In The Name of God

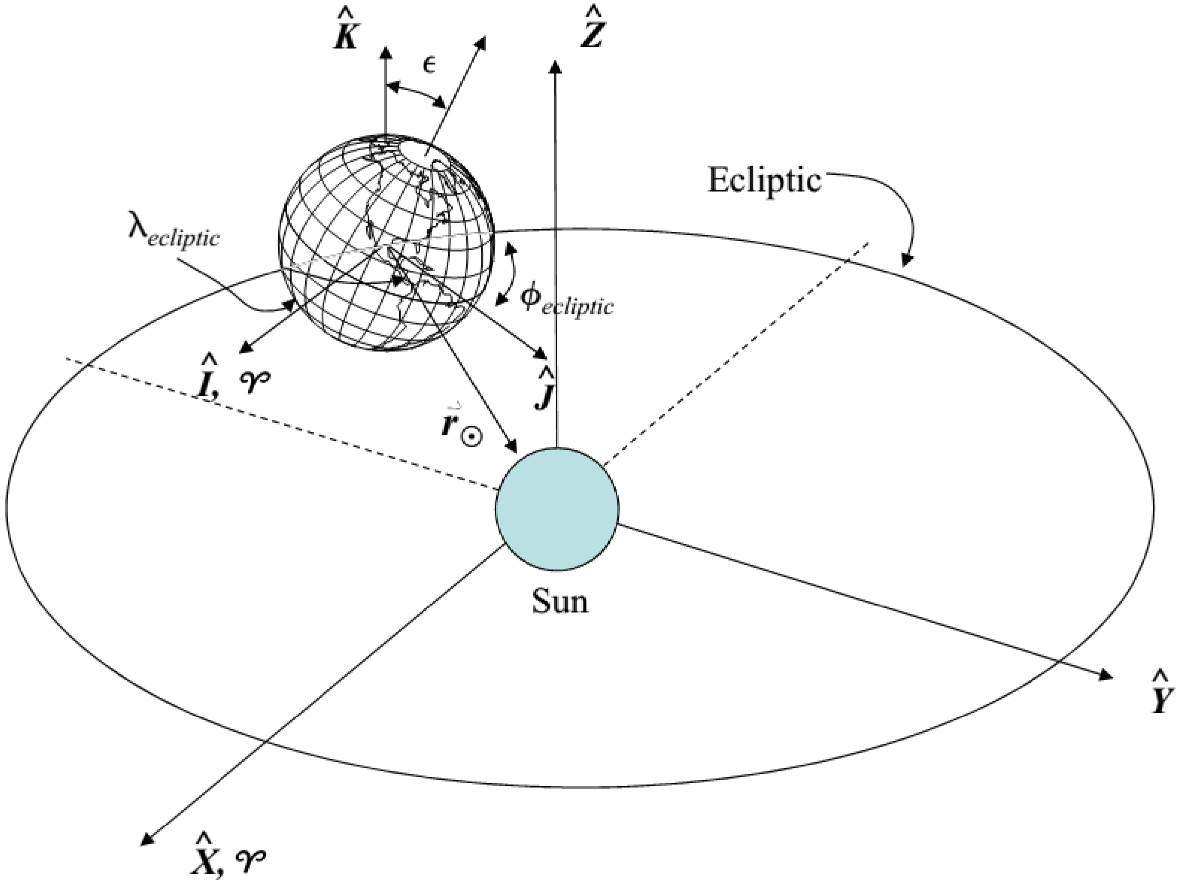
Sharif University of Technology
Department of Aerospace Engineering

45-784: Advanced Orbital Mechanics

CH#3: Celestial Phenomena

Solar Phenomena

- The position vector of the Sun is needed for
  - analysis of perturbation forces on satellites
  - sensor view/ determination of solar-panel illumination/…



Solar Phenomena

- ***ALGORITHM 29:*** $\textit{SUN}\,(JD_{UT1} \Rightarrow \vec{r}_{\odot})$

$$T_{UT1} = \frac{JD_{UT1} - 2{,}451{,}545.0}{36{,}525}$$ (epoch J2000)

$$\lambda_{M_{\odot}} = 280.460^{\circ} + 36{,}000.771\ T_{UT1}$$

LET $T_{TDB} \cong T_{UT1}$

$$M_{\odot} = 357.529\,109\,2^{\circ} + 35{,}999.050\,34\,T_{TDB}$$

$$\lambda_{ecliptic} = \lambda_{M_{\odot}} + 1.914\,666\,471^{\circ}\,\mathrm{SIN}(M_{\odot}) + 0.019\,994\,643\,\mathrm{SIN}(2M_{\odot})$$

$$\phi_{ecliptic} = 0^{\circ}$$ (< 0.000333 *deg*)

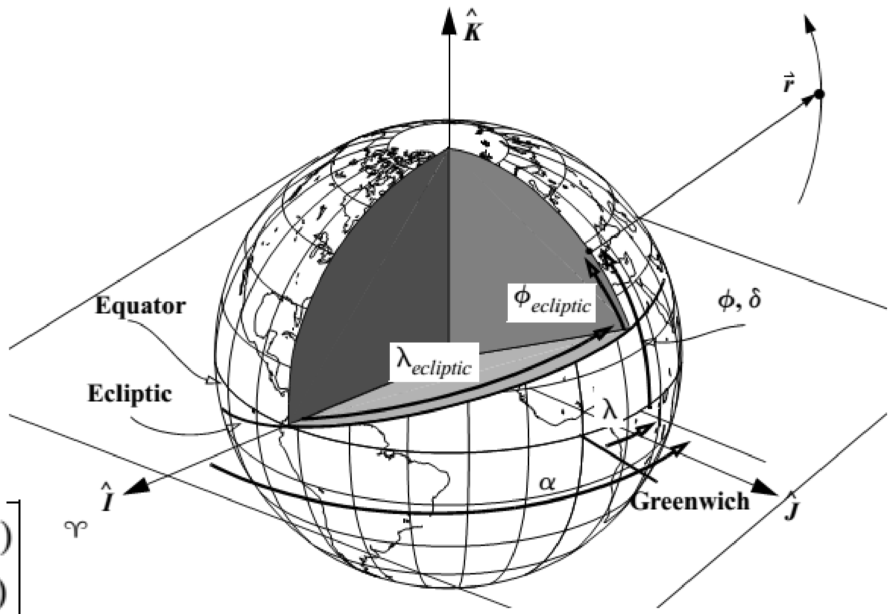
$$r_{\odot} = 1.000\,140\,612 - 0.016\,708\,617\,\mathrm{COS}(M_{\odot}) - 0.000\,139\,589\,\mathrm{COS}(2M_{\odot})$$

$$\epsilon = 23.439\,291^{\circ} - 0.013\,004\,2\,T_{TDB}$$

$$\vec{r}_{\odot} = \begin{bmatrix} r_{\odot}\,\mathrm{COS}(\lambda_{ecliptic}) \\ r_{\odot}\,\mathrm{COS}(\epsilon)\,\mathrm{SIN}(\lambda_{ecliptic}) \\ r_{\odot}\,\mathrm{SIN}(\epsilon)\,\mathrm{SIN}(\lambda_{ecliptic}) \end{bmatrix} \mathrm{AU}$$

