

# Home Work #3

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## 1 Question 1

$$z = f(x, y) = y \sin(x + y) - x \sin(x - y)$$

Gradient of  $f(x, y)$ :

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{bmatrix} y \cos(x + y) - \sin(x - y) - x \cos(x - y) \\ y \cos(x + y) + \sin(x + y) + x \cos(x - y) \end{bmatrix}$$

### 1.1 part a

#### 1.1.1 figures

$$\vec{X}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

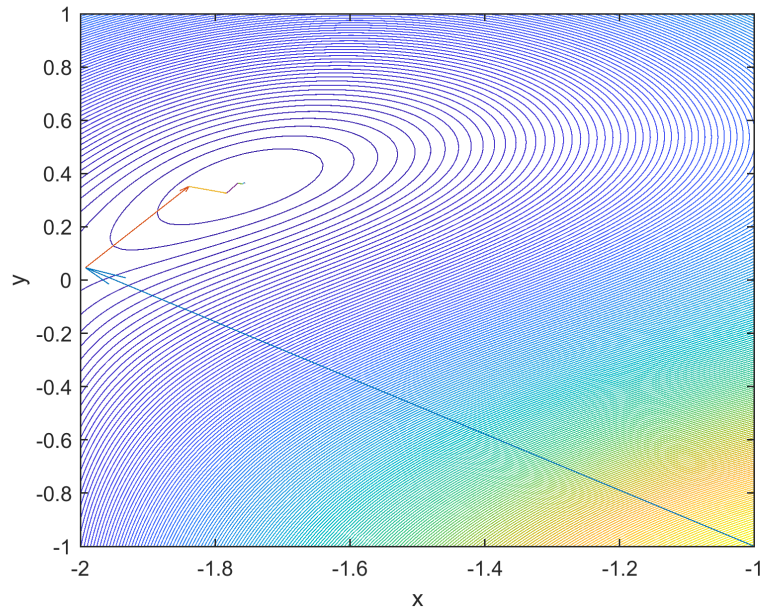
Tolerance is:  $10^{-7}$

Answer is:

$$\vec{X}_{ans} = \begin{bmatrix} -1.7556 \\ 0.3655 \end{bmatrix}$$

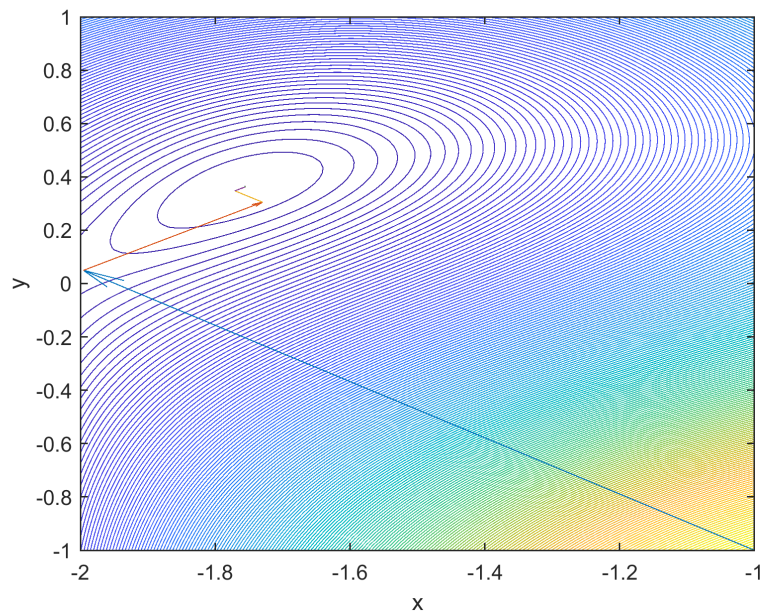
- Steepest Descent
  - Quadratic Interpolation

