An Intelligent Hybrid Algorithm for Urban Waste Collection Problem using Rank based Ant Colony System

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Abstract

This paper presents the application of ant heuristics to solve an urban waste collection problem in the real world scale. The basic nature of the considered problem is that of a capacitated arc routing problem, although it has several specific characteristics, mainly derived from traffic regulations. The PILP model of the problem is defined in this paper; considering numerous variables and constraints involved in real size problems and many practical limitations such as calling close and one way streets which are difficult to include in classic models, we propose a practical solution method which consists of three components: Zoning Algorithm, Route Generating Algorithm and Combination Algorithm, where ant heuristics are applied in the last two algorithms. The ant heuristics type is that of ACS_{rank} *which uses elite ants to rank the best tours in order to increase the probability of selecting the most suitable routes in each iteration and to reduce time to reach the best solution. The experimental results on real world problems, using data from 3 separate areas of Tehran, showed the superiority of the solution generated by the proposed intelligent algorithm to the solution generated by human experts.*

1- **Introduction:**

Urban waste management is a critical issue in urban management so that any deficiency in which can cause irreparable damages to the environment, hygiene and aesthetics of cities. Waste management consists of six processes: waste generation, local movement, collection, transportation, recycling and land filling. Waste generation process deals with factors affecting the generation of wastes. It also examines different types and combination of urban solid waste. The second process, local movement, includes three main activities of displacing (moving wastes to storing places), holding (time and way of keeping wastes before collection) and processing (doing any type of physical, chemical or biological process on wastes such as separation of paper and glass from waste). Waste collection consists of picking up wastes and loading them into collection vehicles. In Tehran present waste management system, which uses transition stations, waste collection consists of picking up wastes, loading them into collection vehicles and transferring and discharging them to transition stations. The main purpose of constructing transition stations is to collect wastes by means of smaller vehicles and transferring them to much bigger vehicles in these stations. Also, some operations such as compression and recycling are done in these stations. Now, 447 vehicles with a daily 2500 trips collect 12 to 18 tons of waste per vehicle throughout Tehran. Transportation is the forth process of waste management and applied to equipment used to transfer wastes to a longer distance for the process of recycling or land filling. Recycling is any method that causes physical or chemical transformation of wastes. Recycling can also be done in collection or storing stages. In the land filling process, wastes are buried or incinerated in a predetermined area. Considering the environmental factors is of major importance in this process.

Based on latest available data, the amount of waste generated in Tehran has an increasing trend during the past 15 years and amount of waste in 2004 was 2,626,519 tons, of which 2,570, 988 tons (97%) are urban solid waste. Each night 7000 tons of wastes are generated in 128 districts of Tehran with the population of about 8 million. These wastes are collected from houses by three types of vehicles and transferred to transition stations and then to landfill areas in southern Tehran, Kahrizak. Tehran area, mainly in residential streets, is equipped with 90Kg containers located in 150-meter distances of each other. Vehicles are allocated according to the capacity of vehicles and the amount of waste generated in each district. It should be noted that reducing the costs of waste collection will have a considerable impact on total waste management system costs. It can be gained through reducing the total distance traversed by the collection vehicles that finally leads to reduction in fuel and labor costs and environmental damages caused by air pollution.

The problem being considered is of arc routing type since in the actual circumstances roads are traversed completely, even if collection points are located in specific points of the roads. In addition, the capacity of the trucks is limited. Thus, the basic nature of the problem is that of a capacitated arc routing problem (CARP). Although CARP has not been so extensively studied as its node routing counterpart, there is still a rather vast literature on arc routing problems (Dror, 2001) and also on CARP in particular (Golden, 1981; Laporte, 1983; Belenguer, 1998).

Nevertheless, the problem at hand presents several characteristics that make it different from classical arc routing problems. These characteristics are mainly derived from traffic regulations. The underlying graph of the problem is a mixed one with directed arcs, since in reality some streets can be traversed in both directions and some in only one direction. At the junctions street some turns are forbidden, and some are allowed. Bautista (2007) has taken into accounts these constraints. These issues have been addressed theoretically by Laccomme, Prins and Ramdane-Chérif (2001) and by Belenguer et al. (2006). Problems without forbidden turns have been considered for routing and scheduling street sweepers in Bodin and Kursh (1979) and for the Rural Postman Problem on mixed graphs with turn penalties in Corberán et al. (2002), where the authors propose a solution approach based on penalizing forbidden turns and post-processing illegal solutions. Roy et al. (1989) customized the formulation of the Capacitated Chinese Postman Problem for the context of the Canada Post Corporation and a general heuristic was presented to solve the problem under several hypotheses with penalties assigned to forbidden turns.

