MATHEMATICAL QUESTIONS

Question 1

Show that if the in-phase $x_i(t)$ and quadrature $x_q(t)$ components of the passband digital signal

$$x(t) = A_c[x_i(t)\cos(2\pi f_c t + \theta) - x_q(t)\sin(2\pi f_c t + \theta)]$$

are statistically independent and at least one has zero mean, the corresponding power spectral density equals

$$S_x(f) = \frac{A_c^2}{4}[S_{x_i}(f - f_c) + S_{x_i}(f + f_c) + S_{x_q}(f - f_c) + S_{x_q}(f + f_c)]$$

, where $S_{x_i}(f)$ and $S_{x_q}(f)$ are the power spectral densities of the in-phase and quadrature components, respectively.

$$R_x(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x(t)x^*(t-\tau)\}dt$$

$$= \lim_{T\to\infty}\frac{A_c^2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)\cos(2\pi f_c t+\theta)x_i^*(t-\tau)\cos(2\pi f_c(t-\tau)+\theta)\}dt$$

$$- \lim_{T\to\infty}\frac{A_c^2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)\cos(2\pi f_c t+\theta)x_q^*(t-\tau)\sin(2\pi f_c(t-\tau)+\theta)\}dt$$

$$- \lim_{T\to\infty}\frac{A_c^2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)\sin(2\pi f_c t+\theta)x_i^*(t-\tau)\cos(2\pi f_c(t-\tau)+\theta)\}dt$$

$$+ \lim_{T\to\infty}\frac{A_c^2}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)\sin(2\pi f_c t+\theta)x_q^*(t-\tau)\sin(2\pi f_c(t-\tau)+\theta)\}dt$$

1st term:

$$\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_i^*(t-\tau)\}(\cos(2\pi f_c\tau)+\cos(4\pi f_c t-2\pi f_c\tau+2\theta))dt$$

$$= \lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_i^*(t-\tau)\}\cos(2\pi f_c\tau)dt$$

$$+\underbrace{\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_i^*(t-\tau)\}\cos(4\pi f_c t-2\pi f_c\tau+2\theta)dt}_{=0} = \frac{A_c^2}{2}\cos(2\pi f_c\tau)R_{x_i}(\tau)$$

2nd term:

$$\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_q^*(t-\tau)\}(-\sin(2\pi f_c\tau)+\sin(4\pi f_c t-2\pi f_c\tau+2\theta))dt$$

Question 1 continued on next page...

$$= -\frac{A_c^2}{2}\sin(2\pi f_c\tau)\underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_q^*(t-\tau)\}dt}_{=R_{x_ix_q}(\tau)}$$

$$+\underbrace{\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_q^*(t-\tau)\}\sin(4\pi f_ct-2\pi f_c\tau+2\theta)dt}_{=0} = -\frac{A_c^2}{2}\sin(2\pi f_c\tau)R_{x_ix_q}(\tau)$$

3rd term:

$$= \lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)x_i^*(t-\tau)\}(\sin(2\pi f_c\tau)+\sin(4\pi f_ct-2\pi f_c\tau+2\theta))dt$$

$$= \frac{A_c^2}{2}\sin(2\pi f_c\tau)\underbrace{\lim_{T\to\infty}\frac{1}{T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)x_i^*(t-\tau)\}dt}_{=R_{x_qx_i}(\tau)}$$

$$+\underbrace{\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)x_i^*(t-\tau)\}\sin(4\pi f_ct-2\pi f_c\tau+2\theta)dt}_{=0} = \frac{A_c^2}{2}\sin(2\pi f_c\tau)R_{x_qx_i}(\tau)$$

4th term:

$$= \lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_i(t)x_i^*(t-\tau)\}(\cos(2\pi f_c\tau)+\cos(4\pi f_ct-2\pi f_c\tau+2\theta))dt$$

$$= \lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)x_q^*(t-\tau)\}\cos(2\pi f_c\tau)dt$$

$$+\underbrace{\lim_{T\to\infty}\frac{A_c^2}{2T}\int_{\frac{-T}{2}}^{\frac{T}{2}}\mathcal{E}\{x_q(t)x_q^*(t-\tau)\}\cos(4\pi f_ct-2\pi f_c\tau+2\theta)dt}_{=0} = \frac{A_c^2}{2}\cos(2\pi f_c\tau)R_{x_q}(\tau)$$