Figure 1: Steepest Descent and Quadratic Interpolation



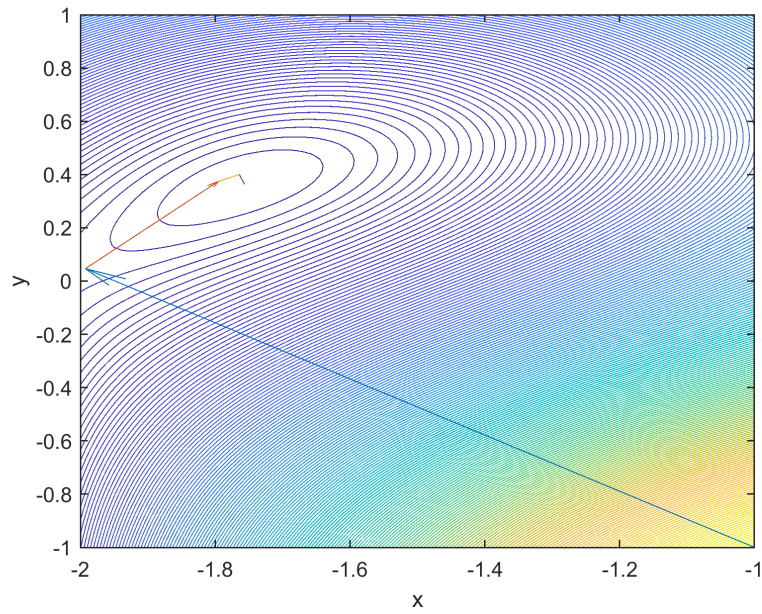
- Golden Section

Figure 2: Steepest Descent and Golden Section



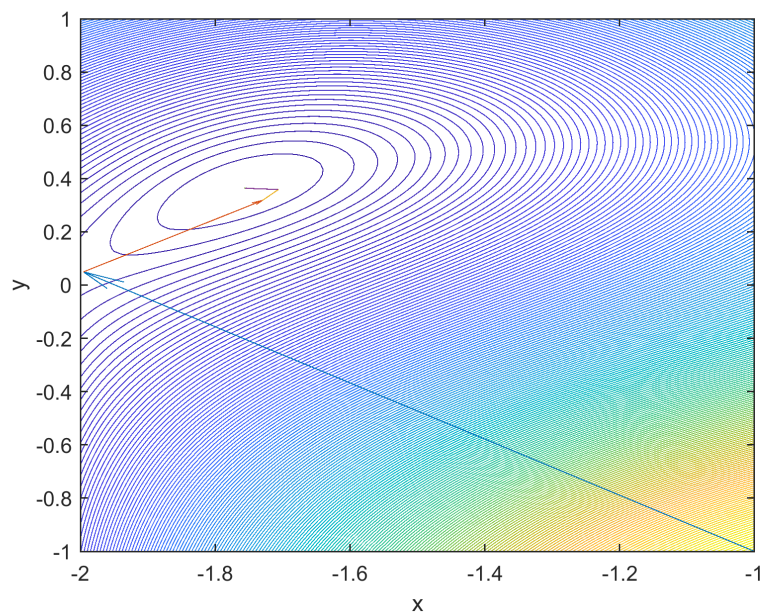
- BFGS
  - Quadratic Interpolation

Figure 3: BFGS and Quadratic Interpolation



- Golden Section

Figure 4: BFGS and Golden Section



**1.1.2 result**

- Time

Table 1: Time compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
0.238 sec	0.183 sec	0.164 sec	0.102 sec

- Number of Cost calculation

Table 2: Number of Cost calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
360	336	242	213

- Number of Gradient calculation

Table 3: Number of Gradient calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
19	13	13	9

