

A REVIEW OF KALMAN FILTER WITH ARTIFICIAL INTELLIGENCE TECHNIQUES

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Abstract

Kalman filter (KF) is a widely used estimation algorithm for many applications. However, in many cases, it is not easy to estimate the exact state of the system due to many reasons such as an imperfect mathematical model, dynamic environments, or inaccurate parameters of KF. Artificial intelligence (AI) techniques have been applied to many estimation algorithms thanks to the advantage of AI techniques that have the ability of mapping between the input and the output, the so-called “black box”. In this paper, we found and reviewed 55 papers that proposed KF with AI techniques to improve its performance. Based on the review, we categorised papers into four groups according to the role of AI as follows: 1) Methods tuning parameters of KF, 2) Methods compensating errors in KF, 3) Methods updating state vector or measurements of KF, and 4) Methods estimating pseudo-measurements of KF. In the concluding section of this paper, we pointed out the directions for future research that suggestion to focus on more research for combining the categorised groups. In addition, we presented the suggestion of beneficial approaches for representative applications.

Introduction

Kalman filter (KF) is a popular and efficient recursive estimator actively used in many applications such as multi-target tracking, sensor fusion (or integration), and many others [1]–[8]. The advantages of the KF as recursive estimators are well known and widely appreciated: they don’t require entire previous data for the estimation but require only present measurement and previously calculated data. Despite the effectiveness of KF, several problems exist which are affecting their performance. For instance, KF requires accurate information on system dynamics and filter parameters such as error covariance matrices, for its optimality. An imperfect mathematical model or nonlinearity of model, and non-Gaussian error which cannot be fully described within the model causes performance degradation.

Recently, many artificial intelligence (AI) techniques, such as neural network (NN), including feedforward neural network (FFNN), radial basis function neural network (RBFNN), recurrent neural network (RNN), or reinforcement learning (RL), etc. are proposed and utilised for many research fields because of their capability of mapping between the input and the output [9]–[21]. Several researchers have proposed applying AI techniques to enhance KF and address their specific disadvantages. The heterogeneity of these approaches makes it difficult to systematically assess current progress and formulate the research challenges related to the combination of AI-based approaches and KF for performance improvement. To the best of the authors’ knowledge review that would provide a systematic overview of AI-based improvements for KF has not been previously performed. In this work, we present a systematic overview of approaches to integrating AI with KF.

According to the undertaken literature survey the bulk of the approaches can be categorised into four groups according to the role of AI techniques as follow: 1) Methods tuning parameters of KF, 2) Methods compensating errors in KF, 3) Methods updating state vector or measurements of KF, and 4) Methods estimating pseudo-measurements of KF. For these categories that have been used in different applications, we have summarised the characteristics of each approach. It was found that most of the proposed hybrid systems showed essential improvements in performance compared to the KF or AI standalone estimation system. Through the review, the following open issues are found: 1) More systematic validation of the proposed algorithm, e.g., comparing with the previously proposed approach and computational complexity analysis, are needed and 2) More studies are required to overcome the nonlinearity or non-Gaussian error of the KF. In addition, there were not many previous approaches that showed the combination of categorised groups which is seen to be a possible solution to improve the performance of KF.

The rest of this paper is composed as follow: Section 2 introduces the KF and NN briefly which are most widely applied. Section 3 reviews proposed research on conference and journal papers that focus on KF with AI techniques categorised into four groups. Section 4 discusses the remaining challenges, research gap, and future research direction. Finally, in section 5 we conclude this paper.

Kalman Filters and Neural Networks

Kalman Filter

Kalman Filter (KF) is a widely used recursive estimator, first introduced, as known as linear Kalman filter (LKF), by Kalman [22]. To apply this technique to various systems and improve the performance, many variations of KF such as extended Kalman filter (EKF), unscented Kalman filter (UKF), indirect Kalman filter (IKF), cubature Kalman filter (CKF), adaptive EKF (AEKF), adaptive UKF (AUKF), etc. are proposed [23]–[26]. Here we introduce LKF which can be considered as the most basic of any other various KF.

