In the Name of GOD

Guidance and Navigation I: Three-point Guidance Laws

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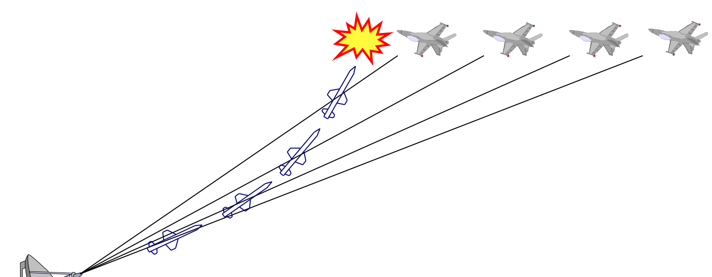
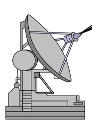
Tactical Guidance Laws

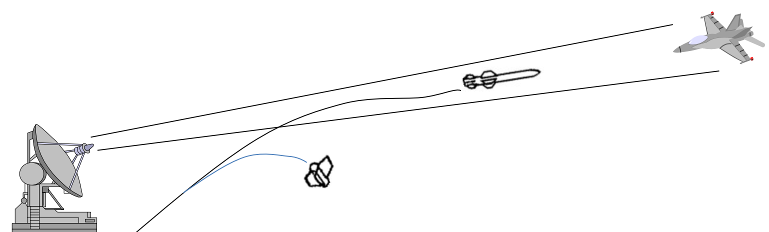
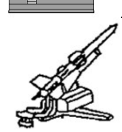
- Three-point Guidance
  - Basic Line-of-Sight Guidance
  - Command to Line-of-Sight Guidance
  - Optimal LOS guidance
  - Implementation Issues
  - CLOS Guidance with Lead Angle
  - Three-dimensional Implementation of CLOS Guidance

LOS Guidance Law

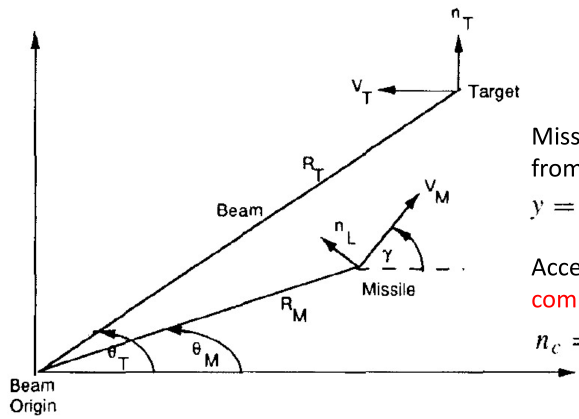
- GV intercept the target if it keeps the LOS between the external tracker and the target

Applications of LOS Guidance Law

- LOS Guidance Laws can be implemented in Beam Rider and Command systems
- Guidance commands are functions of the deviations from the beam centerline.
- Capture, controllability, aiming

Basic LOS guidance



Missile Distance from the LOS:

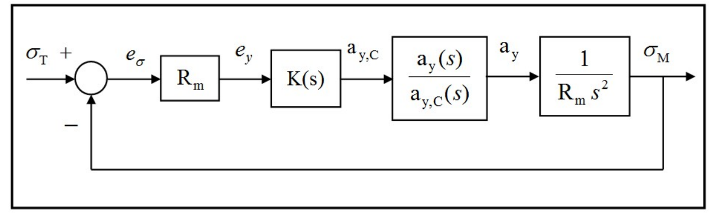
$$y = R_M(\theta_T - \theta_M)$$

Acceleration command:

$$n_c = K\, y$$

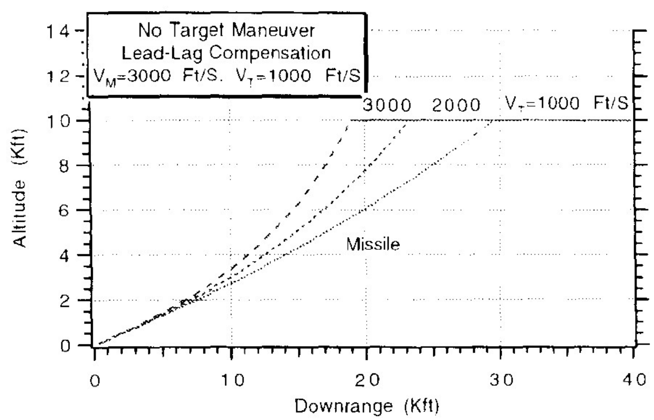
Block Diagram of Basic LOS Guidance

- The LOS guidance can be formulated as a linear regulator problem and K(s) can be designed based on the linear control theory.