**1.2 part b****1.2.1 figures**

$$\vec{X}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

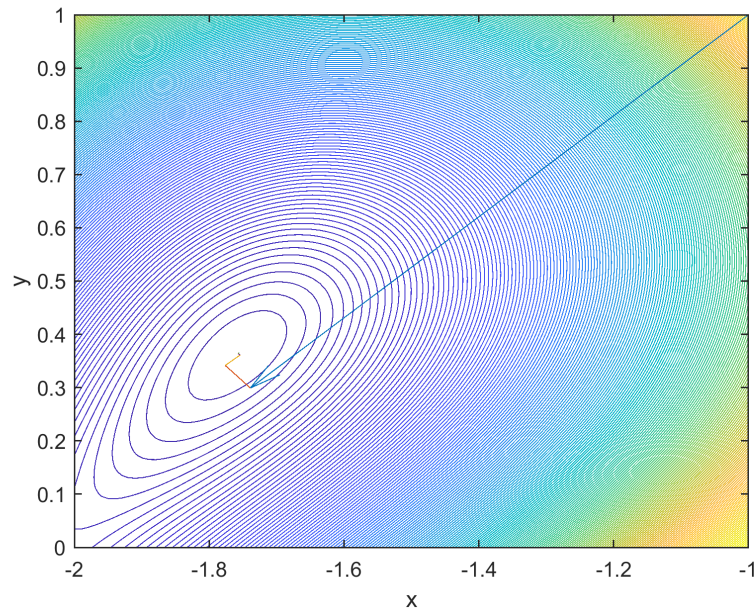
Tolerance is:  $10^{-7}$

$$\vec{X}_{ans} = \begin{bmatrix} -1.7556 \\ 0.3655 \end{bmatrix}$$



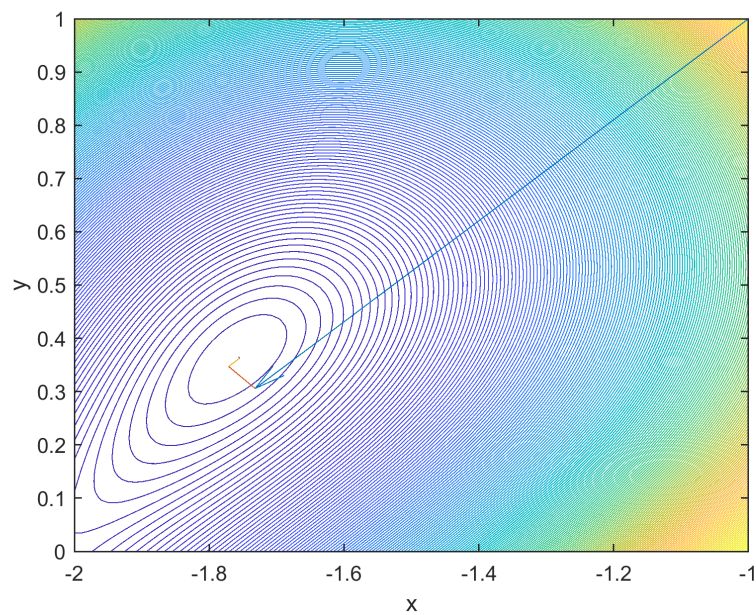
- Steepest Descent
  - Quadratic Interpolation

Figure 5: Steepest Descent and Quadratic Interpolation



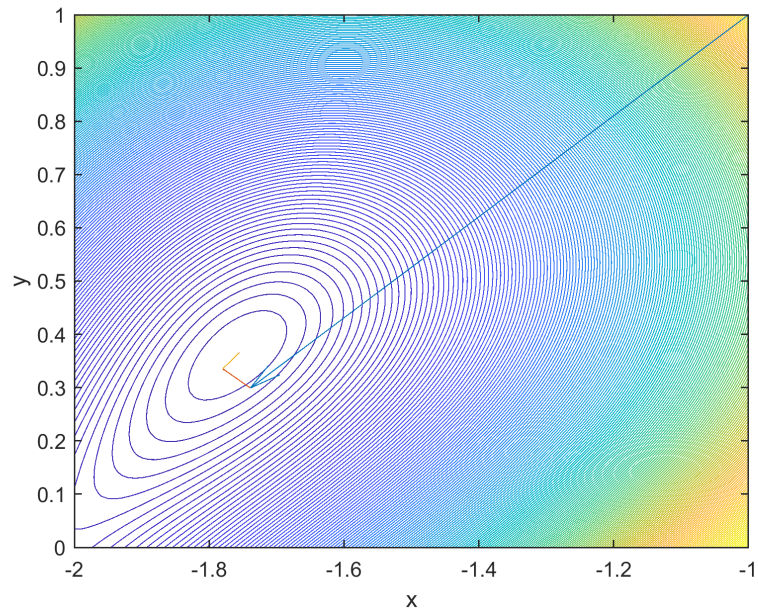
- Golden Section

Figure 6: Steepest Descent and Golden Section



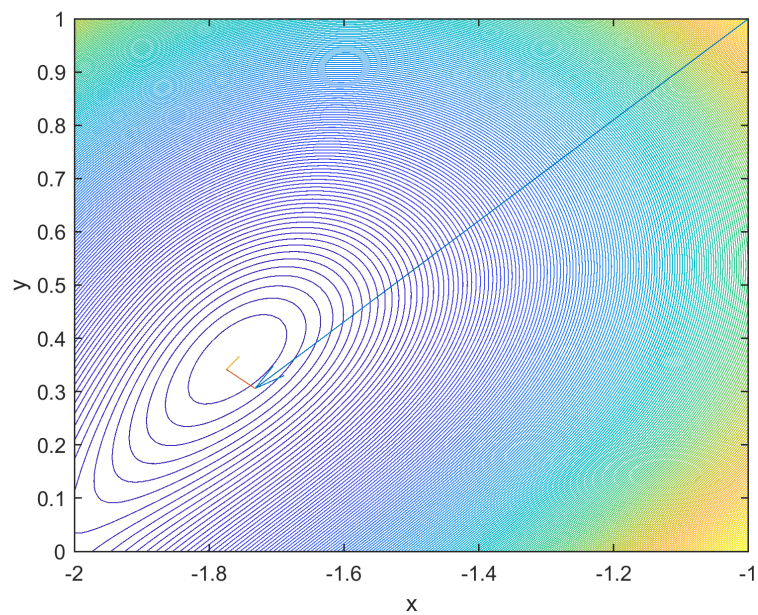
- BFGS
  - Quadratic Interpolation

Figure 7: BFGS and Quadratic Interpolation



- Golden Section

Figure 8: BFGS and Golden Section



### 1.2.2 result

- Time

Table 4: Time compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
0.208 sec	0.146 sec	0.106 sec	0.142 sec

