Home Work #1

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1 Question 1

1.1 a

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$$\boldsymbol{C} = \dot{\boldsymbol{r}} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\boldsymbol{C}}{\mu} = \boldsymbol{e} \rightarrow \boldsymbol{e} = \frac{\boldsymbol{C}}{\mu} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$$

$$\boldsymbol{h}.\boldsymbol{e} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix} . \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} = 0$$

1.2 b, c

$$r = \frac{P}{1 + e\cos(\theta)} \xrightarrow{P = \frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right)$$

Beacuse $\boldsymbol{r}.\boldsymbol{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and $\boldsymbol{h} = \text{constant}$, then v and θ calculated as below:

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \ DU/TU$$

$$\theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right) = 2.7862_{rad}$$

2 Question 2

3 Question 3

$$R_e = 6378_{km}$$

$$r_p = R_e + 500$$

$$r_a = R_e + 5000$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.2465$$

$$a = \frac{r_a + r_p}{2} = 9128$$

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 8679.1_{\text{sec}}$$

3.1 a

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1 - e^2)\frac{a}{R_e}\sin(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 2.5021 & 1.0022 \end{bmatrix} rad$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan(\theta/2)\right)$$

$$M_e = E - e\sin(E)$$

$$M_1 = 2.1587, \quad M_2 = 0.6275$$

$$t = \frac{M}{2\pi}\tau$$

$$\Delta t = |t_1 - t_2| = 2981.8 - 866.7 = 2115.0\,\text{sec}$$

3.2 b

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1 - e^2)\frac{a}{R_e}\cos(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 0.4251 & 2.2506 \end{bmatrix} rad$$

$$M_1 = 0.2521, \quad M_2 = 1.8202$$

$$\Delta t = |t_1 - t_2| = 2514.3 - 348.2 = 2166.13\,\text{sec}$$

4 Question 4

4.1 a

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6570$$

For first circular orbit $r_a = r_p$.

$$h = 51174 \rightarrow v = 7.7891_{km/\sec}$$

Second orbit (elliptical):

$$r_p = 6570, \quad r_a = 42160$$

$$h = 67316 \rightarrow v_a = 10.2460_{km/\sec}, \quad v_p = 1.5967_{km/\sec}$$

Third orbit (circular):

$$h = 129634 \rightarrow v = 3.0748_{km/\sec}$$

Total delta change in velocity:

$$\Delta v_{\text{total}} = \Delta v_1 + \Delta v_2 = |10.2460 - 7.7891| + |1.5967 - 3.0748| = 3.9350_{km/\sec}$$

4.2 b

Time is half of the period.

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 37850_{\text{sec}} \rightarrow t = 18925_{\text{sec}}$$

5 Question 5

First, change foot to km.

$$\boldsymbol{r} = \begin{bmatrix}1.2756 & 1.9135 & 3.1891\end{bmatrix} km$$

$$\boldsymbol{v} = \begin{bmatrix}7.9053 & 0 & 15.8106\end{bmatrix} km/\sec$$

5.1 a

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 146.0963 \rightarrow \text{hyperbolic}$$

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{bmatrix}302531.3 & 50421.9 & -151265.7\end{bmatrix}$$

5.2 b

$$P = \frac{h^2}{\mu} = 293399$$

$$e = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r} = \begin{bmatrix} -2.3244 & 14.5133 & 0.1889 \end{bmatrix}, \quad e = 14.6995$$

$$r = \frac{P}{1 + e\cos(\theta)} = \frac{293399}{1 + 14.6995\cos(\theta)}$$

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