Home Work #1

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1 Question 1

1.1 a

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$$\boldsymbol{C} = \dot{\boldsymbol{r}} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\boldsymbol{C}}{\mu} = \boldsymbol{e} \rightarrow \boldsymbol{e} = \frac{\boldsymbol{C}}{\mu} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$$

$$\boldsymbol{h}.\boldsymbol{e} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix} . \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} = 0$$

1.2 b, c

$$r = \frac{P}{1 + e\cos(\theta)} \xrightarrow{P=\frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right)$$

Beacuse $\boldsymbol{r}.\boldsymbol{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and $\boldsymbol{h} =$ constant, then v and θ calculated as below:

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \ DU/TU$$

$$\theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right) = 2.7862_{rad}$$

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