MATHEMATICAL QUESTIONS

Question 1

For the circuit of Fig. 1,

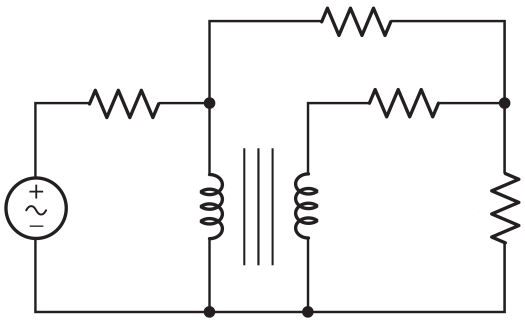

Figure 1: A sample circuit.

(a) Draw the circuit graph.

(b) Find a reduced node-to-branch incident matrix $\mathbf{A}$.

(c) Find the reduced mesh-to-branch incident matrix $\mathbf{M}$.

(d) Find a fundamental cut-set matrix $\mathbf{Q}$.

(e) Find a fundamental loop matrix $\mathbf{B}$.

(f) Can you introduce a tree for which the matrices $\mathbf{A}$ *and* $\mathbf{Q}$ *are equal?*

Question 1 continued on next page...

(g) Can you introduce a tree for which the matrices $\mathbf{M}$ *and* $\mathbf{B}$ *are equal?*

Question 2

Prove that the branch voltages of a tree of a given circuit graph provide a set of linearly independent voltages.

Question 3

The circuit of Fig. 2 includes LTI resistors and a voltage source. In an experimental measurement, we set $R_1 = 1\,\Omega$**, and find that** $v_1 = 4$ **V,** $i_1 = 1$ **A, and** $v_2 = 1$ **V. In a second measurement, we set** $R_1 = 2\,\Omega$**, and find that** $v_1 = 2$ **V and** $i_1 = 1.2$ **A, but we forget to measure** v_2**. Can you determine the value of** v_2 **in the second experiment? The inside of the sub-circuit** N **remains unchanged for the two experiments.**

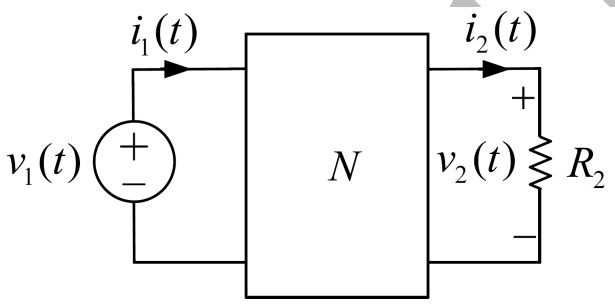

Figure 2: An LTI resistive network with a driving voltage source.

Question 4

Draw the dual circuit of the circuit shown in Fig. 3 and write at least two dual circuit equations for the two circuits.

Question 4 continued on next page...

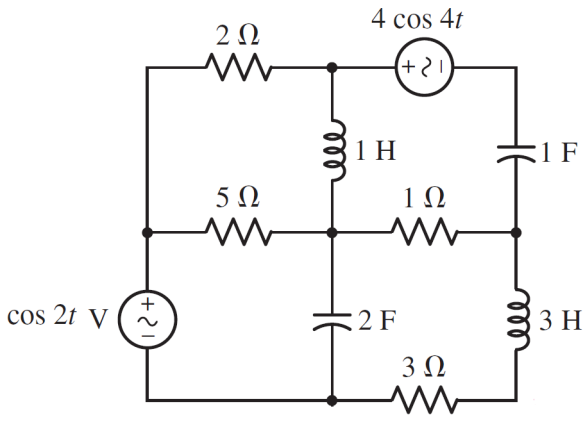

Figure 3: A circuit for which the dual network is required.

Question 5

Write the KCL and KVL equations corresponding to the fundamental cut sets and loops of the circuit graph shown in Fig. 4 having the highlighted tree.

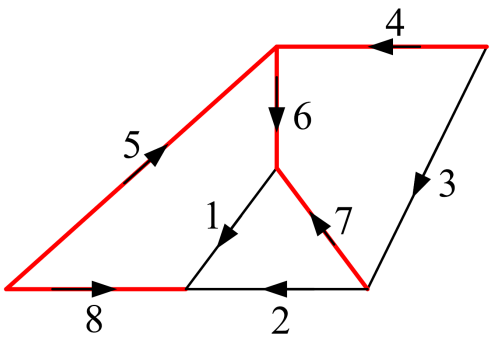

Figure 4: A circuit graph and one of its associated trees.

Question 6 continued on next page...

Question 6

Draw a directed graph whose node-to-branch incidence matrix $\mathbf{A}_a$ is given by

$$\mathbf{A}_a = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

SOFTWARE QUESTIONS

Question 7

Dijkstra's conventional algorithm is a systematic method to find the shortest path between two given nodes of a weighted graph. However, a more common variant of the algorithm fixes a single node as the reference node and finds shortest paths from the source to all other nodes in the graph, producing a shortest-path tree. Implement Dijkstra's algorithm as a MATLAB function and use it to find a tree of a given connected circuit graph.
Note: A circuit graph is a special weighted graph, where all the edges have a same weight.
Note: A graph can be represented by a matrix. In fact, for the graph $G(\mathbf{N} = \{1, 2, \cdots, n\}, \mathbf{E})$ with n node, the representing matrix of the graph is $A_{n\times n|} = [a_{ij}]$, where a_{ij} is 1 if $(i, j) \in \mathbf{E}$, and 0 otherwise.

BONUS QUESTIONS

Question 8

Return your answers by filling the LaTeXtemplate of the assignment. If you want to add a circuit schematic, you can draw it directly using TikZ package, or draw it in a secondary application such as Microsoft Visio and then, import it as a figure.

EXTRA QUESTIONS

Question 9

Feel free to solve the following questions from the book *"Basic Circuit Theory"* by C. Desoer and E. Kuh.

1. **Chapter 9, question 1.**
2. **Chapter 9, question 3.**
3. **Chapter 9, question 4.**
4. **Chapter 9, question 9.**