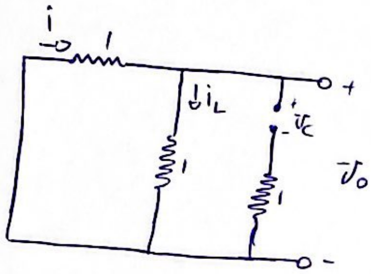
سئوال 1)

الف)



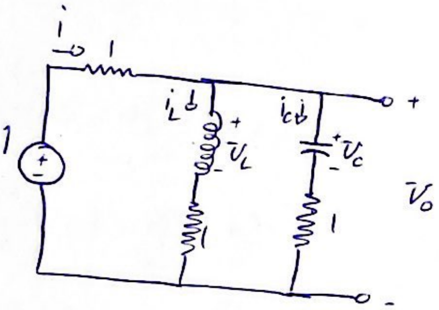
تحلیل مدار در $t=0^-$:

$$i(0^-) = i_L(0^-) = 0$$

$$v_C(0^-) = v_o(0^-) = 0$$

$$\frac{di}{dt}(0^-) = \frac{d^2 i}{dt^2}(0^-) = \frac{di_L}{dt}(0^-) = \frac{d^2 i_L}{dt^2}(0^-) = 0$$

$$\frac{dv_C}{dt}(0^-) = \frac{d^2 v_C}{dt^2}(0^-) = \frac{dv_o}{dt}(0^-) = \frac{d^2 v_o}{dt^2}(0^-) = 0$$



تحلیل مدار در $t=0^+$:

$$\boxed{i_L(0^+) = i_L(0^-) = 0}$$

$$\boxed{v_C(0^+) = v_C(0^-) = 0}$$

KVL: $-1 + i + v_C + i_C = 0$

$$i_C = i - i_L \to i(0^+) + v_C(0^+) + i(0^+) - i_L(0^+) = 1 \to \boxed{i(0^+) = \frac{1}{2}}$$

$$v_o(0^+) = v_C(0^+) + i_C(0^+) = \cancel{v_C(0^+)} + \overset{\frac{1}{2}}{i(0^+)} - \cancel{i_L(0^+)} = \boxed{\frac{1}{2}}$$

KVL: $-1 + i + v_L + i_L = 0 \to \underbrace{i(0^+)}_{\frac{1}{2}} + \frac{di_L}{dt}(0^+) + \underbrace{i_L(0^+)}_{0} = 1 \to \boxed{\frac{di_L}{dt}(0^+) = \frac{1}{2}}$

$$v_o(0^+) = v_C(0^+) + i_C(0^+) = \cancel{v_C(0^+)} + \frac{dv_C}{dt}(0^+) \to \boxed{\frac{1}{2} = \frac{dv_C}{dt}(0^+)}$$

$$i = i_L + i_C$$

$$i + v_C + i_C = 1 \to i_L + i_C + v_C + i_C = 1$$

$$\to i_L + 2\frac{dv_C}{dt} + v_C = 1 \to \underbrace{\frac{di_L}{dt}(0^+)}_{\frac{1}{2}} + 2\frac{d^2 v_C}{dt^2}(0^+) + \overset{\frac{1}{2}}{\frac{dv_C}{dt}(0^+)} = 0$$

$$\to \boxed{\frac{d^2 v_C}{dt^2}(0^+) = -\frac{1}{2}}$$

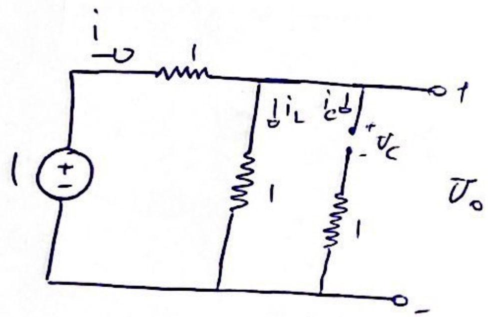
$$\to i_C = i - i_L \to \frac{d^2 v_C}{dt^2}(0^+) = \frac{di}{dt}(0^+) - \frac{di_L}{dt}(0^+) \to \boxed{\frac{di}{dt}(0^+) = 0}$$

$$v_o = v_C + i_C \to \frac{dv_o}{dt}(0^+) = \frac{dv_C}{dt}(0^+) + \frac{d^2 v_C}{dt^2}(0^+) \to \boxed{\frac{dv_o}{dt}(0^+) = 0}$$

ادامه سؤال 1)

الف)

$$v_o = v_L + i_L \to \frac{dv_o}{dt}(0^+) = \frac{d^2 i_L}{dt}(0^+) + \frac{di_L}{dt}(0^+) \to \boxed{\frac{d^2 i_L}{dt^2}(0^+) = -\frac{1}{2}}$$



تحلیل مدار در $t \to \infty$:

$$\boxed{i(\infty) = \frac{1}{2}}$$

$$\boxed{i_L(\infty) = \frac{1}{2}}$$

$$\boxed{v_C(\infty) = v_o(\infty) = \frac{1}{2}}$$

$$\frac{d^2 i_L}{dt^2}(\infty) = \frac{di_L}{dt}(\infty) = \boxed{0}$$

$$\frac{d^2 i_C}{dt^2}(\infty) = \frac{di_C}{dt}(\infty) = \boxed{0}$$

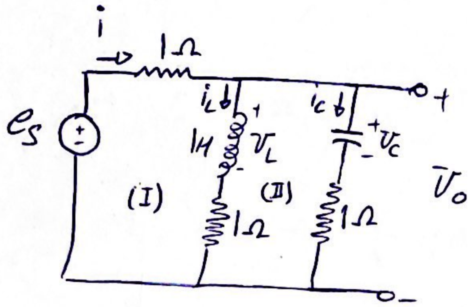
$$\frac{d^2 v_o}{dt^2}(\infty) = \frac{d^2 v_C}{dt^2}(\infty) = \boxed{0}$$

$$\frac{dv_C}{dt}(\infty) = \frac{dv_o}{dt}(\infty) = \boxed{0}$$

Scanned with CamScanner

ادامه سؤال 1)

ب)



$$V_o = v_C + i_C = v_C + \frac{dv_C}{dt}$$

$$i = \frac{e_s - V_o}{1} \longrightarrow i = e_s - v_C - \frac{dv_C}{dt}$$

$$i = i_L + i_C \longrightarrow i_L = i - i_C = e_s - v_C - 2\frac{dv_C}{dt}$$

$$KVL@(I): \; -e_s + i + \frac{di_L}{dt} + i_L = 0 \longrightarrow \cancel{-e_s} + \cancel{e_s} - v_C - \frac{dv_C}{dt} + \frac{de_s}{dt} - \frac{dv_C}{dt} - 2\frac{d^2v_C}{dt^2} + e_s - v_C - 2\frac{dv_C}{dt} = 0$$

$$\longrightarrow \boxed{\frac{d^2v_C}{dt^2} + 2\frac{dv_C}{dt} + v_C = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s}$$

$$V_o = v_L + i_L = \frac{di_L}{dt} + i_L$$

$$i = e_s - V_o = e_s - \frac{di_L}{dt} - i_L$$

$$i = i_L + i_C \longrightarrow i_C = i - i_L = e_s - \frac{di_L}{dt} - 2i_L$$

$$KVL@(I,II): \; -e_s + i + v_C + i_C = 0 \longrightarrow i + \int i_C + i_C = e_s \longrightarrow \frac{di}{dt} + i_C + \frac{di_C}{dt} = \frac{de_s}{dt}$$

$$\longrightarrow \frac{de_s}{dt} - \frac{d^2i_L}{dt^2} - \frac{di_L}{dt} + e_s - \frac{di_L}{dt} - 2i_L + \frac{de_s}{dt} - \frac{d^2i_L}{dt^2} - 2\frac{di_L}{dt} = \frac{de_s}{dt}$$

$$\longrightarrow \boxed{\frac{d^2i_L}{dt^2} + 2\frac{di_L}{dt} + i_L = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s}$$

