Home Work #2

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1 Question 1

Spacecraft position in ITRF coordinates is given by

$$\mathbf{r} = \begin{bmatrix} 6789 & 6893 & 7035 \end{bmatrix}^{\mathrm{T}}_{km}$$

1.1 part a

Find Latitude and Longitude. For this purpose used algorithm 12 of Valado's book. This algorithm is implemented in the function 'latlon.py' in the 'code/Q1' folder. The function takes the spacecraft position vector as input and returns the latitude and longitude in degrees. The iteration ended when the difference is smaller than 1e-10. The results are:

Table 1: Results of part a

Variables	**Values**
Latitude	36.12°
Longitude	45.43°
h_{ellp}	5591.51_{km}

1.2 part b

In this part, we used the astropy package to find the position vector in the GCRF coordination system. The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. Position vector in GCRF:

$$\mathbf{r} = \begin{bmatrix} -862.54 & -9634.75 & 7037.25 \end{bmatrix}^{\mathrm{T}}_{km}$$

1.3 part c

In this part, we used the astropy package to find GMST(θ_{GMST}) and LST(θ_{LST}) The Python code for this can be found in the 'code/Q1' folder in the Jupyter Notebook file. The results are:

Table 2: Results of part c

GMST(θ_{GMST})	**LST(θ_{LST})**
112.78°	149.78°

2 Question 2

Satellite position and velocity vectors in the Earth-Centered Inertial (ECI) coordinate system:

$$\vec{r}_{ECI} = \begin{bmatrix}-346 & 8265 & 4680\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$

$$\vec{v}_{ECI} = \begin{bmatrix}-5.657 & -1.73 & 2.703\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

2.1 part a

Algorithm for converting from ECI to orbital x-y plane coordinates is described below:

1. Calculate the angular momentum vector $\vec{h}$ by taking the cross product of the position vector $\vec{r}$ and the velocity vector $\vec{v}$ in the ECI coordinate system.
2. Calculate the unit vector $\hat{z}$ in the direction of $\vec{h}$.
3. Calculate the unit vector $\hat{x}$ in the x-direction of the satellite position.
4. Calculate the unit vector $\hat{y}$ in the y-direction of the orbital plane by taking the cross product of $\hat{z}$ and $\hat{x}$.
5. Express the ECI position and velocity vectors $\vec{r}$ and $\vec{v}$ in the new coordinate system by taking their dot products with $\hat{x}$, $\hat{y}$, and $\hat{z}$.
6. Project the position and velocity vectors onto the x-y plane by setting the z-component of each vector to zero.

Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.
results:

$$r_{\text{x, y plane}} \begin{bmatrix}9504.3327488 & 0 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$

$$\vec{v}_{\text{x, y plane}} \begin{bmatrix}0 & 4.16567802 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

2.2 part b

To calculate satellite position after 30 minutes, the differential equation for the satellite solved for 30 minutes. The differential equation is:

$$\vec{r}_{\text{x, y plane}} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = -\frac{\mu}{r^3}\vec{r}$$

where μ is the gravitational parameter of the Earth, and r is the magnitude of the position vector $\vec{r}$. Note: Above algorithm is implemented in the jupyter notebook file Q2.ipynb.

results:

$$\vec{r}_{\text{x, y plane}} = \begin{bmatrix}1379.53 & 4493.87 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km}}$$

$$\vec{v}_{\text{x, y plane}} = \begin{bmatrix}-9.72 - 2.96 & 0\end{bmatrix}^{\mathrm{T}}_{\mathrm{km/\sec}}$$

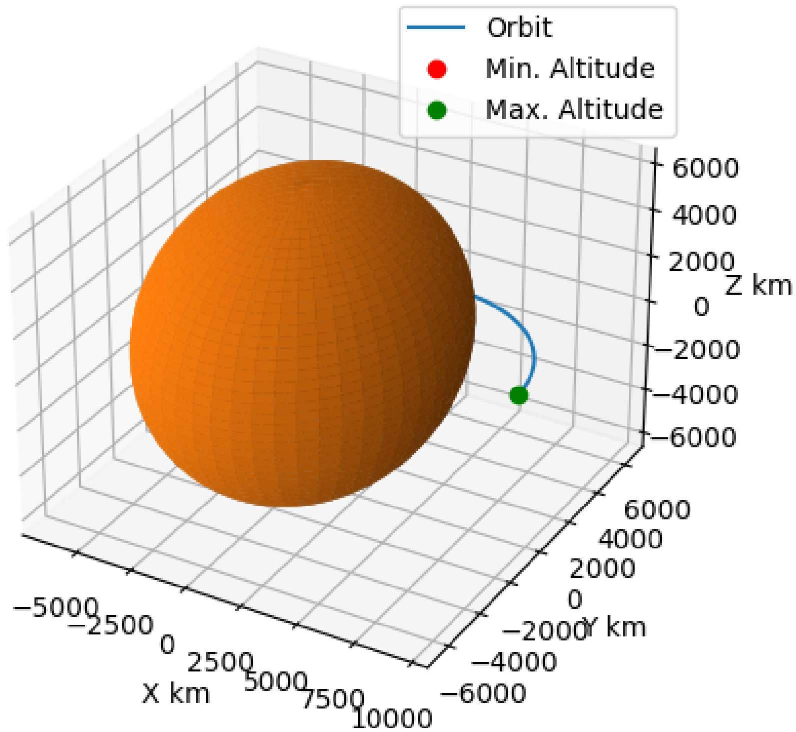

Figure 1: The position of the spacecraft in the GCRF coordinate system.

