Home Work #3

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1 Question 1

In this equation we discuses about perturbation in classical orbital element. Formulas for the Gaussian form of the VOP equations using the disturbing force with specific force components resolved in the RSW system:

$$
\begin{aligned}
\frac{da}{dt} =& \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right) \\
\frac{de}{dt} =& \frac{\sqrt{1-e^2}}{na}\left(\sin(\theta)F_R + \left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S\right) \\
\frac{di}{dt} =& \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W \\
\frac{d\Omega}{dt} =& \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W \\
\frac{d\omega}{dt} =& \frac{\sqrt{1-e^2}}{nae}\left(-\cos(\theta)F_R + \sin(\omega)\left(1+\frac{1}{p}\right)F_S\right) - \frac{r\cot(i)\sin(u)}{h}F_W \\
\frac{M_0}{dt} =& \frac{1}{na^2e}\left((p\cos(\theta) - 2er)F_R - (p+r)\sin(\theta)F_S\right) - \frac{dn}{dt}(t-t_0)
\end{aligned}
\tag{1}
$$

1.1 part a

If we want to change a, we need to have force in R or S direction. If there is a force in R and S direction other parameters like eccentricity, ω, and M_0 will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "a" without the change of other parameters.

$$
\begin{aligned}
\sin(\theta)F_R &= -\left(\cos(\theta) + \frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)F_S \\
\cos(\theta)F_R &= \sin(\omega)\left(1+\frac{1}{p}\right)F_S \\
(p\cos(\theta) - 2er)F_R &= (p+r)\sin(\theta)F_S
\end{aligned}
\tag{2}
$$

1.2 part b

If we want to change inclination, we need to have force in W direction. If there is a force in W direction other parameters like ω, and Ω will change and others will be constant. If we can solve the below equations and find the answer (if exist), we can change parameter "eccentricity" without the change of other parameters.

$$\begin{aligned} \frac{r\cos(u)}{na^2\sqrt{1-e^2}} &= 0 \\ \frac{r\cot(i)\sin(u)}{h} &= 0 \end{aligned} \tag{3}$$

1.3 part c

From the below equation, we can find the most efficient θ is in $\theta = 90°$ because $\sin(90°) = 1$, and to find the best direction it depends on with of parameters e and $\frac{p}{r}$ is bigger.

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left(e\sin(\theta)F_R + \frac{p}{r}F_S\right) \tag{4}$$

1.4 part d

From below equation we can find most efficient u is when $u = 0°$ because $\cos(0°) = 1$.

$$\frac{di}{dt} = \frac{r\cos(u)}{na^2\sqrt{1-e^2}}F_W \tag{5}$$

so:

$$u = 0 \rightarrow \theta + \omega = 0° \rightarrow \theta = -\omega$$

and for Ω, from below equation we can find most efficient θ is when $\theta = 90°$ because $\sin(90°) = 1$.

$$\frac{d\Omega}{dt} = \frac{r\sin(\theta)}{na^2\sqrt{1-e^2}\sin(i)}F_W$$

2 Question 2

In this question, we investigate the effect of perturbation forces on the orbital elements.

2.1 J_2 perturbation

Forces in RSW system:

$$\begin{aligned} F_R &= -\frac{3\mu J_2 R^2}{2r^4}\left(1 - 3\sin^2(i)\sin^2(u_0)\right) \\ F_S &= -\frac{3\mu J_2 R^2}{2r^4}\sin^2(i)\sin(u_0)\cos(u_0) \\ F_W &= -\frac{3\mu J_2 R^2}{2r^4}\sin(i)\cos(i)\sin(u_0) \end{aligned} \tag{6}$$

From (1) we know the effect of other forces on orbital elements.

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