Home Work #3

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1 Question 1

In this equation we discuses about perturbation in classical orbital element. Formulas for the Gaussian form of the VOP equations using the disturbing force with specific force components resolved in the RSW system:

$$
\begin{aligned}
\frac{da}{dt} =& \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right) \\
\frac{de}{dt} =& \frac{\sqrt{1-e^2}}{na}\left(\sin(\theta)F_R + \left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S\right) \\
\frac{di}{dt} =& \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W \\
\frac{d\Omega}{dt} =& \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W \\
\frac{d\omega}{dt} =& \frac{\sqrt{1-e^2}}{nae}\left(-\cos(\theta)F_R + \sin(\omega)\left(1+\frac{1}{p}\right)F_S\right) - \frac{r\cot(i)\sin(u)}{h}F_W \\
\frac{M_0}{dt} =& \frac{1}{na^2e}\left((p\cos(\theta) - 2er)F_R - (p+r)\sin(\theta)F_S\right) - \frac{dn}{dt}(t-t_0)
\end{aligned}
\tag{1}
$$

1.1 part a

If we want to change a, we need to have force in R or S direction. If there is a force in R and S direction other parameters like eccentricity, ω, and M_0 will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "a" without the change of other parameters.

$$
\begin{aligned}
\sin(\theta)F_R &= -\left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S \\
\cos(\theta)F_R &= \sin(\omega)\left(1+\frac{1}{p}\right)F_S \\
(p\cos(\theta) - 2er)F_R &= (p+r)\sin(\theta)F_S
\end{aligned}
\tag{2}
$$

1.2 part b

If we want to change inclination, we need to have force in W direction. If there is a force in W direction other parameters like ω, and Ω will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "eccentricity" without the change of other parameters.

$$\begin{aligned} \frac{r\cos(u)}{na^2\sqrt{1-e^2}} &= 0 \\ \frac{r\cot(i)\sin(u)}{h} &= 0 \end{aligned} \tag{3}$$

1.3 part c

From the below equation, we can find the most efficient θ is in $\theta = 90°$ because $\sin(90°) = 1$, and to find the best direction it depends on with of parameters e and $\frac{p}{r}$ is bigger.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right) \tag{4}$$

1.4 part d

From below equation we can find most efficient u is when $u = 0°$ because $\cos(0°) = 1$.

$$\frac{di}{dt} = \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W \tag{5}$$

so:

$$u = 0 \rightarrow \theta + \omega = 0° \rightarrow \theta = -\omega$$

and for Ω, from below equation we can find most efficient θ is when $\theta = 90°$ because $\sin(90°) = 1$.

$$\frac{d\Omega}{dt} = \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W$$

2 Question 2

In this question, we investigate the effect of perturbation forces on the orbital elements.

2.1 J_2 perturbation

Forces in RSW system:

$$\begin{aligned} F_R &= -\frac{3\mu J_2 R^2}{2r^4}\left(1 - 3\sin^2(i)\sin^2(u_0)\right) \\ F_S &= -\frac{3\mu J_2 R^2}{2r^4}\sin^2(i)\sin(u_0)\cos(u_0) \\ F_W &= -\frac{3\mu J_2 R^2}{2r^4}\sin(i)\cos(i)\sin(u_0) \end{aligned} \tag{6}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