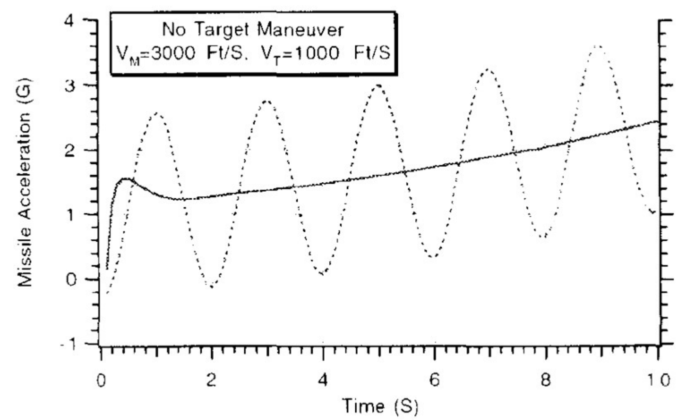


Simulink Model

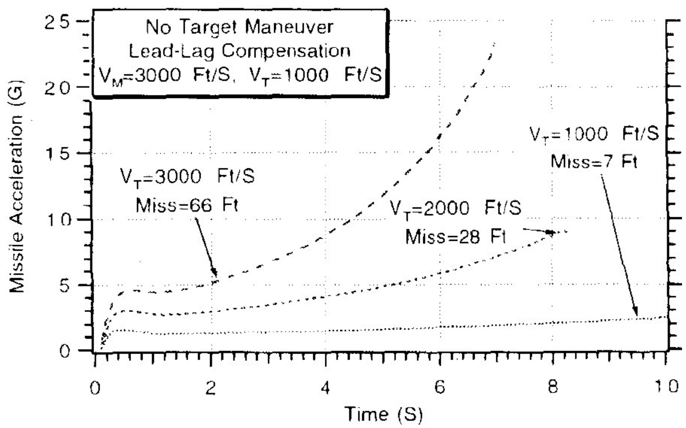
Basic LOS Guidance: Approaching Target: Target Velocity



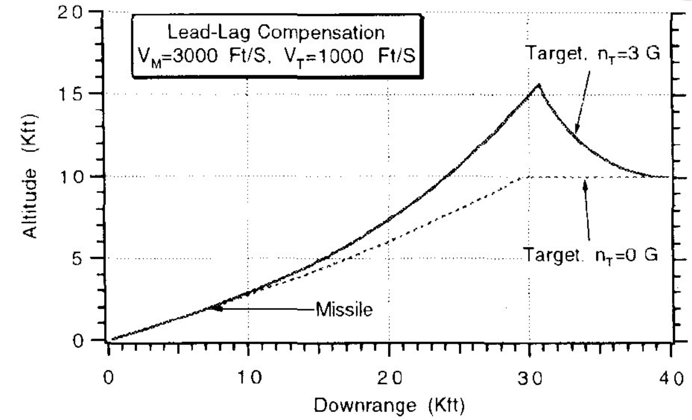
Basic LOS Guidance: Approaching Target: Lead Compensation



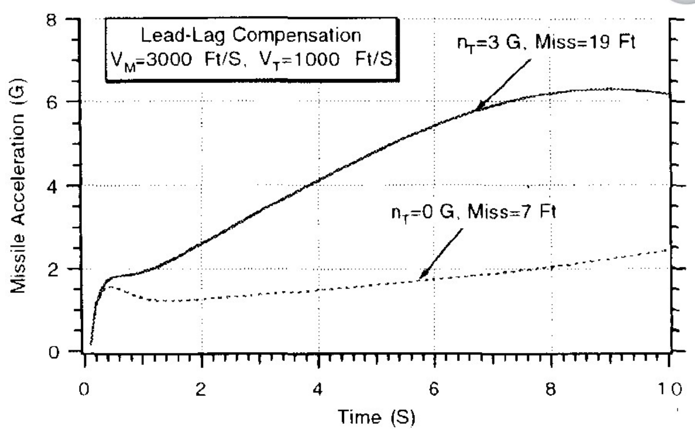
Basic LOS Guidance: Approaching Target: Acceleration Commands



