Home Work #2

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1 Question 1

$$G(s) = \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

Steady state error:

$$e_{ss} = \lim_{s \to 0} s \times \frac{1}{G(s)+1} R(s)$$

When input is step $R(s)$ is $\frac{1}{s}$.

$$e_{ss} = \lim_{s \to 0} \frac{1}{G(s)+1}$$

1.1 lead compensation

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}, \quad 0 < \alpha < 1$$

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1}$$

Define:

$$K_c \alpha = K$$

then:

$$G_c(s) = K \frac{Ts+1}{\alpha Ts+1}$$

The open-loop transfer function of the compensated system is:

$$G_c(s)G(s) = G_c(s) = K \frac{Ts+1}{\alpha Ts+1} G(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1} KG(s) = G_c(s) = \frac{Ts+1}{\alpha Ts+1} G_1(s)$$

Where:

$$KG(s) = G_1(s)$$

Steady state error must be less than 0.05%

$$e_{ss} = \lim_{s \to 0} = \frac{1}{1+G_1(0)} = 0.05 \to 0.05 + 0.05G_1(0) = 1 \to 0.95 = 0.05G_1(0) \to G_1(0) = 19$$

$$\lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)} = \frac{50 \times 0.5}{1 \times 1.5^3 \times 2} = 3.7037 \xrightarrow[G_1 = KG]{G_1(0)=19} K = 5.1300$$

The amplitude ratio:

$$|G_1(j\omega)| = |KG(j\omega)| = 5.1300\frac{50\sqrt{\omega^2+0.5^2}}{\sqrt{\omega^2+1^2}\times(\sqrt{\omega^2+1.5^2})^3\times\sqrt{\omega^2+2^2}}$$

Gain Crossover frequency:

$$|G_1(j\omega_g)| = 1$$

$$5.1300\frac{50\sqrt{\omega_g^2+0.5^2}}{\sqrt{\omega_g^2+1^2}\times(\sqrt{\omega_g^2+1.5^2})^3\times\sqrt{\omega_g^2+2^2}} = 1$$

$$26.3169\times 2500(\omega_g^2+0.25) = (\omega_g^2+1)(\omega_g^2+2.25)^3(\omega_g^2+4)$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

$$\omega_g = 3.6233$$

The phase angle:

$$\angle G(j\omega) = 0^\circ + \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\frac{\omega}{1} - 3\tan^{-1}\frac{\omega}{1.5} - \tan^{-1}\frac{\omega}{2}$$

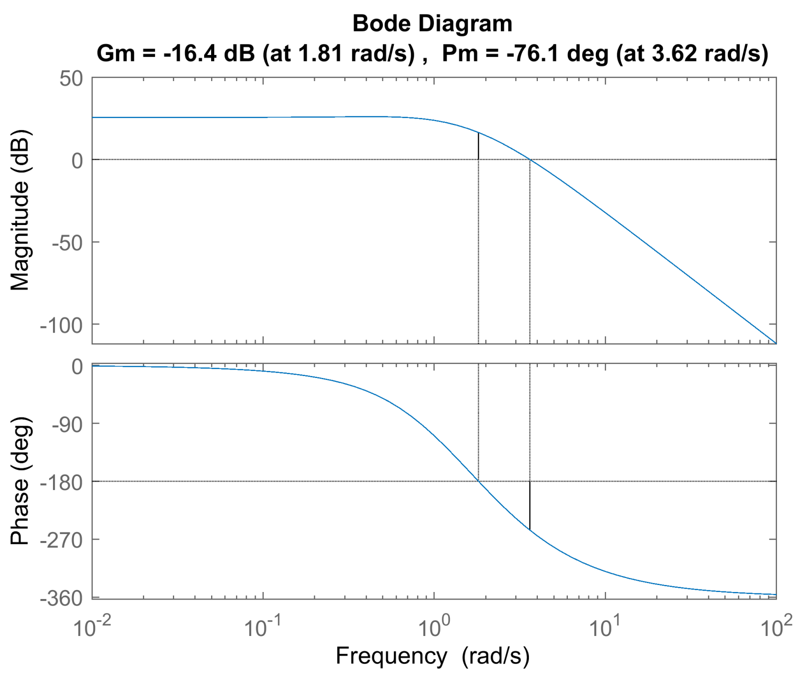
$$\angle G(j\omega_g) = \tan^{-1}\frac{3.6233}{0.5} - \tan^{-1}\frac{3.6233}{1} - 3\tan^{-1}\frac{3.6233}{1.5} - \tan^{-1}\frac{3.6233}{2} = -4.46917_{rad} = -256.064^\circ$$

Phase margin:

$$\gamma = 180 + \angle G(j\omega_g) = 180 - 256.064 = -76.064$$

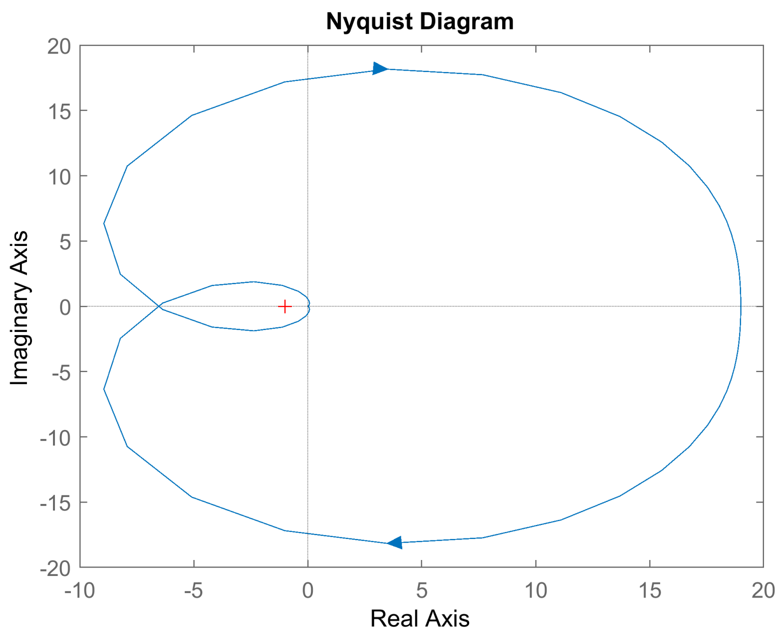
Now we check above calculation with MATLAB margin function.

Figure 1: System Bode diagram using MATLAB



Nyquist plot for system.