$R_x(\tau)$ equals sum of these 4 terms and because $x_i(t)$ and $x_q(t)$ are statistically independent and at least one has zero-mean, then $R_{x_q,x_i}(\tau) = R_{x_i,x_q}(\tau) = 0$. So,

$$\Rightarrow R_x(\tau) = \frac{A_c^2}{2}\cos(2\pi f_c\tau)(R_{x_i}(\tau)+R_{x_q}(\tau))$$

$$\Rightarrow S_x(f) = F\{R_x(\tau)\} = \frac{A_c^2}{2}F\{\cos(2\pi f_c\tau)(R_{x_i}(\tau)+R_{x_q}(\tau))\}$$

$$= \frac{A_c^2}{4}[S_{x_i}(f-f_c)+S_{x_i}(f+f_c)+S_{x_q}(f-f_c)+S_{x_q}(f+f_c)]$$

Question 2

Calculate the bit error rate of the binary polar NRZ signaling with amplitudes $\pm\frac{A}{2}$ polluted by a zero-mean Gaussian noise with variance σ^2. Assume that ISI is perfectly canceled, no syn-

Question 2 continued on next page...

chronization mismatch is imposed, transmitted bits are equally-probable, and the decision threshold is V. Find the optimum value of the decision threshold and the corresponding minimum bit error rate.

$$\begin{aligned}
P_e =& P_0 P_{e|0} + P_1 P_{e|1} = \frac{1}{2}(P_{e|0} + P_{e|1}) \\
=& \frac{1}{2}\Big(P\big(y(t_k) > V\big|a_k = -\frac{A}{2}\big) + P\big(y(t_k) \leq V\big|a_k = \frac{A}{2}\big)\Big) \\
=& \frac{1}{2}\Big(P\big(n(t_k) - \frac{A}{2} > V\big|a_k = -\frac{A}{2}\big) + P\big(n(t_k) + \frac{A}{2} \leq V\big|a_k = \frac{A}{2}\big)\Big) \\
=& \frac{1}{2}\Big(Q\big(\frac{V + \frac{A}{2}}{\sigma}\big) + Q\big(\frac{-V + \frac{A}{2}}{\sigma}\big)\Big), \qquad Q(x) = P(X > x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt
\end{aligned}$$

$$\begin{aligned}
\frac{dP_e}{dV} = 0 &\to \frac{1}{2}\Big(\frac{d}{dV}Q\big(\frac{V + \frac{A}{2}}{\sigma}\big) + \frac{d}{dV}Q\big(\frac{-V + \frac{A}{2}}{\sigma}\big)\Big) = 0 \\
&\to \frac{1}{2\sigma}\Big(Q'\big(\frac{V + \frac{A}{2}}{\sigma}\big) - Q'\big(\frac{-V + \frac{A}{2}}{\sigma}\big)\Big) = 0, \qquad Q'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \\
&\to \frac{1}{2\sigma}\Big(\frac{1}{\sqrt{2\pi}} e^{-\frac{(\frac{V + \frac{A}{2}}{\sigma})^2}{2}} - \frac{1}{\sqrt{2\pi}} e^{-\frac{(\frac{-V + \frac{A}{2}}{\sigma})^2}{2}}\Big) = 0 \\
&\to \frac{1}{2\sigma\sqrt{2\pi}}\Big(e^{-\frac{(\frac{V + \frac{A}{2}}{\sigma})^2}{2}} - e^{-\frac{(\frac{-V + \frac{A}{2}}{\sigma})^2}{2}}\Big) = 0 \\
&\to \frac{1}{2\sigma\sqrt{2\pi}}\Big(e^{-\frac{A^2}{8\sigma^2}} e^{-\frac{V^2}{2\sigma^2}}\Big(e^{-\frac{AV}{2\sigma^2}} - e^{\frac{AV}{2\sigma^2}}\Big)\Big) = 0 \\
&\to e^{-\frac{AV}{2\sigma^2}} - e^{\frac{AV}{2\sigma^2}} = 0 \to -\frac{AV}{2\sigma^2} = \frac{AV}{2\sigma^2} \to -V = V \to V = 0 \\
&\Rightarrow P_{e_{min}} = Q\big(\frac{A}{2\sigma}\big)
\end{aligned}$$