Basic LOS Guidance: Maneuvering Target: Engagement Trajectory

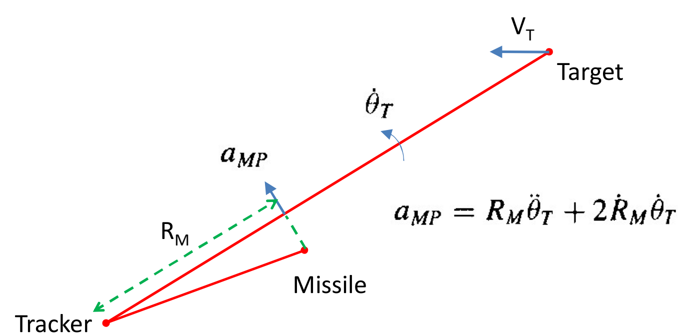


Basic LOS Guidance: Maneuvering Target: Acceleration Commands



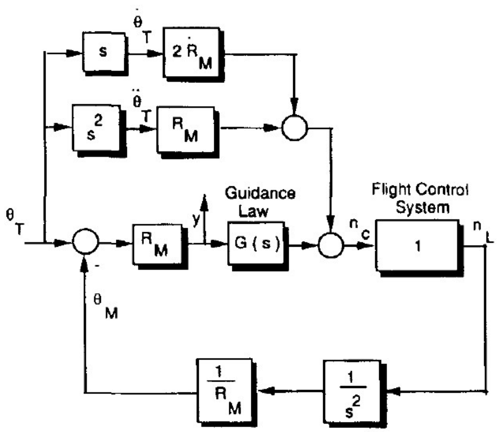
Command to Line-of-Sight (CLOS) Guidance

- Performance of the basic LOS guidance can be significantly improved by taking the beam motion into account.

