Sharif University of Technology
Institute of Aerospace

Homework #1

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Course: *Optimal Control I* – Professor: *Dr. Assadian*
Due date: *March 28th, 2025*

Problem 1

(a) $z = f(x, y) = y\sin(x + y) - x\sin(x - y)$
Gradient of f(x, y):

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{bmatrix} y\cos(x + y) - \sin(x - y) - x\cos(x - y) \\ y\cos(x + y) + \sin(x + y) + x\cos(x - y) \end{bmatrix}$$

non linear equations with two unknowns. We use MATLAB to solve this equations. MATLAB file is attached. Answers are provided in table 1

Table 1: Answers

x	y
-3.41877	-1.82764
-2.88904	1.84693
-2.02875	0.00000
-1.84693	-2.88904
-1.82764	3.41877
-1.75560	0.36547
-0.36547	-1.7556
0.00000	-2.02875
0.00000	0.00000
0.00000	2.02875
0.36547	1.7556
1.75560	-0.36547
1.82764	-3.41877
1.84693	2.88904
2.02875	0.00000
2.88904	-1.84693
3.41877	1.82764

Hessian matrix:

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial xy} \\ \frac{\partial^2 f}{\partial yx} & \frac{\partial f}{\partial y^2} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{matrix} -y\sin(x + y) - 2\cos(x - y) + x\cos(x - y) & \cos(x + y) - y\sin(x + y) + cos(x - y) - x\sin(x - y) \\ cos(x + y) - y\sin(x + y) + \cos(x - y) - x\sin(x - y) & x\sin(x - y) + 2\cos(x + y) - y\sin(x + y) \end{matrix}$$

Hessian matrix and eigenvalues have calculated in MATLAB and attached. Maximum Minimum Saddle Point

Table 2: Answers With Conditions

x	y	Point Condition
-3.41877	-1.82764	Maximum
-2.88904	1.84693	Saddle Point
-2.02875	0.00000	Saddle Point
-1.84693	-2.88904	Saddle Point
-1.82764	3.41877	Minimum
-1.75560	0.36547	Maximum
-0.36547	-1.7556	Minimum
0.00000	-2.02875	Saddle Point
0.00000	0.00000	Saddle Point
0.00000	2.02875	Saddle Point
0.36547	1.7556	Minimum
1.75560	-0.36547	Saddle Point
1.82764	-3.41877	Minimum
1.84693	2.88904	Saddle Point
2.02875	0.00000	Saddle Point
2.88904	-1.84693	Saddle Point
3.41877	1.82764	Maximum

