

$$P_0(1,2), P_1(3,5), P_2(6,2), P_3(9,4)$$

جواب:  
(الف)

$$n = 3, P = 2, P = m - n - 1 \rightarrow m = 6$$

$$U = \{0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, 1\}$$

$$u_0 = 0, u_1 = \frac{1}{6}, u_2 = \frac{2}{6}, u_3 = \frac{3}{6}, u_4 = \frac{4}{6}, u_5 = \frac{5}{6}, u_6 = 1$$

$$N_{0,0} = \begin{cases} 1 & u_0 \leq u < u_1 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{1,0} = \begin{cases} 1 & u_1 \leq u < u_2 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{2,0} = \begin{cases} 1 & u_2 \leq u < u_3 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{3,0} = \begin{cases} 1 & u_3 \leq u < u_4 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{4,0} = \begin{cases} 1 & u_4 \leq u < u_5 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{5,0} = \begin{cases} 1 & u_5 \leq u < u_6 \\ 0 & \text{Otherwise} \end{cases}$$

$$N_{(i,P)}(u) = \frac{u - u_i}{u_{i+P} - u_i} N_{i,P-1}(u) + \frac{u_{i+P+1} - u}{u_{i+P+1} - u_{i+1}} N_{i+1,P-1}(u)$$

$$N_{0,1} = \begin{cases} 6u & u_0 \leq u < u_1 \\ 2 - 6u & u_1 \leq u < u_2 \end{cases}$$

$$N_{1,1} = \begin{cases} 6u - 1 & u_1 \leq u < u_2 \\ 3 - 6u & u_2 \leq u < u_3 \end{cases}$$

$$N_{2,1} = \begin{cases} 6u - 2 & u_2 \leq u < u_3 \\ 4 - 6u & u_3 \leq u < u_4 \end{cases}$$

$$N_{3,1} = \begin{cases} 6u - 3 & u_3 \leq u < u_4 \\ 5 - 6u & u_4 \leq u < u_5 \end{cases}$$

$$N_{4,1} = \begin{cases} 6u - 4 & u_4 \leq u < u_5 \\ 6(1 - u) & u_5 \leq u < u_6 \end{cases}$$

$$N_{0,2} = 3uN_{0,1} + 3\left(\frac{3}{6} - u\right)N_{1,1}$$

$$N_{0,2} = \begin{cases} 18u^2 & u_0 \leq u < u_1 \\ -36u^2 + 18u - 1.5 & u_1 \leq u < u_2 \\ 18u^2 - 18u + 4.5 & u_2 \leq u < u_3 \end{cases}$$

$$N_{1,2} = (3u - 0.5)N_{1,1} + (2 - 3u)N_{2,1}$$

$$N_{1,2} = \begin{cases} 18u^2 - 6u + 0.5 & u_1 \leq u < u_2 \\ -36u^2 + 30u - 5.5 & u_2 \leq u < u_3 \\ 18u^2 - 24u + 8 & u_3 \leq u < u_4 \end{cases}$$