Scanned with CamScanner

ادامه سئوال 1)

ب)

$$V_0 = V_C + i_C = V_C + \frac{dV_C}{dt} \rightarrow \frac{dV_0}{dt} = \frac{d^2V_C}{dt^2} + \frac{dV_C}{dt}$$

$$\xrightarrow{\text{از معادله دیفرانسیل } V_C} \frac{d^2V_C}{dt^2} + 2\frac{dV_C}{dt} + V_C = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s$$

$$\rightarrow \frac{d^2V_C}{dt^2} + \frac{dV_C}{dt} + \frac{dV_C}{dt} + V_C = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s$$

$$\rightarrow \boxed{\frac{dV_0}{dt} + V_0 = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s}$$

$$i = e_s - V_0 \rightarrow V_0 = e_s - i$$

$$\xrightarrow{\text{از معادله دیفرانسیل } V_0} \frac{dV_0}{dt} + V_0 = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s \rightarrow \frac{de_s}{dt} - \frac{di}{dt} + e_s - i = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s$$

$$\rightarrow \boxed{\frac{di}{dt} + i = \frac{1}{2}\frac{de_s}{dt} + \frac{1}{2}e_s}$$

پ)

$$\frac{d^2V_C}{dt^2} + 2\frac{dV_C}{dt} + V_C = \frac{1}{2}\delta(t) + \frac{1}{2}u(t) = \frac{1}{۲} \quad t > ۰$$

برای V_C:

چون طرف دوم معادله δ' نداریم می‌شود فهمید که $V_C(0^+) = V_C(0^-)$:

$$\int_{0^-}^{0^+} \frac{d^2V_C}{dt^2} + 2\int_{0^-}^{0^+} \frac{dV_C}{dt} + \int_{0^-}^{0^+} V_C = \frac{1}{2}\int_{0^-}^{0^+} \delta(t) + \int_{0^-}^{0^+} \frac{1}{2}u(t)$$

$$\rightarrow \frac{dV_C}{dt}(0^+) - \cancel{\frac{dV_C}{dt}(0^-)}^{0} + \cancel{2V_C(0^+) - 2V_C(0^-)}^{0} = \frac{1}{2} \rightarrow \boxed{\frac{dV_C}{dt}(0^+) = \frac{1}{2}}$$

$$\boxed{V_C(0^+) = V_C(0^-) = 0}$$

$$\xrightarrow{\text{معادله مشخصه}} S^2 + 2S + 1 = 0 \rightarrow S_1 = S_2 = -1$$

$$\xrightarrow{\text{پاسخ عمومی}} k_1 e^{-t} + k_2 t e^{-t}$$

$$\xrightarrow{\text{پاسخ ویژه}} k_3 = \frac{1}{2} \rightarrow V_C(t) = k_1 e^{-t} + k_2 t e^{-t} + \frac{1}{2}$$

$$\xrightarrow{V_C(0^+) = 0} k_1 + \frac{1}{2} = 0 \rightarrow \boxed{k_1 = -\frac{1}{2}}$$

$$\xrightarrow{\frac{dV_C}{dt}(0^+) = \frac{1}{2}} \left(-k_1 e^{-t} + k_2 e^{-t} - k_2 t e^{-t}\right)\Big|_{0^+} = \frac{1}{2} \rightarrow -k_1 + k_2 = \frac{1}{2} \rightarrow \boxed{k_2 = 0}$$

$$\rightarrow \boxed{V_C(t) = \left(-\frac{1}{2}e^{-t} + \frac{1}{2}\right)u(t)}$$

Scanned with CamScanner

ادامه سؤال 1)

پ)

برای i_L : $\frac{d^2 i_L}{dt^2} + 2\frac{d i_L}{dt} + i_L = \frac{\delta(t)}{2} + \frac{1}{2}u(t) = \frac{1}{2} \quad t>0$

$\xrightarrow{\text{مانند } v_C}$ پاسخ طبیعی $= K_1 e^{-t} + K_2 t e^{-t}$

پاسخ واداشته $= \frac{1}{2}$ $\quad \longrightarrow i_L(t) = K_1 e^{-t} + K_2 t e^{-t} + \frac{1}{2} \quad , \quad t>0$

$\xrightarrow[\text{در } v_C]{\text{مشابه استدلال}} i_L(0^+) = i_L(0^-) = 0 \longrightarrow K_1 + \frac{1}{2} = 0 \longrightarrow \boxed{K_1 = -\frac{1}{2}}$

$\xrightarrow[\text{در } v_C]{\text{مشابه روش}} \frac{di_L}{dt}(0^+) = \frac{1}{2} \longrightarrow -K_1 + K_2 = \frac{1}{2} \longrightarrow \boxed{K_2 = 0}$

$\longrightarrow \boxed{i_L(t) = \left(-\frac{1}{2}e^{-t} + \frac{1}{2}\right)u(t)}$

برای v_o : $\frac{dv_o}{dt} + v_o = \frac{1}{2}\delta(t) + \frac{1}{2}u(t) = \frac{1}{2} \quad t>0$

$\xrightarrow{\text{معادله مشخصه}} s+1=0 \longrightarrow$ پاسخ طبیعی $= K_1 e^{-t}$

پاسخ واداشته $= \frac{1}{2} \longrightarrow v_o(t) = K_1 e^{-t} + \frac{1}{2} \quad , \quad t>0$

$\int_{0^-}^{0^+} \frac{dv_o}{dt} + \int_{0^-}^{0^+} v_o = \frac{1}{2}\int_{0^-}^{0^+}\delta(t) + \int_{0^-}^{0^+}\frac{1}{2}u(t)$

$\longrightarrow v_o(0^+) - \cancelto{0}{v_o(0^-)} = \frac{1}{2} \longrightarrow v_o(0^+) = \frac{1}{2}$

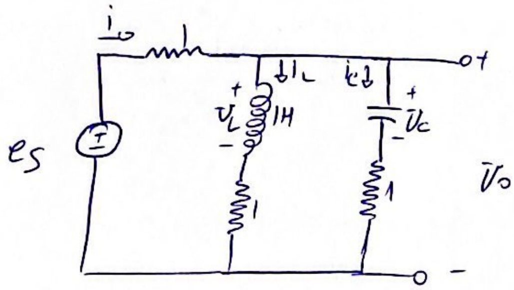
$\longrightarrow K_1 + \frac{1}{2} = \frac{1}{2} \longrightarrow K_1 = 0 \longrightarrow \boxed{v_o(t) = \frac{1}{2}u(t)}$

برای $i(t)$: $\frac{di}{dt} + i = \frac{1}{2}\delta(t) + \frac{1}{2}u(t)$

$\xrightarrow{\text{تشابه کامل معادله با } v_o}$ $\boxed{i(t) = \frac{1}{2}u(t)}$

ادامه سؤال 1)

ت)



