Home Work #2

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1 Question 1

$$G(s) = \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

Steady state error:

$$e_{ss} = \lim_{s\to 0} s \times \frac{1}{G(s)+1} R(s)$$

When input is step $R(s)$ is $\frac{1}{s}$.

$$e_{ss} = \lim_{s\to 0} \frac{1}{G(s)+1}$$

1.1 lead compensation

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \quad 0 < \alpha < 1$$

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1}$$

Define:

$$K_c \alpha = K$$

then:

$$G_c(s) = K \frac{Ts+1}{\alpha Ts+1}$$

The open-loop transfer function of the compensated system is:

$$G_c(s)G(s) = G_c(s) = K\frac{Ts+1}{\alpha Ts+1}G(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1}KG(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1}G_1(s)$$

Where:

$$KG(s) = G_1(s)$$

Steady state error must be less than 0.05%

$$e_{ss} = \lim_{s\to 0} = \frac{1}{1+G_1(0)} = 0.05 \to 0.05 + 0.05G_1(0) = 1 \to 0.95 = 0.05G_1(0) \to G_1(0) = 19$$

$$\lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)} = \frac{50 \times 0.5}{1 \times 1.5^3 \times 2} = 3.7037 \xrightarrow[G_1 = KG]{G_1(0)=19} K = 5.1300$$

The amplitude ratio:

$$|G_1(j\omega)| = |KG(j\omega)| = 5.1300 \frac{50\sqrt{\omega^2 + 0.5^2}}{\sqrt{\omega^2 + 1^2} \times (\sqrt{\omega^2 + 1.5^2})^3 \times \sqrt{\omega^2 + 2^2}}$$

Gain Crossover frequency:

$$|G_1(j\omega_g)| = 1$$

$$5.1300 \frac{50\sqrt{\omega_g^2 + 0.5^2}}{\sqrt{\omega_g^2 + 1^2} \times (\sqrt{\omega_g^2 + 1.5^2})^3 \times \sqrt{\omega_g^2 + 2^2}} = 1$$

$$26.3169 \times 2500(\omega_g^2 + 0.25) = (\omega_g^2 + 1)(\omega_g^2 + 2.25)^3(\omega_g^2 + 4)$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

$$\omega_g = 3.6233$$

The phase angle:

$$\angle G(j\omega) = 0^\circ + \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\frac{\omega}{1} - 3\tan^{-1}\frac{\omega}{1.5} - \tan^{-1}\frac{\omega}{2}$$

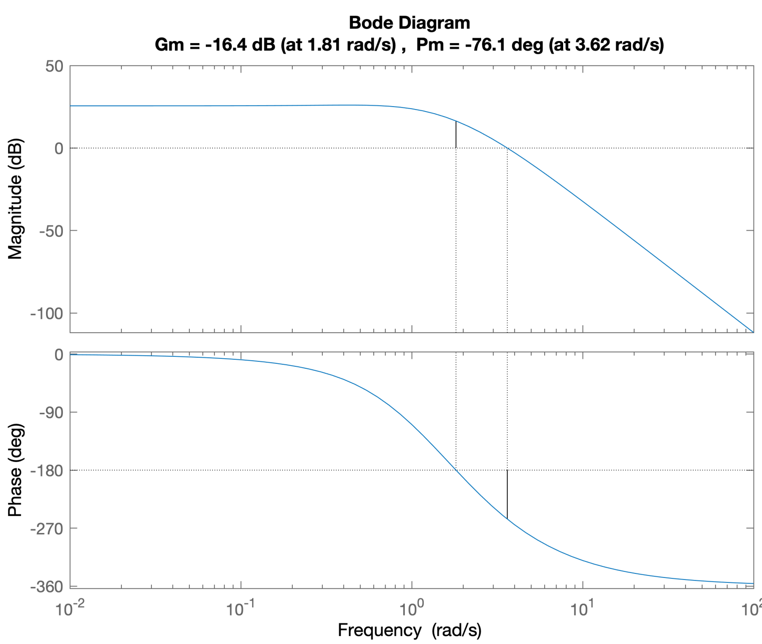
$$\angle G(j\omega_g) = \tan^{-1}\frac{3.6233}{0.5} - \tan^{-1}\frac{3.6233}{1} - 3\tan^{-1}\frac{3.6233}{1.5} - \tan^{-1}\frac{3.6233}{2} = -4.46917_{rad} = -256.064^\circ$$

Phase margin:

$$\gamma = 180 + \angle G(j\omega_g) = 180 - 256.064 = -76.064$$

Now we check above calculation with MATLAB margin function.

