Home Work #3

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1 Question 1

In this equation we discuses about perturbation in classical orbital element. Formulas for the Gaussian form of the VOP equations using the disturbing force with specific force components resolved in the RSW system:

$$
\begin{aligned}
\frac{da}{dt} =& \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right)\\
\frac{de}{dt} =& \frac{\sqrt{1-e^2}}{na}\left(\sin(\theta)F_R + \left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S\right)\\
\frac{di}{dt} =& \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W\\
\frac{d\Omega}{dt} =& \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W\\
\frac{d\omega}{dt} =& \frac{\sqrt{1-e^2}}{nae}\left(-\cos(\theta)F_R + \sin(\omega)\left(1+\frac{1}{p}\right)F_S\right) - \frac{r\cot(i)\sin(u)}{h}F_W\\
\frac{M_0}{dt} =& \frac{1}{na^2e}\left((p\cos(\theta) - 2er)F_R - (p+r)\sin(\theta)F_S\right) - \frac{dn}{dt}(t-t_0)
\end{aligned}
\tag{1}
$$

1.1 part a

If we want to change a, we need to have force in R or S direction. If there is a force in R and S direction other parameters like eccentricity, ω, and M_0 will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "a" without the change of other parameters.

$$
\begin{aligned}
\sin(\theta)F_R &= -\left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S\\
\cos(\theta)F_R &= \sin(\omega)\left(1+\frac{1}{p}\right)F_S\\
(p\cos(\theta) - 2er)F_R &= (p+r)\sin(\theta)F_S
\end{aligned}
\tag{2}
$$

1.2 part b

If we want to change inclination, we need to have force in W direction. If there is a force in W direction other parameters like ω, and Ω will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "eccentricity" without the change of other parameters.

$$\begin{aligned}\frac{r\cos(u)}{na^2\sqrt{1-e^2}} &= 0\\ \frac{r\cot(i)\sin(u)}{h} &= 0\end{aligned} \tag{3}$$

1.3 part c

From the below equation, we can find the most efficient θ is in $\theta = 90^\circ$ because $\sin(90^\circ) = 1$, and to find the best direction it depends on with of parameters e and $\frac{p}{r}$ is bigger.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right) \tag{4}$$

1.4 part d

From below equation we can find most efficient u is when $u = 0^\circ$ because $\cos(0^\circ) = 1$.

$$\frac{di}{dt} = \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W \tag{5}$$

so:

$$u = 0 \to \theta + \omega = 0^\circ \to \theta = -\omega$$

and for Ω, from below equation we can find most efficient θ is when $\theta = 90^\circ$ because $\sin(90^\circ) = 1$.

$$\frac{d\Omega}{dt} = \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W$$

2 Question 2

In this question, we investigate the effect of perturbation forces on the orbital elements.

2.1 J_2 perturbation

Forces in RSW system:

$$\begin{aligned}F_R &= -\frac{3\mu J_2 R^2}{2r^4}\left(1 - 3\sin^2(i)\sin^2(u_0)\right)\\ F_S &= -\frac{3\mu J_2 R^2}{2r^4}\sin^2(i)\sin(u_0)\cos(u_0)\\ F_W &= -\frac{3\mu J_2 R^2}{2r^4}\sin(i)\cos(i)\sin(u_0)\end{aligned} \tag{6}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

