

Home Work #2

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1 Question 1

Spacecraft position in ITRF coordinates is given by

$$\mathbf{r} = [6789 \quad 6893 \quad 7035]_{km}^T$$

1.1 part a

Find Latitude and Longitude. For this purpose used algorithm 12 of Valado's book. This algorithm is implemented in the function 'latlon.py' in the 'code/Q1' folder. The function takes the spacecraft position vector as input and returns the latitude and longitude in degrees. The iteration ended when the difference is smaller than $1e-10$. The results are:

Table 1: Results of part a

Variables	Values
Latitude	36.12°
Longitude	45.43°
h_{ellp}	5591.51_{km}

1.2 part b

In this part, we used the astropy package to find the position vector in the GCRF coordination system. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. Position vector in GCRF:

$$\mathbf{r} = [-862.54 \quad -9634.75 \quad 7037.25]_{km}^T$$

1.3 part c

In this part, we used the astropy package to find $GMST(\theta_{GMST})$ and $LST(\theta_{LST})$. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. The results are:

Table 2: Results of part c

$GMST(\theta_{GMST})$	$LST(\theta_{LST})$
112.78°	149.78°

2 Question 2

Satellite position and velocity vectors in the Earth-Centered Inertial (ECI) coordinate system:

$$\vec{r}_{ECI} = [-346 \quad 8265 \quad 4680]_{\text{km}}^T$$

$$\vec{v}_{ECI} = [-5.657 \quad -1.73 \quad 2.703]_{\text{km/sec}}^T$$

2.1 part a

Algorithm for converting from ECI to orbital x-y plane coordinates is described below:

1. Calculate the angular momentum vector \vec{h} by taking the cross product of the position vector \vec{r} and the velocity vector \vec{v} in the ECI coordinate system.
2. Calculate the unit vector \hat{z} in the direction of \vec{h} .
3. Calculate the unit vector \hat{x} in the x-direction of the satellite position.
4. Calculate the unit vector \hat{y} in the y-direction of the orbital plane by taking the cross product of \hat{z} and \hat{x} .
5. Express the ECI position and velocity vectors \vec{r} and \vec{v} in the new coordinate system by taking their dot products with \hat{x} , \hat{y} , and \hat{z} .
6. Project the position and velocity vectors onto the x-y plane by setting the z-component of each vector to zero.

Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.
results:

$$r_{x, y \text{ plane}} [9504.3327488 \quad 0 \quad 0]_{\text{km}}^T$$

$$\vec{v}_{x, y \text{ plane}} [0 \quad 4.16567802 \quad 0]_{\text{km/sec}}^T$$

2.2 part b

To calculate satellite position after 30 minutes, the differential equation for the satellite solved for 30 minutes. The differential equation is:

$$\begin{aligned} \vec{r}_{x, y \text{ plane}} &= \vec{v} \\ \frac{d\vec{v}}{dt} &= -\frac{\mu}{r^3} \vec{r} \end{aligned}$$

where μ is the gravitational parameter of the Earth, and r is the magnitude of the position vector \vec{r} . Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.

results:

$$\vec{r}_{x, y \text{ plane}} = [1379.53 \quad 4493.87 \quad 0]_{\text{km}}^T$$

$$\vec{v}_{x, y \text{ plane}} = [-9.72 \quad -2.96 \quad 0]_{\text{km/sec}}^T$$

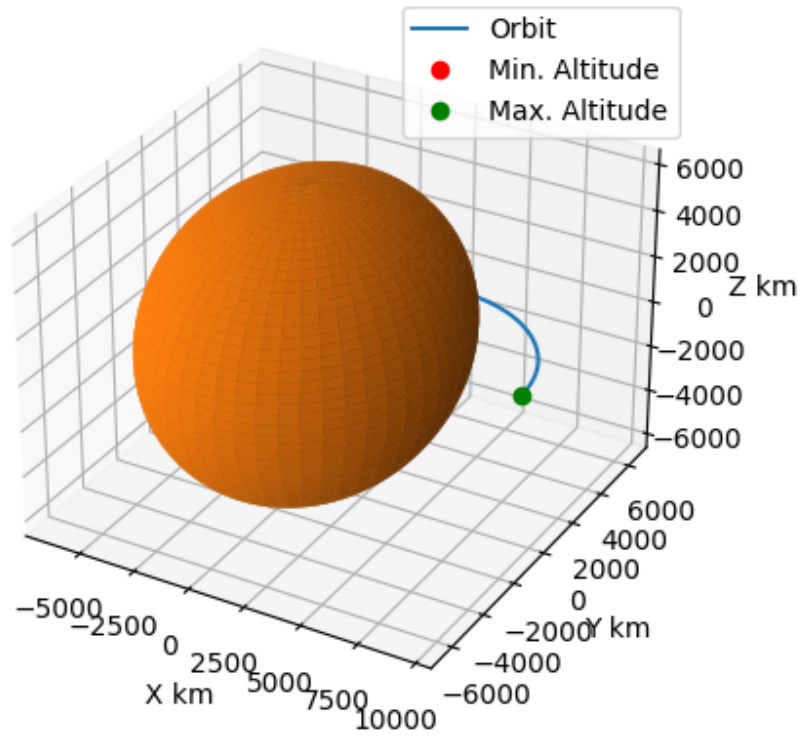


Figure 1: The position of the spacecraft in the GCRF coordinate system.

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