Transformations for Ecliptic Lat. & Long.

$$\vec{r}_{XYZ} = \begin{bmatrix} r\cos(\phi_{ecliptic})\cos(\lambda_{ecliptic}) \\ r\cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) \\ r\sin(\phi_{ecliptic}) \end{bmatrix} \qquad \vec{r}_{IJK} = \mathrm{ROT1}[-\epsilon]\vec{r}_{XYZ}$$



$$\vec{r}_{IJK} = r\begin{bmatrix} \cos(\phi_{ecliptic})\cos(\lambda_{ecliptic}) \\ \cos(\epsilon)\cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) - \sin(\epsilon)\sin(\phi_{ecliptic}) \\ \sin(\epsilon)\cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) + \cos(\epsilon)\sin(\phi_{ecliptic}) \end{bmatrix}$$

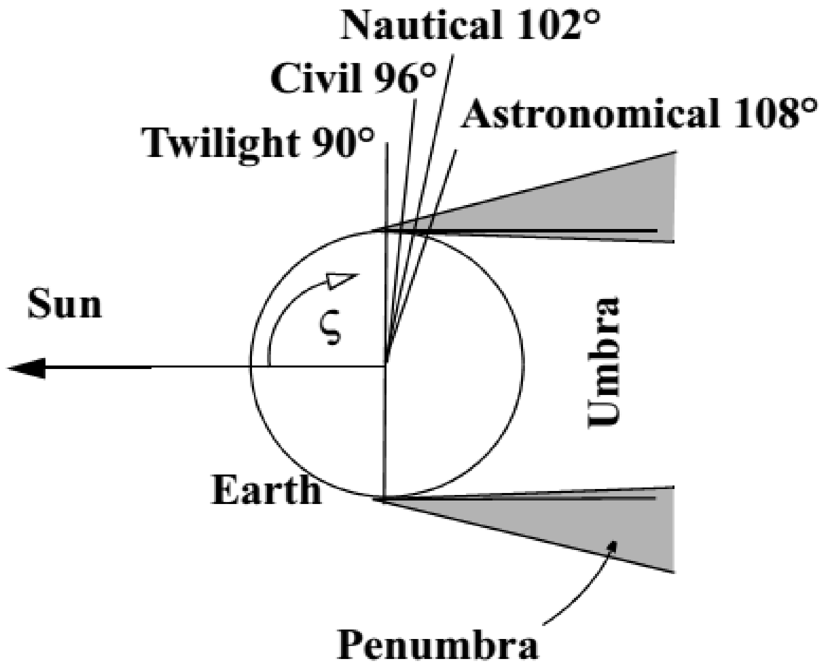
$$\hat{L}_{XYZ} = \begin{bmatrix} \cos(\phi_{ecliptic})\cos(\lambda_{ecliptic}) \\ \cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) \\ \sin(\phi_{ecliptic}) \end{bmatrix} \qquad \hat{L}_{IJK} = \begin{bmatrix} \cos(\delta)\cos(\alpha) \\ \cos(\delta)\sin(\alpha) \\ \sin(\delta) \end{bmatrix}$$

$$\hat{L}_{XYZ} = \mathrm{ROT1}(\epsilon)\hat{L}_{IJK} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\epsilon) & \sin(\epsilon) \\ 0 & -\sin(\epsilon) & \cos(\epsilon) \end{bmatrix}\begin{bmatrix} \cos(\delta)\cos(\alpha) \\ \cos(\delta)\sin(\alpha) \\ \sin(\delta) \end{bmatrix}$$

$$\begin{cases} \sin(\delta) = \sin(\phi_{ecliptic})\cos(\epsilon) + \cos(\phi_{ecliptic})\sin(\epsilon)\sin(\lambda_{ecliptic}) \\ \sin(\alpha) = \dfrac{-\sin(\phi_{ecliptic})\sin(\epsilon) + \cos(\phi_{ecliptic})\cos(\epsilon)\sin(\lambda_{ecliptic})}{\cos(\delta)} \\ \cos(\alpha) = \dfrac{\cos(\phi_{ecliptic})\cos(\lambda_{ecliptic})}{\cos(\delta)} \end{cases}$$

Sunrise, Sunset, and Twilight Times

- **Sunrise** and **sunset** are the times when the apparent upper limb of the Sun is seen on the horizon by an observer on the Earth, while **twilight** is the time at which the Sun has a particular angular separation from the observer.
- A key parameter in this area is the angle between the site and the Sun, ς.
- rays from the upper limb of the Sun extend about 15' 45" beyond 90° $\longrightarrow$ 90° 50'



Sunrise, Sunset, and Twilight Times

- ***ALGORITHM 30:*** **SUNRISESET** $(JD_{UT1}, \phi_{gc}, \lambda, \varsigma \Rightarrow UT_{sunrise}, UT_{sunset})$

$$JD_{sunrise} = JD_{0^h} + 6/24 - \lambda/360, \quad JD_{sunset} = JD_{0^h} + 18/24 - \lambda/360$$

