Home Work #2

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1 Question

Data:

$$\bar{x}_{ac} - \bar{x}_{cg} = 0.05, \quad C_{m_{ac_{wb}}} = -0.016, \quad C_{L_{wb}} = 0.45$$
$$C_m = C_{m_{ac_{wb}}} + C_{L_{wb}}(\bar{x}_{ac} - \bar{x}_{cg}) \tag{1}$$

From equation 1 we have:

$$C_m = -0.016 + 0.45(0.05) = 0.0065 \rightarrow C_m = 0.0065$$

2 Question

2.1 part

We assume that $C_{L_{wb}}$ have linear behavior between -1.5° and 8°.

At (-1.5) degree $C_{L_{wb}}$ is 0 and at 5 degree $C_{L_{wb}}$ is 0.52.

$$\frac{\Delta C_{L_{wb}}}{\Delta \alpha} = C_{L_{\alpha_{wb}}} \rightarrow C_{L_{\alpha_{wb}}} = \frac{0.52 - 0.0}{5 - (-1.5)} = 0.08$$
$$C_{L_{0_{wb}}} = 0.08 \times 1.5 = 0.12$$
$$C_{L_{wb}} = C_{L_{0_{wb}}} + C_{L_{\alpha_{wb}}}\alpha = 0.12 + 0.08\alpha \tag{2}$$

From equation 2 at 1 degree angle of attack $C_{L_{wb}} = 0.2$ and at 7.88 degree angle of attack $C_{L_{wb}} = 0.75$.

$$C_m = C_{m_0} + C_{m_\alpha}\alpha \tag{3}$$
$$C_{m_0} = C_{m_{ac_{wb}}} + C_{L_{0_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}}) \tag{4}$$
$$C_{m_\alpha} = C_{L_{\alpha_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}}) \tag{5}$$

$$C_{m_{wb}} = (C_{m_{ac_{wb}}} + C_{L_{0_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}})) + C_{L_{\alpha_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}}) \tag{6}$$

From equation 6:

1. at 1 degree angle of attack:
$$-0.01 = C_{m_{ac_{wb}}} + 0.2(0.35 - \bar{x}_{ac_{wb}})$$
2. at 7.88 degree angle of attack:
$$0.05 = C_{m_{ac_{wb}}} + 0.75(0.35 - \bar{x}_{ac_{wb}})$$

There is 2 equations and 2 unknowns.

Answers:

$$x_{ac_{wb}} = 0.24c, \quad C_{m_{ac_{wb}}} = -0.032$$

2.2 part

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{i_H}} i_H \tag{7}$$

Now define every parameter in equation 7.

$$C_{m_0} = C_{m_{ac_{wb}}} + C_{L_{0_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}}) \tag{8}$$

$$C_{m_\alpha} = C_{L_{\alpha_{wb}}}(\bar{x}_{cg} - \bar{x}_{ac_{wb}}) - C_{L_{\alpha_H}}\eta_H \frac{S_H}{S}(1 - \frac{\partial\varepsilon}{\partial\alpha})(\bar{x}_{ac_H} - \bar{x}_{cg}) \tag{9}$$

$$C_{m_{i_H}} = -C_{L_{\alpha_H}}\eta_H \frac{S_H}{S}(\bar{x}_{ac_H} - \bar{x}_{cg}) \tag{10}$$

Data:

$$\alpha = 7.88^\circ, \quad i_H = 2.7^\circ, \quad C_{m_{ac_{wb}}} = -0.032, \quad C_{L_{\alpha_{wb}}} = 0.08_{1/\deg}, \quad C_{L_{0_{wb}}} = 0.12, \quad \bar{x}_{cg} = 0.35, \quad \bar{x}_{ac} = 0.24$$

$$C_{L_{\alpha_H}} = 0.1_{1/\deg}, \quad S_H = 0.02_{m^2}, \quad S = 0.1_{m^2}, \quad \varepsilon_0 = 0, \quad \frac{\partial\varepsilon}{\partial\alpha} = 0.35, \quad \bar{x}_{ac_H} - \bar{x}_{cg} = 1.7$$

We assume that $\eta_H = 1$.

$$C_{m_0} = -0.032 + 0.12(0.35 - 0.24) = -0.0188$$

$$C_{m_\alpha} = 0.08(0.35 - 0.24) - 0.1\frac{0.02}{0.1}(1 - 0.35)(1.7) = -0.0133$$

$$C_{m_{i_H}} = -0.1\frac{0.02}{0.1}(1.7) = -0.0340$$

$$C_m = -0.0188 - 0.0133 \times 7.88 - 0.0340 \times 2.7 = -0.2154$$

$$C_{m_{cg}} = -0.2154$$

2.3 part

From equation 9 we know that $C_{m_\alpha} < 0$ so airplane has static stability.

From equation 8 we know that $C_{m_0} \neq 0$ so airplane is not balance.

3 Question

$$C_{l_\beta} = C_{l_{\beta_W}} + C_{l_{\beta_H}} = 0$$
$$C_{L_\beta} = -\frac{\Gamma}{2} C_{L_\alpha} \tag{11}$$

From equation 11 we calculate C_{L_β} for wing and horizontal tail.

1. Wing:
$$C_{L_{\beta_W}} = -\frac{\Gamma_W}{2} C_{L_{\alpha_W}}$$

2. Horizontal tail:
$$C_{L_{\beta_H}} = \bar{C}_{L_{\beta_H}} \eta_H \frac{S_H}{S_W} \frac{b_H}{b_W}$$

From problem we know $AR_W = AR_H$ and $S_W = 4S_H$, so $b_W = 2b_H$.

We assume $\eta_H = 1$.

$$C_{L_{\beta_H}} = \frac{\bar{C}_{L_{\beta_H}}}{8}$$
$$C_{l_\beta} = C_{l_{\beta_W}} + C_{l_{\beta_H}} = -\frac{\Gamma_W}{2} C_{L_{\alpha_W}} - \frac{\Gamma_H}{2} \frac{C_{l_{\alpha_H}}}{8} \xrightarrow{C_{l_{\alpha_H}} = C_{l_{\alpha_W}}} \Gamma_H = -8\Gamma_W$$
$$\Gamma_H = -8\Gamma_W \tag{12}$$

From equation 12 we calculate horizontal tail dihedral.

$$\Gamma_H = -8 \times 3 = -24^\circ$$

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