Linear Kalman Filter

The state-space model for the LKF can be simply denoted as follows:

$$x_k = F_k x_{k-1} + w_k \quad (1)$$

where x_k is the state vector at time step k , F_k is the state transition matrix, and $w_k \sim \mathcal{N}(0, Q_k)$ is the process noise assumed to be Gaussian noise with a known covariance matrix Q_k .

The measurement model for the LKF can be simply denoted as follows:

$$z_k = H_k x_k + v_k \quad (2)$$

where z_k is the measurement vector at time step k , H_k is the measurement matrix, and $v_k \sim \mathcal{N}(0, R_k)$ is the measurement noise assumed to be Gaussian noise with a known covariance matrix R_k .

LKF can be divided into two main steps: Prediction and update step. First, in the prediction step, the predicted state estimate and the predicted error covariance matrix can be denoted follows:

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} \quad (3)$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \quad (4)$$

where $\hat{x}_{k|k-1}$ is the predicted state estimate at time step k with measurement up to time step $k-1$ and $P_{k|k-1}$ is the predicted error covariance matrix.

In the update step, KF updates Kalman gain, state vector, and its covariance as follows:

$$y_k = z_k - H_k \hat{x}_{k|k-1} \quad (5)$$

$$S_k = H_k P_{k|k-1} H_k^T + R_k \quad (6)$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1} \quad (7)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k y_k \quad (8)$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \quad (9)$$

where y_k is the innovation (or measurement residual), S_k is the innovation covariance matrix, K is the Kalman gain, $\hat{x}_{k|k}$ is the updated state estimate, and $P_{k|k}$ is the updated error covariance matrix.

However, since the LKF assumes a linear system, it can be degraded or diverged in many real applications which are usually nonlinear. To overcome this drawback of LKF, EKF and UKF are widely used. EKF linearises a higher-order nonlinear system to 1st-order using Jacobian. However, since EKF uses linearisation to make a nonlinear system to be partially linear, the performance can be degraded in a highly nonlinear case. To overcome this, UKF uses unscented transformation (UT), a sampling technique using sigma points, to be used in nonlinear systems. In this paper, hybrid approaches of KF with AI techniques that considered using EKF and UKF to handle the nonlinearity of the system are included.

Neural Network

Neural network (NN) is one kind of AI that are widely used because of their ability that can map between the input and the output without prior knowledge of the model. Various NN such as FFNN, RNN, radial basis function neural network (RBFNN), fuzzy neural network (FNN), etc. has been proposed [13]–[16], [27]. As it is most widely applied, we briefly introduce FFNN.

Feedforward Neural Network

Feedforward neural network (FFNN) is composed of three parts, input layers, hidden layers, and output layers and an example is shown in the below figure (Figure 1).

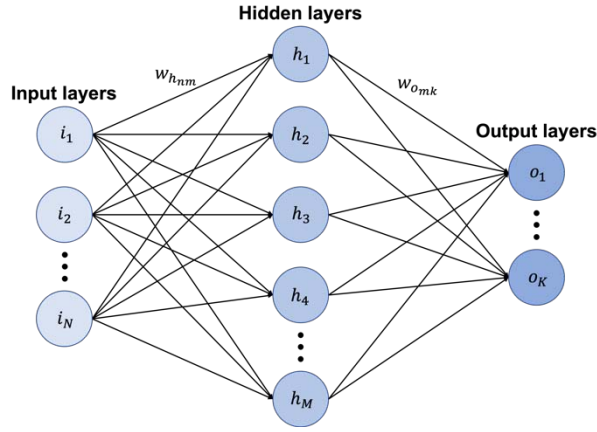


Figure 1. Neural Network Architecture Example

Outputs are obtained by weights, which connect the input layer to the hidden layer and the hidden layer to the output layer as follows:

$$h_m = f\left(\sum_{n=1}^N w_{hnm} i_n\right) \quad (10)$$

$$o_k = \sum_{m=1}^M w_{omk} h_m \quad (11)$$

where i_n is the n^{th} input layer node, h_m is the m^{th} hidden layer node, o_k is the k^{th} output layer node, w_{hnm} is the weight that connects i_n to h_m , w_{omk} is the weight that connects h_m to o_k , and $f(\cdot)$ is the activation function. A common example of activation functions is the sigmoid function which can be denoted as:

$$f(x) = \frac{1}{1 + e^{-x}} \quad (12)$$

From equations (10) and (11), the output can be represented as follows:

$$o_k = \sum_{m=1}^M w_{omk} f\left(\sum_{n=1}^N w_{hnm} i_n\right) \quad (13)$$

where weights can be trained using backpropagation for FFNN, the so-called backpropagation neural network (BPNN).

