1)}{6} = \frac{(M-1)(2M-1)}{6}$$

$$R_a[n] = E\{a_{n+k} a_k\} = E\{a_{n+k}\}E\{a_k\} = (\sum_{i=0}^{M-1} \frac{1}{M} i)^2 = (\frac{M(M-1)}{2M})^2 = \frac{(M-1)^2}{4}, \quad n \neq 0$$

$$\mathcal{F}\{\text{rect}(\frac{t}{D})\} = D\text{sinc}(Df)$$

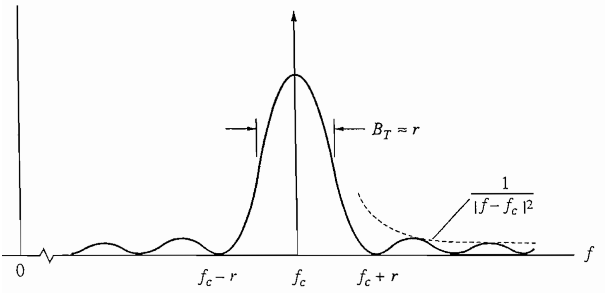
$$S_{x_i}(f) = \frac{|P(f)|^2}{D}\sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD} = D\text{sinc}^2(Df)[\frac{(M-1)(2M-1)}{6} + \frac{(M-1)^2}{4}\sum_{n\neq 0} e^{-j2\pi nfD}]$$

$$S_{lp}(f) = D\text{sinc}^2(Df)[\frac{M^2-1}{12} + \frac{(M-1)^2}{4D}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{D})] = \frac{M^2-1}{12r}\text{sinc}^2(\frac{f}{r}) + \frac{(M-1)^2}{4}\delta(f)$$

ASK

Example (PSD of ASK)

For ASK, the equivalent lowpass spectrum is $S_{lp}(f) = \frac{M^2-1}{12r}\text{sinc}^2(\frac{f}{r}) + \frac{(M-1)^2}{4}\delta(f)$ with effective bandwidth of $B_T \approx r$ and spectral efficiency $r_b/B_T \approx \log_2(M)$.



Statement (ASK Modulator)

An ASK modulator can be directly implemented or can be indirectly implemented by driving an analog amplitude modulator using a suitable line code signal.

ASK Generation method

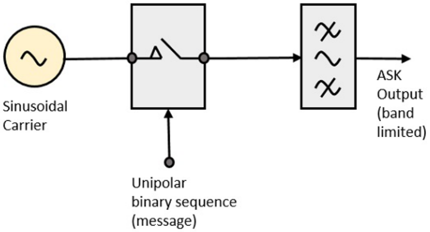


Figure: Binary ASK (BASK) modulator.

ASK

Statement (ASK Demodulator)

An ASK demodulator can be directly implemented or can be indirectly implemented by extracting a suitable line code signal from an analog amplitude demodulator.

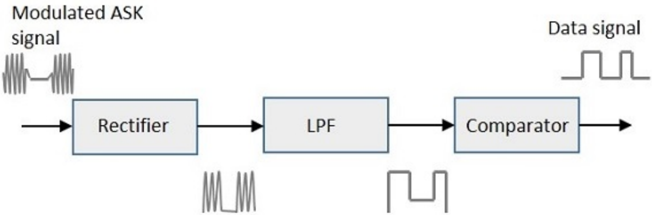


Figure: Binary ASK (BASK) demodulator.

PSK

Statement (PSK)

In PSK,

$$x_i(t) = \sum_k \cos(\phi_k) p(t - kD), \quad x_q(t) = \sum_k \sin(\phi_k) p(t - kD)$$

$$\phi_k = \pi(2a_k + 1)/M, \quad a_k = 0, 1, \cdots, M - 1$$

$$\textit{or} \quad \phi_k = \pi(2a_k)/M, \quad a_k = 0, 1, \cdots, M - 1$$

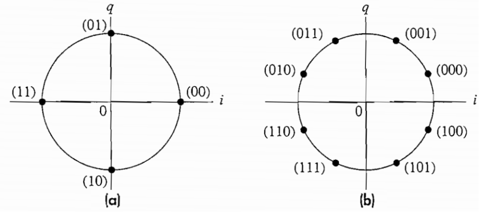


Figure: PSK constellation for (a) $M = 4$ and (b) $M = 8$.