Calculate T_{UT1}, $\lambda_{M_\odot}$, $M_\odot$, $\lambda_{ecliptic}$, ϵ

$$\text{TAN}(\alpha_\odot) = \text{COS}(\epsilon)\text{TAN}(\lambda_{ecliptic})$$

$$\text{SIN}(\delta_\odot) = \text{SIN}(\epsilon)\text{SIN}(\lambda_{ecliptic})$$

$$\text{COS}(LHA_{sunset}) = \frac{\text{COS}(\varsigma) - \text{SIN}(\delta_\odot)\text{SIN}(\phi_{gc})}{\text{COS}(\delta_\odot)\text{COS}(\phi_{gc})}$$

$$LHA_{sunrise} = 360° - LHA_{sunset}$$

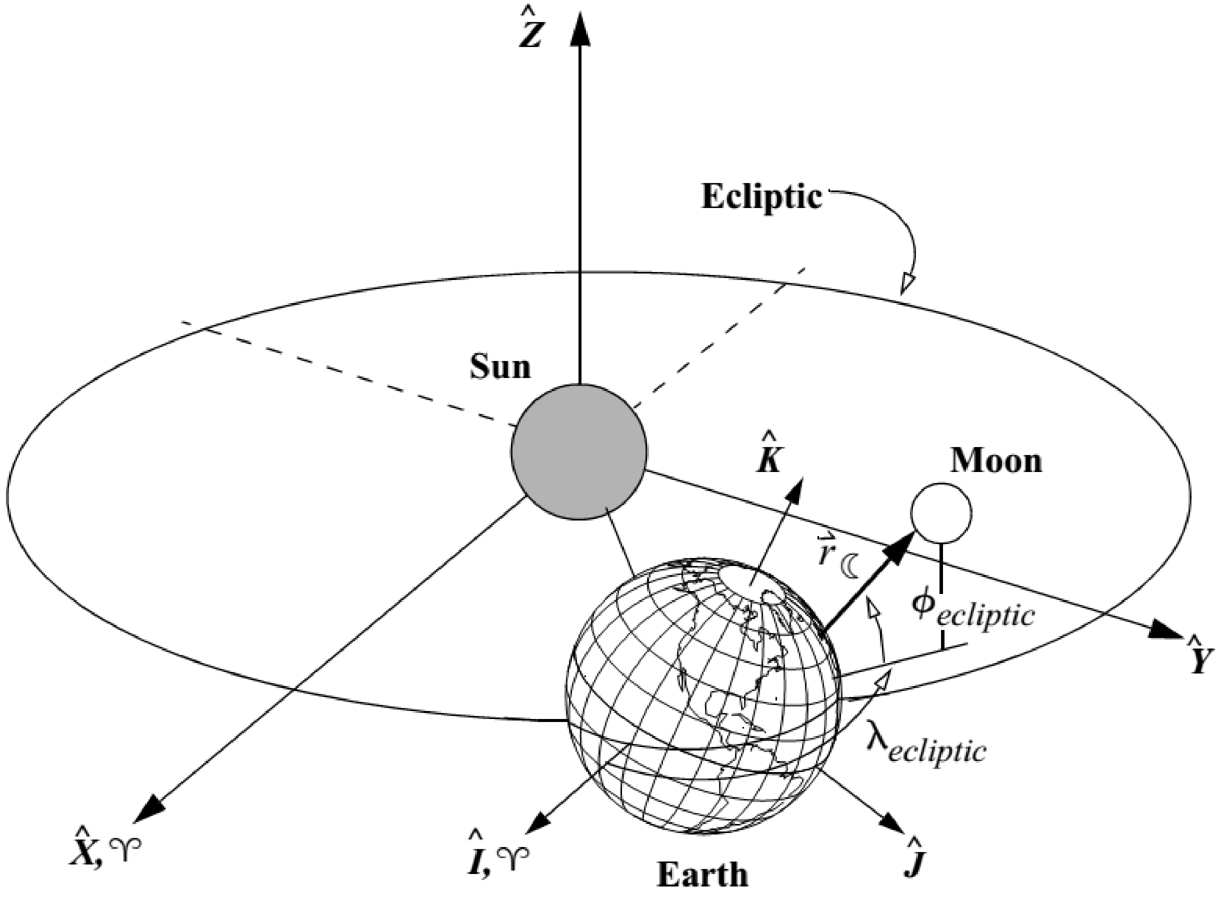
$$UT = LHA_\odot + \alpha_\odot - GMST$$

$$h = trunc(\frac{UT°}{15}), \quad min = trunc\left((\frac{UT°}{15} - h)60\right), \quad s = (\frac{UT°}{15} - h - \frac{m}{60})3600$$

Lunar Phenomena

- Knowledge of the Moon’s location and illumination is required to determine optimum observation times .
- Moon Position Vector

$$T_{TDB} = \frac{JD_{TDB} - 2{,}451{,}545.0}{36{,}525}$$



Lunar Phenomena

$$
\begin{aligned}
\lambda_{ecliptic} &= 218.32^\circ + 481{,}267.8813\,T_{TDB} + 6.29\,\mathrm{SIN}(134.9 + 477{,}198.85\,T_{TDB}) \\
&\quad - 1.27\,\mathrm{SIN}(259.2 - 413{,}335.38\,T_{TDB}) + 0.66\,\mathrm{SIN}(235.7 + 890{,}534.23\,T_{TDB}) \\
&\quad + 0.21\,\mathrm{SIN}(269.9 + 954{,}397.70\,T_{TDB}) - 0.19\,\mathrm{SIN}(357.5 + 35{,}999.05\,T_{TDB}) \\
&\quad - 0.11\,\mathrm{SIN}(186.6 + 966{,}404.05\,T_{TDB}) \\
\phi_{ecliptic} &= 5.13^\circ\,\mathrm{SIN}(93.3 + 483{,}202.03\,T_{TDB}) + 0.28\,\mathrm{SIN}(228.2 + 960{,}400.87\,T_{TDB}) \\
&\quad - 0.28\,\mathrm{SIN}(318.3 + 6003.18\,T_{TDB}) - 0.17\,\mathrm{SIN}(217.6 - 407{,}332.20\,T_{TDB}) \\
\wp &= 0.9508^\circ + 0.0518\,\mathrm{COS}(134.9 + 477{,}198.85\,T_{TDB}) \\
&\quad + 0.0095\,\mathrm{COS}(259.2 - 413{,}335.38\,T_{TDB}) + 0.0078\,\mathrm{COS}(235.7 + 890{,}534.23\,T_{TDB}) \\
&\quad + 0.0028\,\mathrm{COS}(269.9 + 954{,}397.70\,T_{TDB}) \\
\bar{\epsilon} &= 23.439\,291^\circ - 0.013\,004\,2\,T_{TDB} - 1.64\times10^{-7}\,T_{TDB}^2 + 5.04\times10^{-7}\,T_{TDB}^3 \\
r_{\leftmoon} &= \frac{1}{\mathrm{SIN}(\wp)} \quad \text{(DU)}
\end{aligned}
$$

$$
\Rightarrow \vec{r}_{\leftmoon} = r_{\leftmoon}\begin{bmatrix} \mathrm{COS}(\phi_{ecliptic})\,\mathrm{COS}(\lambda_{ecliptic}) \\ \mathrm{COS}(\epsilon)\,\mathrm{COS}(\phi_{ecliptic})\,\mathrm{SIN}(\lambda_{ecliptic}) - \mathrm{SIN}(\epsilon)\,\mathrm{SIN}(\phi_{ecliptic}) \\ \mathrm{SIN}(\epsilon)\,\mathrm{COS}(\phi_{ecliptic})\,\mathrm{SIN}(\lambda_{ecliptic}) + \mathrm{COS}(\epsilon)\,\mathrm{SIN}(\phi_{ecliptic}) \end{bmatrix}
$$