$$N_{2,2} = (3u - 1)N_{2,1} + (2.5 - 3u)N_{3,1}$$

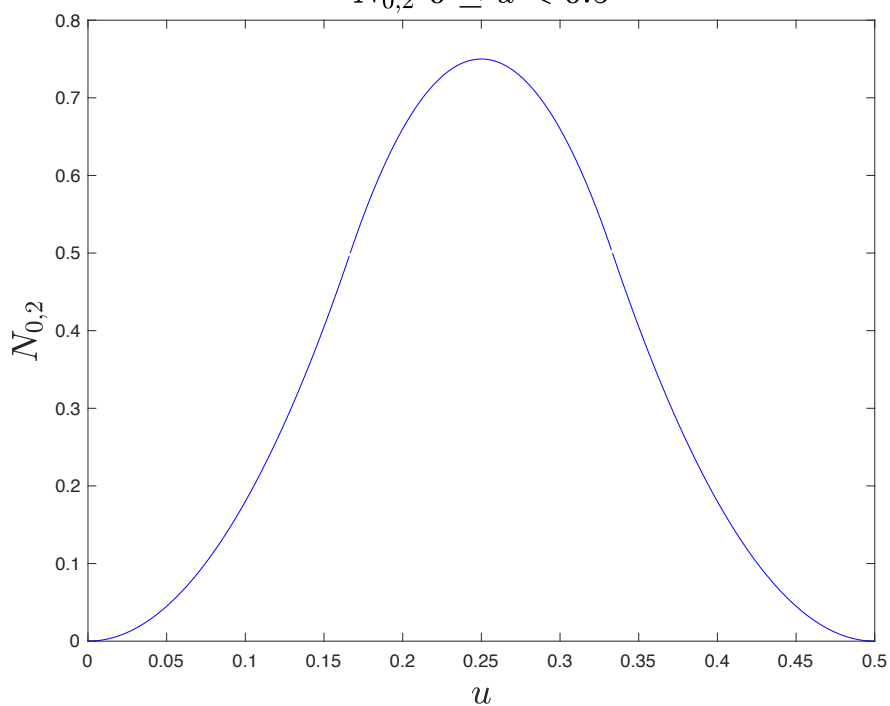
$$N_{2,2} = \begin{cases} 18u^2 - 12u + 2 & u_2 \leq u < u_3 \\ -36u^2 + 42u - 11.5 & u_3 \leq u < u_4 \\ 18u^2 - 30u + 12.5 & u_4 \leq u < u_5 \end{cases}$$

$$N_{3,2} = (3u - 1.5)N_{3,1} + (3 - 3u)N_{4,1}$$

$$N_{3,2} = \begin{cases} 18u^2 - 18u + 4.5 & u_3 \leq u < u_4 \\ -36u^2 + 54u - 19.5 & u_4 \leq u < u_5 \\ 18u^2 - 36u + 18 & u_3 \leq u < u_5 \end{cases}$$