برای : $t = 0^+$

$$v_C(0^-) = i_L(0^-) = v_o(0^-) = i(0^-) = 0$$

$$KVL: \; -e_s + i + v_C + i_C = 0$$

$$\rightarrow i + v_C + \frac{dv_C}{dt} = \delta(t)$$

$$i = i_L + i_C = i_L + \frac{dv_C}{dt} \rightarrow i_L + v_C + 2\frac{dv_C}{dt} = \delta(t)$$

$$\rightarrow \int_{0^-}^{0^+} i_L + \int_{0^-}^{0^+} v_C + 2\int_{0^-}^{0^+} \frac{dv_C}{dt} = \int_{0^-}^{0^+} \delta(t) \rightarrow 2(v_C(0^+) - \cancel{v_C(0^-)}) = 1$$

$$\rightarrow \boxed{v_C(0^+) = \frac{1}{2}}$$

$$KVL: \quad i + v_L + i_L = \delta(t) \rightarrow i_L + i_C + \frac{di_L}{dt} + i_L = \delta(t)$$

$$\rightarrow \frac{di_L}{dt} + 2i_L + \frac{dv_C}{dt} = \delta(t) \rightarrow \int_{0^-}^{0^+} \frac{di_L}{dt} + 2\int_{0^-}^{0^+} i_L + \int_{0^-}^{0^+} \frac{dv_C}{dt} = \int_{0^-}^{0^+} \delta(t)$$

$$\rightarrow i_L(0^+) - \cancel{i_L(0^-)}^{0} + \cancel{v_C(0^+)}^{\frac{1}{2}} - \cancel{v_C(0^-)}^{0} = 1 \rightarrow \boxed{i_L(0^+) = \frac{1}{2}}$$

$$i_L + v_C + 2\frac{dv_C}{dt} = \delta(t) \rightarrow \cancel{i_L(0^+)}^{\frac{1}{2}} + \cancel{v_C(0^+)}^{\frac{1}{2}} + 2\frac{dv_C}{dt}(0^+) = 0$$

$$\rightarrow \boxed{\frac{dv_C}{dt}(0^+) = -\frac{1}{2}}$$

$$v_o = v_C + i_C = v_C + \frac{dv_C}{dt}$$

$$\rightarrow v_o(0^+) = \cancel{v_C(0^+)}_{\frac{1}{2}} + \cancel{\frac{dv_C}{dt}}_{-\frac{1}{2}}(0^+) = \boxed{0}$$

$$i = i_L + i_C = i_L + \frac{dv_C}{dt} \rightarrow i(0^+) = i_L(0^+) + \frac{dv_C}{dt}(0^+) = \boxed{0}$$

$$v_o = v_L + i_L = \frac{di_L}{dt} + i_L \rightarrow \cancel{v_o(0^+)}^{0} = \frac{di_L}{dt}(0^+) + \cancel{i_L(0^+)}^{\frac{1}{2}}$$

$$\rightarrow \boxed{\frac{di_L}{dt}(0^+) = -\frac{1}{2}}$$

$$\frac{di_L}{dt} + \frac{dv_C}{dt} + 2\frac{d^2v_C}{dt^2} = \delta'(t)$$

$$\rightarrow \cancel{\frac{di_L}{dt}(0^+)}^{-\frac{1}{2}} + \cancel{\frac{dv_C}{dt}(0^+)}^{-\frac{1}{2}} = -2\frac{d^2v_C}{dt^2}(0^+) \rightarrow \boxed{\frac{d^2v_C}{dt^2} = +\frac{1}{2}} \rightarrow \boxed{\frac{d^2i_L}{dt^2} = \frac{1}{2}}$$

$$\rightarrow v_o = v_L + i_L \rightarrow \frac{dv_o}{dt} = \frac{d^2i_L}{dt^2} + \frac{di_L}{dt} \rightarrow \boxed{\frac{dv_o}{dt}(0^+) = 0}, \quad i = i_C + i_L \rightarrow \boxed{\frac{di}{dt}(0^+) = \frac{d^2v_C}{dt^2}(0^+) + \frac{di_L}{dt}(0^+) = 0}$$

Scanned with CamScanner

ادامه سؤال 1)

$$\frac{d^2 v_C}{dt^2} + 2\frac{dv_C}{dt} + v_C = \frac{1}{2}\delta'(t) + \frac{1}{2}\delta(t)$$

ث)

برای v_C :

$$\rightarrow \int_{0^-}^{0^+}\int\left(\frac{d^2 v_C}{dt^2} + \frac{dv_C}{dt} + v_C\right)dt = \int_{0^-}^{0^+}\int\left(\frac{1}{2}\delta'(t) + \frac{1}{2}\delta(t)\right)dt$$

$$\rightarrow v_C(0^+) - \cancel{v_C(0^-)}^{0} = \frac{1}{2} \rightarrow \boxed{v_C(0^+) = \frac{1}{2}}$$

$$\int_{0^-}^{0^+}\frac{d^2 v_C}{dt^2}dt + \int_{0^-}^{0^+} 2\frac{dv_C}{dt}dt + \int_{0^-}^{0^+} v_C dt = \frac{1}{2}\int_{0^-}^{0^+}\delta'(t) + \frac{1}{2}\int_{0^-}^{0^+}\delta(t)$$

$$\rightarrow \frac{dv_C}{dt}(0^+) - \cancel{\frac{dv_C}{dt}(0^-)}^{0} + \overset{2}{\overset{\frac{1}{2}}{\cancel{v_C(0^+)}}} - \overset{2}{\cancel{v_C(0^-)}}^{0} = \frac{1}{2} \rightarrow \boxed{\frac{dv_C}{dt}(0^+) = -\frac{1}{2}}$$

حال معادله را حل می‌کنیم:

معادله مشخصه

$$\rightarrow s^2 + 2s + 1 = 0 \rightarrow \text{پاسخ عمومی} = K_1 e^{-t} + K_2\, t e^{-t}$$

$$\text{پاسخ خاص} = 0$$

$$\rightarrow v_C(t) = K_1 e^{-t} + K_2 t e^{-t} \quad , \quad t > 0$$

$$v_C(0^+) = \frac{1}{2} \rightarrow \boxed{K_1 = \frac{1}{2}}$$

$$\frac{dv_C}{dt}(0^+) = -\frac{1}{2} \rightarrow \left(-K_1 e^{-t} + K_2 e^{-t} - K_2 t e^{-t}\right)\Big|_{0^+} = -\frac{1}{2} \rightarrow -K_1 + K_2 = -\frac{1}{2}$$

$$\rightarrow \boxed{K_2 = 0} \rightarrow \boxed{v_C(t) = \frac{1}{2}e^{-t}u(t)}$$

Scanned with CamScanner

ادامه سؤال 1)

الف)

برای i_L :

$$\frac{d^2 i_L}{dt^2} + 2\frac{di_L}{dt} + i_L = \frac{1}{2}\delta'(t) + \frac{1}{2}\delta(t)$$

کلیه مراحل این قسمت هم مشابه با v_C می‌باشد و اگر این مراحل را طی کنید :

$$\boxed{i_L(t) = \frac{1}{2}e^{-t}u(t)}$$

بله برای دو متغیر دیگر هم می‌شود.