However, using previous data (memory) in the case of the data having sequential form is unable with the feedforward structure. To overcome this drawback, RNN uses recurrent units which enable the network to capture sequential information. In this

paper, hybrid approaches of KF with AI techniques that considered using RNN for sequential information are included.

KF with AI Techniques

An Overview of KF with AI Techniques

Total 55 papers are reviewed in this paper. Among 55 papers, 17 papers (31%) were group 1, parameter tuning, 14 papers (25%) were group 2, error compensation, 15 papers (27%) were group 3, state vector or measurement update, 7 papers (13%) were group 4, pseudo-measurement estimation, and the rest of papers, 2 papers (4%) were combined approaches as shown in below figure (Figure 2).

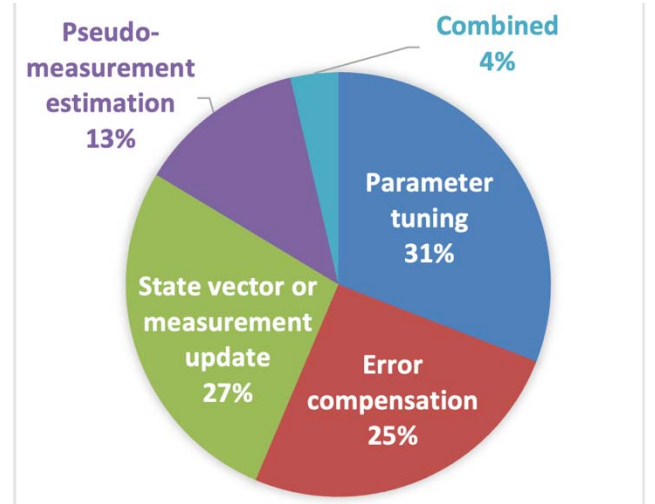


Figure 2. Group Composition Ratio for Review

In addition, from the papers we reviewed, it was can be seen that the number of papers is increasing recently as shown in the below figure (Figure 3).

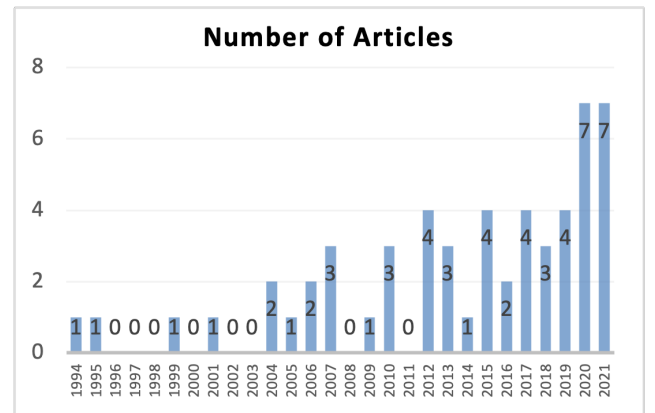


Figure 3. Number of Papers by Year

We reviewed papers that are trying to combine KF with AI techniques, and reviewed approaches are categorised into four groups according to the role of AI techniques as follow:

- A. Methods tuning parameters of KF,
- B. Methods compensating errors in KF,
- C. Methods updating state or measurements vector of KF, and
- D. Methods estimating pseudo-measurements of KF.

Detailed explanations about the four groups are explained in the next section.

Tuning Parameters of KF

The first group is the methods that tuning the parameters of KF such as process noise covariance matrix Q_k , measurement noise covariance matrix R_k , weighting factors, etc., where those parameters are predicted by AI techniques [28]–[44]. Especially, many studies in this group are focused on tuning noise covariance matrices. In many cases, noise covariance matrices have been assumed to be constant during the whole process of KF estimation. However, in dynamic environments, the performance of the KF can be degraded by using constant noise covariance matrices. To overcome this drawback, methods that tuning the noise covariance matrices using the AI techniques are proposed. One example, Ullah, et al. proposed a hybrid method to improve the accuracy of the KF using NN that can predict and update measurement noise covariance matrix R_k for temperature estimation which is heavily affected by humidity level [40]. This approach is advantageous in dynamic environments by tuning the noise covariance matrices, as its purpose, and by tuning the noise covariance matrices in a very short period, it could be applied for the nonlinear estimation cases as well. Most of the studies are focused on tuning the noise covariance matrices in dynamic environments such as vehicle localisation and navigation application.

