Home Work #2

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1 Question 1

Spacecraft position in ITRF coordinates is given by

$$\mathbf{r} = \begin{bmatrix} 6789 & 6893 & 7035 \end{bmatrix}^{\mathrm{T}}_{km}$$

1.1 part a

Find Latitude and Longitude. For this purpose used algorithm 12 of Valado's book. This algorithm is implemented in the function 'latlon.py' in the 'code/Q1' folder. The function takes the spacecraft position vector as input and returns the latitude and longitude in degrees. The iteration ended when the difference is smaller than 1e-10. The results are:

Table 1: Results of part a

Variables	**Values**
Latitude	36.12°
Longitude	45.43°
h_{ellp}	5591.51_{km}

1.2 part b

In this part, we used the astropy package to find the position vector in the GCRF coordination system. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. Position vector in GCRF:

$$\mathbf{r} = \begin{bmatrix} -862.54 & -9634.75 & 7037.25 \end{bmatrix}^{\mathrm{T}}_{km}$$

1.3 part c

In this part, we used the astropy package to find GMST(θ_{GMST}) and LST(θ_{LST}) The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. The results are:

Table 2: Results of part c

GMST(θ_{GMST})	**LST(θ_{LST})**
112.78°	149.78°

2 Question 2

Satellite position and velocity vectors in the Earth-Centered Inertial (ECI) coordinate system:

$$\vec{r}_{ECI} = \begin{bmatrix}-346 & 8265 & 4680\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$

$$\vec{v}_{ECI} = \begin{bmatrix}-5.657 & -1.73 & 2.703\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

2.1 part a

Algorithm for converting from ECI to orbital x-y plane coordinates is described below:

1. Calculate the angular momentum vector $\vec{h}$ by taking the cross product of the position vector $\vec{r}$ and the velocity vector $\vec{v}$ in the ECI coordinate system.
2. Calculate the unit vector $\hat{z}$ in the direction of $\vec{h}$.
3. Calculate the unit vector $\hat{x}$ in the x-direction of the satellite position.
4. Calculate the unit vector $\hat{y}$ in the y-direction of the orbital plane by taking the cross product of $\hat{z}$ and $\hat{x}$.
5. Express the ECI position and velocity vectors $\vec{r}$ and $\vec{v}$ in the new coordinate system by taking their dot products with $\hat{x}$, $\hat{y}$, and $\hat{z}$.
6. Project the position and velocity vectors onto the x-y plane by setting the z-component of each vector to zero.

Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.
results:

$$r_{\text{x, y plane}} \begin{bmatrix}9504.3327488 & 0 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$
$$\vec{v}_{\text{x, y plane}} \begin{bmatrix}0 & 4.16567802 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

2.2 part b

To calculate satellite position after 30 minutes, the differential equation for the satellite solved for 30 minutes. The differential equation is:

$$\vec{r}_{\text{x, y plane}} = \vec{v}$$
$$\frac{d\vec{v}}{dt} = -\frac{\mu}{r^3}\vec{r}$$

where μ is the gravitational parameter of the Earth, and r is the magnitude of the position vector $\vec{r}$. Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.

results:

$$\vec{r}_{\text{x, y plane}} = \begin{bmatrix}1379.53 & 4493.87 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$
$$\vec{v}_{\text{x, y plane}} = \begin{bmatrix}-9.72 - 2.96 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

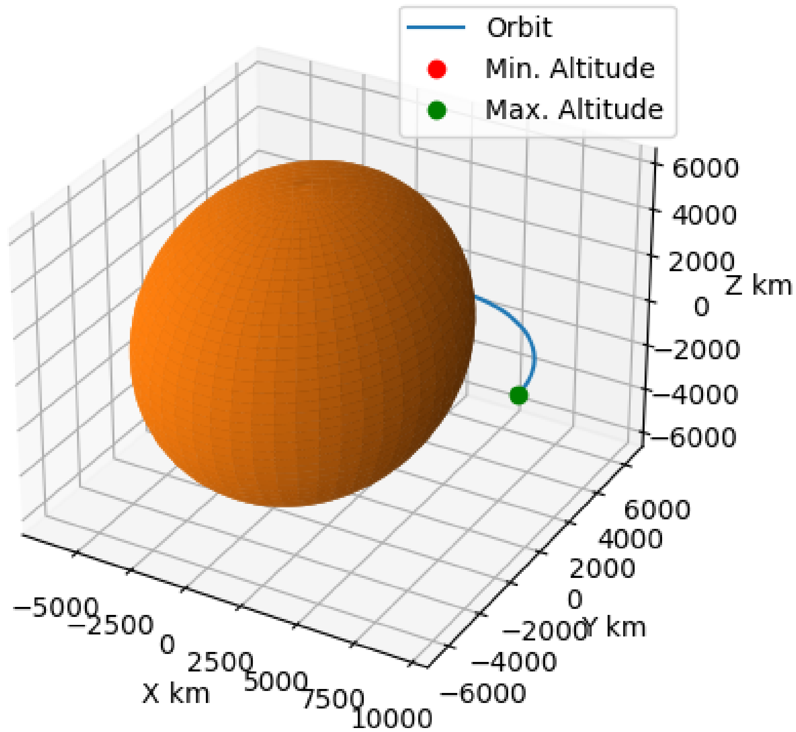

Figure 1: The position of the spacecraft in the GCRF coordinate system.

3 Question 3

The observation time is on August 1, 2023, at 15:00:00 UTC.

3.1 part a

In this to calculate Earth and Sun location used JPL Horizons On-Line Ephemeris System. The code used the "de440s" version which is the last and most accurate version of the DE series. The code used a short version because it doesn't need all the data time that was provided and the complete one has 3 gigabyte size. The code is in Q3.ipynb jupyter notebook.

3.2 part b

To check if the satellite has a clear view of the Sun, we need to calculate the angle between the Sun vector and the satellite vector. If the angle is less than the half-angle of the satellite's field of view (FOV), then the satellite has a clear view of the Sun. The calculation can be done as follows:

Find the position vector of the Sun in the ECI coordinate system, denoted as $\vec{r}_{Sun}$. This can be obtained from an ephemeris or by using an orbital model. Find the position vector of the satellite in the ECI coordinate system, denoted as $\vec{r}_{sat}$. Find the unit vector in the direction of the Sun from the satellite, denoted as $\hat{s}$: $\hat{s} = \frac{\vec{r}Sun - \vec{r}sat}{|\vec{r}Sun - \vec{r}sat|}$ Find the unit vector in the direction of the satellite from the Earth's center, denoted as $\hat{sat}$: $\hat{sat} = \frac{\vec{r}sat}{|\vec{r}sat|}$ Find the angle between $\hat{s}$ and $\hat{sat}$ using the dot product: $\cos\theta = \hat{s} \cdot \hat{sat}$ Compare the angle θ with the half-angle of the satellite's FOV, denoted as $\theta_{FOV}/2$. If $\theta \leq \theta_{FOV}/2$, then the satellite has a clear view of the Sun. The calculation is done using Python in Q3.ipynb jupyter notebook. The result is that satellite is in the umbra.

4 Question 4

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