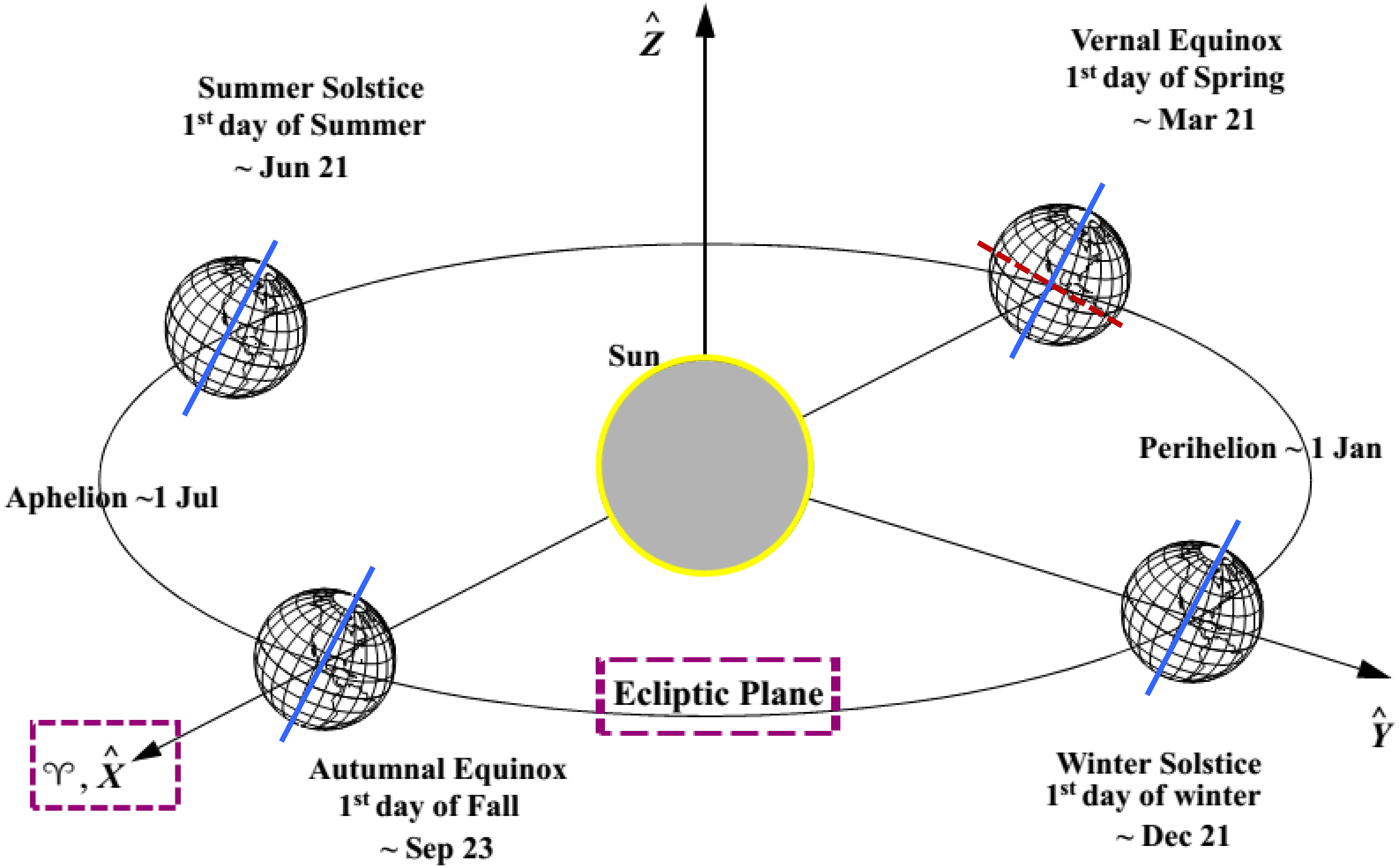
Interplanetary Systems

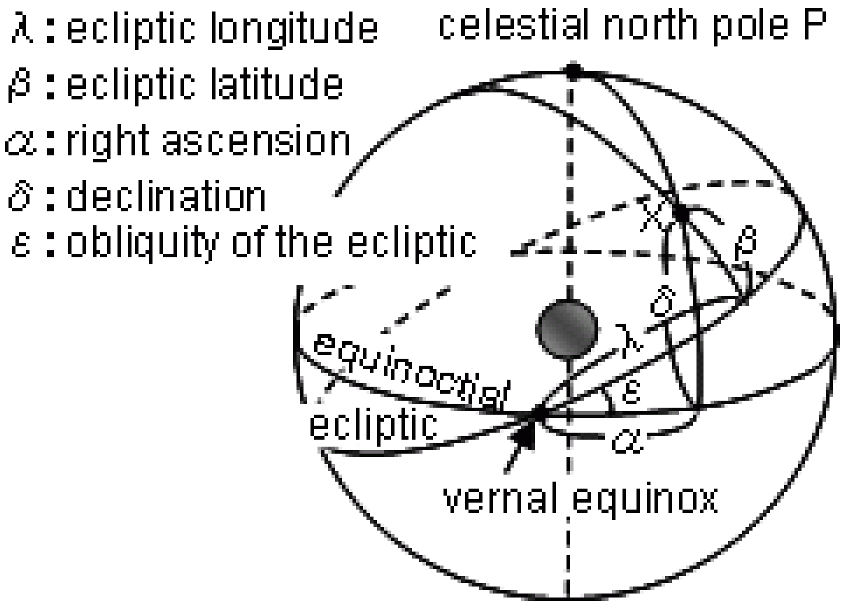
- Heliocentric Coordinate System (HRF)



