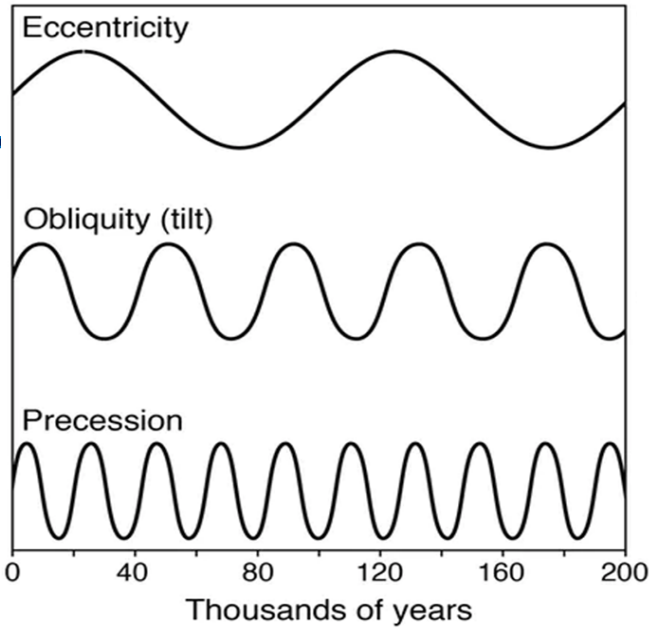
Transformation of ITRF to GCRF



- Orbit determination requires both celestial reference frames and terrestrial reference frames.

reduction formula (Earth orientation model)

$$\vec{r}_{GCRF} = [\boldsymbol{P}(t)][\boldsymbol{N}(t)][\boldsymbol{R}(t)][\boldsymbol{W}(t)]\vec{r}_{ITRF}$$

