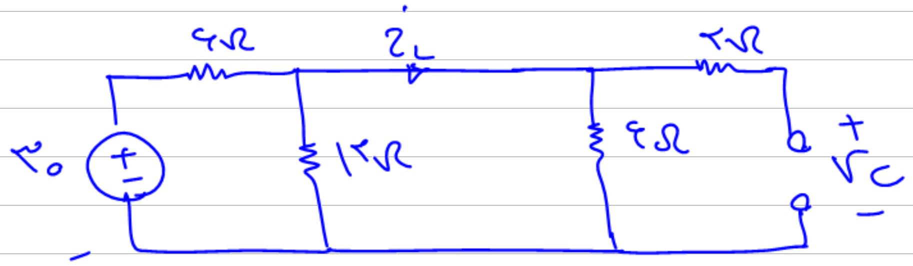
پاسخنامه تمرین سری ۴م - مدارهای مرتبه اول

سوال ۱)

شکل ۱ :

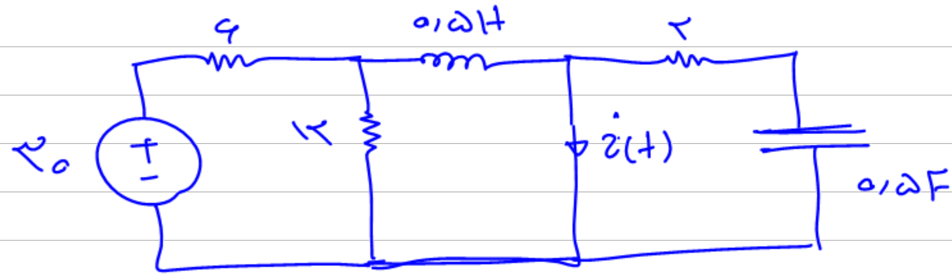
$t=0^-$



تقسیم ولتاژ
$$\Rightarrow v_C = 30 \times \frac{4}{4+6} = 12 , \quad i_L = \frac{v_C}{6} = 2A$$

$$\Rightarrow v_C(0^-) = 12 , \; i_L(0^-) = 2A \Rightarrow \begin{cases} v_C(0^+) = 12v \\ i_L(0^+) = 2A \end{cases}$$

$t>0$:



$$v_C(t) = (12-0)e^{-t} + 0 = 12e^{-t} \Rightarrow v_C(t) = 12e^{-t} \quad t>0$$

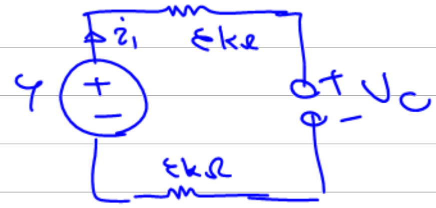
$$i_L(t) = (2-5)e^{-8t} + 5 = 5 - 3e^{-8t} \Rightarrow i_L(t) = (5-3e^{-8t}) \quad t>0$$

$$\Rightarrow i(t) = i_L - C\frac{dv_C}{dt} = (5-3e^{-8t}) - 6e^{-t}$$

$$\Rightarrow i(t) = 5 - 3e^{-8t} + 6e^{-t} \quad t>0$$

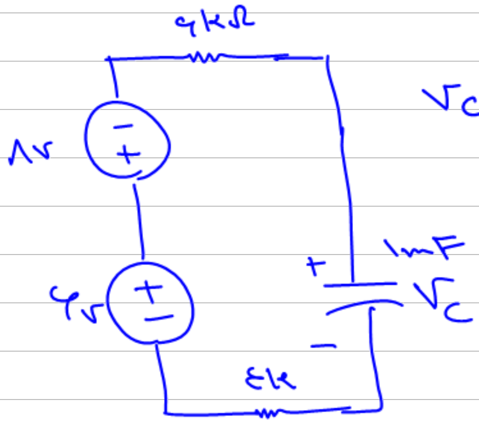
حل ۲:

$t<1$:



$$\Rightarrow v_C(1^-) = v_C(1^+) = 6\,v$$

$t>1$:

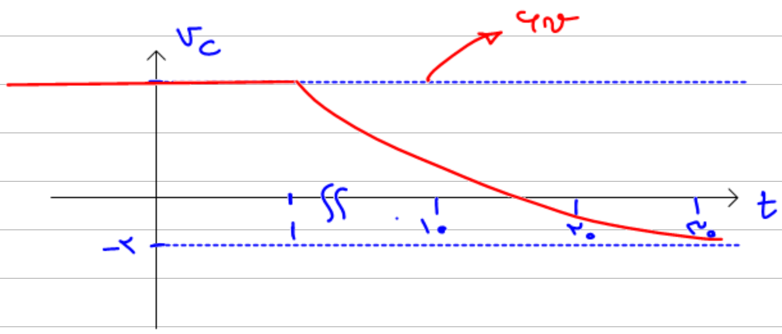


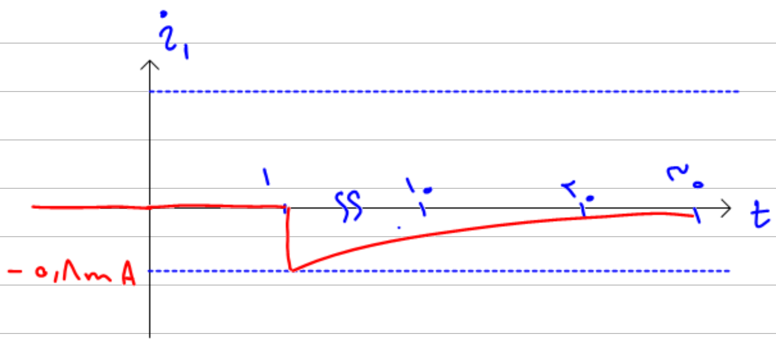
$$v_C = \left[v_C(1^+) - v_C(\infty)\right] e^{-\frac{t-1}{\tau}} + v_C(\infty)$$

$$\begin{cases} v_C(\infty) = -2 \\ \tau = RC = 10 \times 1 = 10\ s \end{cases}$$