Answers and conditions are provided in table 2

Figure 1: 3D figure of function

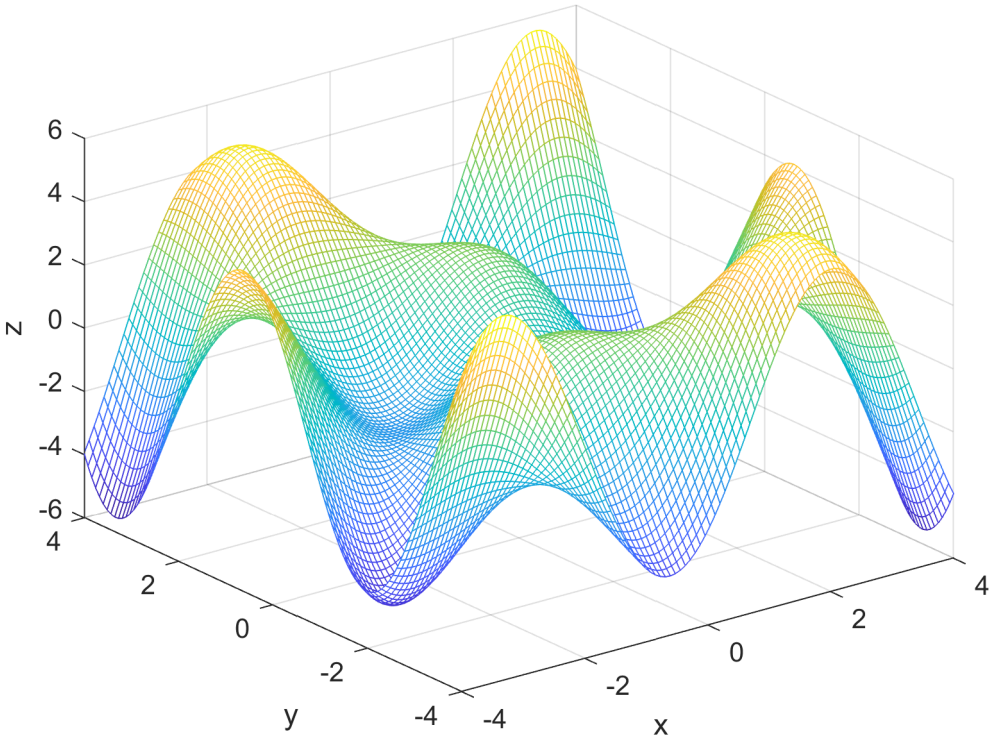


Figure 2: 3D figure of function with Points

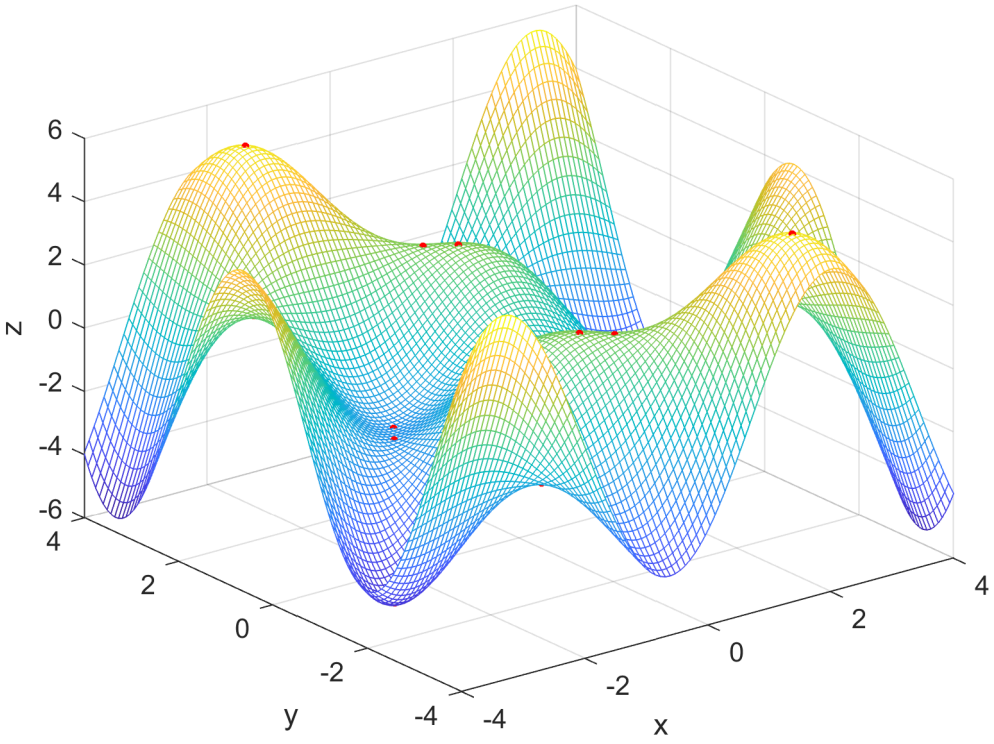