Application: Moon Rise and Set Times

- Moonrise and moonset are the times when the upper limb of the Moon is on the horizon for an observer on the Earth.
- The procedure to find the moonrise/moonset times is similar to the sunrise/sunset algorithm.

$$JD_{temp} = JD_{0^h}$$

WHILE $(\Delta UT) > 30^s$

$$n = n + 1$$

$$T_{UT1} = \frac{JD_{Temp} - 2{,}451{,}545.0}{36{,}525} \qquad \text{Let } T_{TDB} \cong T_{UT1}$$

Calculate $\lambda_{ecliptic}, \phi_{ecliptic}, \epsilon$

$$l = \cos(\phi_{ecliptic})\cos(\lambda_{ecliptic})$$

$$m = \cos(\epsilon)\cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) - \sin(\epsilon)\sin(\phi_{ecliptic})$$

$$n = \sin(\epsilon)\cos(\phi_{ecliptic})\sin(\lambda_{ecliptic}) + \cos(\epsilon)\sin(\phi_{ecliptic})$$

Application: Moon Rise and Set Times

$$\alpha_{☾} = \operatorname{ATAN2}(m, l)$$

$$\operatorname{SIN}(\delta_{☾}) = n$$

$$\textbf{LSTIME}\,(JD_{Temp}, \lambda \Rightarrow LST, GMST)$$

$$GHA_{☾_n} = GMST - \alpha_{☾}$$

$$LHA = GHA_{☾_n} + \lambda$$

IF first time

$$\Delta GHA = 347.81°$$

ELSE

$$\Delta GHA = \frac{GHA_{☾_n} - GHA_{☾}}{\Delta UT}$$

$$\text{IF } \Delta GHA < 0 \text{ THEN } \Delta GHA = \Delta GHA + \frac{360°}{\Delta UT}$$

$$x_n = \frac{0.00233 - \operatorname{SIN}(\delta_{☾})\operatorname{SIN}(\phi_{gc})}{\operatorname{COS}(\delta_{☾})\operatorname{COS}(\phi_{gc})}$$

Application: Moon Rise and Set Times

IF $x_n > 1$, *next day phenomena*

IF Moonrise calculation THEN $LHA_n = 360° - \cos^{-1}(x_n)$

ELSE $LHA_n = \cos^{-1}(x_n)$

$$\Delta UT = \frac{LHA_n - LHA}{\Delta GHA}$$

IF $\Delta UT < -0.5$ {event is on the day before}

$$\Delta UT = \Delta UT + \frac{360°}{\Delta GHA}$$

ELSE IF $\Delta UT > 0.5$ {event is on the day after}

$$\Delta UT = \Delta UT - \frac{360°}{\Delta GHA}$$

$$\Delta t = \Delta t + \Delta UT$$

$$JD_{temp} = JD_{temp} + \Delta UT$$

$$GHA_{☾} = GHA_{☾_n}$$

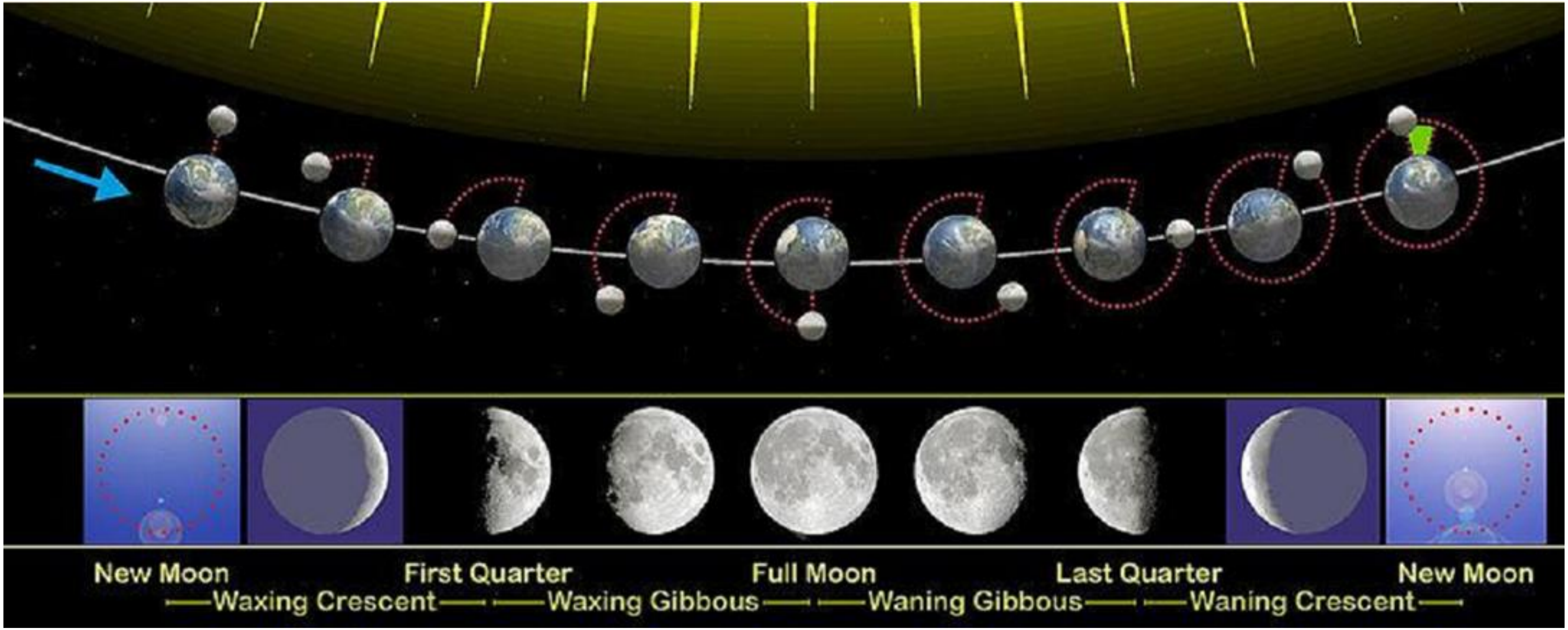
END WHILE LOOP → $UT_{moonrise}, UT_{moonset} = \Delta t(24)$