Scanned with CamScanner

ادامه سؤال 1)

ث)

برای V_o:

$$\frac{dV_o}{dt} + V_o = \frac{1}{2}\delta'(t) + \frac{1}{2}\delta(t)$$

با توجه به معادله و اینکه V_o حتماً حاوی ضربه است می‌شود گفت:

$$V_o(t) = K_1 e^{-t}u(t) + K_2\delta(t)$$

حال برای پیدا کردن ضرایب:

$$\int_{0^-}^{0^+}\int \frac{dV_o}{dt}dt + \int_{0^-}^{0^+}\int V_o\,dt = \frac{1}{2}\int_{0^-}^{0^+}\int \delta'(t)dt + \frac{1}{2}\int_{0^-}^{0^+}\int \delta(t)dt$$

$$\to \int_{0^-}^{0^+} V_o\,dt + \int_{0^-}^{0^+} \underbrace{0}_{\text{تابعی ناپیوسته فاقد ضربه}}\,dt = \frac{1}{2}\int_{0^-}^{0^+}\delta(t)dt + \frac{1}{2}\int_{0^-}^{0^+}u(t)dt$$

$$\to K_2 + 0 = \frac{1}{2} \to \boxed{K_2 = \frac{1}{2}}$$

$$\int_{0^-}^{0^+}\frac{dV_o}{dt}dt + \int_{0^-}^{0^+}V_o\,dt = \frac{1}{2}\int_{0^-}^{0^+}\delta'(t) + \frac{1}{2}\int_{0^-}^{0^+}\delta(t) \to V_o(0^+) + \overset{\frac{1}{2}}{\cancel{K_2}} = \frac{1}{2}$$

$$\to \boxed{V_o(0^+) = 0}$$

$$\to K_1 = 0 \to \boxed{V_o(t) = \frac{1}{2}\delta(t)}$$

به همین ترتیب هم برای $i(t)$ جلو می‌رویم و نتیجه می‌شود که:

$$\boxed{i(t) = \frac{1}{2}\delta(t)}$$

Scanned with CamScanner

(ج

$$V_o = V_C + i_C = V_C + \frac{dV_C}{dt}$$

طبق شکل مدار، میدانیم که :

$$\xrightarrow{\text{از قسمت ث}} V_C(t) = \frac{1}{2}e^{-t}u(t) \Rightarrow V_o = \cancel{\frac{1}{2}e^{-t}u(t)} - \cancel{\frac{1}{2}e^{-t}u(t)} + \frac{1}{2}e^{-t}\delta(t)$$

$$\Rightarrow V_o = \frac{1}{2}e^{-t}\delta(t) \Rightarrow$$ پس ضربه دارد

$$i = i_s - V_o \Rightarrow i = \delta - \frac{1}{2}e^{-t}\delta(t) \Rightarrow$$ این متغیر هم ضربه دارد

$$i_C = \frac{dV_C}{dt} = -\frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-t}\delta(t) \Rightarrow$$ حاوی ضربه است

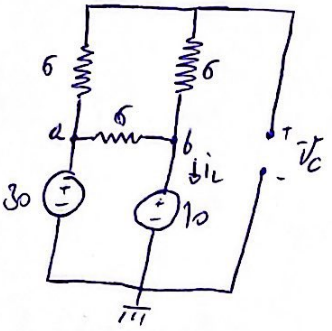
در واقع میتوان گفت جریان ضربه از شاخه شامل سلف عبور نمی‌کند و از شاخه حامل خازن عبور می‌کند که این باعث می شود و لتاژ خروجی ما هم شامل ضربه بشود، به طور خلاصه متغیرهایی که حاوی ضربه هستند بدین صورت می باشد :

V_L ، V_o ، i_C ، i ، V_{R_1} ، V_{R_2} $\Rightarrow$ مقاومت سری شاخه خازن

V_{R_1} $\Rightarrow$ مقاومت افقی سمت چپ

Scanned with CamScanner

سوال 2)



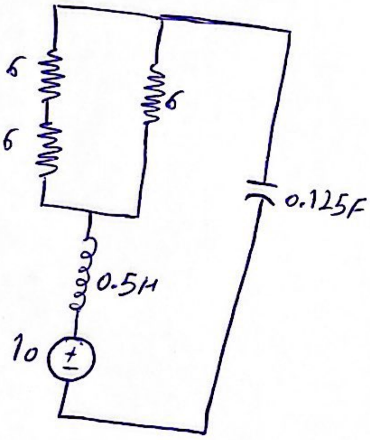
در $t=0^-$:

$$\frac{V_c - V_b}{6} + \frac{V_c - V_a}{6} = 0$$

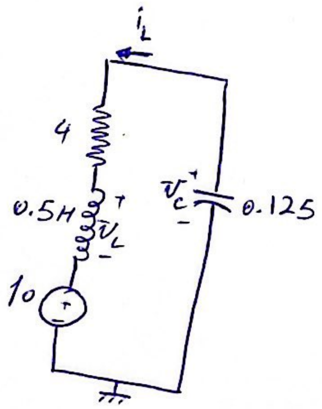
$$\longrightarrow 2V_c - V_b - V_a = 0, \quad V_b = 10, \quad V_a = 20 \longrightarrow \boxed{V_c(0^-) = 20}$$

$$i_L = \frac{V_a - V_b}{6} + \frac{V_c - V_b}{6} = \frac{50-20}{6} = 5$$

$$\longrightarrow \boxed{i_L(0^-) = 5}$$



$\longrightarrow$



برای $t>0$:

KVL:

$$4i_L + V_L + 10 - V_c = 0 \longrightarrow V_c - V_L - 4i_L = 10$$

$$\longrightarrow V_c(0^-) + \frac{1}{C}\int -i_L \; - L\frac{di_L}{dt} - 4i_L = 10$$

$$\longrightarrow 20 - 8\int i_L - \frac{1}{2}\frac{di_L}{dt} - 4i_L = 10$$

$$\longrightarrow -\frac{1}{2}\frac{d^2 i_L}{dt^2} - 4\frac{di_L}{dt} - 8i_L = 0 \longrightarrow \boxed{\frac{d^2 i_L}{dt^2} + 8\frac{di_L}{dt} + 16 i_L = 0} \quad (*)$$

$$\longrightarrow 4i_L(0^+) + \frac{1}{2}\frac{di_L(0^+)}{dt} + 10 - V_c(0^+) = 0 \longrightarrow 20 + \frac{1}{2}\frac{di_L}{dt}(0^+) + 10 - 20 = 0$$

$$\longrightarrow \boxed{\frac{di_L}{dt}(0^+) = -20} \quad \boxed{i_L(0^+) = 5}$$

$$\xrightarrow{(*)} i_L = K_1 e^{-4t} + K_2 t e^{-4t}$$

Scanned with CamScanner

ادامه سئوال 2)

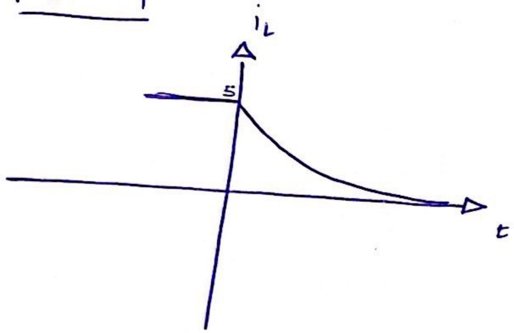
$i_L(0^+) = 5 \longrightarrow \boxed{K_1 = 5}$

$\frac{di_L}{dt}(0^+) = -20 \longrightarrow \left(-4K_1 e^{-4t} + K_2 e^{-4t} - 4K_2 t e^{-4t}\right)\Big|_{0^+} = -20$

$\longrightarrow -20 + K_2 = -20 \longrightarrow \boxed{K_2 = 0}$

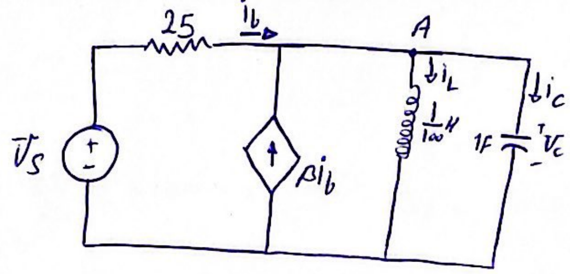
$\longrightarrow i_L(t) = 5e^{-4t} \quad , \quad t > 0$

$i_L(t) = 5 \quad , \quad t < 0 \longrightarrow$



سئوال 3)

