

Home Work #1

Ali BaniAsad 401209244

March 16, 2023

1 Question 1

$$h = 200_{km} \rightarrow r = R_e + h = 6378.137 + 200 = 6578.1137, \quad \mu = 3.986 \times 10^{14}_{m^3/s^2} = 3.986 \times 10^5_{km^3/s^2}$$

The orbit is circular.

1.1 Part a

$$T = 2\pi \sqrt{\frac{r^3}{\mu}} = 2\pi \sqrt{\frac{6578.1137^3}{3.986 \times 10^5}} = 5309.62_{sec} \quad (1)$$

$$T\omega = 2\pi \rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{5309.62} = 0.00118_{rad/sec} \quad (2)$$

1.2 Part b

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{6578.1137}} = 7.78_{km/sec} \quad (3)$$

The new velocity is calculated as:

$$v_{new} = v + 0.5_{km/sec} = 8.28_{km/sec} \quad (4)$$

Assume the new orbit is circular just changed altitude and has a new velocity.

$$v_{new} = \sqrt{\frac{\mu}{r_{new}}} \rightarrow r_{new} = \frac{\mu}{v_{new}^2} = 5808_{km} \quad (5)$$

$$T = 2\pi \sqrt{\frac{r_{new}^3}{\mu}} = 2\pi \sqrt{\frac{5808^3}{3.986 \times 10^5}} = 4405.08_{sec} \quad (6)$$

$$T_{new}\omega_{new} = 2\pi \rightarrow \omega_{new} = \frac{2\pi}{T_{new}} = \frac{2\pi}{4405.08} = 0.00143_{rad/sec} \quad (7)$$

r_{new} is smaller than the earth's radius.

2 Question 2

Assume \mathbf{r}_0 and \mathbf{v}_0 as:

$$\mathbf{r}_0 = [1600 \quad 5310 \quad 3800]_{km}^T, \quad \mathbf{v}_0 = [-7.350; \quad 0.4600 \quad 2.470]_{km/sec}^T$$

2.1 Part a

the n-body problem was solved with MATLAB with the n-body function in the question directory. The results are illustrated below.