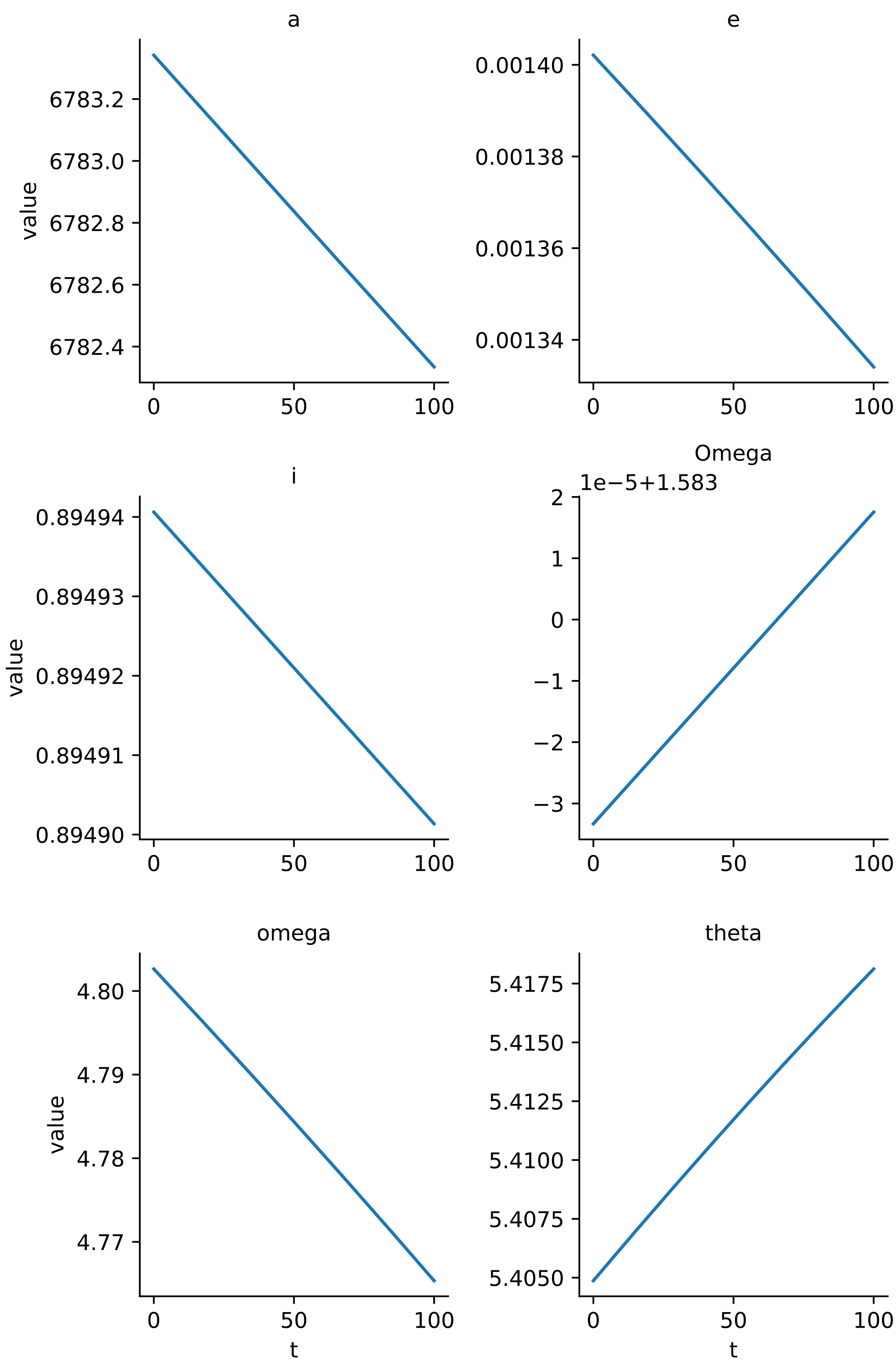

Figure 1: Variation of Parameter due to J_2 perturbation for 100 seconds

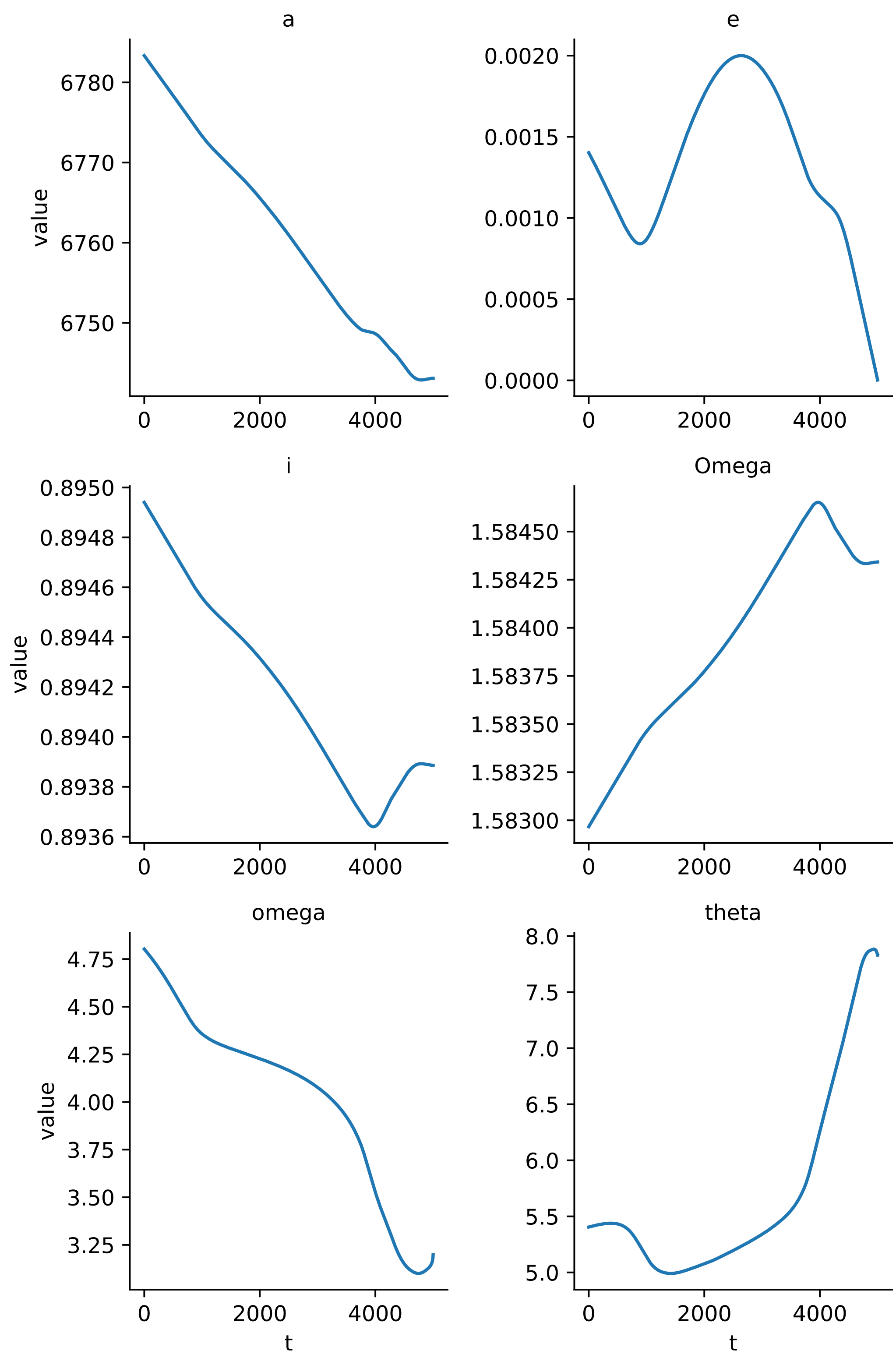

Figure 2: Variation of Parameter due to J_2 perturbation for 5000 seconds

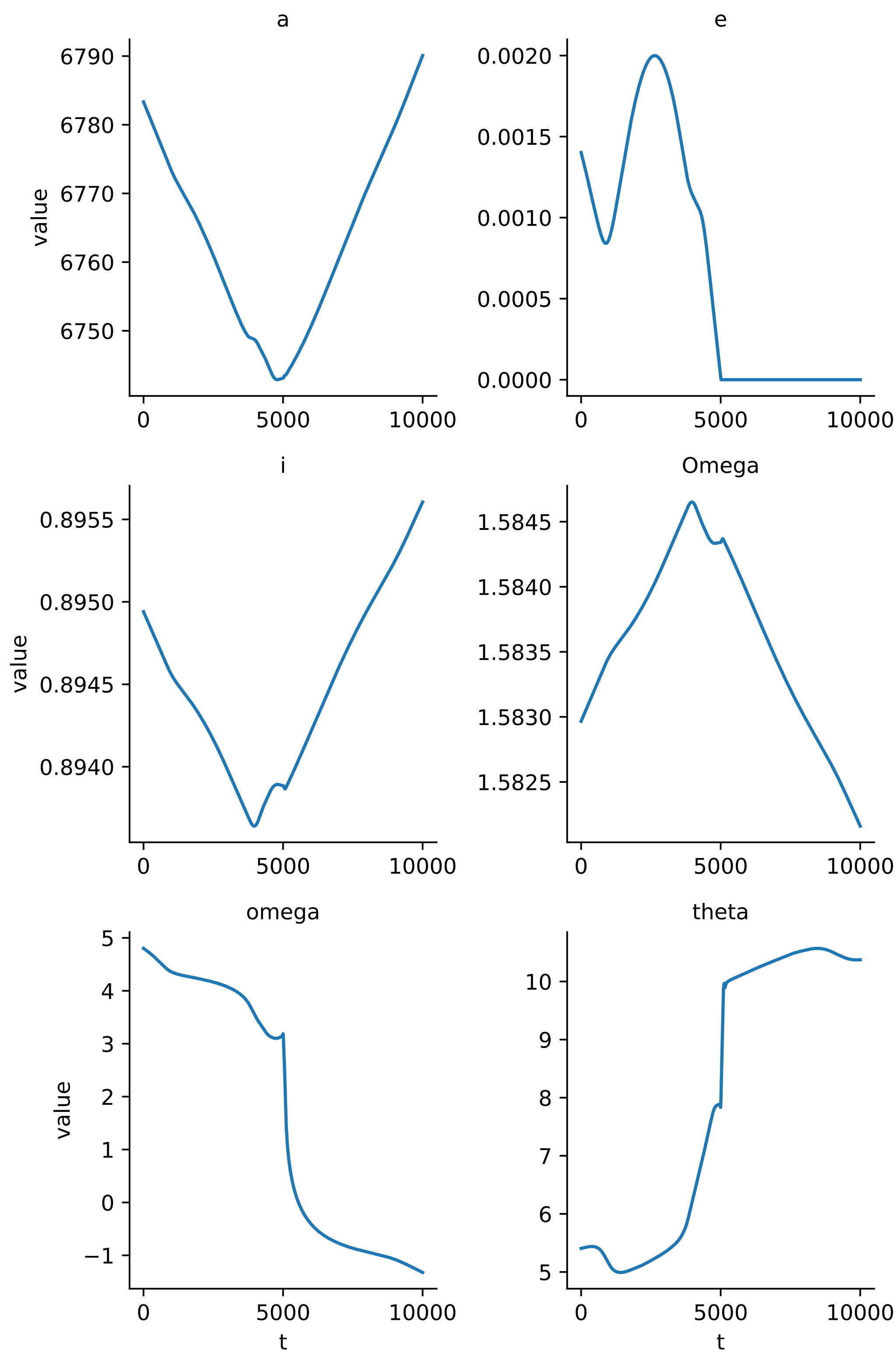

Figure 3: Variation of Parameter due to J_2 perturbation for 10000 seconds

2.2 Drag perturbation

In this section, there is simplifying the assumption that drag force is in the S direction.

$$F_{drag} = \frac{1}{2}\rho v^2 s C_D \rightarrow \mathbf{F_{drag}} = -\frac{1}{2}\rho v^2 s C_D \vec{S}$$

$$\mathbf{a_{drag}} = -\frac{1}{2m_s}\rho v^2 s C_D \vec{S}$$

From (1) we know the effect of other forces on orbital elements. Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