Figure 1: Bode digram using MATLAB



MATLAB bode digram and our calculation are exactly the same.

$$\bar{\gamma} = 45 + 5 = 50°, \quad \phi_m = \bar{\gamma} - \gamma = 126.0646°.$$

$$\phi_m = 85°$$

This is too much! best we can do is about 90° in lead compensation so we add 85°

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)} = 0.0019$$

α is very low.

$$K_c = \frac{K}{\alpha} = \frac{5.1300}{0.0019} = 2691.1$$

$$|G_1(j\omega_m)| = \sqrt{\alpha} = |KG(j\omega_m)| = 5.13\frac{50\sqrt{\omega_m^2 + 0.5^2}}{\sqrt{\omega_m^2 + 1^2} \times (\sqrt{\omega_m^2 + 1.5^2})^3 \times \sqrt{\omega_m^2 + 2^2}} = 0.0019$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

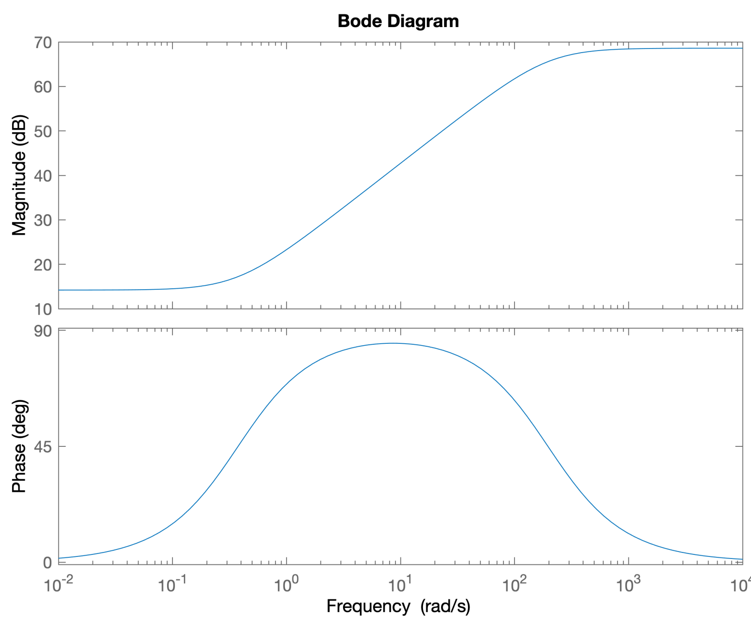
$$\omega_m = 8.58898$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \rightarrow T = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{8.58898\sqrt{0.0019}} = 2.6666$$

$$G_c(s) = K_c\frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = 2691.1\frac{s + 0.3750}{s + 196.720}$$

Bode digram for lead compensation using MATLAB.

Figure 2: lead compensation Bode digram using MATLAB

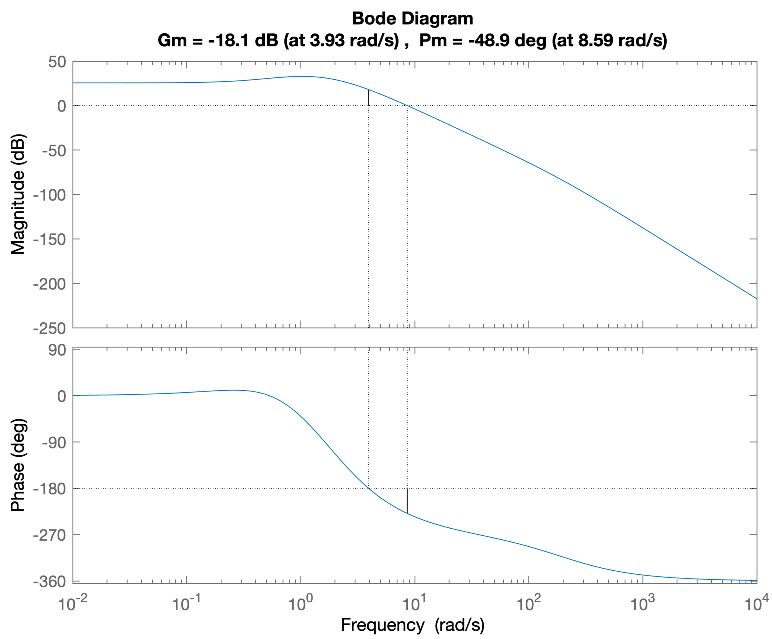


Now add lead controller to system.

$$G_c(s)G(s) = 2691.1\frac{s+0.3750}{s+196.720}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

