Home Work #1

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1 Question 1

$$f_X(x) = \frac{ab}{b^2 + x^2}, \quad b > 0$$

1.1 part a

$$\int_{-\infty}^{+\infty} f(x)\mathrm{d}x = 1 \quad \rightarrow \quad \int_{-\infty}^{+\infty} \frac{ab}{b^2 + x^2}\mathrm{d}x = 1 \rightarrow a \arctan(\frac{x}{b})\Big|_{-\infty}^{+\infty} = 1 \rightarrow a\pi = 1 \rightarrow a = \frac{1}{\pi}$$

$$f_X(x) = \frac{1}{\pi}\frac{b}{b^2 + x^2}, \quad b > 0$$

1.2 part b

$$E(X) = \mu_X = \int_{-\infty}^{+\infty} xf(x)\mathrm{d}x$$

Because $xf(x)$ is an odd function, the result of the integrator between ∞ and $-\infty$ is zero.

$$\int_{-\infty}^{+\infty} xf(x)\mathrm{d}x = 0 \rightarrow \mu_X = 0$$

$$\sigma_X^2 = E((X-\mu)^2) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\mathrm{d}x = \int_{-\infty}^{+\infty} x^2 f(x)\mathrm{d}x = \frac{b}{\pi}(x - b\arctan(\frac{x}{b}))\Big|_{-\infty}^{+\infty} \neq \text{finite}$$

2 Question 3

A positive test is A event:$P(A)$, $\quad$ Having the flu is B event: $P(B) = 0.05$

2.1 part a

The probability of a positive test if someone has flu:

$$P(A|B) = 0.99$$

The probability of a positive test if someone doesn't have flu:

$$P(A|\bar{B}) = 0.01$$

$$P(A) = P(A \cap B) + P(A \cap \bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B}) = 0.99 \times 0.05 + 0.01 \times 0.95 = 0.059$$

$$P(A|B)P(B) = P(B|A)P(A) \rightarrow P(B|A) = \frac{P(A|B)P(B)}{P(A)} = 0.84$$

2.2 part b

C is the event when two positive tests happen. The probability of two positive tests if someone has flu:

$$P(B)P(A|B)P(A|B) = 0.049$$

The probability of two positive tests if someone doesn't have flu:

$$P(\bar{B})P(A|\bar{B})P(A|\bar{B}) = 9.5 \times 10^{-5}$$

$$P(C) = P(B)P(A|B)P(A|B) + P(\bar{B})P(A|\bar{B})P(A|\bar{B}) = 0.0491$$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = 0.998$$

Contents

1 Question 1 **1**
1.1 part a 1
1.2 part b 1

2 Question 3 **1**
2.1 part a 2
2.2 part b 2

List of Figures

List of Tables