

# Home Work #3

Ali BaniAsad 401209244

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## 1 Question 1

In this equation we discuss about perturbation in classical orbital element. Formulas for the Gaussian form of the VOP equations using the disturbing force with specific force components resolved in the RSW system:

$$\begin{aligned}\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left( e \sin(\theta) F_R + \frac{p}{r} F_S \right) \\ \frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na} \left( \sin(\theta) F_R + \left( \cos(\theta) + \frac{e + \cos(\theta)}{1 + e \cos(\theta)} \right) F_S \right) \\ \frac{di}{dt} &= \frac{r \cos(u)}{na^2 \sqrt{1-e^2}} F_W \\ \frac{d\Omega}{dt} &= \frac{r \sin(\theta)}{na^2 \sqrt{1-e^2} \sin(i)} F_W \\ \frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{nae} \left( -\cos(\theta) F_R + \sin(\omega) \left( 1 + \frac{1}{p} \right) F_S \right) - \frac{r \cot(i) \sin(u)}{h} F_W \\ \frac{M_0}{dt} &= \frac{1}{na^2 e} ((p \cos(\theta) - 2er) F_R - (p + r) \sin(\theta) F_S) - \frac{dn}{dt} (t - t_0)\end{aligned}\tag{1}$$

### 1.1 part a

If we want to change  $a$ , we need to have force in R or S direction. If there is a force in R and S direction other parameters like eccentricity,  $\omega$ , and  $M_0$  will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "a" without the change of other parameters.

$$\begin{aligned}\sin(\theta) F_R &= - \left( \cos(\theta) + \frac{e + \cos(\theta)}{1 + e \cos(\theta)} \right) F_S \\ \cos(\theta) F_R &= \sin(\omega) \left( 1 + \frac{1}{p} \right) F_S \\ (p \cos(\theta) - 2er) F_R &= (p + r) \sin(\theta) F_S\end{aligned}\tag{2}$$

### 1.2 part b

If we want to change inclination, we need to have force in W direction. If there is a force in W direction other parameters like  $\omega$ , and  $\Omega$  will change and others will be constant. If we can solve the below equations and

find the answer (if exist), we can change parameter "eccentricity" without the change of other parameters.

$$\begin{aligned}\frac{r \cos(u)}{na^2\sqrt{1-e^2}} &= 0 \\ \frac{r \cot(i) \sin(u)}{h} &= 0\end{aligned}\tag{3}$$

### 1.3 part c

From the below equation, we can find the most efficient  $\theta$  is in  $\theta = 90^\circ$  because  $\sin(90^\circ) = 1$ , and to find the best direction it depends on with of parameters  $e$  and  $\frac{p}{r}$  is bigger.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \left( e \sin(\theta) F_R + \frac{p}{r} F_S \right)\tag{4}$$

### 1.4 part d

From below equation we can find most efficient  $u$  is when  $u = 0^\circ$  because  $\cos(0^\circ) = 1$ .

$$\frac{di}{dt} = \frac{r \cos(u)}{na^2\sqrt{1-e^2}} F_W\tag{5}$$

so:

$$u = 0 \rightarrow \theta + \omega = 0^\circ \rightarrow \theta = -\omega$$

and for  $\Omega$ , from below equation we can find most efficient  $\theta$  is when  $\theta = 90^\circ$  because  $\sin(90^\circ) = 1$ .