الف)



$$KCL @ A : \quad (\beta+1) i_b = i_L + i_C$$

$$i_b = \frac{V_S - V_C}{25}$$

$$i_L = 100\int V_C dt \longrightarrow \frac{(\beta+1)(V_S - V_C)}{25} = 100\int V_C + \frac{dV_C}{dt}$$

$$\longrightarrow \frac{\beta+1}{25}\frac{dV_S}{dt} = \left(\frac{\beta+1}{25}\right)\frac{dV_C}{dt} + \frac{d^2V_C}{dt^2} + 100V_C$$

$$\longrightarrow \boxed{\frac{d^2V_C}{dt^2} + \left(\frac{\beta+1}{25}\right)\frac{dV_C}{dt} + 100V_C = \frac{\beta+1}{25}\frac{dV_S}{dt}}$$

$$S^2 + 2\alpha S + \omega_0^2 = 0$$

ب) میدانیم در معادله ای با معادله مشخصه:

$$\alpha = 0$$

برای اینکه مدار نوسان ساز باشد باید که: $\Delta = \alpha^2 - \omega_0^2 < 0$

$$\omega_0 = 10 \ , \ \alpha = \frac{\beta+1}{50}$$

در این جا میبینیم که:

برای نوسان ساز بودن $\longrightarrow \frac{\beta+1}{50} = 0 \longrightarrow \beta = -1 \longrightarrow \Delta = 0 - 100 < 0 \checkmark \longrightarrow \boxed{\beta = -1}$

$$\Delta < 0$$

پ) برای اینکه مدار میرای ضعیف باشد باید که:

$$\longrightarrow \left(\frac{\beta+1}{50}\right)^2 - 100 < 0 \longrightarrow$$

$$-10 < \frac{\beta+1}{50} < 10 \longrightarrow \boxed{-501 < \beta < 499}$$

ت)

$$\frac{d^2V_C}{dt^2} + \frac{501}{25}\frac{dV_C}{dt} + 100V_C = \frac{501}{25}\delta(t)$$

Scanned with CamScanner

ادامه سوال 3)

استا)

$$\xrightarrow{\text{معادله مشخصه}} s^2+\frac{501}{25}s+100=0 \longrightarrow \begin{array}{l} s_1=-9.38 \\ s_2=-10.65 \end{array}$$

$$\xrightarrow[\text{پاسخ عمومی}]{} v_C(t)=K_1e^{-9.38t}+K_2e^{-10.65t} \quad , t>0$$

$$\longrightarrow \text{پاسخ ویژه} = 0 \longrightarrow v_C(t)=\left(K_1e^{-9.38t}+K_2e^{-10.65t}\right)u(t)$$

چون در ورودی ضربه نداریم پس $v_C(0^+)=v_C(0^-)=0$ است و اکنون با تکنیک انتگرال گیری $\frac{dv_C}{dt}(0^+)$ را پیدا می کنیم:

$$\int_{0^-}^{0^+}\frac{d^2v_C}{dt^2}dt+\int_{0^-}^{0^+}\frac{501}{25}\frac{dv_C}{dt}dt+100\underbrace{\int_{0^-}^{0^+}v_C\,dt}_{0}=\frac{501}{25}\int_{0^-}^{0^+}\delta(t)$$

$$\longrightarrow \frac{dv_C}{dt}(0^+)-\cancel{\frac{dv_C}{dt}(0^-)}^{\,0}+\frac{501}{25}\left(\cancel{v_C(0^+)-v_C(0^-)}^{\,0}\right)=\frac{501}{25}$$

$$\longrightarrow \boxed{\frac{dv_C}{dt}(0^+)=\frac{501}{25}=20.04}$$

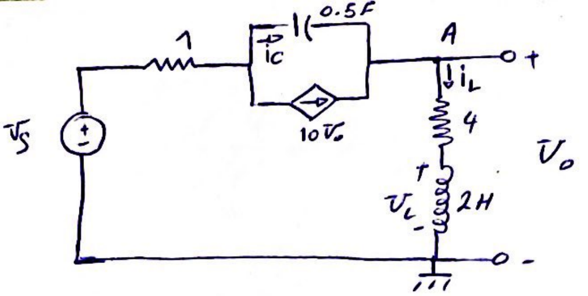
$$v_C(0^+)=0 \longrightarrow \boxed{K_1+K_2=0}$$

$$\frac{dv_C}{dt}(0^+)=20.04 \longrightarrow \boxed{-9.38K_1-10.65K_2=20.04}$$

$$\xrightarrow{\text{حل دو معادله دو مجهول}} \begin{array}{l} K_1=15.78 \\ K_2=-15.78 \end{array} \longrightarrow \boxed{v_C(t)=15.78\left(e^{-9.38t}-e^{-10.65t}\right)u(t)}$$

Scanned with CamScanner

سئوال 4)



الف)

KVL: $-v_S + i_C + 10v_o + v_C + 4i_L + v_L = 0$

$v_o = 4i_L + v_L$

$v_L = 2\frac{di_L}{dt}$

$i_C = \frac{1}{2}\frac{dv_C}{dt}$

$$\longrightarrow \boxed{-v_S + \frac{1}{2}\frac{dv_C}{dt} + 44i_L + 22\frac{di_L}{dt} + v_C = 0} \quad *$$

KCL@A: $i_L = i_C + 10v_o \longrightarrow -39i_L - 20\frac{di_L}{dt} = \frac{1}{2}\frac{dv_C}{dt}$

$$\longrightarrow \boxed{\frac{dv_C}{dt} = -78i_L - 40\frac{di_L}{dt}} \quad **$$

$$\xrightarrow{*,\,**} -\frac{dv_S}{dt} + \frac{1}{2}\left(-78\frac{di_L}{dt} - 40\frac{d^2i_L}{dt^2}\right) + 44\frac{di_L}{dt} + 22\frac{d^2i_L}{dt^2} + \left(-78i_L - 40\frac{di_L}{dt}\right) = 0$$

$$\longrightarrow \boxed{\frac{d^2i_L}{dt^2} - \frac{35}{2}\frac{di_L}{dt} - 39i_L = \frac{1}{2}\frac{dv_S}{dt}}$$

$$(*) \longrightarrow (22D+44)i_L + (\tfrac{1}{2}D+1)v_C = v_S \longrightarrow i_L = \frac{v_S - (\frac{1}{2}D+1)v_C}{22D+44}$$

$$(**) \longrightarrow Dv_C = -(78+40D)\left(\frac{v_S - (\frac{1}{2}D+1)v_C}{22D+44}\right)$$

$$\longrightarrow (22D^2 + 44D)v_C = (39D + 20D^2 + 78 + 40D)v_C - (40D+78)v_S$$

$$\longrightarrow (2D^2 - 35D - 78)v_C = (-40D - 78)v_S$$

$$\longrightarrow \boxed{\frac{d^2v_C}{dt^2} - \frac{35}{2}\frac{dv_C}{dt} - 39v_C = -20\frac{dv_S}{dt} - 39v_S}$$

Scanned with CamScanner

ادامه سؤال 4)

ب)

$$\boxed{\frac{d^2 v_C}{dt^2} - \frac{35}{2}\frac{dv_C}{dt} - 39 v_C = -20\delta - 39u(t)}$$

$v_C(0^+) = v_C(0^-) = 0$

$\frac{dv_C}{dt}(0^-) = 0$

$$\int_{0^-}^{0^+} \frac{d^2 v_C}{dt^2} dt - \frac{35}{2}\int_{0^-}^{0^+} \frac{dv_C}{dt} dt - 39\int_{0^-}^{0^+} v_C dt = -20\int_{0^-}^{0^+} \delta(t) dt - 39\int_{0^-}^{0^+} u(t) dt$$

$$\longrightarrow \boxed{\frac{dv_C}{dt}(0^+) = -20}$$

معادله مشخصه

$\longrightarrow s^2 - \frac{35}{2}s - 39 = 0 \longrightarrow s_1 = 19.5$

$s_2 = -2 \longrightarrow$ پاسخ عمومی $= K_1 e^{19.5t} + K_2 e^{-2t}$

پاسخ ویژه $\longrightarrow = \boxed{1} \longrightarrow v_C(t) = (K_1 e^{19.5t} + K_2 e^{-2t} + 1)u(t)$