Figure 1: 3D trajectory

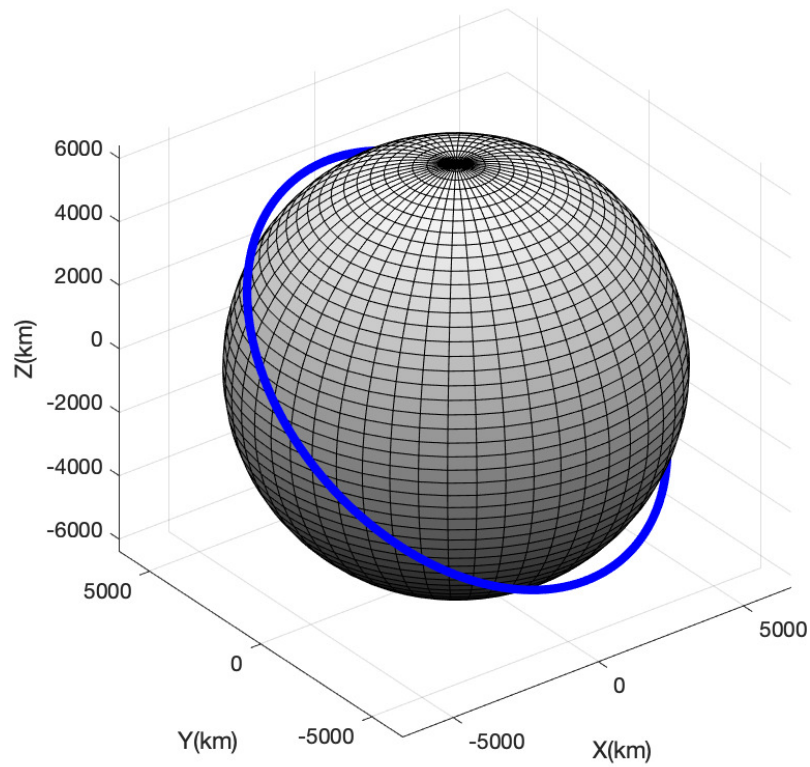


Figure 2: 3D trajectory in zx axis

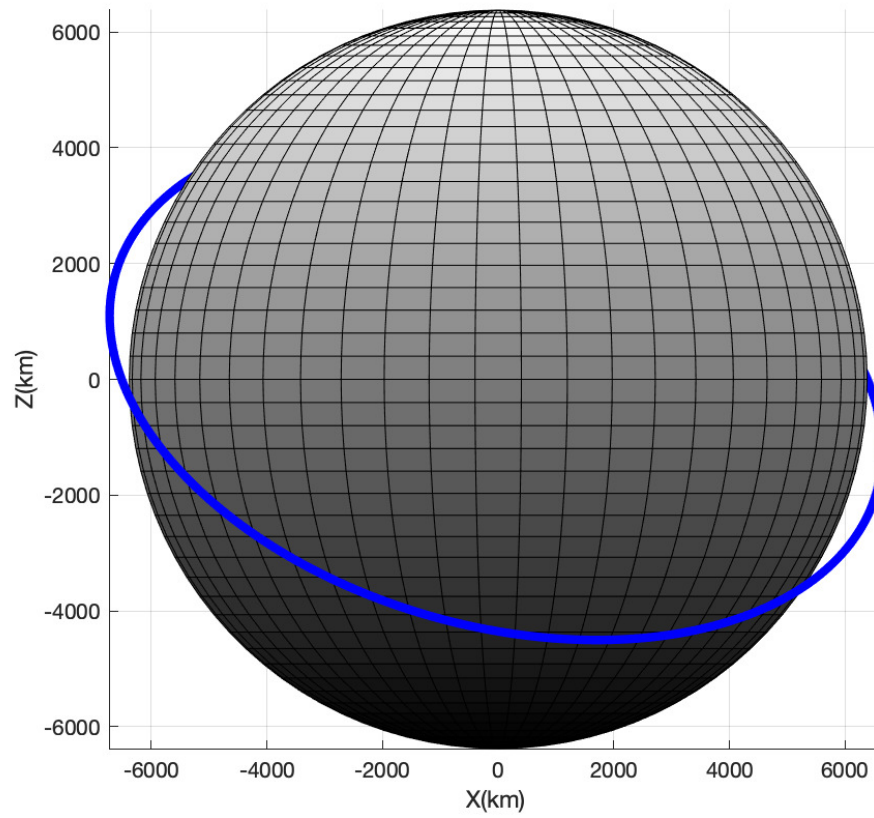