- Number of Cost calculation

Table 5: Number of Cost calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
246	285	142	142

- Number of Gradient calculation

Table 6: Number of Gradient calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
14	12	7	7

## 2 Question 2

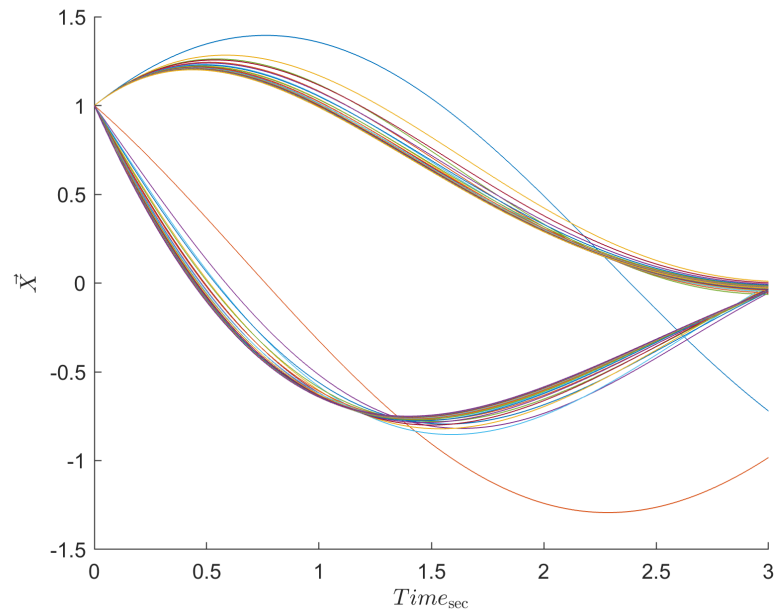
### 2.1 part a

Tolerance is:  $10^{-4}$

#### 2.1.1 figures

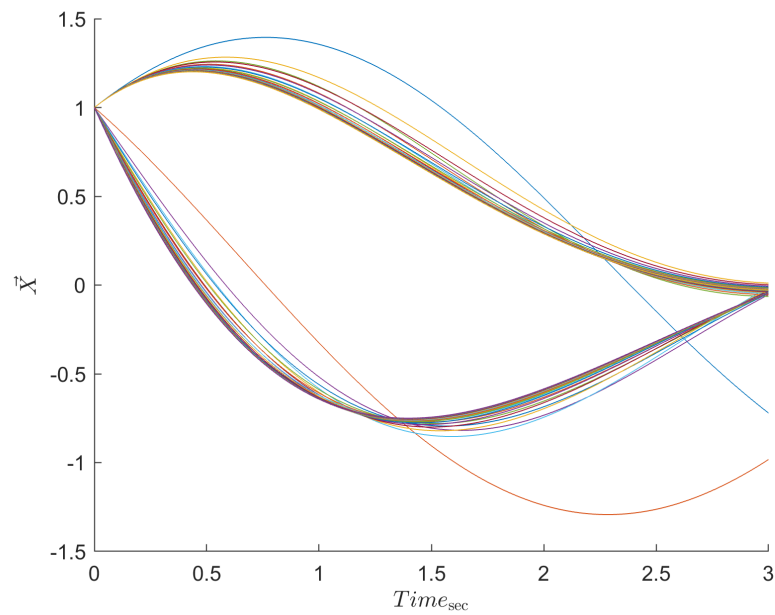
- Steepest Descent
  - Quadratic Interpolation