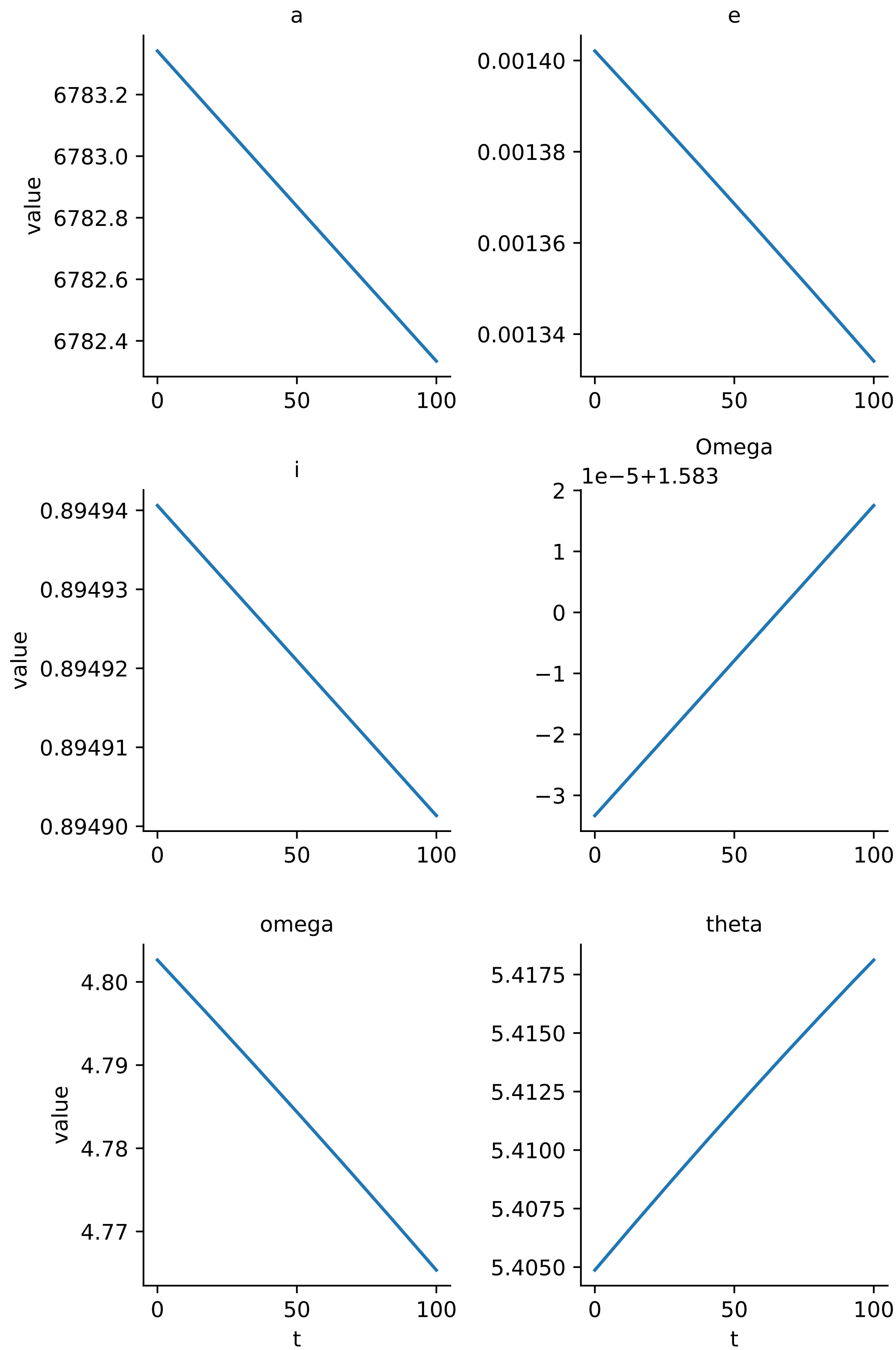

Figure 1: Variation of Parameter due to J_2 perturbation for 100 seconds

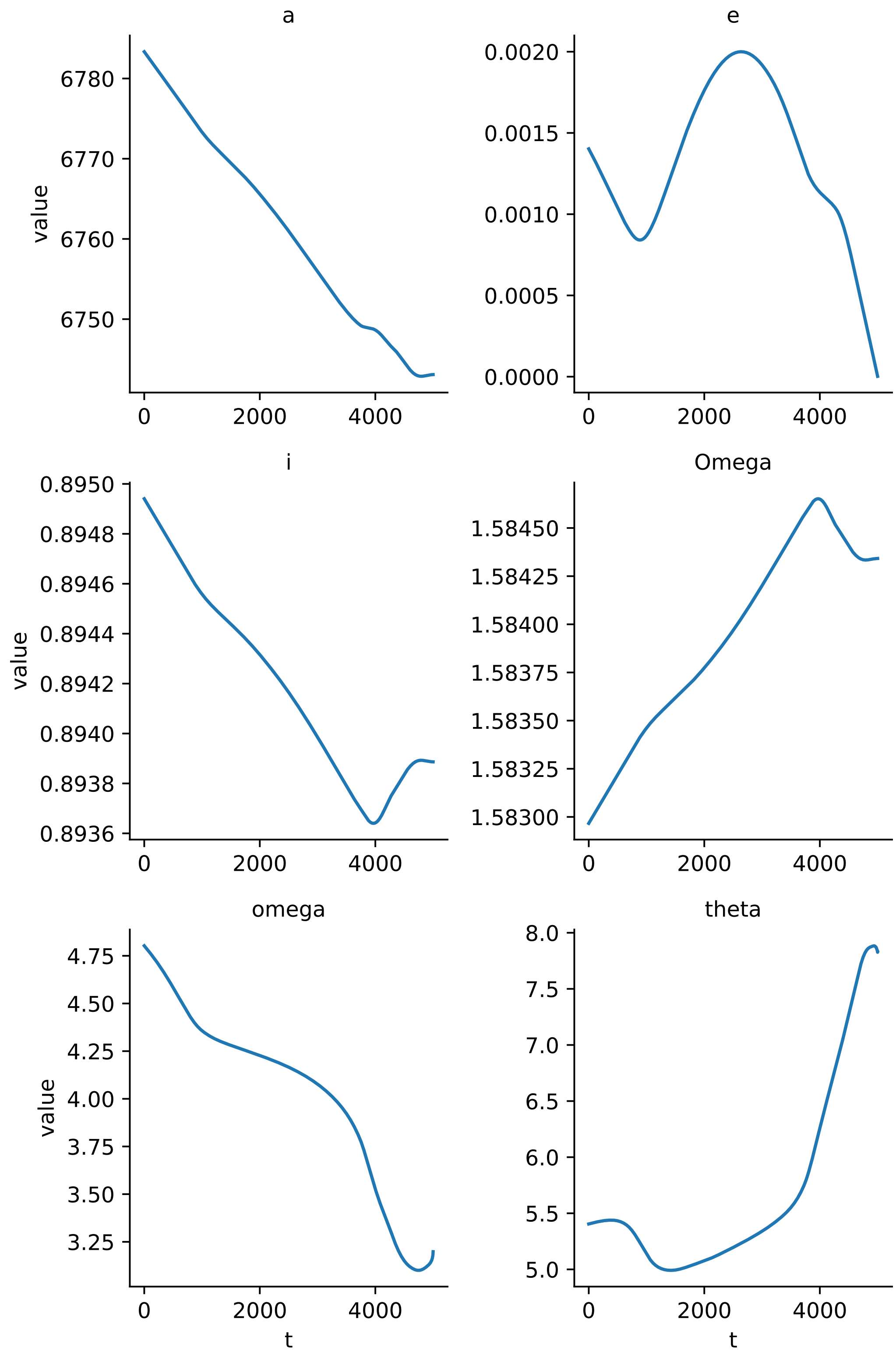

Figure 2: Variation of Parameter due to J_2 perturbation for 5000 seconds

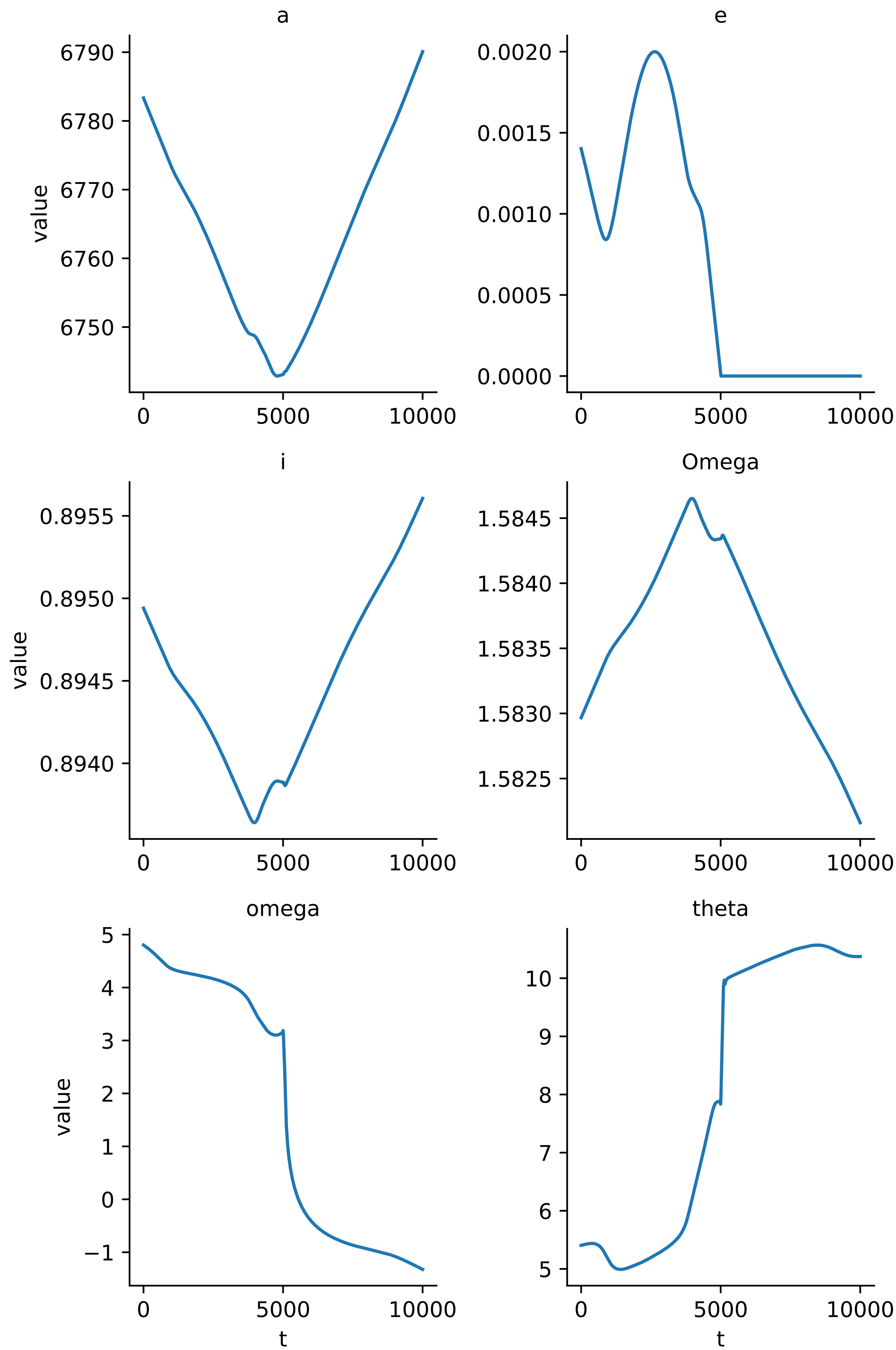

Figure 3: Variation of Parameter due to J_2 perturbation for 10000 seconds

2.2 Drag perturbation

In this section, there is simplifying the assumption that drag force is in the S direction.

$$F_{drag} = \frac{1}{2}\rho v^2 s C_D \rightarrow \mathbf{F_{drag}} = -\frac{1}{2}\rho v^2 s C_D \vec{S}$$

$$\mathbf{a_{drag}} = -\frac{1}{2m_s}\rho v^2 s C_D \vec{S}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