Interplanetary Systems

- Heliocentric Coordinate System

  - ecliptic latitude ($\phi_{ecliptic}$)
    ecliptic longitude ($\lambda_{ecliptic}$)



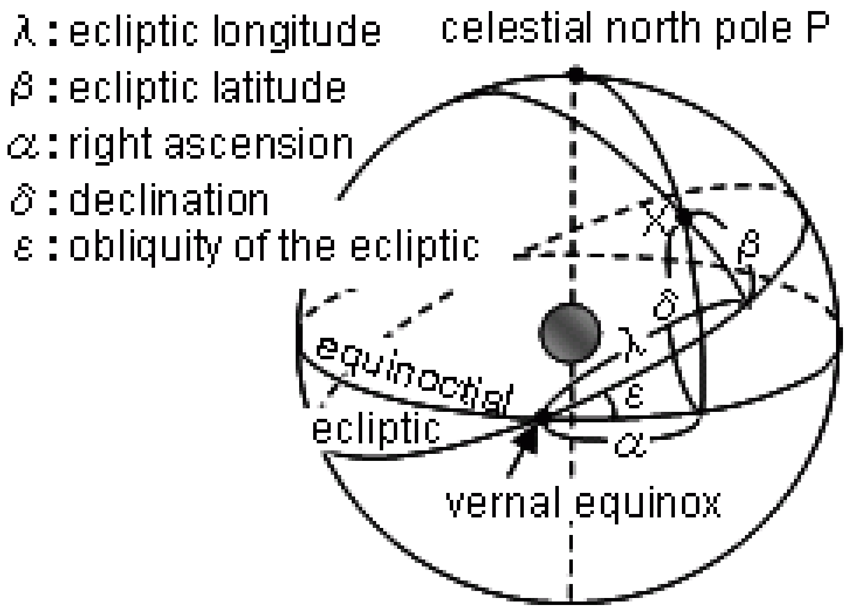
ecliptic and equatorial coordinate system

- Barycentric Celestial Reference Frame (BCRF):

  HRF with origin located at barycenter of solar system.

Interplanetary Systems

- What is the relationship between α , δ and $\phi_{ecl.}, \lambda_{ecl}$?



ecliptic and equatorial coordinate system

Interplanetary Systems

- International Celestial Reference Frame (ICRF)
  - Origin: barycenter of the solar system
  - ICRF is realized by observations of 3414 extragalactic radio sources from Very Long Baseline Interferometry (VLBI) measurements.

- Earth-based Systems
  - Origin: Earth's center (geocentric), or at a site on the Earth's surface (topocentric)

Earth-based Systems

- Geocentric Equatorial Coordinate System, IJK

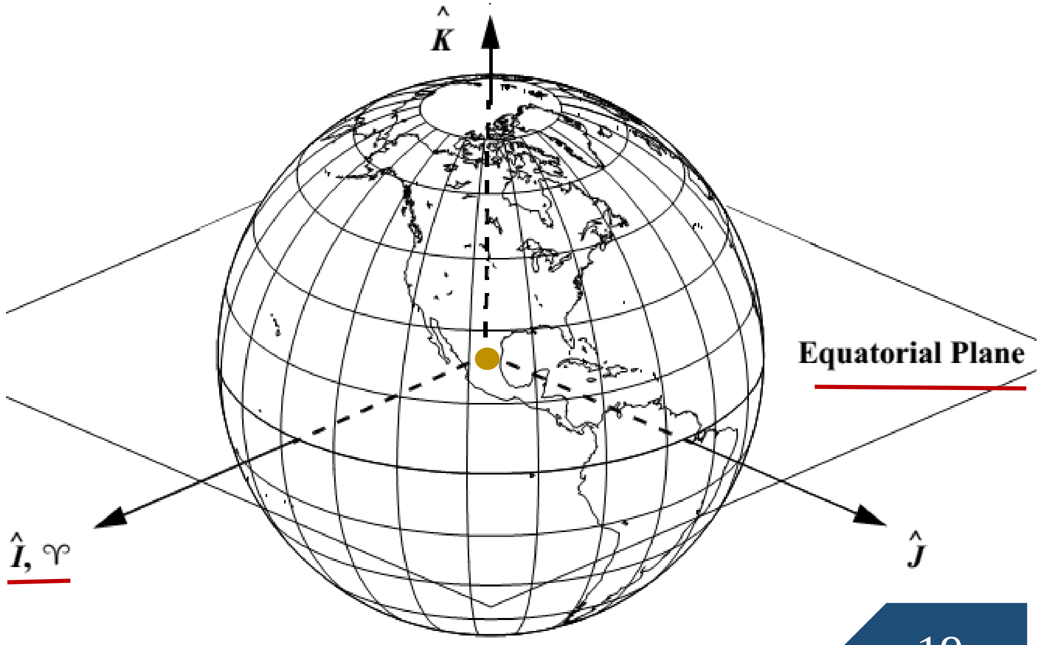
  Earth Centered Inertial (ECI)/Conventional Inertial System (CIS)

  ↓

  equator and equinox's motion ?!

