Home Work #2

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1 Question 1

$$G(s) = \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

Steady state error:

$$e_{ss} = \lim_{s \to 0} s \times \frac{1}{G(s)+1} R(s)$$

When input is step $R(s)$ is $\frac{1}{s}$.

$$e_{ss} = \lim_{s \to 0} \frac{1}{G(s)+1}$$

1.1 lead compensation

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \quad 0 < \alpha < 1$$

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1}$$

Define:

$$K_c \alpha = K$$

then:

$$G_c(s) = K \frac{Ts+1}{\alpha Ts+1}$$

The open-loop transfer function of the compensated system is:

$$G_c(s)G(s) = G_c(s) = K \frac{Ts+1}{\alpha Ts+1} G(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1} KG(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1} G_1(s)$$

Where:

$$KG(s) = G_1(s)$$

Steady state error must be less than 0.05%

$$e_{ss} = \lim_{s \to 0} = \frac{1}{1+G_1(0)} = 0.05 \to 0.05 + 0.05G_1(0) = 1 \to 0.95 = 0.05G_1(0) \to G_1(0) = 19$$

$$\lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)} = \frac{50 \times 0.5}{1 \times 1.5^3 \times 2} = 3.7037 \xrightarrow[G_1=KG]{G_1(0)=19} K = 5.1300$$

The amplitude ratio:

$$|G(j\omega)| = \frac{50\sqrt{\omega^2+0.5^2}}{\sqrt{\omega^2+1^2}\times(\sqrt{\omega^2+1.5^2})^3\times\sqrt{\omega^2+2^2}}$$

Gain Crossover frequency:

$$|G(j\omega_g)| = 1$$

$$\frac{50\sqrt{\omega_g^2+0.5^2}}{\sqrt{\omega_g^2+1^2}\times(\sqrt{\omega_g^2+1.5^2})^3\times\sqrt{\omega_g^2+2^2}} = 1$$

$$2500(\omega_g^2+0.25) = (\omega_g^2+1)(\omega_g^2+2.25)^3(\omega_g^2+4)$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

$$\omega_g = 2.037$$

The phase angle:

$$\angle G(j\omega) = 0^\circ + \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\frac{\omega}{1} - 3\tan^{-1}\frac{\omega}{1.5} - \tan^{-1}\frac{\omega}{2}$$

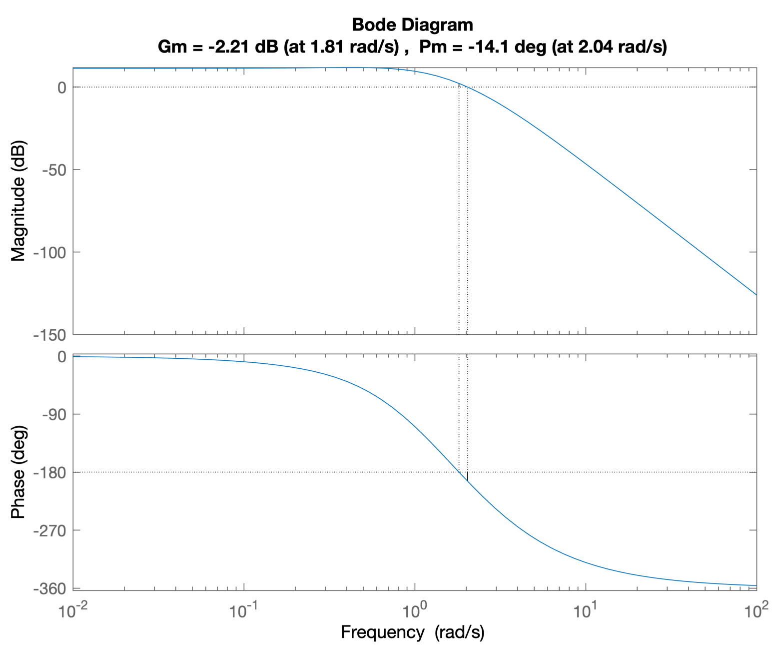
$$\angle G(j\omega_g) = \tan^{-1}\frac{2.037}{0.5} - \tan^{-1}\frac{2.037}{1} - 3\tan^{-1}\frac{2.037}{1.5} - \tan^{-1}\frac{2.037}{2} = -3.3871_{rad} = -194.0702^\circ$$

Phase margin:

$$\gamma = 180 + \angle G(j\omega_g) = 180 - 194.0702 = -14.0702$$

Now we check above calculation with MATLAB margin function.

Figure 1: Bode digram using MATLAB



MATLAB bode digram and our calculation are exactly the same.

$$\bar{\gamma} = 45 + 5 = 50^\circ, \quad \phi_m = \bar{\gamma} - \gamma = 64.070^\circ$$

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)} = 0.0530$$

$$K_c = \frac{K}{\alpha} = \frac{5.1300}{0.0530} = 96.787$$

$$|G_1(j\omega_m)| = \sqrt{\alpha} = |KG(j\omega_m)| = 5.13\frac{50\sqrt{\omega_m^2 + 0.5^2}}{\sqrt{\omega_m^2 + 1^2} \times (\sqrt{\omega_m^2 + 1.5^2})^3 \times \sqrt{\omega_m^2 + 2^2}} = 0.230$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

$$\omega_m = 5.5226$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \rightarrow T = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{5.5226\sqrt{0.0530}} = 0.7865$$

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = 96.787\frac{s+1.2714}{s+23.9880}$$

Contents

1 **1**
1.1 lead compensation 1

List of Figures

1 Bode digram using MATLAB 2