Figure 9: Steepest Descent and Quadratic Interpolation



- Golden Section

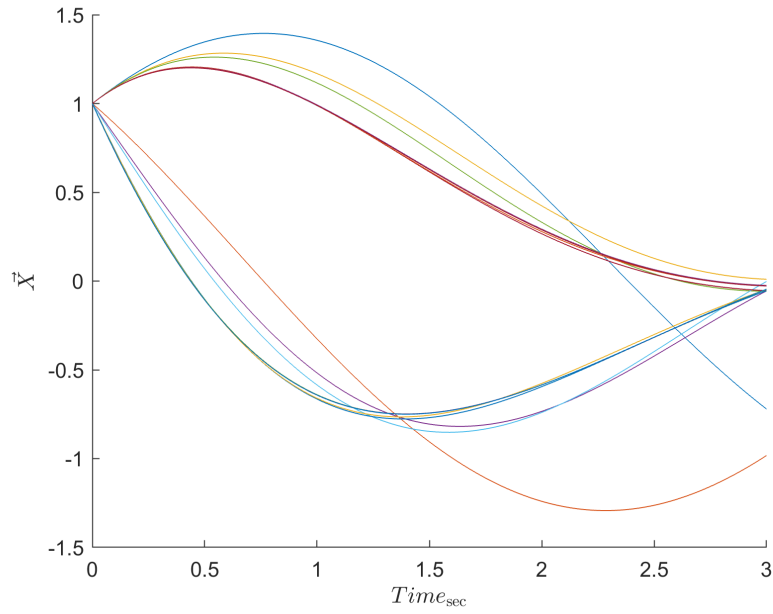
Figure 10: Steepest Descent and Golden Section





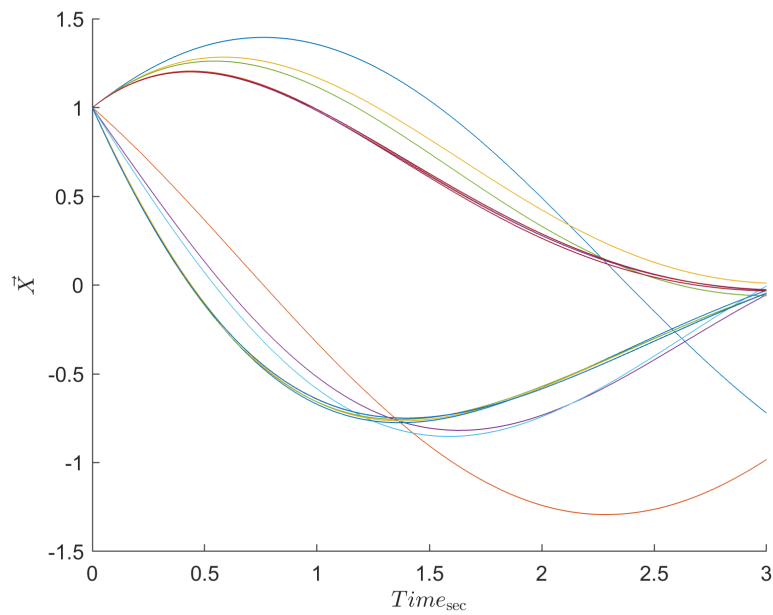
- BFGS
  - Quadratic Interpolation

Figure 11: BFGS and Quadratic Interpolation



- Golden Section

Figure 12: BFGS and Golden Section



**2.1.2 result**

- Time

Table 7: Time compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
17.000 sec	24.353 sec	3.905 sec	4.985 sec

- Number of Cost calculation

Table 8: Number of Cost calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
1285	1922	273	373

- Number of Gradient calculation

Table 9: Number of Gradient calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
51	51	11	11

### 2.1.3 Four iteration for BFGS and Quadratic interpolation

Figure 13: BFGS and Quadratic Interpolation with four iteration

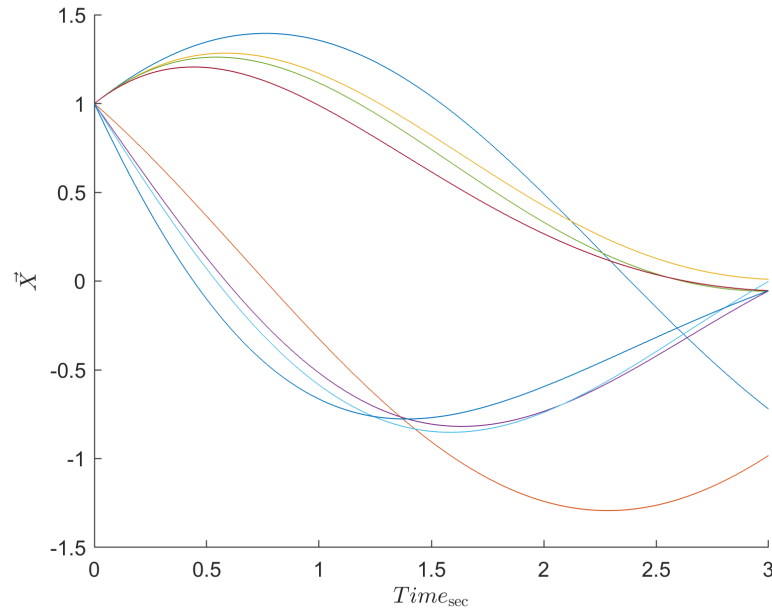


Table 10: four iteration and gradient tolerance compare

	Time	Number of Cost calculation	Number of Gradient calculation
Four iteration	3.905 <sub>sec</sub>	273	11
Gradient tolerance	1.586 <sub>sec</sub>	100	4

## 2.2 part b

Tolerance is:  $10^{-16}$  for  $\lambda S_i$  or  $10^{-4}$  for norm of gradient.