Figure 2: System Nyquist plot using MATLAB



MATLAB bode diagram and our calculation are exactly the same.

$$\bar{\gamma} = 45 + 5 = 50^\circ, \quad \phi_m = \bar{\gamma} - \gamma = 126.0646^\circ.$$

$$\phi_m = 85^\circ$$

This is too much! best we can do is about 90° in lead compensation so we add 85°.

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)} = 0.0019$$

α is very low.

$$K_c = \frac{K}{\alpha} = \frac{5.1300}{0.0019} = 2691.1$$

$$|G_1(j\omega_m)| = \sqrt{\alpha} = |KG(j\omega_m)| = 5.13\frac{50\sqrt{\omega_m^2 + 0.5^2}}{\sqrt{\omega_m^2 + 1^2} \times (\sqrt{\omega_m^2 + 1.5^2})^3 \times \sqrt{\omega_m^2 + 2^2}} = \sqrt{0.0019}$$

This equation solved with MATLAB and code has attacked (Q1_a.m).

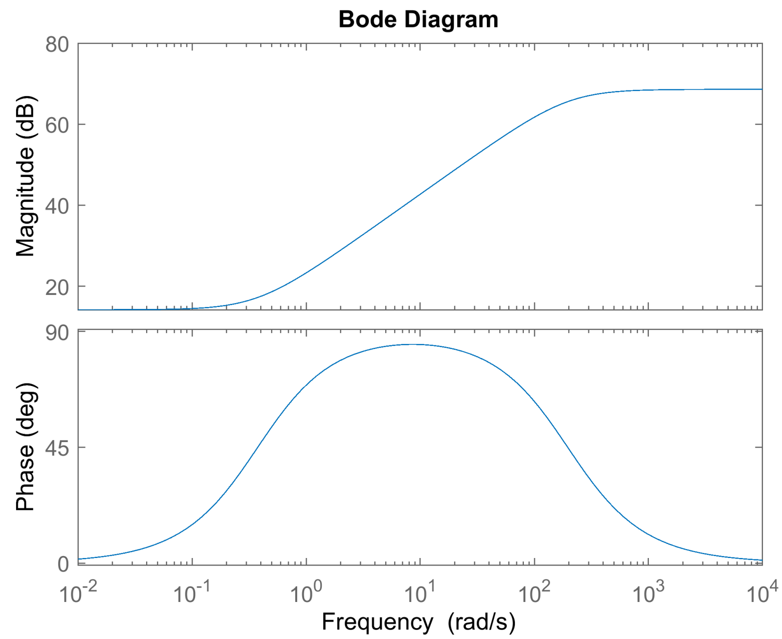
$$\omega_m = 8.58898$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \rightarrow T = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{8.58898\sqrt{0.0019}} = 2.6666$$

$$G_c(s) = K_c\frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = 2691.1\frac{s + 0.3750}{s + 196.720}$$

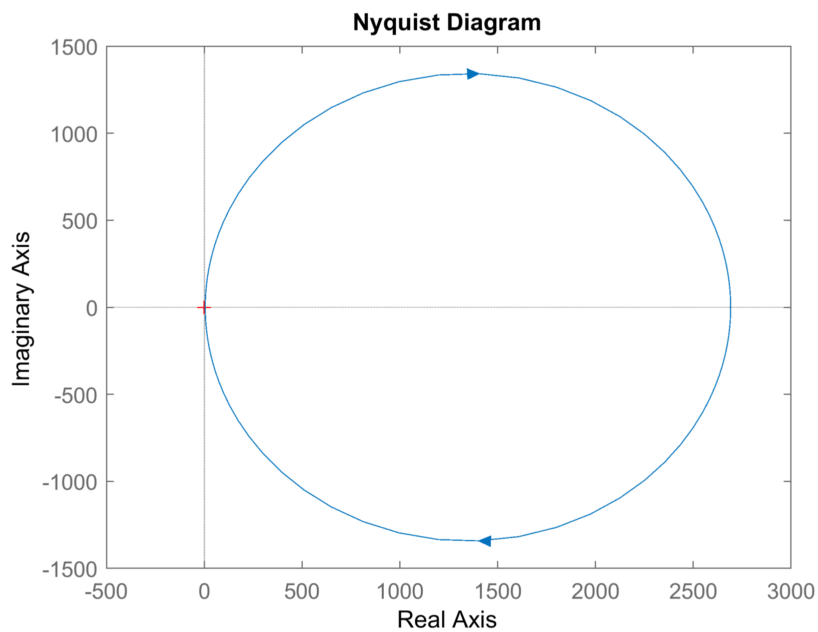
Bode diagram for lead compensation using MATLAB.

Figure 3: lead compensation Bode diagram using MATLAB



Nyquist plot for lead compensation using MATLAB.

Figure 4: lead compensation nyquist plot using MATLAB

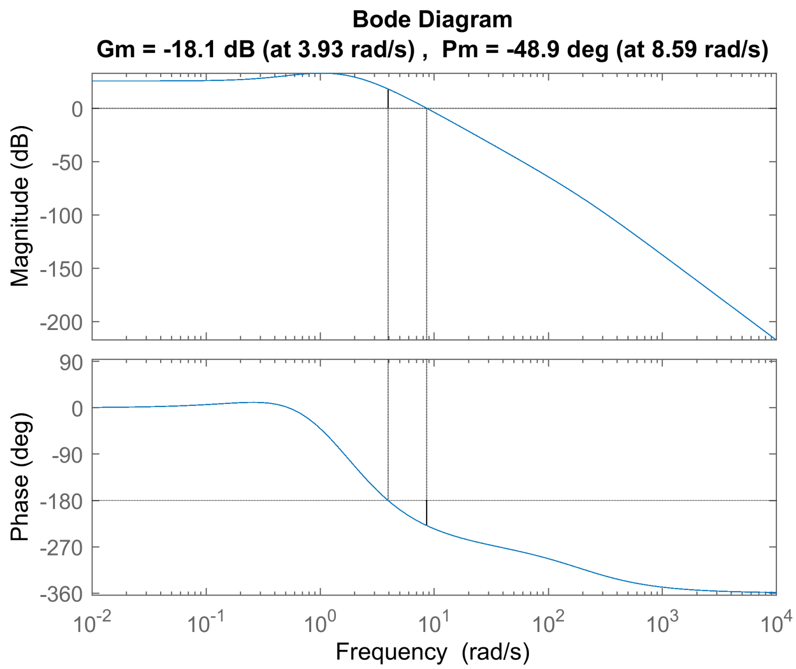


Now add lead compensation to system.