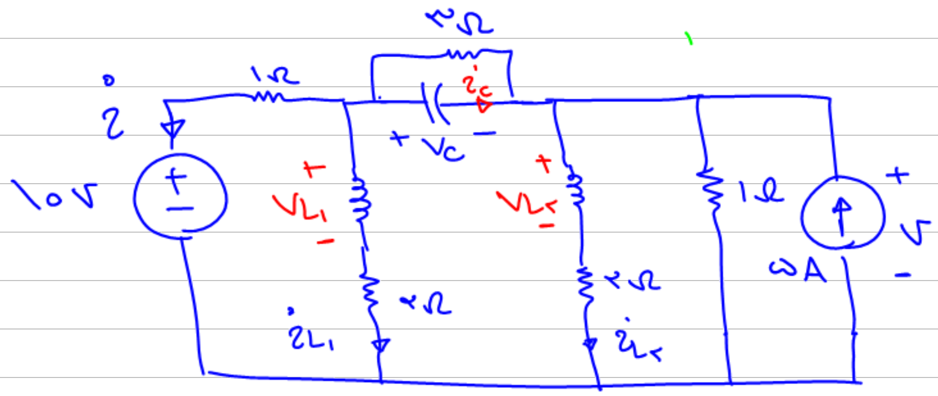
$$\Rightarrow v_C = (6-(-2))\, e^{-\frac{t-1}{10}} - 2$$

$$= 8 e^{-\frac{t-1}{10}} - 2$$





سوال ۲)

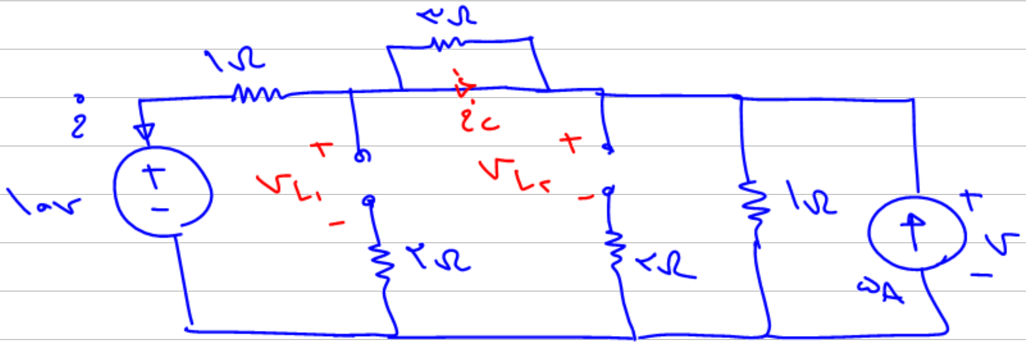


ورودی ضربه نداریم $\longrightarrow$

$$i_{L_1}(0^+) = \boxed{i_{L_1}(0^-) = 0}$$

$$i_{L_2}(0^+) = \boxed{i_{L_2}(0^-) = 0}$$

$$v_C(0^+) = \boxed{v_C(0^-) = 0}$$



$$i = -i_C = \underbrace{\frac{5-10}{2}}_{\text{تقسیم}} = -2.5 \Rightarrow \begin{cases} i_C = +2.5\,A \\ i = -2.5\,A \end{cases}$$

$$v = \underbrace{(10-5)\times 0.5}_{\text{تقسیم}} = 2.5\,v \Rightarrow v_{L_1} = v_{L_2} = 7.5\,v$$

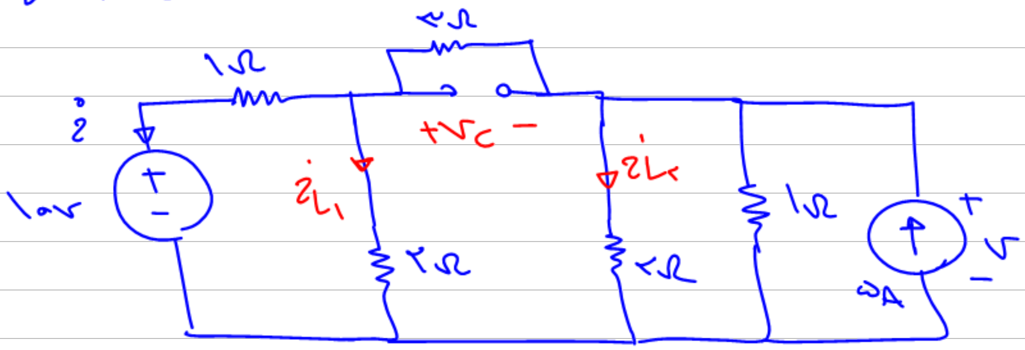
$$\Rightarrow v_{L_1} = L_1\frac{di_{L_1}}{dt} = \frac{di_{L_1}}{dt} = 7.5 \Rightarrow \boxed{\frac{di_{L_1}}{dt}(0^+) = 7.5\,v}$$

به طور مشابه $\longrightarrow$ $\boxed{\frac{di_{L_2}}{dt}(0^+) = 7.5\,v}$

$$i_C = C\frac{dv_C}{dt} \Rightarrow \boxed{\frac{dv_C}{dt}(0^+) = 2.5\,A}$$

$$\Rightarrow \boxed{\frac{dv}{dt}(0^+) = 0},\ \boxed{\frac{di}{dt}(0^+) = 0}$$

$t = +\infty$



$$\Rightarrow \begin{cases} v_{L_1}=0 \longrightarrow \dfrac{di_{L_1}}{dt}(\infty)=0 \\ v_{L_2}=0 \longrightarrow \dfrac{di_{L_2}}{dt}(\infty)=0 \\ i_C=0 \longrightarrow \dfrac{dv_C}{dt}(\infty)=0 \end{cases}$$

$$\frac{dv}{dt}(\infty)=0,\quad \frac{di}{dt}(\infty)=0$$

از قضیه جمع آثار و تقسیم جریان استفاده می‌کنیم:

$$i_{L_1}(\infty) = \frac{[(1\|2)+3]\|1}{[(1\|2)+3]\|1+2}\,10A + \left(\frac{1\|2}{1\|2+1\|2+3}\right)\left(\frac{1}{1+2}\right)5A = \frac{120}{39}A$$

$$i_{L_2}(\infty) = \frac{[(1\|2)+3]\|1}{[(1\|2)+3]\|1+2}\,5A + \left(\frac{1\|2}{1\|2+1\|2+3}\right)\left(\frac{1}{1+2}\right)10A = \frac{75}{39}A$$

$$v_C(\infty) = 2i_{L_1} - 2i_{L_2} = \frac{90}{39}$$

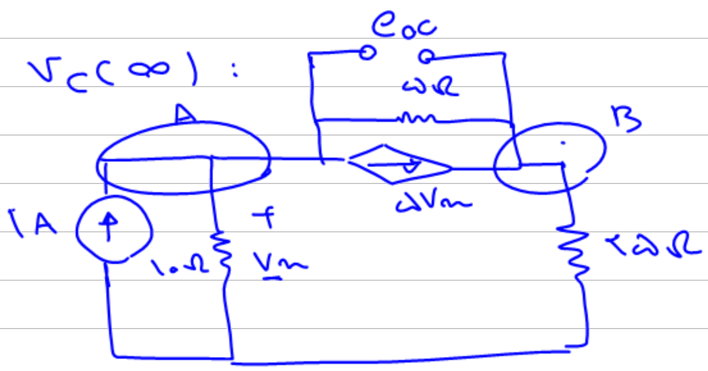
$$\Rightarrow i(\infty) = -i_{L_1} - \frac{v_C}{3} = \frac{-150}{39}$$

$$v(\infty) = 1\times\left(5 - i_{L_2} + \frac{v_C}{3}\right) = \frac{150}{39}$$

سوال ۳)