Section 2, proposes the mathematical representation of the problem. Section 3, overviews the concept of ant system; and section 4 proposes the intelligent solution method, which is a rank-based ant colony system. Then, in section 5, the proposed method is applied to three areas (districts) of Tehran and the results are used to estimate total improvement and savings which can be achieved if the proposed algorithm is applied to all (128) districts of Tehran.

2- **Problem Definition**

Introduction

Major part of waste collection costs consists of operational costs e.g. labor, fuel and maintenance costs. The aim of this modeling work is to determine the routes of collection in such a way that:

1- Total distance traversed in each district is reduced; this can lead to reduction in collection time and fuel costs.
2- Each vehicle collects waste up to its maximum possible capacity.

Mathematical Representation:

Each zone can be presented as in figure 1, in which nodes i, j and k indicate the inner nodes (waste containers) of zones and nodes O and D indicate depot and transition station respectively. These nodes can also represent the neighboring nodes in other zones. Two entrant arcs to node j indicate that the vehicle which collects this node comes either from the depot or another node such as i. Two exiting arcs, similarly to the entrant arcs, indicate that the vehicle which already collected node j, will depart to transition station or another neighboring node such as k. We assume that only one depot and one transition station are used to start and end waste collection operations.

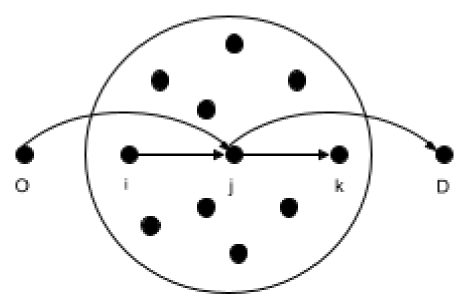

Figure 1- schematic presentation of zones

Thus, decision variables and parameters are defined as follows:

$$X_{ijt} = \begin{cases} 1 & \text{vehicle t travels from i to j.} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{Ojt} = \begin{cases} 1 & \text{vehicle t comes from O and starts collecting from j} \\ 0 & \text{otherwise} \end{cases}$$

$$X_{iDt} = \begin{cases} 1 & \text{vehicle t finishes collecting in node i and travels to D} \\ 0 & \text{otherwise} \end{cases}$$

d_{ij} travel cost between i,j
m No. of vehicles
C_t vehicle capacity in terms of the No. of containers
J No. of containers in the area

The objective function minimizes total distance traversed by collecting vehicles. Other operational costs such as labor and fuel can also be considered in the objective function. Therefore, the mathematical model can be formulated as below:

$$\text{Min } \left(\sum_i \sum_j d_{ij} \sum_t X_{ijt} + \sum_t \sum_j X_{Ojt}.d_{Oj} + \sum_t \sum_i X_{iDt}.d_{iD} \right)$$

s.t.

$$\sum_i \sum_j X_{ijt} = C_t \qquad \forall \ t = 1, \dots , m \ \text{ and } \ i \neq j \qquad (1)$$

$$\sum_t \left(\sum_i X_{ijt} + X_{Ojt} \right) = 1 \qquad \forall \ i,j=1,\dots,J \ \text{ and } \ i \neq j \qquad (2)$$

$$\sum_t \left(\sum_k X_{jkt} + X_{jDt} \right) = 1 \qquad \forall \ i,j=1,\dots,J \qquad (3)$$

$$\sum_i X_{ijt} + X_{Ojt} = \sum_k X_{jkt} + X_{jDt} \qquad \forall \ i,j=1,\dots,J \ \text{ and } \ \forall t=1,\dots,m \qquad (4)$$

Constraint (1) takes into account the limited capacity of vehicles. It is worth mentioning that we assume that the waste generation rate of households is constant. According to the capacity of vehicles (3, 6 and 9 tons) and the average weight of waste in each container (80 Kg), the smallest group of containers to be covered by a 3-ton vehicle is made up of 40 containers. If 6-ton and 9-ton vehicles are used, then 80 and 120 containers can be covered by a vehicle respectively.
Constraint (2), (3) and (4) ensures us that all nodes are met by the vehicles in a continuous route. The first two parts indicate the continuity and the third part indicates that each node is met only by one vehicle.