Figure 1

$$N_{0,2} \quad 0 \leq u < 0.5$$



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Figure 2

$$N_{1,2} \quad 1/6 \leq u < 4/6$$

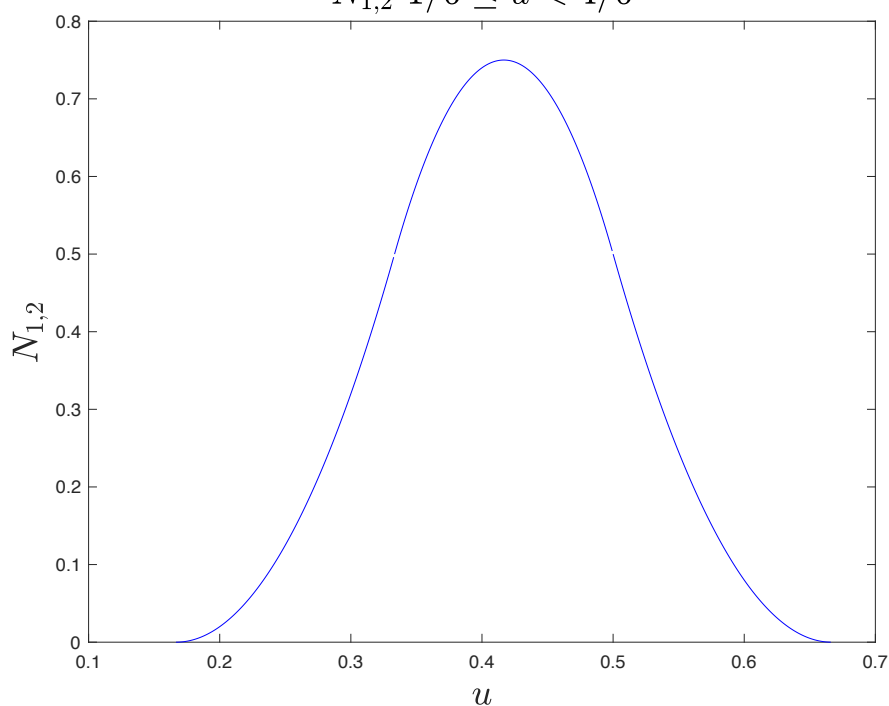


Figure 3

$$N_{2,2} \quad 2/6 \leq u < 5/6$$

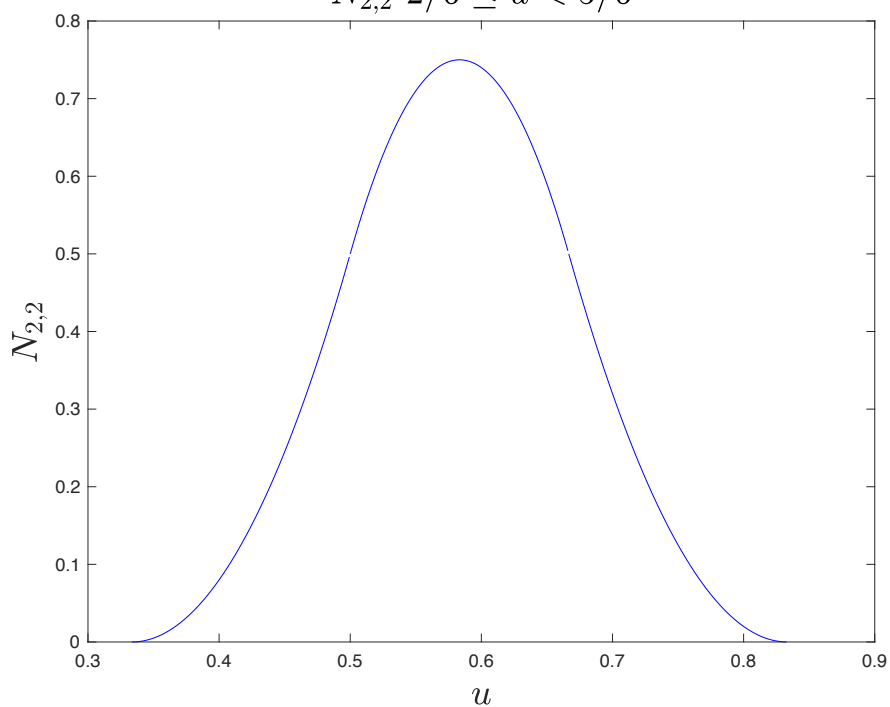
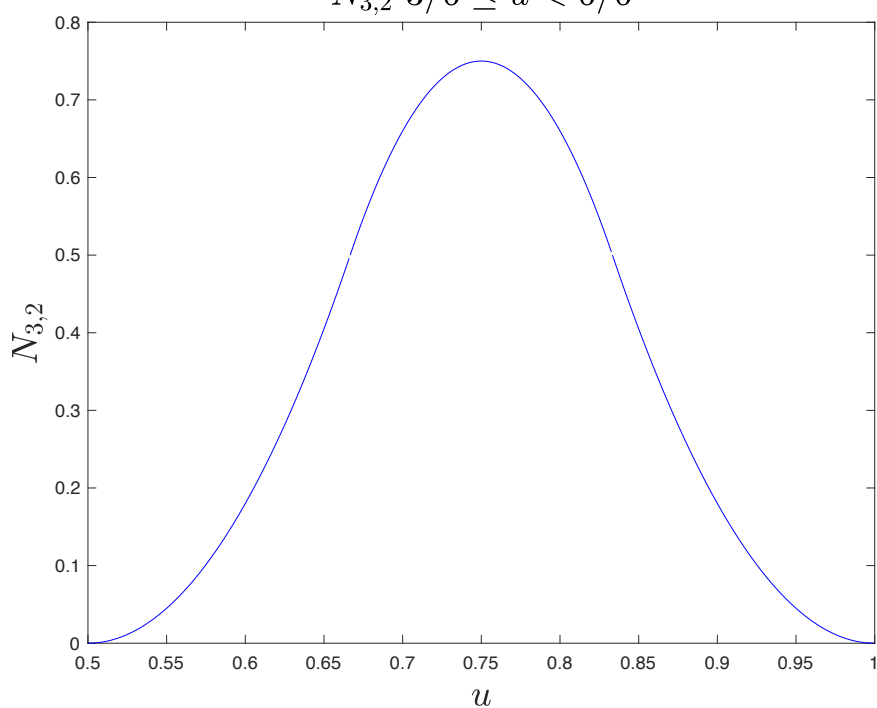