Precession matrix of date t

Nutation matrix of date t

sidereal-rotation matrix of date t

polar-motion matrix of date t

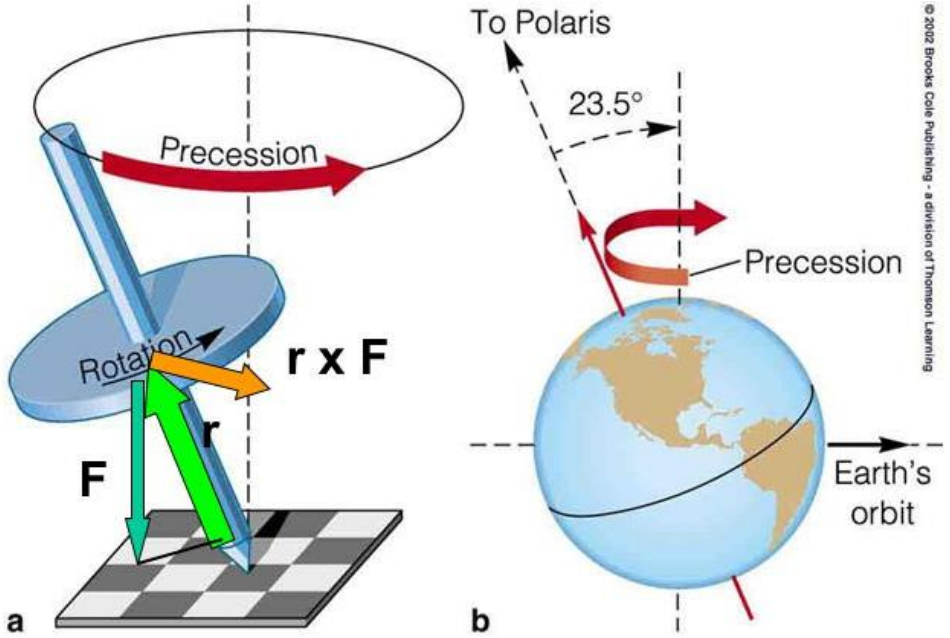
- IAU Resolutions : IAU-2006/2000

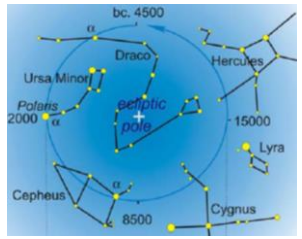
precession

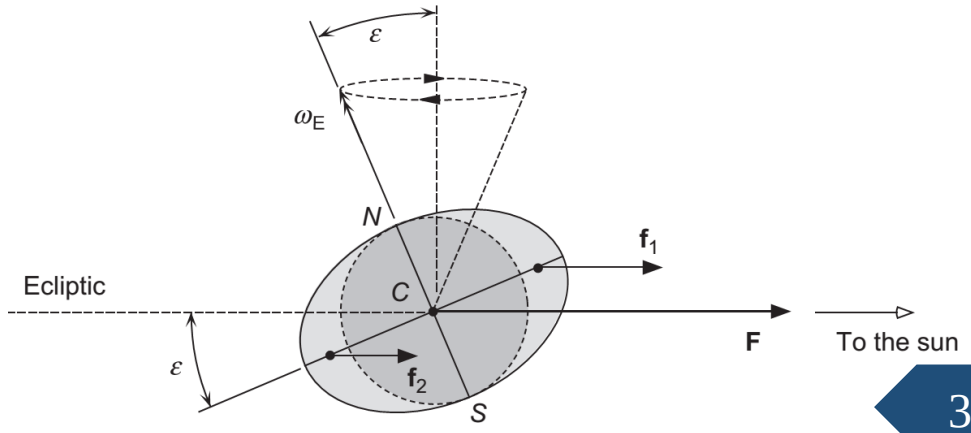
nutation

Precession

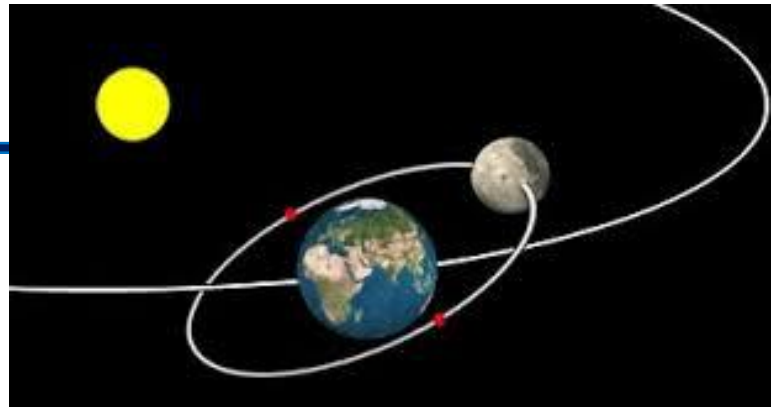
- Precession
  - ❑ precession of the ecliptic
    - equinox precession to the west (12 "/century)
    - Ecliptic obliquity reduction(47 "/century)
  - ❑ luni-solar precession
    - Period: 26000 year (49.846 "/year)
  - ❑ general precession
    - 50 "/year