Phases of the Moon

- The percentage of the Moon's surface that reflects light to the Earth changes constantly.
- phases of the Moon: $phase_{☾} = \lambda_{ecliptic_{\odot}} - \lambda_{ecliptic_{☾}}$
  - new, 0°
  - first quarter, 90°
  - full, 180°
  - Last quarter, 270°
- Percentage of the Moon's surface that's illuminated

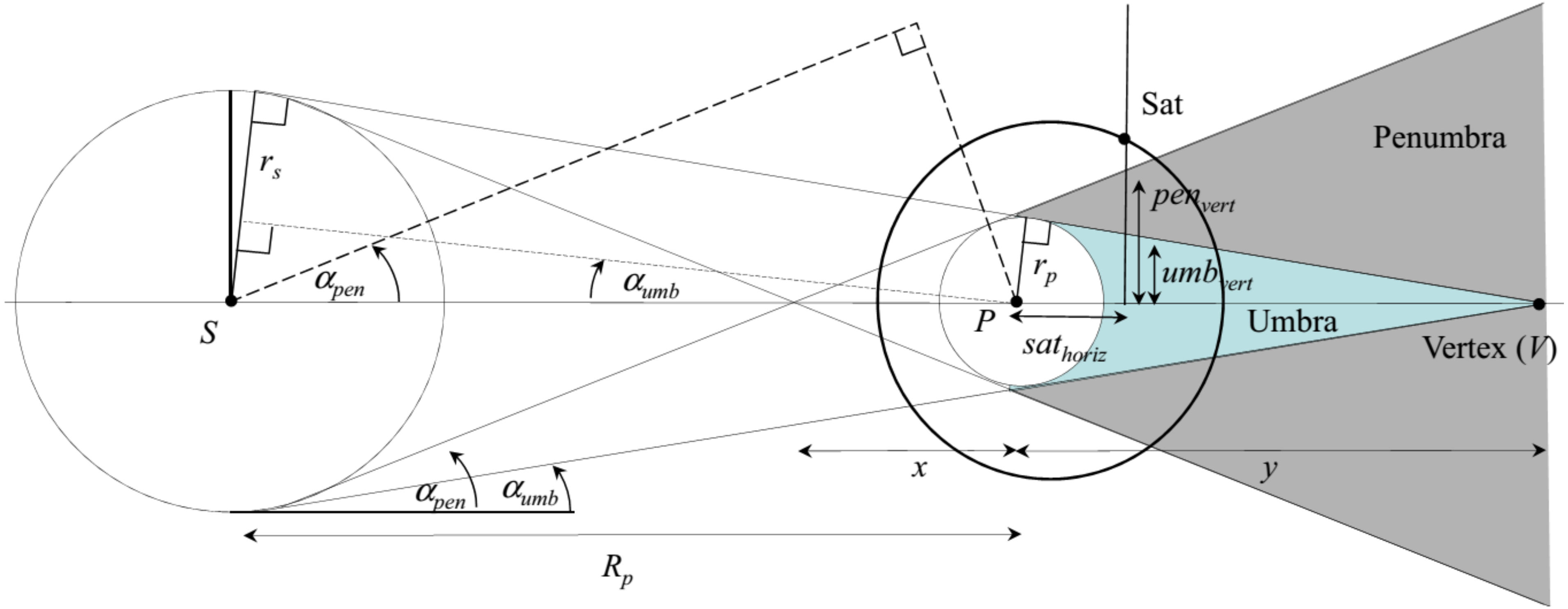
$$\%\,\text{disk} = \frac{100\%}{2}(1 - \cos(phase_{☾}))$$

Phases of the Moon



Eclipses

- Eclipse Geometry



For the Sun-Earth system, PV ≈ 1.384e6 km, or about four times the distance to the Moon

$$\mathrm{SIN}(\alpha_{umb}) = \frac{r_s - r_p}{R_p} = \frac{696000 - 6378}{149599870} \Rightarrow 0.264\ 121\ 687°$$

$$\mathrm{SIN}(\alpha_{pen}) = \frac{r_s + r_p}{R_p} = \frac{696000 + 6378}{149599870} \Rightarrow 0.269\ 007\ 205°$$

Eclipses

- a geosynchronous satellite (r ~ 42163 km) traverses about 13098 km in the penumbral regions. At its orbital velocity, the satellite spends about 71 min in these regions. This doesn't include the time in the umbral region, which is about 12412 km (about 67 min).

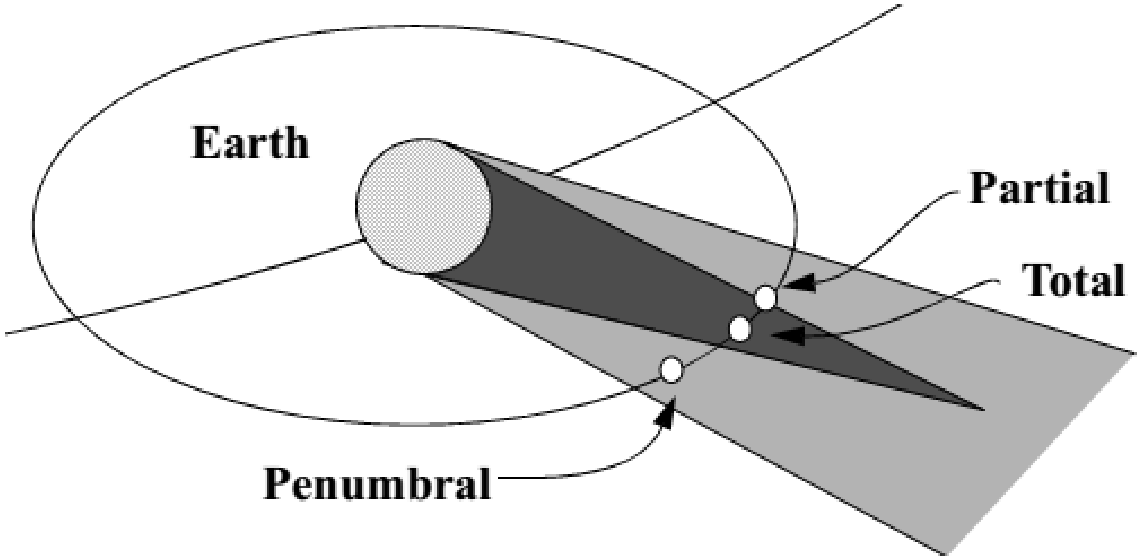
$$\frac{PV}{r_p} = \frac{SV}{r_s} \qquad PV = \frac{SVr_p}{r_s}\ , \ SV = PV + R_p \longrightarrow PV = \frac{PVr_p}{r_s} + \frac{R_p r_p}{r_s}$$