Complexity
The mathematical model of the problem is of PILP type with three decision variable ad four constraint types. Assume that 500 containers (nodes) are covered by 5 collection vehicles; then the model includes 1,255,000 decision variables and 2,505,000 constraints. On the one hand, solving this problem using traditional methods in a reasonable time is not possible. On the other hand, the mathematical model does not represent certain practical constraints such as one-way streets and dead-ends. Solving the problem, which is NP-hard in nature, for many nodes is time-consuming task which results in impractical solutions.
Consequently, in this paper, we introduce a solution method in which a specific type of ant colony heuristic is used to yield appropriate practical solution.

3- The Ant Heuristics

The Ant System (AS) is a new approach for NP-hard optimization problems. AS is a population-based approach that exploits from positive feedbacks generated in each iteration. It was first proposed to solve the Traveling Salesman Problem (TSP), but has been successfully applied to problems such as quadratic assignment, job-shop scheduling and arc routing. The basic idea of AS algorithm is inspired by observation of a colony of Argentinean ants. In their first experiments, Goss et al. (1989) found out that ants leave a trail of pheromone behind so that other ants can observe the trail and follow the path. Moving ants will follow the pheromone trail with high probability and this is the way how a trial is reinforced and more ants follow that trail. As a result, in the constant time interval, a colony of ants deposits a greater amount of pheromone on shorter path between two points. AS algorithm was first introduced by Dorigo et al. (1991) for solving combinatorial optimization problems. This method uses a colony of artificial ants which choose their ways in a network by using a probability that is higher on shorter paths. Also, there is an evaporation mechanism in this algorithm, as in real world, which corrects previous pheromone data along the time. Evaporation let the ants to look for better new paths by gradually clearing past history.

State transition rule, which makes balance between exploitation of pheromone trail data and exploring for new paths, was assumed in Ant Colony System (ACS) algorithm. According to this rule, by defining a q_0 q_0 and generating a random number between 0 and 1 and comparing the random number to q_0 q_0, one of exploitation or exploration approaches is selected. q_0 q_0 indicates the relative advantage of exploitation over exploration. The closer is q_0 q_0 to 1, the greater is the probability of exploiting from previous good solutions and the closer is q_0 q_0 to 0, the lesser is the probability of searching for better solutions is. In this paper we have used these formulas for the state transition rule:

$$a_{ij} = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}} \qquad \forall_j \in N_i$$

$$S=\begin{cases}\arg Max\left\{[\tau(r,u)].[\eta(r,u)]^{\beta}\right\} & if\ q<q_0 \\ s & O.W.\end{cases} \qquad \text{euquation (1)}$$

In this paper, rank-based ant colony systems (ACS_{rank}) is used that emphasizes on the best tours in each iteration. This method allocates more pheromone to the tours traversed by elite ants when updating pheromone trails. Ants are ranked according to their degree of elitism that itself depends on the length of the ants` tours. The basic assumption is that probably some arcs of elite ants` tours are part of optimum route. Ranking can be used as follows: After all *m* ants finish their tours, ants will be put in order according to the length of their tour: $L_1 \leq L_2 \leq \ldots \leq L_m$. The impact of each ant in updating pheromone trails is determined by the rank (μ) of that ant. Additionally, only *w* of ants is considered in each iteration. As a result, the danger of excessive emphasis on pheromone trail of the ants traveled sub-optimum routes will be restrained. After all, this method utilized the combination of ranking and elitism to reach the optimum solution in a shorter time. The formulation and parameters of this method are as follows:

Attractiveness

can be written by this formula:

$$\eta_{ij}=\frac{1}{d_{ij}}$$

Trail Update

$$\tau_{ij}(t+1)=\rho\tau_{ij}(t)+\Delta\tau_{ij}+\Delta\tau_{ij}{}^{*} \qquad \text{euquation (2)}$$

$$\Delta\tau_{ij}=\sum_{\mu=1}^{\sigma-1}\Delta\tau_{ij}^{\mu}$$

$$\Delta\tau_{ij}^{\mu}=\begin{cases}(\sigma-\mu)\dfrac{Q}{L_{\mu}} & if\ edge\ (i,j) is\ part\ of\ the\ best\ solution\ found \\ 0 & O.W.\end{cases}$$

$$\Delta\tau_{ij}{}^{*}=\begin{cases}\sigma\dfrac{Q}{L^{*}} & if\ edge\ (i,j) is\ part\ of\ the\ best\ solution\ found \\ 0 & O.W.\end{cases}$$

