MATHEMATICAL QUESTIONS

Question 1

First-order hold (FOH) performs a linear interpolation to generate the sampled signal

$$y(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\Lambda(\frac{t-kT_s}{T_s})$$

from the band-limited signal $x(t)$ with bandwidth W.

(a) Find the spectrum of the sampled signal $Y(f)$.

(b) Propose a condition on the sampling period T_s for the perfect reconstruction of the original signal $x(t)$ from the sampled signal $y(t)$.

(c) Obtain an expression for the reconstruction filter $H(f)$.

Question 2

The frequency-domain sampling theorem says that if $x(t)$ is a time-limited signal such that $x(t) = 0$ for $|t| \geq T$, then $X(f)$ is completely determined by its sample values $X(nf_0)$ with $f_0 \leq 1/2T$. Prove this theorem.

Question 3

The analog signal $x(t)$ which takes the values of $[-x_m, +x_m]$ uniformly passes a uniform midrise quantizer with $N = 2^\nu$ levels. Find the SQNR of the quantizer.

Question 4

The lowpass signal $x(t)$ with a bandwidth of W is sampled with a sampling interval of T_s and the signal

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT_s)p(t-nT_s)$$

is reconstructed from the samples, where $p(t)$ is an arbitrary-shaped pulse (not necessarily time limited to the interval $[0, T_s]$).

(a) *Find the Fourier transform of $x_p(t)$.*

(b) *Find the conditions for perfect reconstruction of $x(t)$ from $x_p(t)$.*

(c) *Determine the required reconstruction filter.*

Question 5

The lowpass signal $x(t)$ with a bandwidth of W is sampled at the Nyquist rate and the signal

$$y(t) = \sum_{n=-\infty}^{\infty} (-1)^n x(nT_s)\delta(t - nT_s)$$

is generated.

(a) *Find the Fourier transform of $y(t)$.*

(b) *Can $x(t)$ be reconstructed from $y(t)$ by using a linear time-invariant system? Why?*

(c) *Can $x(t)$ be reconstructed from $y(t)$ by using a linear time-varying system? How?*

Question 6

A stationary source is distributed according to the triangle probability density function $f_X(x) = 0.5\Lambda(0.5x)$. This source is quantized using the four-level uniform quantizer

$$Q(x) = \begin{cases} 1.5, & 1 < x \leq 2 \\ 0.5, & 0 < x \leq 1 \\ -0.5, & -1 < x \leq 0 \\ -1.5, & -2 \leq x \leq -1 \end{cases}$$

Determine the probability density function of the random variable representing the quantizer error $X - Q(X)$.

SOFTWARE QUESTIONS

Question 7

A quantizer with 2^ν quantized levels working over the input range $[-1, 1]$ is fed with a zero-mean Gaussian random variable having the variance σ^2. Develop a MATLAB/Python code to calculate the signal to quntization noise ratio when the quntization intervals are uniformly distributed and when the quantization intervals are nonuniformly distributed according to A-law companding method with the parameter A. Discuss the results for different values of ν, σ^2, and A. Feel free to plot any suitable curve to better describe the observations.

BONUS QUESTIONS

Question 8

The DCT of an $N \times N$ picture with luminance function $x(m, n), 0 \leq m, n \leq N-1$ can be obtained as

$$X(0,0) = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} x(k,l)$$

$$X(u,v) = \frac{2}{N}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} x(k,l)\cos\left[\frac{(2k+1)u\pi}{2N}\right]\cos\left[\frac{(2l+1)v\pi}{2N}\right], \quad u, v \neq 0$$

The $X(0, 0)$ coefficient is usually called the DC component, and the other coefficients are called the AC components. Find the DCT of a constant picture having $x(m, n) = C, 0 \leq m, n \leq N - 1$.

Question 9

Return your answers by filling the LaTeXtemplate of the assignment.