- Geocentric Celestial Coordinate System, GCRF
  - modified ECI for precession and nutation

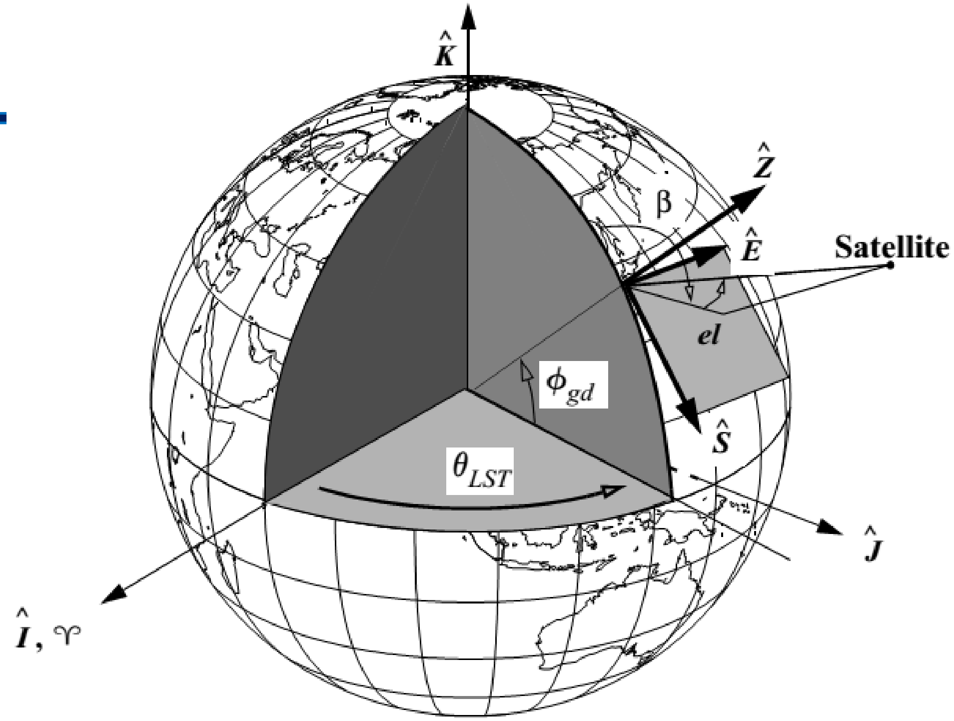
- Body-Fixed Coordinate System, ITRF
  - ECEF (neglects polar motion)
  - $\mathbf{T}^{ITRF-GCRF} = \mathbf{R}_3(\theta_{GMST})$?

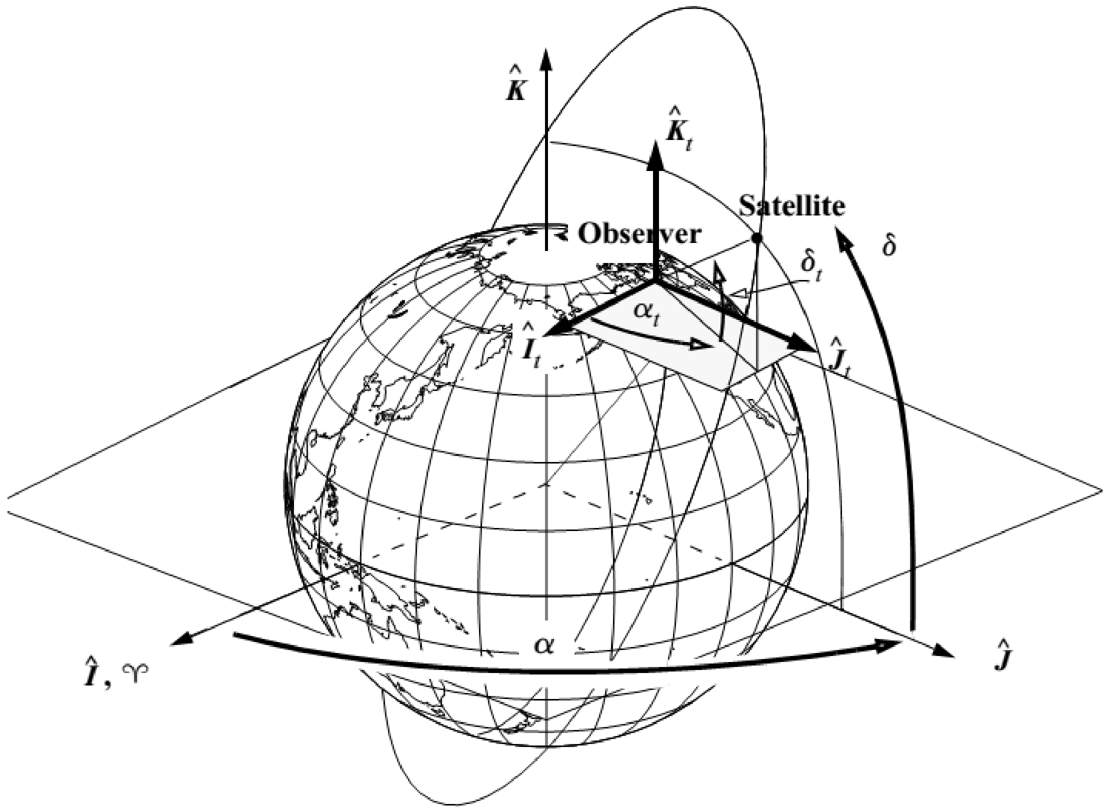


Earth-based Systems

- Topocentric Horizon Coordinate System, SEZ

$$\mathbf{T}^{SEZ-GCRF} = \mathbf{R}_2(90-\phi_{gd})\mathbf{R}_3(\theta_{LST})$$





- Topocentric Equatorial Coordinate System

Interplanetary Systems

- What is the relationship between α_t , δ_t and Az, el ?