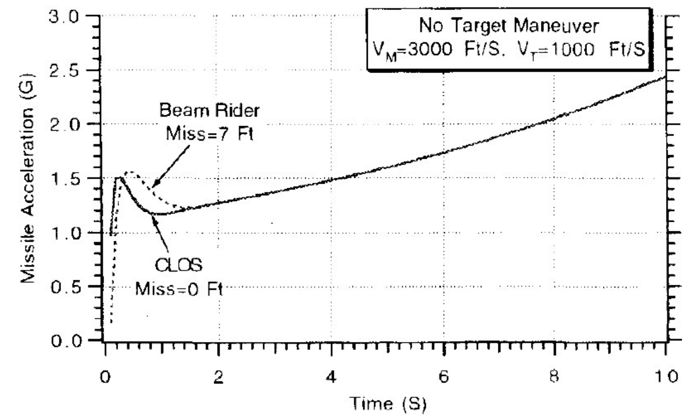


$$a_{MP} = R_M\ddot{\theta}_T + 2\dot{R}_M\dot{\theta}_T$$

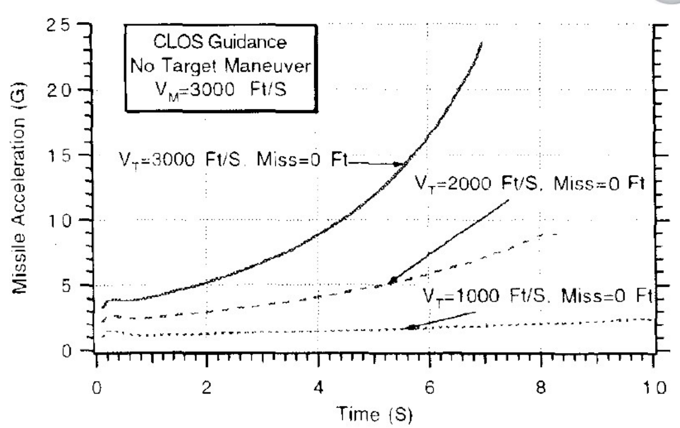
CLOS Guidance: Block Diagram



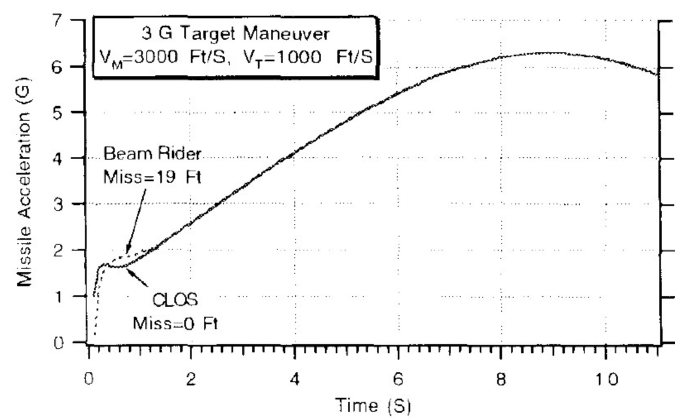
Comparison of CLOS and Basic LOS (Beam Rider)



CLOS Guidance: Target Velocity

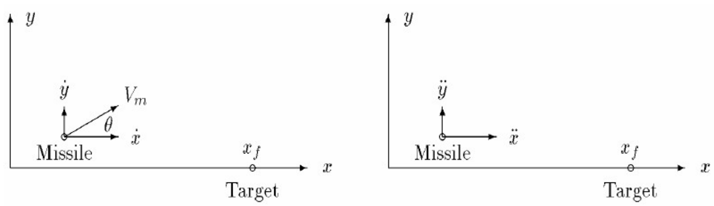


Comparison of CLOS Guidance and Beam Rider: Target maneuver: Acceleration Commands



Optimal LOS Guidance

- Assumptions:
  - Fixed target
  - Ideal Dynamics: $a_m(s)/a_c(s)=1$
  - Θ is a small angle => $\ddot{y} = a_c$



Pourtakdoust and Nobahari, Optimal LOS Guidance, 2000.

Optimal LOS Guidance

- Model

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_c \end{cases} \qquad \begin{cases} x_1 = y \\ x_2 = \dot{y} \end{cases}$$

- Initial and Final Condition

$$\begin{cases} x_1(t_0) = y_0 \\ x_2(t_0) = \dot{y}_0 \end{cases} \qquad \begin{cases} x_1(t_f) = 0 \\ x_2(t_f) = free \end{cases}$$

- Performance Measure

$$J = \frac{1}{2}\int_{t_0}^{t_f} (b\,y^2 + a_c^2)\,dt$$

Optimal LOS Guidance

- Using the optimal control theory, it can be shown that:

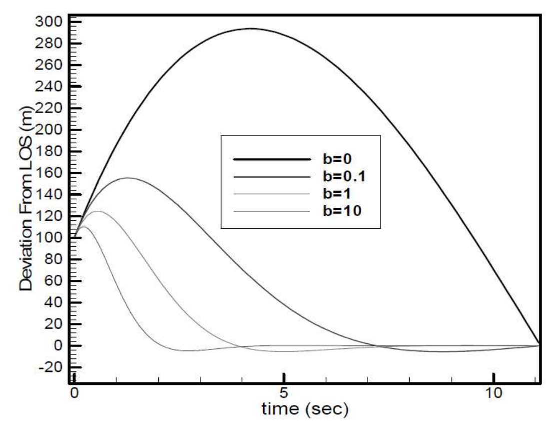
$$a_c = \frac{\left(\phi_3(t_{go})\,y + \phi_2(t_{go})\,\dot{y}\right)\phi_2(t_{go}) + \left(b\phi_1(t_{go})\,y + b\phi_0(t_{go})\,\dot{y}\right)\phi_0(t_{go})}{\phi_2(t_{go})\,\phi_1(t_{go}) - \phi_0(t_{go})\,\phi_3(t_{go})}$$