In these formulas, $\Delta\tau_{ij}^{\mu}$ $\Delta\tau_{ij}^{\mu}$ is the increased amount of pheromone on arc (i,j) deposited by the μ μ-th best ant, L_{μ} L_{μ} is the tour length of μ μ-th best ant, $\Delta\tau_{ij}{}^{*}$ $\Delta\tau_{ij}{}^{*}$ is the increased amount of pheromone on arc (i,j) deposited by all elite ants, σ is the number of elite ants and L^* L^* is the length of best tour traveled by elite ants.

In all ants methods and algorithms, there are parameters which determine the direction of those methods. In addition to those parameters, there are also some strategies, such as the methods of updating pheromone trails, that affect the nature of the solution method. As mentioned before, in this paper, the strategy of ranking elite ants is applied to reach the solution. Following some of parameters used in the proposed method of paper are defined:

- *Number of Ants*: In this method the number of ants was assumed proportional to the number of containers.
- α, β α, β: These two parameters respectively demonstrate the attractiveness of η and τ to each other.
- *Number of Iterations*: In this method $I^{\max}$ $I^{\max}$ is also multiple of the number of containers.
- q_0 q_0: This parameter indicates the relative advantage of exploitation over exploration.

4- The Proposed Solution Method:

The solution method presented in this paper solves the problem in three stages and thus it comprises of three algorithms: zoning, route generating and combination algorithm (Figure-2). The relationship of these algorithms are as follows:

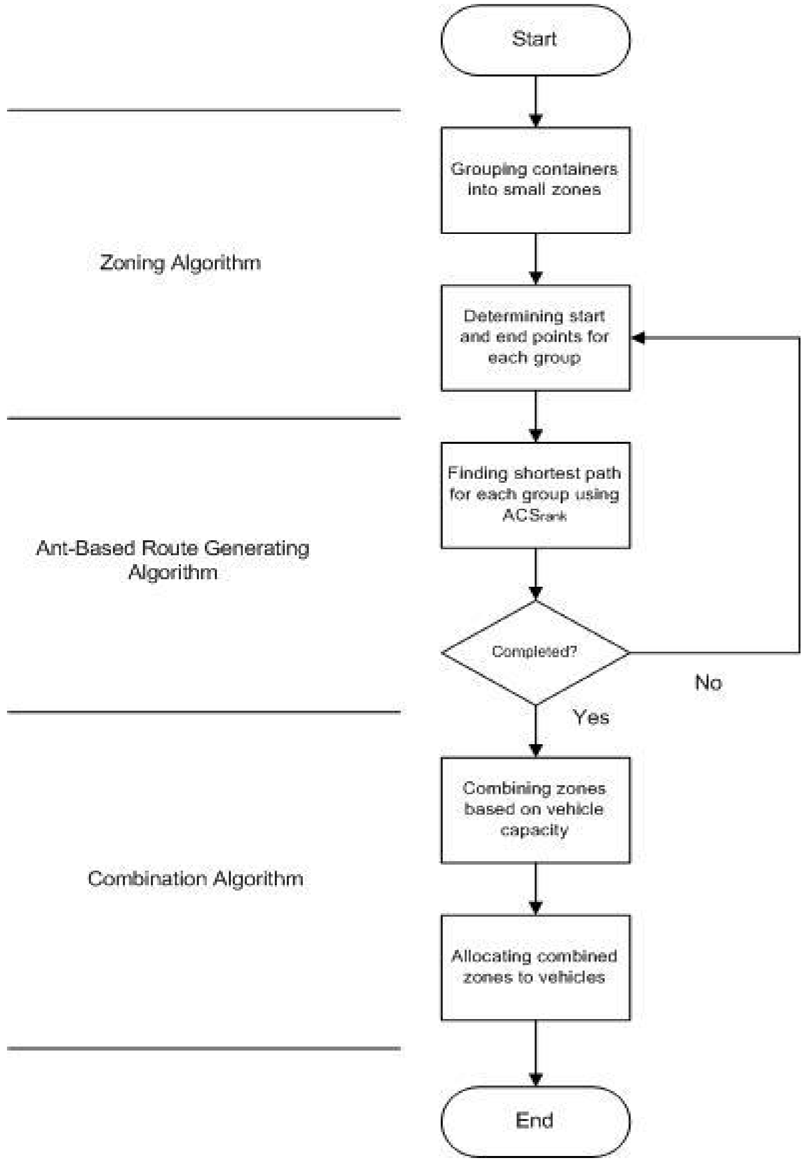

Figure 2- The Structure of Proposed Intelligent Algorithm

Zoning Algorithm: Considering the NP-hard nature of the problem, this algorithm divides each area to small zones in order to reduce the computation time for solving the problem. To do so, first, all the nodes and their neighboring points are determined and stored in a database (neighboring database). Two points are neighbors only if there exists a connecting route between them in real world. Then, each area is divided into small zones according to the neighboring database. Each small zone is distinguished by the RGB function which allocates a specific RGB color to each zone. At last, the boarder points of each zone are defined. A boarder point is a candidate for start or end of the waste collection process within the zone.

Route Generating Algorithm: This algorithm, for each zone, finds the shortest path between each pair of potential start and end points of the operation. The rank-based ant colony system is used here to find the shortest path in each zone. For generating routes, all the paths in each zone should start from a start or end candidate node. A scheme of the ant heuristic algorithm applied in this phase is given below:

The stopping criteria of this algorithm are as follows:

- When a path meet an end candidate node, it is asked whether all the zone`s nodes are included in the path. If so, this path is valid and stored to be used in the next step and If not, the algorithm continues to complete the path.
- When the length of the path reaches to 2.5 times as much as the number of nodes in each zone, the path will be deleted.
- When a loop of nodes like 123123 or 1234512334512345 is repeated, the path is deleted.
- If the paths reaches to its starting node and length of the path is more than twice as much as the number of nodes in each zone, the path will be deleted. If not, the algorithm continues.

At last, this algorithm gives the shortest path in each zone between a pair of start and end candidate node.

Combination Algorithm: Considering the capacity of each given vehicle in terms of the maximum number of bins to cover, the Combination algorithm determines an appropriate combination of the neighboring zones to be covered by a given vehicle. To do so, Zones are assumed as cells which are going to put together according to the best tour reached in the previous stage. This algorithm, as the previous algorithm, uses rank-based ant colony system to combine the zones. The main difference is that the length of the strings is constant, because the zones are combined in proportion to the capacity of the vehicle such that the total distance traversed is minimized. As a result, the stopping criteria are length of the strings. To reach a better answer, this algorithm first finds combinations for the vehicles with the most capacity and then for the lighter ones respectively. This results in more fuel cost reduction, because heavier vehicles needs less to discharge their loads in transition stations and less distance is traversed as a result. Therefore, finding longer strings first, gives in much improved solutions. Finally, this algorithm gives the shortest path in each area which consists of some smaller zones.

5- **Computational Results:**

In this section, the experimental results of applying the proposed solution method to three districts of Tehran, with an approximate population of 152,000 inhabitants and area of 17.77 km^2 are presented. The results are briefly illustrated under some indices mostly related to operational costs of urban waste collection. Also, the indices are compared with those of the real world practice. It should be noted that real world solution for the waste collection process is based on a combination of experience and intuition where the aim is to minimize efforts and resources required to accomplish a specified task.

The geographical map representing the test problems is depicted in Fig. 3. These areas totally include 795 waste containers (nodes) and 5 collection vehicles with the capacities of 3, 6 and 9 tons are used in each area. The specifications of the problems being condsidered are briefly introduced in Table 1.

Table 2- specifications of the problems solved

	District ID No.	Area(km^2)	% (of Tehran)	No. of containers	No. of collection vehicles	population
P_1	1	11.5	18	400	5	98558
P_2	3	4.00	6.25	210	5	34261
P_3	4	2.27	3	185	5	19454