Figure 3: 3D trajectory in zy axis

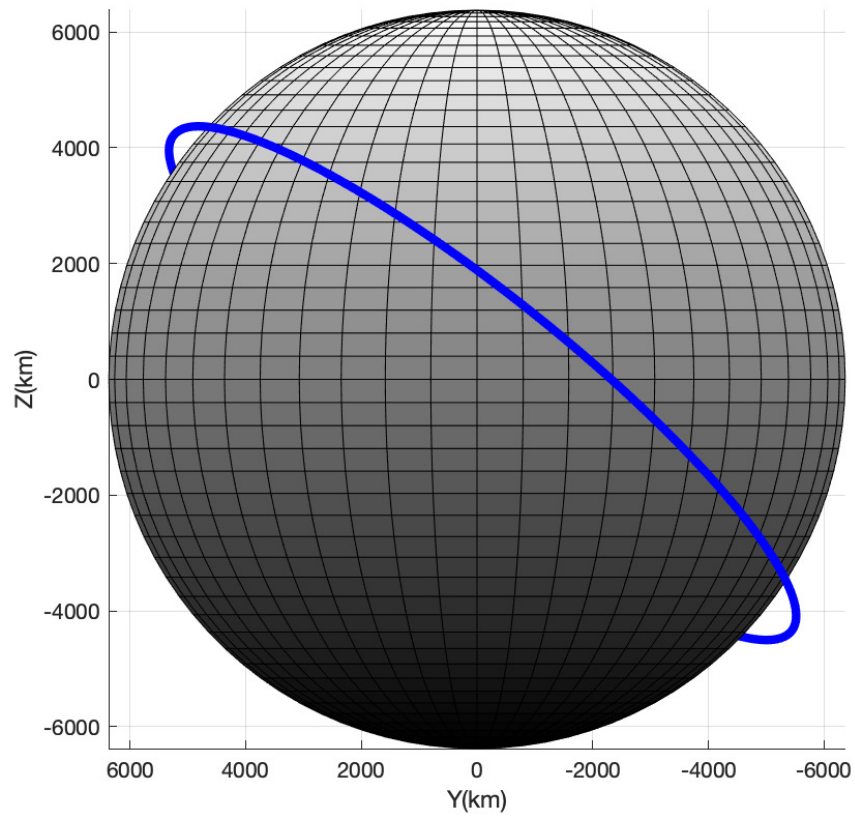
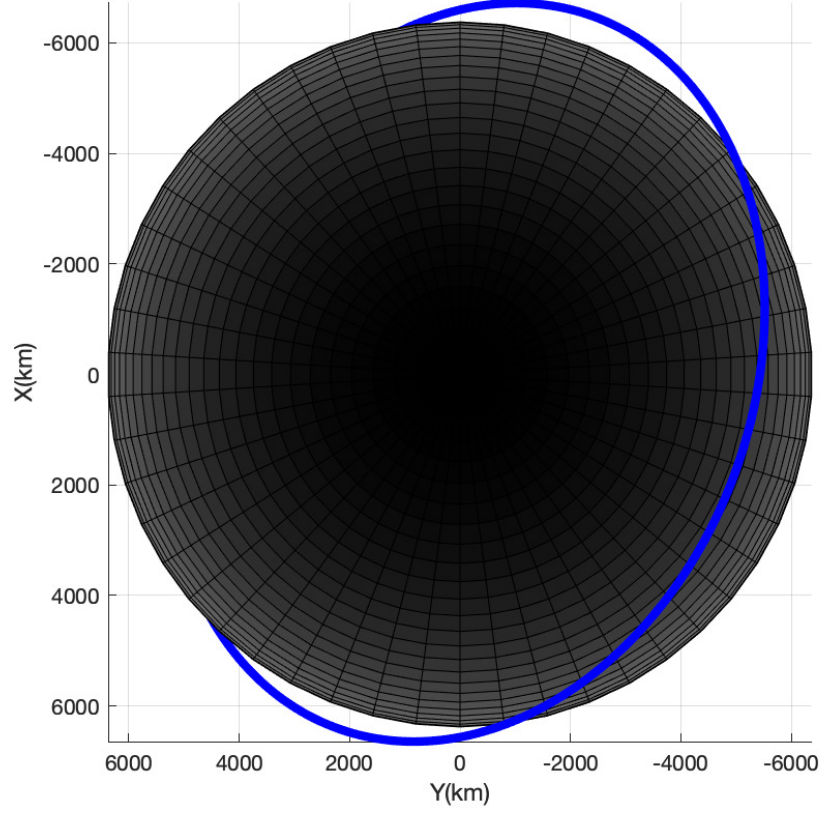


