Home Work #1

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1 Question 1

$$h = 200_{km} \rightarrow r = R_e + h = 6378.137 + 200 = 6578.1137, \quad \mu = 3.986 \times 10^{14}_{m^3/s^2} = 3.986 \times 10^{5}_{km^3/s^2}$$

The orbit is circular.

1.1 Part a

$$T = 2\pi\sqrt{\frac{r^3}{\mu}} = 2\pi\sqrt{\frac{6578.1137^3}{3.986 \times 10^5}} = 5309.62_{\sec} \tag{1}$$

$$T\omega = 2\pi \rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{5309.62} = 0.00118_{rad/\sec} \tag{2}$$

1.2 Part b

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{6578.1137}} = 7.78_{km/\sec} \tag{3}$$

The new velocity is calculated as:

$$v_{new} = v + 0.5_{km.\sec} = 8.28_{km.\sec} \tag{4}$$

Assume the new orbit is circular just changed altitude and has a new velocity.

$$v_{new} = \sqrt{\frac{\mu}{r_{new}}} \rightarrow r_{new} = \frac{\mu}{v_{new}^2} = 5808_{km} \tag{5}$$

$$T = 2\pi\sqrt{\frac{r_{new}^3}{\mu}} = 2\pi\sqrt{\frac{5808^3}{3.986 \times 10^5}} = 4405.08_{\sec} \tag{6}$$

$$T_{new}\omega_{new} = 2\pi \rightarrow \omega_{new} = \frac{2\pi}{T_{new}} = \frac{2\pi}{4405.08} = 0.00143_{rad/\sec} \tag{7}$$

r_{new} is smaller than the earth's radius.

2 Question 2

Assume $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ as:

$$\boldsymbol{r}_0 = \begin{bmatrix}1600 & 5310 & 3800\end{bmatrix}^{\mathrm{T}}_{km}, \quad \boldsymbol{r}_0 = \begin{bmatrix}-7.350; & 0.4600 & 2.470\end{bmatrix}^{\mathrm{T}}_{km/\sec}$$

2.1 Part a

the n-body problem was solved with MATLAB with the n-body function in the question directory. The results are illustrated below.

