MATHEMATICAL QUESTIONS

Question 1

Calculate the power content of the SSB signal $u(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t)$.

Question 2

Find expressions for the in-phase and quadrature components, $x_c(t)$ and $x_s(t)$, as well as the envelope and phase, $V(t)$ and $\Theta(t)$ for DSB and SSB signals.

Question 3

The message signal $m(t)$ is applied to the system shown in Fig. 1 to generate the signal $y(t)$.

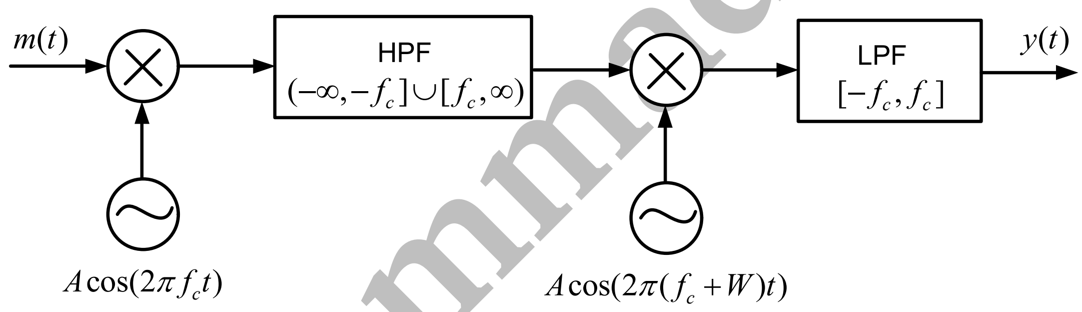

Figure 1: A sample scrambler.

(a) Find the spectrum of $y(t)$, i.e. $Y(f)$.

(b) Show that if $y(t)$ is transmitted, the receiver can pass it through a replica of the system shown in Fig. 1 to obtain $m(t)$ back.

(c) How can the system be used as a simple scrambler to enhance communication privacy?

Question 4 continued on next page...

Question 4

In Fig. 2, the transmitted power is 125 mW and the minimum acceptable SNR at the receiver is 30 dB. Assuming the noise power spectral density $N_0 = 4 \times 10^{-21}$ W/Hz and the bandwidth $B = 400$ kHz, answer the following questions.

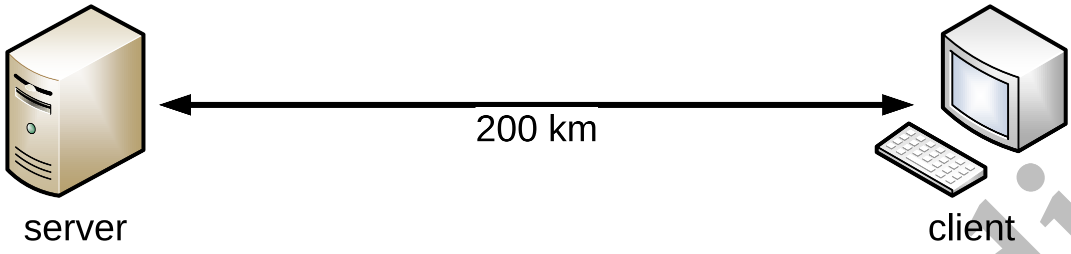

Figure 2: Server and Client.

(a) Suppose the server is connected to the client with a copper cable, and the transmission loss of the line is 2 dB/km. Is it possible to have a successful connection without using any amplifier?

(b) Assume that we have m identical amplifiers, each with a gain of 10 dB. If your answer to the last part is no, find the minimum of m to have a successful connection.

(c) Repeat the previous parts assuming the server and the client are connected with an optical fiber, and the transmission loss of the line is 0.3 dB/km.

Question 5

Fig. 3 shows the schematic of a ring modulator.

Question 5 continued on next page...