الف)

$\Rightarrow v_C(\infty):$

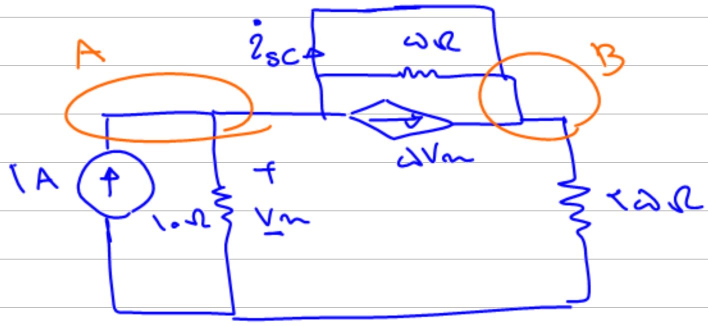


$$A\ \text{kcl}: \quad -1 + \frac{V_x}{10} + 5V_x + \frac{e_{oc}}{5} = 0 \Rightarrow 51V_x + 2e_{oc} = 10$$

$$B\ \text{kcl}: \quad \frac{V_x - e_{oc}}{25} - \frac{e_{oc}}{5} - 5V_x = 0 \Rightarrow -124V_x - 6e_{oc} = 0$$

$$\begin{bmatrix} V_x \\ e_{oc} \end{bmatrix} = \begin{bmatrix} 51 & 2 \\ -124 & -6 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} V_x = 1,03 \\ e_{oc} = -21,38 \end{cases}$$

ثابت زمانی τ



$$A\ \text{kcl}: \quad -1 + \frac{V_x}{10} + i_{sc} + 5V_x = 0 \qquad 51V_x + 10i_{sc} = 10$$

$$B\ \text{kcl}: \quad -5V_x - i_{sc} + \frac{V_x}{25} = 0 \Rightarrow -124V_x - 25i_{sc} = 0$$

$$\begin{bmatrix} V_x \\ i_{sc} \end{bmatrix} = \begin{bmatrix} 51 & 10 \\ -124 & -25 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 7,14 \\ -35,43 \end{bmatrix}$$

$$\left.\begin{aligned} R_{eq} &= \frac{e_{oc}}{i_{sc}} = \frac{-21,38}{-35,43} = 0,6034\ \Omega \\ C &= 2\,mF \end{aligned}\right\} \tau = 1,206\ ms$$

$$y(t)=\left(y(0^{+})-y(\infty)\right)e^{-\frac{t}{\tau}}+y(\infty)$$

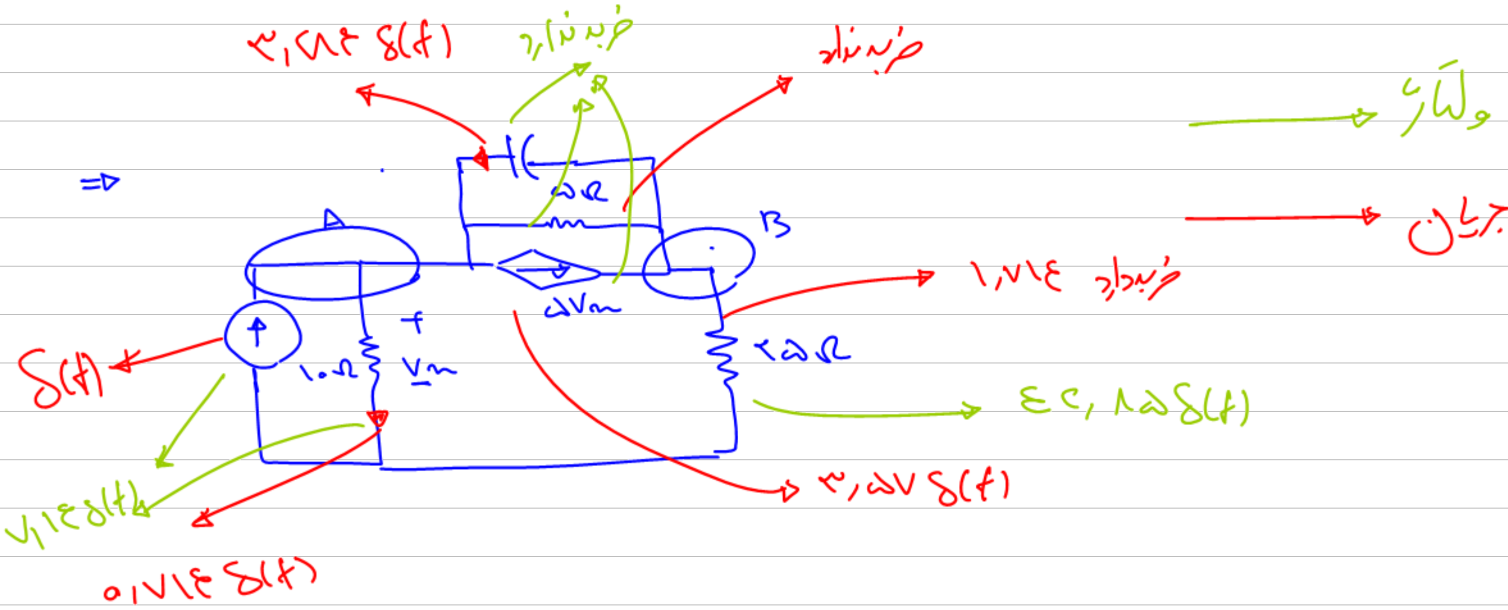
$$\Rightarrow \begin{cases} V_c(t)=(0+۲۱,۳۸)\,e^{-۸۲۹t}-۲۱,۳۸ \Rightarrow V_c(t)=۲۱,۳۸\left(e^{-۸۲۹t}-۱\right)u(t) \\ V_x(t)=(۷,۱۴-۱,۰۳)\,e^{-۸۲۹t}+۱,۰۳ \Rightarrow V_x(t)=\left(۶,۱۱\,e^{-۸۲۹t}+۱,۰۳\right)u(t) \end{cases}$$