3 Question 3

The observation time is on August 1, 2023, at 15:00:00 UTC.

3.1 part a

In this to calculate Earth and Sun location used JPL Horizons On-Line Ephemeris System. The code used the "de440s" version which is the last and most accurate version of the DE series. The code used a short version because it doesn't need all the data time that was provided and the complete one has 3 gigabyte size. The code is in Q3.ipynb jupyter notebook.

3.2 part b

To check if the satellite has a clear view of the Sun, we need to calculate the angle between the Sun vector and the satellite vector. If the angle is less than the half-angle of the satellite's field of view (FOV), then the satellite has a clear view of the Sun. The calculation can be done as follows:

Find the position vector of the Sun in the ECI coordinate system, denoted as $\vec{r}_{Sun}$. This can be obtained from an ephemeris or by using an orbital model. Find the position vector of the satellite in the ECI coordinate system, denoted as $\vec{r}_{sat}$. Find the unit vector in the direction of the Sun from the satellite, denoted as $\hat{s}$: $\hat{s} = \frac{\vec{r}Sun - \vec{r}sat}{|\vec{r}Sun - \vec{r}sat|}$ Find the unit vector in the direction of the satellite from the Earth's center, denoted as $\hat{sat}$: $\hat{sat} = \frac{\vec{r}sat}{|\vec{r}sat|}$ Find the angle between $\hat{s}$ and $\hat{sat}$ using the dot product: $\cos\theta = \hat{s} \cdot \hat{sat}$ Compare the angle θ with the half-angle of the satellite's FOV, denoted as $\theta_{FOV}/2$. If $\theta \leq \theta_{FOV}/2$, then the satellite has a clear view of the Sun. The calculation is done using Python in Q3.ipynb jupyter notebook. The result is that satellite is in the umbra.

4 Question 4

ISS spacecraft observation orbital elements and Ground Station Location location is provided in 4, and 4.4, respectively. The orbital elements are used to calculate the position of the ISS spacecraft at the time of observation. The position is then used to calculate the line of sight vector from the ground station to the ISS spacecraft. The line of sight vector is then used to calculate the elevation and azimuth angles of the ISS spacecraft. The elevation and azimuth angles are then used to calculate spacecraft visibility.

Orbital Element	Value
Eccentricity	0.0005771
Inclination	51.6409°
Perigee Height	415km
Apogee Height	423km
RAAN	88.8414°
Argument of Perigee	75.2083°
True Anomaly	0

Table 3: ISS Observation

Latitude	Longitude
34°13'28.9"N	118°03'26.3"W

Table 4: Ground Station Location

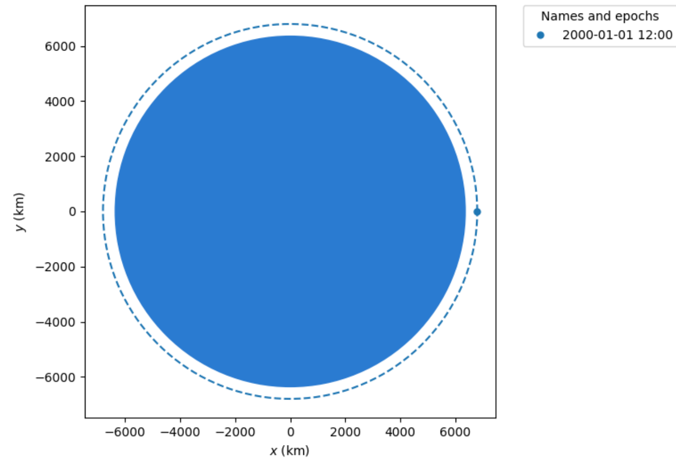

Figure 2: ISS spacecraft orbit.

4.1 part a

The spacecraft differential equations were solved for 48 hours to find the position of the ISS spacecraft at the time of observation in GCRF. The result is shown in 3.