- Where ($b=4a^4$):

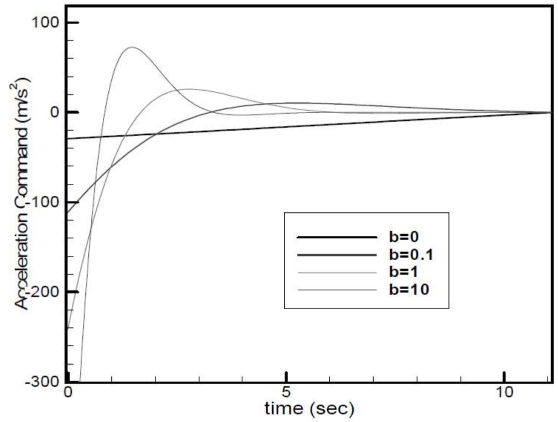
$$\phi_0(t) = \frac{\sin at \cosh at - \cos at \sinh at}{4a^3} \qquad \phi_1(t) = \frac{\sin at \sinh at}{2a^2}$$

$$\phi_2(t) = \frac{\sin at \cosh at + \cos at \sinh at}{2a} \qquad \phi_3(t) = \cos at \cosh at$$

Optimal LOS Guidance



Optimal LOS Guidance

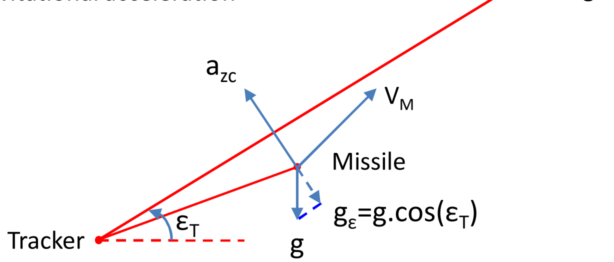


CLOS Guidance Law: Implementation Issues

- Saturation of Acceleration Commands
  - Structural limitations
  - Aerodynamic limitations
- Measurement of R_M and d/dt(R_M)
  - Jamming
  - One may use a look up table
- The need to use numerical differentiation from LOS angles.
  - Sensitivity to noise
- Max permitted angle between x_B axis and LOS
- The interceptor should be Skid-to-Turn (STT)
- Limited Commands Rate

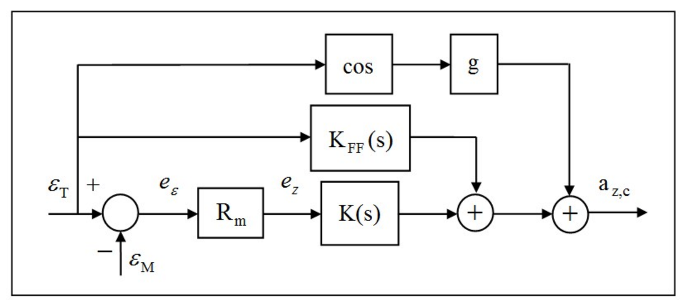
CLOS Guidance Law: Gravity Compensation

- Gravity is applied to the interceptor in addition to the acceleration commands.
- Accelerometers can not measure the gravitational acceleration



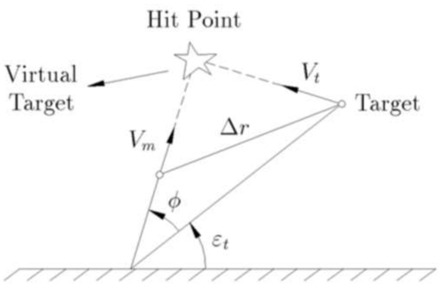
CLOS Guidance Law: Gravity Compensation

- The component of the gravity acceleration which is perpendicular to the LOS should be compensated.



CLOS Guidance Law with Lead Angle

- Missile is kept on the LOS of PIP instead of the real target!
- Predicted Impact Point (PIP) is continuously updated.

