

# Homework #1

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Course: *Optimal Control I* – Professor: *Dr. Assadian*  
Due date: *March 28th, 2025*

## Problem 1

(a)  $z = f(x, y) = y \sin(x + y) - x \sin(x - y)$

Gradient of  $f(x, y)$ :

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{bmatrix} y \cos(x + y) - \sin(x - y) - x \cos(x - y) \\ y \cos(x + y) + \sin(x + y) + x \cos(x - y) \end{bmatrix}$$

Two nonlinear equations with two unknowns. We use MATLAB to solve this equations. MATLAB file is attached. Answers are provided in table 1

Table 1: Answers

x	y
-3.41877	-1.82764
-2.88904	1.84693
-2.02875	0.00000
-1.84693	-2.88904
-1.82764	3.41877
-1.75560	0.36547
-0.36547	-1.7556
0.00000	-2.02875
0.00000	0.00000
0.00000	2.02875
0.36547	1.7556
1.75560	-0.36547
1.82764	-3.41877
1.84693	2.88904
2.02875	0.00000
2.88904	-1.84693
3.41877	1.82764

Hessian matrix:

$$H = \frac{\partial^2 f}{\partial \vec{x}^2} = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{bmatrix} -y \sin(x + y) - 2 \cos(x - y) + x \cos(x - y) & \cos(x + y) - y \sin(x + y) + \cos(x - y) - x \sin(x - y) \\ \cos(x + y) - y \sin(x + y) + \cos(x - y) - x \sin(x - y) & x \sin(x - y) + 2 \cos(x + y) - y \sin(x + y) \end{bmatrix}$$

Hessian matrix and eigenvalues have calculated in MATLAB and attached. Maximum Minimum Saddle Point

Table 2: Answers With Conditions

x	y	Point Condition
-3.41877	-1.82764	Maximum
-2.88904	1.84693	Saddle Point
-2.02875	0.00000	Saddle Point
-1.84693	-2.88904	Saddle Point
-1.82764	3.41877	Minimum
-1.75560	0.36547	Maximum
-0.36547	-1.7556	Minimum
0.00000	-2.02875	Saddle Point
0.00000	0.00000	Saddle Point
0.00000	2.02875	Saddle Point
0.36547	1.7556	Minimum
1.75560	-0.36547	Saddle Point
1.82764	-3.41877	Minimum
1.84693	2.88904	Saddle Point
2.02875	0.00000	Saddle Point
2.88904	-1.84693	Saddle Point
3.41877	1.82764	Maximum