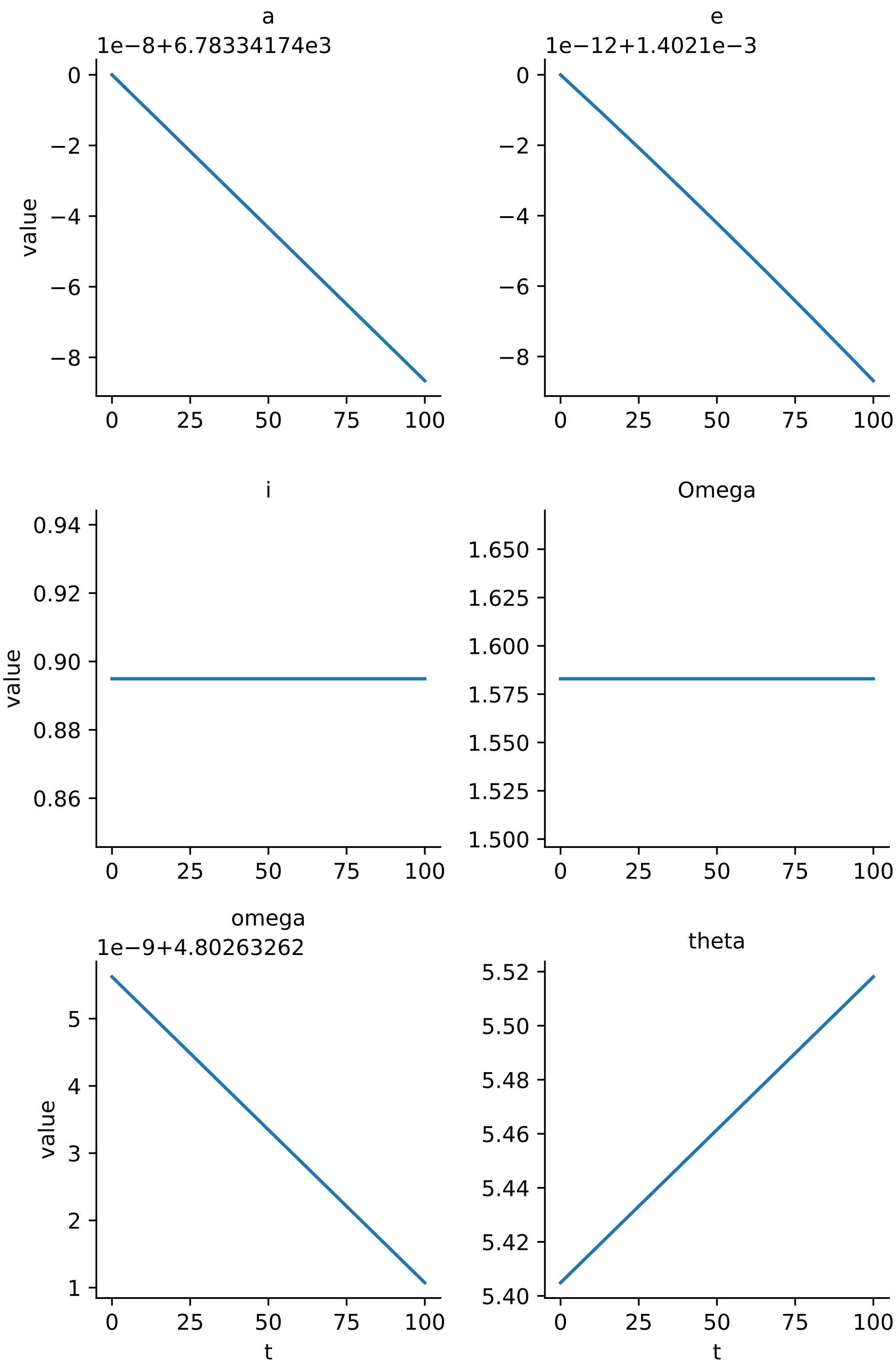

Figure 4: Variation of Parameter due to drag perturbation for 100 seconds

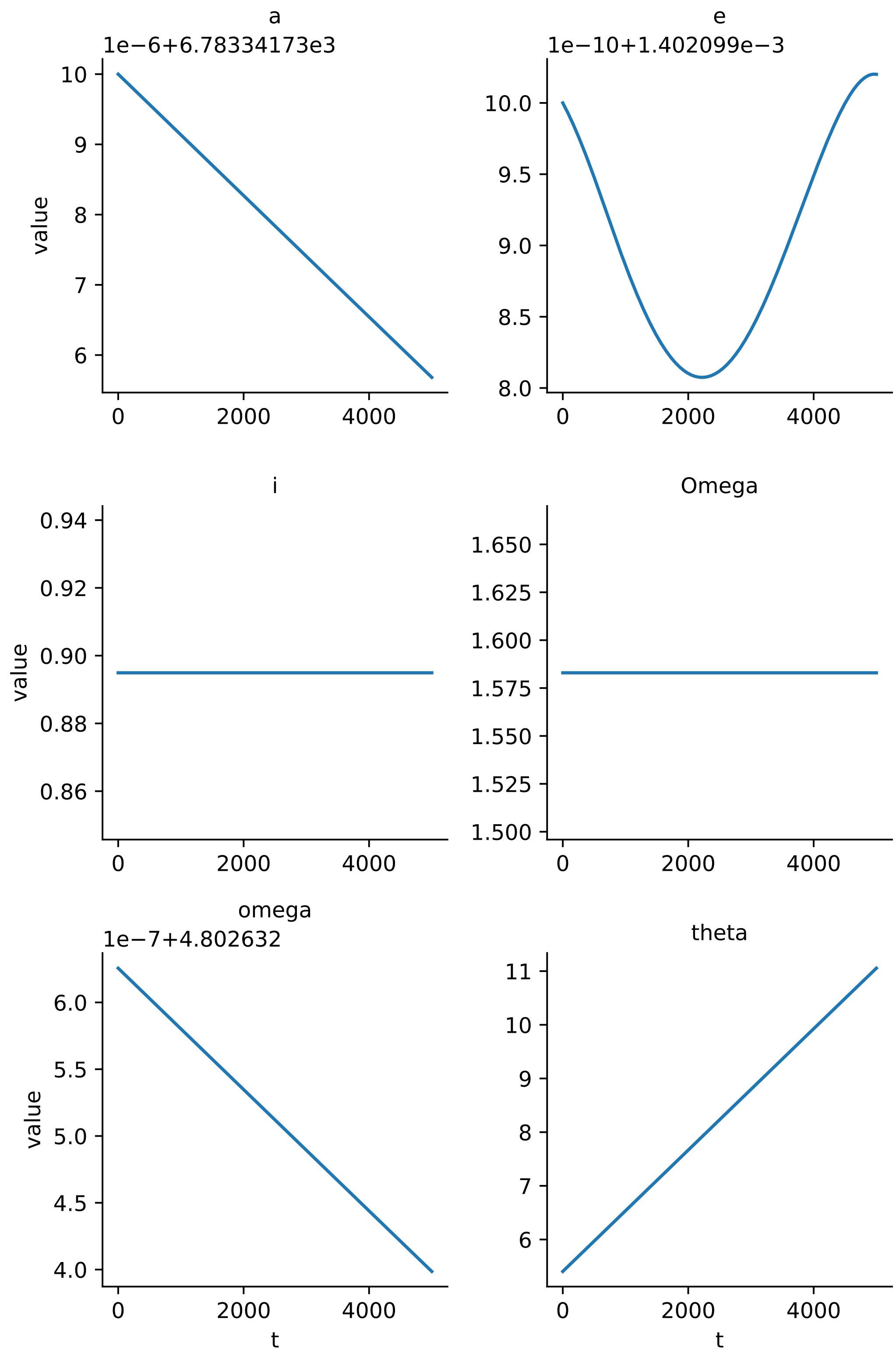

Figure 5: Variation of Parameter due to drag perturbation for 5000 seconds

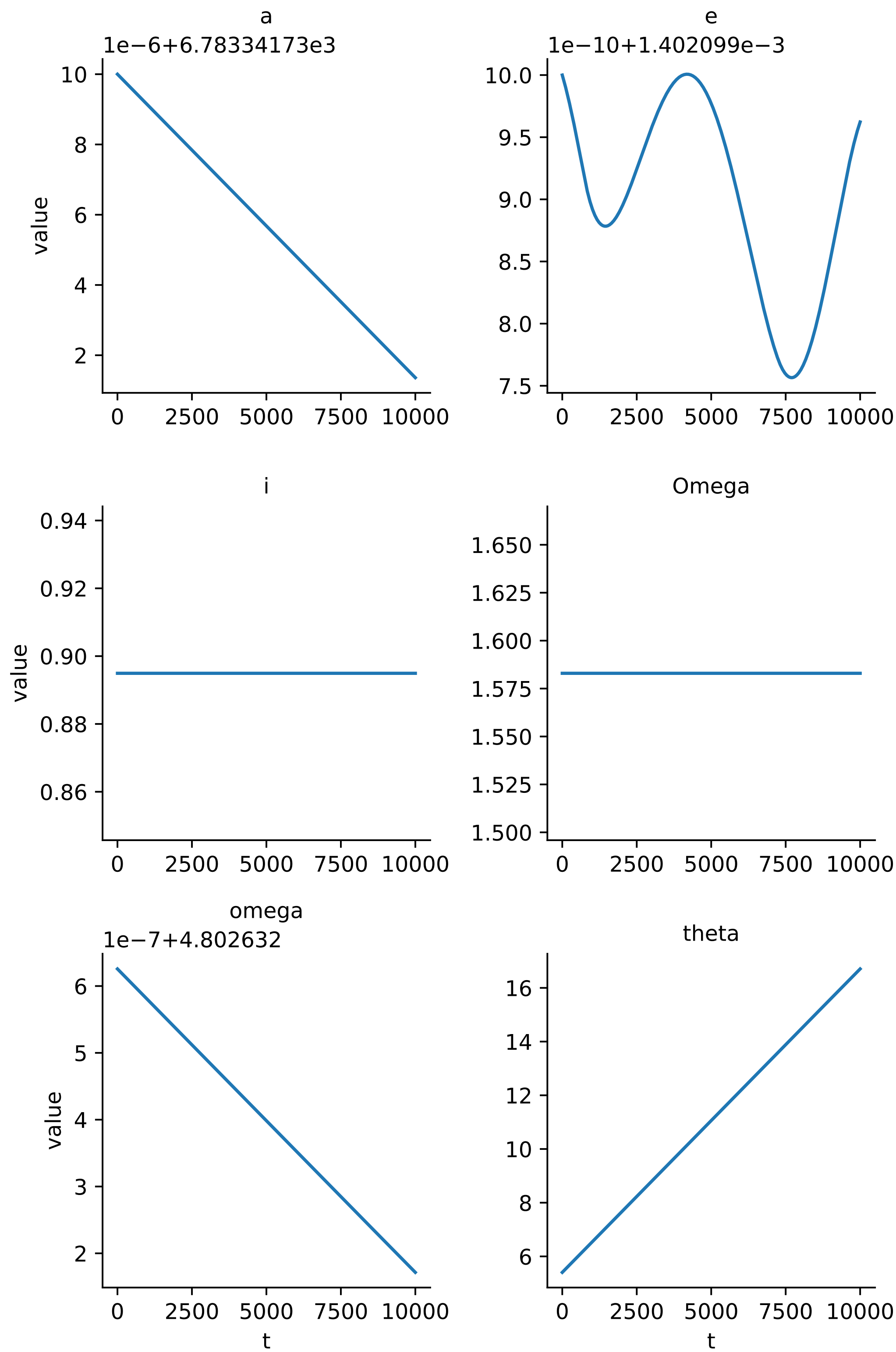

Figure 6: Variation of Parameter due to drag perturbation for 10000 seconds

2.3 Moon gravity perturbation

In this section, we investigate the effect of the moon's third body on satellite orbital elements. In this problem we calculate forces of moon and earth, then, integrate from differential equation for several time to see effects in short, long period and secular.