Question 3

Calculate the power spectral density of an Mary twined line coding with the pulse

$$p(t) = \frac{A}{2} \sqcap \big(\frac{t}{D/2}\big) - \frac{A}{2} \sqcap \big(\frac{t - D/2}{D/2}\big)$$

and levels $a_k = \pm 1, \pm 3, \cdots, \pm(M - 1)$. Find the corresponding bandwidth, average power, average DC, power spectral density at zero frequency, baud rate, and bit rate.

$$\begin{aligned}
P(f) &= \frac{AD}{4} \operatorname{sinc}\big(\frac{D}{2} f\big)(1 - e^{-j\pi f D}) = \frac{AD}{2} \operatorname{sinc}\big(\frac{D}{2} f\big) e^{-j\pi f \frac{D}{2}} \frac{e^{j\pi f \frac{D}{2}} - e^{-j\pi f \frac{D}{2}}}{2} \\
&= \frac{AD}{2} j \operatorname{sinc}\big(\frac{D}{2} f\big) e^{-j\pi f \frac{D}{2}} \sin\big(\pi f \frac{D}{2}\big)
\end{aligned}$$

Question 3 continued on next page. . .

$$\Rightarrow |P(f)| = \frac{AD}{2}\operatorname{sinc}(\frac{D}{2}f)\sin(\pi f\frac{D}{2}) = \frac{AD^2\pi f}{4}\operatorname{sinc}^2(f\frac{D}{2})$$

$$R_a[0] = E\{a_k a_k\} = \sum_k \frac{1}{M}a_k^2 = \frac{2}{M}\sum_{k=1}^{\frac{M}{2}}(2k-1)^2 = \frac{2}{M}(\frac{M}{6}(M-1)(M+1)) = \frac{M^2-1}{3}$$

$$R_a[n] = E\{a_{n+k}a_k\} = E\{a_{n+k}\}E\{a_k\} = (\sum_k \frac{1}{M}a_k)^2 = 0$$

$$\Rightarrow S_x(f) = \frac{|P(f)|^2}{D}\sum_{n=-\infty}^{\infty} R_a[n]e^{-j2\pi nfD} = \frac{A^2D^3\pi^2f^2}{16}\operatorname{sinc}^4(f\frac{D}{2})\frac{M^2-1}{3}$$

$$\Rightarrow S_x(f) = \frac{M^2-1}{48}A^2D^3\pi^2f^2\operatorname{sinc}^4(f\frac{D}{2})$$

Baud rate:

$$r = \frac{1}{D}$$

Bit rate:

$$r_b = r\log_2(M)$$

Bandwidth:

$$\frac{2}{D} = 2r$$

Power spectral density at zero frequency:

$$S_x(0) = 0$$

Average DC:

$$DC = E\{a_k\frac{\frac{A}{2}\frac{D}{2} - \frac{A}{2}\frac{D}{2}}{D}\} = 0$$

Average Power:

$$\bar{P} = \frac{2}{M}\sum_{k=1}^{\frac{M}{2}}(2k-1)^2\frac{\frac{A^2}{4}D}{D} = \frac{A^2}{12}(M^2-1)$$

Question 4

Prove that the power spectral density of the Mary PSK is

$$S_x(f) = \frac{A_c^2}{4r}\left[\textbf{sinc}^2(\frac{f-f_c}{r}) + \textbf{sinc}^2(\frac{f+f_c}{r})\right]$$

Let $\cos(\phi_k) = I_k$ and $\sin(\phi_k) = \Phi_k$.

$$\phi_k = \frac{\pi}{M}(2a_k + N) \Rightarrow \begin{cases} x_i(t) = \sum_k \cos(\phi_k)p(t-KD) = \sum_k I_k p(t-KD) \\ x_q(t) = \sum_k \sin(\phi_k)p(t-KD) = \sum_k \Phi_k p(t-KD) \end{cases}$$