ب)

$\xrightarrow{\text{پاسخ ضربه}}$ $h_{v_c}(t)=۲۱,۳۸\times(-۸۲۹)\,e^{-۸۲۹t}u(t)=-۱۷,۷۲۴\times۱۰^{۳}\,e^{-۸۲۹t}\,u(t)$

$h_{v_x}(t)=۶,۱۱\times(-۸۲۹)\,e^{-۸۲۹t}u(t)+۷,۱۴\delta(t)=-۵,۰۶۵\,e^{-۸۲۹t}\,u(t)+۷,۱۴\,\delta(t)$

$\Rightarrow$



سوال ۴)

الف)

$$\begin{cases} v_C + v - v_S = 0 & (KVL) \quad \mathrm{I} \\ v + 2i - \frac{1}{2}\frac{dv_C}{dt} = 0 & (KCL) \quad \mathrm{II} \\ i = \frac{2v - v_S}{5} & \quad \mathrm{III} \end{cases}$$

$$\left.\begin{array}{l} \mathrm{II},\mathrm{III} \longrightarrow \frac{5}{18}v_C' + \frac{2}{9}v_S = v \\ \mathrm{I} \end{array}\right\} \Rightarrow \frac{5}{18}\frac{dv_C}{dt} + v_C = \frac{7}{9}v_S$$

$$\Rightarrow \frac{dv_C}{dt} + \frac{18}{5}v_C = \frac{14}{5}v_S$$

$$\mathrm{I} \longrightarrow v_C = v_S - v \Rightarrow \frac{dv_S}{dt} - \frac{dv}{dt} + \frac{18}{5}v_S - \frac{18}{5}v = \frac{14}{5}v_S$$

$$\Rightarrow \frac{dv}{dt} + \frac{18}{5}v = \frac{dv_S}{dt} + \frac{4}{5}v_S$$

ب)

v_C: $\frac{dv_C}{dt} + \frac{18}{5}v_C = \frac{14}{5} \quad t>0 \,,\; v_C(0^+)=0$

$$\Rightarrow v_C = \underbrace{\frac{7}{9}}_{\text{دائمی}} - \underbrace{\frac{7}{9}e^{-\frac{18}{5}t}}_{\text{گذرا}} \,,\; t>0 \Rightarrow v_C(t) = \frac{7}{9}\left(1-e^{-\frac{18}{5}t}\right)u(t)$$

v: $\quad , \; v(0^-)=0$

$$t>0 \Rightarrow \frac{dv}{dt} + \frac{18}{5}v = \frac{4}{5} \,,\; v(0^+) = ??$$

$$\int_{0^-}^{0^+} \longrightarrow v(0^+) - v(0^-) = 1 \Rightarrow v(0^+) = 1$$

$$\Rightarrow v(t) = \left(1-\frac{2}{9}\right)e^{-\frac{18}{5}t} + \frac{2}{9} \Rightarrow v(t) = \Big(\underbrace{\frac{7}{9}e^{-\frac{18}{5}t}}_{\text{گذرا}} + \underbrace{\frac{2}{9}}_{\text{دائمی}}\Big)u(t)$$

ب)

$$v_C: \quad \frac{dv_C}{dt} + \frac{18}{5} v_C = \frac{14}{5}\delta(t) \quad , \quad v_C(0^-) = 0 \ , \ v_C(0^+) = ??$$

$$\int_{0^-}^{0^+} \longrightarrow \quad v_C(0^+) - v_C(0^-) = \frac{14}{5} \Rightarrow v_C(0^+) = \frac{14}{5}$$

$$t > 0 \quad \frac{dv_C}{dt} + \frac{18}{5} v_C = 0 \Rightarrow v_C = \frac{14}{5} e^{-\frac{18}{5}t} \quad t > 0$$

$$\Rightarrow \boxed{v_C(t) = \underbrace{\frac{14}{5} e^{-\frac{18}{5}t} u(t)}_{\text{گذرا}}}$$

$$(2) \longrightarrow \quad v = v_S - v_C = \underbrace{\delta(t) - \frac{14}{5} e^{-\frac{18}{5}t}}_{\text{گذرا}}$$

* پاسخ ضربه ~~یک~~ سیستم پایدار، قسمت دائمی ندارد.

ت)

$v_C: \quad s(t) = \frac{۷}{۹}(۱-e^{-\frac{۱۸}{۵}t})u(t) \Rightarrow s' = \frac{۱۴}{۵}e^{-\frac{۱۸}{۵}t}u(t)$

$h(t) = \frac{۱۴}{۵}e^{-\frac{۱۸}{۵}t}u(t) \Rightarrow h(t) = s'(t)$

$v: \quad s(t) = (\frac{۷}{۹}e^{-\frac{۱۸}{۵}t} + \frac{۲}{۹})u(t) \Rightarrow s'(t) = \frac{-۱۴}{۵}e^{-\frac{۱۸}{۵}t}u(t) + (\frac{۷}{۹}+\frac{۲}{۹})\delta(t)$

$\Rightarrow h(t) = -\frac{۱۴}{۵}e^{-\frac{۱۸}{۵}t} + \delta(t) \Rightarrow h(t) = s'(t)$

$\Leftarrow$ در یک سیستم LTI پاسخ ضربه برابر با مشتق پاسخ پله است.

ث)

$v_s(t) = ۴[u(t) - u(t-۲)]$

$\leftarrow$ از خاصیت LTI بودن مدار استفاده می‌کنیم:

$\Rightarrow v_C(t) = ۴[s(t) - s(t-۲)]$

$\Rightarrow v_C(t) = \left[\frac{۲۸}{۹}(۱-e^{-\frac{۱۸}{۵}t})u(t) - \frac{۲۸}{۹}(۱-e^{-\frac{۱۸}{۵}(t-۲)})u(t-۲)\right]$

$v(t) = v_s - v_C = (\frac{۸}{۹} + \frac{۲۸}{۹}e^{-\frac{۱۸}{۵}t})u(t) - (\frac{۸}{۹} + \frac{۲۸}{۹}e^{-\frac{۸}{۵}(t-۲)})u(t-۲)$

$\Rightarrow v(t) =$

سؤال ۵)

در هر مدار خطی تغییرناپذیر با زمان (LTI) صرف نظر از مرتبه مدار موارد زیر برقرار است:

۱) پاسخ حالت صفر نسبت به ورودی خطی است.

۲) پاسخ ورودی صفر نسبت به شرایط اولیه خطی است.