$$\sum F_{satellite} = \frac{-Gm_{earth}}{r_{e2s}^3}\mathbf{r_{e2s}} + \frac{-Gm_{moon}}{r_{m2s}^3}\mathbf{r_{m2s}}$$

Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

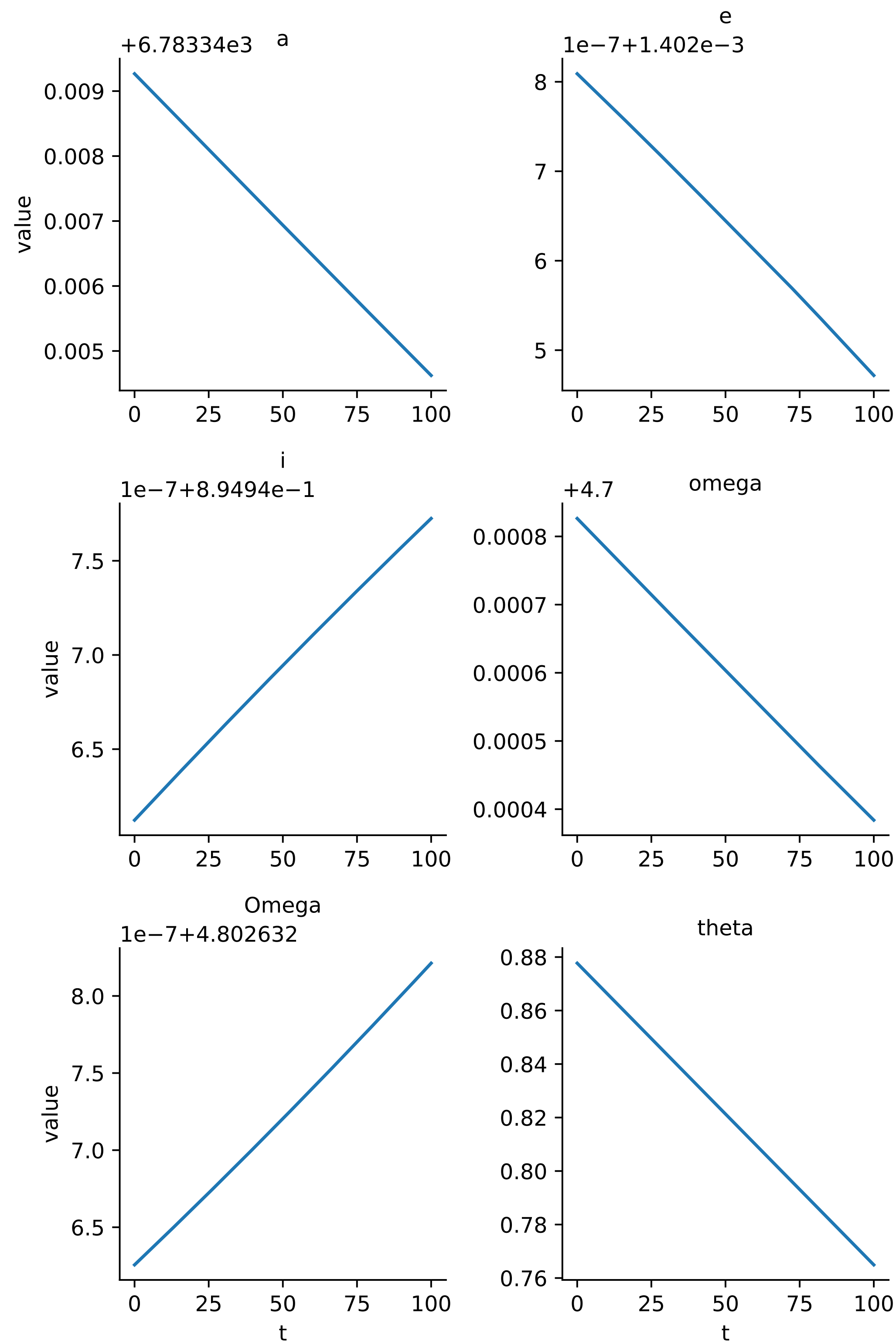

Figure 7: Variation of Parameter due to moon gravity perturbation for 100 seconds

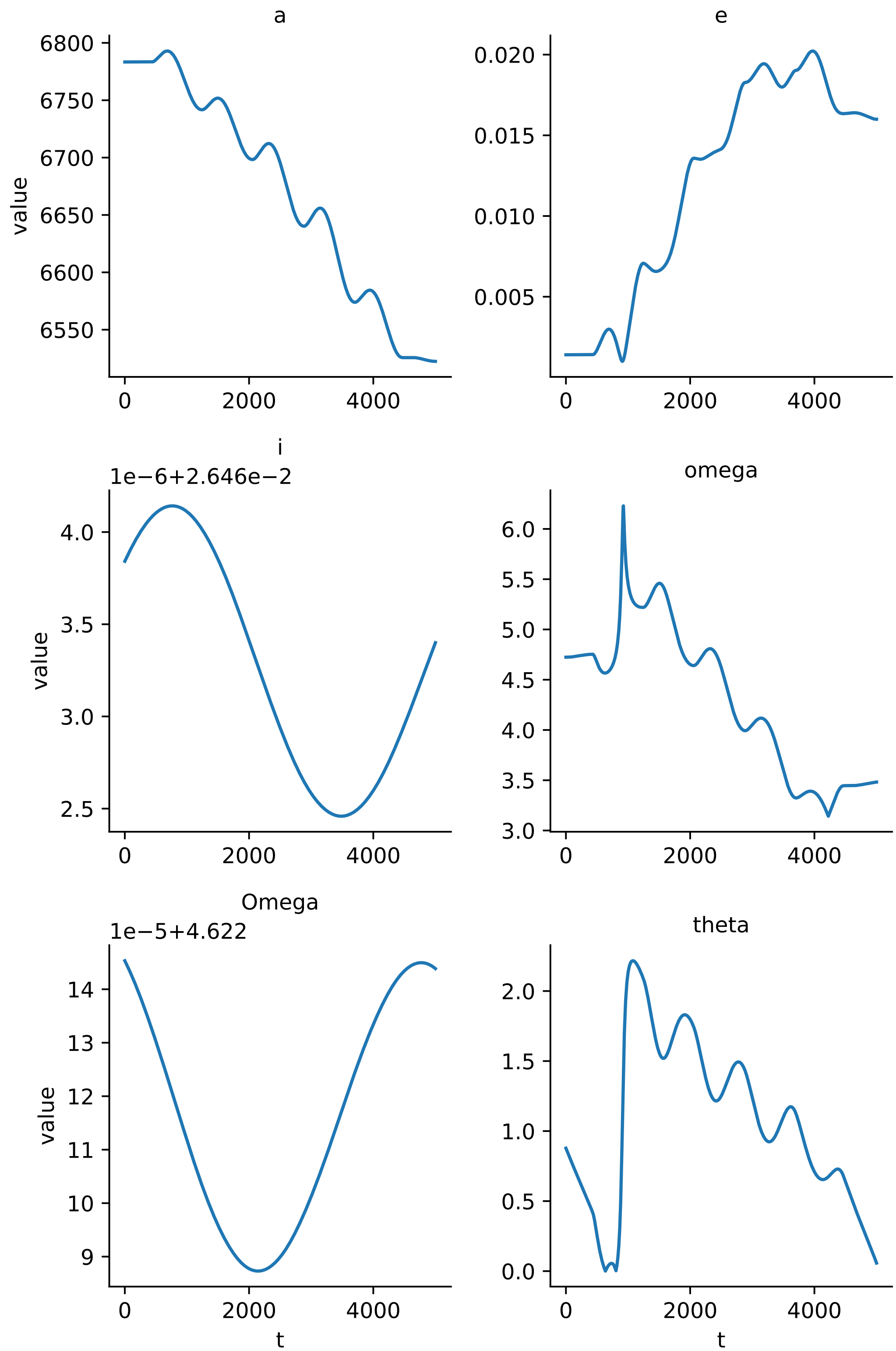

Figure 8: Variation of Parameter due to moon gravity perturbation for 5000 seconds