### 2.2.1 figures

- Steepest Descent
  - Quadratic Interpolation

Figure 14: Steepest Descent and Quadratic Interpolation

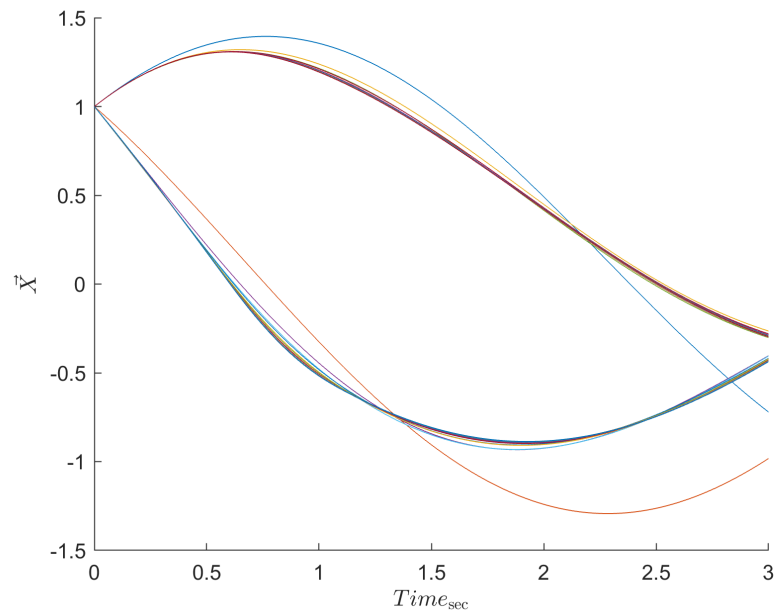
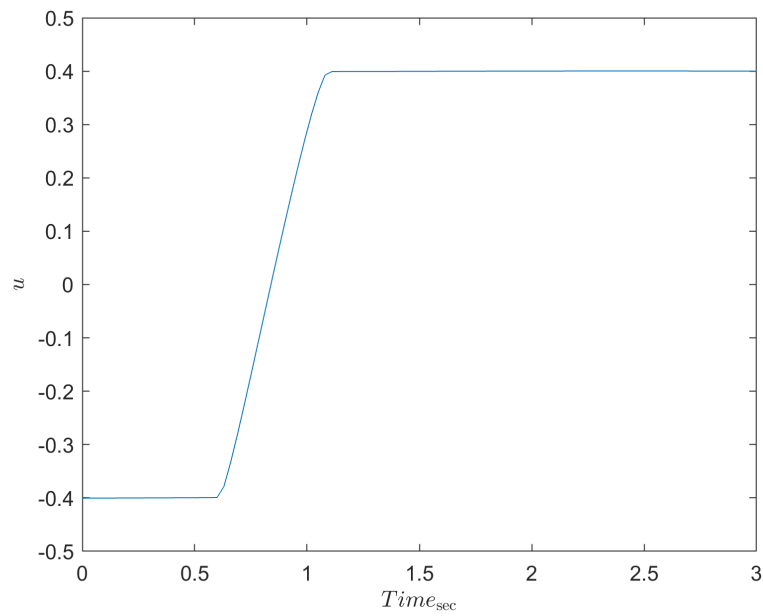