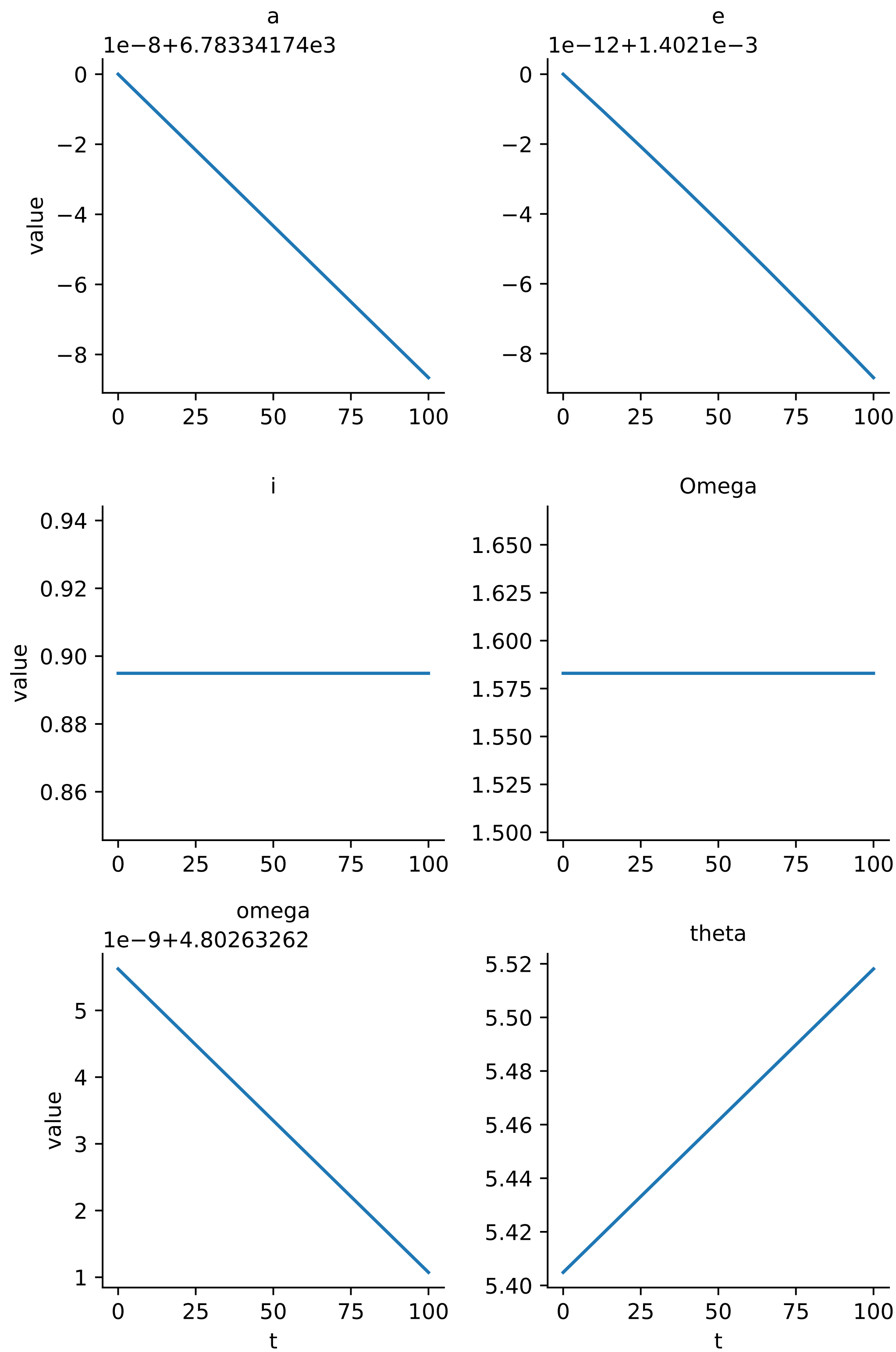

Figure 4: Variation of Parameter due to drag perturbation for 100 seconds

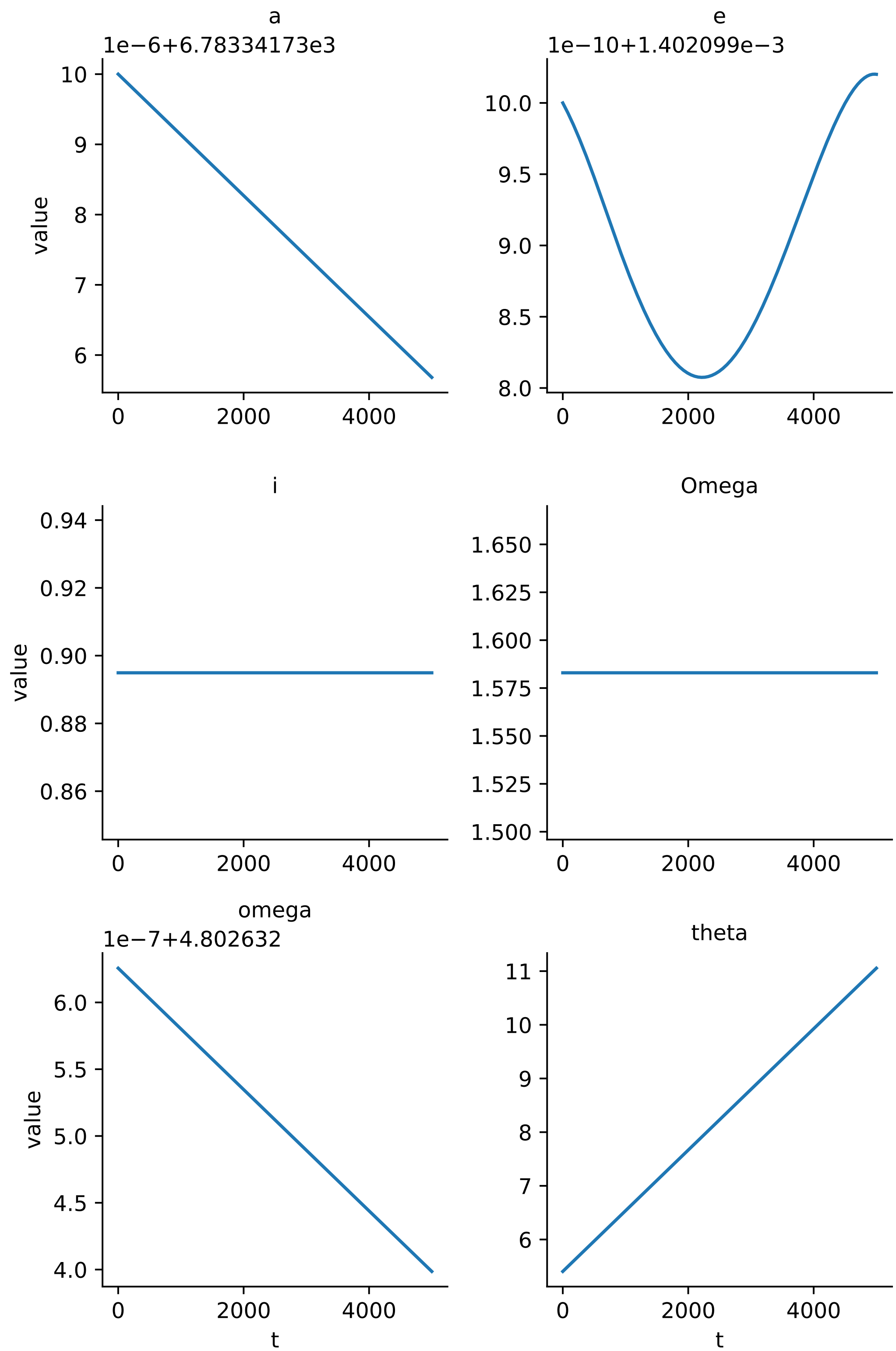

Figure 5: Variation of Parameter due to drag perturbation for 5000 seconds

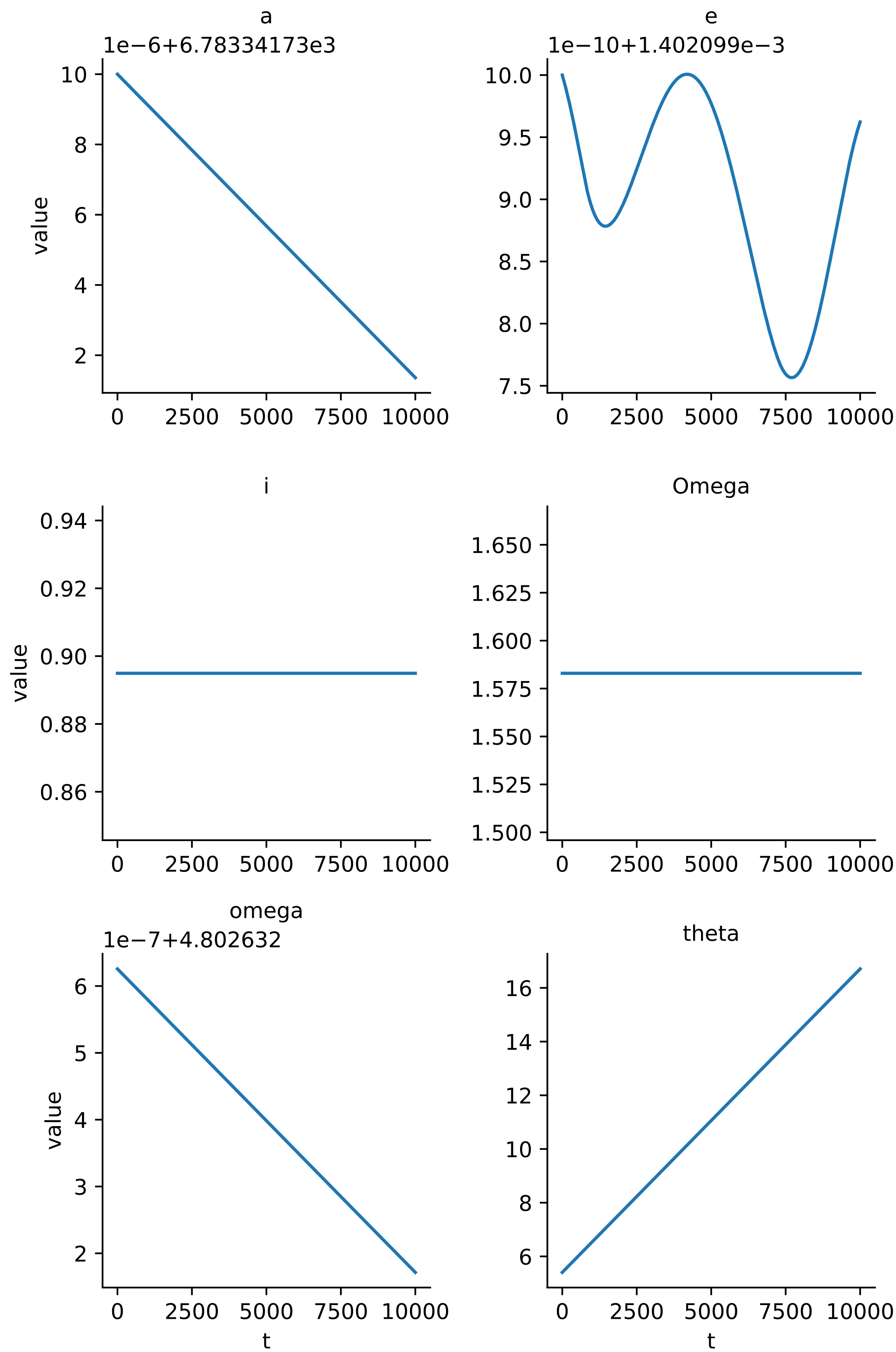

Figure 6: Variation of Parameter due to drag perturbation for 10000 seconds

2.3 Moon gravity perturbation

In this section, we investigate the effect of the moon's third body on satellite orbital elements. In this problem we calculate forces of moon and earth, then, integrate from differential equation for several time to see effects in short, long period and secular.

$$\sum F_{satellite} = \frac{-Gm_{earth}}{r_{e2s}^3}\mathbf{r_{e2s}} + \frac{-Gm_{moon}}{r_{m2s}^3}\mathbf{r_{m2s}}$$

Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

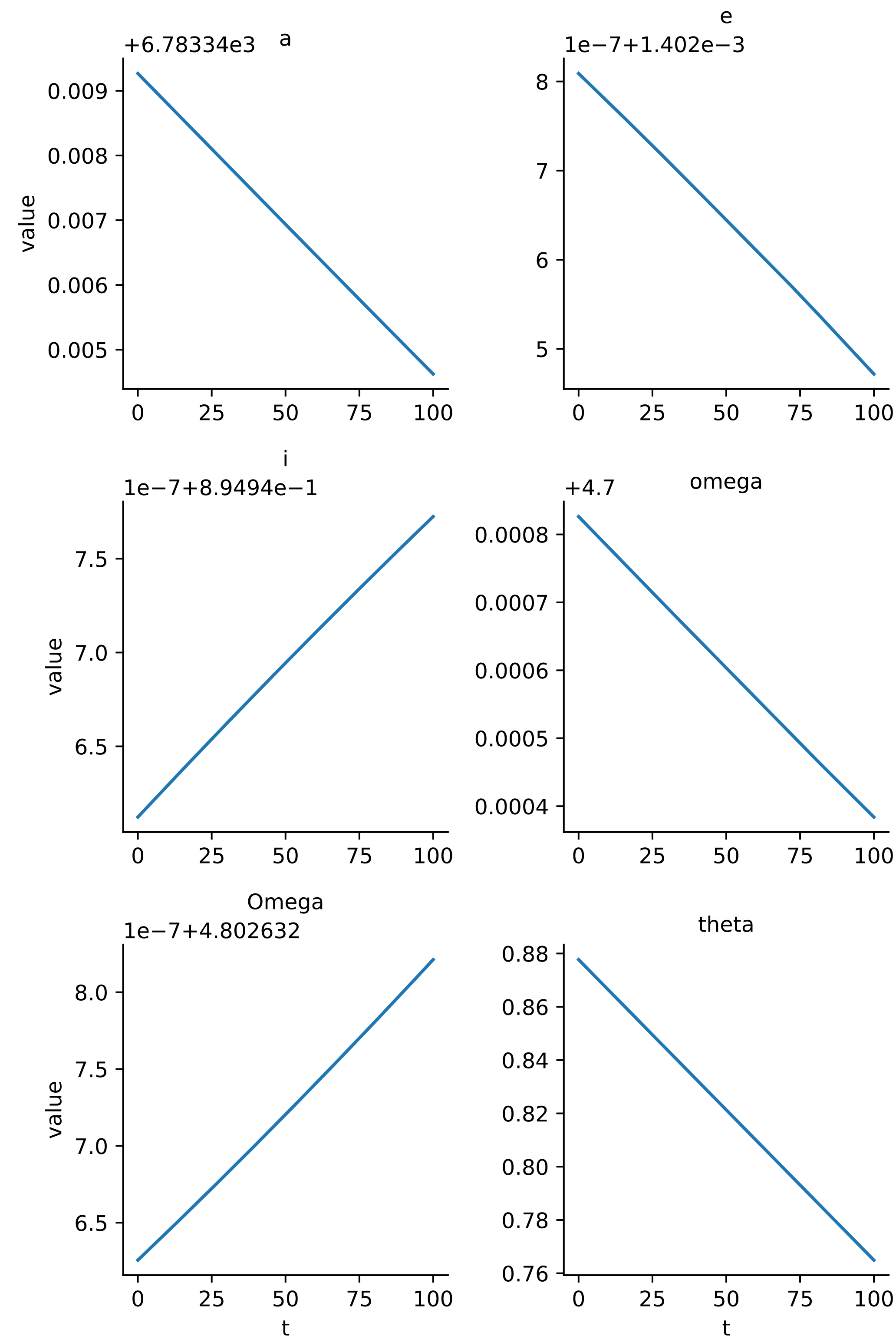

Figure 7: Variation of Parameter due to moon gravity perturbation for 100 seconds

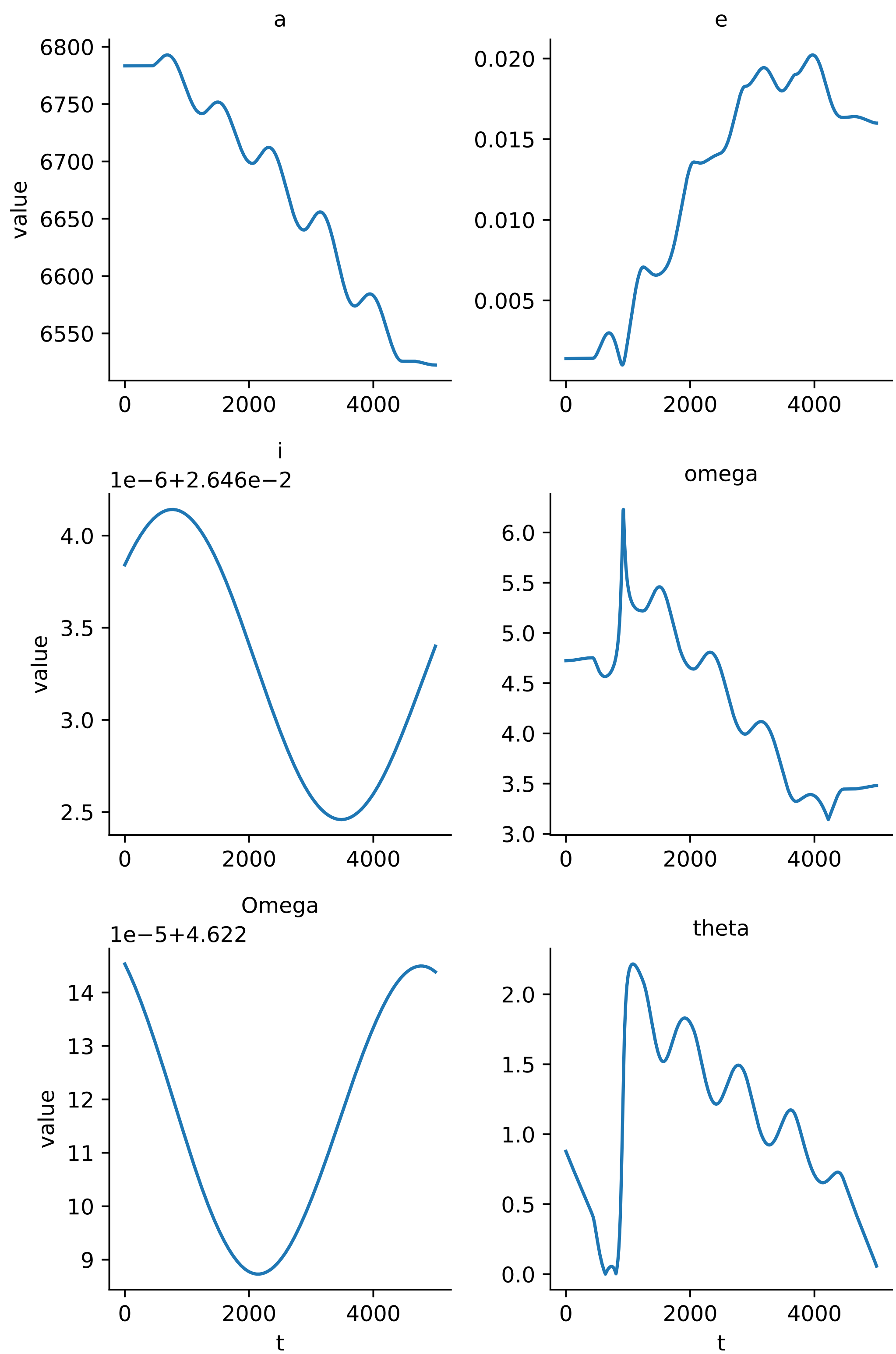

Figure 8: Variation of Parameter due to moon gravity perturbation for 5000 seconds