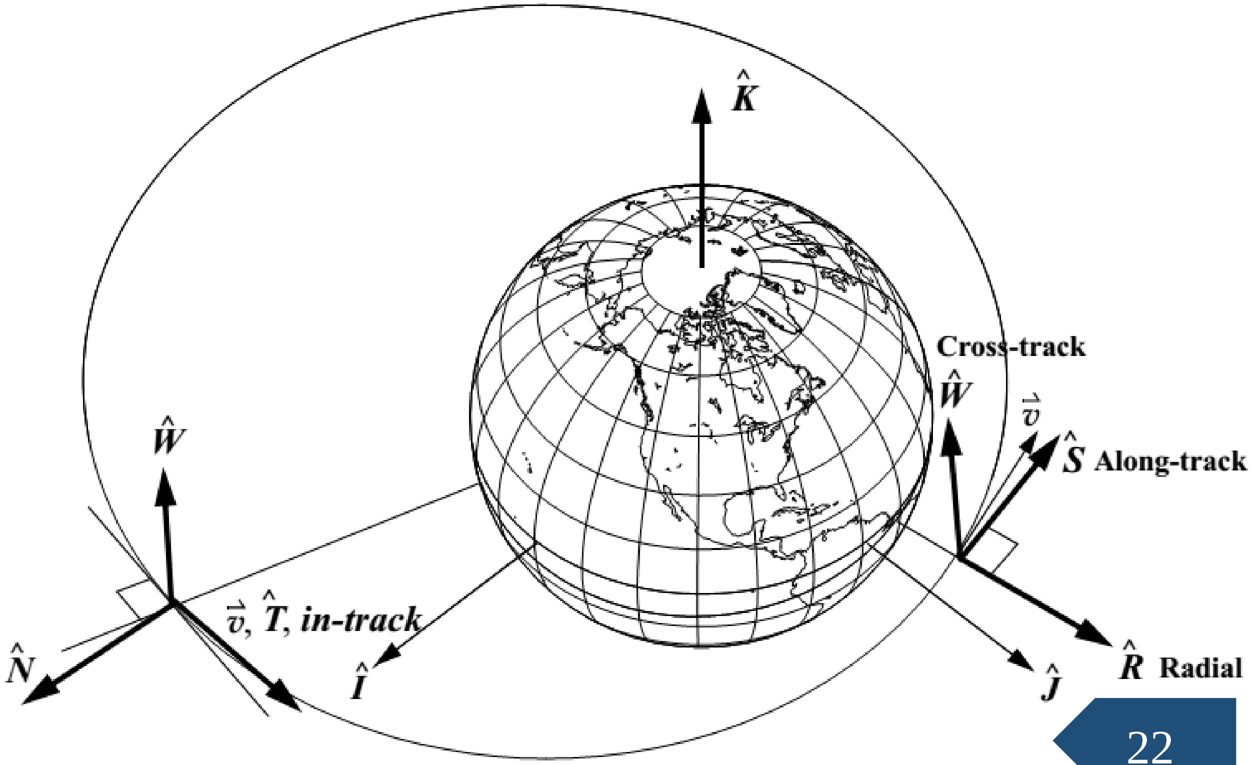
Satellite-based Systems

- Perifocal Coordinate System, PQW
- Nodal coordinate system: $\mathbf{T}^{Nodal-GCRF} = \mathbf{R}_1(i)\mathbf{R}_3(\Omega)$
- Satellite Coordinate System, RSW (Gaussian coordinate system: RTN, LVLH)