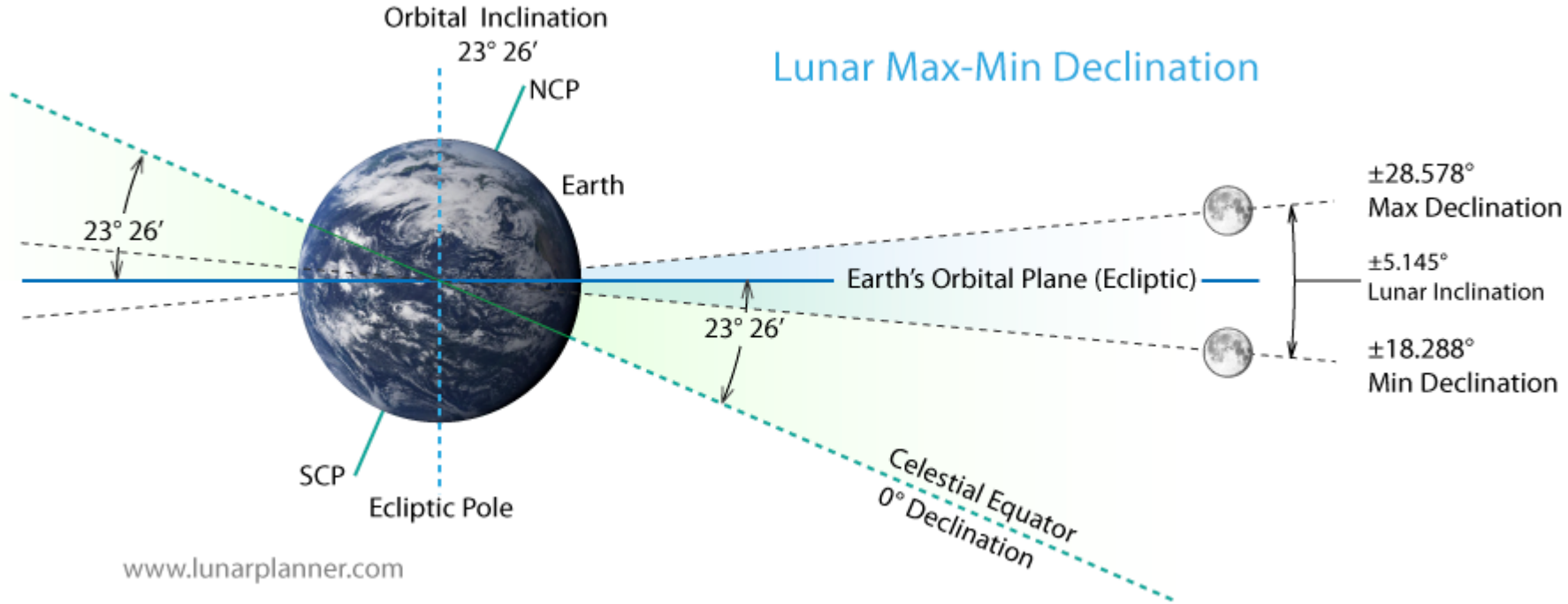




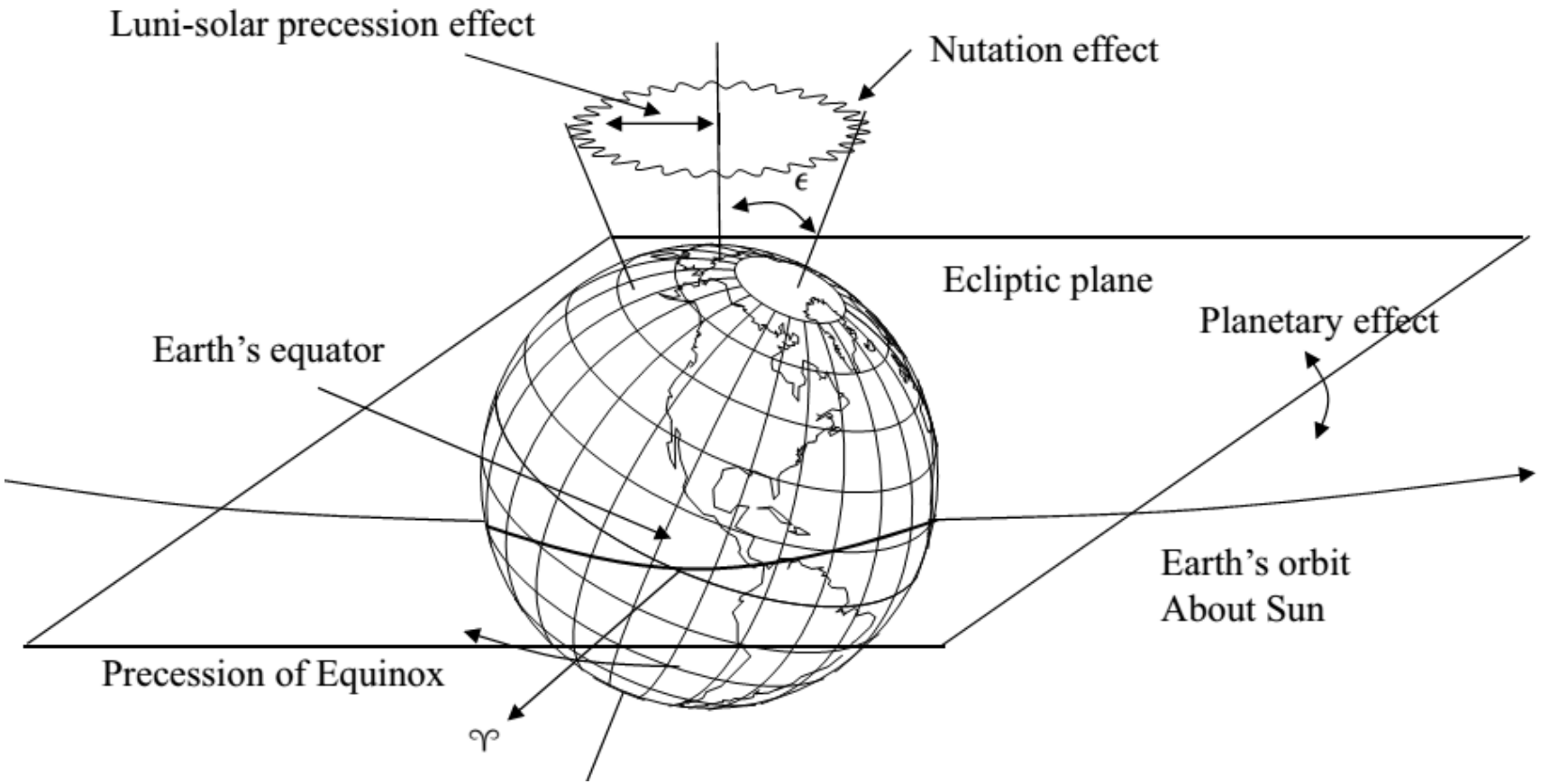
Nutation



- Nutation
  - period about 18.6 years
  - Precession of the equator/regression of the lunar node
  - Variation of the max torque direction
  - Max amplitude of 9" in obliquity of the ecliptic and 17" in longitude



Precession-Nutation

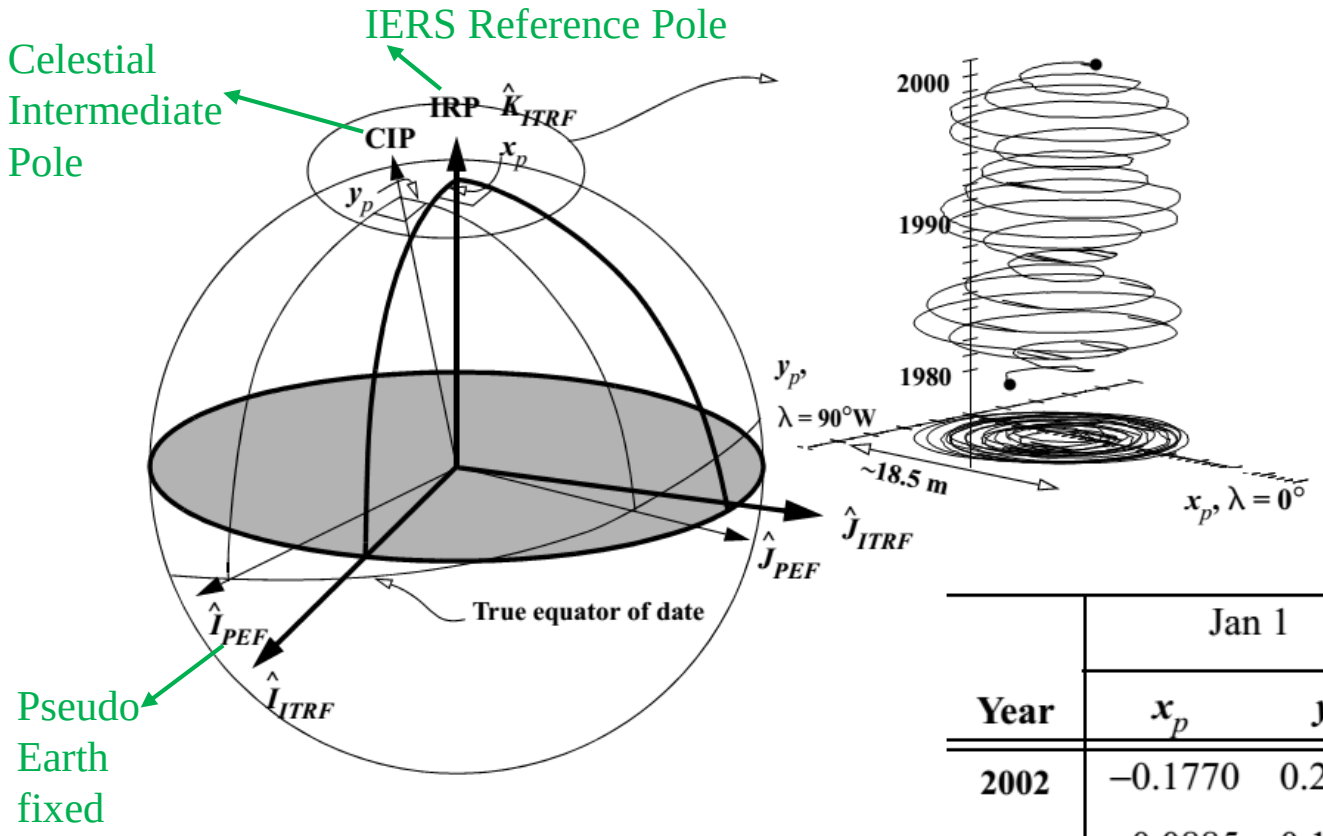


Some definitions

- ecliptic of date: ecliptic plane orientation (variable due to general precession) with an ideal inertial frame on a specific date.
- mean equator of date: modeled equator wobbling around the ecliptic
- mean equinox of date: the intersection of equator and ecliptic planes on a given date
- true equator of date: including the effect of nutation