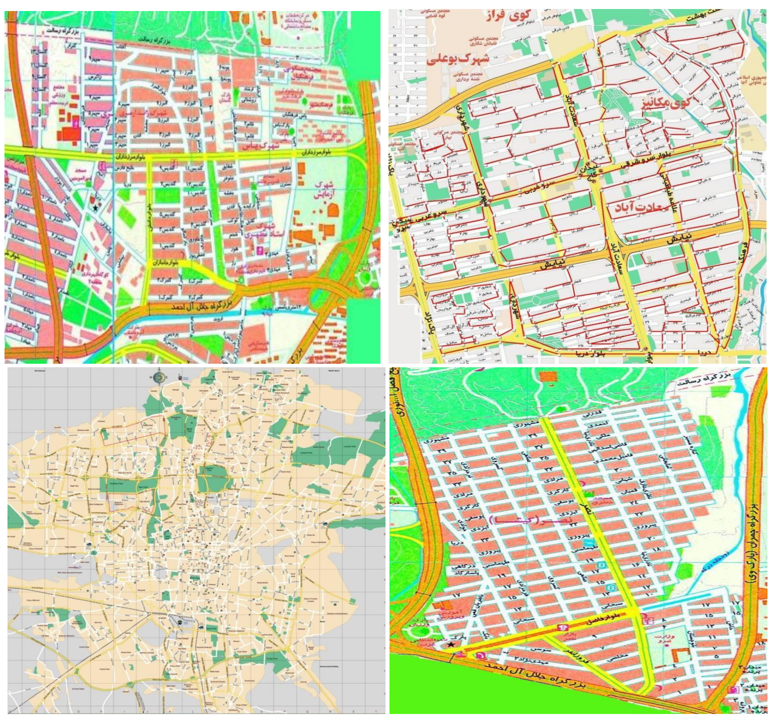

Figure 3- Geographical map of the areas studied

The economic performance of the proposed method is measured using the distance traversed, objective function of the mathematical model as well as reduction in fuel costs and number of vehicles required where these two are caused by improvement in the former one. This is elaborated as follows:

1- *Distance traversed each year:* this index compare total distance travelled by each vehicle in real practice with that of the proposed algorithm.
2- *Fuel consumption in one year:* assuming the fuel consumption of heavy vehicles used in waste collection process is 40 liters per each 100 kilometers, we calculate total reduction in fuel consumption for total distance travelled by each vehicle.
3- *Vehicle purchasing costs:* formulas introduced below are used to calculate reduction in vehicle purchasing costs:

average yearly distance covered by a vehicle= (distance travelled in a year)/(No. of existing vehicles in the area)

optimum No. of vehicles= (optimum distance travelled in a year)/ (average yearly distance covered by a vehicle)

improvement ratio= (No. of vehicles in the area – optimum No. of vehicles)× vehicle price

4- *Human resource costs:* we assume that the average yearly salary of each worker is 36 million Rials and 5 workers are allocated to each vehicle; total reduction in labor costs is derived from improvement in the number vehicles needed to be purchased.

The results of the proposed solution method for the three areas and the improvement obtained through running the algorithm compared with the real world practice are shown in Table 3, below.

Table3- A - Experimental results for the areas

	Distance Travelled (meter)			Fuel Consumption(liter)		
	Real World	ACS_{rank}	Improvement(%)	Real World	ACS_{rank}	Improvement
P_1	148,687	127,342	14.3	21,708	18,592	3116
P_2	51,271	44,041	14.1	7,486	6,429	1056
P_3	40,888	35,327	13.6	5,790	5,157	812

Table 3- B - Numerical Results for sample areas

Vehicle Purchasing Costs (Rial)	Labor Costs (Rial)

	Real World	ACS_{rank}	Improvement	Real World	ACS_{rank}	Improvement
P_1	$5 \times$ vp*	$4.28 \times$ vp	$0.72 \times$ vp	$5 \times 36\times10^6$	$4.28 \times 36\times10^6$	$0.72 \times 36\times10^6$
P_2	$5 \times$ vp	$4.3 \times$ vp	$0.70 \times$ vp	$5 \times 36\times10^6$	$4.30 \times 36\times10^6$	$0.70 \times 36\times10^6$
P_3	$5 \times$ vp	$4.31 \times$ vp	$0.69 \times$ vp	$5 \times 36\times10^6$	$4.31 \times 36\times10^6$	$0.69 \times 36\times10^6$

* vp: vehicle price

This algorithm has been coded in VB version 6.0 and the program has been run on a PC with CPU AMD Athlon 64, 3000+ MHz and 2 GB RAM. With this configuration, the required CPU time in seconds are depicted in Table 3. According to Table 3, the required CPU time increases as the number of containers (nodes) increases. This obviously shows the NP-hard nature of the problem.