Question 4 continued on next page…

, where $p(t) = \text{rect}(\frac{t}{D})$. Now,

$$\begin{cases} \overline{I_k} = E\{\cos(\phi_k)\} = E\{\cos(\frac{\pi}{M}(2a_k+N))\} = \frac{1}{M}\Sigma_{k=0}^{M-1}\cos(\frac{\pi}{M}(2k+N)) = 0 \\ \overline{\Phi_k} = E\{\sin(\phi_k)\} = 0 \\ \overline{I_k^2} = E\{\cos^2(\phi_k)\} = E\{\frac{1}{2}+\frac{1}{2}\cos(2\phi)\} = \frac{1}{2}+\frac{1}{2}E\{\cos(2\phi)\} = \frac{1}{2} \\ \overline{\Phi_k^2} = E\{\sin^2(\phi_k)\} = \frac{1}{2} \\ \overline{I_k\Phi_k} = E\{\cos(\phi_k)\sin(\phi_k)\} = E\{\frac{1}{2}\sin(2\phi_k)\} = 0 \end{cases}$$

$$S_{x_i}(f) = \frac{1}{D}|P(f)|^2\Sigma_{n=-\infty}^{\infty}R_I[n]\exp(-j2\pi fnD) = \frac{1}{2D}|P(f)|^2 = \frac{1}{2D}D^2\,\text{sinc}^2(fD)$$

$$S_{x_q}(f) = \frac{1}{D}|P(f)|^2\Sigma_{n=-\infty}^{\infty}R_\Phi[n]\exp(-j2\pi fnD) = \frac{1}{2D}|P(f)|^2 = \frac{1}{2D}D^2\,\text{sinc}^2(fD)$$

$$\to S_{x_i}(f) = S_{x_q}(f) = \frac{1}{2r}\,\text{sinc}^2(\frac{f}{r})$$

$$S_x(f) = \frac{A_c^2}{4}\{S_{x_i}(f+f_c)+S_{x_i}(f-f_c)+S_{x_q}(f+f_c)+S_{x_q}(f-f_c)\}$$

$$= \frac{A_c^2}{4}\frac{1}{2r}\{2\,\text{sinc}^2(\frac{f-f_c}{r})+2\,\text{sinc}^2(\frac{f+f_c}{r})\} = \frac{A_c^2}{4r}\{\text{sinc}^2(\frac{f-f_c}{r})+\text{sinc}^2(\frac{f+f_c}{r})\}$$

Question 5

Show that the spectrum of the Nyquist pulse

$$p(t) = \frac{\cos(2\pi\beta t)}{1-(4\beta t)^2}\textbf{sinc}(rt)$$

is

$$P(f) = \begin{cases} \frac{1}{r}, & |f| < \frac{r}{2}-\beta \\ \frac{1}{r}\cos^2\left(\frac{\pi}{4\beta}(|f|-\frac{r}{2}+\beta)\right), & \frac{r}{2}-\beta < |f| < \frac{r}{2}+\beta \\ 0, & |f| > \frac{r}{2}+\beta \end{cases}$$

Evaluate the result for the special case of $\beta = \frac{r}{2}$.

$$P(f) = \begin{cases} \frac{1}{r}, & |f| < \frac{r}{2}-\beta \\ \frac{1}{r}\cos^2\left(\frac{\pi}{4\beta}(|f|-\frac{r}{2}+\beta)\right) = \frac{1}{2r}[1+\cos\left(\frac{\pi}{2\beta}(|f|-\frac{r}{2}+\beta)\right)], & \frac{r}{2}-\beta < |f| < \frac{r}{2}+\beta \\ 0, & |f| > \frac{r}{2}+\beta \end{cases}$$

$$= \begin{cases} \frac{1}{r}, & |f| < \frac{r}{2}-\beta \\ \frac{1}{2r}[1-\sin\left(\frac{\pi}{2\beta}(|f|-\frac{r}{2})\right)], & \frac{r}{2}-\beta < |f| < \frac{r}{2}+\beta \\ 0, & |f| > \frac{r}{2}+\beta \end{cases}$$

$$\Rightarrow P''(f) = \begin{cases} \frac{\pi^2}{8r\beta^2}\sin\left(\frac{\pi}{2\beta}(|f|-\frac{r}{2})\right), & \frac{r}{2}-\beta < |f| < \frac{r}{2}+\beta \\ 0, & O.W. \end{cases}$$

Question 5 continued on next page...