Answers and conditions are provided in table 2

Figure 1: 3D figure of function

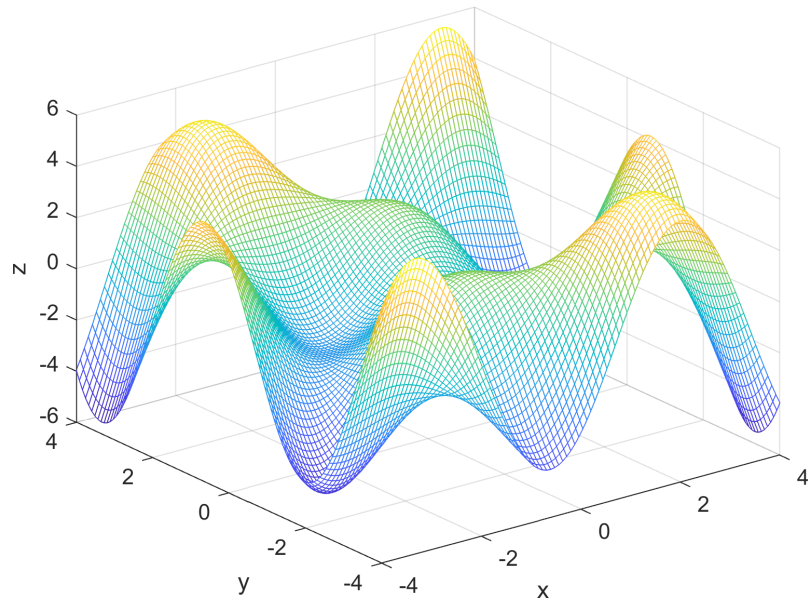


Figure 2: 3D figure of function with Points

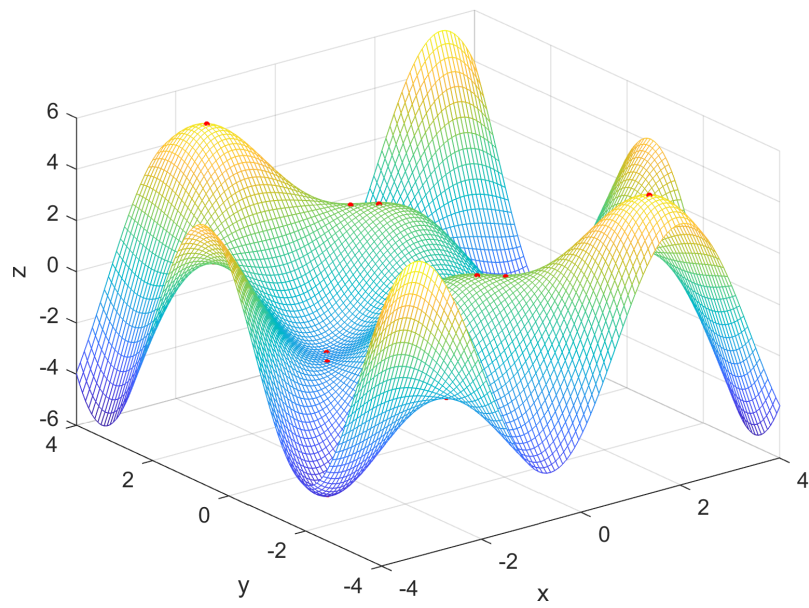


Figure 3: Contour figure of function

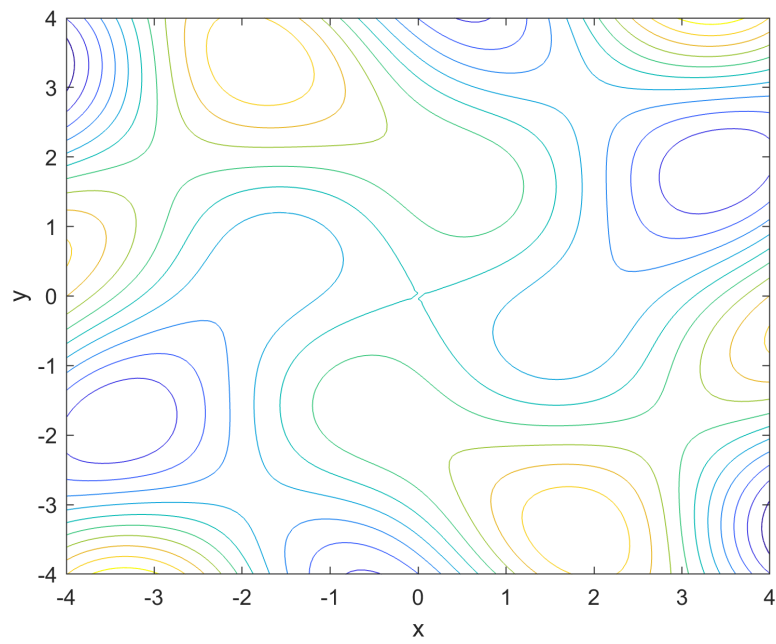
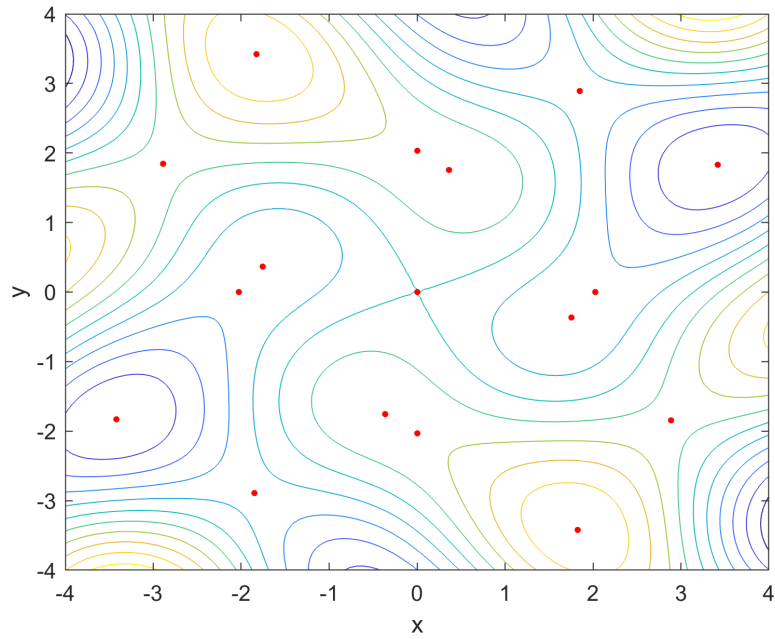


Figure 4: Contour figure of function with Points



- (b)  $z = f(x, y) = x^3 - 3xy^2$   
 Gradient of  $f(x, y)$ :

$$\vec{\nabla} f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\vec{\nabla} f = \begin{bmatrix} 3x^2 - 3y^2 \\ -6xy \end{bmatrix}$$

Two linear equations with two unknowns.

$$3x^2 - 3y^2 = 0$$

$$-6xy = 0$$

Answers is  $x = 0$  and  $y = 0$ .

Hessian matrix:

$$H = \frac{\partial^2 f}{\partial \vec{X}^2} = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial xy} \\ \frac{\partial^2 f}{\partial yx} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$H = \begin{bmatrix} 6x & -6y \\ -6y & -6x \end{bmatrix}$$

In  $x = 0$  and  $y = 0$  Hessian matrix in :

$$H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

so this point is saddle point.