$$\frac{d\Omega}{dt} = \frac{r \sin(\theta)}{na^2\sqrt{1-e^2} \sin(i)} F_W$$

## 2 Question 2

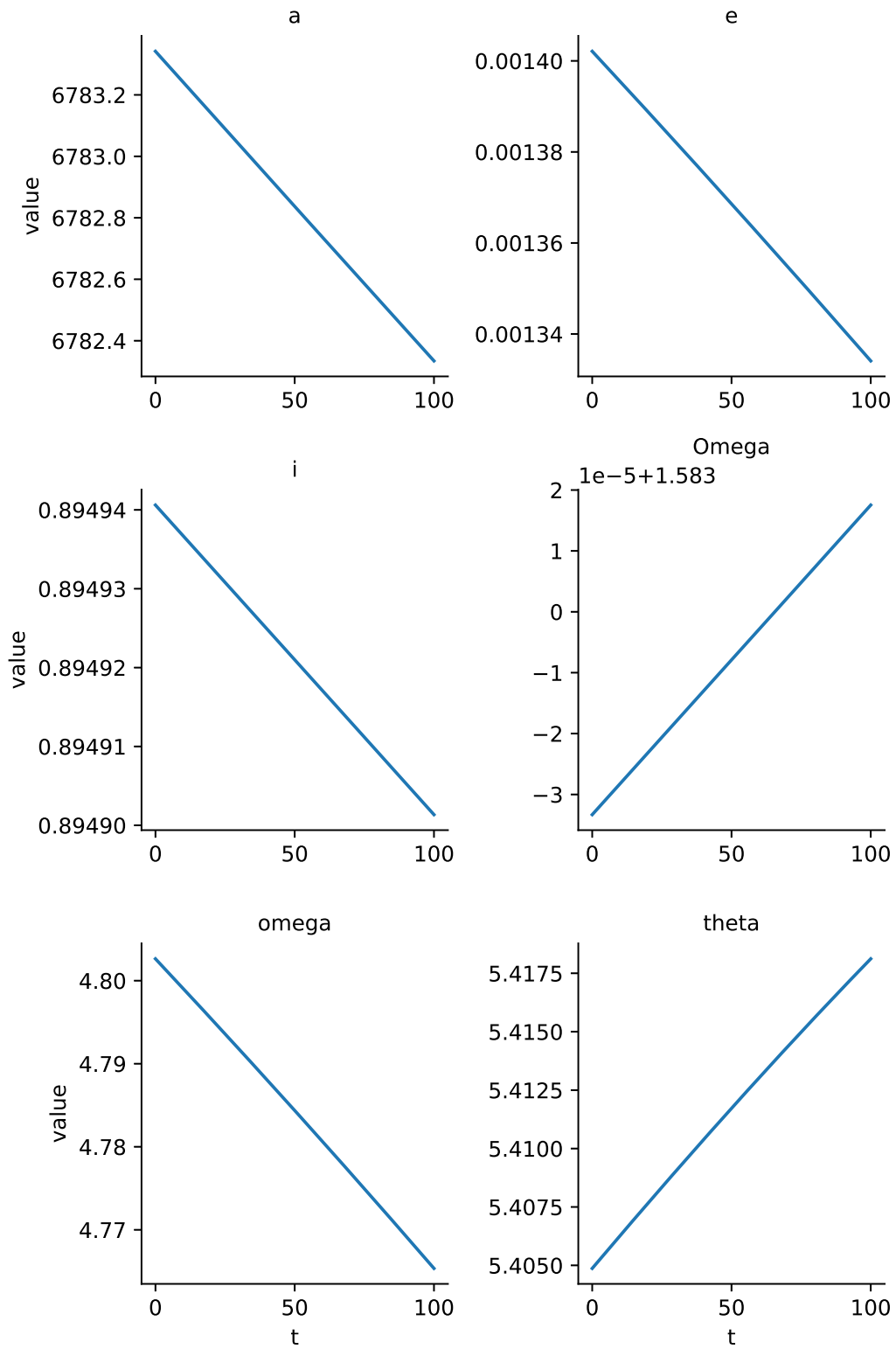
In this question, we investigate the effect of perturbation forces on the orbital elements.