$$G_c(s)G(s) = 2691.1\frac{s+0.3750}{s+196.720}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

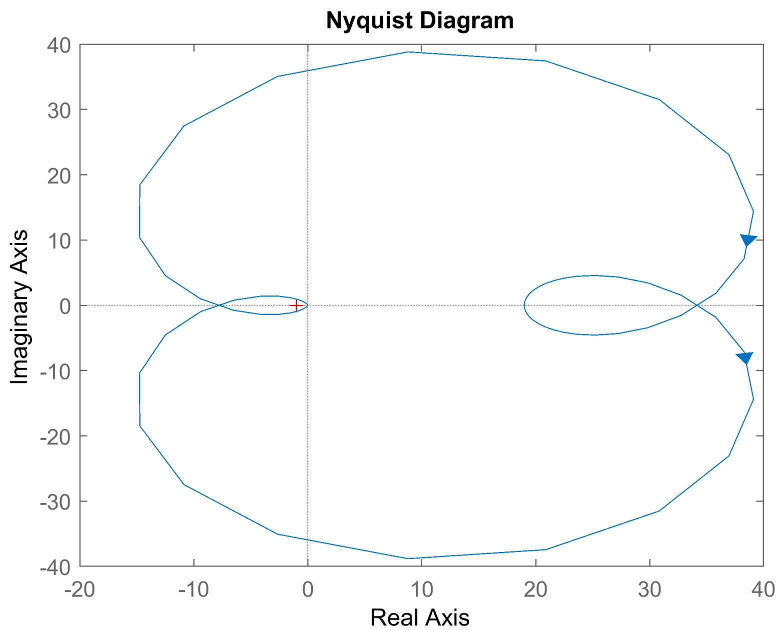
Bode diagram for system with adding lead compensation.

Figure 5: Bode diagram for system with lead compensation using MATLAB



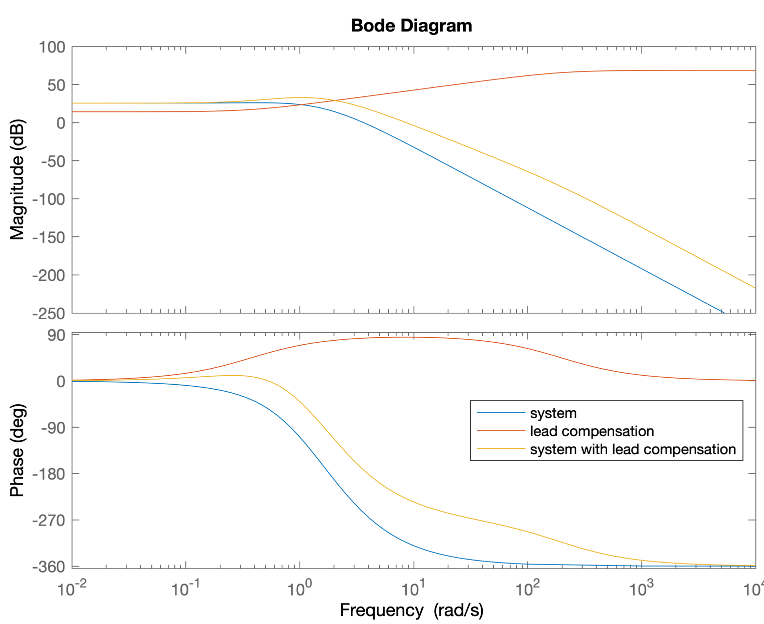
Nyquist plot for system with adding lead compensation.

Figure 6: Nyquist plot for system with lead compensation using MATLAB



That was the best we can do with lead compensation! All bode diagram in one figure:

Figure 7: all bode diagram using MATLAB



We didn't get requirement so we add new lead compensation. we skip previous calculation and use

MATLAB margin function in figure 5.

$$\bar{\gamma} = 45 + 5 = 50^\circ, \quad \phi_m = \bar{\gamma} - \gamma = 98.9000^\circ.$$

$$\phi_m = 85^\circ$$

This is too much! best we can do is about 90° in lead compensation so we add 85°.

$$\sin(\phi_m) = \frac{1-\alpha}{1+\alpha} \rightarrow \alpha = \frac{1-\sin(\phi_m)}{1+\sin(\phi_m)} = 0.0019$$

α is very low. With before lead compensation we currect steady state error so we assume $K = 1$.

$$K_c = \frac{K}{\alpha} = \frac{1}{0.0019} = 524.5825$$

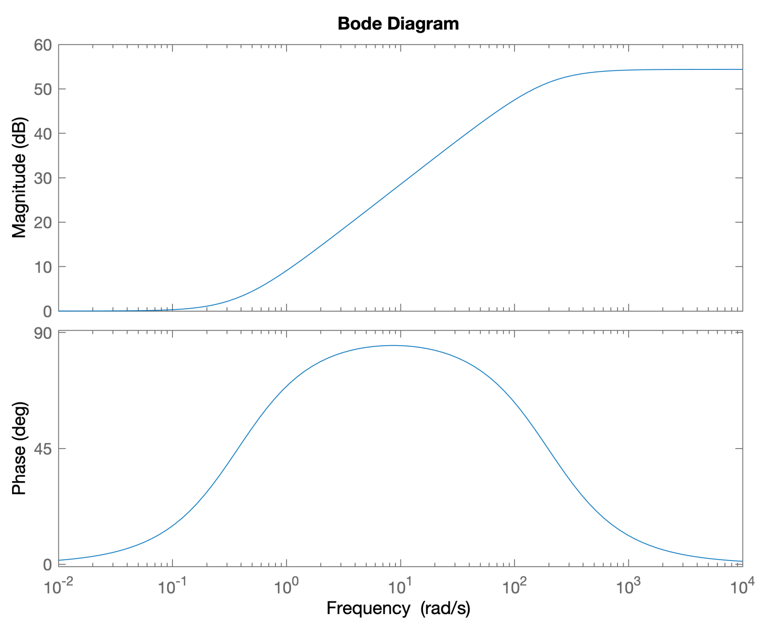
From previous calculation and MATLAB calculation figure 5 $\omega_m = 8.58898$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \rightarrow T = \frac{1}{\omega_m\sqrt{\alpha}} = \frac{1}{8.58898\sqrt{0.0019}} = 2.6666$$