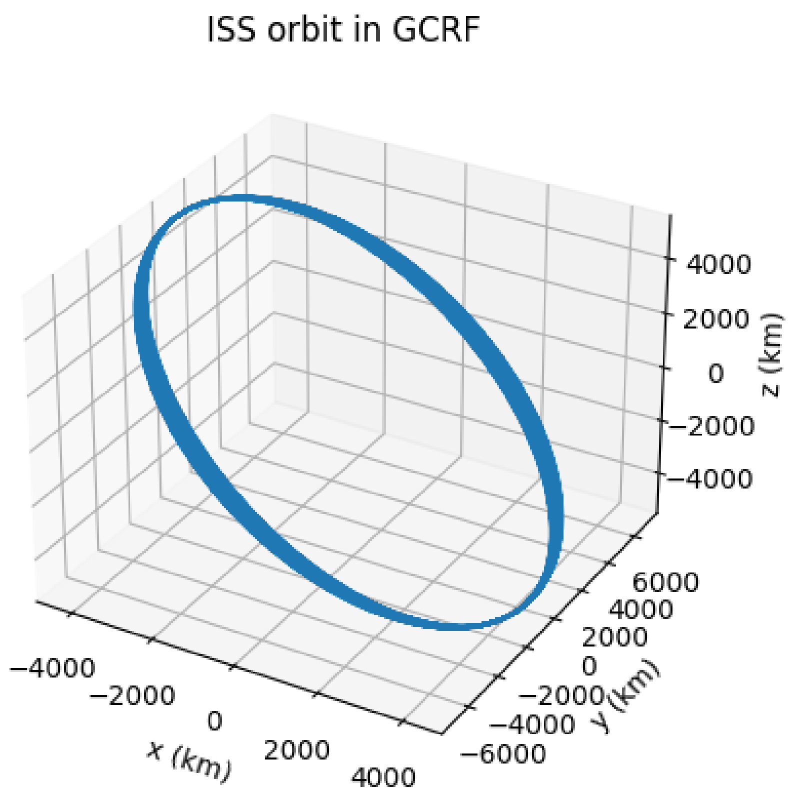

Figure 3: ISS spacecraft orbit in GCRF.

4.2 part b

The position of the ISS spacecraft at the time of observation in GCRF is then converted to ITRF with function in Q4.ipynb Jupyter notebook. The result is shown in 4.

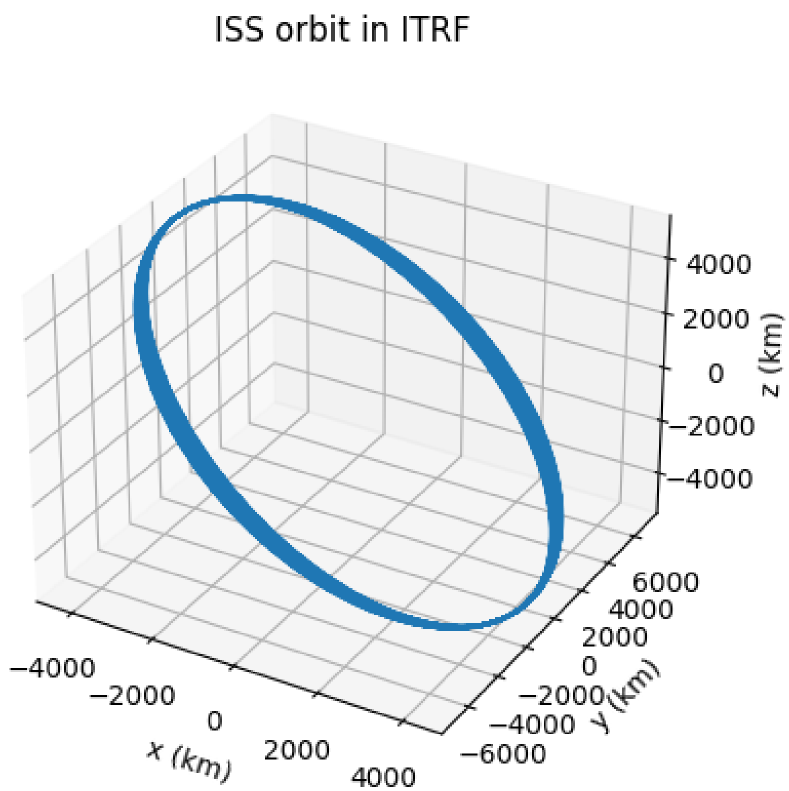

Figure 4: ISS spacecraft orbit in ITRF.

4.3 part c

In this part of the homework, we used the Astropy coordinate library to calculate the line of sight vector from a ground station to the International Space Station (ISS) spacecraft. To determine if the spacecraft is visible, we calculated the attitude and azimuth and checked if the elevation angle is greater than 10°. The implementation of this algorithm can be found in the Q4.ipynb Jupyter Notebook.

Our calculations showed that the spacecraft is visible for a total of 169,351 seconds, which is equivalent to 48 hours. We used a step time of 1 second in our calculations.

4.4 part d

In this section, we implemented the above algorithm to calculate the visibility of the International Space Station (ISS) for all latitudes from $-90°$ to $90°$ and longitudes from $-180°$ to $180°$. After performing the calculations, we found that the location with the most observation time is:

[Insert coordinates and location information here] This information can be useful for planning future observations of the ISS. We performed these calculations using a step time of 1 minute and a step size of 30 degrees for both latitudes and longitudes. This level of granularity allowed us to obtain accurate results while minimizing computational time. The best location for the ground station is at latitude $-30.00°$ degrees and longitude -60.00° degrees, with a total view time of 47.15 hours over the 48 hours.

Latitude	Longitude
-30°N	-60°W

Table 5: Ground Station Location

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