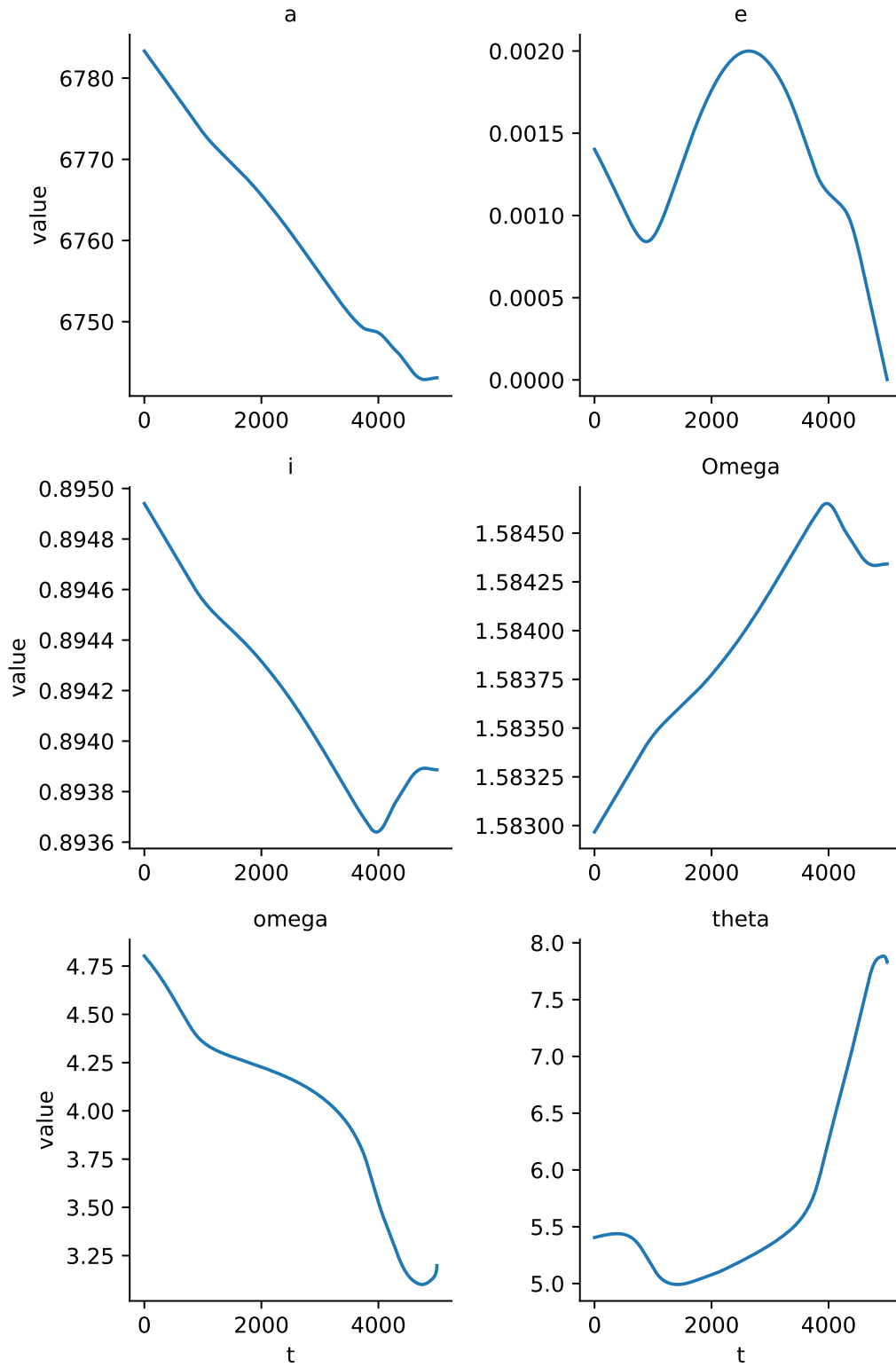
### 2.1 $J_2$ perturbation

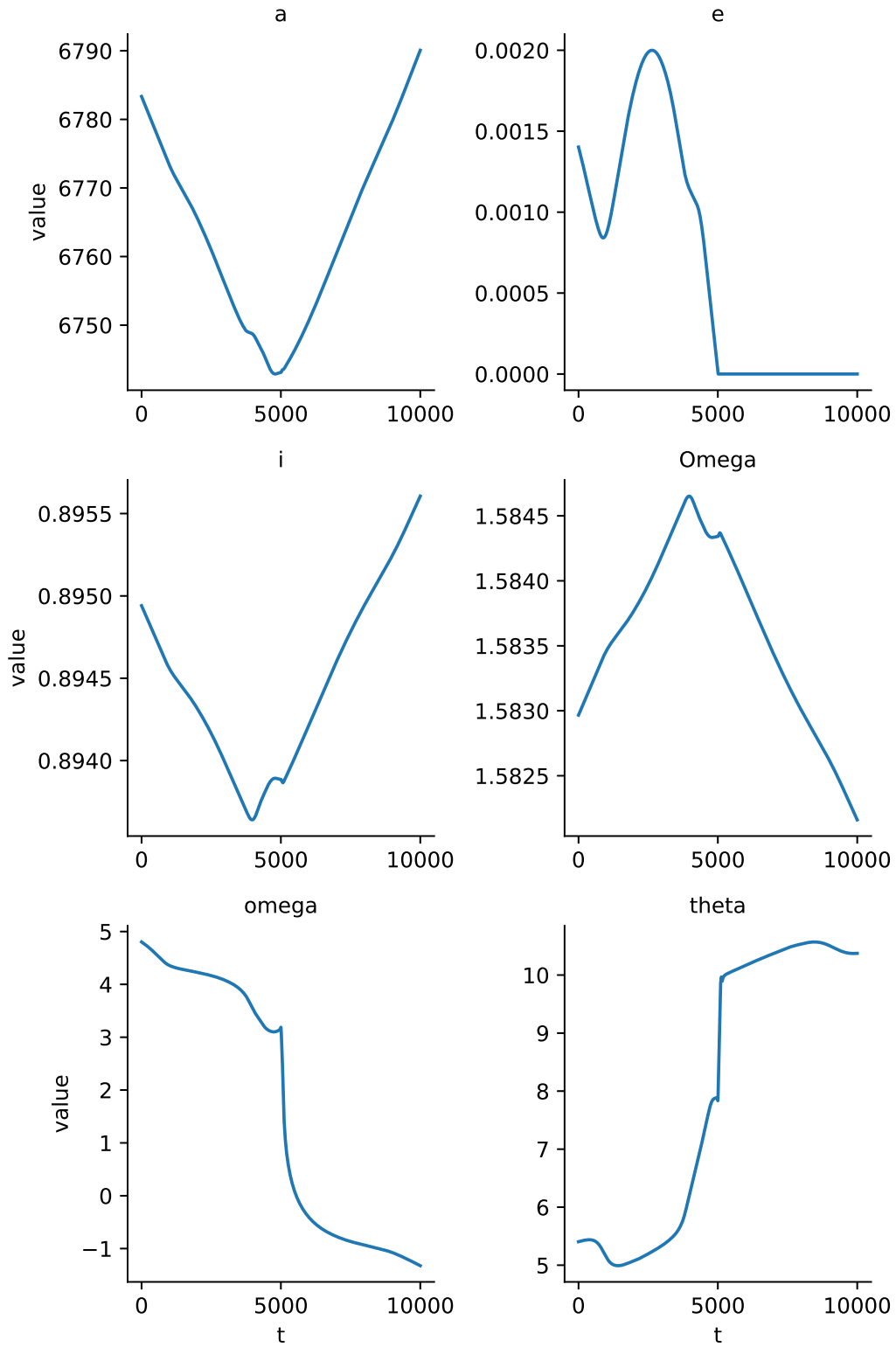
Forces in RSW system:

$$\begin{aligned}F_R &= -\frac{3\mu J_2 R^2}{2r^4} (1 - 3\sin^2(i) \sin^2(u_0)) \\ F_S &= -\frac{3\mu J_2 R^2}{2r^4} \sin^2(i) \sin(u_0) \cos(u_0) \\ F_W &= -\frac{3\mu J_2 R^2}{2r^4} \sin(i) \cos(i) \sin(u_0)\end{aligned}\tag{6}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

