

- 1) The Hubble Space Telescope is about to be released from the Space Shuttle, which is in a circular orbit at 590 km altitude. The relative velocity (from the Space Shuttle bay) at the instance of the ejection is 0.1 m/s down, 0.04 m/s backwards, and 0.02 m/s to the right. In other words:

*Hubble Initial Condition:*

$$x(0) = y(0) = z(0) = 0$$

$$\dot{x}(0) = -0.1 \text{ m/s}; \dot{y}(0) = -0.04 \text{ m/s}; \dot{z}(0) = -0.02 \text{ m/s}$$

- 1-1 Determine and plot the position and velocity of the Hubble Space Telescope for 320 minutes after separation from the Shuttle.  
1-2 This part is considered as a bonus (optional). See B2 at the end.



- 2) The problem is to investigate the effect of **direct** solar radiation pressure on the variation of orbital elements. Consider a spherical Earth orbiting satellite whose initial conditions are given below. Assume the satellite has a totally reflecting area-to-mass ratio of  $2 \text{ m}^2/\text{kg}$  and that its absorptivity is equal to 2. The initial Sun corresponds to a Julian date of  $JD_0 = 2438400.5$ ,

2-1 Plot the variation of orbital elements ( $h, e, \theta, \Omega, i, \omega$ ) over the next 5 years.

2-2 Plot the satellite ground track for 2 periods.

**Notes:** The attached MATLAB script can be used to find the Sun initial ephemeris.

Eclipse intervals should be considered.

*Satellite Initial Condition:*

$$\vec{r}(0) = [1600 \quad 5310 \quad 3800]^T (\text{km}) \quad , \quad \vec{v}(0) = [-7.35 \quad 0.46 \quad 2.47]^T (\text{km/s})$$

- 3) Assume that the Earth third Earth zonal harmonic of its perturbing gravitational potential is given as  $\phi = \frac{J_3}{2} \frac{\mu}{r} \left( \frac{R}{r} \right)^3 (5 \cos^3 \varphi - 3 \cos \varphi)$ . Use the method of General Perturbation (GPE) to find an expression for  $\frac{de}{dt}$  due to  $J_3$  effect.

- 4) Consider the CRTBP in Canonical form for Earth-Moon system. Using the given initial condition of the spacecraft at time zero:

4-1 Determine the orbital energy or the Jacobi constant,

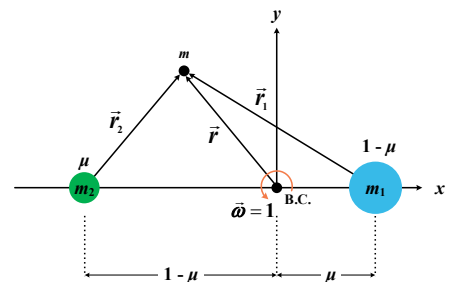
4-2 Compute and plot the planar orbit trajectory, and determine whether or not you get a closed (repeating pattern) orbit in the xy plane,

4-3 If the orbit is closed, what is its period (for one closed trajectory) in canonical TU as well as in seconds.

*Spacecraft Initial Condition in the rotating frame:*

$$\vec{r}(0) = [0.994 \quad 0 \quad 0]^T \quad , \quad \vec{v}(0) = [0 \quad -2.001585106 \quad 0]^T$$

$$m_{\text{Earth}} = 5.974 \times 10^{24} \text{ kg}; m_{\text{Moon}} = 7.348 \times 10^{22} \text{ kg}; r_{12} = 3.844 \times 10^5 \text{ km}$$



**Bonus Problem: (Optional)**

The bonus problem is optional, but must be typed as a report if one chooses to do it.

**B2:** Consider part 1-2 of the first problem. In this case the Hubble is in a neighboring orbit at a relative position and velocity with respect to the SpaceX (in the same circular orbit as Shuttle) given below. The goal is to use the relative motion EOM with thruster effects to determine the control action required bring back the Hubble to rendezvous with the SpaceX. You can use any classical or modern control methodology accomplish this rendezvous mission. Any additional information can be logically assumed or obtained from available research with proper referencing. The report should include:

B2-1 The EOMs, plus discussion of the control method used and its parameters,

B2-2 The 3D plot of the Hubble trajectory,

B2-3 The plot of the Hubble velocity for the rendezvous back to the SpaceX,

B2-3 The time history of the thruster acceleration components and magnitude needed to perform the mission.

*Hubble Initial Condition:*

$$x(0) = 143m; y(0) = 137.279m; z(0) = 17.766m$$

$$\dot{x}(0) = -0.1m/s; \dot{y}(0) = 0.27m/s; \dot{z}(0) = -0.01m/s$$