Figure 1: 3D trajectory

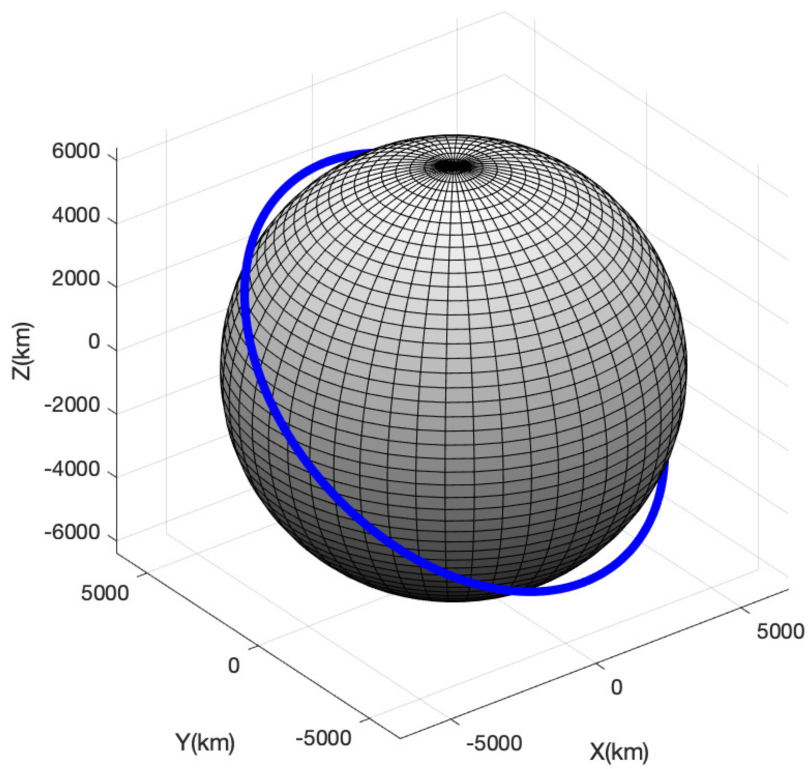


Figure 2: 3D trajectory in zx axis

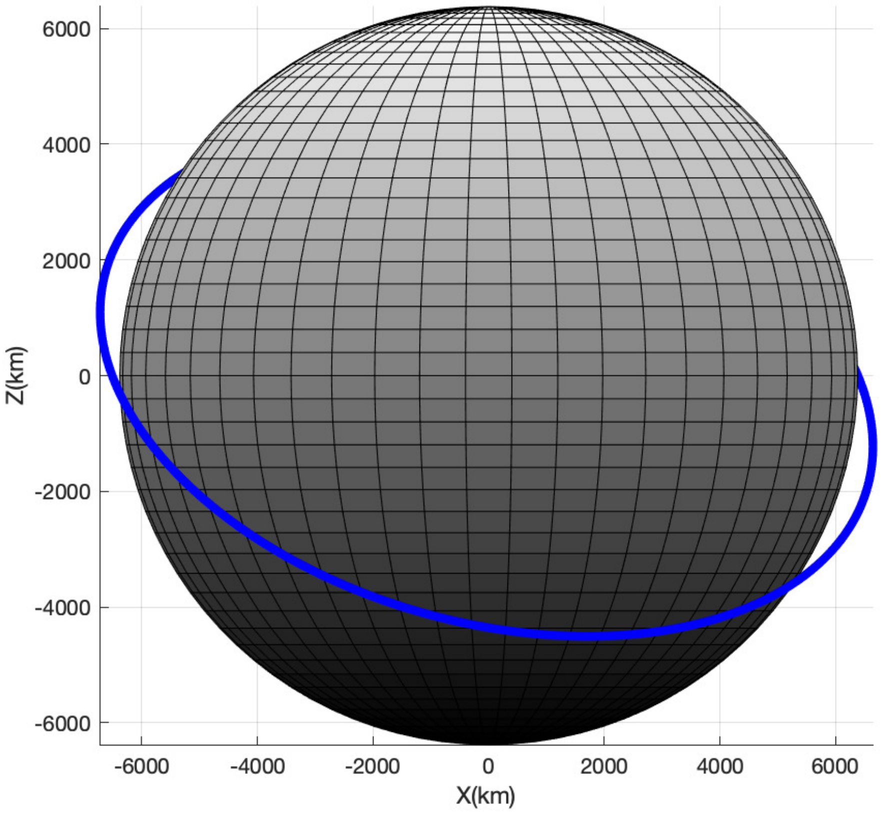