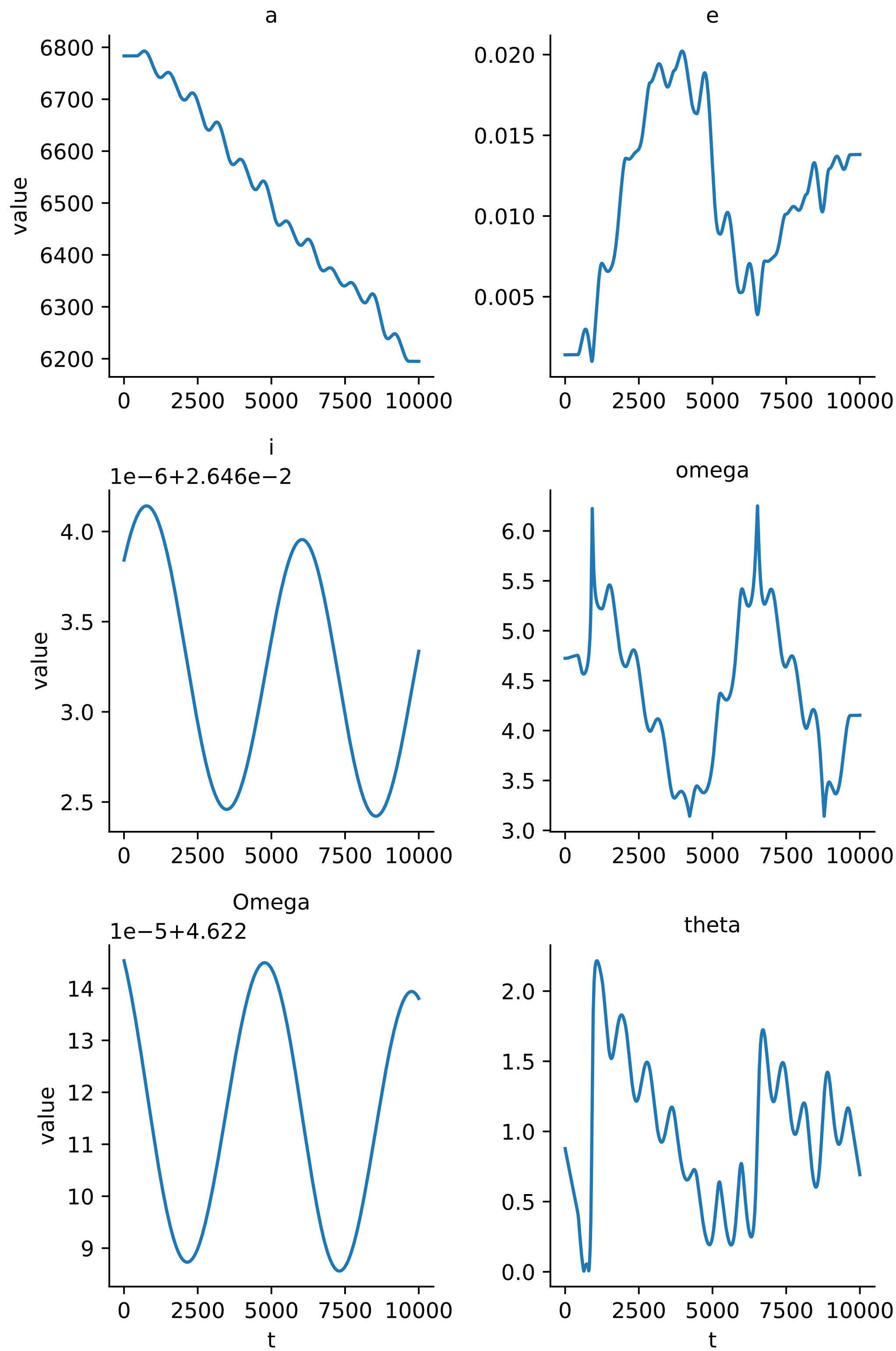

Figure 9: Variation of Parameter due to moon gravity perturbation for 10000 seconds

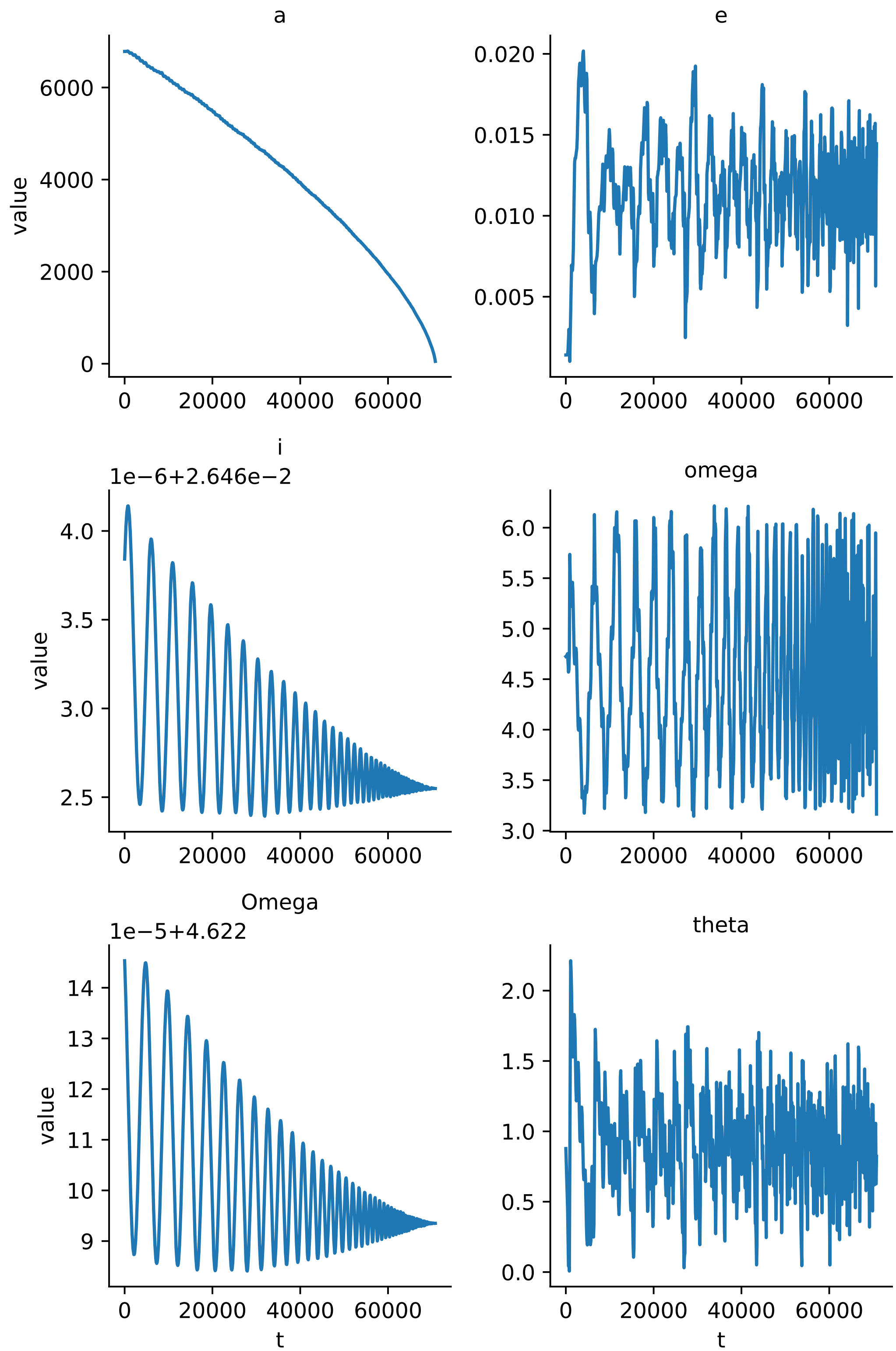

Figure 10: Variation of Parameter due to moon gravity perturbation for 100000 seconds

2.4 Sun gravity perturbation

In this section, we investigate the effect of the sun's third body on satellite orbital elements. In this problem we calculate forces of sun and earth, then, integrate from differential equation for several time to see effects in short, long period and secular.

$$\sum F_{satellite} = \frac{-Gm_{earth}}{r_{e2s}^3}\mathbf{r_{e2s}} + \frac{-Gm_{sun}}{r_{s2s}^3}\mathbf{r_{s2s}}$$

Here is the result of perturbation forces on orbital elements. The simulation has been in the Jupyter notebook. the results are presented here.