Polar Motion

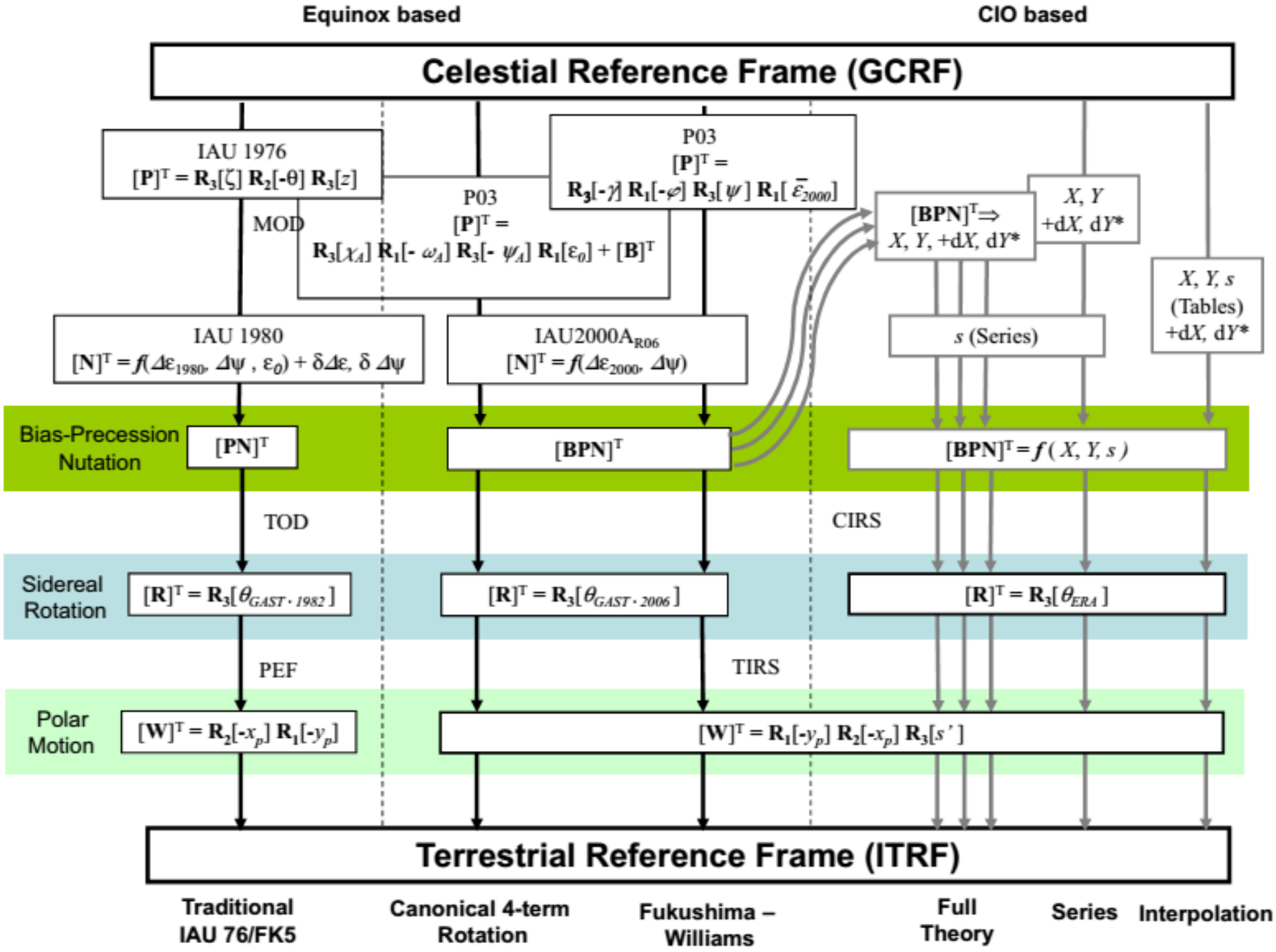
- movement of the rotation axis with respect to the crust of the Earth



Year	Jan 1		Oct 1	
	x_p	y_p	x_p	y_p
2002	–0.1770	0.2940	0.1985	0.2005
2003	–0.0885	0.1880	0.2585	0.3040
2004	0.0313	0.1538	0.1985	0.4315
2005	0.1492	0.2380	0.0588	0.4169
2006	0.0527	0.3834	0.0327	0.2524