۳) $h(t)=\dfrac{ds}{dt}(t)$

$h(t) \longrightarrow$ پاسخ ضربه

$s(t) \longrightarrow$ پاسخ پله

$z(t) \longrightarrow$ پاسخ ورودی صفر

$$\Rightarrow \begin{cases} y_1(t)=h(t)+z(t) & t>0 \\ y_2(t)=s(t)+3z(t) & t>0 \end{cases}$$

$$\left.\begin{aligned} y_1-\frac{y_2}{3} &= h-\frac{s}{3} \\ h&=\frac{ds}{dt} \end{aligned}\right\} \Rightarrow y_1-\frac{y_2}{3}=s'-\frac{s}{3}$$

$$\Rightarrow -\frac{1}{3}-\frac{4}{3}e^{-t}+\frac{5}{3}e^{-\frac{t}{2}}=\frac{ds}{dt}-\frac{s}{3} \quad , \quad s(0^-)=0 \Rightarrow s(0^+)=0$$

$$\Rightarrow s(t)=s_h+s_p$$

$$\begin{cases} s_h=ke^{\frac{t}{3}} \\ s_p=A+Be^{-t}+Ce^{-\frac{t}{2}} \end{cases} \Rightarrow A=1,\ B=1,\ C=-2$$

$$\Rightarrow s(t)=ke^{\frac{t}{3}}+1+e^{-t}-2e^{-\frac{t}{2}}$$

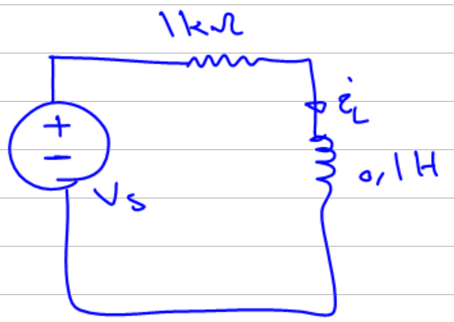
$$s(0^+)=0 \Rightarrow k+1+1-2=0 \Rightarrow k=0$$

$$\Rightarrow s(t)=1+e^{-t}-2e^{-\frac{t}{2}} \Rightarrow h(t)=(-e^{-t}+e^{-\frac{t}{2}})u(t)$$

$$y_1(t)=h(t)+z(t) \Rightarrow z(t)=(-5e^{-t}+8e^{-\frac{t}{2}}) \quad t>0$$

سؤال ۶)

الف)



$$\frac{di_L}{dt} + 10^4 i_L = 10 V_s$$

$$v_s = 3e^{-10^4 t} \longrightarrow i_L = i_h + i_p \begin{cases} i_h = ke^{-10^4 t} \\ i_p = Ate^{-10^4 t} \end{cases}$$

(فرم کلی پاسخ کامل)

برابر ریشه با توان

$$\Rightarrow \frac{d(Ate^{-10^4 t})}{dt} + 10^4 \times Ate^{-10^4 t} = 30e^{-10^4 t}$$

$$Ae^{-10^4 t} - 10^4 Ate^{-10^4 t} + 10^4 Ate^{-10^4 t} = Ae^{-10^4 t} = 30e^{-10^4 t}$$

$$\Rightarrow A = 30$$

$$\left.\begin{array}{l} \Rightarrow i_L(t) = ke^{-10^4 t} + 30te^{-10^4 t} \\ i_L(0^+) = 0 \end{array}\right\} i_L(t) = 30(1 - te^{-10^4 t})u(t)$$

ب)

$$v_s = 1000 \sin(\omega t + 30^\circ)$$

$$i_L(t) = i_h + i_p \longrightarrow \begin{cases} i_h = ke^{-10^4 t} \\ i_p = A\sin(\omega t + \varphi) \end{cases}$$

$$A\omega \cos(\omega t + \varphi) - 10^4 A \sin(\omega t + \varphi) = 10^4 \sin(\omega t + 30^\circ)$$

$$\omega = 10^4 \longrightarrow 10^4 A(\cos(\omega t + \varphi) - \sin(\omega t + \varphi)) = 10^4 \sin(\omega t + 120^\circ)$$

$$\Rightarrow A[(\cos\varphi - \sin\varphi)\cos\omega t - (\sin\varphi + \cos\varphi)\sin\omega t] = [\frac{\sqrt{3}}{2}\cos\omega t - \frac{1}{2}\sin\omega t]$$

$$\begin{cases} \cos\varphi = \dfrac{\sqrt{3}+1}{4A} \\ \sin\varphi = \dfrac{1-\sqrt{3}}{4A} \end{cases} \Rightarrow \begin{cases} \cos^2\varphi + \sin^2\varphi = 1 \Rightarrow A = \dfrac{\sqrt{2}}{2} \\ tg\varphi = \dfrac{1-\sqrt{3}}{1+\sqrt{3}} \Rightarrow \varphi = -15^\circ \end{cases}$$

۳۲

۳ ۲۱۷

$$\Rightarrow i_L(t) = Ke^{-10^4 t} + \frac{\sqrt{2}}{2}\sin(10^4 t - 15^\circ)$$

$$i_L(0^+) = 0 \Rightarrow K + \frac{\sqrt{2}}{2}\sin(-15) = 0 \Rightarrow K = \frac{\sqrt{2}}{2}\sin(15)$$

$$i_L(t) = \frac{\sqrt{2}}{2}\sin(15)\, e^{-10^4 t} + \frac{\sqrt{2}}{2}\sin(10^4 t - 15^\circ)$$

(ج)

$$v_s = 1000\sin(\omega t + \theta), \quad \omega = 10kHz$$

$$\frac{di_L}{dt} + 10^4 i_L = v_s \Rightarrow i_p = A\sin(\omega t + \varphi)$$

مشابه قسمت ب $\longrightarrow$

$$\begin{cases} \sin\varphi = \dfrac{\sin\theta + \cos\theta}{2A} \\ \cos\varphi = \dfrac{\cos\theta - \sin\theta}{2A} \end{cases} \Rightarrow tg\varphi = \frac{1 + tg\theta}{1 - tg\theta} = tg(\theta - 45)$$

$$\Rightarrow \varphi = \theta - 45^\circ \quad (I)$$

$$\left.\begin{aligned} &\Rightarrow i = i_h + i_p = Ke^{-\frac{t}{\tau}} + A\sin(\omega t + \varphi) \\ &i(0^+) = 0 \end{aligned}\right\} \Rightarrow K = -A\sin(\varphi)$$