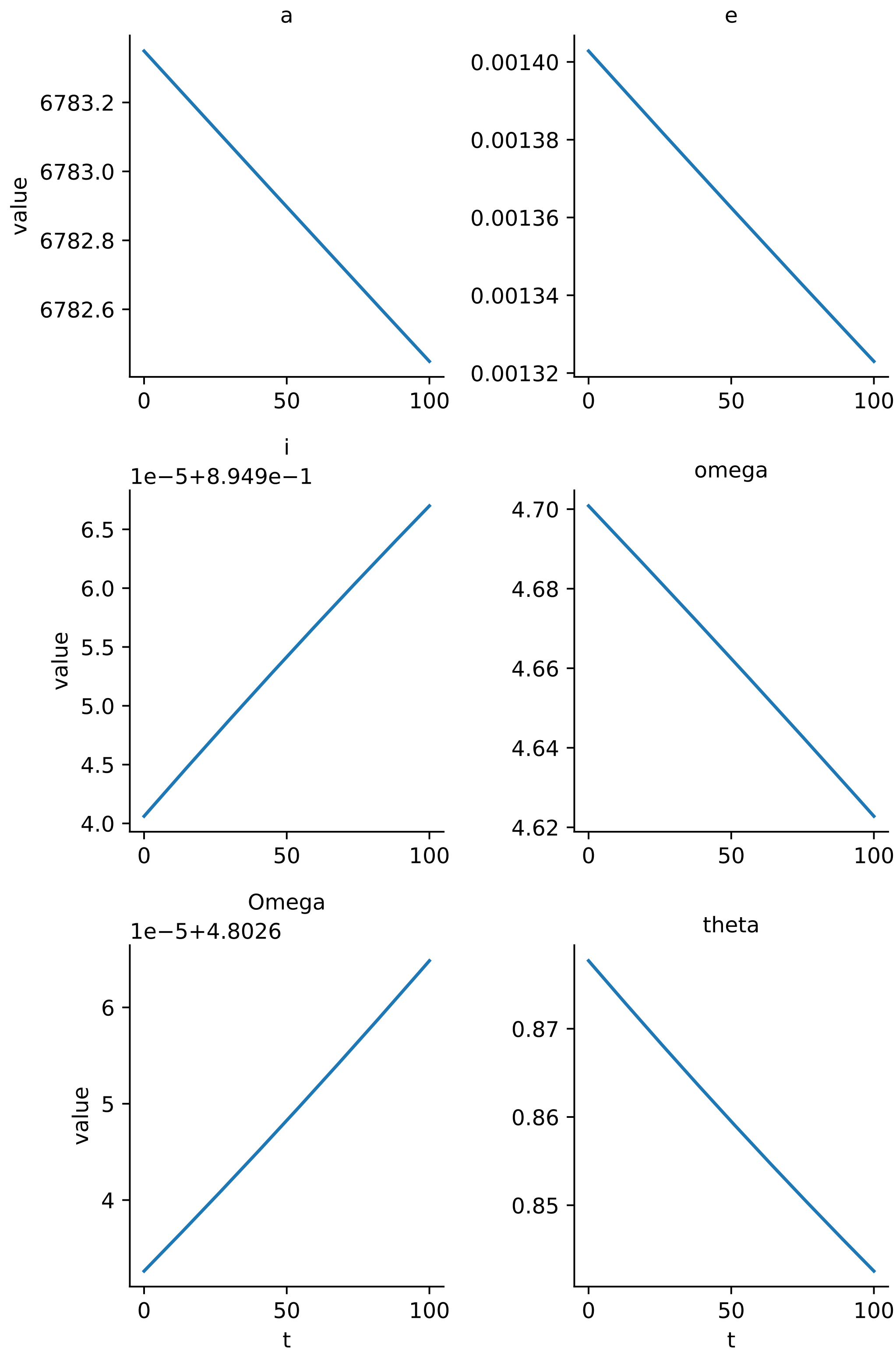

Figure 11: Variation of Parameter due to sun gravity perturbation for 100 seconds

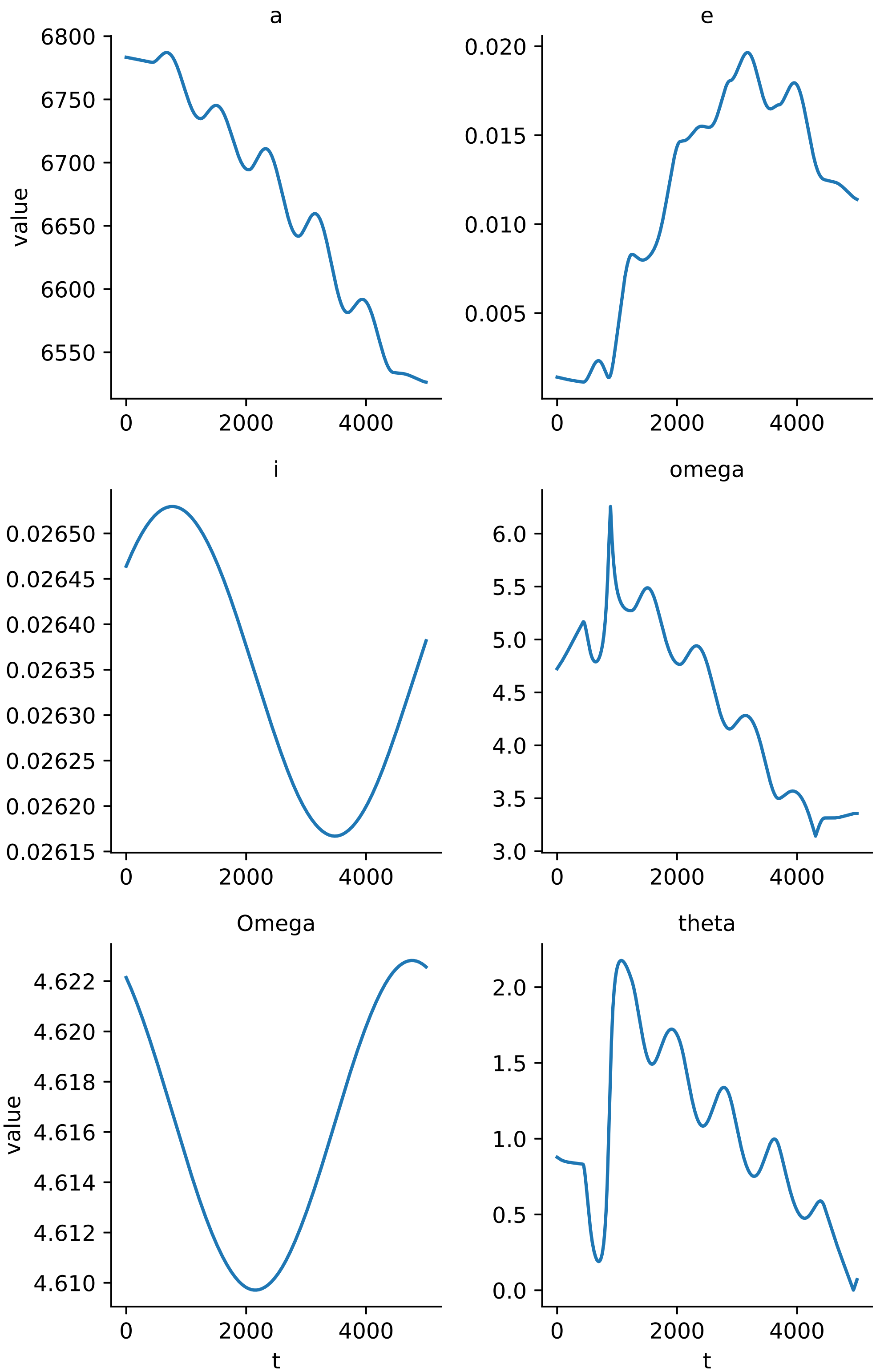

Figure 12: Variation of Parameter due to sun gravity perturbation for 5000 seconds

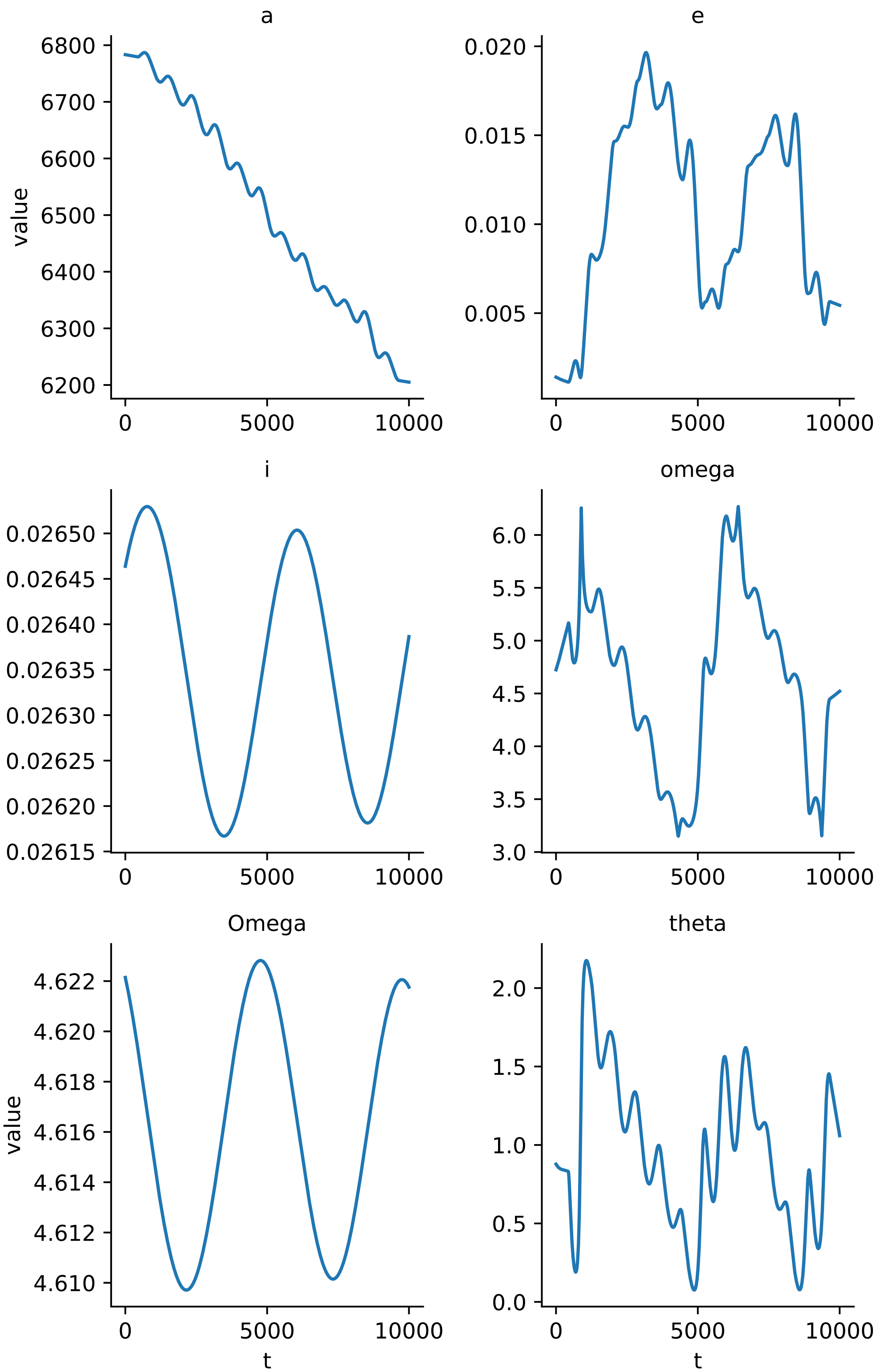

Figure 13: Variation of Parameter due to sun gravity perturbation for 10000 seconds