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = 524.5825 \frac{s + 0.3750}{s + 196.720}$$

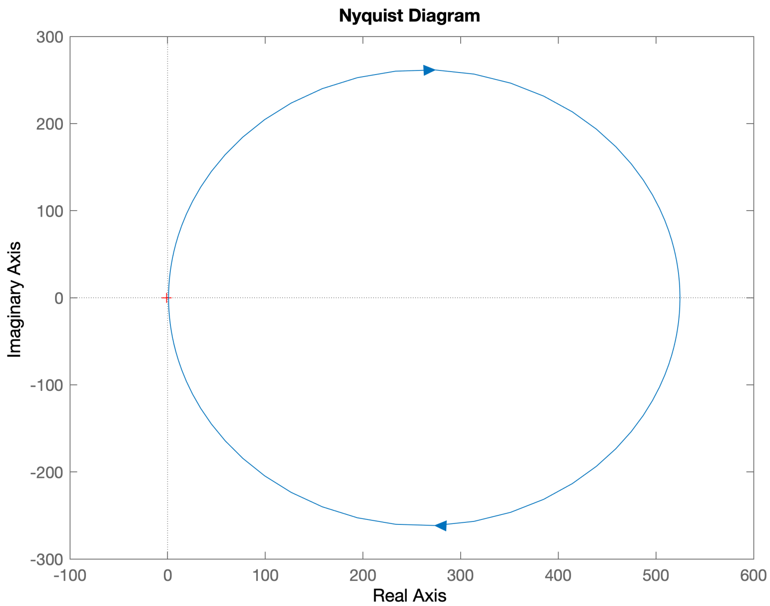
Bode diagram for second lead compensation using MATLAB.

Figure 8: second lead compensation Bode diagram using MATLAB



Nyquist plot for second lead compensation using MATLAB.

Figure 9: second lead compensation nyquist plot using MATLAB

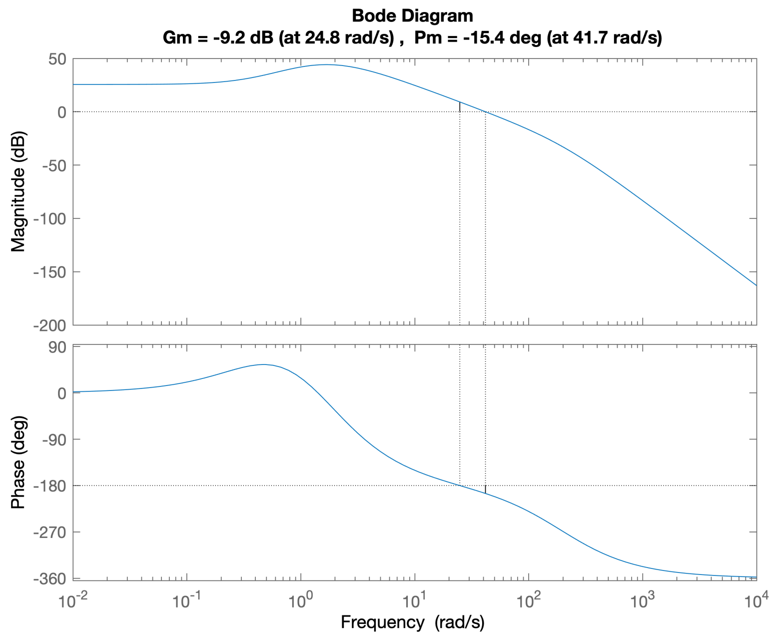


Now add second lead compensation to system and lag compensation.

$$C_{c_1}(s)G_c(s)G(s) = 524.5825\frac{s+0.3750}{s+196.720}2691.1\frac{s+0.3750}{s+196.720}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

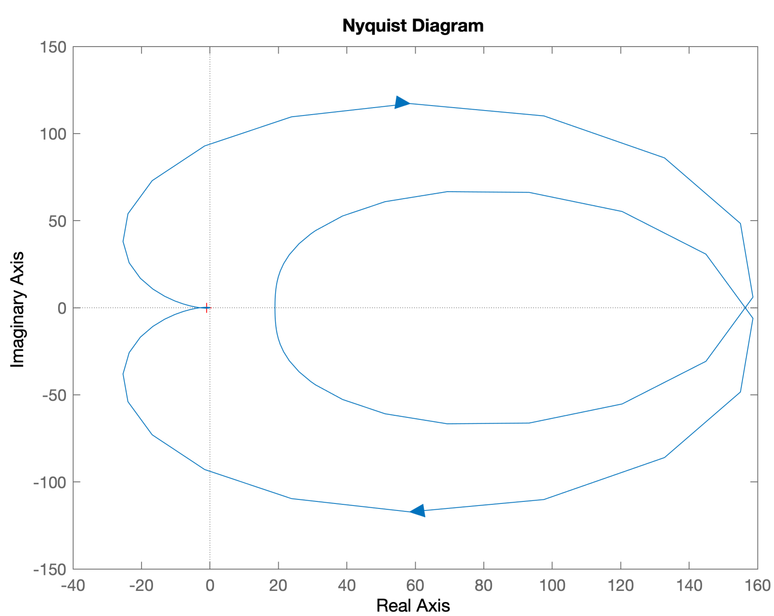
Bode diagram for system with adding lead compensation.

Figure 10: Bode diagram for system with lead compensation using MATLAB



Nyquist plot for system with adding lead compensation.

