Home Work #4

Ali BaniAsad 401209244

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1 Question 1

1.1 Part a

Satellite is in 600 km altitude circular orbit. Sun-synchronous orbit is desired. Find the inclination of the orbit.
Solution:

$$i = \cos^{-1}\left(\frac{-2}{3}\frac{a^{7/2}\Delta\Omega(1-e^2)^2}{R^2 J_2\sqrt{\mu}}\right) = \cos -1\left(\frac{-2}{3}\frac{(6978.14)^{7/2}0.9856(1-0.0000)^2}{(6398.14)^2 0.00108263\sqrt{398600.4415}}\right) = 97.788^\circ \tag{1}$$

1.2 Part b

Satellite is in 600 km altitude elliptical orbit. Sun-synchronous orbit is desired. Find the inclination of the orbit with e = 0.1.
Solution:

$$i = \cos^{-1}\left(\frac{-2}{3}\frac{a^{7/2}\Delta\Omega(1-e^2)^2}{R^2 J_2\sqrt{\mu}}\right) = \cos^{-1}\left(\frac{-2}{3}\frac{(6978.14)^{7/2}0.9856(1-0.1)^2}{(6398.14)^2 0.00108263\sqrt{398600.4415}}\right) = 97.632^\circ \tag{2}$$

1.3 Part c

Here is 3D chart indicating the relation between inclination, eccen- tricity, and semi major axis of the orbit such that the orbit is Sun-Synchronous.

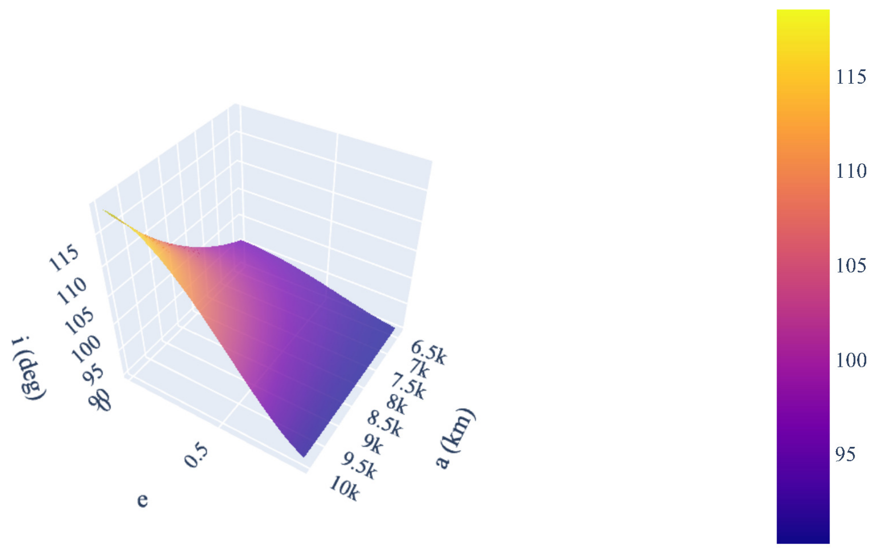

Figure 1: 3D plot of inclination, eccentricity and semi major axis

1.4 Part d

Here is the result for venus orbit.

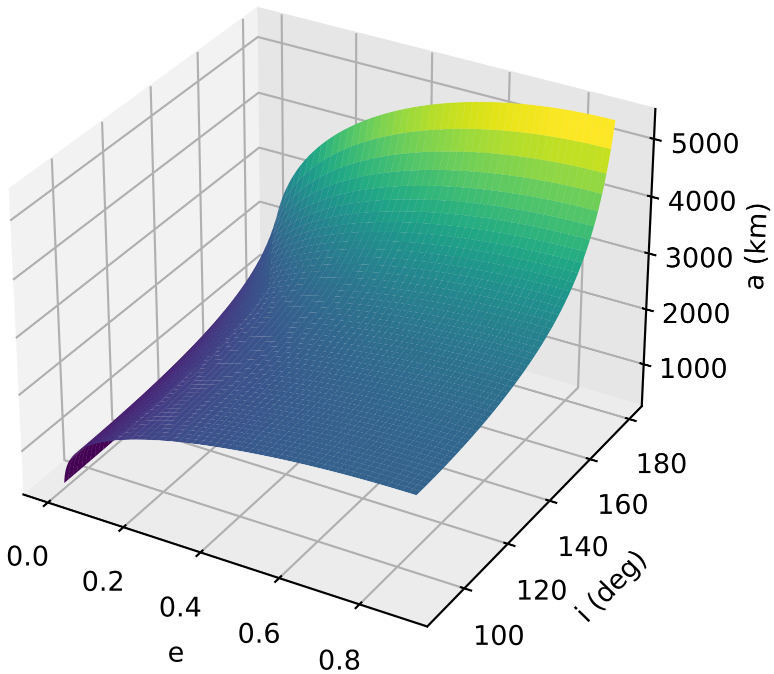

Figure 2: 3D plot of inclination, eccentricity and semi major axis for venus

and here is result for pregee of orbits and venus orbit.