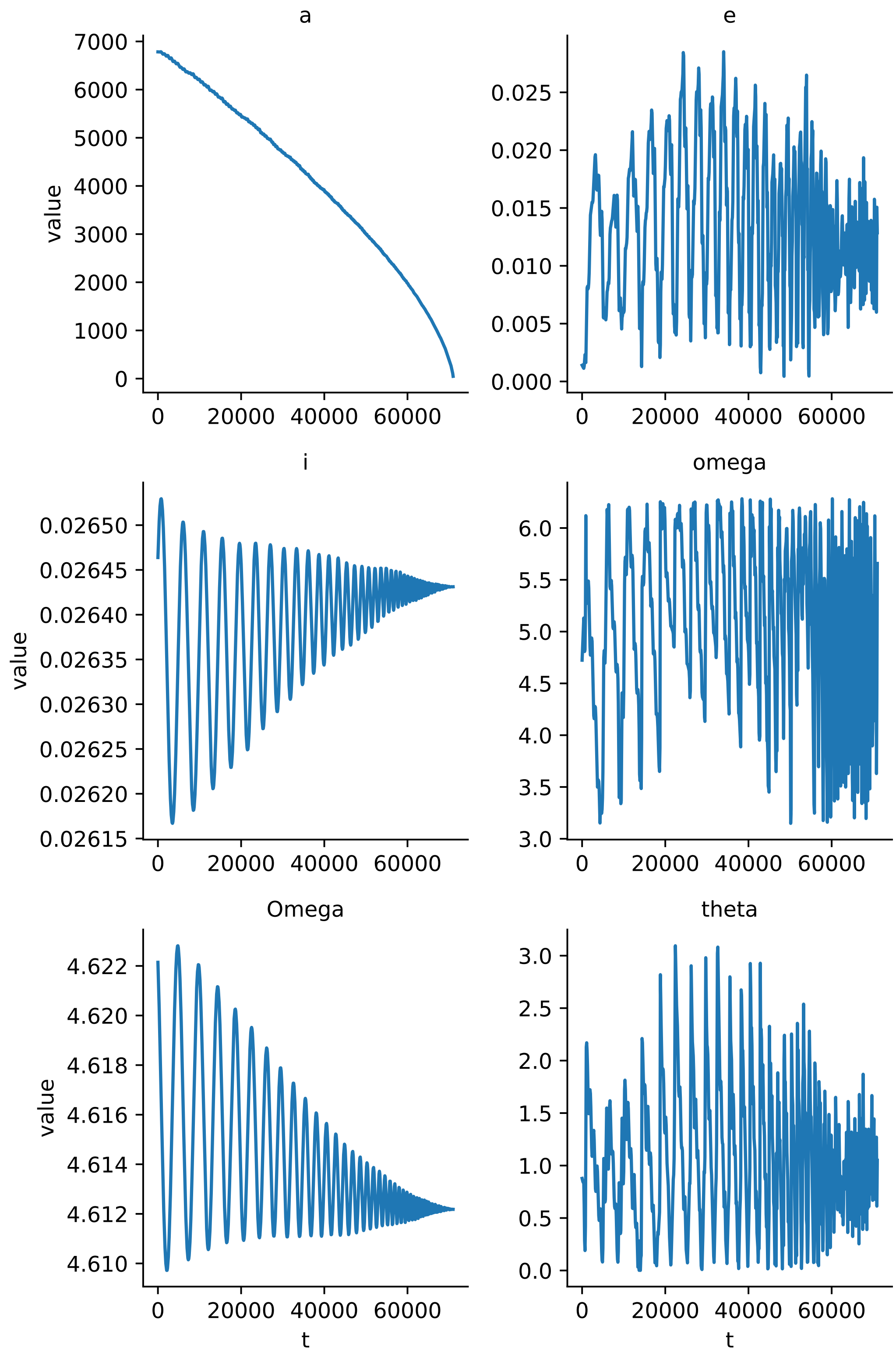

Figure 14: Variation of Parameter due to sun gravity perturbation for 100000 seconds

2.5 Solar radiation perturbation

The below equation shows the force of solar radiation.

$$P_{SRP} = \nu \frac{S}{c} C_R \frac{A_s}{m}$$

where ν calculates if the satellite is in the earth's shadow or not. Then used the below equations for rate changes.

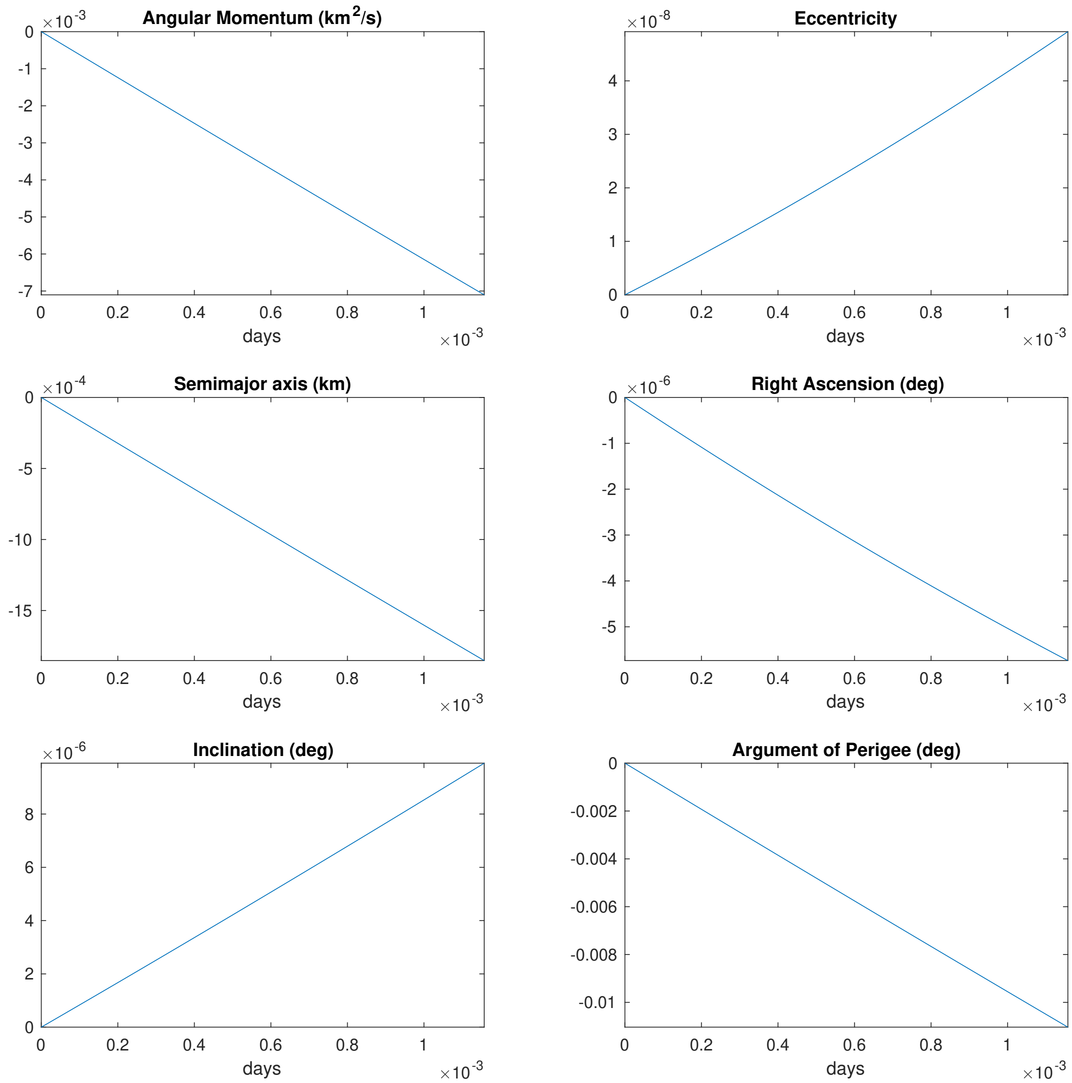

Figure 15: Variation of Parameter due to solar radiation perturbation for 100 seconds

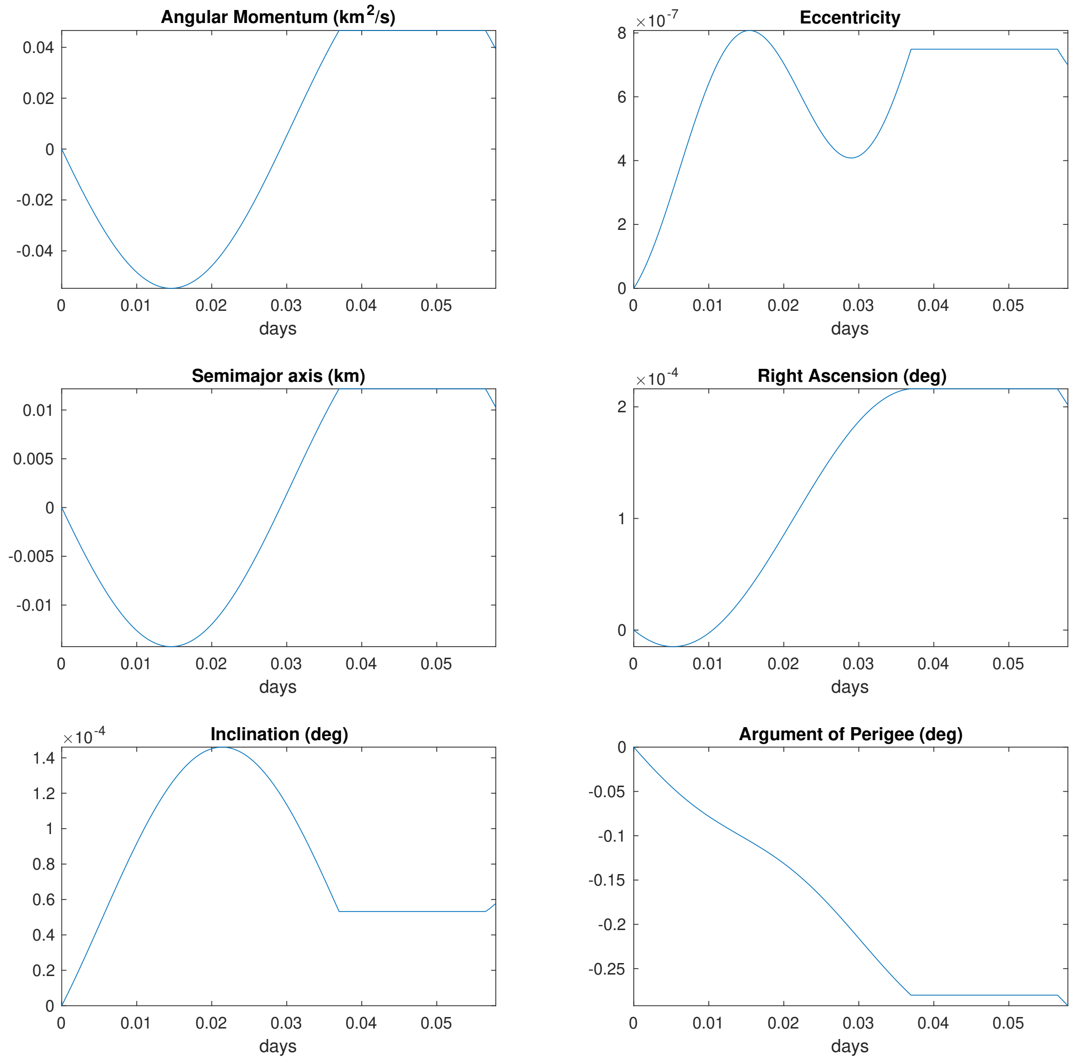

Figure 16: Variation of Parameter due to solar radiation perturbation for 5000 seconds