PSK

Statement (PSK)

In PSK,

$$x_i(t) = \sum_k \cos(\phi_k)p(t - kD), \quad x_q(t) = \sum_k \sin(\phi_k)p(t - kD)$$

$$\phi_k = \pi(2a_k + 1)/M, \quad a_k = 0, 1, \cdots, M - 1$$

$$\text{or} \quad \phi_k = \pi(2a_k)/M, \quad a_k = 0, 1, \cdots, M - 1$$

$$\begin{aligned} x(t) &= A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)] \\ &= A_c[\cos(2\pi f_c t + \theta)\sum_k \cos(\phi_k)p(t - kD) - \sin(2\pi f_c t + \theta)\sum_k \sin(\phi_k)p(t - kD)] \\ &= A_c \sum_k \cos(2\pi f_c t + \theta + \phi_k)p(t - kD) \end{aligned}$$

PSK

Statement (PSD of PSK)

For PSK, the equivalent lowpass spectrum is $S_{lp}(f) = \frac{1}{r} sinc^2(\frac{f}{r})$ with effective bandwidth of $B_T \approx r$ and spectral efficiency $r_b/B_T \approx \log_2(M)$.

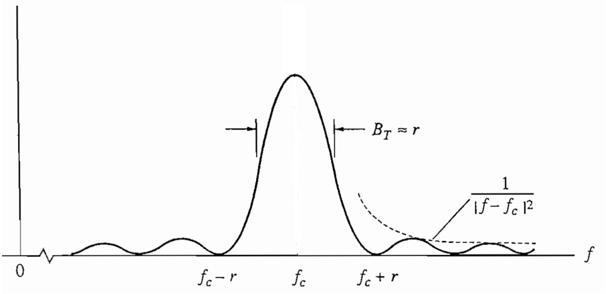


Figure: PSD of PSK.

Statement (PSK Modulator)

A PSK modulator can be directly implemented or can be indirectly implemented by driving an analog PM modulator using a suitable line code signal.

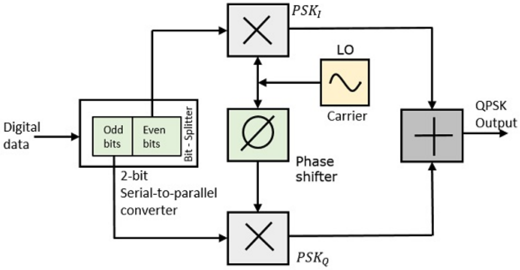


Figure: Quadrature PSK (QPSK) modulator.

PSK

Statement (PSK Demodulator)

A PSK demodulator can be directly implemented or can be indirectly implemented by extracting a suitable line code signal from an analog PM demodulator.

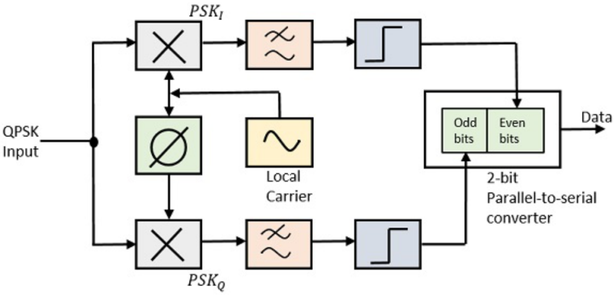


Figure: Quadrature PSK (QPSK) demodulator.