Figure 1: Variation of Parameter due to  $J_2$  perturbation for 100 seconds

Figure 2: Variation of Parameter due to  $J_2$  perturbation for 5000 seconds

Figure 3: Variation of Parameter due to  $J_2$  perturbation for 10000 seconds

## 2.2 Drag perturbation

In this section, there is simplifying the assumption that drag force is in the S direction.

$$F_{drag} = \frac{1}{2}\rho v^2 s C_D \rightarrow \mathbf{F}_{drag} = -\frac{1}{2}\rho v^2 s C_D \vec{S}$$

$$\mathbf{a}_{drag} = -\frac{1}{2m_s}\rho v^2 s C_D \vec{S}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

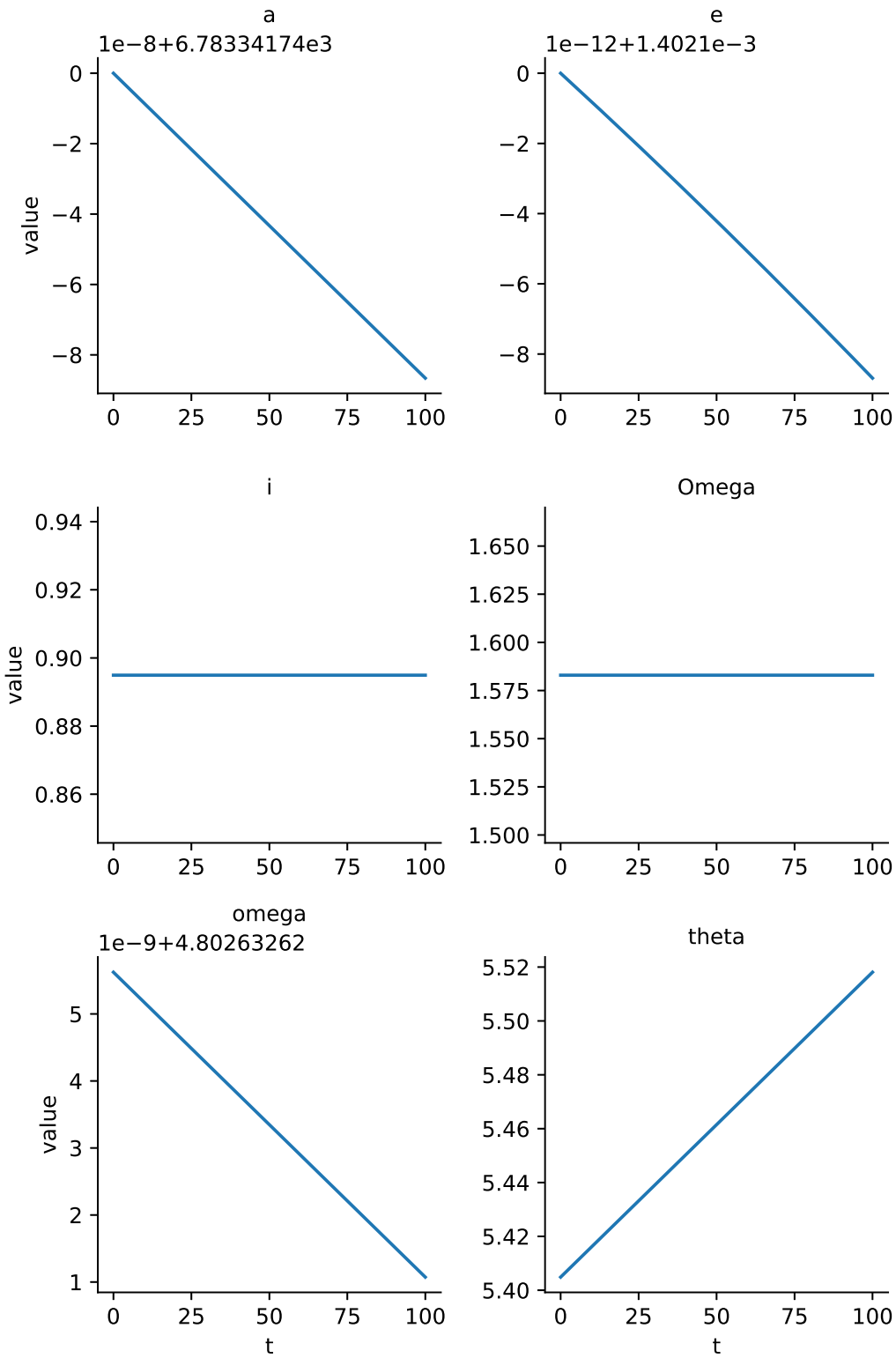


Figure 4: Variation of Parameter due to drag perturbation for 100 seconds

