

In the Name of GOD



Guidance and Navigation I: INS Tests and Calibration

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Objectives



- To **check the performance** of the overall system
- To see if the system **behaves as predicted** from the knowledge of the components.
- To see if the system can work well in **real environment**.
- To investigate the adverse **interactions** between the components.
- To **calibrate** the INS (***)
- A **manufacturer** will wish to check that the units built on a **production line** will **meet the design specification**
- The **customer** will wish to **confirm** that the inertial system will fulfill his or her **requirements**.

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Components of each error terms



Each error term include some or all of the following components:

- fixed or repeatable terms
- temperature induced variations
- switch-on to switch-on variations
- in-run variations

The first two ones can be calibrated.

Switch-on to switch-on variation and **in-run** effects, that is, the random effects caused by instabilities within the gyroscope, **can not be calibrated before the operation** of the system

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Typical test rotations for a strapdown inertial measurement unit (IMU)



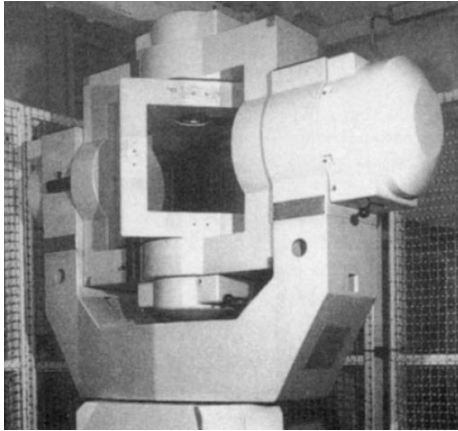
Rotation number	Start position	Turn	Test position
1		90° about y	
2		90° about x	
3		-180° about x	
4		90° about x	

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Three-axis Test Table



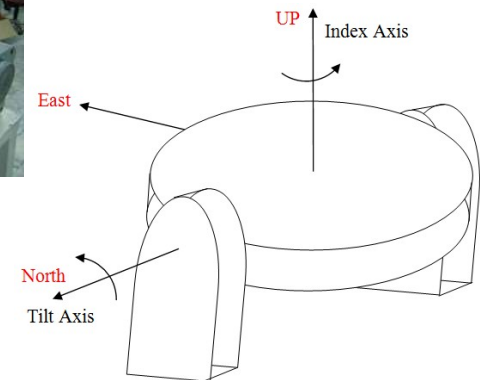
- [Video 1](#)
- [Video 2](#)

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Two-axis test table



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Two-axis test table

- Tilt axis is always fixed WRT the earth (toward the geographic **North**)
 - Index axis is always perpendicular to the rotating plane
 - **Initially** the **rotating plane** is aligned WRT the local **level**
 - After installation of the IMU on the table, **$[\mathbf{T}]^{B_0N}$ is known**
 - **After any** tilt and index **motion** of the table **$[\mathbf{T}]^{BN}$ is known** as a function of the tilt angle (θ) and the index angle (ψ)
- => the expected **accelerometer output** (**$-g^B$**) is known
- **During any** tilt and index **motion** of the table, **ω_{NB} is known** as a function of tilt rate ($d\theta/dt$) and index rate ($d\psi/dt$)

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The expected rate gyro outputs

$$\omega_{IB}^B = \omega_{IE}^B + \omega_{EN}^B + \omega_{NB}^B$$

- Position of the table is fixed WRT the earth =>

$$\omega_{IE}^N = \begin{bmatrix} \omega_e \cos L \\ 0 \\ -\omega_e \sin L \end{bmatrix} \quad \omega_{EN} = 0$$

- The instantaneous **ω_{NB} is known** as a function of the tilt rate ($d\theta/dt$) and the index rate ($d\psi/dt$)
- => The expected **rate gyro outputs** (**$[\omega_{IB}]^B$**) are known

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The expected accelerometer outputs



$$\mathbf{a}^N = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$

- After any tilt and index motion of the table, $[\mathbf{T}]^{BN}$ is known as a function of the tilt angle (θ) and the index angle (ψ)
- the expected accelerometer outputs can be calculated from:

$$\mathbf{a}^B = [\mathbf{T}]^{BN} \mathbf{a}^N$$

Calibration of the INS accelerometers



- Since there is always installation **misalignments**, the expected acceleration in x direction is a function of all accelerometers outputs:

$$\hat{a}_x = B_{ax} + S_{axx} a_x + M_{axy} a_y + M_{axz} a_z + n_{ax}$$

- It can also be written as:

$$\hat{a}_x = C^T x_x + n_{ax} \quad \text{where: } C = \begin{bmatrix} 1 \\ a_x \\ a_y \\ a_z \end{bmatrix}, \quad x_x = \begin{bmatrix} B_{ax} \\ S_{axx} \\ M_{axy} \\ M_{axz} \end{bmatrix}$$

- x_x : the unknown vector

Calibration of INS accelerometers



- In the same way:

$$\hat{a}_y = B_{ay} + S_{ayy} a_y + M_{ayx} a_x + M_{ayz} a_z + n_{ay}$$

$$\hat{a}_z = B_{az} + S_{azz} a_z + M_{azx} a_x + M_{azy} a_y + n_{az}$$

- 12 unknown parameter can be defined for 3 accelerometers.
- 3 equation can be written for each tilt and index (each test)
- ⇒ At least 4 independent test must be run to obtain a system of 12 equations and 12 unknowns (4 different sets of tilt and index)
- ⇒ More tests can result better accuracies using the method of **least square**.

Calibration of INS accelerometers



- the expected acceleration in x direction may also be considered to depend to higher order terms of the accelerometers outputs:

$$a_{xind} = B_{ax} + S_{axx} a_x + S_{2x} a_x^2 + M_{axy} a_y + M_{axz} a_z + n_{ax}$$

- Here, at least 5 independent test must be run to obtain a system of 15 equations and 15 unknowns

Calibration of the INS rate gyros



- Since there is always installation **misalignments**, the **expected angular rate around x_b axis** is a function of all rate gyros outputs:

$$\hat{\omega}_x = B_{gx} + S_{gxx}\omega_x + M_{gxy}\omega_y + M_{gxz}\omega_z + n_{gx}$$

- Considering higher-order error terms and g-dependent errors:

$$\begin{aligned} \hat{\omega}_x = & B_{gx} + B_{ax}a_x + B_{ay}a_y + B_{az}a_z + S_{gxx}\omega_x + S_{gxx2}\omega_x^2 + S_{gxx3}\omega_x^3 \\ & + M_{gxy}\omega_y + M_{gxz}\omega_z + n_{gx} \end{aligned}$$

- Some gyros (e.g. **FOGs**) are not sensitive to accelerations