0.1" = 0.000 028°= 3.09 m
for the surface of the Earth

GCRF to ITRF


Equinox based
CIO based
Celestial Reference Frame (GCRF)
IAU 1976
[P]T = R3[ζ] R2[-θ] R3[z]
P03
[P]T =
R3[-γ] R1[-φ] R3[ψ] R1[ε̄2000]
P03
[P]T =
R3[χA] R1[- ωA] R3[- ψA] R1[ε0] + [B]T
MOD
[BPN]T⇒
X, Y, +dX, dY*
X, Y
+dX, dY*
X, Y, s
(Tables)
+dX, dY*
s (Series)
IAU 1980
[N]T = f(Δε1980, Δψ , ε0) + δΔε, δ Δψ
IAU2000AR06
[N]T = f(Δε2000, Δψ)
Bias-Precession Nutation
[PN]T
[BPN]T
[BPN]T = f(X, Y, s)
TOD
CIRS
Sidereal Rotation
[R]T = R3[θGAST · 1982]
[R]T = R3[θGAST · 2006]
[R]T = R3[θERA]
PEF
TIRS
Polar Motion
[W]T = R2[-xp] R1[-yp]
[W]T = R1[-yp] R2[-xp] R3[s ']
Terrestrial Reference Frame (ITRF)
Traditional IAU 76/FK5
Canonical 4-term Rotation
Fukushima – Williams
Full Theory
Series
Interpolation

ITRF to GCRF

- Polar Motion (IAU-2006/2000, CIO based)

$$\vec{r}_{TIRS} = \mathrm{ROT3}(-s')\mathrm{ROT2}(x_p)\mathrm{ROT1}(y_p)\vec{r}_{ITRF} = [W]_{\text{ITRF-TIRS}}\vec{r}_{ITRF}$$

$$s' = -0.0015''\left(\frac{a_c^2}{1.2} + a_a^2\right)T_{TT} \cong -0.000\,047''\,T_{TT}$$

Chandler wobble $(a_c = 0.26'')$ ***annual wobble*** $(a_a = 0.12'')$

$$[W]_{\text{ITRF-TIRS}} = \begin{bmatrix} \cos(x_p)\cos(s') & -\cos(y_p)\sin(s') + \sin(y_p)\sin(x_p)\cos(s') & -\sin(y_p)\sin(s') - \cos(y_p)\sin(x_p)\cos(s') \\ \cos(x_p)\sin(s') & \cos(y_p)\cos(s') + \sin(y_p)\sin(x_p)\sin(s') & \sin(y_p)\cos(s') - \cos(y_p)\sin(x_p)\sin(s') \\ \sin(x_p) & -\sin(y_p)\cos(x_p) & \cos(y_p)\cos(x_p) \end{bmatrix}$$

- Earth Rotation Angle (IAU-2006/2000, CIO based)

$$\theta_{ERA} = 280.460\,618\,375\,04^\circ + 360.985\,612\,288\,808(JD_{UT1} - 2{,}451{,}545.0)$$
$$= 2\pi(0.779\,057\,273\,264\,0 + 1.002\,737\,811\,911\,354\,48(JD_{UT1} - 2{,}451{,}545.0))$$

$$\vec{r}_{CIRS} = \mathrm{ROT3}(-\theta_{ERA})\vec{r}_{TIRS}$$

$$\vec{v}_{CIRS} = \mathrm{ROT3}(-\theta_{ERA})\vec{v}_{TIRS} + \vec{\omega}_{\oplus} \times \vec{r}_{TIRS}$$

ITRF to GCRF

- Precession-Nutation (IAU-2006/2000, CIO based)

$$\vec{r}_{GCRF} = [\boldsymbol{PN}]\vec{r}_{CIRS}$$

$$= \begin{bmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{bmatrix} \mathrm{ROT3}(s)\vec{r}_{CIRS}$$