Figure 11: Nyquist plot for system with lead compensation using MATLAB



We didn't get requirement so we add new lead compensation. we skip previous calculation and use

MATLAB margin function in figure 5.

$$\bar{\gamma} = 45 + 10 = 55^\circ, \quad \phi_m = \bar{\gamma} - \gamma = 98.9000^\circ.$$

1.2 lag compensation

$$G_c(s) = K_c \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\beta T}}, \quad \beta > 1$$

In lag compensation we find out K for steady state error.

$$K = 5.1300$$

$$\gamma_d = 45^\circ \rightarrow \bar{\gamma_d} = 45 + 5 = 50^\circ$$

$$\phi(\omega_g) = 50 - 180 = -130^\circ$$

The phase angle:

$$\angle G(j\omega) = 0^\circ + \tan^{-1}\frac{\omega}{0.5} - \tan^{-1}\frac{\omega}{1} - 3\tan^{-1}\frac{\omega}{1.5} - \tan^{-1}\frac{\omega}{2}$$

$$\angle G(j\omega_g) = 0^\circ + \tan^{-1}\frac{\omega_g}{0.5} - \tan^{-1}\frac{\omega_g}{1} - 3\tan^{-1}\frac{\omega_g}{1.5} - \tan^{-1}\frac{\omega_g}{2} = -130^\circ$$

This equation solved with MATLAB and code has attacked (Q1_b.m)

$$\omega_g = 1.2025$$

The amplitude ratio:

$$|G_1(j\omega)| = |KG(j\omega)| = 5.1300 \frac{50\sqrt{\omega^2 + 0.5^2}}{\sqrt{\omega^2 + 1^2} \times (\sqrt{\omega^2 + 1.5^2})^3 \times \sqrt{\omega^2 + 2^2}}$$

$$\beta = |G_1(j\omega_g)| = 5.1300 \frac{50\sqrt{\omega_g^2 + 0.5^2}}{\sqrt{\omega_g^2 + 1^2} \times (\sqrt{\omega_g^2 + 1.5^2})^3 \times \sqrt{\omega_g^2 + 2^2}}$$

$$\beta = 5.1300 \frac{50\sqrt{1.2025^2 + 0.5^2}}{\sqrt{1.2025^2 + 1^2} \times (\sqrt{1.2025^2 + 1.5^2})^3 \times \sqrt{1.2025^2 + 2^2}} = 12.8807$$

Assume:

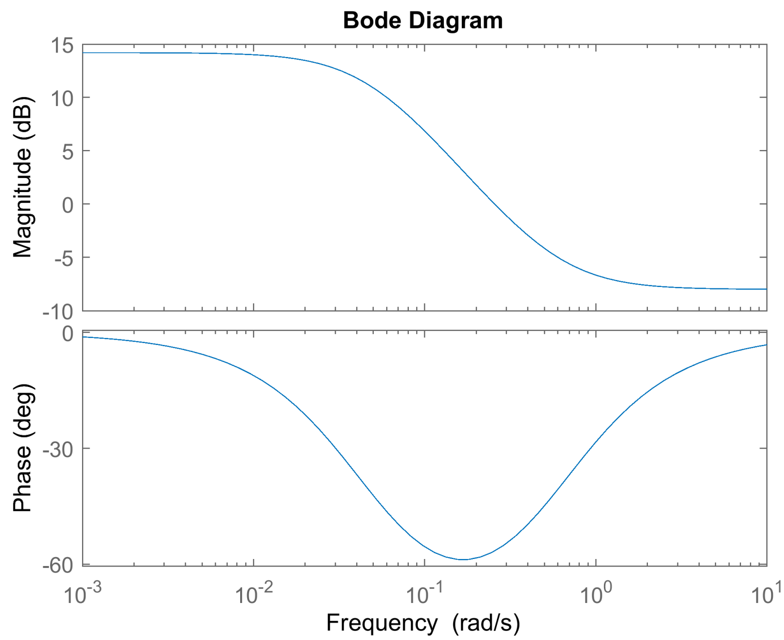
$$\frac{1}{T} = \frac{\omega_g}{2} \rightarrow \frac{1}{T} = 0.6012 \rightarrow T = 1.6632$$

$$\rightarrow \frac{1}{\beta T} = 0.0467, \quad K_c = \frac{K}{\beta} = \frac{5.1300}{12.8807} = 0.3983$$

$$G_c(s) = K_c \frac{s + \dfrac{1}{T}}{s + \dfrac{1}{\beta T}} = 0.3983 \frac{s + 0.6012}{s + 0.0467}$$

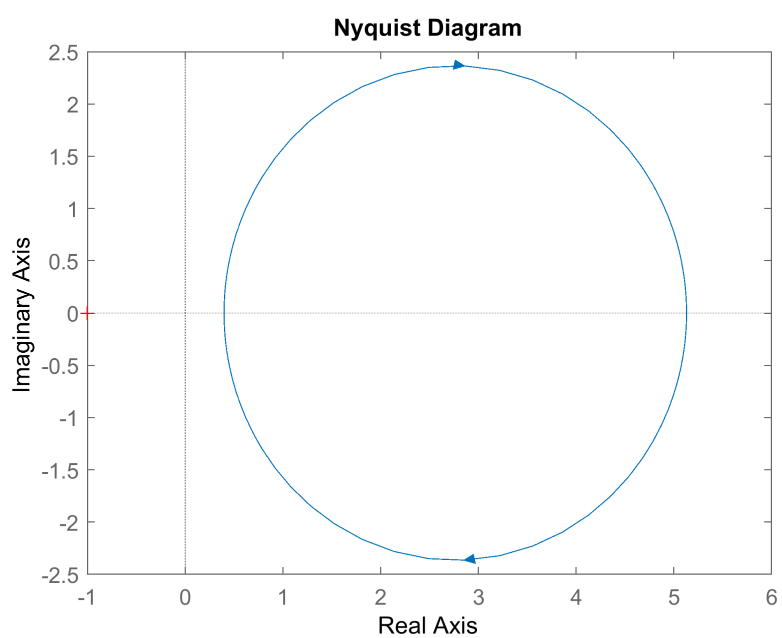
Bode diagram for lag compensation using MATLAB.

Figure 12: lag compensation Bode diagram using MATLAB



Nyquist plot for lag compensation using MATLAB.

Figure 13: lag compensation Nyquist plot using MATLAB

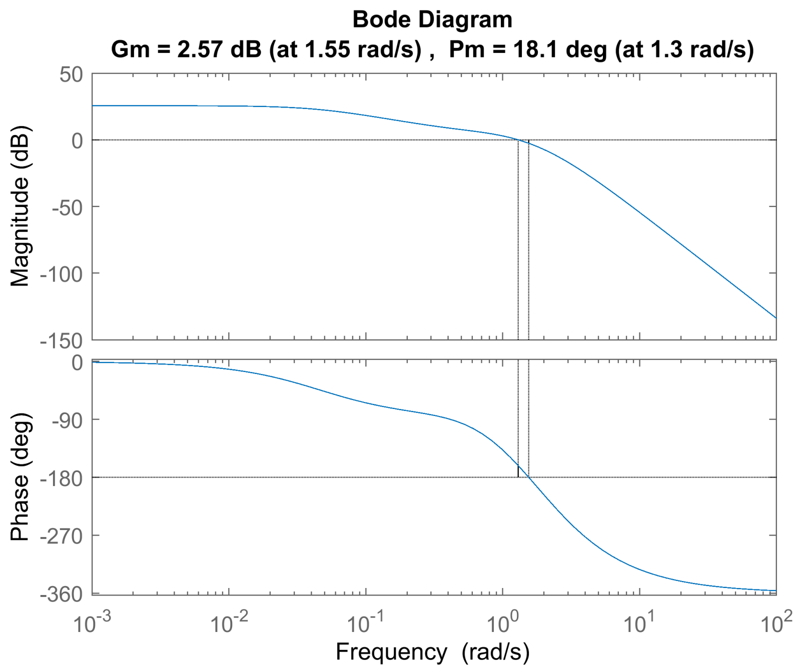


Now add lag compensation to system.