Figure 3: 3D trajectory in zy axis

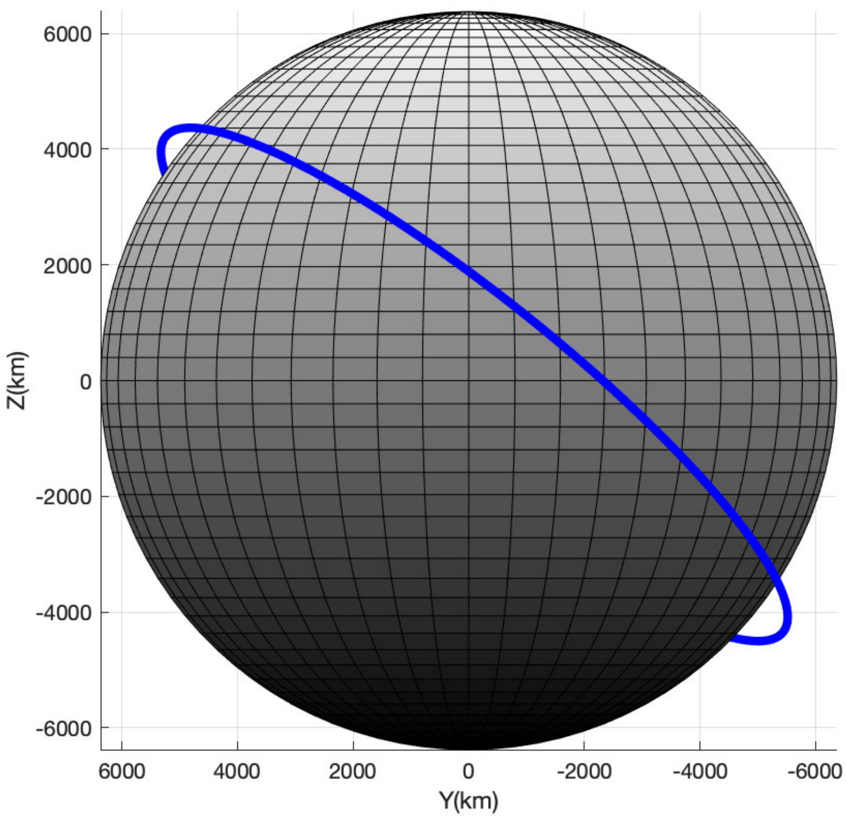
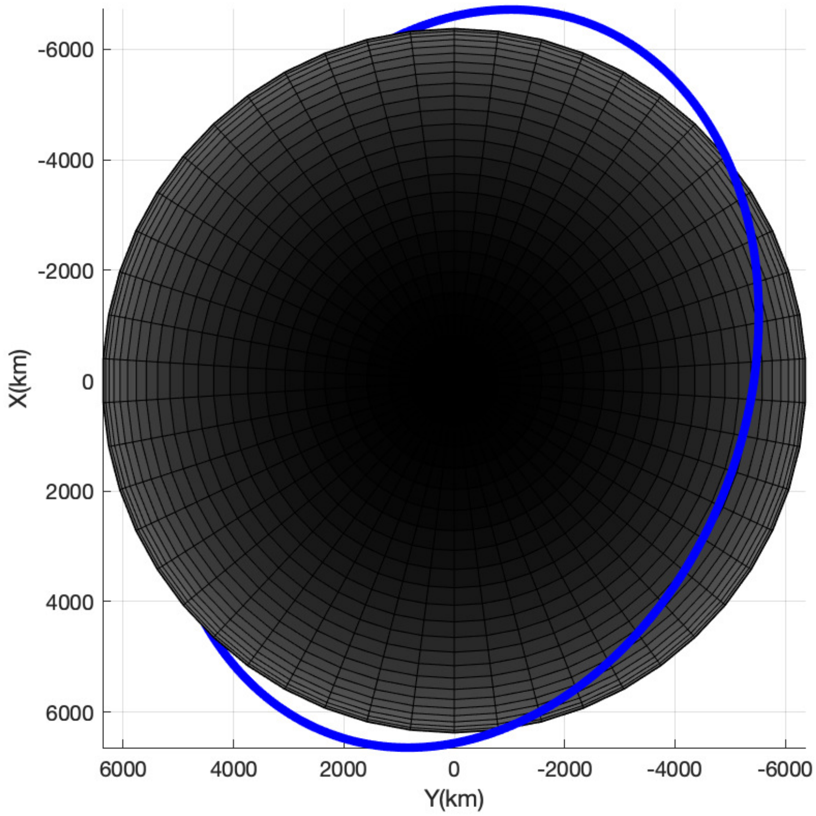