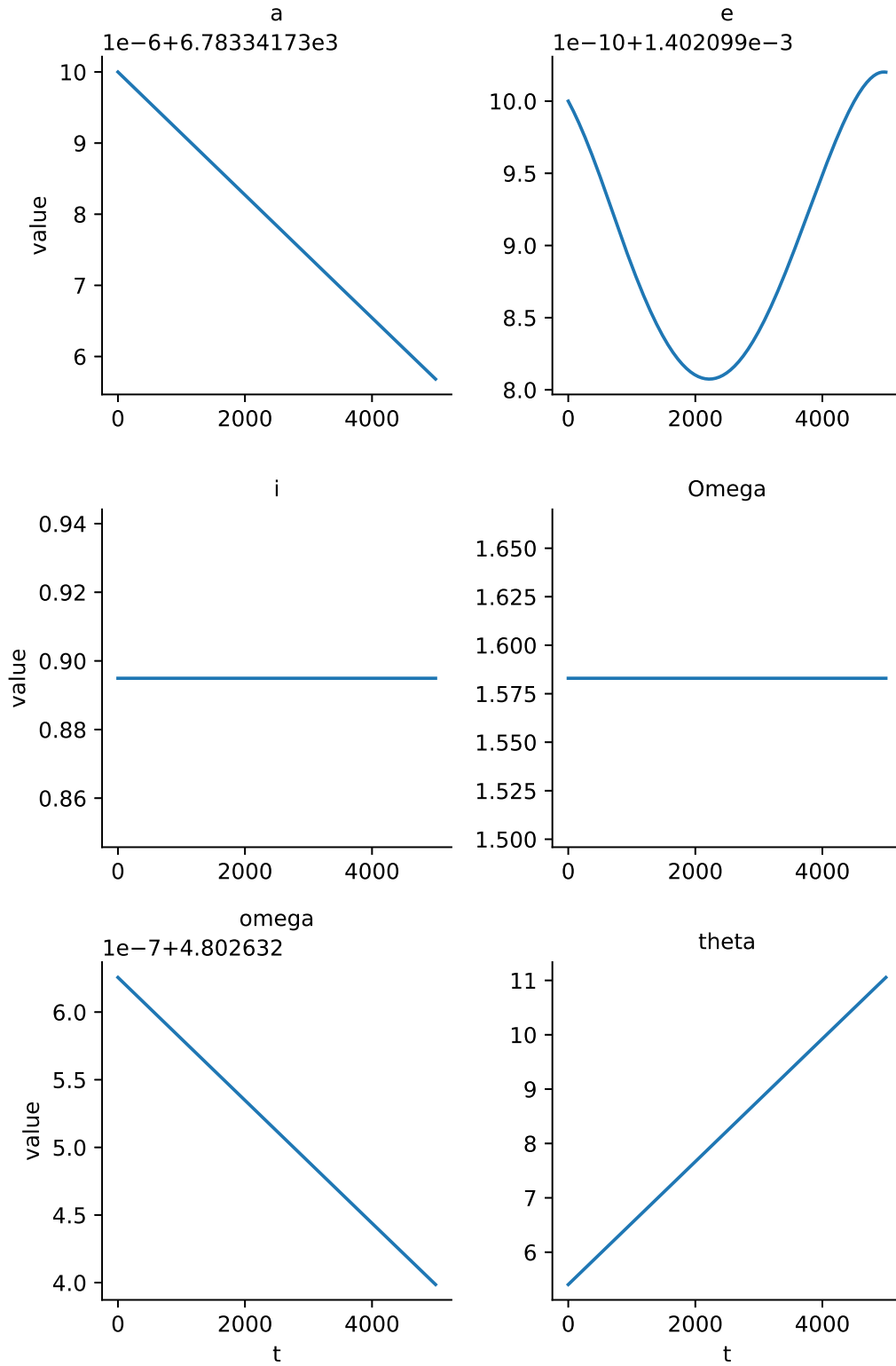


Figure 5: Variation of Parameter due to drag perturbation for 5000 seconds



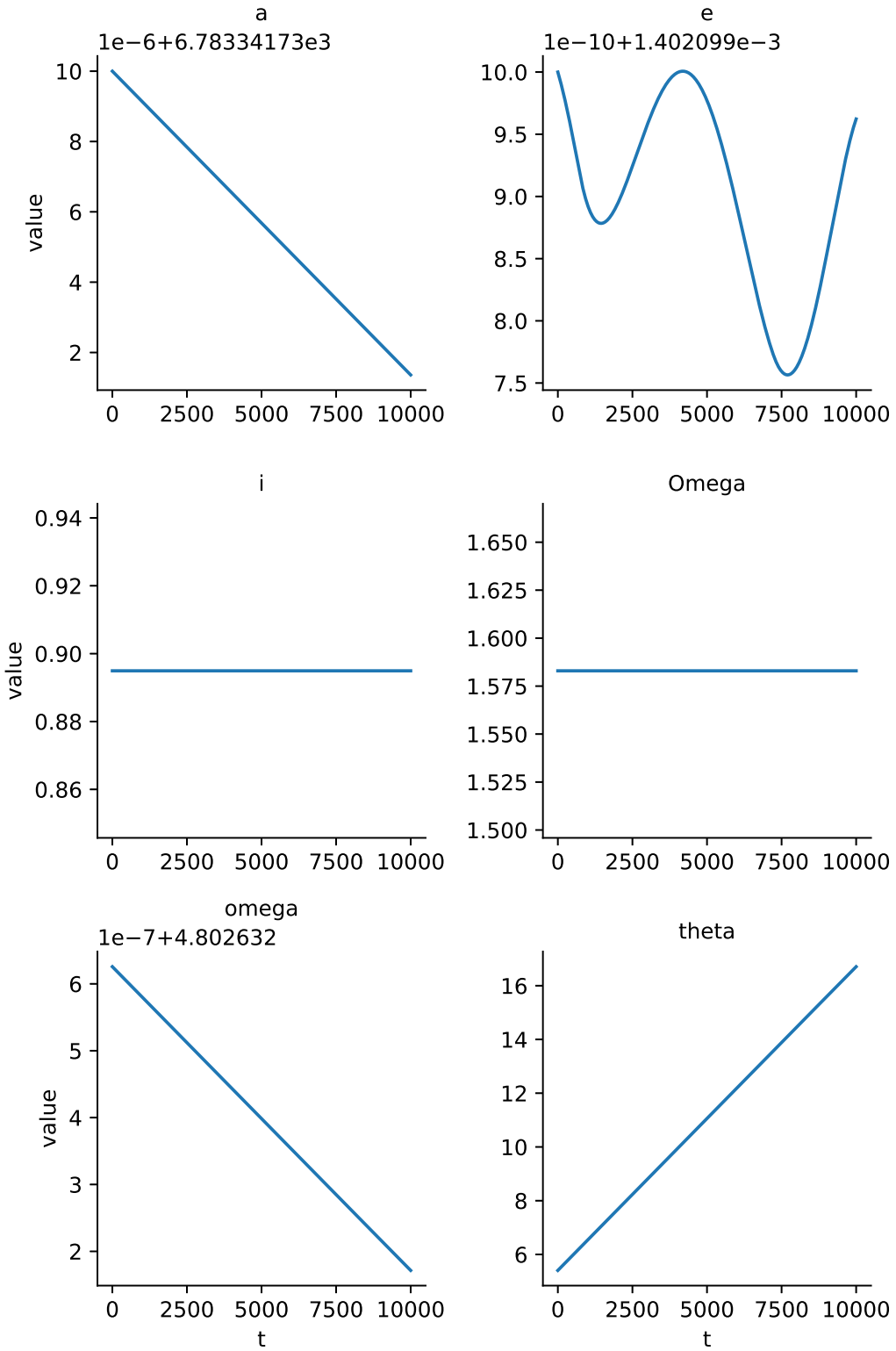


Figure 6: Variation of Parameter due to drag perturbation for 10000 seconds

## 2.3 Moon gravity perturbation

In this section, we investigate the effect of the moon's third body on satellite orbital elements. In this problem we calculate forces of moon and earth, then, integrate from differential equation for several time to see effects in short, long period and secular.

$$\sum F_{satellite} = \frac{-Gm_{earth}}{r_{e2s}^3} \mathbf{r}_{e2s} + \frac{-Gm_{moon}}{r_{m2s}^3} \mathbf{r}_{m2s}$$

Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

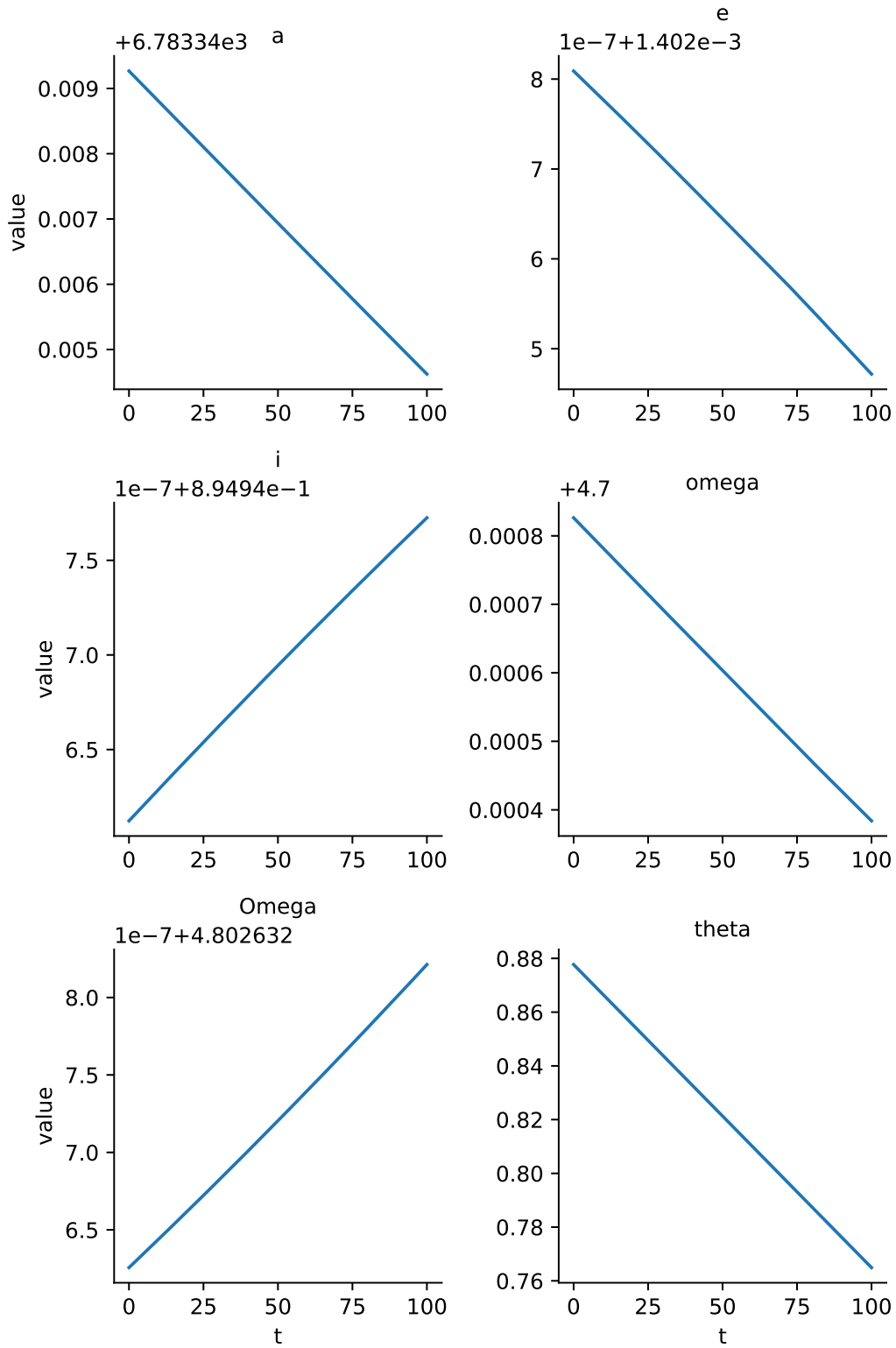


Figure 7: Variation of Parameter due to moon gravity perturbation for 100 seconds