Reviewed approaches in this group are mostly using AEKF for state estimation and BPNN or RNN to predict the parameters of KF. AEKF was utilised in this group most commonly because of the advantage that it has an ability that can tune the noise covariance matrices adaptively. BPNN or RNN takes innovation (Equation 5) or measurements directly

from the sensors as an input to predict the parameters of KF. A general conceptual flow of the algorithms is described in the figure as shown below (Figure 4).

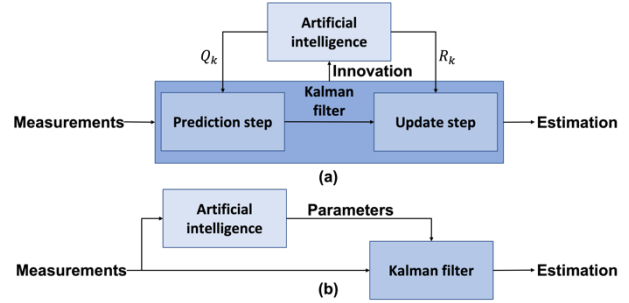


Figure 4. General Concept Flow of Algorithms for Tuning the Parameters of KF with (a) Innovation as Input and (b) Measurements as Input

Compensating Errors in KF

The second group is the methods that compensate the errors in the result of KF using errors predicted by AI techniques [45]–[58]. The performance of KF can be degraded due to various reasons such as an imperfect mathematical model, errors in parameters or sensors, etc. as mentioned before. To correct the error, methods that compensate the errors in the result of KF by using AI techniques were proposed. One example, Nguyen, et al. proposed a hybrid method for enhancing robot accuracy using EKF that estimates modelled error and NN that can predict unmodelled error to compensate the result of EKF [51]. This approach is advantageous because of its simple framework that can be utilised in many applications. This approach compensates the error between the KF estimated data and true data, which is predicted using AI techniques, so it is available to be implemented for many applications relatively easily compared to other approaches. Owing to its simple framework, proposed approaches in this group are applied to many applications.

Reviewed approaches in this group are mostly using LKF for state estimation and BPNN to predict error in the result of KF. LKF and BPNN were utilised most commonly because of their simplicity. The input of the BPNN cannot be simply generalised because approaches in this group are applied in various models. One thing clear is that most of the approaches in this group are used the NN to predict the error between the result of KF and the true data.

A general conceptual flow of the algorithms is described in the figure as shown below (Figure 5).

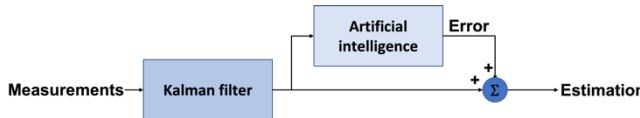


Figure 5. General Concept Flow of Algorithms for Compensating the Errors in the Result of KF

Updating State Vector or Measurements of KF

The third group is the methods that update the state vector or the measurements of KF using the errors predicted by AI techniques [59]–[73]. The performance of KF can be degraded because of the imperfect mathematical model or the error of the sensor caused by its characteristic (e.g., INS drift). To overcome this drawback, methods that update the state vector or the measurements of KF using AI techniques are proposed. One example, Zhang and Xu proposed a hybrid method for the global positioning system (GPS)/inertial navigation system (INS) integration using KF and NN that can predict the error of the INS to be updated to measurement during GPS outages [64]. This approach is advantageous when a priori model is imperfect, nonlinear, or when the sensors have errors. In addition, many studies in this group proposed the techniques that can train the NN using KF by utilising augmented state vector which includes weights of NN. However, the GPS/INS integration examples still suffer from divergence problems in case of long-term GPS outages. Most of the studies are focused on updating the state vector in simultaneous localisation and mapping (SLAM) or updating the measurements in sensor fusion (or integration) applications.