Figure 4: 3D trajectory in xy axis



2.2 Part b

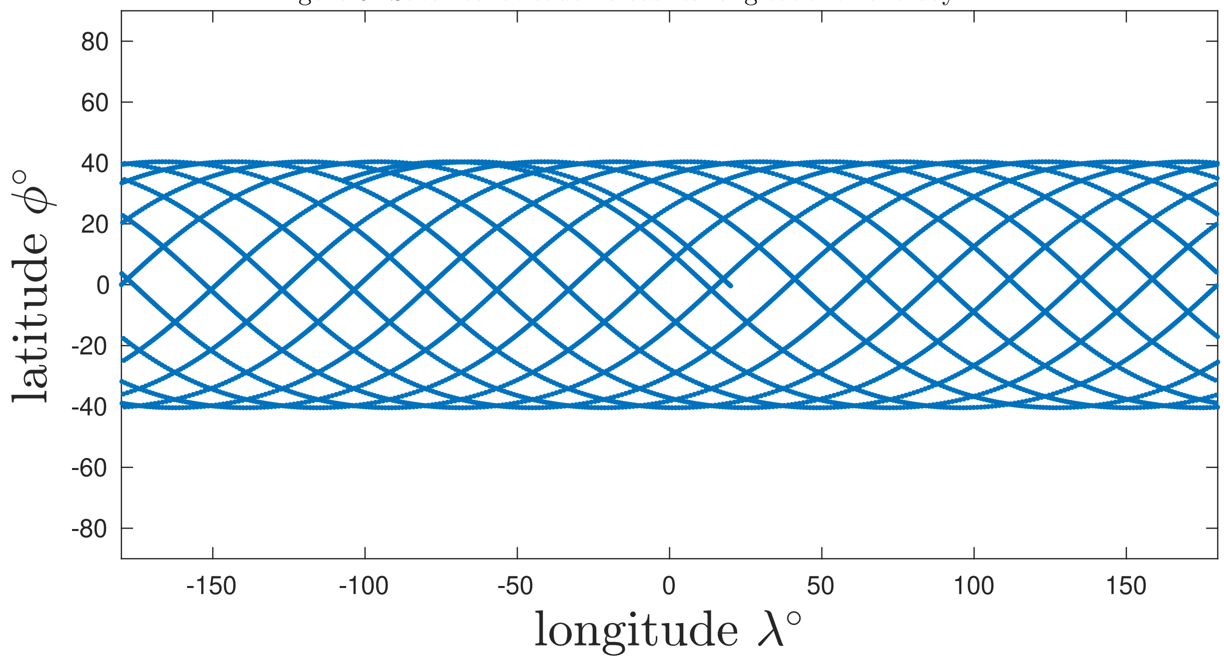
Using the below transfer matrix to transfer from ECI coordinate to the ECEF coordinate.

$$\boldsymbol{T}^{ECCF-ECI} = \begin{bmatrix} \cos(\omega_E t) & -\sin(\omega_E t) & 0 \\ \sin(\omega_E t) & \cos(\omega_E t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \arccos(\frac{\boldsymbol{r}(3)}{r})$$

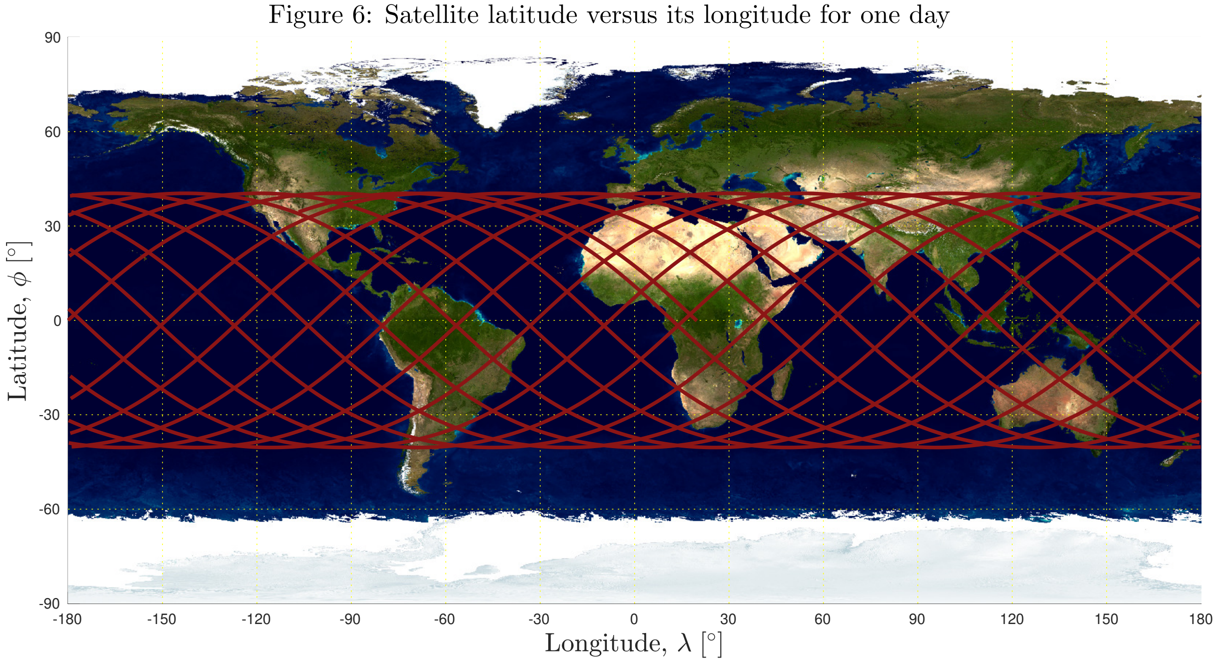
$$\lambda = \begin{cases} \arctan(\frac{\boldsymbol{r}(1)}{r_{xy}}), & \boldsymbol{r}(2) > 0 \\ 2\pi - \arctan(\frac{\boldsymbol{r}(1)}{r_{xy}}), & \boldsymbol{r}(2) \leq 0 \end{cases}$$