$$\Rightarrow P''(f) = -\frac{\pi^2}{4\beta^2}\Big[P(f) - \frac{1}{2r}\mathrm{rect}(\frac{f}{r-2\beta}) - \frac{1}{2r}\mathrm{rect}(\frac{f}{r+2\beta})\Big]$$

$$\Rightarrow -4\pi^2 t^2 p(t) = -\frac{\pi^2}{4\beta^2}\Big[p(t) - \frac{1}{2r}\frac{1}{\pi t}\sin((\frac{r}{2}-\beta)2\pi t) - \frac{1}{2r}\frac{1}{\pi t}\sin((\frac{r}{2}+\beta)2\pi t)\Big]$$

$$p(t) = \frac{1}{1-(4\beta t)^2}\Big[\frac{1}{2r\pi t}\Big(\sin((\frac{r}{2}-\beta)2\pi t) + \sin((\frac{r}{2}+\beta)2\pi t)\Big)\Big]$$

$$= \frac{1}{1-(4\beta t)^2}\Big[\frac{1}{r\pi t}\sin(r\pi t)\cos(2\pi\beta t)\Big] = \frac{\cos(2\pi\beta t)}{1-(4\beta t)^2}\,\mathrm{sinc}(rt)$$

Now for $\beta = \frac{r}{2}$,

$$p(t) = \frac{\text{sinc}(2rt)}{1-(2rt)^2}, \quad P(f) = \frac{1}{r}\cos^2\big(\frac{\pi f}{2r}\big), \quad |f| < r$$

SOFTWARE QUESTIONS

Question 6

Fig. 1 shows an additive Linear Feedback Shift Register (LFSR) scrambler used in Digital Video Broadcasting (DVB). At the beginning of transmission, the shift register is loaded by the specified sync word and then, for each bit transmission, a feedback bit is calculated. The feedback bit is XORed with the transmitted bit and also used to shift the register. Implement the scrambler in MATLAB/Python and demonstrate its performance for several sample bit sequences. How can the scrambler be used for descrambling to recover the original bit sequence?

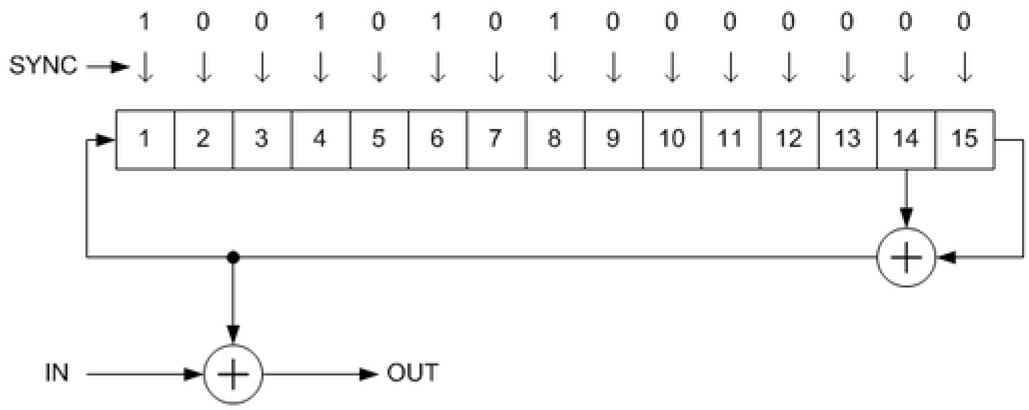

Figure 1: An additive scrambler used in DVB.

Question 6 continued on next page. . .

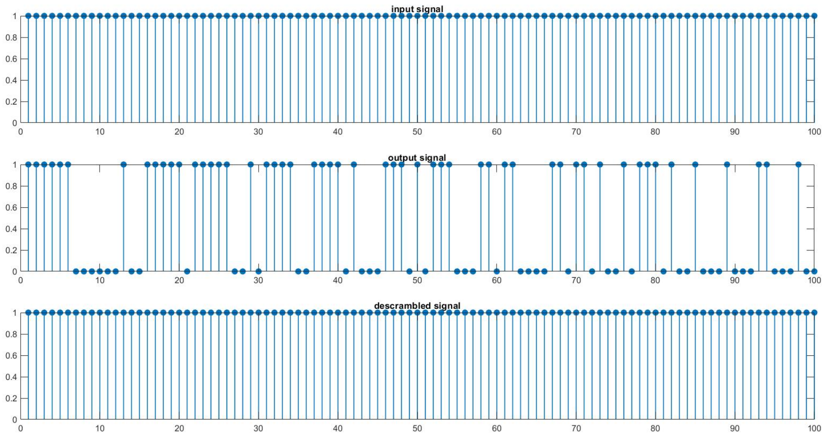

Figure 2: Scrambling and descrambling an all 1 bit stream.