$$\longrightarrow PV\left(1 - \frac{r_p}{r_s}\right) = \frac{R_p r_p}{r_s} \longrightarrow PV = \frac{R_p r_p}{r_s - r_p}$$

$$\longrightarrow PV_\oplus = 1.384 \times 10^6 \text{ km}$$

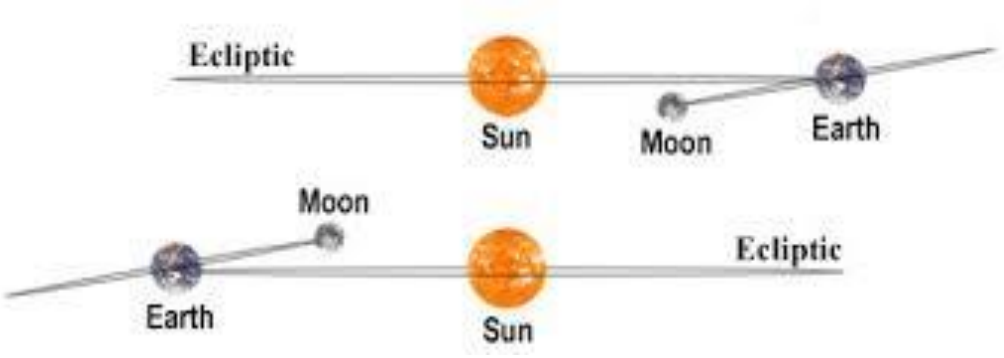
$$= 4R_{\text{Earth-Moon}}$$

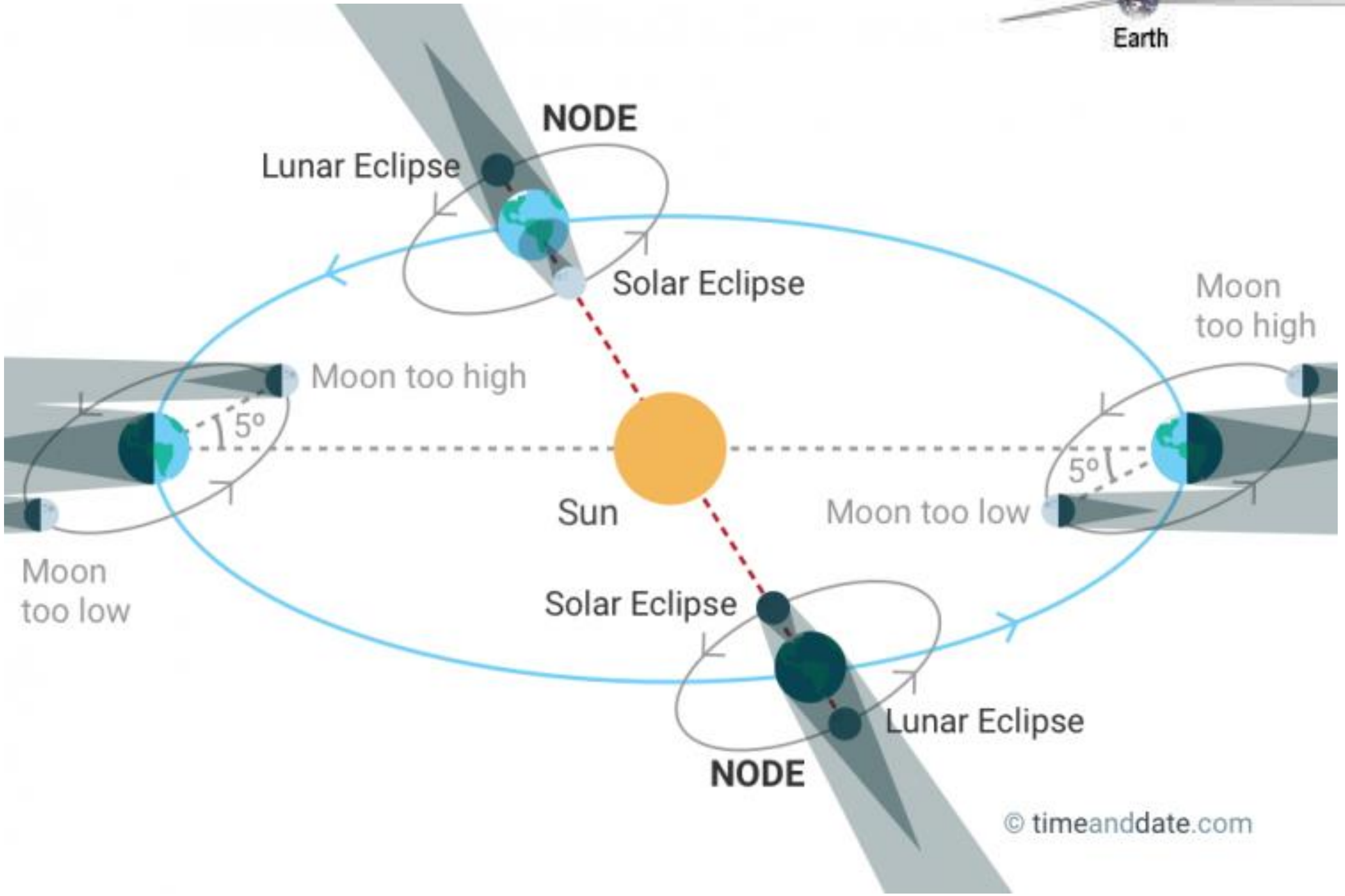
➡ lunar eclipse



Eclipses

- Lunar Eclipse





Geometrical Shadow Analysis

IF $\vec{r}_{\odot} \cdot \vec{r} < 0$

$$\textit{ANGLE} \quad (-\vec{r}_{\odot}, \vec{r}, \varsigma)$$

$$SAT_{horiz} = |\vec{r}| \cos(\varsigma)$$

$$SAT_{vert} = |\vec{r}| \sin(\varsigma)$$

$$x = \frac{r_p}{\sin(\alpha_{pen})}$$

$$PEN_{vert} = \tan(\alpha_{pen})\,(x + SAT_{horiz})$$

IF $SAT_{vert} \le PEN_{vert}$

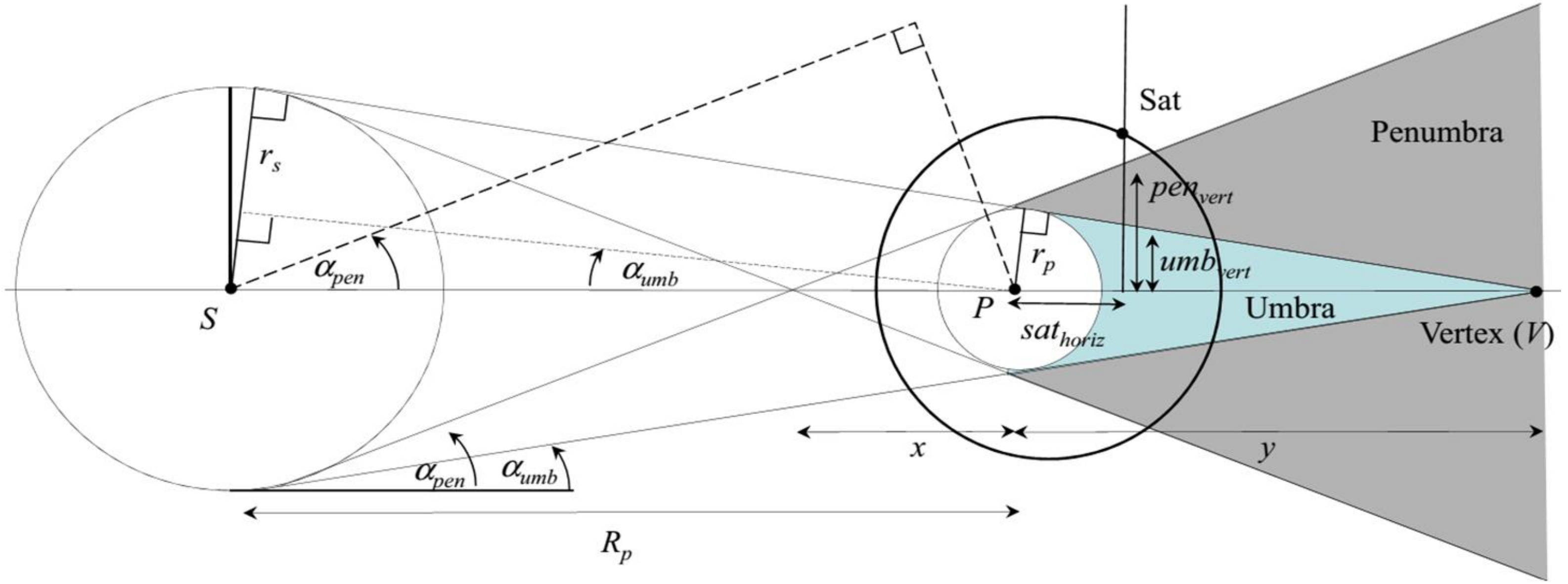