Figure 4: 3D trajectory in xy axis



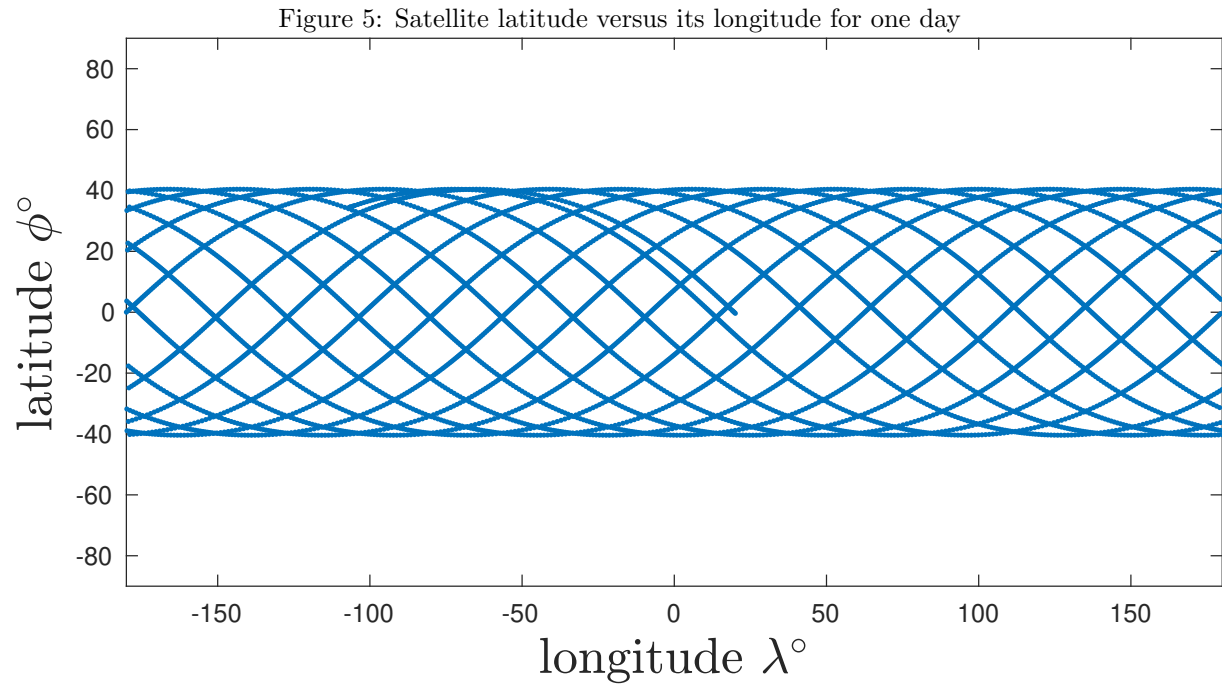
2.2 Part b

Using the below transfer matrix to transfer from ECI coordinate to the ECEF coordinate.

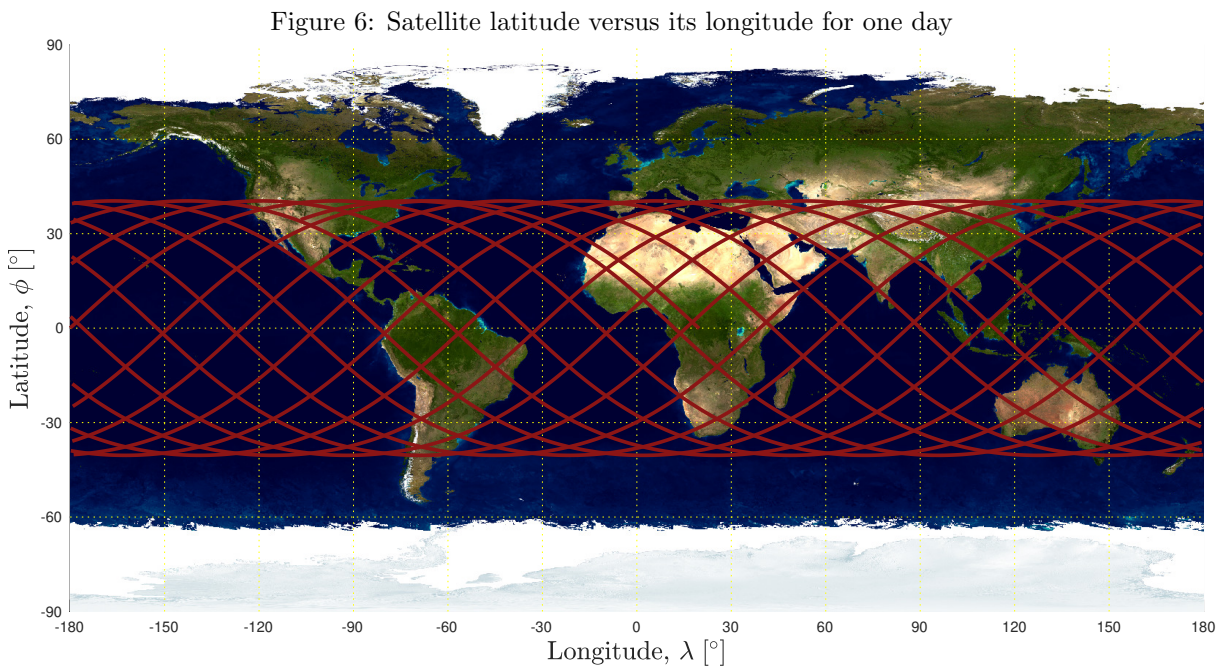
$$\mathbf{T}^{ECCF-ECI} = \begin{bmatrix} \cos(\omega_E t) & -\sin(\omega_E t) & 0 \\ \sin(\omega_E t) & \cos(\omega_E t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \arccos\left(\frac{\mathbf{r}(3)}{r}\right)$$