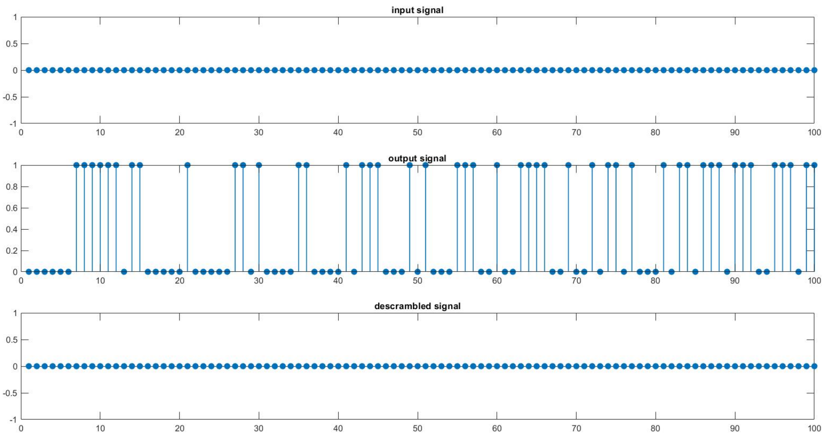

Figure 3: Scrambling and descrambling an all 0 bit stream.

Question 6 continued on next page. . .

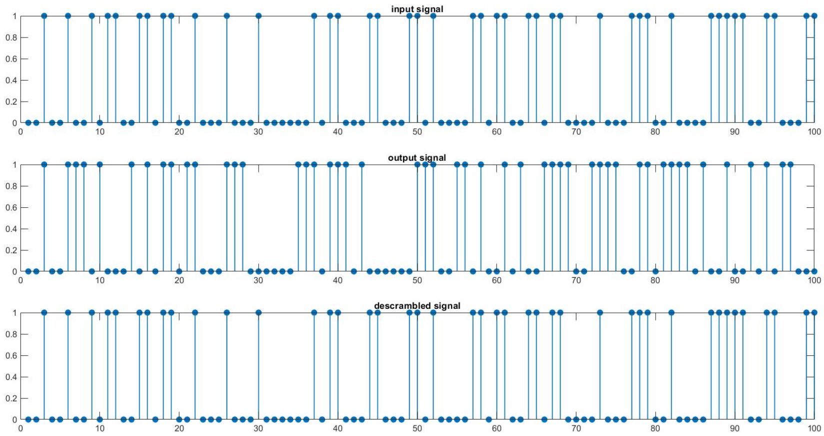

Figure 4: Scrambling and descrambling a random bit stream.

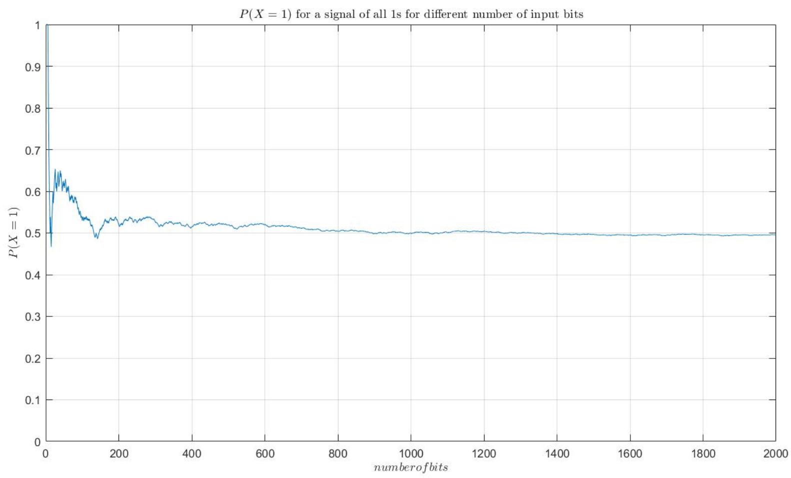

Figure 5: 1-bit occurrence probability for a scrambled all 1 bit stream.

Question 6 continued on next page...