Figure 3: Contour figure of function

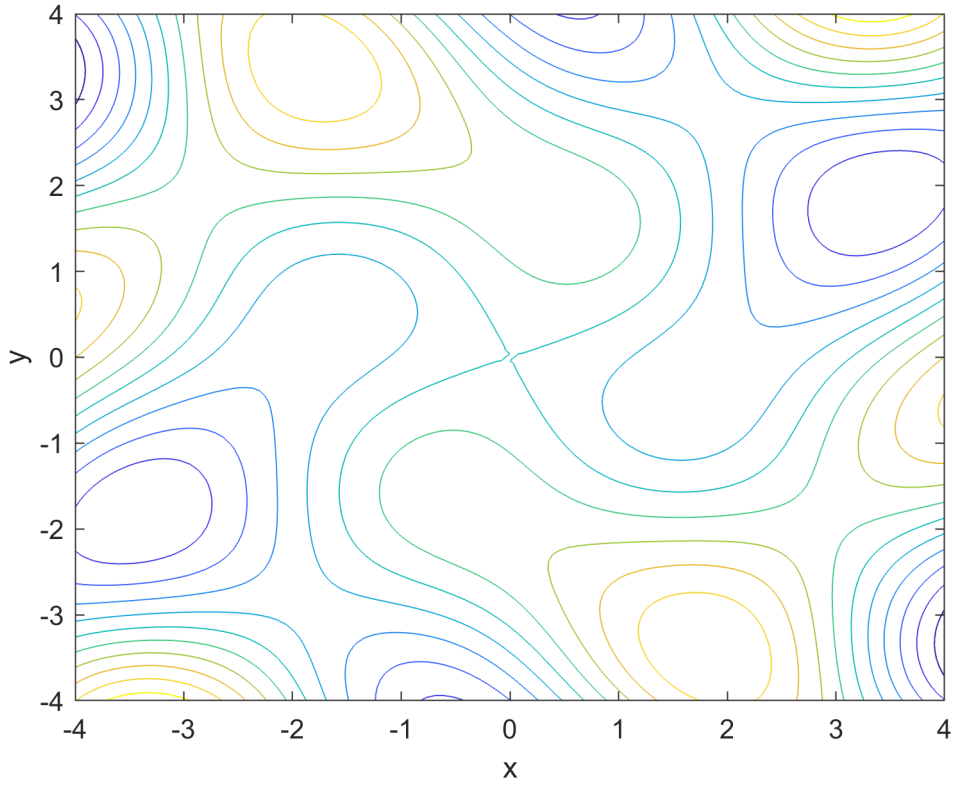
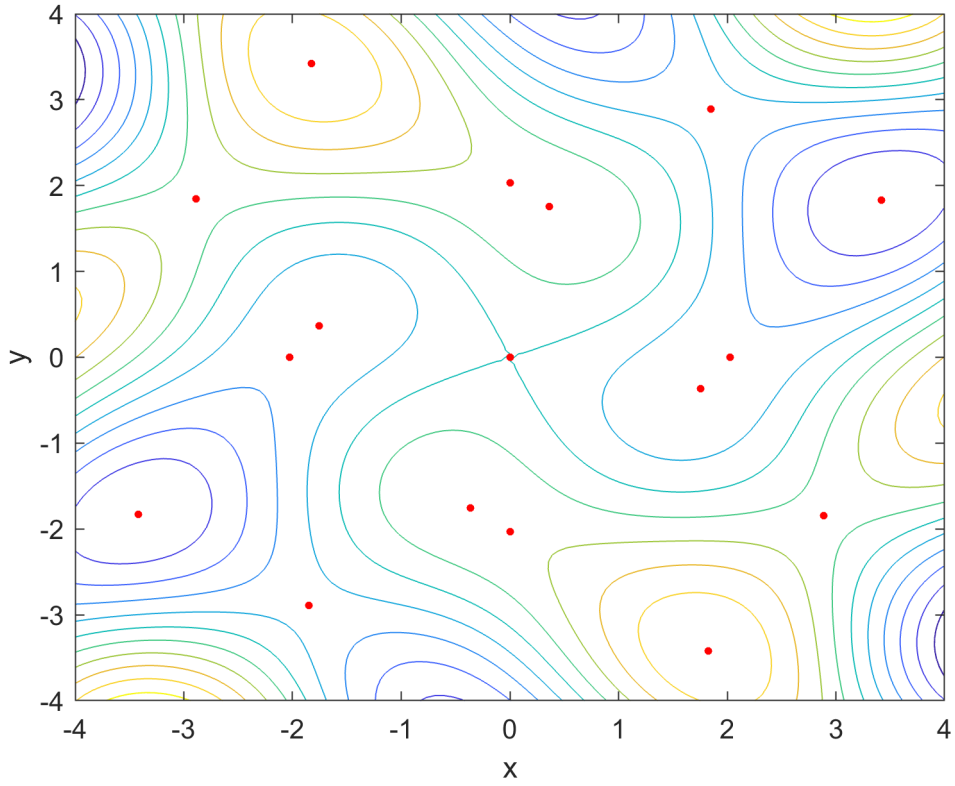


Figure 4: Contour figure of function with Points



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(b) (your solution)