$$G_c(s)G(s) = 0.3983\frac{s+0.6012}{s+0.0467}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

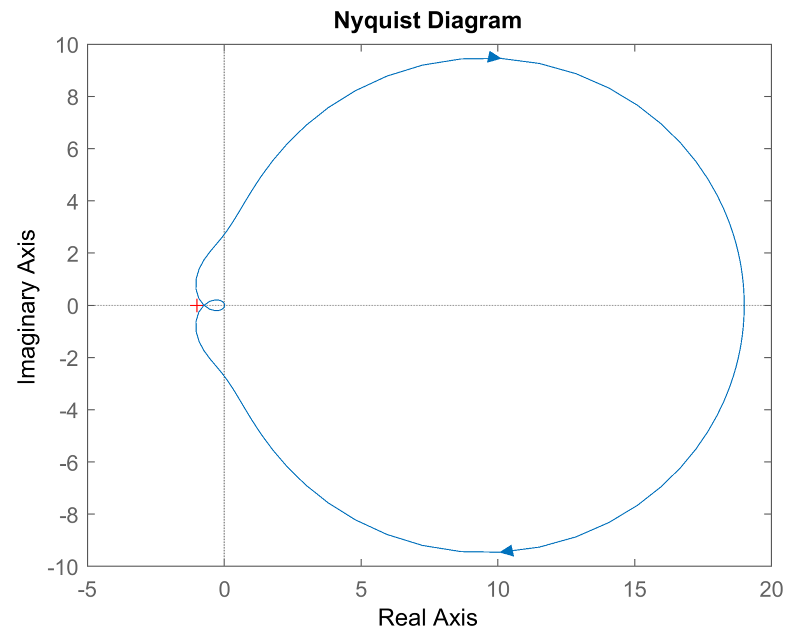
Bode diagram for system with adding lag compensation.

Figure 14: Bode diagram for system with lag compensation using MATLAB



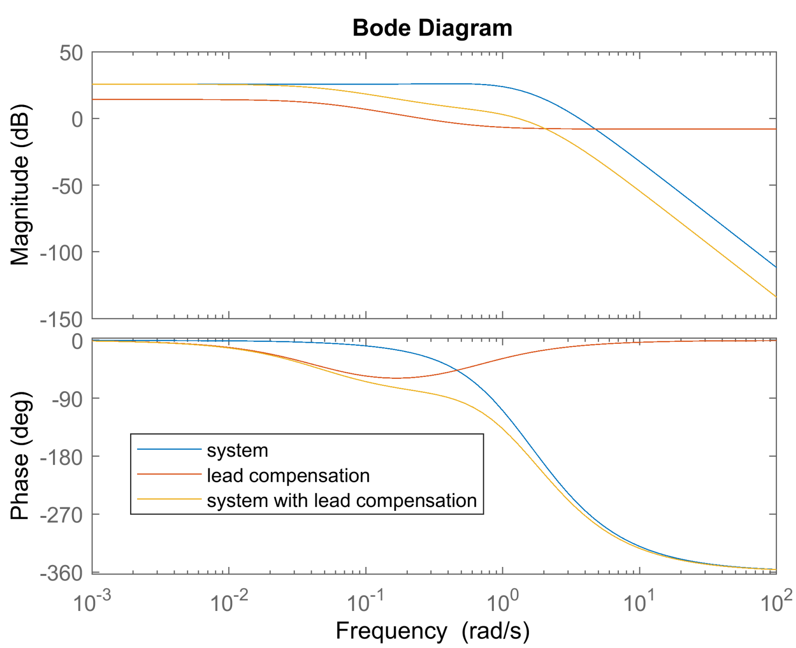
Nyquist plot for system with adding lag compensation.

Figure 15: Nyquist plot for system with lag compensation using MATLAB



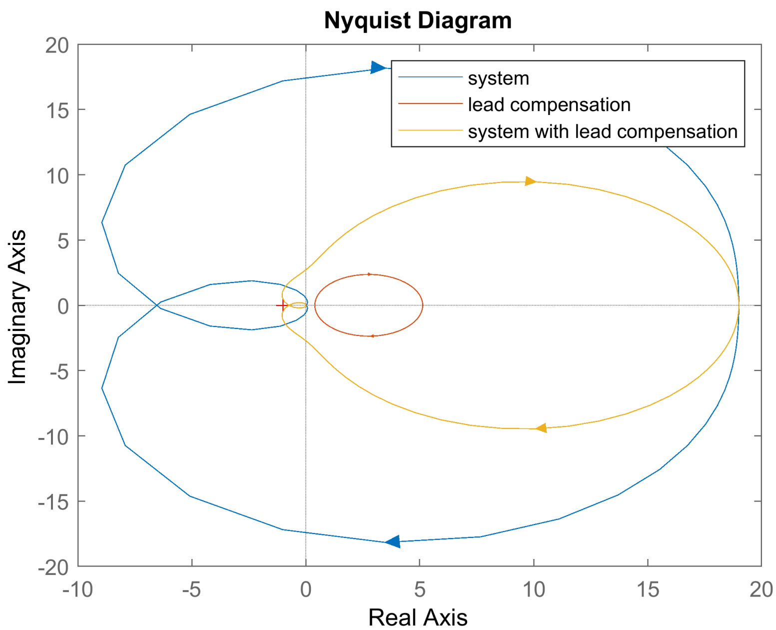
All bode diagram in one figure:

Figure 16: all bode diagram using MATLAB



All Nyquist plot in one figure:

Figure 17: all Nyquist plot using MATLAB



We didn't get what we want in equation so we change our before assumption. assume:

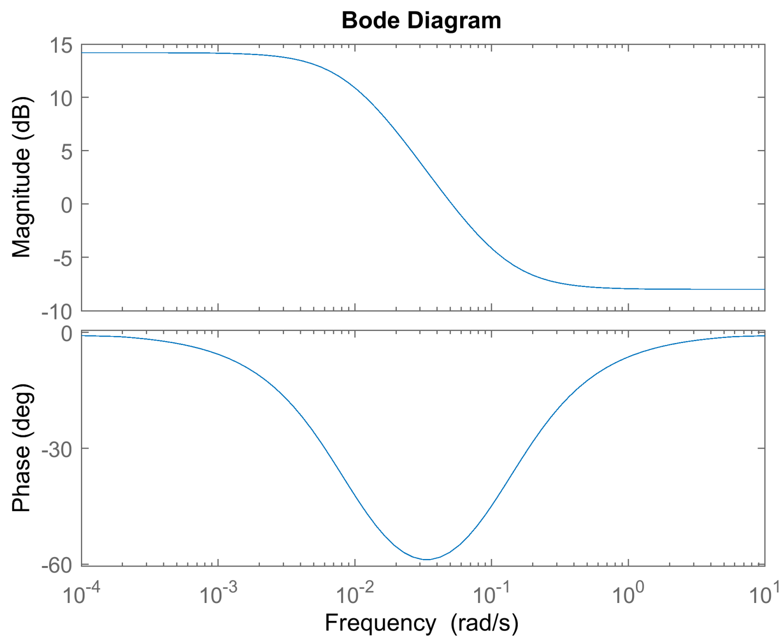
$$\frac{1}{T} = \frac{\omega_g}{10} \to \frac{1}{T} = 0.1202 \to T = 8.3161$$

$$\to \frac{1}{\beta T} = 0.0093, \quad K_c = \frac{K}{\beta} = \frac{5.1300}{12.8807} = 0.3983$$

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} = 0.3983 \frac{s + 0.1202}{s + 0.0093}$$

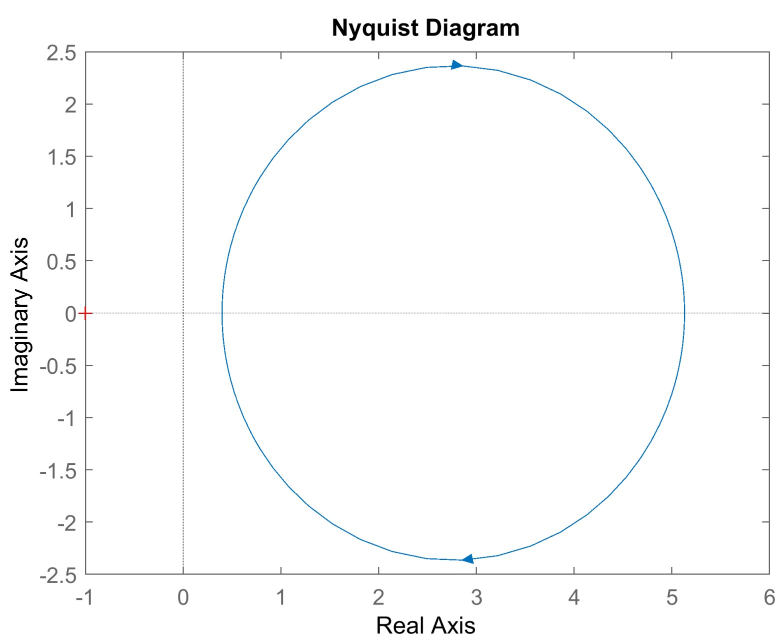
Bode diagram for lag compensation using MATLAB.

Figure 18: new lag compensation Bode diagram using MATLAB



Nyquist plot for lag compensation using MATLAB.

Figure 19: new lag compensation Nyquist plot using MATLAB

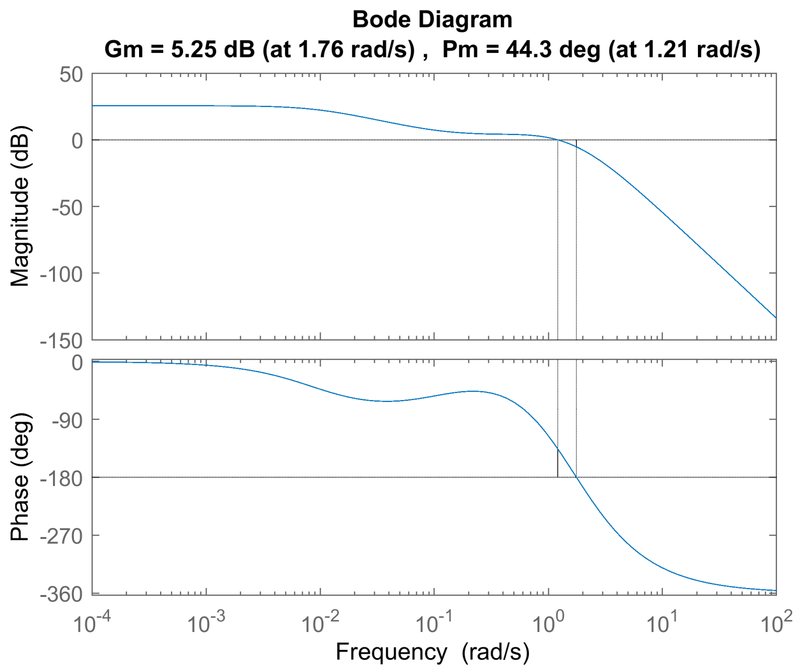


Now add new lag compensation to system.

$$G_c(s)G(s) = 0.3983\frac{s+0.1202}{s+0.0093}\frac{50(s+0.5)}{(s+1)(s+1.5)^3(s+2)}$$