Reviewed approaches in this group are mostly using EKF for state estimation and BPNN or RBFNN to predict error in the state vector or the measurements of KF. EKF was utilised most commonly because of its advantage in the nonlinear case. Especially error state model was commonly used for the sensor integration. BPNN or RBFNN takes previous states as an input to predict the error and difference between a priori (mathematical) model and actual dynamics or the measurements directly from the sensors as an input to predict the difference between one sensor to another sensor (which is considered as ground-truth). After NN finished

predicting the errors, the system updates errors on the state vector or the measurements of KF. A general conceptual flow of the algorithms is described in the figure as shown below (Figure 6).

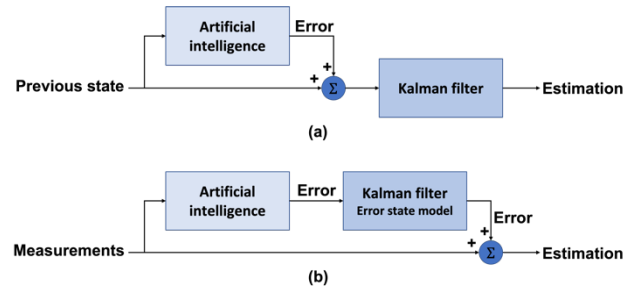


Figure 6. General Concept Flow of Algorithms for Updating (a) the State vector or (b) the Measurements of KF

Estimating Pseudo-measurements of KF

The fourth group is the methods that estimate the pseudo-measurements of KF using AI techniques [74]–[80]. As mentioned above, the performance of KF can be degraded due to the imperfection of the a priori (mathematical) model. To overcome this drawback, updating the state vector was proposed and mentioned in the previous section. Here, another approach, estimating pseudo-measurements of KF using AI techniques are proposed. One example, Kim et al. proposed a hybrid method for ground vehicle sideslip estimation using AUKF and RNN that can predict pseudo-measurements [79]. This approach is advantageous when the dynamic model is nonlinear. In addition, some researchers proposed model-free applications by using kinematics. Approaches in this group are proposed relatively recently than other groups. Most of the studies are focused on estimating roll angle or sideslip angle which have highly nonlinear characteristics but need to be estimated correctly to improve the stability of autonomous ground vehicles.

Reviewed approaches in this group are mostly using UKF for state estimation and BPNN to predict the pseudo-measurements of KF. UKF was utilised in this group most commonly because of the advantage that can be used in the nonlinear case. BPNN takes measurements directly from the sensors as an input and predicts vehicles' roll angle or sideslip angle. A general conceptual flow of the algorithms is described in the figure as shown below (Figure 7).

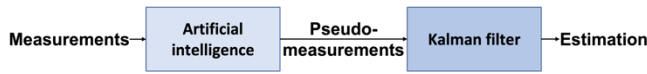


Figure 7. General Concept Flow of Algorithms for Estimating the Pseudo-measurements of KF

Combined Approaches

Apart from the groups mentioned above, some research proposed approaches that can be considered as the combination of previously mentioned groups. The combination can be a good alternative solution to overcome the disadvantages of each group and improve the performance of the KF.

Two approaches are reviewed in this paper. One of two approaches is combining groups 1 and 3 to improve the performance of target tracking problem where NN was utilised to predict error in dynamic model to update state vector and CKF combined variational Bayesian was used to estimate the state and NN weight. In addition, variational Bayesian was utilised to approximate the measurement noise [81]. Another approach is combining groups 1 and 4 to improve the performance of aerial surveillance problems where BPNN and generalised regression NN predict the state as a pseudo-input and parameters of KF, respectively [82].

Discussion

We reviewed and categorised approaches combining KF with AI techniques into four groups in the previous section. A summary of the four groups is described in the table as shown below (see Table 1). All groups showed improved performance compared to using KF or AI standalone cases. Here we discuss the remaining challenges, research gap, and proposal for the future research direction.