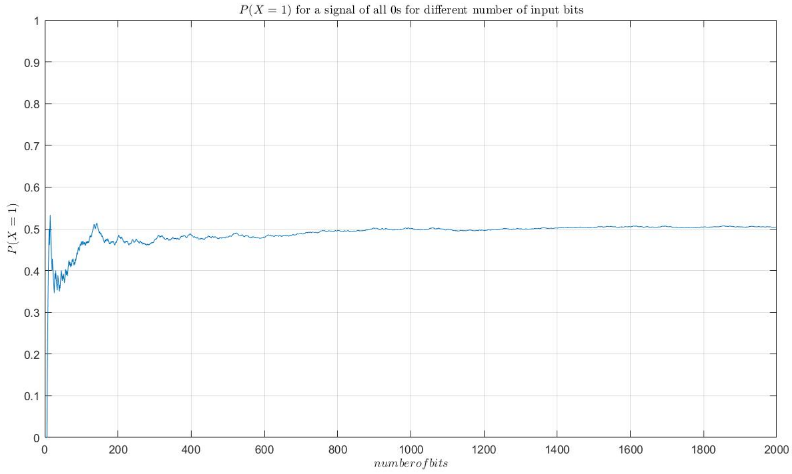

Figure 6: 1-bit occurrence probability for a scrambled all 0 bit stream.

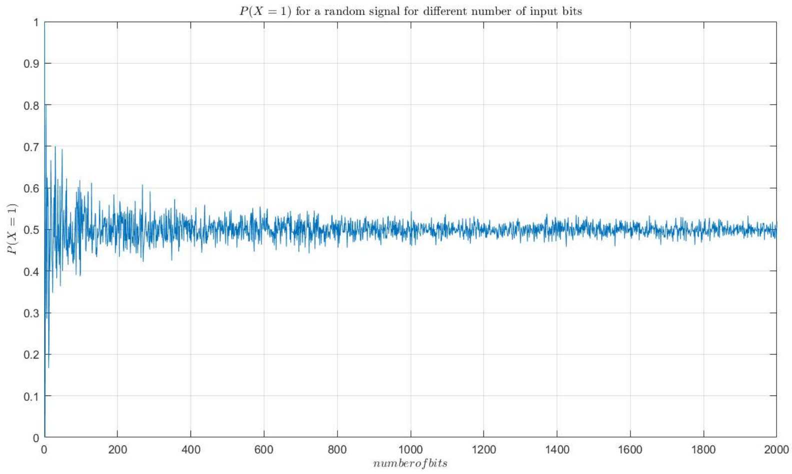

Figure 7: 1-bit occurrence probability for a scrambled random bit stream.

Here is a sample implementation.

```
clc; clear;  close all;
max = 2000 ;
n = 1:max ;
p1=zeros(1,max);
p2=zeros(1,max);
p3=zeros(1,max);
for k = 1:max
    %Signal of all 1s
    x1 = ones(1,k);
    scr1 = scr(x1);
    dscr1 = scr(scr1);
    Ones1=sum(scr1);
    p1(k)=Ones1/k;
    %Signal of all 0s
    x2=zeros(1,k);
    scr2=scr(x2);
    dscr2=scr(scr2);
```