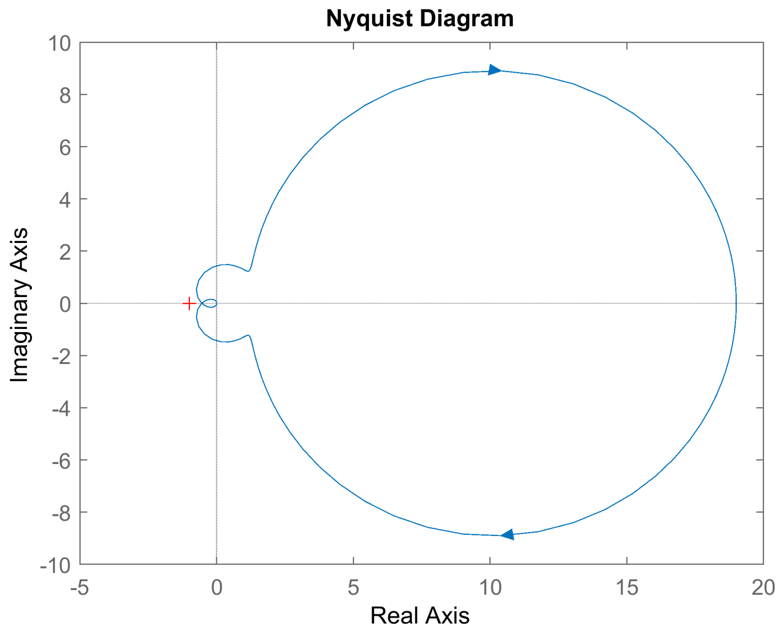
Bode diagram for system with adding lag compensation.

Figure 20: Bode diagram for system with new lag compensation using MATLAB



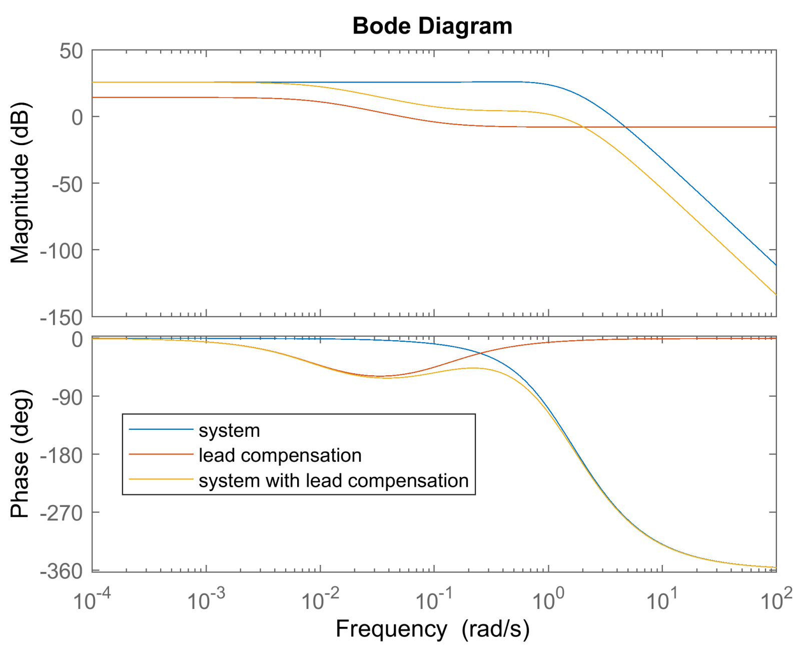
Nyquist plot for system with adding lag compensation.

Figure 21: Nyquist plot for system with new lag compensation using MATLAB



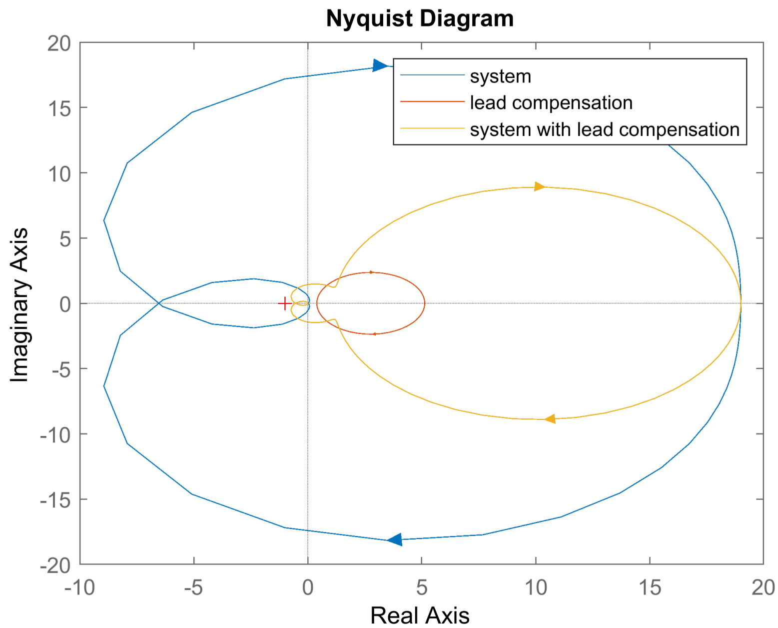
All bode diagram in one figure:

Figure 22: all bode diagram using MATLAB



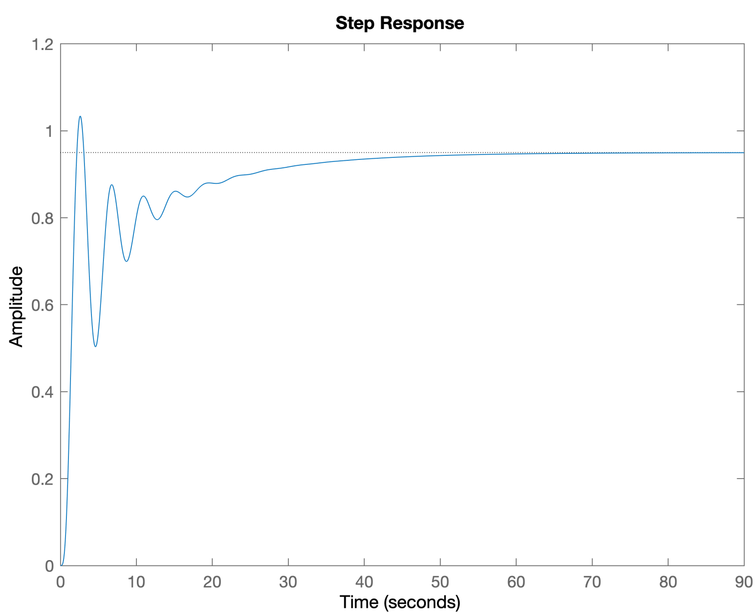
All Nyquist plot in one figure:

Figure 23: all Nyquist plot using MATLAB



In new lag compensation phase margin is 45.3° and we are near to question requirement. Step respond for close loop system.

Figure 24: Step respond with lag compensation



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