پاسخ گذرا $\longrightarrow$

$$K = 0 \Rightarrow \varphi = k\pi \Rightarrow \theta = k\pi + \frac{\pi}{4}$$

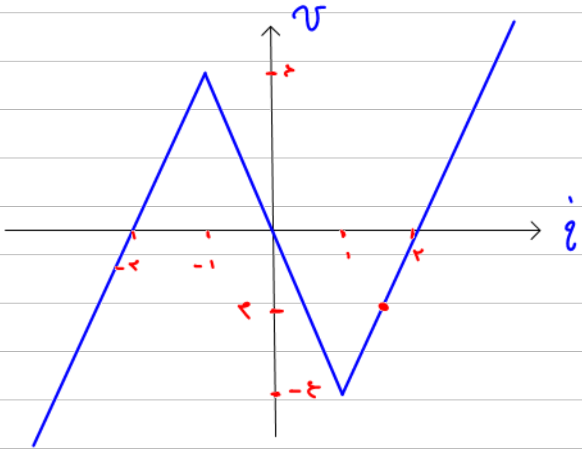
$$\Rightarrow \theta_{new} = 45^\circ \Rightarrow \Delta\theta = 15^\circ$$

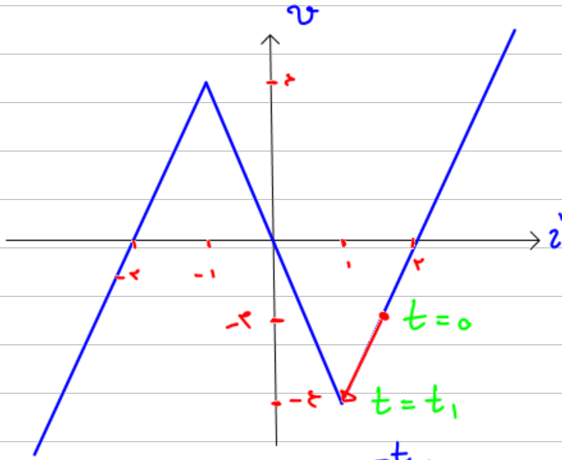
$$K = \text{بیشینه} \Rightarrow \varphi = k\pi + \frac{\pi}{2} \Rightarrow \theta = k\pi + \frac{3\pi}{4}$$

$$\Rightarrow \theta_{new} = -45^\circ \Rightarrow \Delta\theta = -75^\circ$$

سوال ۷)

ضابطه ولتاژ خازن را به ازای یک دوره تناوب به دست می آوریم و رسم می کنیم. بدین منظور مشخصه داده شده را تقریب تکه ای-خطی می زنیم





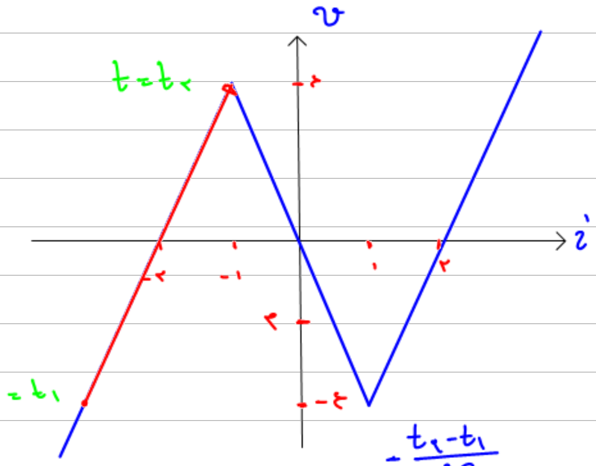
$$v_C(0^-) = v_C(0^+) = -2$$

$$\left.\begin{aligned} C\frac{dv}{dt} + i &= 0 \\ i &= \frac{v}{4} + 2 \end{aligned}\right\} \quad \frac{dv}{dt} + \frac{v}{4C} = \frac{-2}{C}$$

$$\left.\begin{aligned} \Rightarrow v &= ke^{-\frac{t}{4C}} - 8 \\ v_C(0^+) &= -2 \end{aligned}\right\} \quad v = 6e^{-\frac{t}{4C}} - 8$$

$$v(t_1) = -4 \Rightarrow 6e^{-\frac{t_1}{4C}} - 8 = -4 \Rightarrow e^{-\frac{t_1}{4C}} = \frac{2}{3} \Rightarrow t_1 = 4C\ln\left(\frac{3}{2}\right)$$

$$\Rightarrow t_1 = \frac{4\ln\left(\frac{3}{2}\right)}{\ln(9)}$$



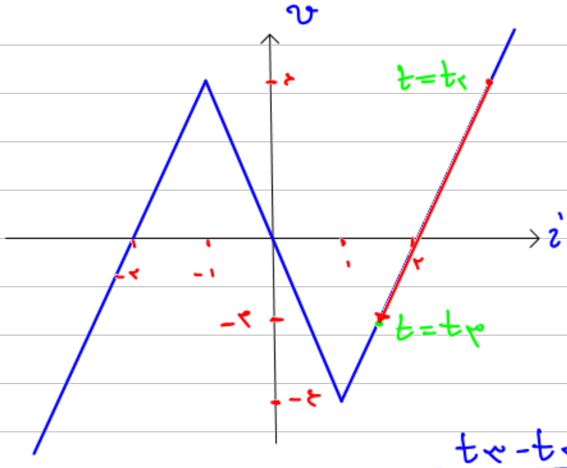
$$v_C(t_1^-) = v_C(t_1^+) = -4$$

$$\left.\begin{aligned} C\frac{dv}{dt} + i &= 0 \\ i &= \frac{v}{4} - 2 \end{aligned}\right\} \quad \frac{dv}{dt} + \frac{v}{4C} = \frac{+2}{C}$$

$$\left.\begin{aligned} \Rightarrow v &= ke^{-\frac{t-t_1}{4C}} + 8 \\ v_C(t_1^+) &= -4 \end{aligned}\right\} \quad v = -12e^{-\frac{t-t_1}{4C}} + 8$$

$$v(t_2) = 4 \Rightarrow -12e^{-\frac{t_2-t_1}{4C}} + 8 = 4 \Rightarrow e^{-\frac{t_2-t_1}{4C}} = +\frac{1}{3} \Rightarrow t_2 - t_1 = 4C\ln(3)$$