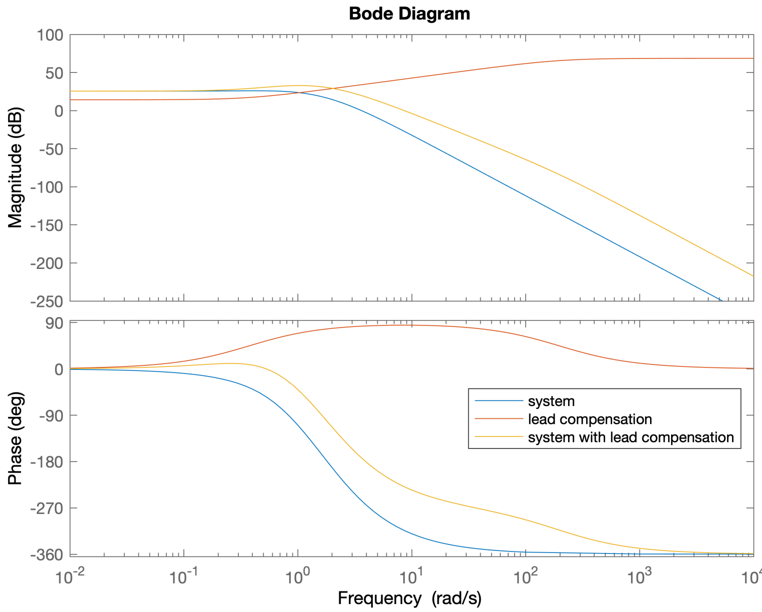
Bode digram for system with adding lead compensation.

Figure 3: Bode digram for system with lead compensation using MATLAB



That was the best we can do with lead compensation! All bode digram in one figure:

Figure 4: all bode digram using MATLAB



1.2 lag compensation

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}, \quad \beta > 1$$

In lag compensation we find out K for steady state error.

$$K = 5.1300$$

$$\gamma_d = 45^\circ \rightarrow \bar{\gamma_d} = 45 + 5 = 50^\circ$$

$$\phi(\omega_g) = 50 - 180 = -130^\circ$$

The phase angle:

$$\angle G(j\omega) = 0^\circ + \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\frac{\omega}{1} - 3\tan^{-1}\frac{\omega}{1.5} - \tan^{-1}\frac{\omega}{2}$$

$$\angle G(j\omega_g) = 0^\circ + \tan^{-1}\frac{\omega_g}{0.5} - \tan^{-1}\frac{\omega_g}{1} - 3\tan^{-1}\frac{\omega_g}{1.5} - \tan^{-1}\frac{\omega_g}{2} = -130^\circ$$

This equation solved with MATLAB and code has attacked (Q1_b.m)

$$\omega_g = 1.2025$$

The amplitude ratio:

$$|G_1(j\omega)| = |KG(j\omega)| = 5.1300 \frac{50\sqrt{\omega^2 + 0.5^2}}{\sqrt{\omega^2 + 1^2} \times (\sqrt{\omega^2 + 1.5^2})^3 \times \sqrt{\omega^2 + 2^2}}$$

$$\beta = |G_1(j\omega_g)| = 5.1300\frac{50\sqrt{\omega_g^2+0.5^2}}{\sqrt{\omega_g^2+1^2}\times(\sqrt{\omega_g^2+1.5^2})^3\times\sqrt{\omega_g^2+2^2}}$$

$$\beta = 5.1300\frac{50\sqrt{1.2025^2+0.5^2}}{\sqrt{1.2025^2+1^2}\times(\sqrt{1.2025^2+1.5^2})^3\times\sqrt{1.2025^2+2^2}} = 12.8807$$

Assume:

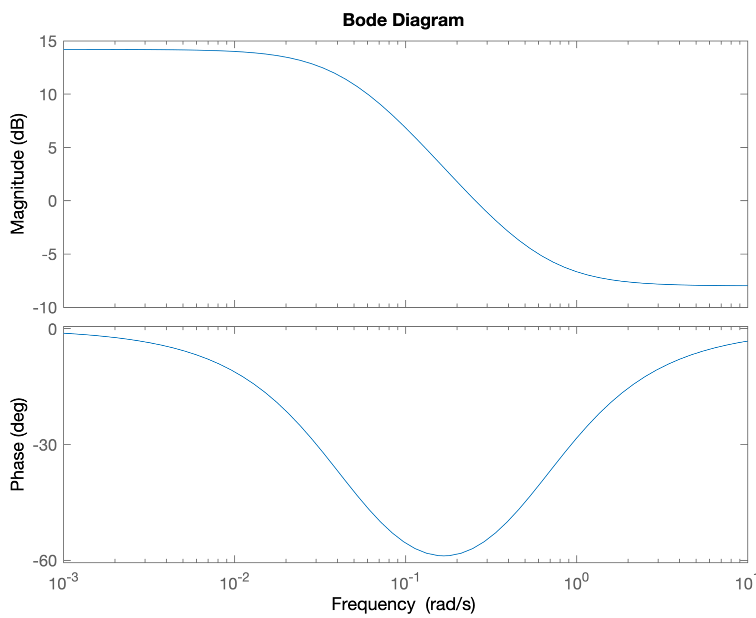
$$\frac{1}{T} = \frac{\omega_g}{2} \to \frac{1}{T} = 0.6012 \to T = 1.6632$$

$$\to \frac{1}{\beta T} = 0.0467, \quad K_c = \frac{K}{\beta} = \frac{5.1300}{12.8807} = 0.3983$$

$$G_c(s) = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} = 0.3983\frac{s+0.6012}{s+0.0467}$$

Bode digram for lag compensation using MATLAB.