$v_C(0^+) = 0 \longrightarrow \boxed{K_1 + K_2 = -1}$

$\frac{dv_C}{dt}(0^+) = -20 \longrightarrow \boxed{19.5K_1 - 2K_2 = -20} \longrightarrow K_1 = -1.023$

$K_2 = 0.023$

$$\longrightarrow \boxed{v_C(t) = (-1.023 e^{19.5t} + 0.023 e^{-2t})u(t)}$$

$$\frac{d^2 i_L}{dt^2} - \frac{35}{2}\frac{di_L}{dt} - 39 i_L = \frac{1}{2}\delta(t)$$

$i_L(0^+) = i_L(0^-) = 0$

$\frac{di_L}{dt}(0^-) = 0$

$$\int_{0^-}^{0^+} \frac{d^2 i_L}{dt^2} dt - \frac{35}{2}\int_{0^-}^{0^+} \frac{di_L}{dt} dt - 39\int_{0^-}^{0^+} i_L dt = \frac{1}{2}\int_{0^-}^{0^+} \delta(t) dt$$

$\longrightarrow \boxed{\frac{di_L}{dt}(0^+) = \frac{1}{2}}$ $\xrightarrow{\text{معادله مشخصه}} s^2 - \frac{35}{2}s - 39 = 0 \longrightarrow$ پاسخ عمومی $= K_1 e^{19.5t} + K_2 e^{-2t}$

پاسخ ویژه $= 0$

$\longrightarrow i_L(t) = K_1 e^{19.5t} + K_2 e^{-2t}$

$i_L(0^+) = 0 \longrightarrow K_1 + K_2 = 0$

$\frac{di_L}{dt}(0^+) = \frac{1}{2} \longrightarrow 19.5K_1 - 2K_2 = \frac{1}{2} \longrightarrow K_1 = 0.023$, $K_2 = -0.023 \longrightarrow \boxed{i_L(t) = 0.023(e^{19.5t} - e^{-2t})u(t)}$

Scanned with CamScanner

ادامه سؤال 4)

پ)

$$\frac{d^2v_C}{dt^2} - \frac{35}{2}\frac{dv_C}{dt} - 39v_C = -39\delta(t) - 20\delta'(t)$$

$$\int\int_{0^-}^{0^+} \frac{d^2v_C}{dt^2}dt - \frac{35}{2}\int\int_{0^-}^{0^+}\frac{dv_C}{dt}dt - 39\int\int_{0^-}^{0^+} v_C\, dt = \int_{0^-}\int \cancel{-39\delta(t)dt} - 20\int_{0^-}^{0^+}\int\delta'(t)$$

$$\rightarrow v_C(0^+) - v_C(0^-) = -20 \rightarrow \boxed{v_C(0^+) = -20}$$

$$\int_{0^-}^{0^+} \frac{d^2v_C}{dt^2}dt - \frac{35}{2}\int_{0^-}^{0^+}\frac{dv_C}{dt}dt - 39\int_{0^-}^{0^+} v_C\, dt = \int_{0^-}^{0^+} -39\delta(t) + \int_{0^-}^{0^+} -20\delta'(t)dt$$

$$\rightarrow \frac{dv_C}{dt}(0^+) - \cancel{\frac{dv_C}{dt}(0^-)} - \frac{35}{2}\left(v_C(0^+) - \cancel{v_C(0^-)}\right) = -39$$

$$\rightarrow \boxed{\frac{dv_C}{dt}(0^+) = -389}$$

طبق قسمت قبل $\rightarrow$ پاسخ عمومی $= k_1 e^{19.5t} + k_2 e^{-2t}$

پاسخ ویژه $= 0 \rightarrow v_C(t) = (k_1 e^{19.5t} + k_2 e^{-2t})u(t)$

$$v_C(0^+) = -20 \rightarrow k_1 + k_2 = -20$$

$$\frac{dv_C}{dt}(0^+) = -389 \rightarrow 19.5k_1 - 2k_2 = -389 \rightarrow \begin{matrix} k_1 = -19.95 \\ k_2 = -0.047 \end{matrix}$$

$$\rightarrow \boxed{v_C(t) = (-19.95e^{19.5t} - 0.047e^{-2t})u(t)}$$

$$\frac{d^2i_L}{dt^2} - \frac{35}{2}\frac{di_L}{dt} - 39i_L = \frac{1}{2}\delta'(t)$$

طبق روش بالا $\rightarrow \boxed{\frac{di_L}{dt}(0^+) = \frac{35}{4}}, \boxed{i_L(0^+) = \frac{1}{2}}$

طبق قسمت های قبل $\rightarrow i_L(t) = (k_1 e^{19.5t} + k_2 e^{-2t})u(t)$

$$i_L(0^+) = \frac{1}{2} \rightarrow k_1 + k_2 = \frac{1}{2}$$

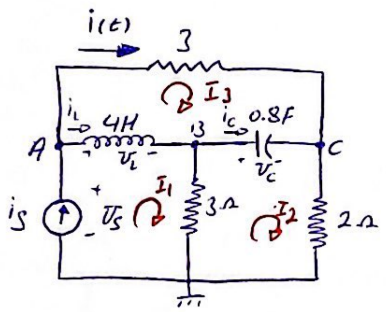
$$\frac{di_L}{dt}(0^+) = \frac{35}{4} \rightarrow 19.5k_1 - 2k_2 = \frac{35}{4} \rightarrow \begin{matrix} k_1 = 0.45 \\ k_2 = 0.05 \end{matrix}$$

$$\rightarrow \boxed{i_L(t) = (0.45e^{19.5t} + 0.05e^{-2t})u(t)}$$

Scanned with CamScanner

سئوال 5)

الف)



$$I_3 = i(t)$$

$$I_1 = i_s$$

$$* \quad v_C + 5I_2 - 3I_1 = 0$$

$$** \quad 3I_3 - v_C - v_L = 0$$

$$v_L = 4\frac{di_L}{dt} \xrightarrow{i_L = i_s - i} v_L = 4\frac{di_s}{dt} - 4\frac{di}{dt}$$

$$v_C = \frac{1}{C}\int i_C = \frac{5}{4}\int I_2 - \frac{5}{4}\int I_3$$

$$\xrightarrow{*} \frac{5}{4}D^{-1}I_2 - \frac{5}{4}D^{-1}I_3 + 5I_2 - 3i_s = 0$$

$$\longrightarrow \boxed{\left(5D + \frac{5}{4}\right)I_2 - \frac{5}{4}i = 3D i_s} \quad (I)$$

$$\xrightarrow{**} 3i - \frac{5}{4}D^{-1}I_2 + \frac{5}{4}D^{-1}i - 4D i_s - 4D i = 0$$

$$\longrightarrow \boxed{i\left(4D^2 + 3D + \frac{5}{4}\right) - \frac{5}{4}I_2 = 4D^2 i_s} \quad (II)$$

$$\xrightarrow[\text{به روش کرامر}]{\text{حل دو معادله دو مجهول}} i = \frac{\begin{vmatrix} 5D+\frac{5}{4} & 3D i_s \\ -\frac{5}{4} & 4D^2 i_s \end{vmatrix}}{\begin{vmatrix} 5D+\frac{5}{4} & -\frac{5}{4} \\ -\frac{5}{4} & 4D^2+3D+\frac{5}{4} \end{vmatrix}} = \frac{20D^3 i_s + 5D^2 i_s + \frac{15}{4}D i_s}{20D^3 + 5D^2 + 15D^2 + \frac{15}{4}D + \frac{25}{4}D + \frac{25}{16} - \frac{25}{16}}$$

$$\longrightarrow i\left(20D^3 + 20D^2 + 10D\right) = i_s\left(20D^3 + 5D^2 + \frac{15}{4}D\right)$$

$$\longrightarrow i\left(D^2 + D + \frac{1}{2}\right) = i_s\left(D^2 + \frac{1}{4}D + \frac{3}{16}\right)$$

$$\longrightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{di}{dt} + \frac{1}{2}i = \frac{d^2 i_s}{dt^2} + \frac{1}{4}\frac{di}{dt} + \frac{3}{16}i_s}$$