$$a = \frac{1}{1 + \cos(d)} \cong \frac{1}{2} + \frac{1}{8}(X^2 + Y^2)$$

$$X = -0.016\,617'' + 2004.191\,898\,T_{TT} - 0.429\,782\,9\,T_{TT}^2 - 0.198\,618\,34\,T_{TT}^3$$
$$+ 0.000\,007\,578\,T_{TT}^4 + 0.000\,005\,928\,5\,T_{TT}^5$$
$$+ \sum_{i=1}^{1306} [A_{xs0_i} \sin(a_{p_i}) + A_{xc0_i} \cos(a_{p_i})] + \sum_{i=1}^{253} [A_{xs1_i} \sin(a_{p_i}) + A_{xc1_i} \cos(a_{p_i})] T_{TT}$$
$$+ \sum_{i=1}^{36} [A_{xs2_i} \sin(a_{p_i}) + A_{xc2_i} \cos(a_{p_i})] T_{TT}^2 + \sum_{i=1}^{4} [A_{xs3_i} \sin(a_{p_i}) + A_{xc3_i} \cos(a_{p_i})] T_{TT}^3$$
$$+ \sum_{i=1}^{1} [A_{xs4_i} \sin(a_{p_i}) + A_{xc4_i} \cos(a_{p_i})] T_{TT}^4$$

ITRF to GCRF

$$
\begin{aligned}
Y = {} & -0.006\,951'' - 0.025\,896\,T_{TT} - 22.407\,274\,7\,T_{TT}^{2} + 0.001\,900\,59\,T_{TT}^{3} \\
& + 0.001\,112\,526\,T_{TT}^{4} + 0.000\,000\,135\,8\,T_{TT}^{5} \\
& + \sum_{i=1}^{962} [A_{ys0_i} \mathrm{SIN}(a_{p_i}) + A_{yc0_i} \mathrm{COS}(a_{p_i})] + \sum_{i=1}^{277} [A_{ys1_i} \mathrm{SIN}(a_{p_i}) + A_{yc1_i} \mathrm{COS}(a_{p_i})]\,T_{TT} \\
& + \sum_{i=1}^{30} [A_{ys2_i} \mathrm{SIN}(a_{p_i}) + A_{yc2_i} \mathrm{COS}(a_{p_i})]\,T_{TT}^{2} + \sum_{i=1}^{5} [A_{ys3_i} \mathrm{SIN}(a_{p_i}) + A_{yc3_i} \mathrm{COS}(a_{p_i})]\,T_{TT}^{3} \\
& + \sum_{i=1}^{1} [A_{ys4_i} \mathrm{SIN}(a_{p_i}) + A_{yc4_i} \mathrm{COS}(a_{p_i})]\,T_{TT}^{4}
\end{aligned}
$$

$$
\begin{aligned}
a_{p_i} = {} & a_{0x1_i} M_{☾} + a_{0x2_i} M_{☉} + a_{0x3_i} u_{M☾} + a_{0x4_i} D_{☉} + a_{0x5_i} \Omega_{☾} \\
& + a_{0x6_i} \lambda_{M☿} + a_{0x7_i} \lambda_{M♀} + a_{0x8_i} \lambda_{M⊕} + a_{0x9_i} \lambda_{M♂} \\
& + a_{0x10_i} \lambda_{M♃} + a_{0x11_i} \lambda_{M♄} + a_{0x12_i} \lambda_{M♅} + a_{0x13_i} \lambda_{M♆} + a_{0x14_i} p_{\lambda}
\end{aligned}
$$