Figure 4

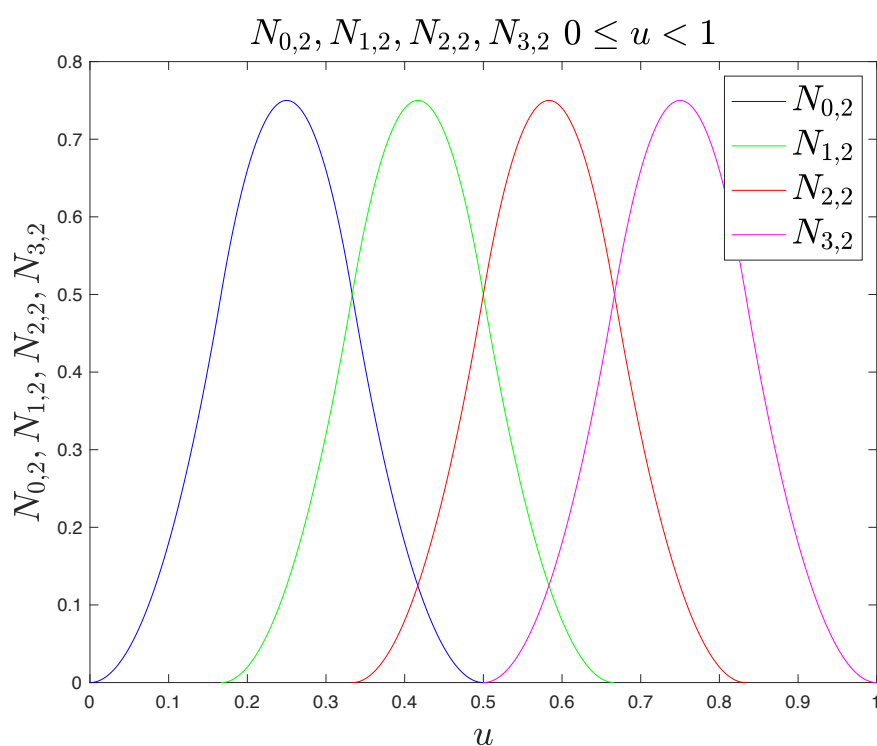
$$N_{3,2} \quad 3/6 \leq u < 6/6$$





All in one figure:

Figure 5



(ت)

این منحنی در بازه گره ای بین  $[u_p, u_{m-p}]$  تعریف می شود.

$$[u_p, u_{m-p}] = [u_2, u_4] = \left[\frac{2}{6}, \frac{4}{6}\right]$$

(ث)

$$C(u) = \sum_{i=0}^n N_{i,p}(u)P_i, \quad u_0 \leq u \leq u_m$$

$$C(u)_x = \begin{cases} 18u^2 & u_0 \leq u < u_1 \\ 18u^2 & u_1 \leq u < u_2 \\ 18u^2 & u_2 \leq u < u_3 \\ 54u^2 + 18u - 4.5 & u_3 \leq u < u_4 \\ -216u^2 + 306u - 250.5 & u_4 \leq u < u_5 \\ 162u^2 - 324u + 162 & u_5 \leq u < u_6 \end{cases}$$

$$C(u)_y = \begin{cases} 36u^2 & u_0 \leq u < u_1 \\ 18u^2 + 6u - 0.5 & u_1 \leq u < u_2 \\ -108u^2 + 90u - 14.5 & u_2 \leq u < u_3 \\ 90u^2 - 108u - 35 & u_3 \leq u < u_4 \\ -108u^2 + 156u - 53 & u_4 \leq u < u_5 \\ 72u^2 - 144u + 72 & u_5 \leq u < u_6 \end{cases}$$