Figure 5: lag compensation Bode digram using MATLAB

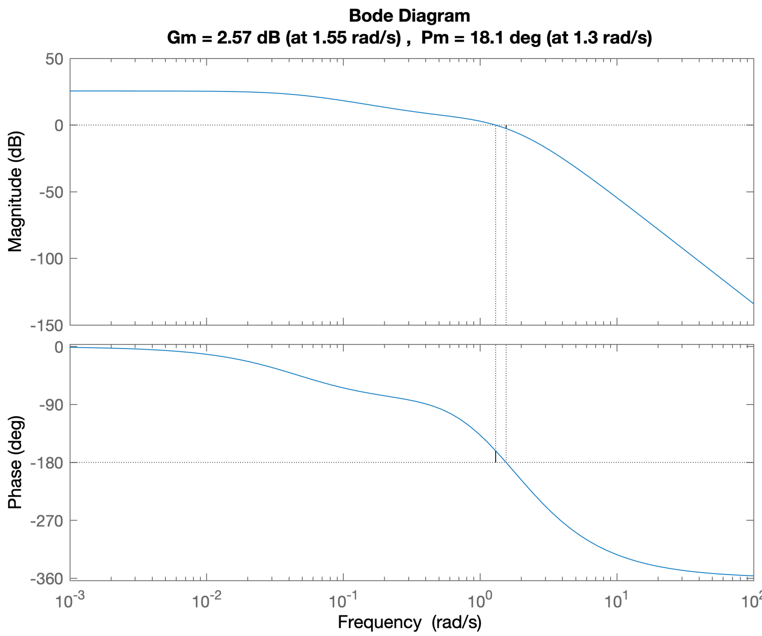


Now add lag controller to system.

$$G_c(s)G(s) = 0.3983\frac{s+0.6012}{s+0.0467}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

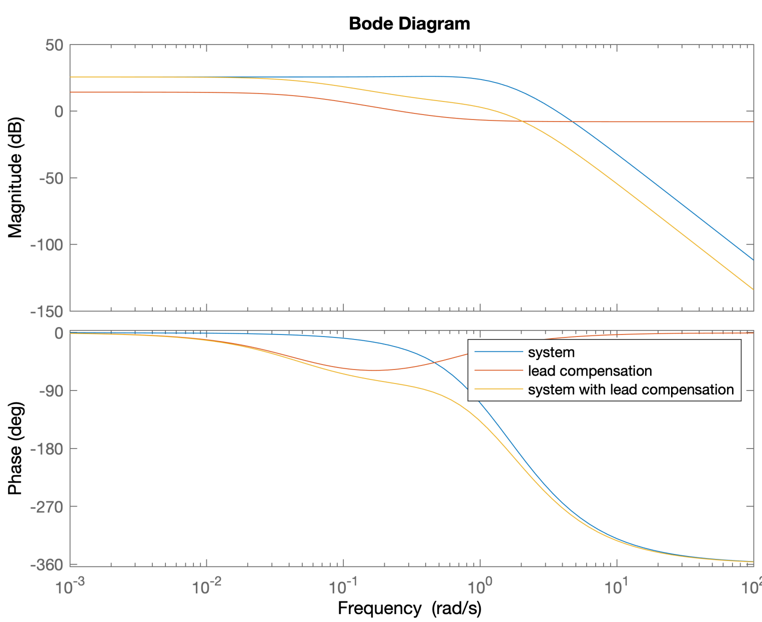
Bode digram for system with adding lag compensation.

Figure 6: Bode digram for system with lag compensation using MATLAB



All bode digram in one figure:

Figure 7: all bode digram using MATLAB



We didn't get what we want in equation so we change our before assumation. assume:

$$\frac{1}{T} = \frac{\omega_g}{10} \rightarrow \frac{1}{T} = 0.1202 \rightarrow T = 1.6632$$

$$\rightarrow \frac{1}{\beta T} = 0.0467, \quad K_c = \frac{K}{\beta} = \frac{5.1300}{12.8807} = 0.3983$$

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = 0.3983 \frac{s + 0.6012}{s + 0.0467}$$

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