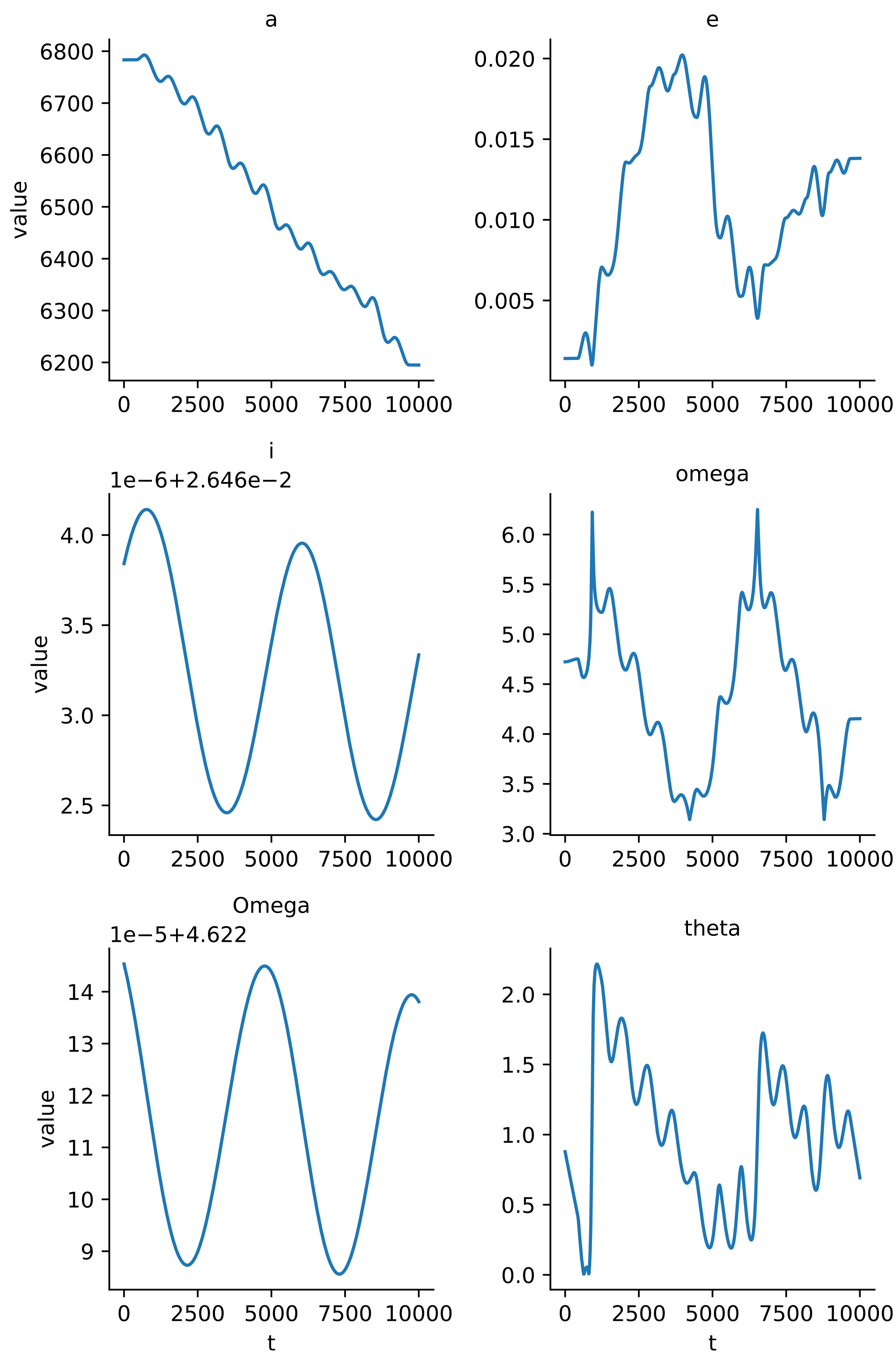

Figure 9: Variation of Parameter due to moon gravity perturbation for 10000 seconds

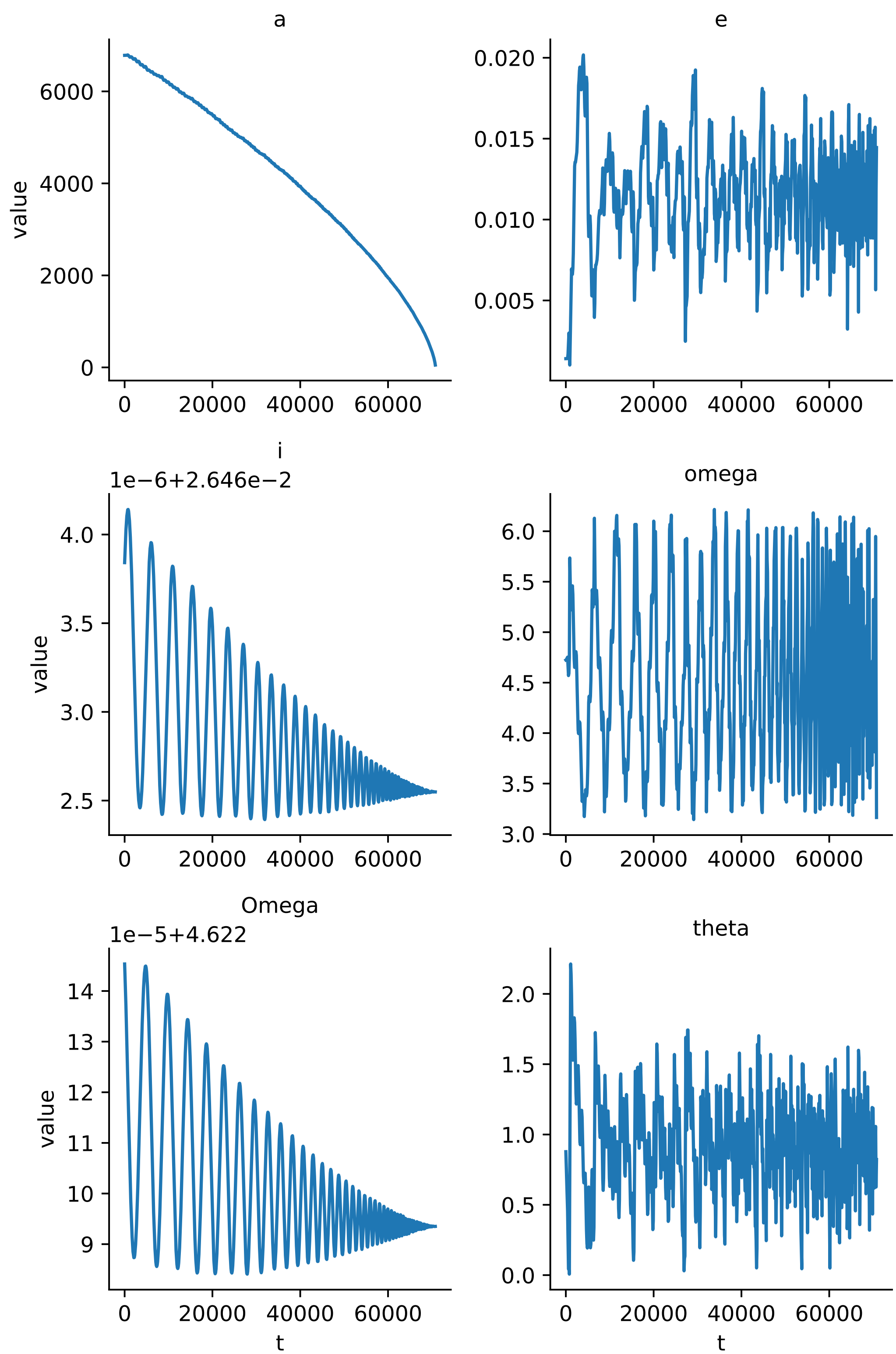

Figure 10: Variation of Parameter due to moon gravity perturbation for 100000 seconds

2.4 Sun gravity perturbation

In this section, we investigate the effect of the sun's third body on satellite orbital elements. In this problem we calculate forces of sun and earth, then, integrate from differential equation for several time to see effects in short, long period and secular.

$$\sum F_{satellite} = \frac{-Gm_{earth}}{r_{e2s}^3}\mathbf{r_{e2s}} + \frac{-Gm_{sun}}{r_{s2s}^3}\mathbf{r_{s2s}}$$

Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

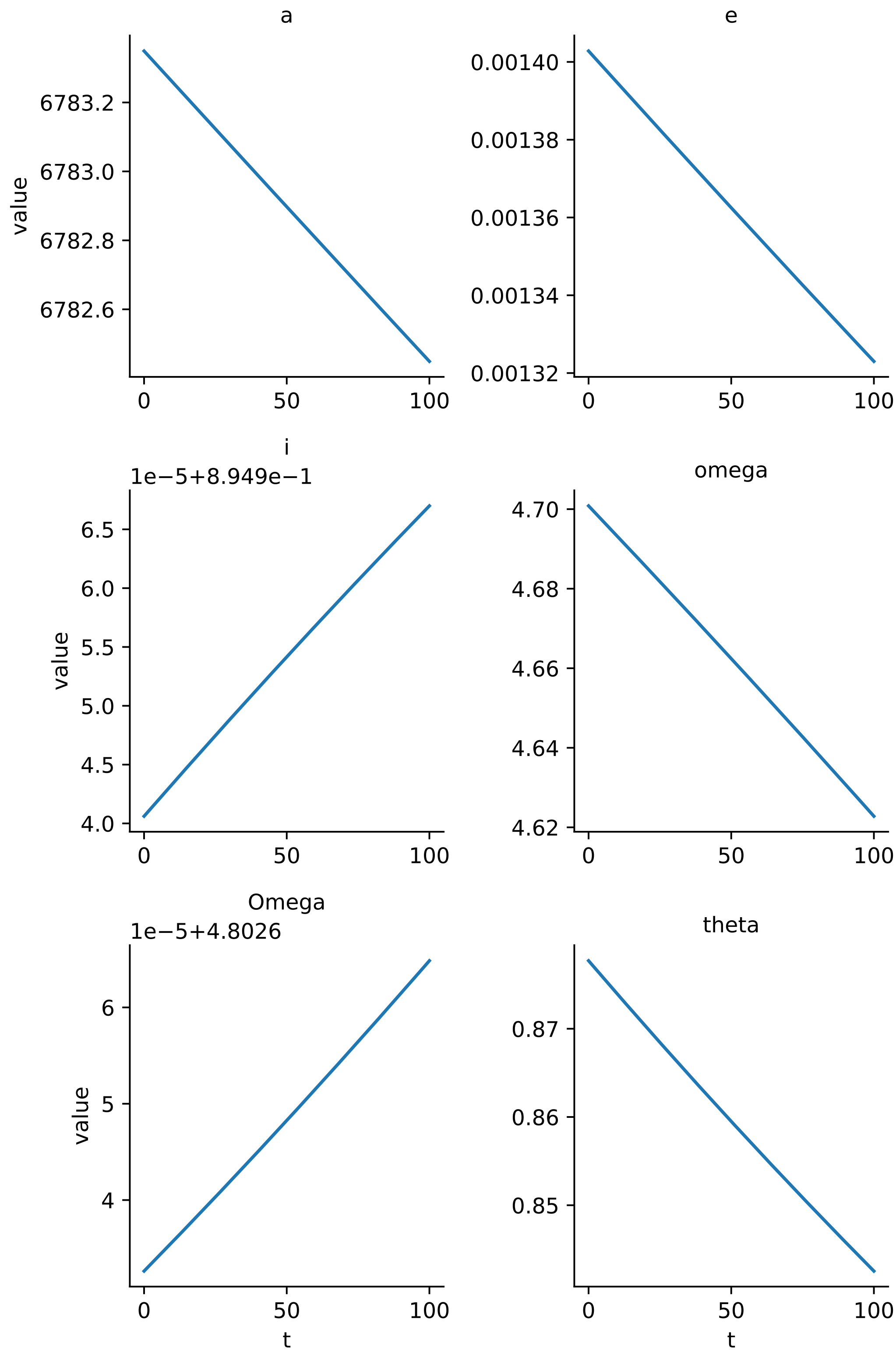

Figure 11: Variation of Parameter due to sun gravity perturbation for 100 seconds

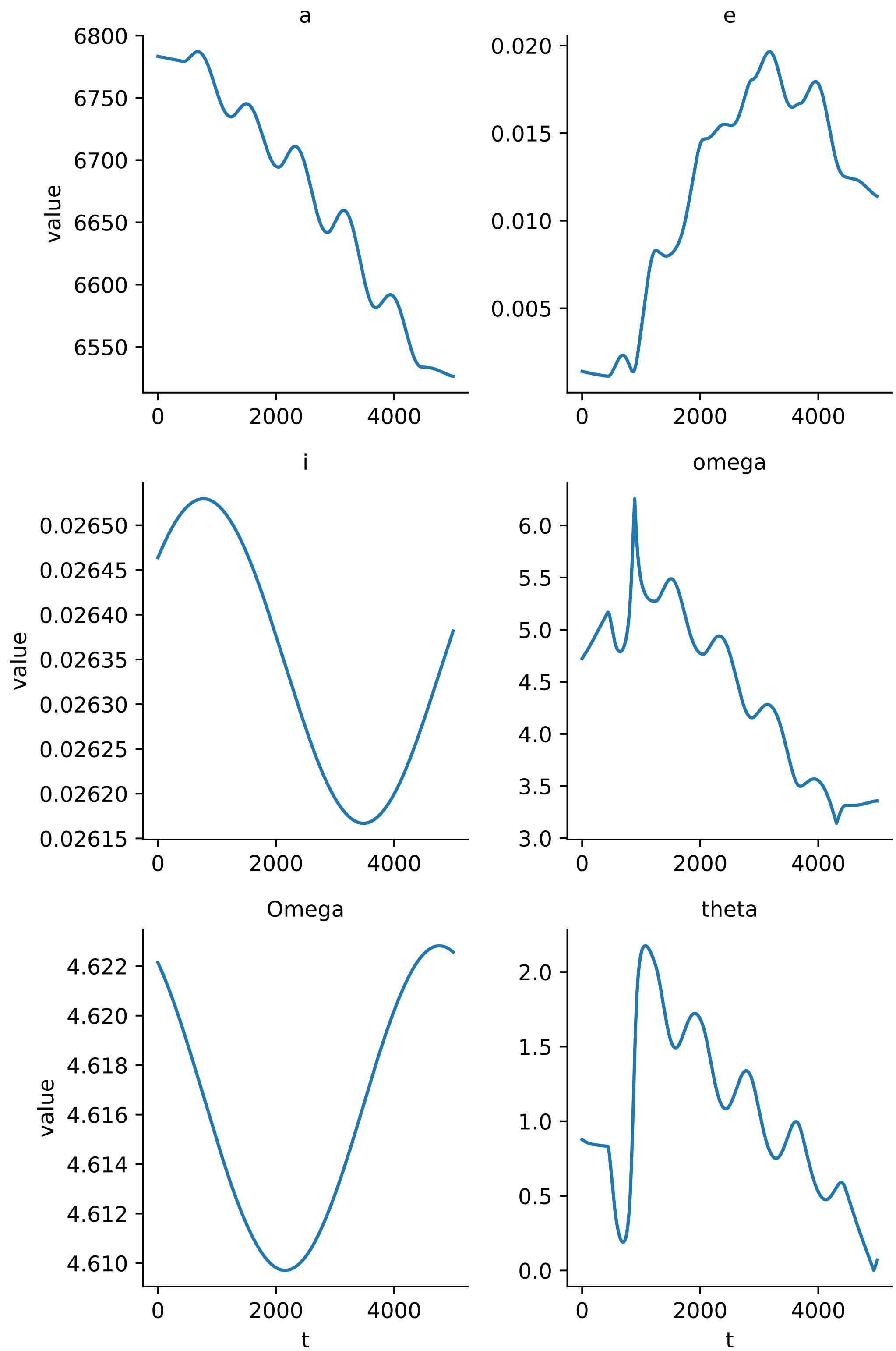

Figure 12: Variation of Parameter due to sun gravity perturbation for 5000 seconds

