MATHEMATICAL QUESTIONS

Question 1

A vestigial-sideband modulation system is depicted in Fig. 1. The bandwidth of the message signal $m(t)$ is W, and the transfer function of the bandpass filter is shown in the figure.

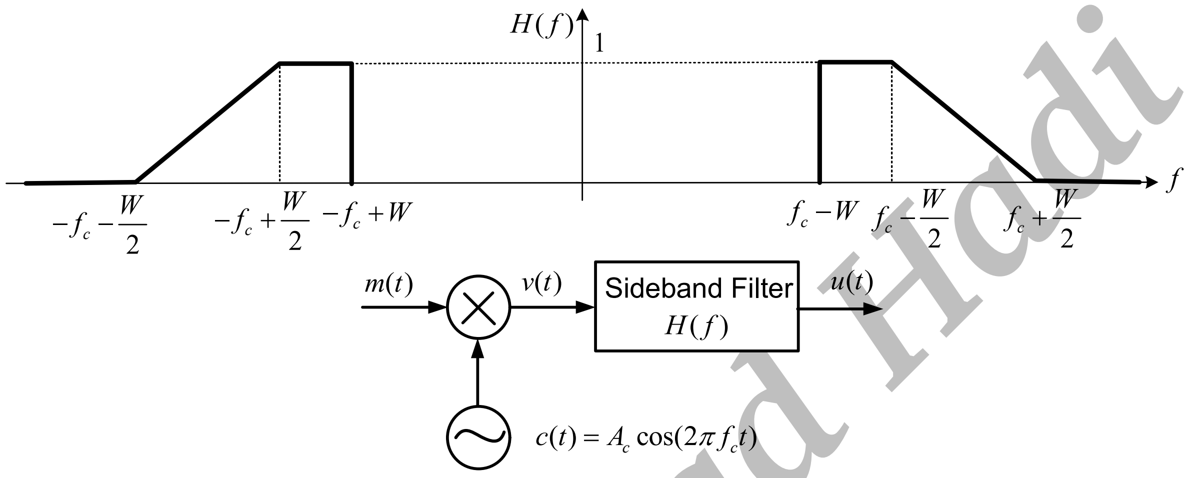

Figure 1: A VSB modulation system.

(a) Determine $h_l(t)$, the lowpass equivalent of $h(t)$, where $h(t)$ represents the impulse response of the bandpass filter.

(b) Derive an expression for the modulated signal $u(t)$.

Question 2

Follow the steps below to show the power of the FM signal $u(t) = A_c \cos(2\pi f_c t + \phi(t))$ is $\frac{A_c^2}{2}$.

(a) Write the power expression for the FM signal and show that the power equals $P = \frac{A_c^2}{2} + I$, where

$$I = \lim_{T\to\infty} \frac{A_c^2}{2T} \int_{-T/2}^{T/2} \cos(4\pi f_c t + 2\phi(t))dt$$

(b) Show that

$$I_\infty = \int_{-\infty}^{\infty} \cos(4\pi f_c t + 2\phi(t))dt$$

relates to the Fourier transforms $\mathcal{F}\{e^{j2\phi(t)}\}$ and $\mathcal{F}\{e^{-j2\phi(t)}\}$ at the frequencies $-2f_c$ and $2f_c$, respectively.

(c) Use Taylor series expansion to show that I_∞ depends to the Fourier transforms $\mathcal{F}\{\phi^n(t)\}, n \in \mathbb{W}$ at the frequency $\pm 2f_c$.

(d) Show that if $f_c \gg W$, where W is the bandwidth of the message-related phase $\phi(t)$, $I_\infty \approx 0$.

(e) Show that the power is approximately equal to $\frac{A_c^2}{2}$.

Question 3

Find the spectrum of the narrowband FM signal

$$u(t) = A_c \cos(2\pi f_c t) - A_c\big[2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\big] \sin(2\pi f_c t)$$

and narrowband PM signal

$$u(t) = A_c \cos(2\pi f_c t) - A_c k_p m(t) \sin(2\pi f_c t)$$

in terms of the message spectrum $M(f)$.