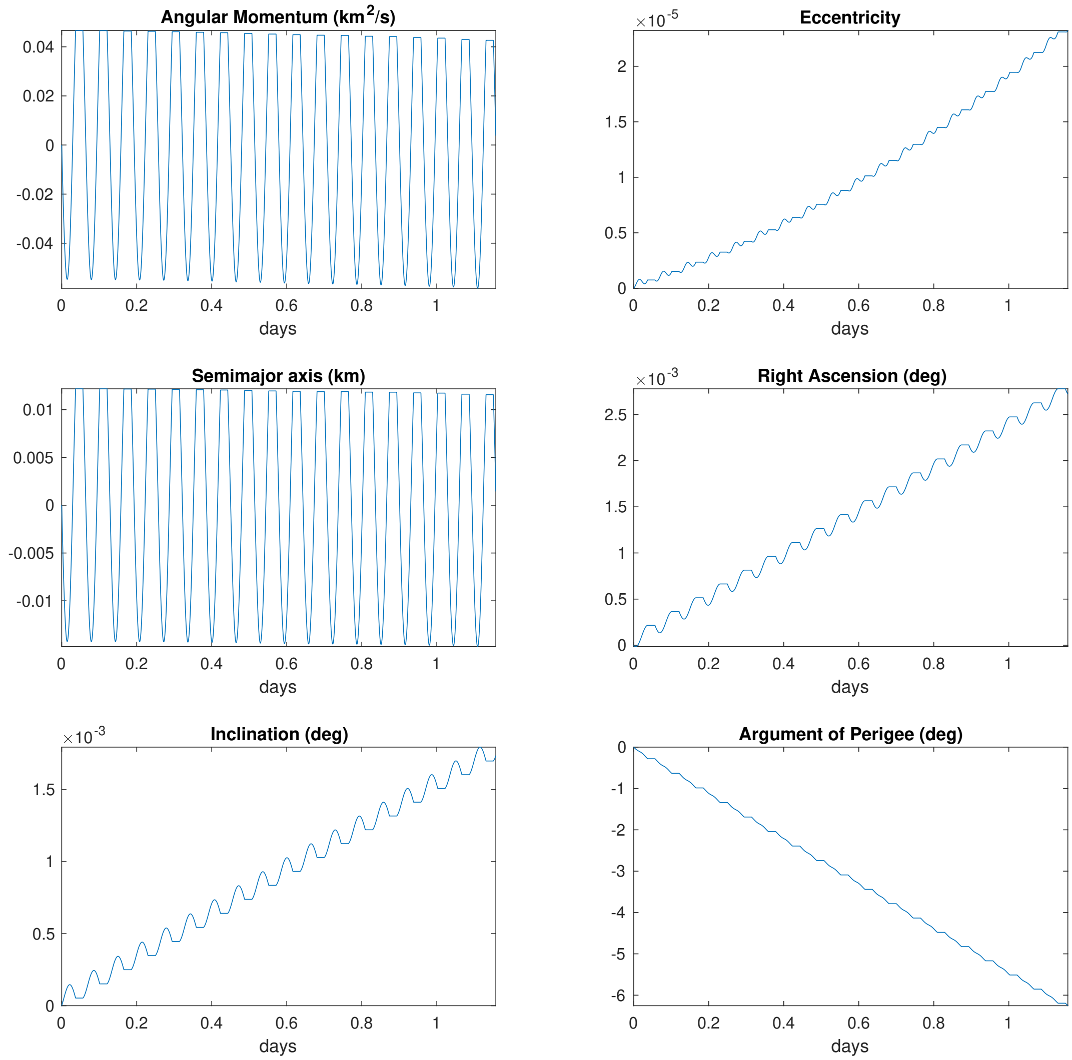

Figure 17: Variation of Parameter due to solar radiation perturbation for 100000 seconds

3 Question 3

ISS spacecraft observation orbital elements and Ground Station Location location is provided in 3, and **??**, respectively. The orbital elements are used to calculate the position of the ISS spacecraft at the time of observation. The position is then used to calculate the line of sight vector from the ground station to the ISS spacecraft. The line of sight vector is then used to calculate the elevation and azimuth angles of the ISS spacecraft. The elevation and azimuth angles are then used to calculate spacecraft visibility.

Orbital Element	Value
Eccentricity	0.0005771
Inclination	51.6409°
Perigee Height	415km
Apogee Height	423km
RAAN	88.8414°
Argument of Perigee	75.2083°
True Anomaly	0

Table 1: ISS Observation

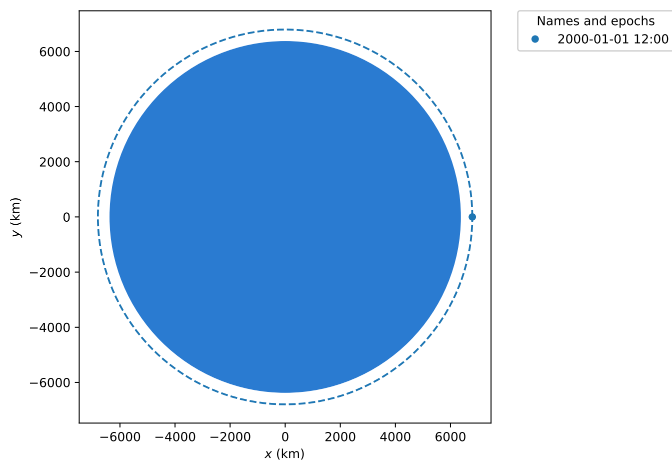

Figure 18: ISS spacecraft orbit.

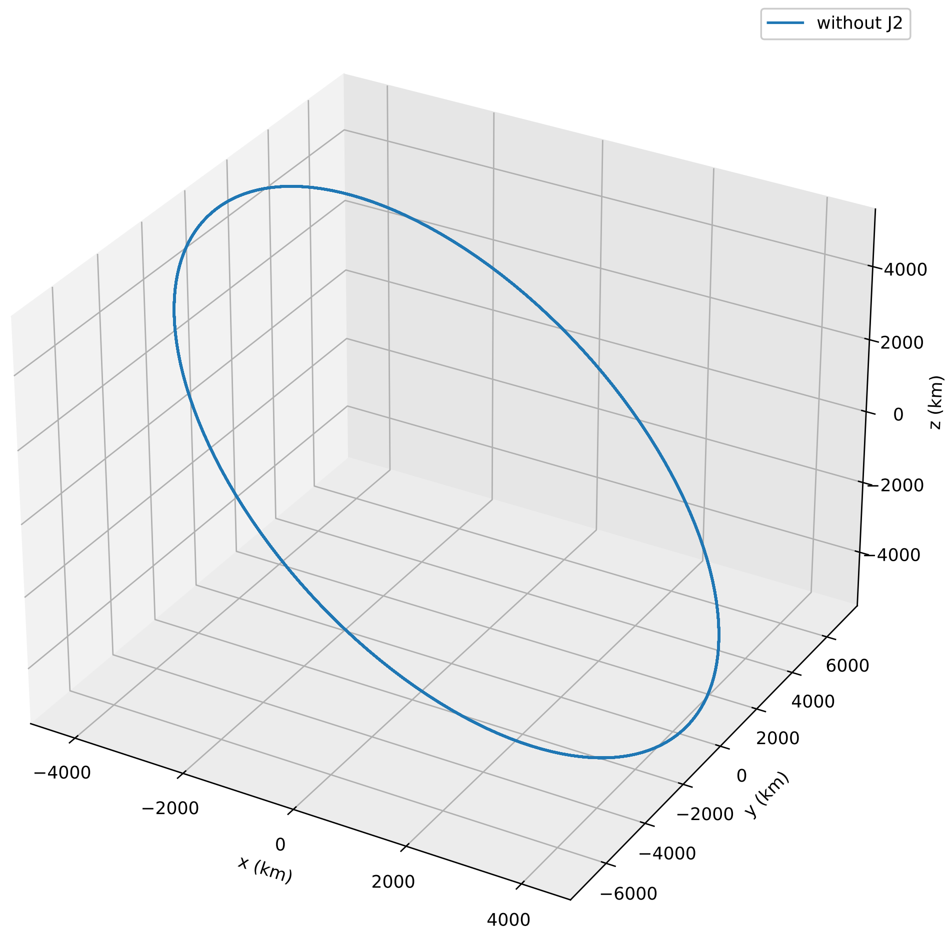

Figure 19: ISS spacecraft orbit trajectory without perturbation

3.1 J_2 perturbation

In this section we add J_2 perturbation to the orbit of the ISS spacecraft. The J_2 perturbation is added to the ISS spacecraft orbit by adding the J_2 perturbation to the mean motion of the ISS spacecraft. The J_2

perturbation is calculated using the following equation:

$$\begin{aligned}
\ddot{x} &= -\frac{\mu}{r^3}x + \frac{3}{2}\frac{J_2\mu R^2}{r^4}\left(1 - 5\frac{z^2}{r^2}\right)x \\
\ddot{y} &= -\frac{\mu}{r^3}y + \frac{3}{2}\frac{J_2\mu R^2}{r^4}\left(1 - 5\frac{z^2}{r^2}\right)y \\
\ddot{z} &= -\frac{\mu}{r^3}z + \frac{3}{2}\frac{J_2\mu R^2}{r^4}\left(3 - 5\frac{z^2}{r^2}\right)z
\end{aligned} \tag{7}$$

ISS trajectory is shown in below figure.

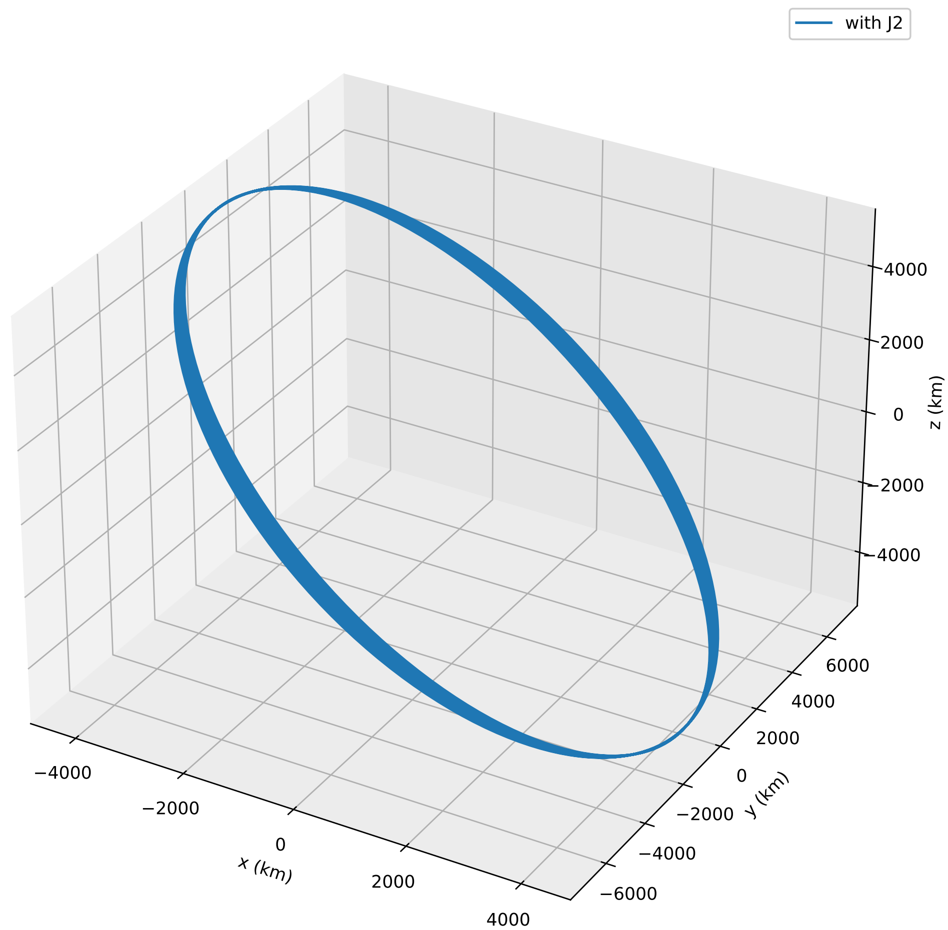

Figure 20: ISS spacecraft orbit trajectory with J_2 perturbation

3.2 Study difference J_2 perturbation orbital

In this section, we study J_2 perturbation in trajectory and error that make with ideal trajectory.