Figure 6

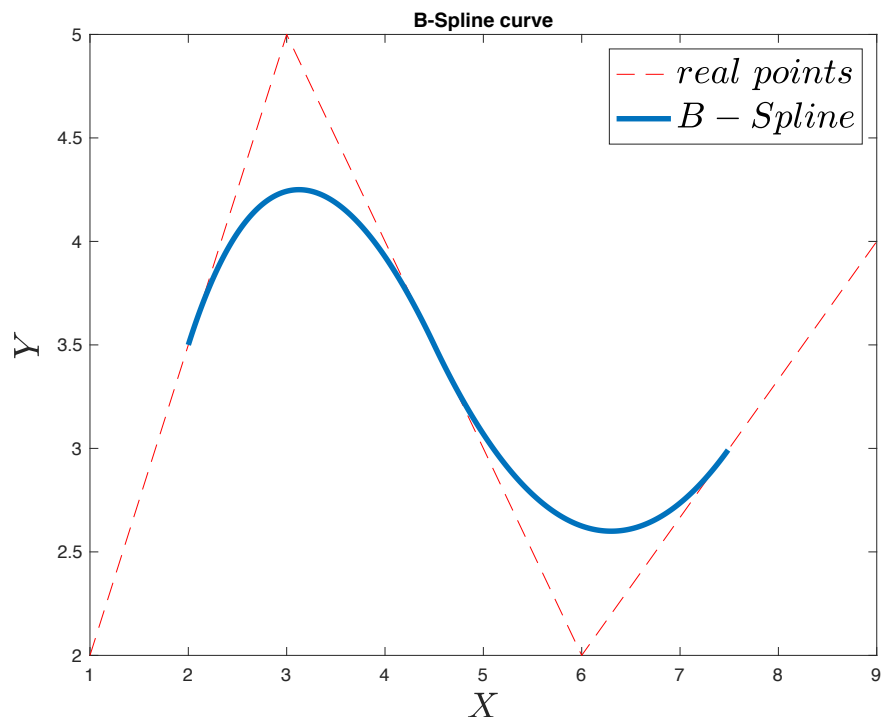
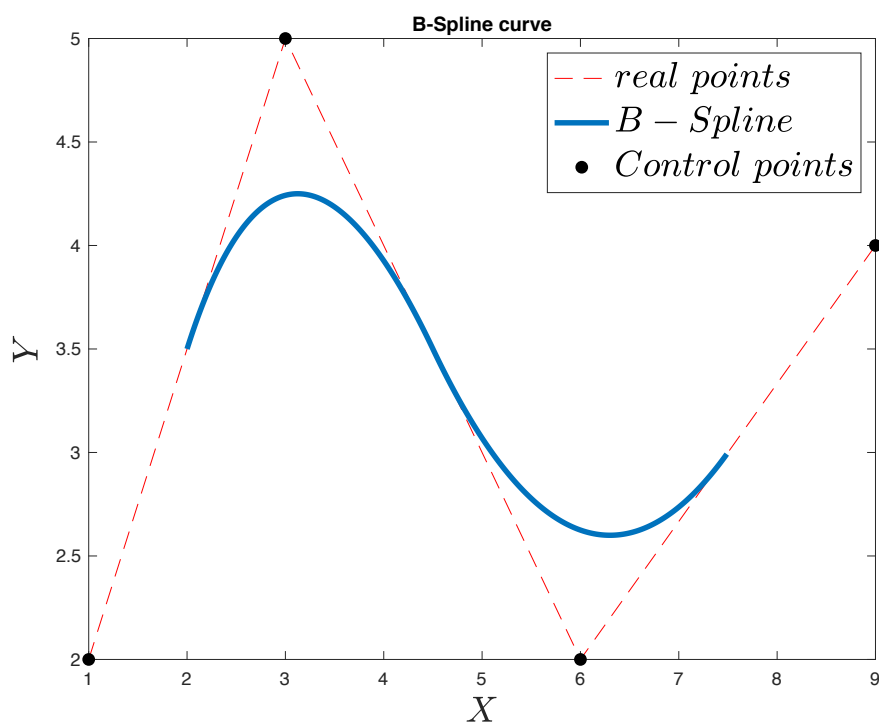


Figure 7





## طراحی به کمک کامپیوتر - نیمسال دوم ۹۹-۱۳۹۸

شماره دانشجویی: ۹۶۱۰۸۳۷۸

نام و نام خانوادگی: علی بنی اسد

عنوان تمرین: B-Spline

پ.ن  
کد های متلب سوال در سایت درس اپلود شده است و کد ها را و فایل را می توانید در [GitHub](https://github.com) ببینید