FSK

Statement (FSK)

In FSK,

$$x_i(t) = \sum_k \cos(2\pi f_d a_k t) p(t - kD), \quad x_q(t) = \sum_k \sin(2\pi f_d a_k t) p(t - kD)$$

$$a_k = \pm 1, \pm 3 \cdots, \pm(M-1)$$

$$\begin{aligned}
x(t) &= A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)] \\
&= A_c[\cos(2\pi f_c t + \theta)\sum_k \cos(2\pi f_d a_k t)p(t - kD) - \sin(2\pi f_c t + \theta)\sum_k \sin(2\pi f_d a_k t)p(t - kD)] \\
&= A_c \sum_k \cos(2\pi f_c t + \theta + 2\pi f_d a_k t)p(t - kD) \\
&= A_c \sum_k \cos(2\pi(f_c + f_d a_k)t + \theta)p(t - kD)
\end{aligned}$$

FSK

Statement (PSD of Sunde's FSK)

For Binary FSK known as Sunde's FSK, the equivalent lowpass spectrum is $S_{lp}(f) = \frac{4}{\pi^2 r_b}\left[\frac{\cos(\frac{\pi f}{r_b})}{(\frac{2f}{r_b})^2 - 1}\right]^2 + \frac{1}{4}\left[\delta(f - \frac{r_b}{2}) + \delta(f + \frac{r_b}{2})\right]$ *with effective bandwidth of* $B_T \approx r_b$ *and spectral efficiency* $r_b/B_T \approx 1$.

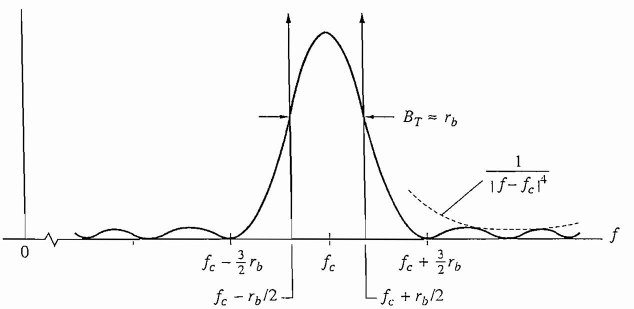


Figure: PSD of Binary FSK.

FSK

Statement (FSK Modulator)

A FSK modulator can be directly implemented or can be indirectly implemented by driving an analog FM modulator using a suitable line code signal.

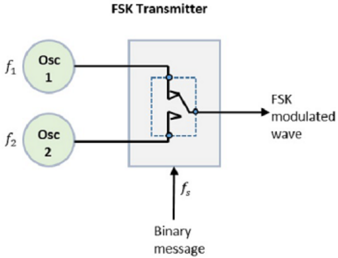


Figure: Binary FSK (BFSK) modulator.

Statement (Synchronous FSK Demodulator)

A FSK demodulator can be directly implemented or can be indirectly implemented by driving an analog FM demodulator using a suitable line code signal.

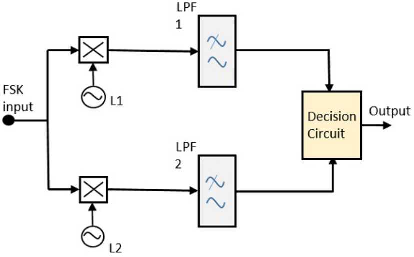


Figure: Binary FSK (BFSK) synchronous demodulator.

FSK

Statement (Asynchronous FSK Demodulator)

A FSK demodulator can be directly implemented or can be indirectly implemented by extracting a suitable line code signal from an analog FM demodulator.

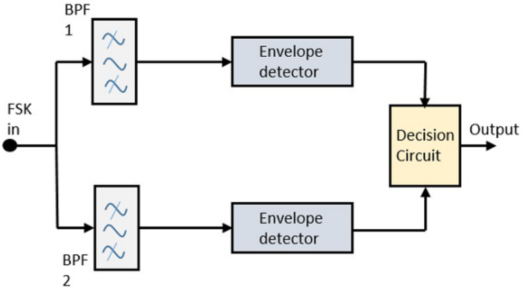


Figure: Binary FSK (BFSK) asynchronous demodulator.

The End