$$\mathbf{T}^{RSW-PCS} = \mathbf{R}_3(\theta)$$

$$\mathbf{T}^{RSW-NQW} = \mathbf{R}_3(\omega+\theta)$$

$$\mathbf{T}^{RSW-EQW} = \mathbf{R}_3(\Omega+\omega+\theta)$$

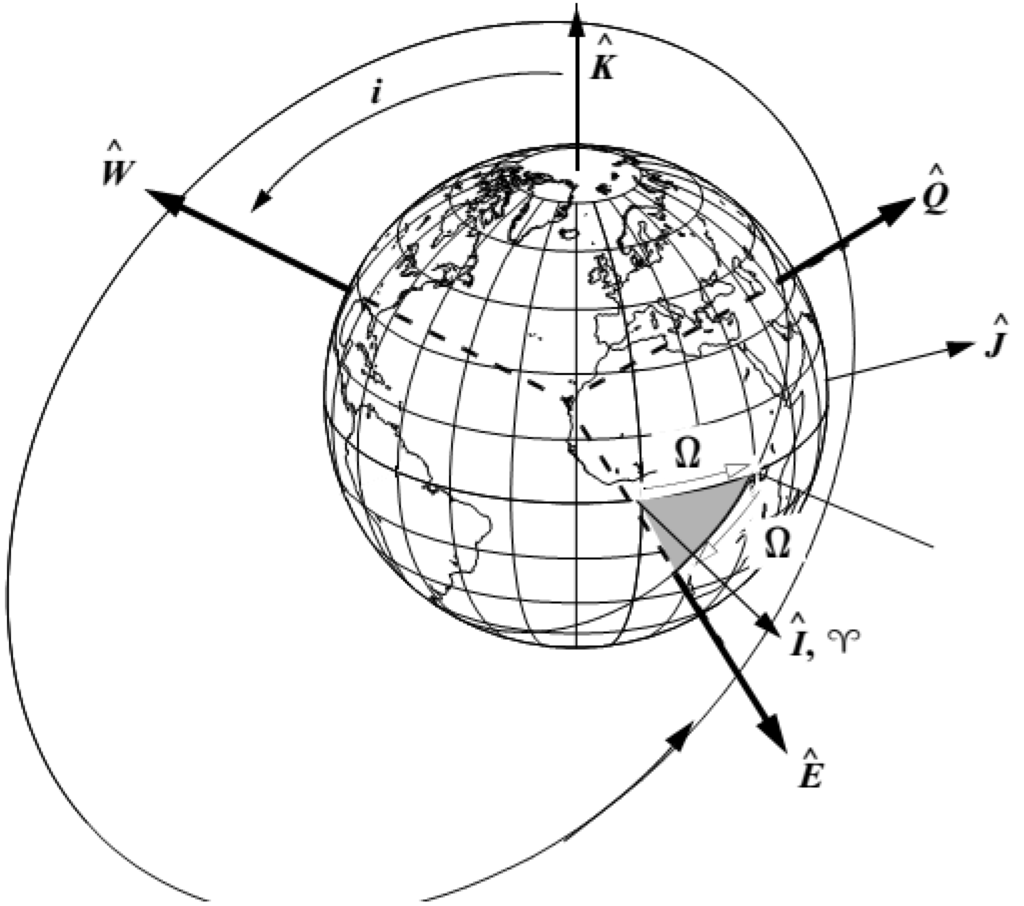


Satellite-based Systems

- Equinoctial coordinate system, EQW

$$\mathbf{T}^{EQW-GCRF} = \mathbf{R}_3(-f_r\Omega)\mathbf{R}_1(i)\mathbf{R}_3(\Omega)$$

$$f_r = \begin{cases} 1 & \textit{prograde orbit} \\ -1 & \textit{retrograde orbit} \end{cases}$$



Time

- Epoch
- Time interval
  - sidereal time
  - solar (universal) time
  - dynamical time
  - Atomic time
- Time & angle units

$$1^h = 60 \text{ minutes}(60^m) = 3600 \text{ seconds}(3600^s)$$

$$1^\circ = 60 \text{ arcminutes}(60') = 3600 \text{ arcseconds}(3600'')$$

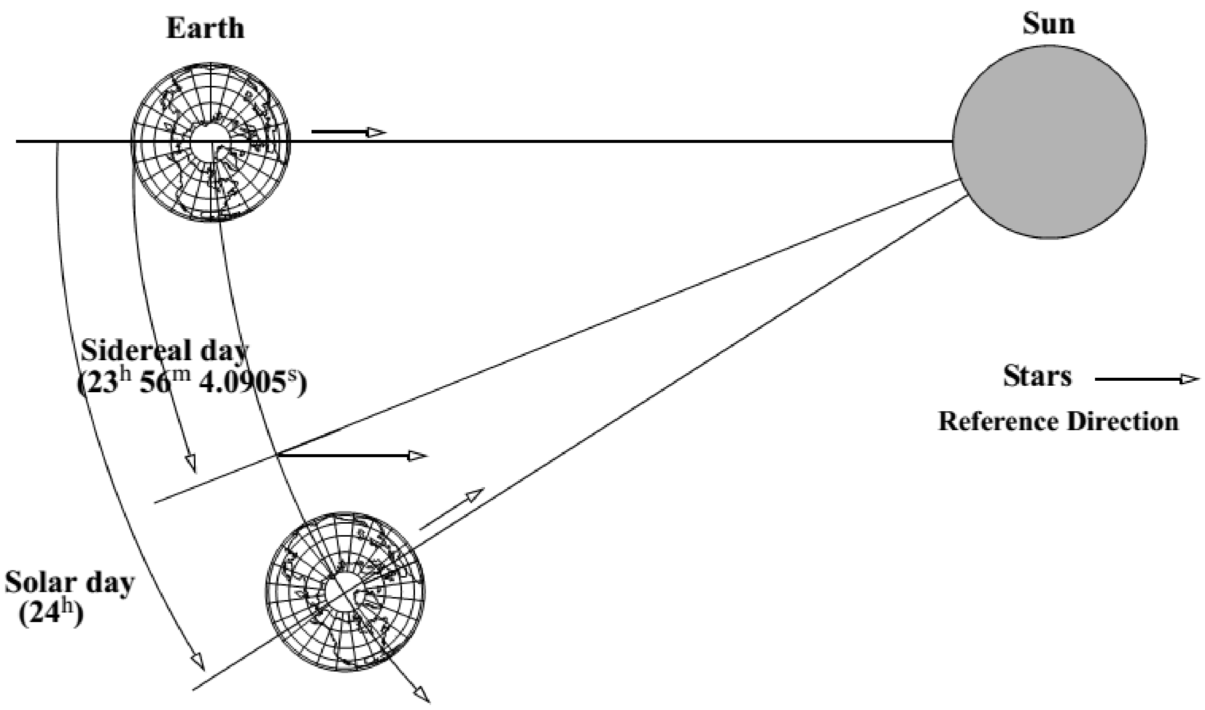
$$1^h = 15^\circ \qquad 1^\circ = \frac{1^h}{15} = 4^m$$

$$1^m = 15' \qquad 1' = \frac{1^m}{15} = 4^s$$

$$1^s = 15'' \qquad 1'' = \frac{1^s}{15}$$

Time

- Solar day vs. sidereal day:



- Solar time is loosely defined by successive transits of the Sun over a local meridian.
- sidereal time is defined by successive transits of the stars over a particular meridian.