$$\Rightarrow t_2 = \frac{4\ln\left(\frac{9}{2}\right)}{\ln(9)}$$



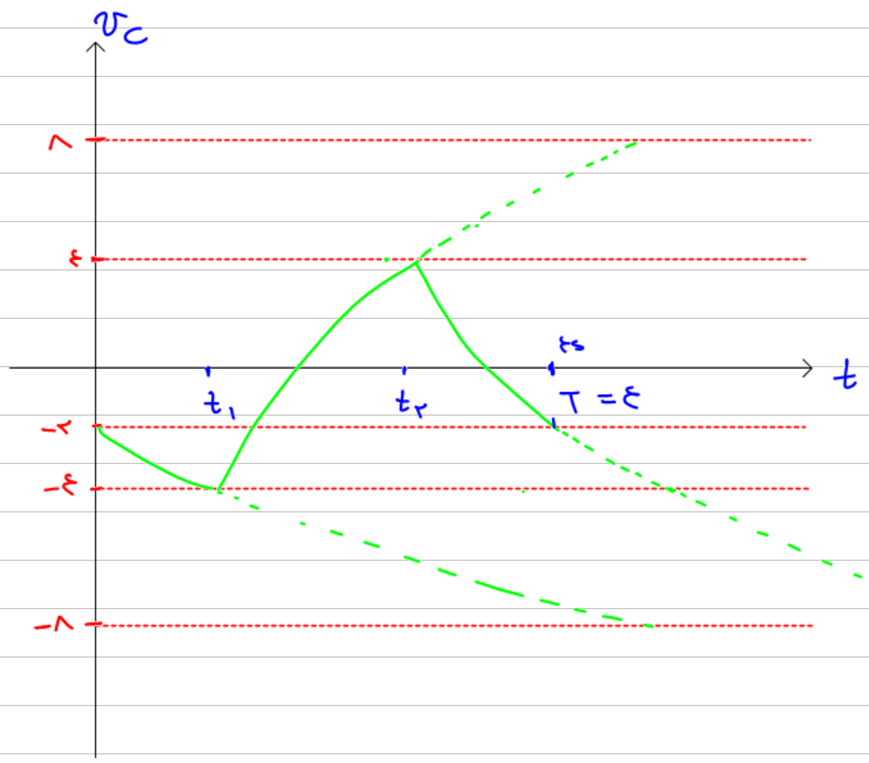
$$v_C(t_2^-) = v_C(t_2^+) = +4$$

$$\left.\begin{aligned} C\frac{dv}{dt} + i &= 0 \\ i &= \frac{v}{4} + 2 \end{aligned}\right\} \quad \frac{dv}{dt} + \frac{v}{4C} = \frac{-2}{C}$$

$$\left.\begin{aligned} \Rightarrow v &= ke^{-\frac{t-t_2}{4C}} - 8 \\ v_C(t_2^+) &= +4 \end{aligned}\right\} \quad v = 12e^{-\frac{t-t_2}{4C}} - 8$$

$$v(t_3) = -2 \Rightarrow 12e^{-\frac{t_3-t_2}{4C}} - 8 = -2 \Rightarrow e^{-\frac{t_3-t_2}{4C}} = \frac{1}{2} \Rightarrow t_3 - t_2 = 4C\ln(2)$$

$$t_3 = \frac{4\ln(9)}{\ln(9)} = 4 \quad \Rightarrow T = 4s$$



سؤال ۸)

الف)

تا زمانی که $V_{C_1}<V_{C_2}$ باشد دیود خاموش است.

$\Rightarrow V_{C_1}<1$

$$V_{C_1}=(0-2)e^{-t}+2=2(1-e^{-t})$$

$$V_{C_1}(t_1)=1 \Rightarrow e^{-t}=\frac{1}{2} \Rightarrow t_1=\ln(2) \Rightarrow V_{C_1}=2(1-e^{-t})\quad 0<t<t_1$$

دیود خاموش است $\Rightarrow V_{C_2}=1V\quad 0<t<t_1$

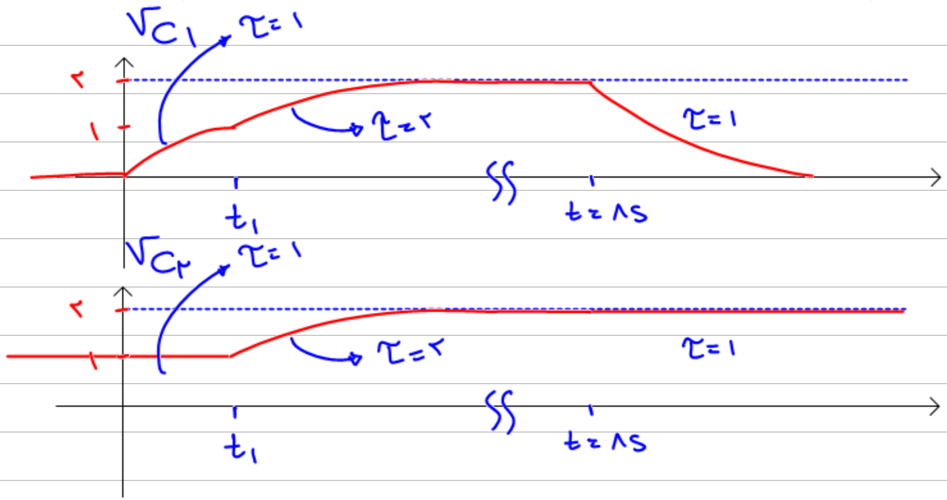
$t>t_1 \Rightarrow V_{C_1}>1$

$$C_{eq}=C_1+C_2=2F$$

$$V_{ceq}(t_1^+)=\frac{C_1V_1(t_1^-)+C_2V(t_2^-)}{C_1+C_2}=\frac{C_1+C_2}{C_1+C_2}=1$$

$$\Rightarrow V_{C_1}=V_{C_2}=(1-2)e^{-t/2}+2=2-e^{-t/2}$$

ب)



سؤال ۴) * شبکه را به صورت یک مدار مقاومتی با مشخصه معادل R_{eq} , e_{oc} , I_{sc} فرض می کنیم

$$S(t)=\frac{1}{20}\left(1-e^{-t/5}\right)u(t)$$

$$\tau=5 \Rightarrow RC=5 \Rightarrow R_{eq}=2.5\,\Omega$$

$$s(\infty)=e_{oc}=\frac{1}{20}$$

$$\Rightarrow i_{sc}=\frac{1}{20}\times\frac{1}{2.5}=\frac{1}{50}\,A$$

$L=1H \Rightarrow$

$$\tau=\frac{L}{R_{eq}}=\frac{1}{2.5}=\frac{2}{5}$$

$$S(\infty)=i_{sc}=\frac{1}{50}\,A$$

$$S(0^+)=0$$

$$\Rightarrow S(t)=\left(S(0^+)-S(\infty)\right)e^{-t/\tau}+S(\infty)$$

$$\Rightarrow s(t)=-\frac{1}{50}e^{-2.5t}+\frac{1}{50}=\frac{1}{50}\left(1-e^{-2.5t}\right)$$

$$\Rightarrow S(t)=\frac{1}{50}\left(1-e^{-2.5t}\right)u(t)$$

$$\Rightarrow h(t)=\frac{ds}{dt}=\frac{1}{20}e^{-2.5t}u(t)+0=\frac{1}{20}e^{-2.5t}u(t)$$