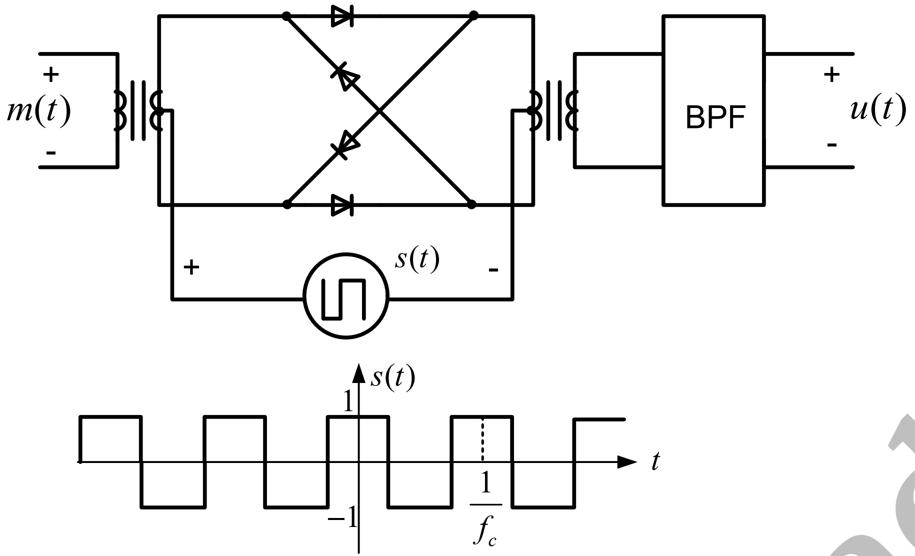

Figure 3: Ring modulator.

(a) Show that if $|m(t)| \ll 1$*, then* $u(t)$ *is a DSB-modulated signal.*

(b) What are the main roles of the three-winding transformers in the ring modulator?

Question 6

An SSB-modulated signal having the bandwidth W and power P_m passes through a distortion-less channel with the attenuation A and delay D, as shown in Fig. 4.

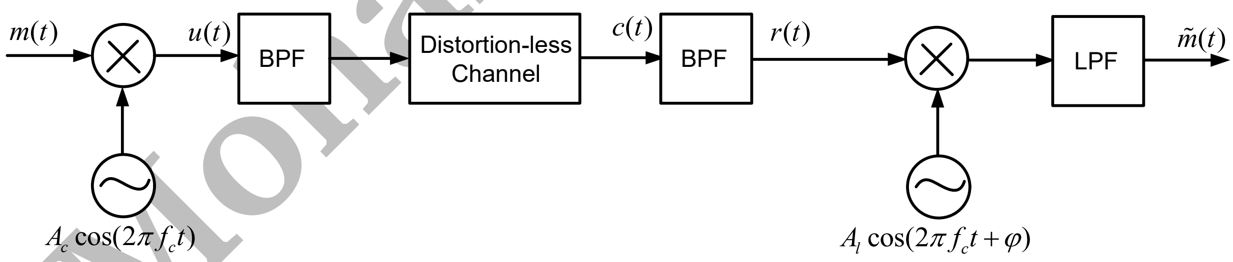

Figure 4: Block diagram of an SSB communication system.

(a) Find the power of the detected signal $\tilde{m}(t)$*.*

(b) Find the power of the signal part in the detected signal $\tilde{m}(t)$.

(c) Calculate the value of the phase shift ϕ resulting in the maximum power of the signal part in the detected signal $\tilde{m}(t)$.

SOFTWARE QUESTIONS

Question 7

Develop a MATLAB/Python code that accepts the message signal $m(t)$ and generates time- and frequency-domain plots of the corresponding DSB-, LSSB-, USSB-, and AM-modulated signals. The input arguments of the function are message signal $m(t)$, carrier frequency f_c, carrier amplitude A_c, AM modulation index a, and modulation type. Investigate the outputs of the function for a sample message signal.

BONUS QUESTIONS

Question 8

Prove that the time duration σ_t and spectral bandwidth σ_f satisfy the inequality

$$\sigma_t \sigma_f \geqslant \frac{1}{4\pi}$$

, where the time duration

$$\sigma_t^2 = \frac{\int_{-\infty}^{\infty} (t-\bar{t})^2 |x(t)|^2 dt}{\int_{-\infty}^{\infty} |x(t)|^2 dt}, \quad \bar{t} = \frac{\int_{-\infty}^{\infty} t|x(t)|^2 dt}{\int_{-\infty}^{\infty} |x(t)|^2 dt}$$

and the spectral bandwidth

$$\sigma_f^2 = \frac{\int_{-\infty}^{\infty} (f-\bar{f})^2 |X(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}, \quad \bar{f} = \frac{\int_{-\infty}^{\infty} f|X(f)|^2 df}{\int_{-\infty}^{\infty} |X(f)|^2 df}$$

such that $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$. For which signal does the equality hold? Can you provide an intuition for this inequality?

Question 9

Return your answers by filling the LaTeX template of the assignment.