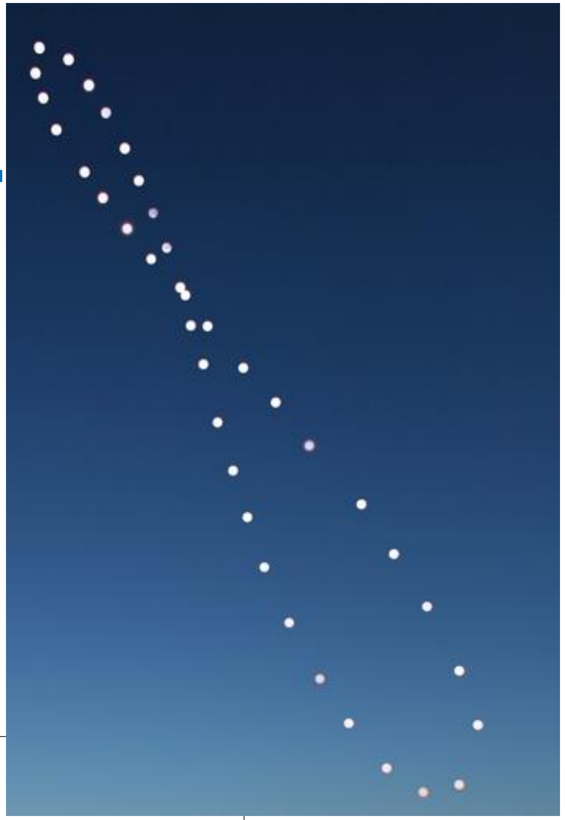
1 solar day = 1.002 737 909 350 795 sidereal day

1 sidereal day = 0.997 269 566 329 084 solar day

1 solar day = 24^h 3^m $56.555\ 367\ 8^s$ sidereal time

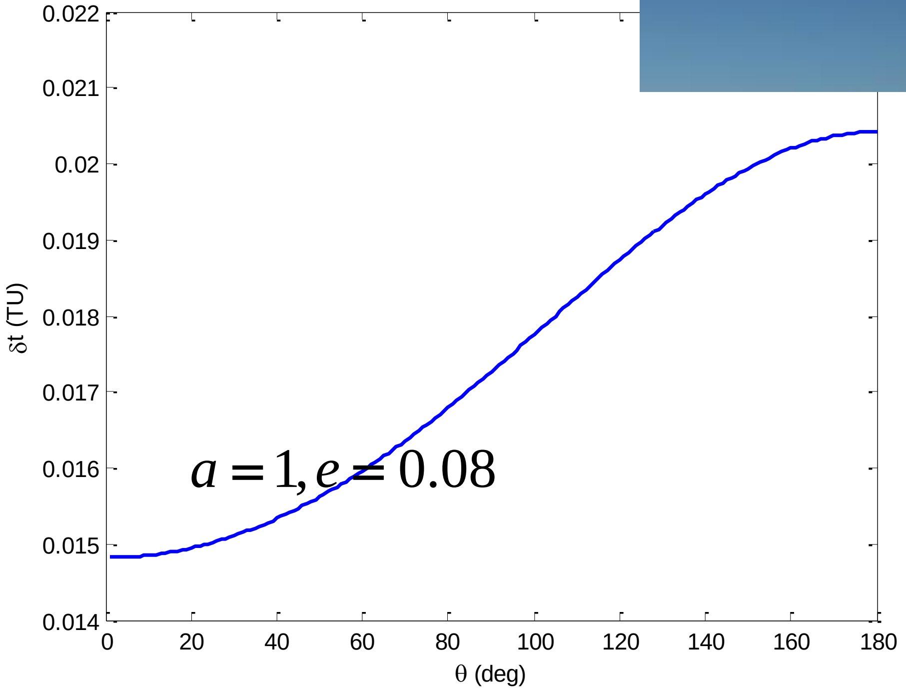
1 sidereal day = 23^h 56^m $4.090\ 524^s$ solar time

Apparent Solar Time



- Sun’s apparent motion results from a combination of the Earth’s rotation on its axis and its annual orbital motion about the Sun.

- The Earth moves with a variable speed in the orbit, causing the Sun to exhibit non-uniform motion along the ecliptic. In addition, the ecliptic is inclined about 23.5° to the celestial equator; thus, the solar motion on the ecliptic appears as a sinusoidal motion when projected on the celestial equator.



Apparent Solar Time

- The Earth's orbit about the Sun has a small eccentricity, causing the length of each day to differ by a small amount

$$local\ apparent\ solar\ time = LHA_{\odot} + 12^{h}$$

$$Greenwich\ apparent\ solar\ time = \underbrace{\theta_{GMST} - \alpha_{\odot}}_{GHA_{\odot}} + 12^{h}$$

- The variation in the Sun's apparent motion in right ascension (along the celestial equator) makes it a poor choice for establishing a precise time system because its observed length varies throughout the year.

Fictitious Mean Sun (1895)

based on the assumption of uniform motion along the celestial equator.