SHADOW = 'PENUMBRA'

$$y = \frac{r_p}{\sin(\alpha_{umb})}$$

$$UMB_{vert} = \tan(\alpha_{umb})\,(y - SAT_{horiz})$$

IF $SAT_{vert} \le UMB_{vert}$

SHADOW = 'UMBRA'



Geometrical Shadow Analysis-2

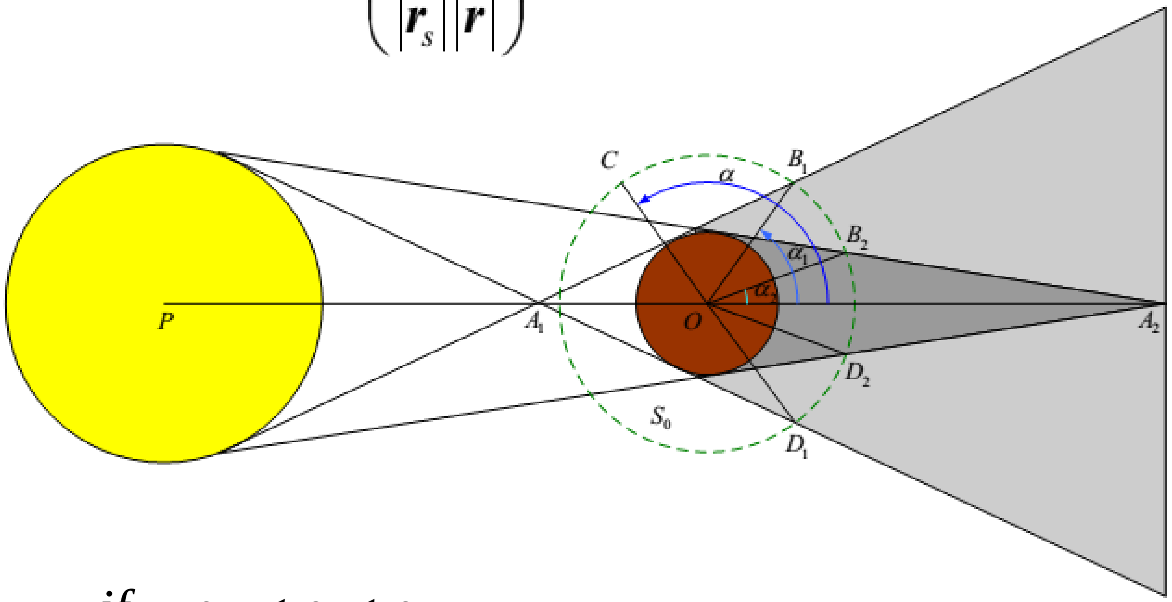
Penumbral cone geometry

$$\begin{cases} \alpha_1 = \pi - \arccos\left(\dfrac{R_e}{x_1}\right) - \arccos\left(\dfrac{R_e}{|\boldsymbol{r}|}\right) \\ x_1 = \dfrac{R_e AU}{R_s + R_e} \end{cases}$$

umbral cone geometry

$$\begin{cases} \alpha_2 = \arccos\left(\dfrac{R_e}{x_2}\right) - \arccos\left(\dfrac{R_e}{|\boldsymbol{r}|}\right) \\ x_2 = \dfrac{R_e AU}{R_s - R_e} \end{cases}$$

$$\alpha = \pi - \arccos\left(\frac{\boldsymbol{r}_s \cdot \boldsymbol{r}}{|\boldsymbol{r}_s||\boldsymbol{r}|}\right)$$