Figure 5: Satellite latitude versus its longitude for one day



Below the figure drawn provided by tamaskis, please click here to see the source code. Please use the mentioned library to run code or skip the part on earth fig.

Figure 6: Satellite latitude versus its longitude for one day



2.3 Part c (Bonus)

In this section find the orbital elements, then, change the inclination to 0.4_{rad} (using oe2ecf and vec2orbElem functions). Then, find $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ by new orbital elements. The results are illustrated below.

Figure 7: 3D trajectory

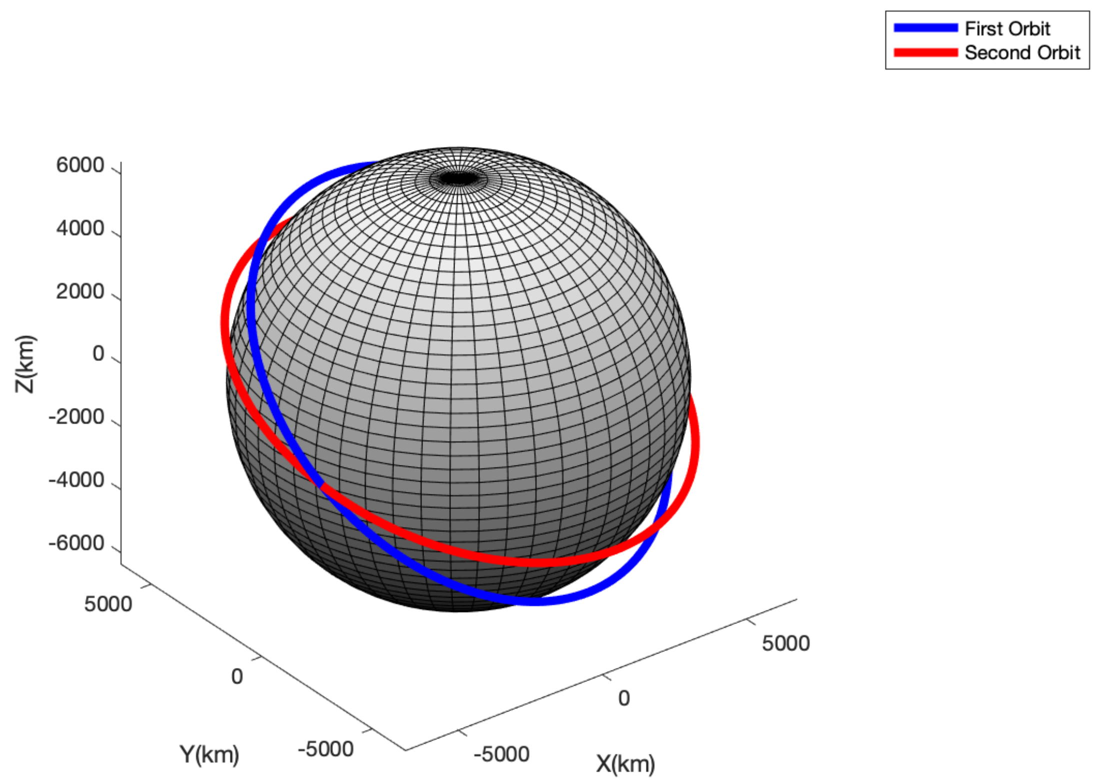