$$\lambda = \begin{cases} \arctan\left(\frac{\mathbf{r}(1)}{\mathbf{r}(2)}\right), & \mathbf{r}(2) > 0 \\ 2\pi - \arctan\left(\frac{\mathbf{r}(1)}{\mathbf{r}(2)}\right), & \mathbf{r}(2) \leq 0 \end{cases}$$



Below the figure drawn provided by tamaskis, please click [here](#) to see the source code. Please use the mentioned library to run code or skip the part on earth fig.



2.3 Part c (Bonus)

In this section find the orbital elements, then, change the inclination to 0.4_{rad} (using `oe2ecf` and `vec2orbElem` functions). Then, find \mathbf{r}_0 and \mathbf{v}_0 by new orbital elements. The results are illustrated below.

Figure 7: 3D trajectory compared to new inclination

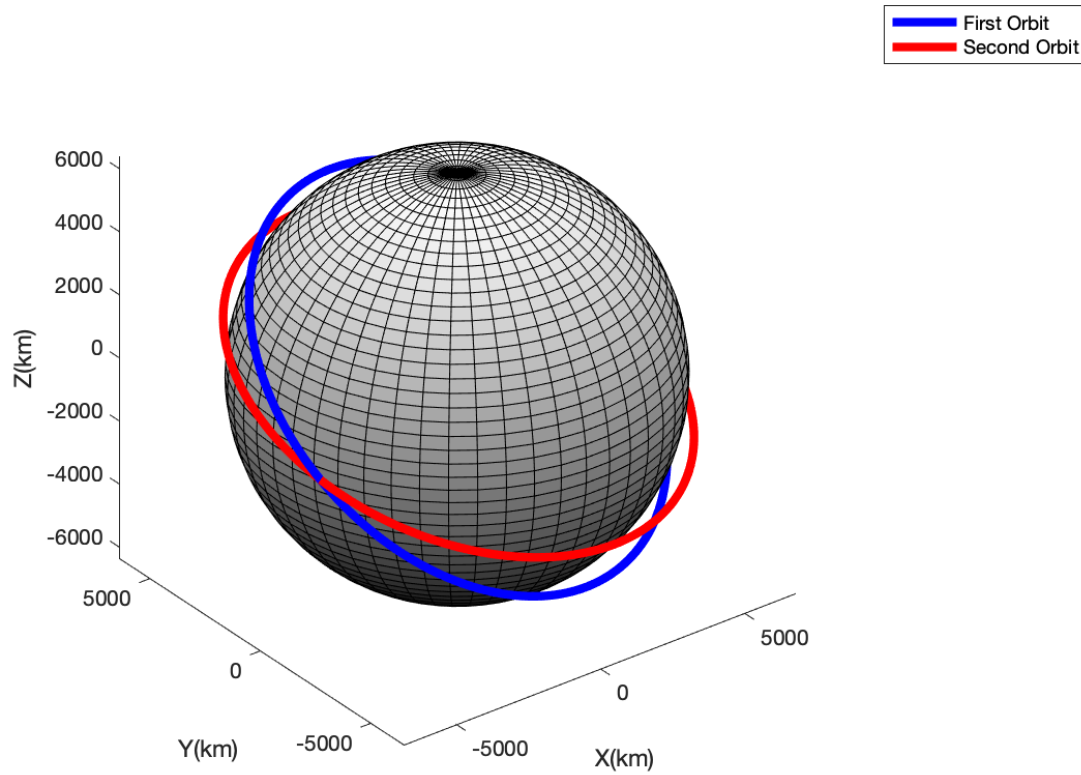


Figure 8: 3D trajectory in zx axis compared to new inclination

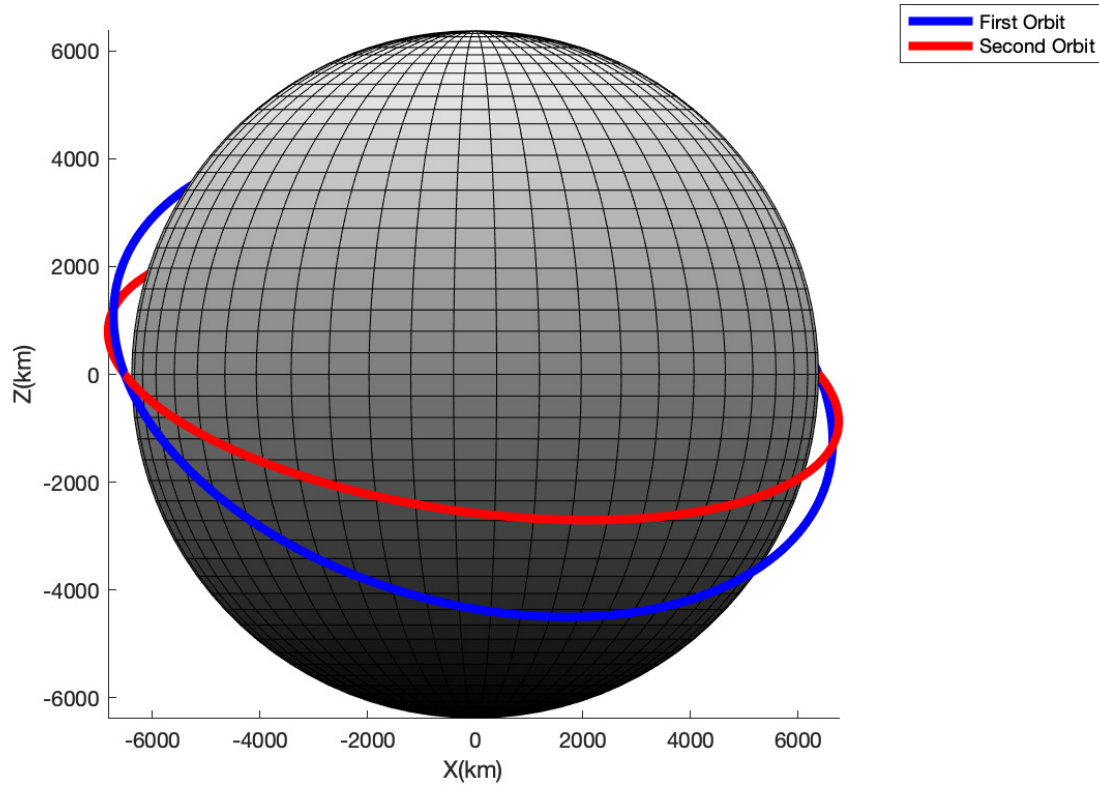


Figure 9: 3D trajectory in zy axis compared to new inclination

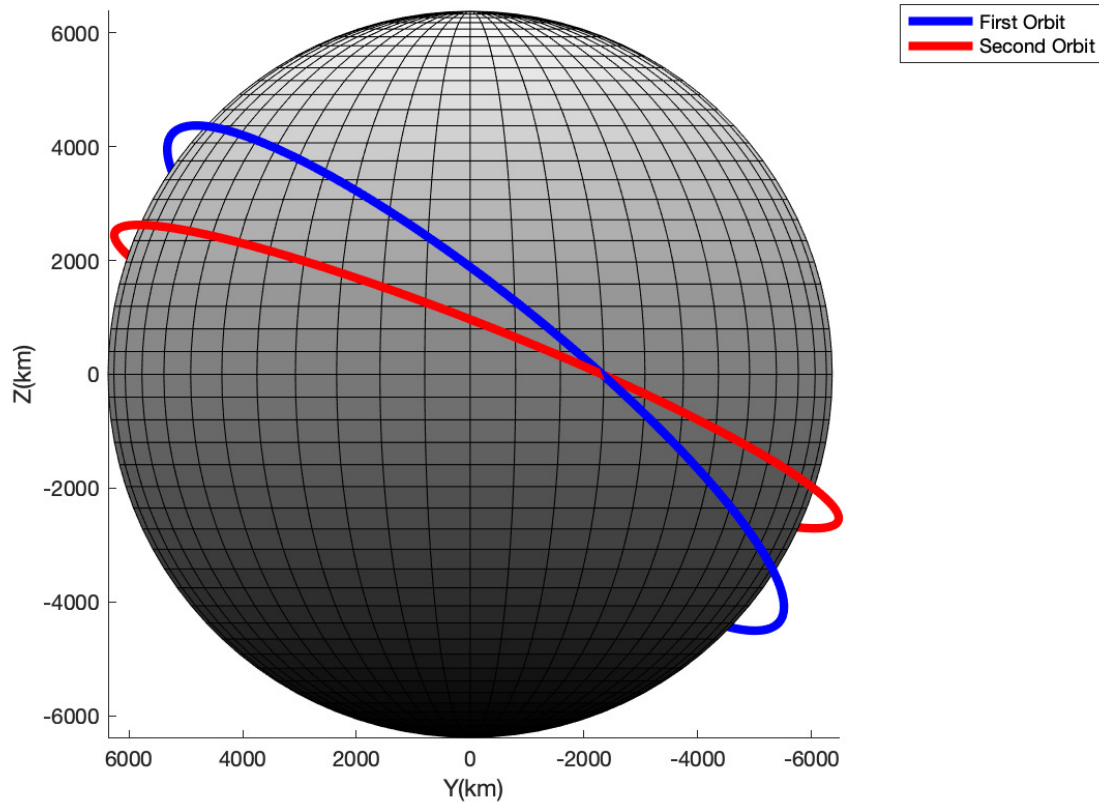
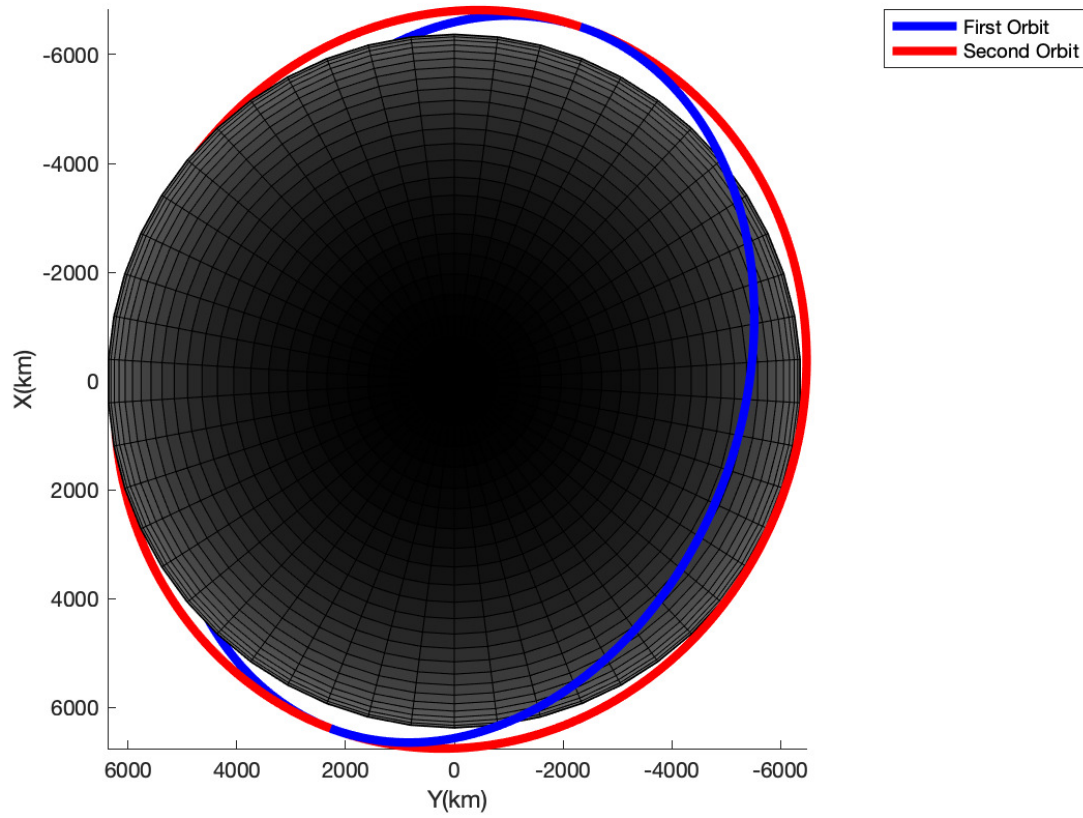
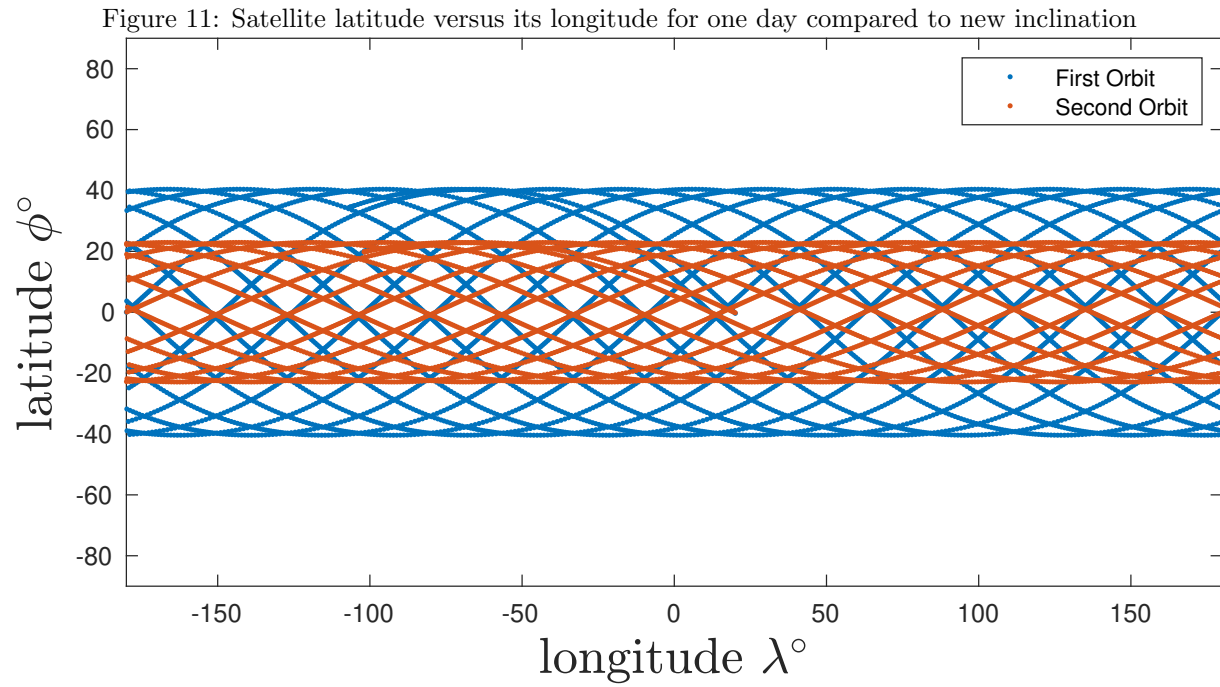


Figure 10: 3D trajectory in xy axis compared to new inclination





In general, changing the inclination of a satellite's orbit will cause its ground track to move to different latitudes and longitudes on the Earth's surface. In this example when the inclination changed from 0.7_{rad} to 0.4_{rad} , the range of longitude decreased.

Contents

1	Question 1	1
1.1	Part a	1
1.2	Part b	1
2	Question 2	1
2.1	Part a	2
2.2	Part b	5
2.3	Part c (Bonus)	7

List of Figures

1	3D trajectory	2
2	3D trajectory in zx axis	3
3	3D trajectory in zy axis	4
4	3D trajectory in xy axis	5
5	Satellite latitude versus its longitude for one day	6
6	Satellite latitude versus its longitude for one day	6
7	3D trajectory compared to new inclination	7
8	3D trajectory in zx axis compared to new inclination	8
9	3D trajectory in zy axis compared to new inclination	9
10	3D trajectory in xy axis compared to new inclination	10
11	Satellite latitude versus its longitude for one day compared to new inclination	11