Scanned with CamScanner

ادامه سئوال 5)

$i_s = u(t) \xrightarrow{t>0} \frac{d^2 i}{dt^2} + \frac{di}{dt} + \frac{1}{2} i = \delta' + \frac{1}{4}\delta + \frac{3}{16}$

ب)

$\xrightarrow{\text{معادله مشخصه}} s^2 + s + \frac{1}{2} = 0 \rightarrow s = -\frac{1}{2} \pm \frac{1}{2}j$

$\xrightarrow{\text{underdamped}} \boxed{i(t) = K_1 e^{-\frac{1}{2}t} \cos(\tfrac{1}{2}t) + K_2 e^{-\frac{1}{2}t} \sin(\tfrac{1}{2}t)} \rightarrow$ جواب عمومی

$\frac{1}{2}K_3 = \frac{3}{16} \rightarrow \boxed{K_3 = \frac{3}{8}} \rightarrow$ جواب خصوصی

حال باید $i(0^+)$, $\frac{di}{dt}(0^+)$ را بدست آوریم:

KCL @ A : $i = i_s - i_L$

$\rightarrow i(0^+) = i_s(0^+) - i_L(0^+)$

$i_L(0^+) = i_L(0^-) = 0 \rightarrow \boxed{i(0^+) = u(0^+) = 1}$

$\xrightarrow{\text{حال از دو طرف معادله انتگرال گیری می کنیم}} \int_{0^-}^{0^+} \frac{d^2 i}{dt^2} + \int_{0^-}^{0^+} \frac{di}{dt} + \frac{1}{2}\int_{0^-}^{0^+} i = \int_{0^-}^{0^+} \delta' + \frac{1}{4}\int_{0^-}^{0^+} \delta + \frac{3}{16}\int_{0^-}^{0^+} \frac{3}{16}$

$\rightarrow \frac{di}{dt}(0^+) - \cancel{\frac{di}{dt}(0^-)}^{0} + \cancel{i(0^+)}^{1} - \cancel{i(0^-)}^{0} = 0 + \frac{1}{4} \rightarrow \boxed{\frac{di}{dt}(0^+) = -\frac{3}{4}}$

$i(t) = K_1 e^{-\frac{1}{2}t} \cos(\tfrac{1}{2}t) + K_2 e^{-\frac{1}{2}t} \sin(\tfrac{1}{2}t) + \frac{3}{8}$

$i(0^+) = 1 \rightarrow K_1 + \frac{3}{8} = 1 \rightarrow \boxed{K_1 = \frac{5}{8}}$

$\frac{di}{dt}(0^+) = -\frac{3}{4} \rightarrow \left(-\frac{1}{2}K_1 e^{-\frac{1}{2}t}\cos(\tfrac{1}{2}t) - \frac{K_1}{2} e^{-\frac{1}{2}t}\sin(\tfrac{1}{2}t) - \frac{1}{2}K_2 e^{-\frac{1}{2}t}\sin(\tfrac{1}{2}t) + \frac{K_2}{2} e^{-\frac{1}{2}t}\cos(\tfrac{1}{2}t)\right)\Big|_{0^+} = -\frac{3}{4}$

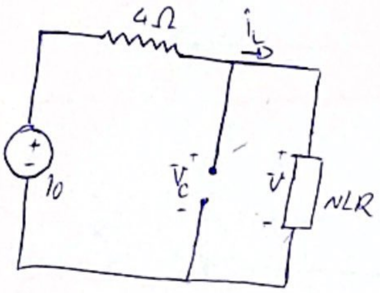
$\rightarrow -\frac{K_1}{2} + \frac{K_2}{2} = -\frac{3}{4} \rightarrow -\frac{5}{16} + \frac{K_2}{2} = -\frac{3}{4} \rightarrow \boxed{K_2 = -\frac{7}{8}} \rightarrow \boxed{i(t) = \frac{5}{8}e^{-\frac{t}{2}}\cos(\tfrac{t}{2}) - \frac{7}{8}e^{-\frac{t}{2}}\sin(\tfrac{t}{2}) + \frac{3}{8}}$

Scanned with CamScanner

سؤال 6)

الف)

در $t=0$ سلف اتصال کوتاه شده است و خازن مدار باز:



$$KVL: -10+4i_L+V=0$$

$$\rightarrow \text{if } i_L<0: -10+4i_L+0=0 \rightarrow i_L=2.5 \times$$

$$\text{if } i_L \geq 0: -10+4i_L+\frac{1}{2}i_L^2=0 \begin{cases} \rightarrow \boxed{i_L=2} \checkmark \\ \rightarrow i_L=-10 \times \end{cases}$$

$\rightarrow$ $\boxed{\text{NLR نقطه کار}: (V,i)=(2,2)}$

$$V_C=V \rightarrow V_C=2 \rightarrow q=\frac{1}{10}2^3+0.8\times 2=2.4$$

$\rightarrow$ $\boxed{\text{NLC نقطه کار}: (q,V)=(2.4,2)}$

$$i_L=2 \rightarrow \varphi=\sqrt[3]{\frac{1}{2}\times 2}=1$$

$\rightarrow$ $\boxed{\text{NLi نقطه کار}: (\varphi,i_L)=(1,2)}$

ب)

تقریب خطی زدن عناصر، حول نقطه کار:

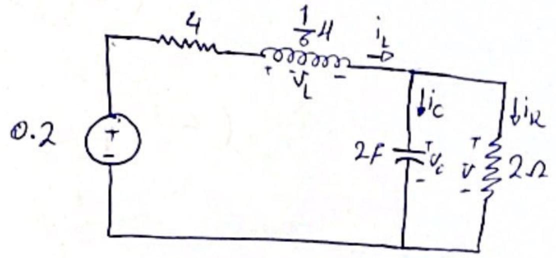
$$R=\frac{dV}{dt}\Big|_{(2,2)}=i\Big|_{(2,2)}=\boxed{2\,\Omega}$$

$$C=\frac{dq}{dt}\Big|_{(2.4,2)}=0.3V^2+0.8\Big|_{(2.4,2)}=\boxed{2F}$$

$$L=\frac{d\varphi}{dt}\Big|_{(1,2)}=\frac{1}{2}\times\frac{1}{3}\times\left(\frac{1}{2}i_L\right)^{-\frac{2}{3}}\Big|_{(1,2)}=\boxed{\frac{1}{6}H}$$

Scanned with CamScanner

ادامه سؤال 6 قسمت ب)



حل مدار برای $t>0$:

(مدل سیگنال کوچک: فقط تاثیر منبع AC $\leftarrow 0.2$

$$KVL:\ -0.2+4i_L+v_L+v_C=0$$

$$\left.\begin{array}{l} \\ i_L=i_C+i_R=C\frac{dv_C}{dt}+\frac{v}{2}\end{array}\right\} \rightarrow -0.2+8\frac{dv_C}{dt}+2v+v_L+v_C=0$$

$$v_L=L\frac{di_L}{dt}=\frac{1}{6}\left(\frac{di_C}{dt}+\frac{dv}{dt}\times\frac{1}{2}\right)=\frac{1}{3}\frac{d^2v_C}{dt^2}+\frac{1}{12}\frac{dv}{dt}$$

$$\rightarrow -0.2+8\frac{dv_C}{dt}+2v+\frac{1}{3}\frac{d^2v_C}{dt^2}+\frac{1}{12}\frac{dv}{dt}+v_C=0$$

$$v_C=v \rightarrow 3v_C+\frac{97}{12}\frac{dv_C}{dt}+\frac{1}{3}\frac{d^2v_C}{dt^2}=0.2$$

$$\rightarrow \boxed{\frac{d^2v_C}{dt^2}+\frac{97}{4}\frac{dv_C}{dt}+9v_C=0.3u(t)=0.6 \quad t>0}$$

$v_C(0^+)=0$

=> $t=0^+$ شرایط اولیه را باید صفر بگیریم (تغییرات حول سیگنال کوچک)

$$i_L(0^+)=0 \rightarrow \frac{dv_C}{dt}(0^+)=i_L(0^+)-\frac{v_C(0^+)}{2}=0$$

$$\rightarrow S^2+\frac{97}{4}S+9=0 \rightarrow S_1\simeq-23.9 \quad S_2\simeq-0.38$$

پاسخ ناهمگن $\rightarrow 9K_3=0.6=\frac{1}{15}$

$$\rightarrow \underset{t>0}{v_C(t)}=K_1e^{-23.9t}+K_2e^{-0.38t}+\frac{1}{15}$$

اثر منبع dc

$$\rightarrow \boxed{\underset{t>0}{v(t)}=0.001e^{-23.9t}-0.068e^{-0.38t}+\frac{1}{15}+2} \quad t>0$$

$$\left.\begin{array}{l} v_C(0^+)=0 \rightarrow K_1+K_2+\frac{1}{15}=0 \rightarrow K_1+K_2=-\frac{1}{15} \\ \frac{dv_C}{dt}(0^+)=0 \rightarrow -23.9K_1-0.38K_2=0 \end{array}\right\} \rightarrow \begin{array}{l}K_1=0.001\\ K_2=-0.068\end{array}$$

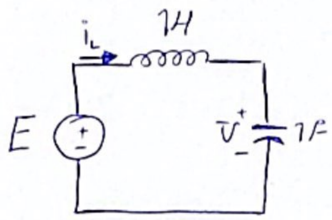
Scanned with CamScanner

سئوال 7)

$t_1 > t > 0$: D:on

t_1 زمانی است که دیود قطع می‌شود

$E > 0$



$i_L = \frac{dV}{dt} \rightarrow V_L = \frac{d^2V}{dt^2}$

KVL: $E = V_L + V = \frac{d^2V}{dt^2} + V$

$\rightarrow V = K_1 \cos t + K_2 \sin t + K_3$

پاسخ ناهمگن $\rightarrow K_3 = E \rightarrow V = K_1 \cos t + K_2 \sin t + E$

$V(0^+) = 0 \rightarrow K_1 + E = 0 \rightarrow K_1 = -E$

$\frac{dV}{dt}(0) = 0 \rightarrow K_2 = 0 \rightarrow \boxed{V(t) = E(1 - \cos t)}$

تا زمانی که دیود روشن می‌باشد

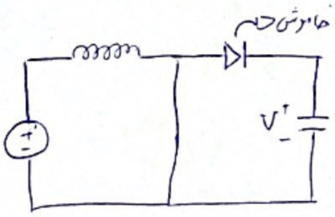
$\boxed{t_1 = \pi}$

D: off $\rightarrow i_D \leq 0 \rightarrow i_L = E \sin t \leq 0 \rightarrow t \geq \pi \rightarrow$ در $t = \pi$ دیود خاموش می‌شود.

$V(t) = 2E$
$i_L(t) = 0$

حال در زمان $\pi \leq t \leq 4$ مدار ثابت مانده است:

در این بازه دیود قطع خواهد بود



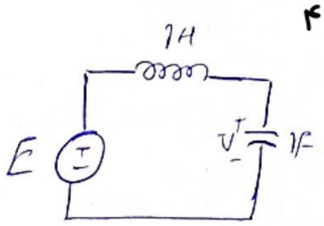
مدار در $4 \leq t \leq 5$:

$v_c(t) = v_c(4^+) = v_c(\pi) \quad 4 < t < 5$

KVL: $-E + \frac{di_L}{dt} = 0 \rightarrow \frac{di_L}{dt} = E \rightarrow i_L(t) = Et + K_4$

$i_L(4^-) = 0 \rightarrow 4E + K_4 = 0 \rightarrow K_4 = -4E$

$\rightarrow \underbrace{i_L(t) = E(t-4)}_{4<t<5} \rightarrow \boxed{i_L(5^-) = E}$



$V(5^-) = 2E$
$i_L(5^-) = E = \frac{dV}{dt}(5^-)$

برای $t_2 > t \geq 5$

t_2 زمانی است که دیود قطع می‌شود و $4 < t < t_2$

$\frac{d^2V}{dt^2} + V = E \rightarrow V_h = K_1 \cos(t-5) + K_2 \sin(t-5)$
$V_p = K_3 \rightarrow K_3 = E$

فرض می‌کنیم بعد از $t=5$ دیود هدایت می‌کند. مسئله را با این فرض حل کنیم. اگر جریان دیود در این بازه مثبت بود فرض هدایت درست است و تا وقتی جریان دیود مثبت است، فرض هدایت برقرار است (وگرنه با فرض قطع دیود حل می‌کنیم)

Scanned with CamScanner

ادامه سؤال 7)

$$v(t) = K_1 \cos(t-5) + K_2 \sin(t-5) + E \qquad \text{۵} < t < t_{\text{۲}}$$

$$v(5^-) = 2E \longrightarrow K_1 + E = 2E \longrightarrow \boxed{K_1 = E}$$

$$\frac{dv}{dt}(5^-) = E \longrightarrow \boxed{K_2 = E}$$

$$\longrightarrow v(t) = E\left(\cos(t-5) + \sin(t-5) + 1\right) \qquad \text{۵} < t < t_{\text{۲}}$$

$$\longrightarrow v(t) = E\left(\sqrt{2}\sin\left(t-5+\frac{\pi}{4}\right) + 1\right) \longrightarrow \frac{dv}{dt} = i_L = \sqrt{2}\,E\cos\left(t-5+\frac{\pi}{4}\right) \geqslant 0$$

$$\longrightarrow \cos\left(t-5+\frac{\pi}{4}\right) \geqslant 0 \xrightarrow{t \geqslant 5} 5 \leqslant t \leqslant 5+\frac{\pi}{4} \longrightarrow \boxed{t_{\text{۲}} = \text{۵} + \frac{\pi}{\text{۴}}}$$

که در این زمان جریان 0 می‌شود و دیود خاموش می‌شود بنابراین بعد آن شرایط ثابت می‌ماند.

جمع بندی نهایی:

$$v(t) \begin{cases} E(1-\cos t) & 0 \leqslant t \leqslant \pi \\ 2E & \pi \leqslant t \leqslant 5 \\ E\left(\sqrt{2}\sin\left(t-5+\frac{\pi}{4}\right)+1\right) & 5 \leqslant t \leqslant 5+\frac{\pi}{4} \\ E(\sqrt{2}+1) & 5+\frac{\pi}{4} \leqslant t \end{cases}$$

Scanned with CamScanner