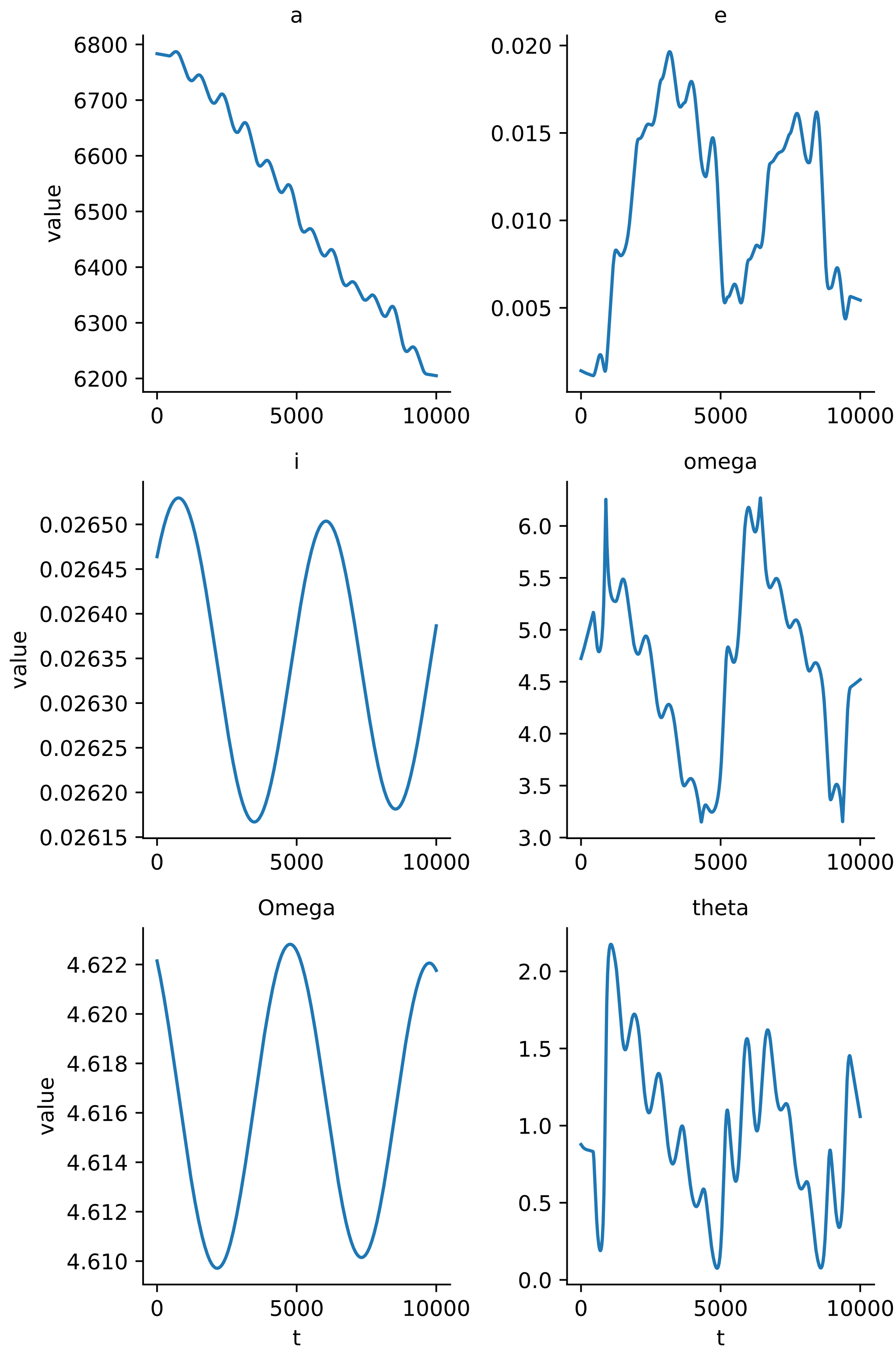

Figure 13: Variation of Parameter due to sun gravity perturbation for 10000 seconds

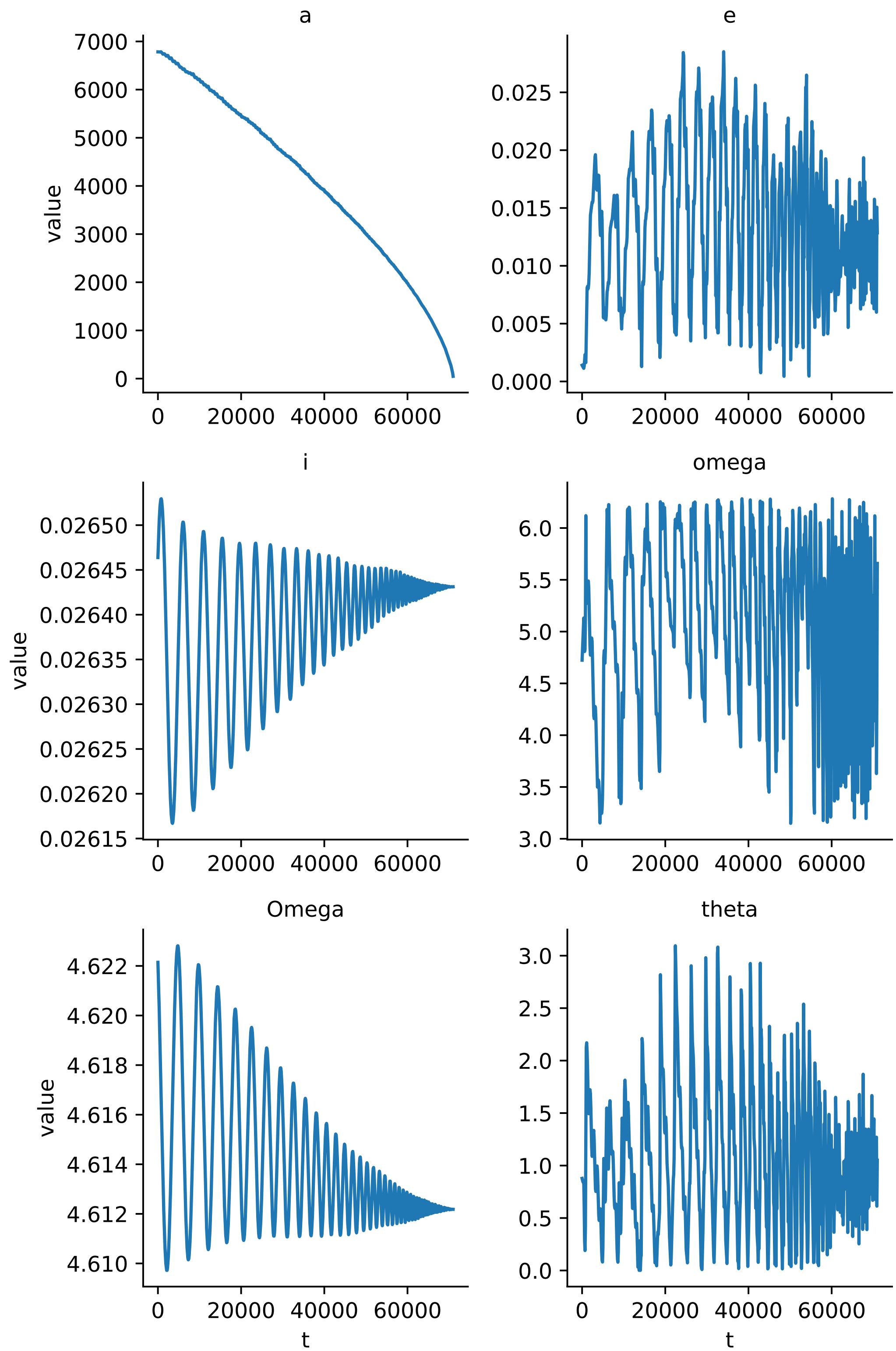

Figure 14: Variation of Parameter due to sun gravity perturbation for 100000 seconds

2.5 Solar radiation perturbation

The below equation shows the force of solar radiation.

$$P_{SRP} = \nu \frac{S}{c} C_R \frac{A_s}{m}$$

where ν calculates if the satellite is in the earth's shadow or not. Then used the below equations for rate changes.

