Home Work #1

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1 Question 1

$$f_X(x) = \frac{ab}{b^2 + x^2}, \quad b > 0$$

1.1 part a

$$\int_{-\infty}^{+\infty} f(x)\mathrm{d}x = 1 \quad \rightarrow \quad \int_{-\infty}^{+\infty} \frac{ab}{b^2 + x^2}\mathrm{d}x = 1 \rightarrow a \arctan(\frac{x}{b})\Big|_{-\infty}^{+\infty} = 1 \rightarrow a\pi = 1 \rightarrow a = \frac{1}{\pi}$$

$$f_X(x) = \frac{1}{\pi}\frac{b}{b^2 + x^2}, \quad b > 0$$

1.2 part b

$$E(X) = \mu_X = \int_{-\infty}^{+\infty} xf(x)\mathrm{d}x$$

Because $xf(x)$ is an odd function, the result of the integrator between ∞ and $-\infty$ is zero.

$$\int_{-\infty}^{+\infty} xf(x)\mathrm{d}x = 0 \rightarrow \mu_X = 0$$

$$\sigma_X^2 = E((X-\mu)^2) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\mathrm{d}x = \int_{-\infty}^{+\infty} x^2 f(x)\mathrm{d}x = \frac{b}{\pi}(x - b\arctan(\frac{x}{b}))\Big|_{-\infty}^{+\infty} \neq \text{finite}$$

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