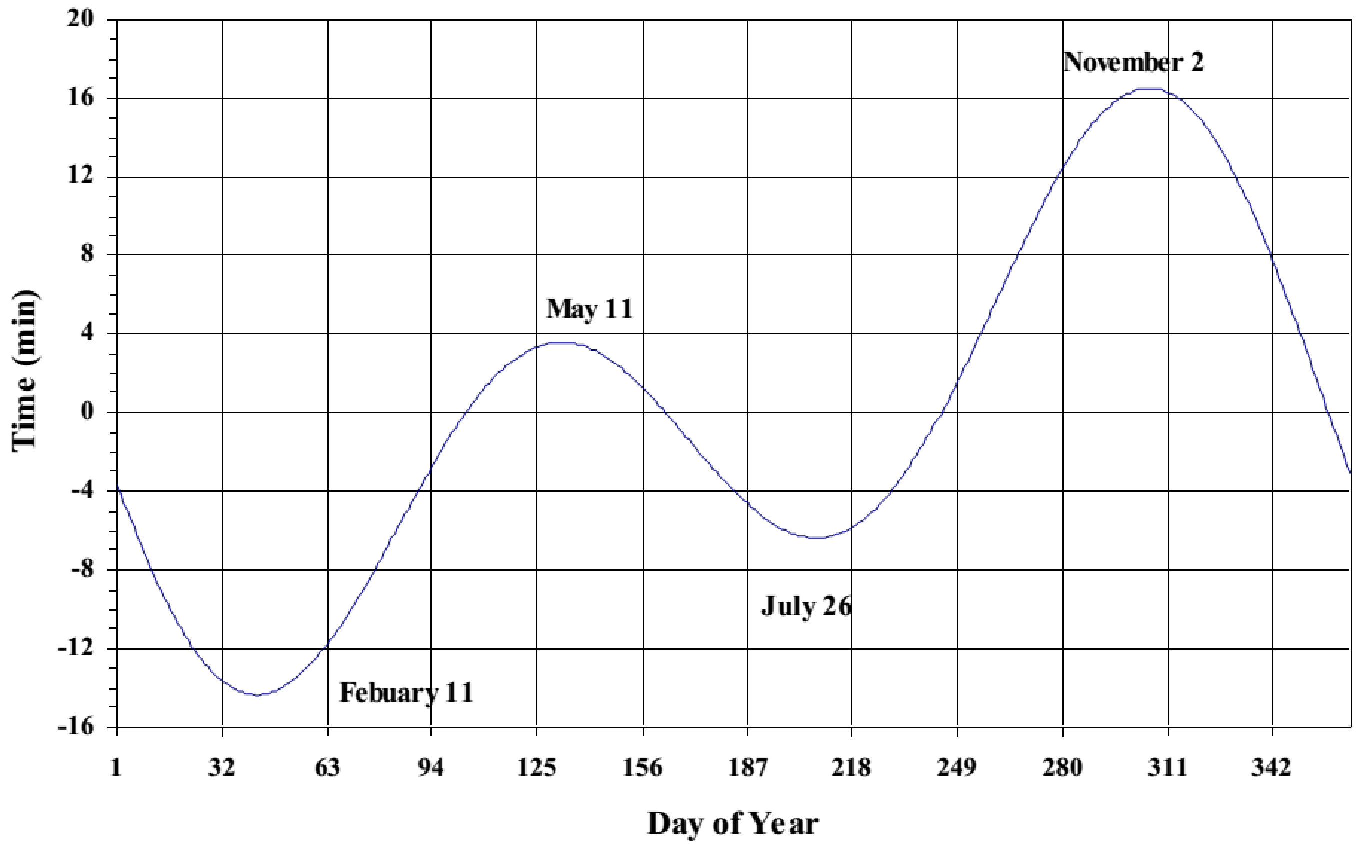
$$\alpha_{FMS} = 18^{h}\ 38^{m}\ 45.836^{s} + 8{,}640{,}184.542^{s}\ T + 0.0929^{s}\ T^{2}$$

Julian centuries from Greenwich at noon, January 1, 1900

Mean Solar Time

- equation of time: difference between apparent and mean solar time

$$EQ_{time} = -1.914\,666\,471^{\circ}\,\mathrm{SIN}(M_{\odot}) - 0.019\,994\,643\,\mathrm{SIN}(2M_{\odot})$$
$$+\,2.466\,\mathrm{SIN}(2\lambda_{ecliptic}) - 0.0053\,\mathrm{SIN}(4\lambda_{ecliptic})$$



Equation of Time Variation

Universal Time

- Universal time, UT, is defined as the mean solar time at Greenwich

$$UT0 = 12^{\mathrm{h}} + GHA_{\odot} = 12^{\mathrm{h}} + LHA_{\odot} - \lambda$$

- UT1: corrected UT0 for polar motion

$$UT1 = UT0 - (x_p \mathrm{SIN}(\lambda) + y_p \mathrm{COS}(\lambda))\,\mathrm{TAN}(\phi_{gc})$$

instantaneous positions of the pole

latitude of the observing site

- Coordinated Universal Time, UTC, is based on atomic clocks. It's designed to follow UT1 within ±0.9s (1972).
- UTC does not change for daylight savings time.