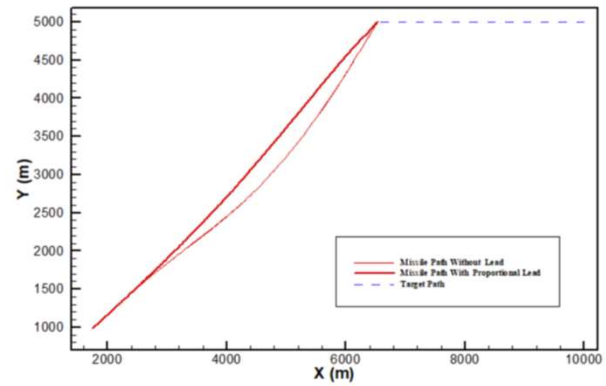


$$\varphi = \dot{\varepsilon}_{\mathrm{t}}\, \mathrm{t}_{\mathrm{go}}$$

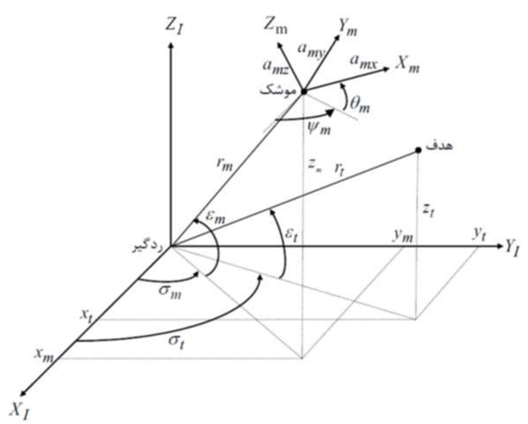
$$\mathrm{t}_{\mathrm{go}} = -\frac{\Delta \mathrm{r}}{\Delta \dot{\mathrm{r}}}$$

- Feed forward acceleration term? Beam limit? Maneuver?

Comparison of CLOS and Lead Guidance Trajectories



3D CLOS Guidance



3D CLOS Guidance: Alternative Error Definitions

- 1

$$\begin{bmatrix} \Delta\varepsilon \\ \Delta\sigma \end{bmatrix} = \begin{bmatrix} \varepsilon_{\mathrm{t}} - \varepsilon_{\mathrm{m}} \\ \sigma_{\mathrm{t}} - \sigma_{\mathrm{m}} \end{bmatrix}$$

- 2

$$\begin{bmatrix} d_\varepsilon \\ d_\sigma \end{bmatrix} = \begin{bmatrix} r_{\mathrm{m}}(\varepsilon_{\mathrm{t}} - \varepsilon_{\mathrm{m}}) \\ r_{\mathrm{m}}(\sigma_{\mathrm{t}} - \sigma_{\mathrm{m}})\cos\varepsilon_{\mathrm{t}} \end{bmatrix}$$

3D CLOS Guidance: LOS rotation and gravity compensation

- Using the spherical coordinate system, the acceleration components of the missile projection on LOS is:

$$\begin{bmatrix} a_{\mathrm{LOS},\varepsilon} \\ a_{\mathrm{LOS},\sigma} \end{bmatrix} = \begin{bmatrix} r_{\mathrm{m}}\ddot{\varepsilon}_{\mathrm{t}} + 2\dot{r}_{\mathrm{m}}\dot{\varepsilon}_{\mathrm{t}} + r_{\mathrm{m}}\dot{\sigma}_{\mathrm{t}}^2 \sin(\varepsilon_{\mathrm{t}})\cos(\varepsilon_{\mathrm{t}}) \\ (r_{\mathrm{m}}\ddot{\sigma}_{\mathrm{t}} + 2\dot{r}_{\mathrm{m}}\dot{\sigma}_{\mathrm{t}})\cos(\varepsilon_{\mathrm{t}}) - 2r_{\mathrm{m}}\dot{\varepsilon}_{\mathrm{t}}\dot{\sigma}_{\mathrm{t}}\sin(\varepsilon_{\mathrm{t}}) \end{bmatrix}$$

- Moreover, the gravity compensation term is:

$$\begin{bmatrix} a_{\mathrm{g},\varepsilon} \\ a_{\mathrm{g},\sigma} \end{bmatrix} = \begin{bmatrix} \mathrm{g}\cos(\varepsilon_{\mathrm{t}}) \\ 0 \end{bmatrix}$$