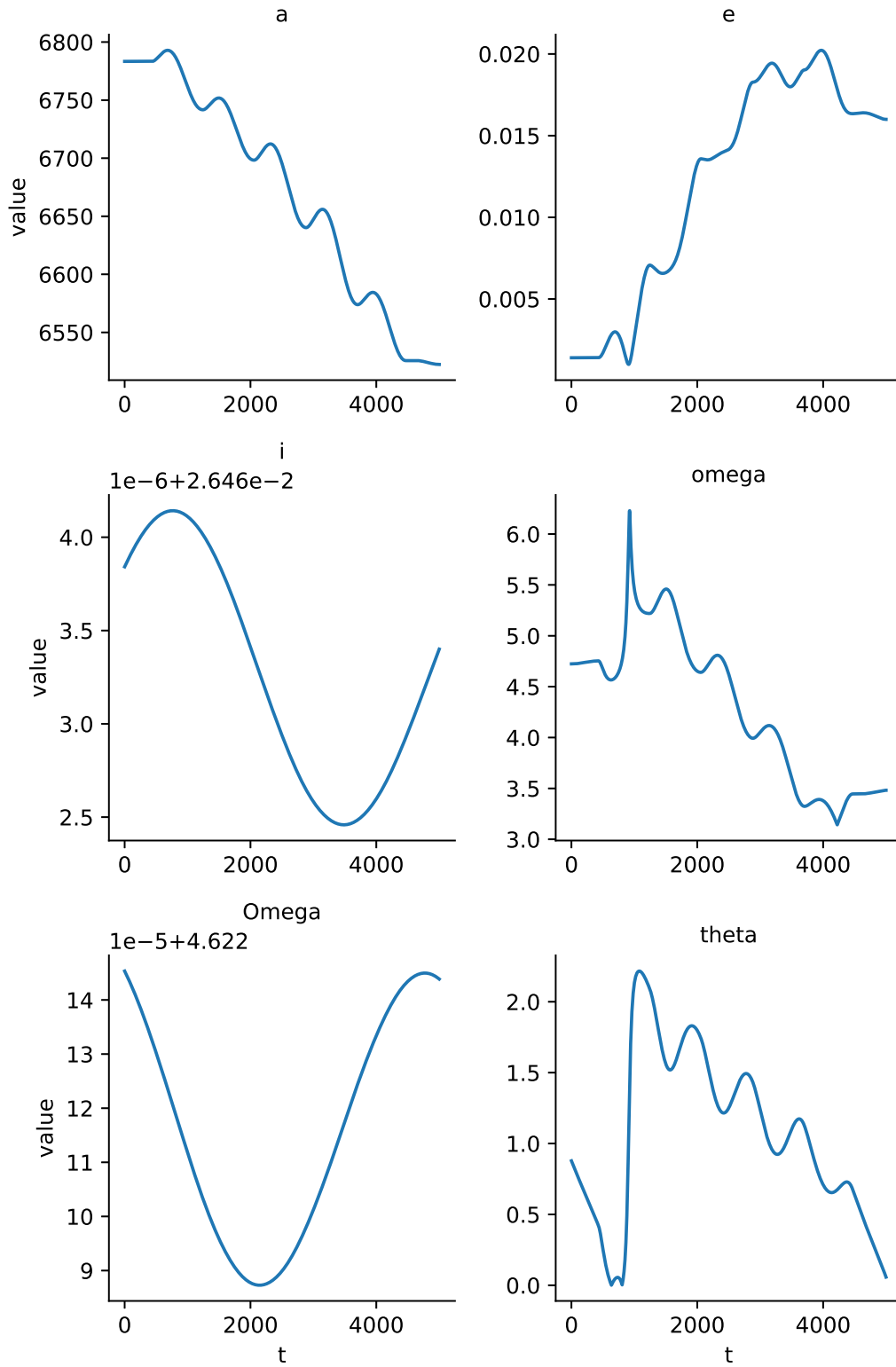


Figure 8: Variation of Parameter due to moon gravity perturbation for 5000 seconds

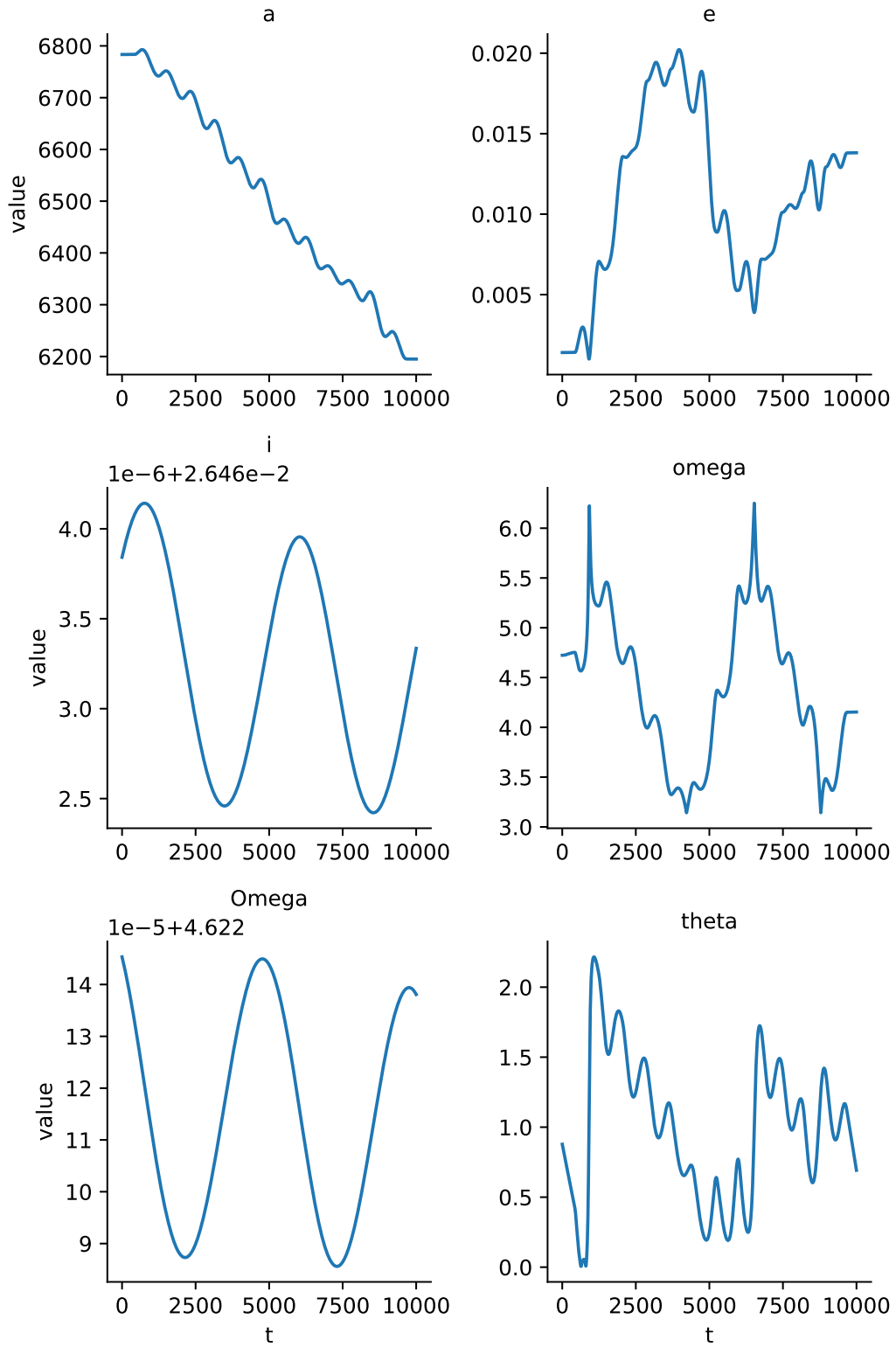


Figure 9: Variation of Parameter due to moon gravity perturbation for 10000 seconds

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