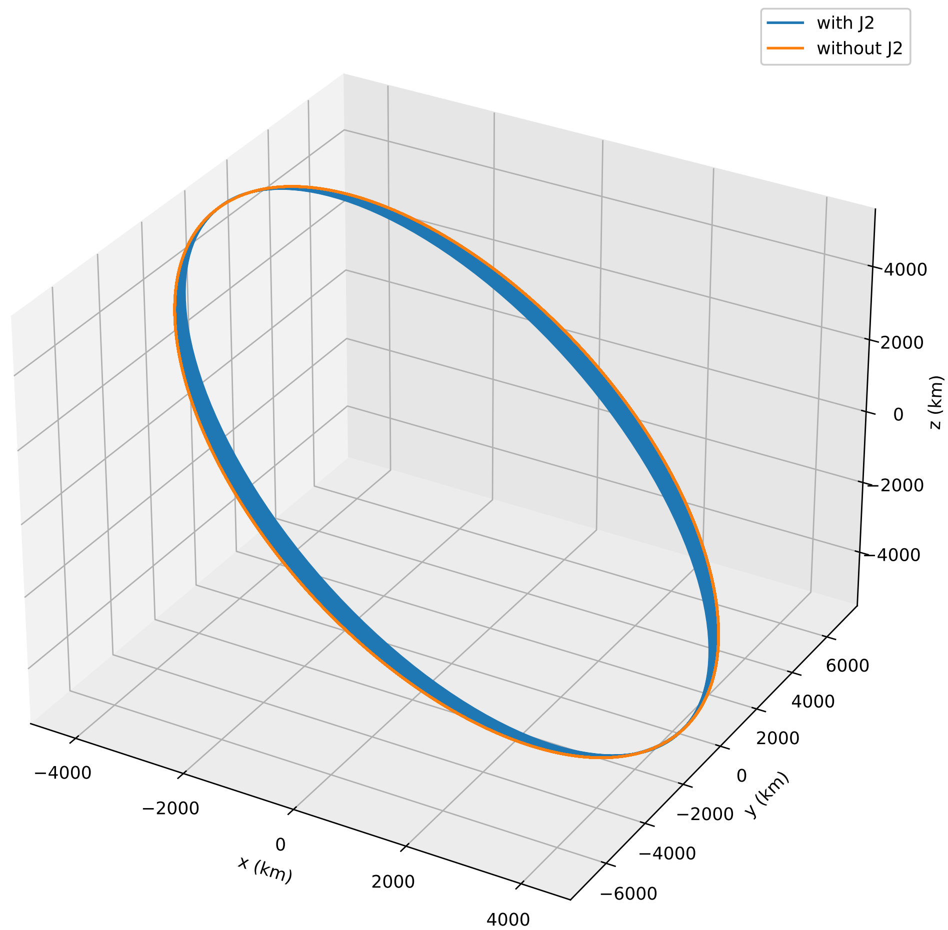

Figure 21: J_2 perturbation effect on ISS spacecraft orbit trajectory

Below figures show error vector with function of time.

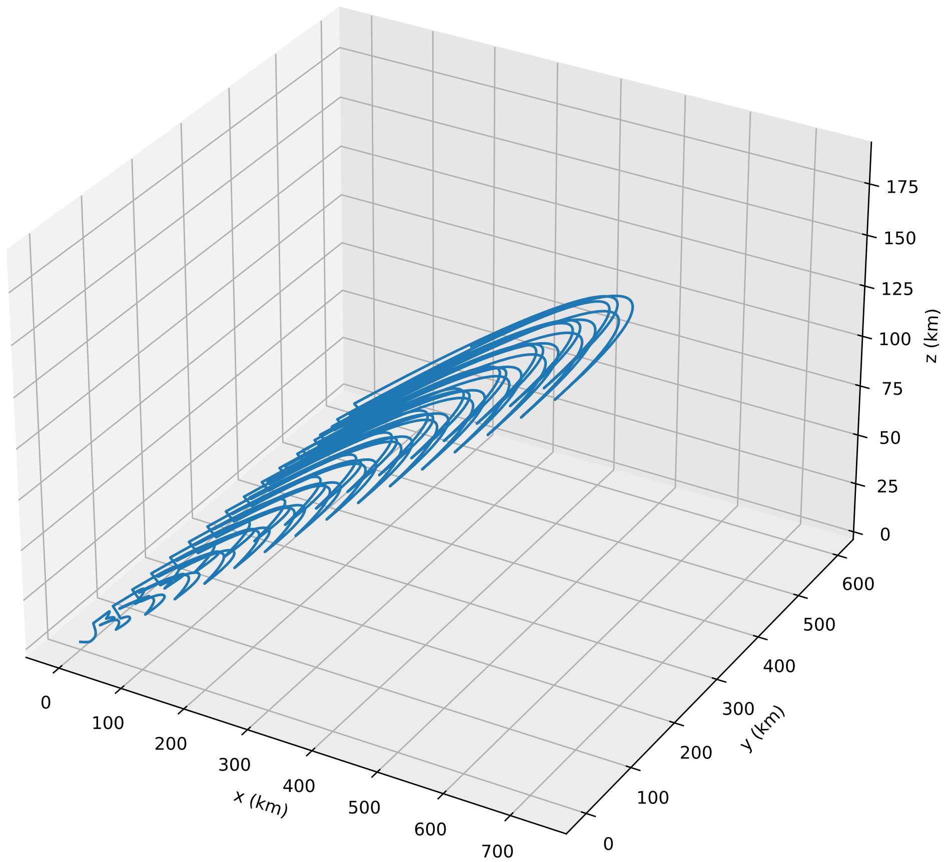

Figure 22: J_2 perturbation error vector with ideal trajectory

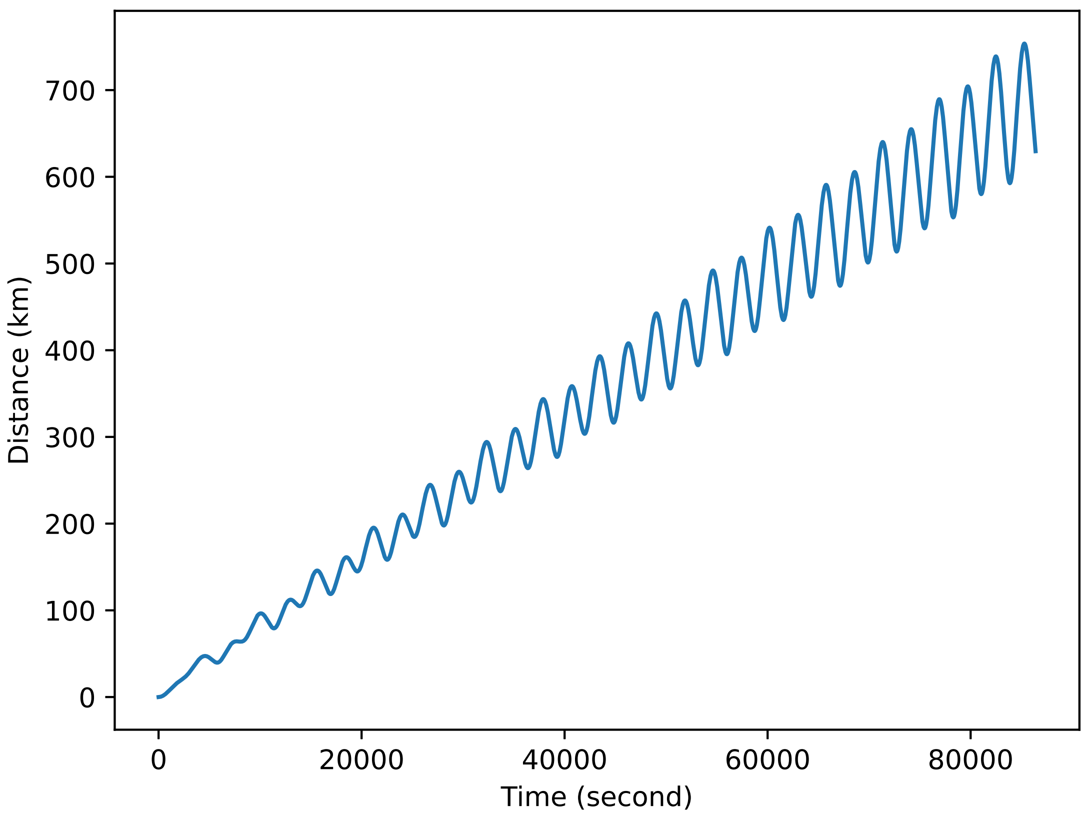

Figure 23: J_2 perturbation error with ideal trajectory

3.3 Allowable error

In the previous part, we see the error but its magnitude is larger than 180 m and we can't see when the error is 180 meters. The below figure shows less time for simulation and finding when the error is 180 meters, and it's about 140 seconds.

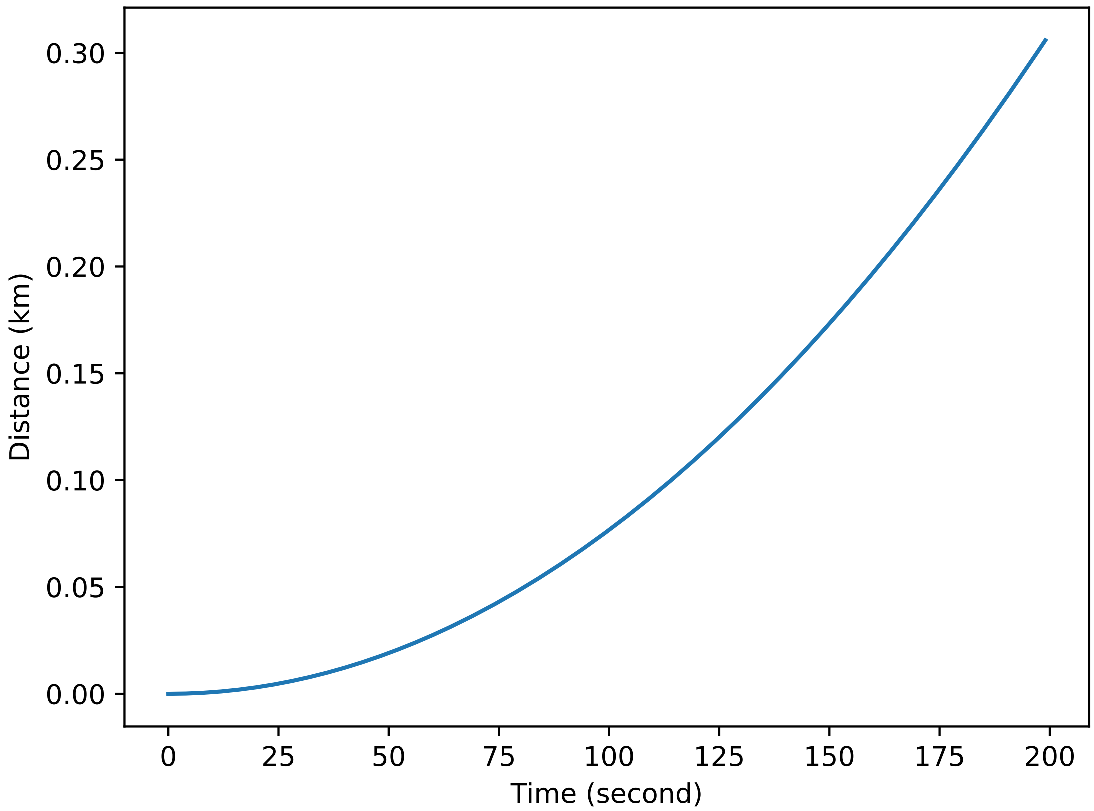

Figure 24: J_2 perturbation error with ideal trajectory till 200 seconds

Contents

List of Figures