Time Zone	Standard	Daylight Savings	Central Meridian
Atlantic	UTC – 4^{h}	UTC – 3^{h}	–60°
Moscow	UTC + 4^{h}		40°
China	UTC + 8^{h}		100°
Japan	UTC + 9^{h}		140°

Julian Date

- The Julian date, JD, is the interval of time measured in days from the epoch January 1, 4713 B.C., 12:00.

JULIAN DATE $(yr, mo, d, h, min, s \Rightarrow JD)$ {1900 to 2100}

$$JD = 367(yr) - \text{INT}\left\{\frac{7\left\{yr + \text{INT}\left(\frac{mo+9}{12}\right)\right\}}{4}\right\} + \text{INT}\left(\frac{275\,mo}{9}\right) + d + 1{,}721{,}013.5 + \frac{\frac{\left(\frac{s}{60*} + min\right)}{60} + h}{24}$$

- This formula is valid for the interval of March 1, 1900 to February 28, 2100. (*use 61 seconds if the day contains a leap second).
- Modified Julian Date: $MJD = JD - \underline{2{,}400{,}000.5}$ JD of Nov. 17,1858, 0:00
- Julian century (from J2000): $T_{xxx} = \dfrac{JD_{xxx} - 2{,}451{,}545.0_{xxx}}{36{,}525}$

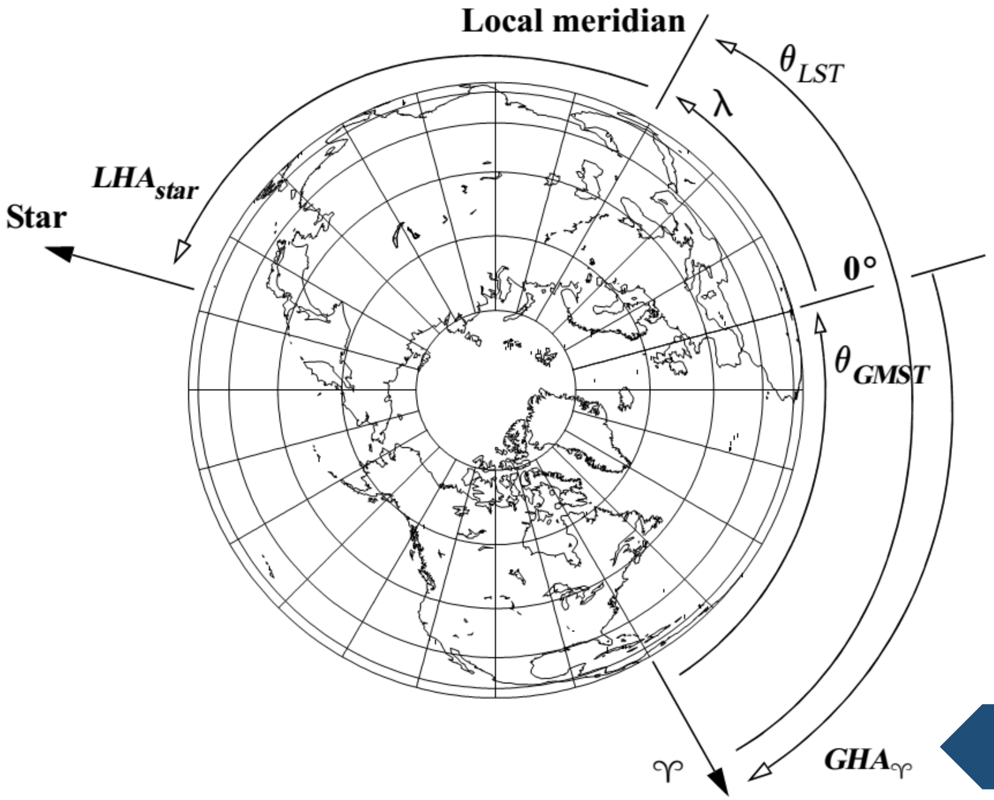
xxx = UT1,…

Sidereal Time

- The changing instantaneous the Earth rotation axis causes station locations to continually changes.

- Sidereal time as the hour angle of the vernal equinox relative to the local meridian.

$$\theta_{LST} = \theta_{GMST} + \lambda$$



- Apparent sidereal time vs. mean sidereal time

$$\theta_{LST} = \alpha_{star} + GHA_{star} + \lambda = \alpha_{star} + LHA_{star}$$

$$LHA_{star} = GHA_{star} + \lambda$$

Sidereal Time

- Greenwich mean sidereal time at midnight

$$\theta_{GMST\,0^h} = 24{,}110.548\,41^s + 8{,}640{,}184.812\,866\,T_{UT1} + 0.093\,104\,T_{UT1}^2 - 6.2\times10^{-6}\,T_{UT1}^3$$

$$\theta_{GMST\,0^h} = 100.460\,618\,4^\circ + 36{,}000.770\,053\,61\,T_{UT1} + 0.000\,387\,93\,T_{UT1}^2 - 2.6\times10^{-8}\,T_{UT1}^3$$

Julian centuries elapsed from the epoch J2000.0 at 0h 0m 0s of the day

- To consider the elapsed UT1 time on the day of interest

$$\theta_{GMST} = \theta_{GMST\,0^h} + \omega_{\oplus prec}\,UT1$$

- The most useful relation for computer software

$$\theta_{GMST} = 67{,}310.548\,41^s + (876{,}600^h + 8{,}640{,}184.812\,866^s)\,T_{UT1} + 0.093\,104\,T_{UT1}^2 - 6.2\times10^{-6}\,T_{UT1}^3$$