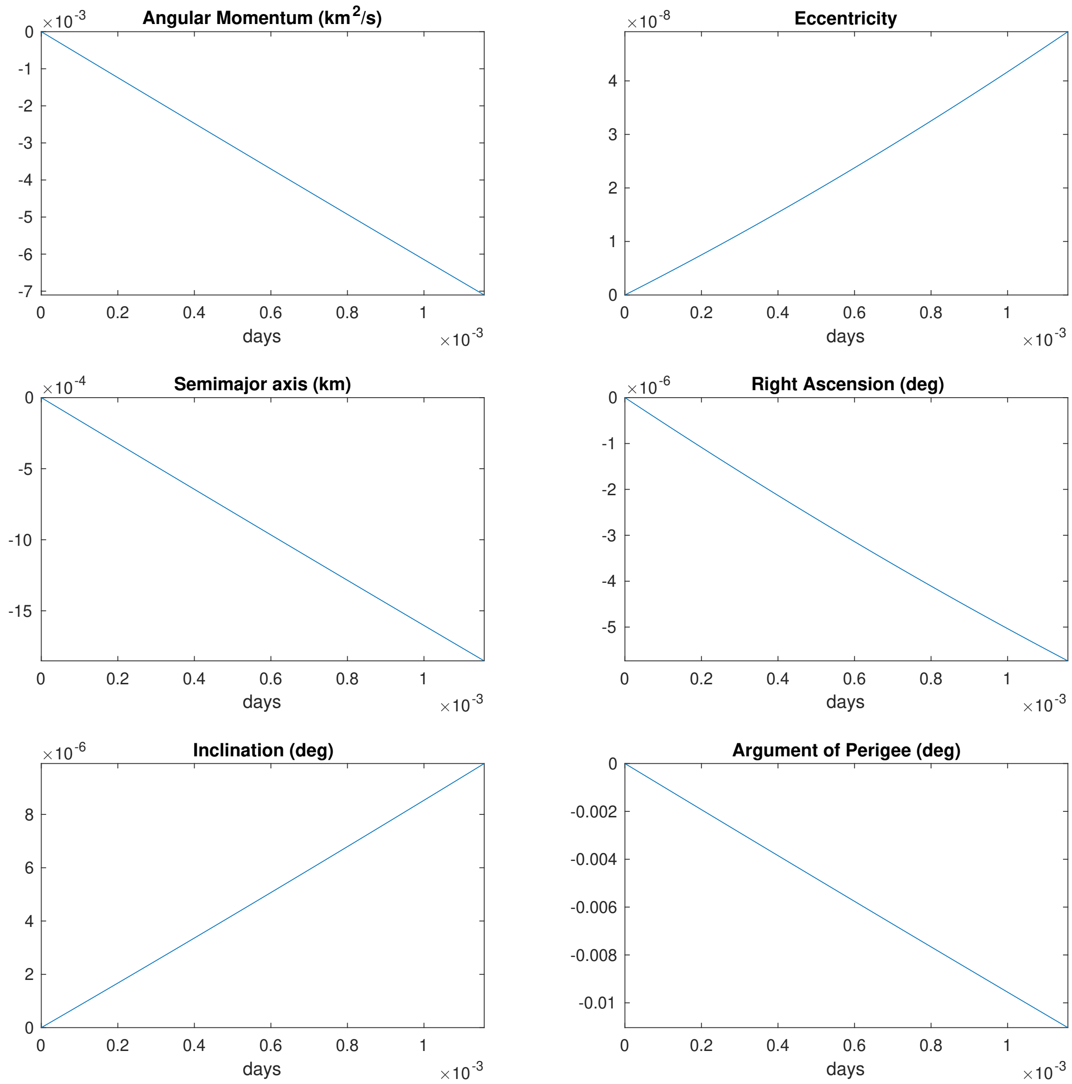

Figure 15: Variation of Parameter due to solar radiation perturbation for 100 seconds

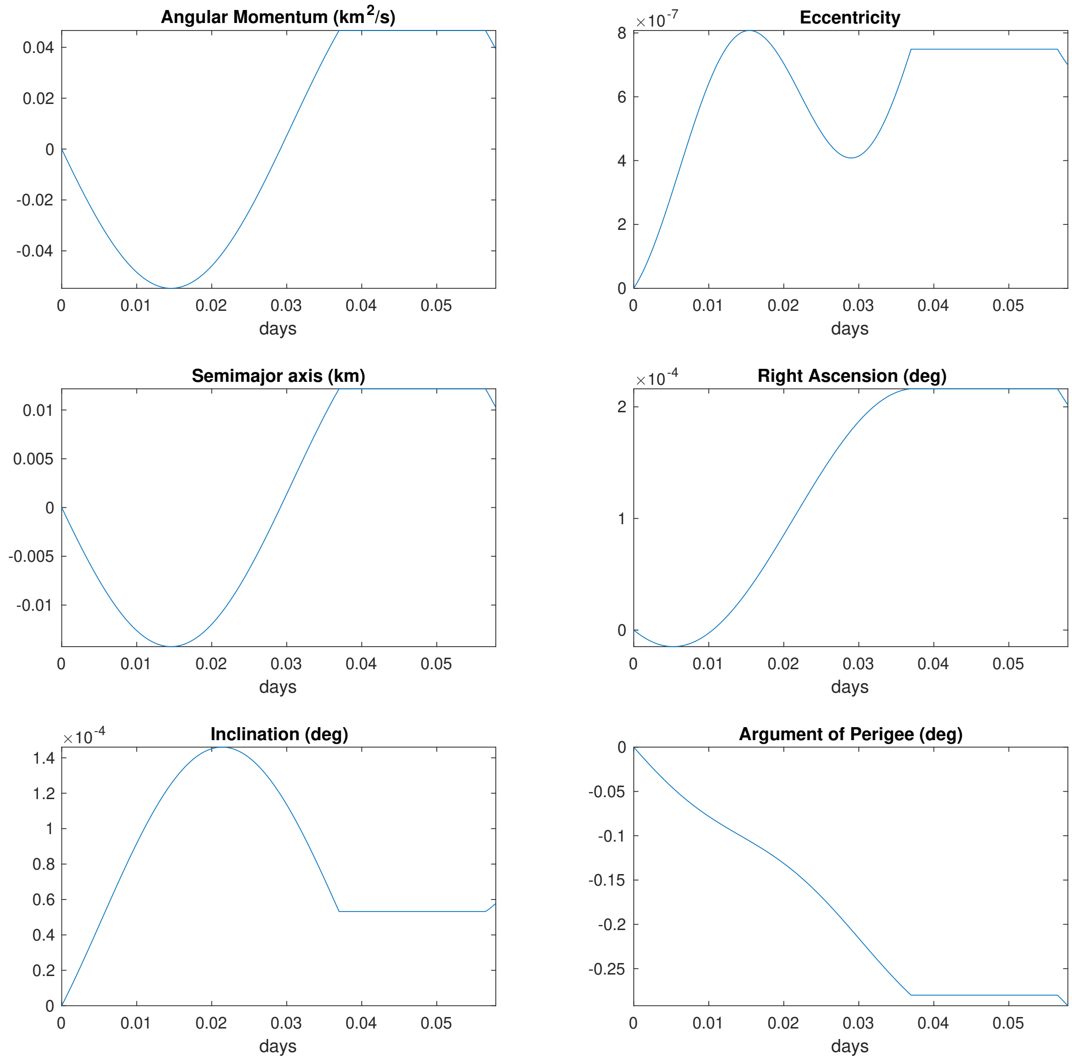

Figure 16: Variation of Parameter due to solar radiation perturbation for 5000 seconds

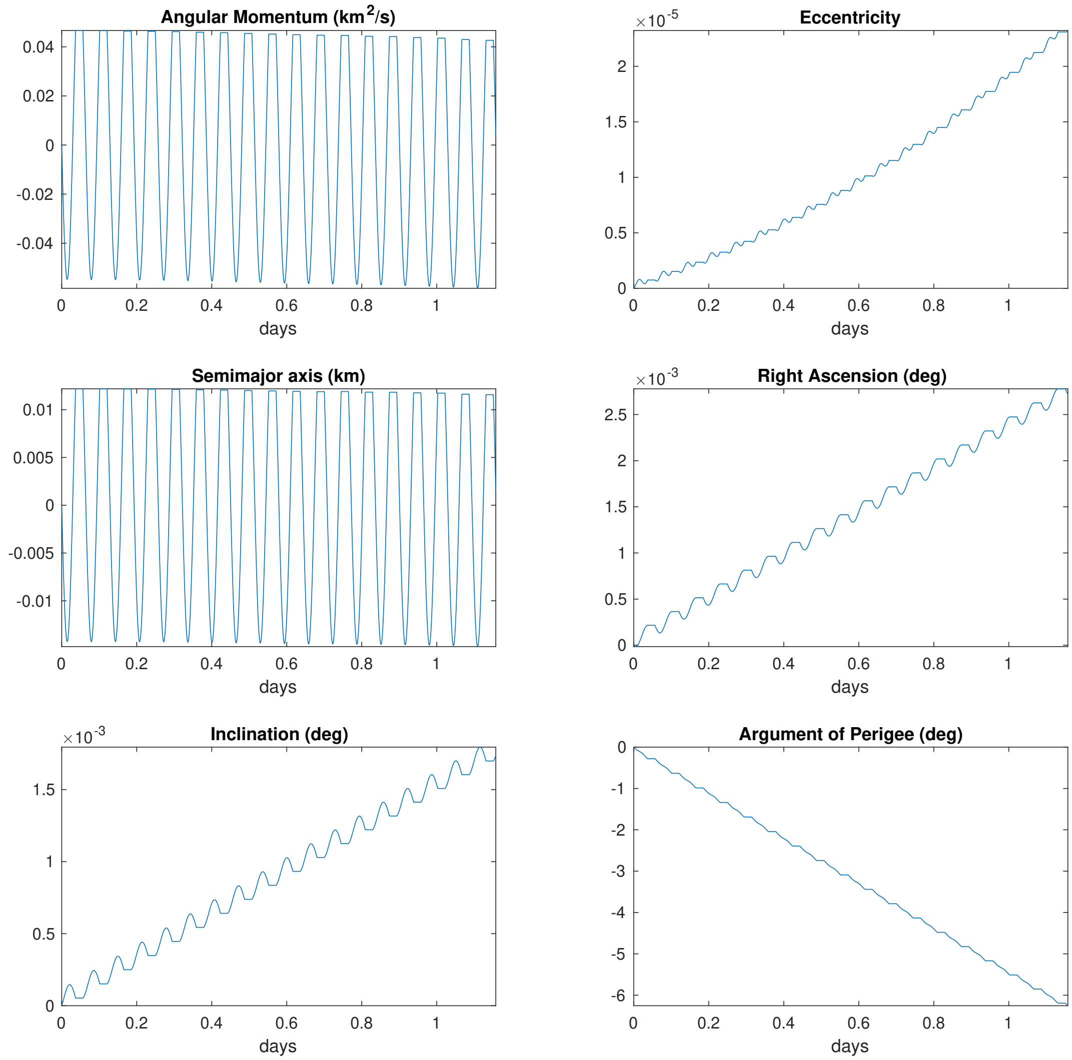

Figure 17: Variation of Parameter due to solar radiation perturbation for 100000 seconds

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