Figure 8: 3D trajectory in zx axis

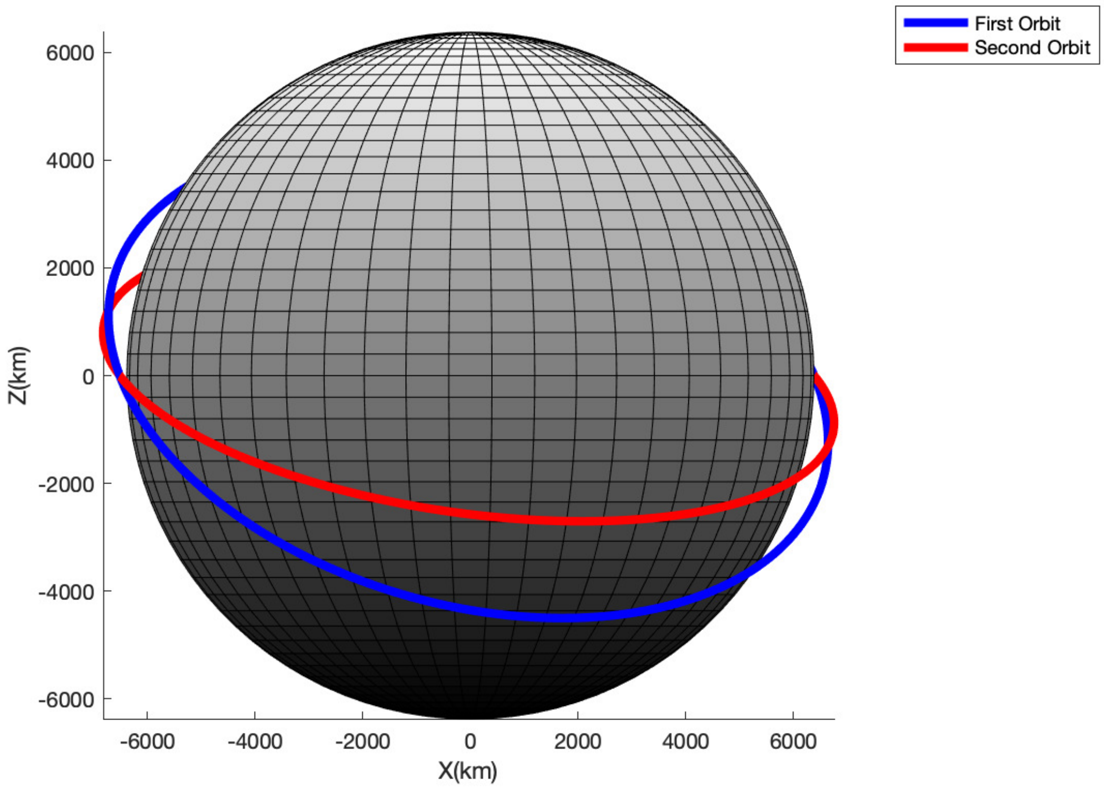


Figure 9: 3D trajectory in zy axis

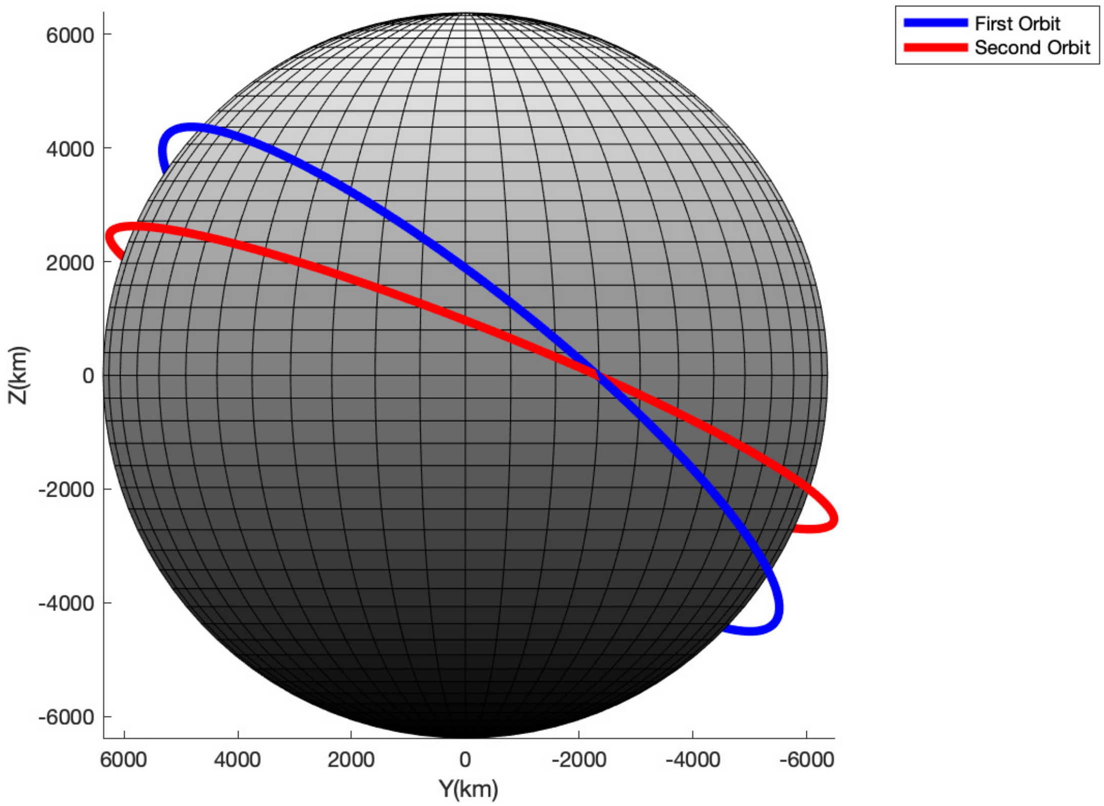


Figure 10: 3D trajectory in xy axis

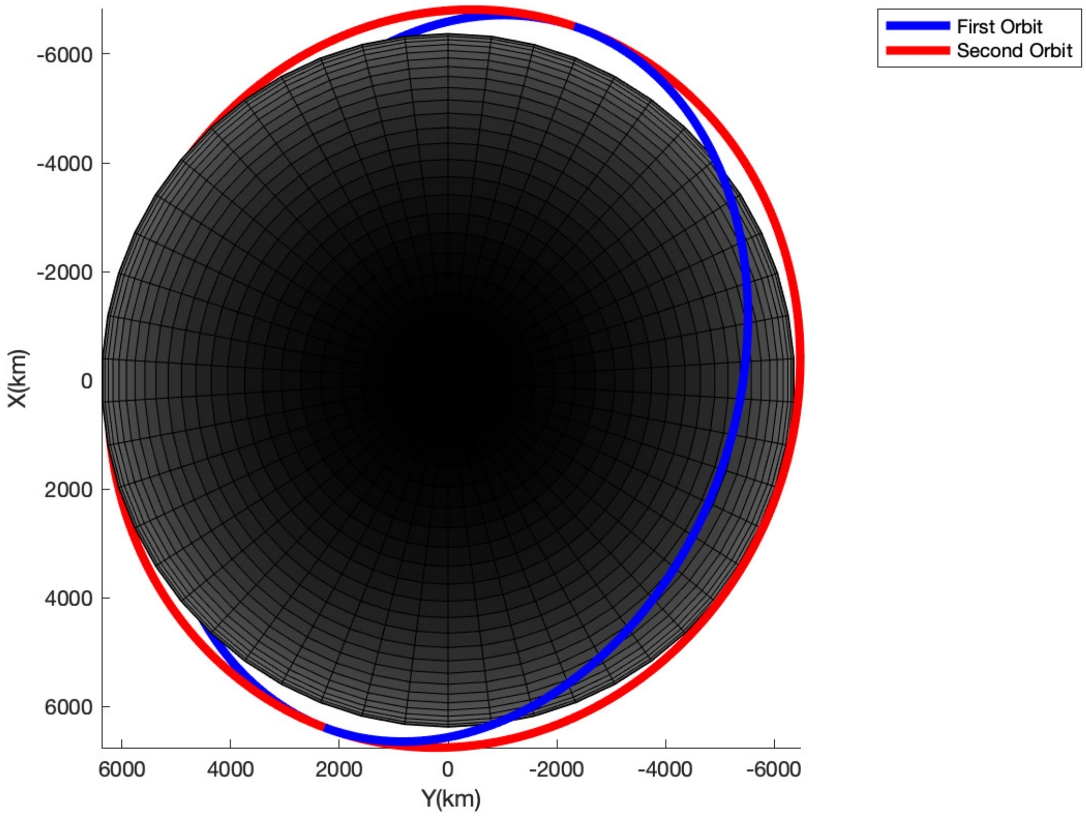
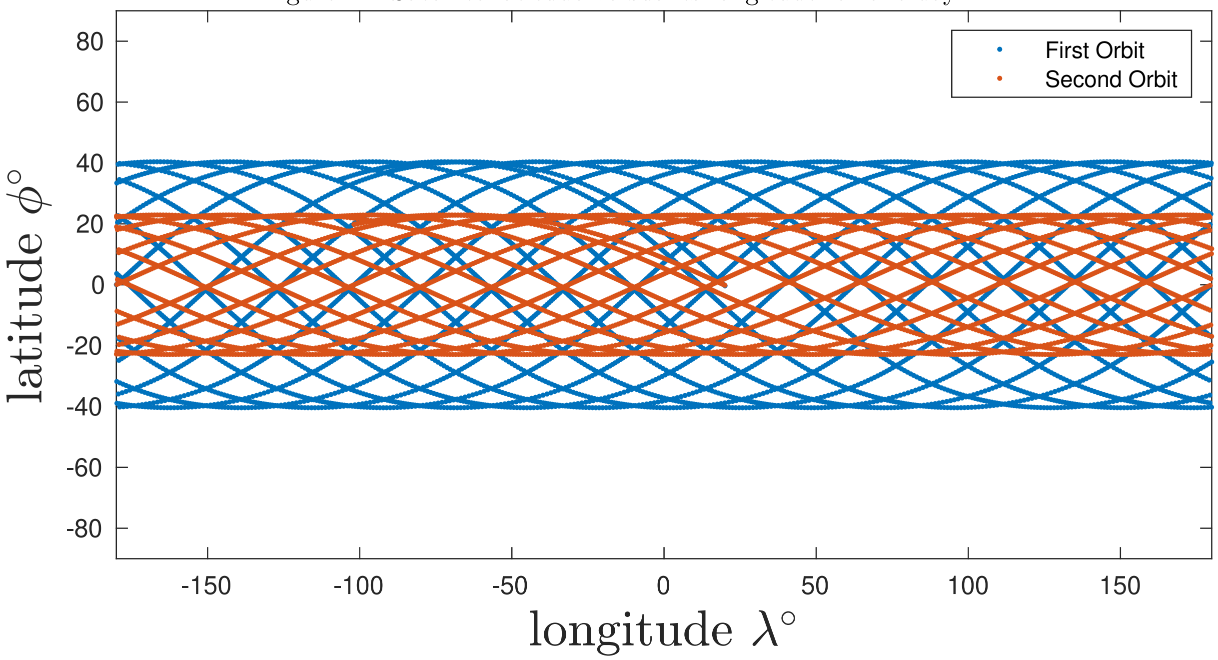


Figure 11: Satellite latitude versus its longitude for one day



In general, changing the inclination of a satellite's orbit will cause its ground track to move to different latitudes and longitudes on the Earth's surface. In this example when the inclination changed from 0.7_{rad} to 0.4_{rad}, the range of longitude decreased.

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