Remaining Challenges and Research Gap

Based on the review of several papers, we found the remaining challenges and research gap in KF with AI techniques. It is necessary to mention the general problems of the previous research first. We found some general problems of research in this topic, and those are explained below. First, many techniques are proposed but most of the studies didn't compare their approach with previously proposed approaches. In addition, the experiment environments or scenarios are diverse in each study even if they considered the same application. Next, many papers didn't mention

the computational cost or computational complexity in their papers. Since many applications are considering real-time environment applications, computational complexity comparison is necessary to be mentioned to show the possibility of real-time applications.

In terms of the technical aspect, although, many previously proposed approaches tried to improve the performance of KF by combining with AI techniques but studies on improving the performance of KF itself, especially nonlinearity or non-Gaussian noise covariances, has not been actively conducted. In addition, the research that combines the groups which can show the further improvements are not conducted yet. Especially for group 3 and group 4, since those groups can overcome the drawback of the nonlinearity problem of KF, it is expected that combining group 3 or 4 with group 1 can improve the performance significantly.

Proposal for the Future Research Direction

Based on the remaining challenges and research gap mentioned above, here we propose following future research directions. First, in the general aspect, it is suggested to researchers to compare the previously proposed algorithm with their proposing algorithm to show the improvement. In addition, in the case of the previously proposed approaches that are applied in the same application exists, it is suggested to compare in the same scenario to show the improvement of proposing algorithm. Also, as mentioned before, it is suggested to include the computational cost or computational complexity analysis to show the improvement of the algorithm or the possibility of real-time applications.

In terms of the technical aspect, it is suggested to research on combined approaches to overcome the limitations of each group. From the previously suggested combined approaches, it is possible to see the potential of the combined approach to solve the problem or improve the performance. Especially it is suggested to combine group 3 or 4 with group 1 to overcome the imperfection of a priori (mathematical) model or nonlinearity and parameter selection problems together. Finally, for future research, we suggest beneficial approaches (groups) depending on the applications based on the review. Owing to the simple framework of groups 1 and 2, those can be utilised in many applications, in other words, the first

two groups are not seemed to be application-specific approaches. For the cases where error state needs to be considered in KF such as GPS/INS integration,

group 3, state vector or measurement update, will be beneficial and for the nonlinear vehicle attitude estimation, group 4 can be beneficial.

Table 1. A Summary of Kalman Filter with Artificial Intelligence Techniques

Groups	Parameter tuning	Error compensation	State vector or measurement update	Pseudo-measurement estimation
Reference	[28]–[44]	[45]–[58]	[59]–[73]	[74]–[80]
No. of papers	17 (31%)	14 (25%)	15 (27%)	7 (13%)
Roles of AI	AI predicts and tunes the parameters of KF, such as error covariances	AI predicts and compensates the errors in the results of KF	AI predicts and updates the errors of the state vector or the measurements	AI predicts and feeds the pseudo-measurements of KF
Characteristics	An ability that can adapt to dynamic environments	A simple framework that can be utilised in many applications	Better performance with an imperfect mathematical (or nonlinear) model or sensor error	An ability that can handle model non-linearity or model-free applications
Common applications	Vehicle localisation and navigation	Multi-target tracking (MTT) and various applications	Simultaneous localisation and mapping (SLAM) and Sensor fusion	Vehicle sideslip or roll angle estimation
Common KF	AEKF	LKF	EKF	UKF
Common AI	BPNN and RNN	BPNN	BPNN and RBFNN	BPNN

Conclusion

In this paper, we reviewed approaches that considered the fusion of KF with AI techniques and categorised the reviewed papers into four groups according to the role of AI as follows: 1) Methods tuning parameters of KF, 2) Methods compensating errors in KF, 3) Methods updating state vector or measurements of KF, and 4) Methods estimating pseudo-measurements of KF. Most of the research showed improved performance of estimation than using KF or AI standalone and it was able to handle nonlinearity or non-Gaussian noise in some cases. From the review, it is found that not much research was conducted for the improvement of KF performance itself and approaches combining mentioned groups. Based on the discussed remaining challenges and research gap, we suggested for the research on combined approaches to overcome the limitations of each group. Also, we suggested that the first two groups (parameter tuning and error compensation) can be used for diverse applications and for the cases of GPS/INS integration and nonlinear vehicle attitude estimation, groups 3 and 4 can be beneficial, respectively. In addition, a future

study will include a benchmark study for the detailed comparison of the prementioned groups.

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