Question 4

The cross-correlation of the power signals $w(t)$ and $v(t)$ is defined as $R_{vw}(\tau) = \langle v(t)w^*(t-\tau)\rangle$, where the time average operator

$$\langle x(t)\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)dt$$

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(a) Show that $R_{vw}(\tau) = R^*_{wv}(-\tau)$.

Question 4 continued on next page...

(b) Prove that $|R_{vw}(\tau)|^2 \leqslant P_v P_w$.

Question 5

Fig. 2 shows the block diagram of an FM to AM demodulator and the schematic of its practical implementation, which is called balanced FM demodulator or FM discriminator.

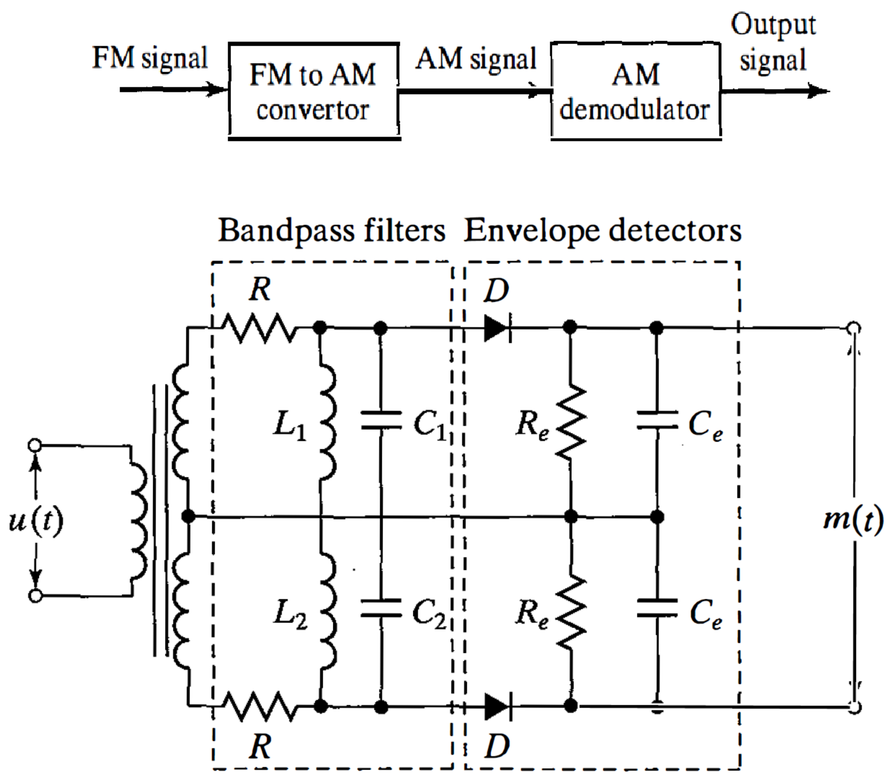

Figure 2: Balanced FM demodulator.

(a) Find the output of the FM to AM converter block if its frequency response is $H(f) = j[V_0 + k(f - f_c), \quad |f - f_c| < 0.5B_c$, where B_c denotes the bandwidth of the input FM modulated signal $u(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau)$. Note that the FM to AM impulse response is real and therefore, $H(-f) = H^*(f)$.

(b) Explain how the shown schematic implements the FM to AM demodulator? Why are there two filters and two envelope detectors in the schematic?

SOFTWARE QUESTIONS

Question 6

Develop a MATLAB/Python code that plots the cross-correlation $R_{vw}(\tau)$ of two power signals $v(t)$ and $w(t)$. Illustrate the output of the code for sample power signals.

BONUS QUESTIONS

Question 7

Two power signals of $v(t)$ and $w(t)$ are called uncorrelated if their cross-correlation $R_{vw}(\tau) = 0, \forall\tau$. Show that for these uncorrelated signals, the power P_z of $z(t) = v(t) + w(t)$ equals $P_z = P_v + P_w$.

Question 8

Return your answers by filling the LaTeXtemplate of the assignment.