# Home Work #2

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## 1 Question 1

Spacecraft position in ITRF coordinates is given by

$$\mathbf{r} = \begin{bmatrix} 6789 & 6893 & 7035 \end{bmatrix}_{km}^{\mathrm{T}}$$

### 1.1 part a

Find Latitude and Longitude. For this purpose used algorithm 12 of Valado's book. This algorithm is implemented in the function 'latlon.py' in the 'code/Q1' folder. The function takes the spacecraft position vector as input and returns the latitude and longitude in degrees. The iteration ended when the difference is smaller than 1e-10. The results are:

Table 1: Results of part a

| **Variables** | **Values** |
|---|---|
| Latitude | 36.12° |
| Longitude | 45.43° |
| $h_{ellp}$ | $5591.51_{km}$ |

### 1.2 part b

In this part, we used the astropy package to find the position vector in the GCRF coordination system. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. Position vector in GCRF:

$$\mathbf{r} = \begin{bmatrix} -862.54 & -9634.75 & 7037.25 \end{bmatrix}_{km}^{\mathrm{T}}$$

### 1.3 part c

In this part, we used the astropy package to find GMST($\theta_{GMST}$) and LST($\theta_{LST}$) The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. The results are:

Table 2: Results of part c

| **GMST($\theta_{GMST}$)** | **LST($\theta_{LST}$)** |
|---|---|
| 112.78° | 149.78° |

# 2 Question 2

Satellite position and velocity vectors in the Earth-Centered Inertial (ECI) coordinate system:

$$\vec{r}_{ECI} = \begin{bmatrix}-346 & 8265 & 4680\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$

$$\vec{v}_{ECI} = \begin{bmatrix}-5.657 & -1.73 & 2.703\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

## 2.1 part a

Algorithm for converting from ECI to orbital x-y plane coordinates is described below:

1. Calculate the angular momentum vector $\vec{h}$ by taking the cross product of the position vector $\vec{r}$ and the velocity vector $\vec{v}$ in the ECI coordinate system.
2. Calculate the unit vector $\hat{z}$ in the direction of $\vec{h}$.
3. Calculate the unit vector $\hat{x}$ in the x-direction of the satellite position.
4. Calculate the unit vector $\hat{y}$ in the y-direction of the orbital plane by taking the cross product of $\hat{z}$ and $\hat{x}$.
5. Express the ECI position and velocity vectors $\vec{r}$ and $\vec{v}$ in the new coordinate system by taking their dot products with $\hat{x}$, $\hat{y}$, and $\hat{z}$.
6. Project the position and velocity vectors onto the x-y plane by setting the z-component of each vector to zero.

Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.
results:

$$r_{\text{x, y plane}} \begin{bmatrix}9504.3327488 & 0 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$
$$\vec{v}_{\text{x, y plane}} \begin{bmatrix}0 & 4.16567802 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

## 2.2 part b

To calculate satellite position after 30 minutes, the differential equation for the satellite solved for 30 minutes. The differential equation is:

$$\vec{r}_{\text{x, y plane}} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = -\frac{\mu}{r^3}\vec{r}$$

where $\mu$ is the gravitational parameter of the Earth, and $r$ is the magnitude of the position vector $\vec{r}$. Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.

results:

$$\vec{r}_{\text{x, y plane}} = \begin{bmatrix}1379.53 & 4493.87 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$
$$\vec{v}_{\text{x, y plane}} = \begin{bmatrix}-9.72 - 2.96 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

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