Question 6 continued on next page. . .

```
    Ones2=sum(scr2);
    p2(k)=Ones2/k;
    %random signal
    x3=randsample(2,k,true)-1;
    scr3=scr(x3);
    dscr3=scr(scr3);
    Ones3=sum(scr3);
    p3(k)=Ones3/k;
end

%Signal of all 1s
figure
subplot(3,1,1)
In = ones(1,100);
stem(In,'filled','LineWidth',1)
title('input signal')
subplot(3,1,2)
Out = scr(In);
stem(Out,'filled','LineWidth',1)
title('output signal')
subplot(3,1,3)
d = scr(Out);
stem(d,'filled','LineWidth',1)
title('descrambled signal')
%Signal of all 0s
figure
subplot(3,1,1)
In = zeros(1,100);
stem(In,'filled','LineWidth',1)
title('input signal')
subplot(3,1,2)
Out = scr(In);
stem(Out,'filled','LineWidth',1)
title('output signal')
subplot(3,1,3)
d = scr(Out);
stem(d,'filled','LineWidth',1)
title('descrambled signal')
%random signal
figure
subplot(3,1,1)
In = randi(2,[1 100]) - 1;
stem(In,'filled','LineWidth',1)
title('input signal')
subplot(3,1,2)
Out = scr(In);
stem(Out,'filled','LineWidth',1)
title('output signal')
subplot(3,1,3)
d = scr(Out);
stem(d,'filled','LineWidth',1)
title('descrambled signal')
figure;
plot(n,p1);
title('$P(X=1)$ for a signal of all 1s for different number of input bits','Interpreter'
    ,'latex')
xlabel('$number of bits$','Interpreter','latex')
ylabel('$P(X=1)$','Interpreter','latex');
ylim([0 1]);
grid on;
figure;
plot(n,p2);
title('$P(X=1)$ for a signal of all 0s for different number of input bits','Interpreter'
    ,'latex')
xlabel('$number of bits$','Interpreter','latex')
ylabel('$P(X=1)$','Interpreter','latex');
ylim([0 1]);
grid on;
```

```
figure;
plot(n,p3);
title('$P(X=1)$ for a random signal for different number of input bits','Interpreter','latex')
xlabel('$number of bits$','Interpreter','latex')
ylabel('$P(X=1)$','Interpreter','latex');
ylim([0 1]);
grid on;


%% Functions:
function [sd] = scr(data)
    sync = [1 0 0 1 0 1 0 1 0 0 0 0 0 0 0];
    sd = zeros(size(data));
    tmp = sync ;
    for i = 1 : length(data)
        a = xor(sync(14),sync(15));
        tmp(2:15) = sync(1:14);
        tmp(1) = a;
        sd(i) = xor(a,data(i));
        sync = tmp;
    end
end
```

Figs. 6-6 show the scrambled bit stream corresponding to an all 1, all 0, and random bit stream. Clearly, the scrambled streams contain a sufficient number of bit fluctuations. Figs. 6-6 show the probability of 1-bit occurrence in the scrambled bit streams. Obviously, the probability of 1- and 0-bit occurrence are equal to 0.5.

BONUS QUESTIONS

Question 7

Consider an Mary ASK digital transmission system with the in-phase component

$$x_i(t) = \sum_k a_k p(t - kD), \quad a_k = 0, 1, \cdots, M-1, \quad p(t) = A \sqcap (\frac{t}{D})$$

, where the symbols a_k have identical and independent transmission probability. At the receiver, a received noisy in-phase component is recovered and sampled at proper time instances t_K. Then, the noisy sample $y(t_K) = n(t_k) + a_k A$ is fed to a decision circuit to decide which symbol is transmitted, where $n(t_K)$ is a zero-mean Gaussian noise distributed as $n(t_k) \sim \mathcal{N}(0, \sigma^2)$. The decision circuits decides on the received symbol $\tilde{a}_K$ as

$$\tilde{a}_K = \begin{cases} 0 & y(t_K) \leq \frac{A}{2} \\ 1 & \frac{A}{2} < y(t_K) \leq \frac{3A}{2} \\ \vdots & \vdots \\ M-2 & \frac{(2M-5)A}{2} < y(t_K) \leq \frac{(2M-3)A}{2} \\ M-1 & \frac{(2M-3)A}{2} < y(t_K) \end{cases}$$

Calculate the symbol bit error rate, i.e., the probability of an error in detecting a symbol.

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For $a_k = 0$, the error occurs when $n(t_k) > \frac{A}{2}$ and for $a_k = M-1$, the error occurs when $n(t_k) < -\frac{A}{2}$ and otherwise, the error occurs when $|n(t_k)| > \frac{A}{2}$. So, we can calculate symbol bit error rate as

$$P_e = \sum_{i=0}^{M-1} P_i P_{e|i} = \frac{1}{M}\sum_{i=0}^{M-1} P_{e|i}$$
$$\Rightarrow P_e = \frac{1}{M}P(n(t_k) > \frac{A}{2}) + (M-2) \times \frac{1}{M}P(|n(t_k)| > \frac{A}{2}) + \frac{1}{M}P(n(t_k) < -\frac{A}{2})$$
$$= (M-1) \times \frac{1}{M}P(|n(t_k)| > \frac{A}{2})$$
$$= \frac{M-1}{M}2Q(\frac{A}{2\sigma})$$

So,

$$P_e = 2\,\frac{M-1}{M}Q\left(\frac{A}{2\sigma}\right)$$

, where

$$Q\left(x\right) = P\left(X > x\right) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\,dt$$

Question 8

Return your answers by filling the LATEXtemplate of the assignment.