Figure 15: Steepest Descent and Quadratic Interpolation Control



- Golden Section



Figure 16: Steepest Descent and Golden Section

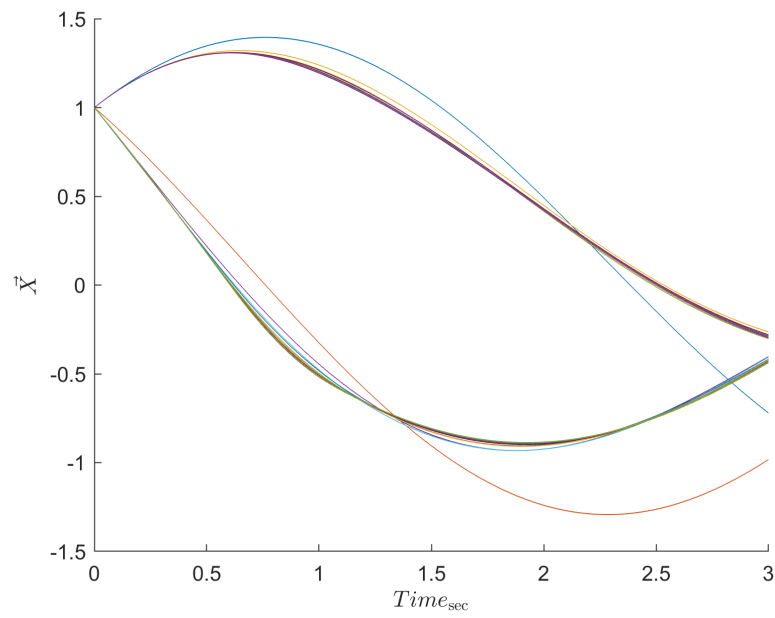
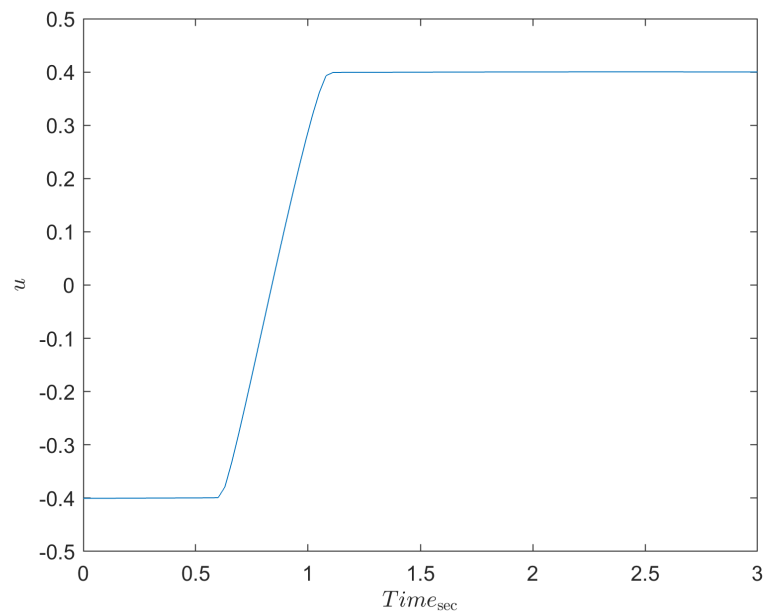


Figure 17: Steepest Descent and Golden Section Control



- BFGS
  - Quadratic Interpolation

Figure 18: BFGS and Quadratic Interpolation

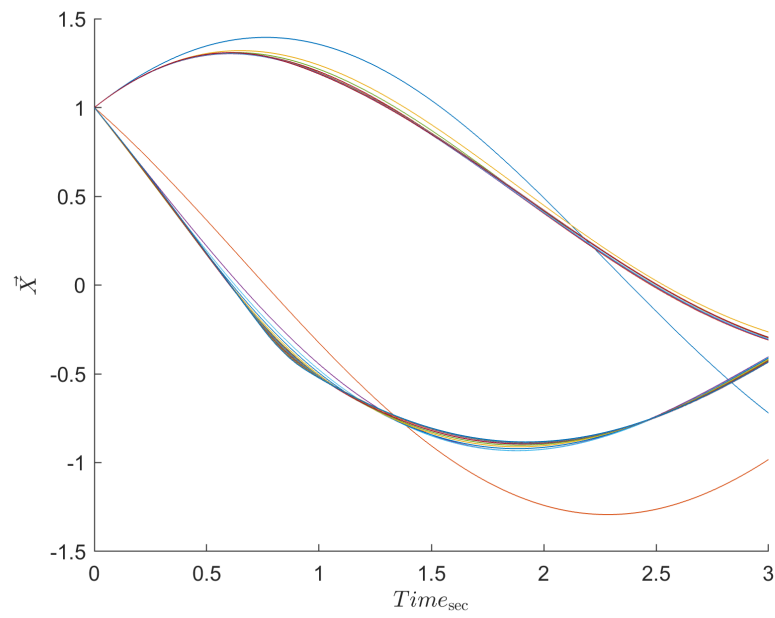
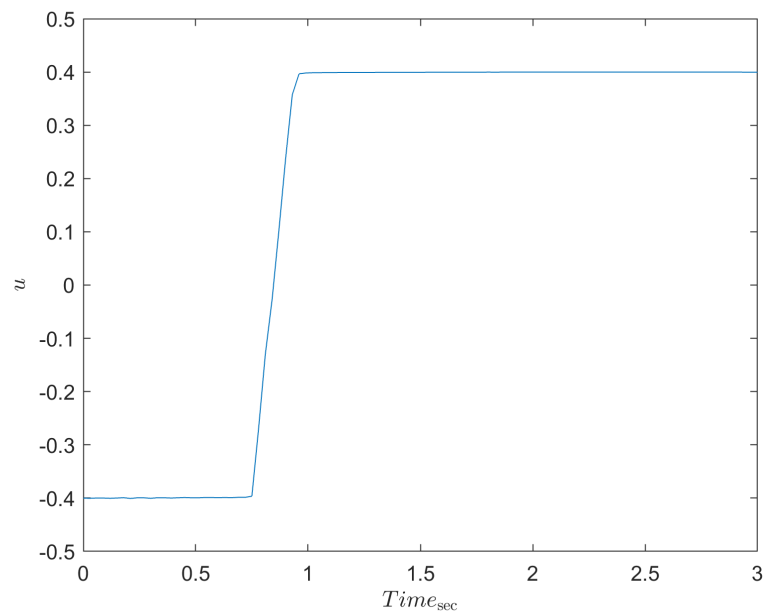