$if \quad \alpha_2 < \alpha < \alpha_1$

$\quad \text{shadow} = \text{"Penaumbra"}$

$if \quad \alpha < \alpha_2$

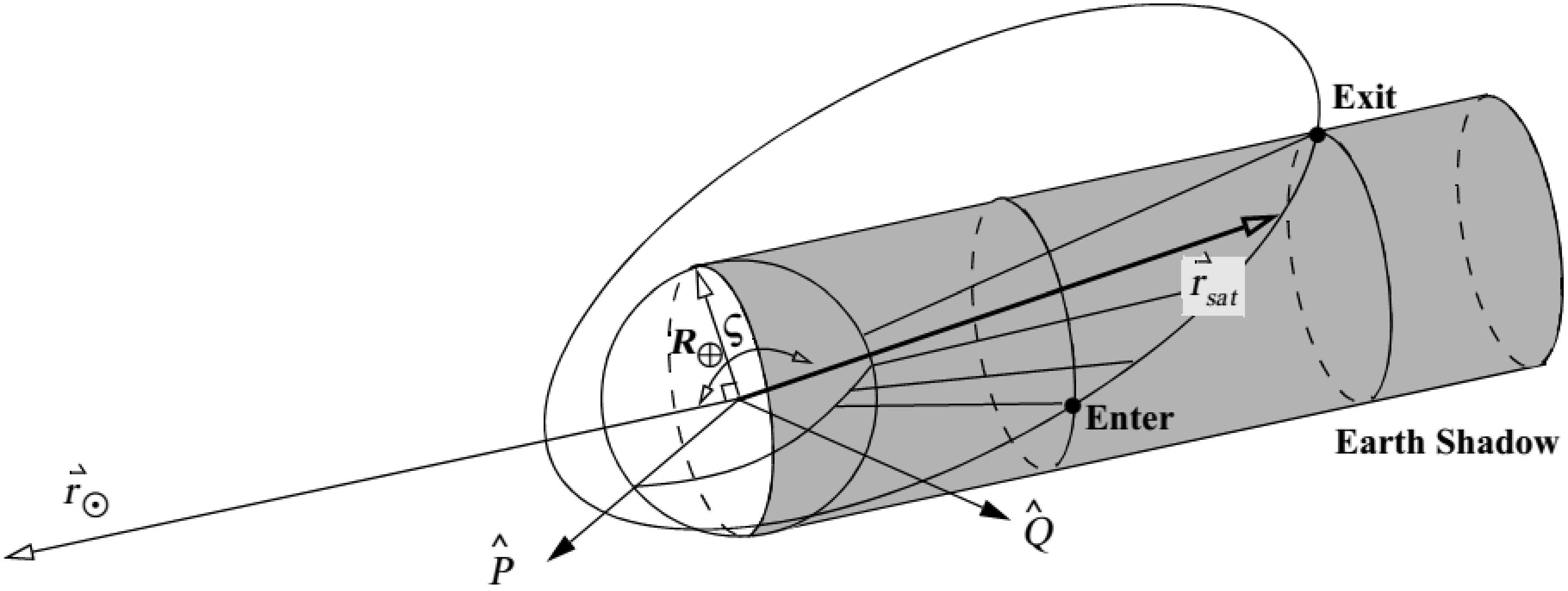
$\quad \text{shadow} = \text{"Umbra"}$

$if \quad \alpha == \alpha_1 \;\; or \;\; \alpha == \alpha_2 : \text{crossing the border}$

$else : sunlight$

Traditional Shadow Analysis

$$\cos(\varsigma) = \frac{\vec{r}_{\odot} \cdot \vec{r}_{sat}}{r_{\odot} r_{sat}}$$



True anomaly

$$\vec{r}_{sat} = x_w \hat{P} + y_w \hat{Q} = r_{sat}\cos(\nu)\hat{P} + r_{sat}\sin(\nu)\hat{Q}$$

$$\cos(\varsigma) = \frac{r_{sat}\cos(\nu)\vec{r}_{\odot} \cdot \hat{P} + r_{sat}\sin(\nu)\vec{r}_{\odot} \cdot \hat{Q}}{r_{\odot} r_{sat}}$$

Traditional Shadow Analysis

$$\beta_1 = \frac{\vec{r}_\odot \cdot \hat{P}}{r_\odot} \qquad \beta_2 = \frac{\vec{r}_\odot \cdot \hat{Q}}{r_\odot} \longrightarrow \cos(\varsigma) = \beta_1 \cos(\nu) + \beta_2 \sin(\nu)$$

- Eclipse condition: $r^2_{sat}\sin(\varsigma)^2 < R_\oplus^2$
- Shadow function: $S = R_\oplus^2(1 + e\cos(\nu))^2 + p^2\{\beta_1\cos(\nu) + \beta_2\sin(\nu)\}^2 - p^2$
- For elliptical orbits, the shadow function vanishes only if

$$1 - \left(\frac{R_\oplus}{a(1-e)}\right)^2 < \beta_1^2 < 1 - \left(\frac{R_\oplus}{a(1+e)}\right)^2$$

Using $\alpha = R_\oplus / p$

$$\alpha_1 = \alpha^4 e^4 - 2\alpha^2(\beta_2^2 - \beta_1^2)e^2 + (\beta_1^2 + \beta_2^2)^2$$

$$\alpha_2 = \alpha^4 4e^3 - 4\alpha^2(\beta_2^2 - \beta_1^2)e$$

$$\alpha_3 = \alpha^4 6e^2 - 2\alpha^2(\beta_2^2 - \beta_1^2) - 2\alpha^2(1 - \beta_2^2)e^2 + 2(\beta_2^2 - \beta_1^2)(1 - \beta_2^2) - 4\beta_2^2\beta_1^2$$

Shadow Analysis

$$\alpha_4 = \alpha^4 4e - 4\alpha^2(1-\beta_2^2)e$$

$$\alpha_5 = \alpha^4 - 2\alpha^2(1-\beta_2^2) + (1-\beta_2^2)^2$$

$$\Rightarrow \quad S = \alpha_1 \cos^4(\nu) + \alpha_2 \cos^3(\nu) + \alpha_3 \cos^2(\nu) + \alpha_4 \cos(\nu) + \alpha_5$$

- **Numerical Shadow Analysis**

$$x = \cos(\nu)$$

set $S = 0$

$$Ax^2 + Bx + Cx\sqrt{1-x^2} + D = 0$$

$$x_{n+1} = x_n + \delta_n = x_n - \frac{f(x_n)}{f'(x_n) - \dfrac{f(x_n)f''(x_n)}{2f'(x_n)}}$$

$$|f(x_n)| \leq \text{tolerance1}$$

$$|x_{n+1} - x_n| \leq \text{tolerance2}$$

Sight and Light

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_A . \mathbf{r}_B}{r_A r_B}\right)$$

$$\theta_1 = \cos^{-1}\frac{R}{r_A} \qquad \theta_2 = \cos^{-1}\frac{R}{r_B}$$

$$\begin{cases} \theta_1 + \theta_2 < \theta & \longrightarrow \text{There is no LOS.} \\ \theta_1 + \theta_2 \geq \theta & \longrightarrow \text{sight} \end{cases}$$