ITRF to GCRF

$$
\begin{aligned}
s = {} & -\frac{XY}{2} + 0.000\,094'' + 0.003\,808\,65\,T_{TT} - 0.000\,122\,68\,T_{TT}^2 - 0.072\,574\,11\,T_{TT}^3 \\
& -0.002\,640\,73\,\mathrm{SIN}(\Omega_{\mathbb{C}}) - 0.000\,063\,53\,\mathrm{SIN}(2\Omega_{\mathbb{C}}) \\
& -0.000\,011\,75\,\mathrm{SIN}(2u_{M\mathbb{C}} - 2D_{\odot} + 3\Omega_{\mathbb{C}}) - 0.000\,011\,21\,\mathrm{SIN}(2u_{M\mathbb{C}} - 2D_{\odot} + \Omega_{\mathbb{C}}) \\
& + 0.000\,004\,57\,\mathrm{SIN}((2u_{M\mathbb{C}} - 2D_{\odot} + 2\Omega_{\mathbb{C}}) - 0.000\,002\,02\,\mathrm{SIN}(2u_{M\mathbb{C}} + 3\Omega_{\mathbb{C}})) \\
& -0.000\,001\,98\,\mathrm{SIN}((2u_{M\mathbb{C}} + \Omega_{\mathbb{C}}) + 0.000\,001\,72\,\mathrm{SIN}(3\Omega_{\mathbb{C}})) \\
& + 0.000\,001\,41\,\mathrm{SIN}((M_{\odot} + \Omega_{\mathbb{C}}) + 0.000\,001\,26\,\mathrm{SIN}(M_{\odot} - \Omega_{\mathbb{C}})) \\
& + 0.000\,000\,63\,\mathrm{SIN}((M_{\mathbb{C}} + \Omega_{\mathbb{C}}) + 0.000\,000\,63\,\mathrm{SIN}(M_{\mathbb{C}} - \Omega_{\mathbb{C}})) \\
& + 0.000\,001\,73\,T_{TT}\mathrm{SIN}(\Omega_{\mathbb{C}}) + 0.000\,003\,57\,T_{TT}\mathrm{COS}(2\Omega_{\mathbb{C}}) + 0.000\,743\,52\,T_{TT}^2\mathrm{SIN}(\Omega_{\mathbb{C}}) \\
& + 0.000\,056\,91\,T_{TT}^2\mathrm{SIN}(2(u_{M\mathbb{C}} - D_{\odot} + \Omega_{\mathbb{C}})) + 0.000\,009\,84\,T_{TT}^2\mathrm{SIN}(2(u_{M\mathbb{C}} + \Omega_{\mathbb{C}})) \\
& - 0.000\,008\,85\,T_{TT}^2\mathrm{SIN}(2\Omega_{\mathbb{C}})
\end{aligned}
$$

$$
\begin{cases}
\vec{r}_{GCRF} = [PN][R][W]\vec{r}_{ITRF} \\
\vec{v}_{GCRF} = [PN][R]\left\{[W]\vec{v}_{ITRF} + \vec{\omega}_{\oplus} \times \vec{r}_{TIRS}\right\}
\end{cases}
$$

$$
\begin{cases}
\vec{r}_{ITRF} = [W]^T[R]^T[PN]^T\vec{r}_{GCRF} \\
\vec{v}_{ITRF} = [W]^T\left\{[R]^T[PN]^T\vec{v}_{GCRF} - \vec{\omega}_{\oplus} \times \vec{r}_{TIRS}\right\}
\end{cases}
$$

ITRF to GCRF

$$M_{☾} = 485868.249\,036'' + 1717915923.2178\,T_{TT} + 31.8792\,T_{TT}^2 + 0.051\,635\,T_{TT}^3 - 0.000\,244\,70\,T_{TT}^4$$

$$M_{☉} = 1287104.79305'' + 129596581.0481\,T_{TT} - 0.5532\,T_{TT}^2 + 0.000\,136\,T_{TT}^3 - 0.000\,011\,49\,T_{TT}^4$$

$$u_{M☾} = 335779.526\,232'' + 1739527262.8478\,T_{TT} - 12.7512\,T_{TT}^2 - 0.001\,037\,T_{TT}^3 + 0.000\,004\,17\,T_{TT}^4$$

$$D_{☉} = 1072260.703\,69'' + 1602961601.2090\,T_{TT} - 6.3706\,T_{TT}^2 + 0.006\,593\,T_{TT}^3 - 0.000\,031\,69\,T_{TT}^4$$

$$\Omega_{☾} = 450160.398\,036'' - 6962890.5431\,T_{TT} + 7.4722\,T_{TT}^2 + 0.007702\,T_{TT}^3 - 0.000\,059\,39\,T_{TT}^4$$

$$\lambda_{M☿} = 4.402\,608\,842 + 2608.790\,314\,157\,4\,T_{TT}$$

$$\lambda_{M♀} = 3.176\,146\,697 + 1021.328\,554\,621\,1\,T_{TT}$$

$$\lambda_{M⊕} = 1.753\,470\,314 + 628.307\,584\,999\,1\,T_{TT}$$

$$\lambda_{M♂} = 6.203\,480\,913 + 334.061\,242\,670\,0\,T_{TT}$$

$$\lambda_{M♃} = 0.599\,546\,497 + 52.969\,096\,264\,1\,T_{TT}$$

$$\lambda_{M♄} = 0.874\,016\,757 + 21.329\,910\,496\,0\,T_{TT}$$

$$\lambda_{M♅} = 5.481\,293\,872 + 7.478\,159\,856\,7\,T_{TT}$$

$$\lambda_{M♆} = 5.311\,886\,287 + 3.813\,303\,563\,8\,T_{TT}$$

$$p_{\lambda} = 0.024\,381\,75\,T_{TT} + 0.000\,005\,386\,91\,T_{TT}^2$$