Table 4 – CPU time spend to solve the problems

Case:	P_1	P_2	P_3
Required CPU time (Sec)	20520	9556	7940

To fix the ant heuristic parameters, the program is tuned and run for different combinations of important effective ACS_{rank} parameters which are α, β α, β and q_0 q_0 (Table 4). Amongst different values assigned to these parameters, the combination of (2,1,0.9) offered the best result. The number of ants is also an important factor which, in this paper, is once valued equal to the number of containers and once to half of those. For the above mentioned parameters, the number of ants equal to the number of containers produced better results. Then the tuning process continues to fix the maximum number of iterations. The upper limit of iteration is set to 500 times; and, as depicted in Fig. 4, there is no significant improvement after iteration 400. One possibility is to stop the algorithm if no improvement achieved for certain number of iterations.

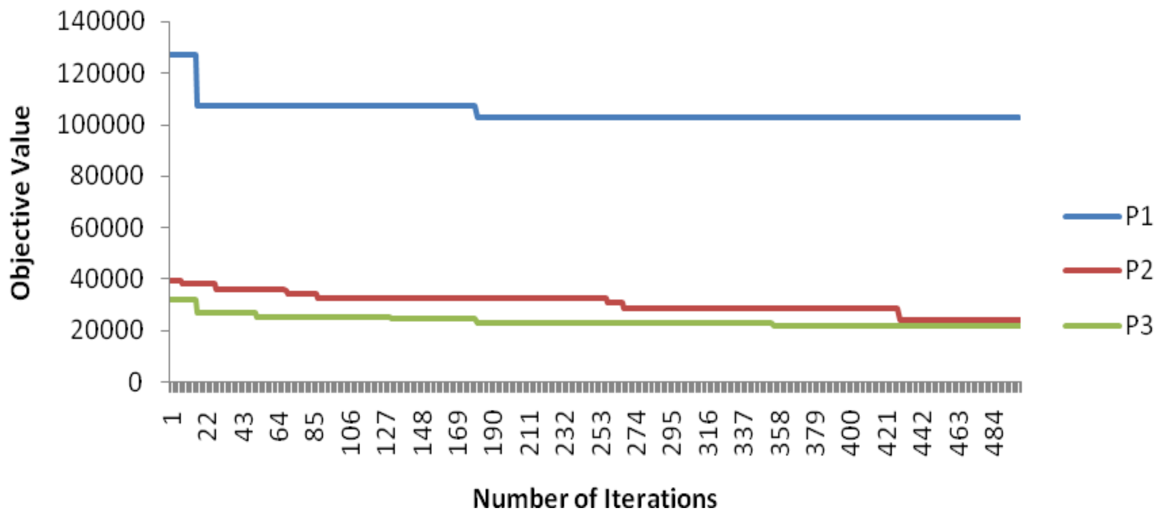

Figure 4- Behavior of the solution method

Table 5- Different values assigned to Ant Heuristic parameters

Parameters	# of Ants		(α, β)					q_0			
Values	$\frac{\# \; of \; containers}{2}$	$\boldsymbol{\# \; of \; containers}$	(1,2)	(4,1)	(3,1)	**(2,1)**	(1,1)	**0.9**	0.8	0.7	0.5

The results can also be estimated for all areas of Tehran, given the amount of improvement and the number of containers in each area. Assuming that Tehran has 128 districts, the amount of improvement for Tehran can be estimated by multiplying 128 to the weighted average of each index. Therefore, total improvements for Tehran can be written as the following table:

Table 6- estimation of results for all areas of Tehran

	Distance (meter)	Fuel (liter)	Purchasing cost (Rial)	Labor (million Rial)
Real World	4,571,990	1,828,984	447* × price of one vehicle	80,460
ACS_{rank}	3,920,830	1,568,351	383 × price of one vehicle	68,940
Improvemet	651,160	260,633	64 × price of one vehicle	11,520

*total number of vehicles according to statistics of municipality of Tehran is 447.

Supposing that serviceable life of each vehicle is 10 years, since the algorithm reduced 64 vehicles, total saving from purchasing vehicles is 6,400 million Rials. Also, assuming the price of gasoline 4000 Rials per liter, total saving on fuel consumption is 1,042,532,000 Rials. If saving on labor is added to these numbers, total saving of applying this algorithm to Tehran is estimated to be 18,962,532,000 Rials.

6- **Conclusions**

The proposed algorithm can reduce fuel consumption and labor costs by finding shorter paths. This algorithm is simply applicable in other routing problem such as money transfer for ATMs and food distribution to supermarkets. Also, time of solving the problem has been significantly reduced. This algorithm can be optimized by improving programming codes or changing programming software. As a result, future researches should be focused on optimizing programming codes and parameters and strategies of ant colony systems.

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