Figure 19: BFGS and Quadratic Interpolation Control



– Golden Section

Figure 20: BFGS and Golden Section

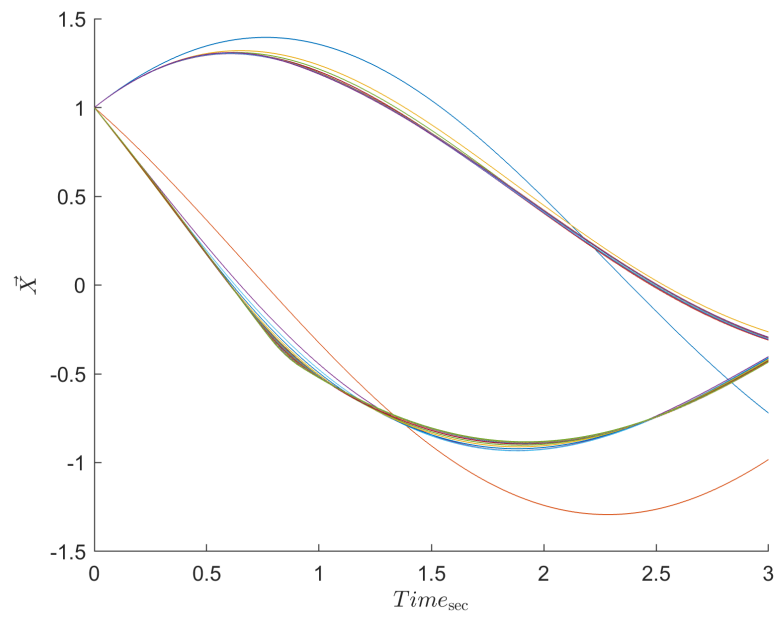
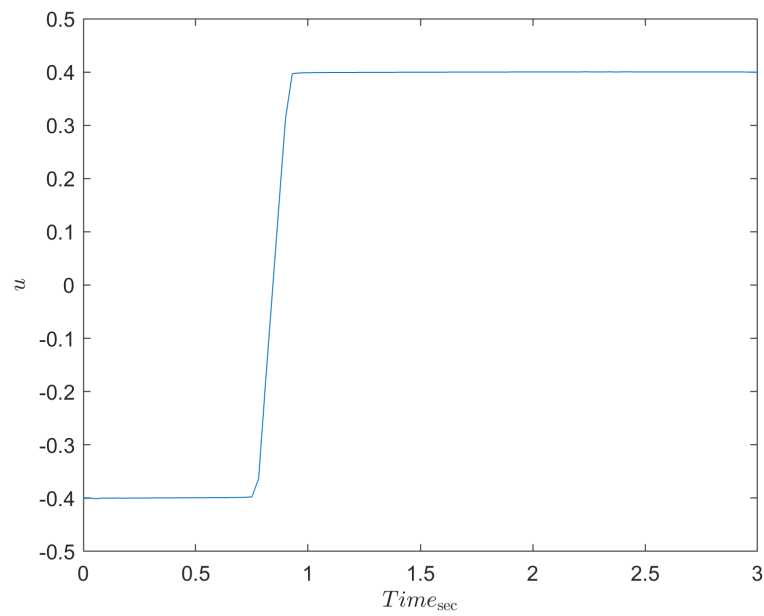


Figure 21: BFGS and Golden Section Control



### 2.2.2 result

- Time

Table 11: Time compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
7.595 sec	33.761 sec	86.730 sec	72.666 sec

- Number of Cost calculation

Table 12: Number of Cost calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
442	1782	2787	2378

- Number of Gradient calculation

Table 13: Number of Gradient calculation compare between four methods

Steepest Descent		BFGS	
Quadratic Interpolation	Golden Section	Quadratic Interpolation	Golden Section
25	55	256	174



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