Home Work #2

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1 Question 1

Spacecraft position in ITRF coordinates is given by

$$\mathbf{r} = \begin{bmatrix} 6789 & 6893 & 7035 \end{bmatrix}_{km}^{\mathrm{T}}$$

1.1 part a

Find Latitude and Longitude. For this purpose used algorithm 12 of Valado's book. This algorithm is implemented in the function 'latlon.py' in the 'code/Q1' folder. The function takes the spacecraft position vector as input and returns the latitude and longitude in degrees. The iteration ended when the difference is smaller than 1e-10. The results are:

Table 1: Results of part a

Variables	**Values**
Latitude	36.12°
Longitude	45.43°
h_{ellp}	5591.51_{km}

1.2 part b

In this part, we used the astropy package to find the position vector in the GCRF coordination system. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. Position vector in GCRF:

$$\mathbf{r} = \begin{bmatrix} -862.54 & -9634.75 & 7037.25 \end{bmatrix}_{km}^{\mathrm{T}}$$

1.3 part c

In this part, we used the astropy package to find GMST(θ_{GMST}) and LST(θ_{LST}) The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. The results are:

Table 2: Results of part c

GMST(θ_{GMST})	**LST(θ_{LST})**
112.78°	149.78°

2 Question 2

Position vector: $\vec{r}_{ECI} = -346\hat{i} + 8265\hat{j} + 4680\hat{k}$ km

Velocity vector: $\vec{v}_{ECI} = -5.657\hat{i} - 1.73\hat{j} + 2.703\hat{k}$ km/s

To find the position and velocity in the orbital x-y plane, we need to first transform the ECI coordinates to the orbital coordinate system (i.e., the perifocal coordinate system). The transformation matrix from ECI to perifocal coordinates is given by:

$[\mathbf{T}] = \begin{bmatrix}\cos\omega\cos\Omega - \sin\omega\cos i\sin\Omega & -\cos\omega\sin\Omega - \sin\omega\cos i\cos\Omega & \sin\omega\sin i & \sin\omega\cos\Omega + \cos\omega\cos i\sin\Omega & -\sin\omega\sin\Omega\end{bmatrix}$

where ω, Ω, and i are the argument of periapsis, right ascension of the ascending node, and inclination of the orbit, respectively.

Assuming that the orbit is circular ($e = 0$), the argument of periapsis and the right ascension of the ascending node are not defined. Hence, we can assume that $\omega = \Omega = 0$. The transformation matrix then simplifies to:

$[\mathbf{T}] = \begin{bmatrix}\cos\Omega & -\sin\Omega & 0 \sin\Omega & \cos\Omega & 0\, 0 & 0 & 1\end{bmatrix}$

Using this transformation matrix, we can find the position and velocity in the perifocal coordinate system as:

$\vec{r}P = [\mathbf{T}]\vec{r}ECI$

$\vec{v}P = [\mathbf{T}]\vec{v}ECI$

Substituting the given values, we get:

$\vec{r}_P = \begin{bmatrix}\cos\Omega & -\sin\Omega & 0 \sin\Omega & \cos\Omega & 0\, 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-346\ 8265\ 4680\end{bmatrix} = \begin{bmatrix}-1254.8 & -3121.6\ 4680\end{bmatrix}$ km

$\vec{v}_P = \begin{bmatrix}\cos\Omega & -\sin\Omega & 0 \sin\Omega & \cos\Omega & 0\, 0 & 0 & 1\end{bmatrix}\begin{bmatrix}-5.657 & -1.73\ 2.703\end{bmatrix} = \begin{bmatrix}-2.302\ 4.529\ 2.703\end{bmatrix}$ km/s

The position and velocity in the x-y plane can be obtained by setting the z-components of the position and velocity vectors to zero, i.e.,

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