Figure 5: 3D figure of function

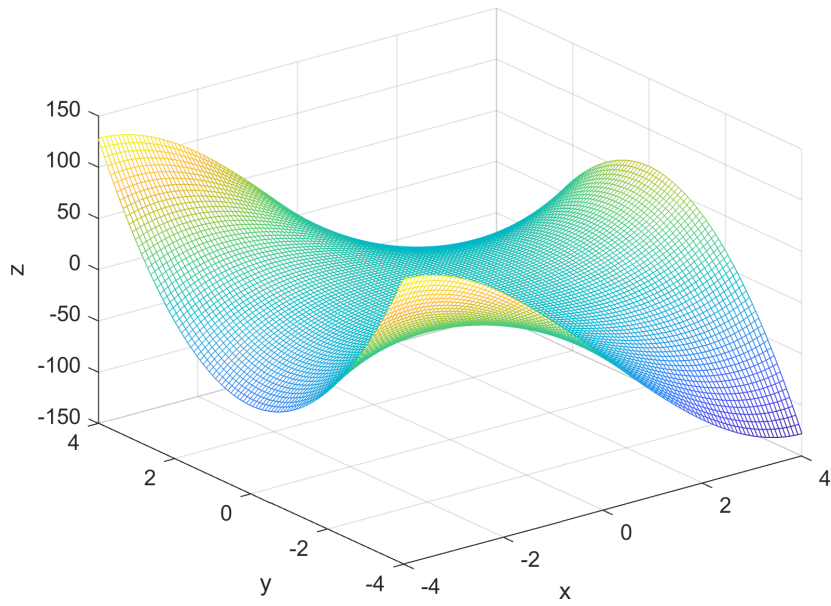


Figure 6: 3D figure of function with Points

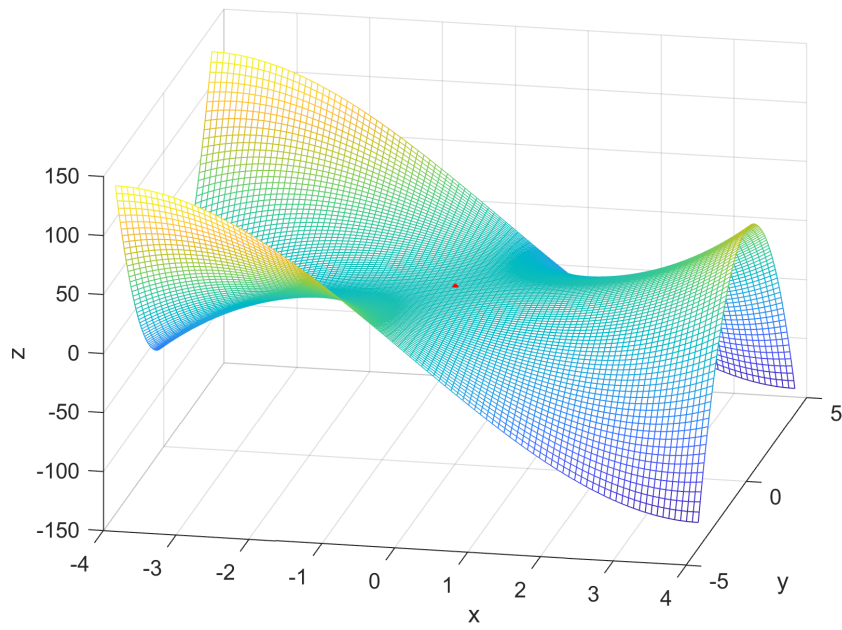


Figure 7: Contour figure of function

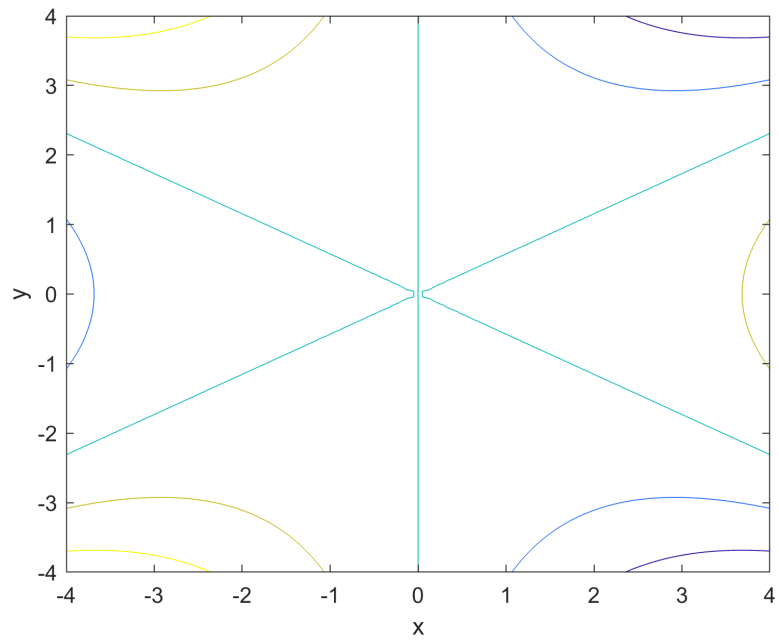


Figure 8: Contour figure of function with Points