سؤال ۱۰)

الف)

$$i_L(0^-) = i_L(0^+) = 0 \Rightarrow i_L = 2(1-e^{-t}) \quad 0<t<1$$

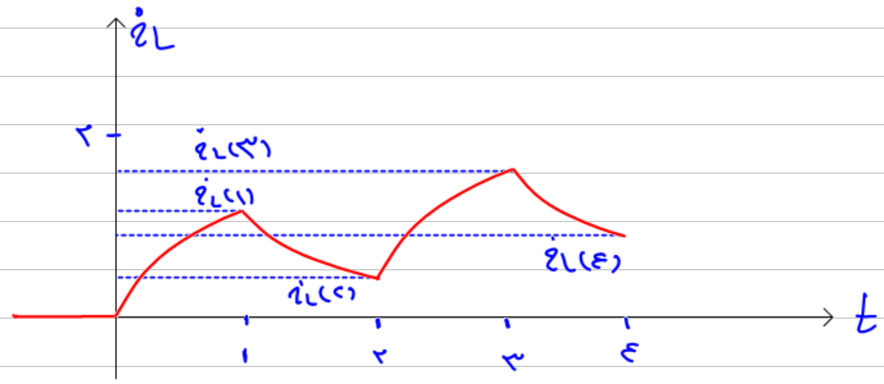
$$\Rightarrow i_L(1^-) = 2(1-e^{-1}) \Rightarrow i_L(1^+) = \frac{2(e-1)}{e}$$

$$\Rightarrow i_L = 2\left(1-\frac{1}{e}\right)e^{-(t-1)} \quad 1<t<2 \Rightarrow i_L(2^-) = i_L(2^+) = \frac{2(e-1)}{e^2}$$

$$\Rightarrow i_L = \frac{2(e-1)}{e^2}e^{-(t-2)} + 2 \quad 2<t<3 \Rightarrow i_L(3^-) = i_L(3^+) = \frac{2(e^3-e^2+e-1)}{e^3}$$

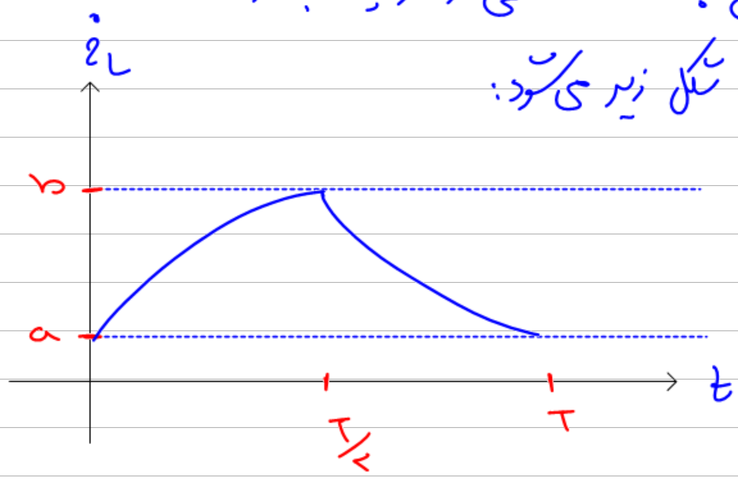
$$\Rightarrow i_L = \left(\frac{2-4e}{e^2}+2\right)e^{-(t-3)} \quad 3<t<4 \Rightarrow i_L(4^-) = \frac{2(e^3-e^2+e-1)}{e^4}$$

$$\Rightarrow \begin{cases} i_L(0) = 0 \\ i_L(1) = \frac{2(e-1)}{e} \\ i_L(2) = \frac{2(e-1)}{e^2} \\ i_L(3) = \frac{2(e^3-e^2+e-1)}{e^3} \\ i_L(4) = \frac{2(e^3-e^2+e-1)}{e^4} \end{cases}$$



ب) فرض کنید پس از مدت زمان طولانی پاسخ به حالت دائمی خود رسیده باشد

⇐ نمودار زمانی پاسخ در هر دوره تناوب به شکل زیر می‌شود:

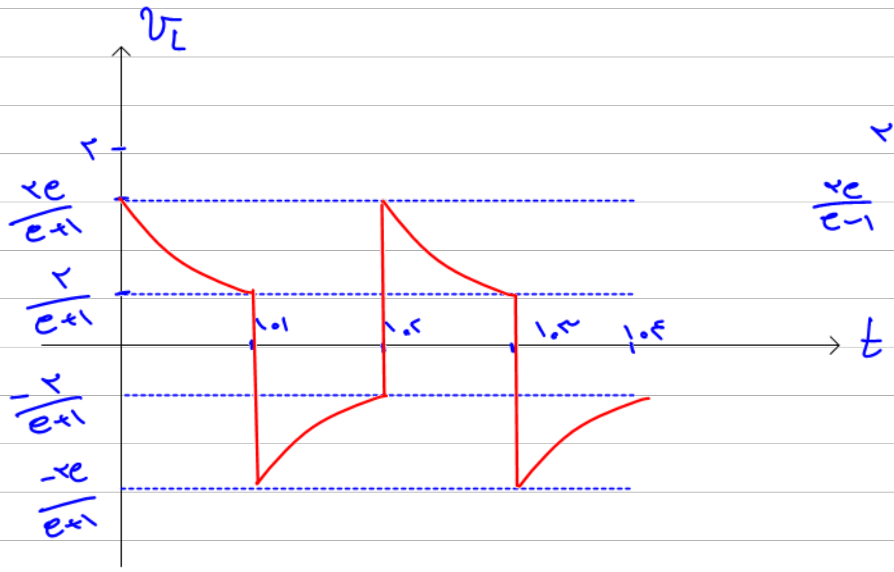


$$\Rightarrow (a-2)e^{-1}+2 = b$$

$$(b-a)e^{-1}+0 = a$$

$$\Rightarrow \begin{cases} a = \frac{2}{e+1} \\ b = \frac{2e}{e+1} \end{cases}$$

$$\Rightarrow v_L = v_s - R i_L = v_s - i_L \Rightarrow$$



$$۲ - \frac{۲}{e+۱}$$

$$\frac{۲e}{e+۱}$$