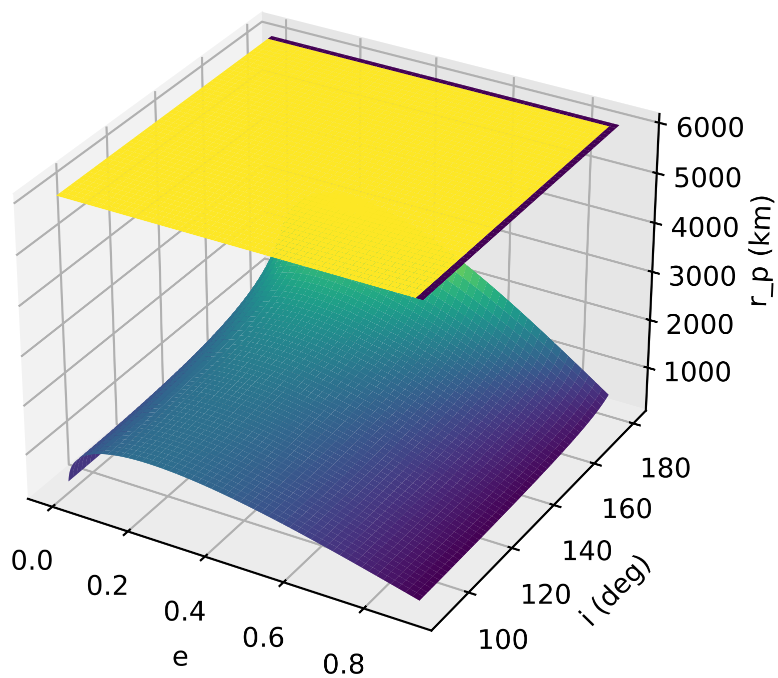

Figure 3: 3D plot of inclination, eccentricity and perigee with venus radius surface

As we can see from above fig is that perigee of orbits are less than venus radius, so it is impossible to have sun-synchronous orbit around venus.

2 Question 2

2.1 Part a

In this section used Algorithm 71 of refrence book. The code is in `Q2.ipynb` file. The result is in `Q2_a.txt` file. The result is:

$$a = 6585.691379586709_{km}$$

2.2 Part b

In repeat ground track assumed the accepted drif error is about 1 persent of semi-major axis. So the error is about 50 km or 0.05 radian. Useng Algorithm 72 of refrence book, the result is in `Q2.ipynb` file. The result is:

$$t_{drift} = 0.0018_{\text{sec}}, \quad \Delta v = -2.31721 \frac{km}{\text{sec}}$$

3 Question 3

3.1 Part a

In this section, we will utilize Algorithm 71 from the reference book to determine the required orbital parameters for a sun-synchronous orbit. The code implementation can be found in the `Q3.ipynb` file. Since the orbit is sun-synchronous, we can directly use the algorithm without the need to calculate the inclination (i). After performing the necessary iterations, the obtained results are as follows:

$$a = 6788.7228\,\text{km}$$

Then, we can calculate the inclination using the formula:

$$i = \frac{-a^{7/2}\dot{\Omega}}{3J_2R_e^2\sqrt{\mu}} = 93.528\,\text{deg}$$

where $\dot{\Omega}$ represents the rate of change of right ascension of the ascending node, J_2 is the second zonal harmonic coefficient, R_e is the Earth's equatorial radius, and μ is the standard gravitational parameter.

3.2 Part b

In this section, we will explore how different initial inclinations affect the results when designing a repeat groundtrack-sun synchronous orbit. We will calculate the semi-major axis for various initial inclinations, assuming a sun-synchronous orbit, and then employ Algorithm 71 to iterate and determine the corresponding inclination (i). The code implementation can be found in the `Q3.ipynb` file.

Upon examining different initial inclination values ranging from 90 to 180 degrees and performing the necessary iterations, we obtained the following results:

$$a = 6788.722\,\text{km} \quad \text{and} \quad i = 93.528\,\text{deg}$$

Interestingly, regardless of the initial inclination value used and the resulting initial semi-major axis, the final outcome remained the same.

Note: For a more detailed analysis and complete code implementation, please refer to the provided `Q3.ipynb` file.

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