

Home Work #1

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1 Question 1

1.1 a

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = [0 \quad 0 \quad -\sqrt{2}]$$
$$\mathbf{C} = \dot{\mathbf{r}} \times \mathbf{h} - \mu \frac{\mathbf{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\mathbf{C}}{\mu} = \mathbf{e} \rightarrow \mathbf{e} = \frac{\mathbf{C}}{\mu} = [-1 \quad -1 \quad 0]$$
$$\mathbf{h} \cdot \mathbf{e} = [0 \quad 0 \quad -\sqrt{2}] \cdot [-1 \quad -1 \quad 0] = 0$$

1.2 b, c

$$r = \frac{P}{1 + e \cos(\theta)} \xrightarrow{P = \frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e \cos(\theta)} \rightarrow \theta = \arccos \left(\left(\frac{h^2}{\mu r} - 1 \right) / e \right)$$

Beacuse $\mathbf{r} \cdot \mathbf{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and \mathbf{h} = constant, then v and θ calculated as below:

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \text{ DU/TU}$$

$$\theta = \arccos \left(\left(\frac{h^2}{\mu r} - 1 \right) / e \right) = 2.7862_{rad}$$

2 Question 2

3 Question 3

4 Question 4

4.1 a

$$h = \sqrt{2\mu} \sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6570$$

For first circular orbit $r_a = r_p$.

$$h = 51174 \rightarrow v = 7.7891_{km/sec}$$

Second orbit (elliptical):

$$r_p = 6570, \quad r_a = 42160$$

$$h = 67316 \rightarrow v_a = 10.2460_{km/sec}, \quad v_p = 1.5967_{km/sec}$$

Third orbit (circular):

$$h = 129634 \rightarrow v = 3.0748_{km/sec}$$

Total delta change in velocity:

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = |10.2460 - 7.7891| + |1.5967 - 3.0748| = 3.9350_{km/sec}$$

4.2 b

Time is half of the period.

$$\tau = 2\pi \sqrt{\frac{a^3}{\mu}} = 37850_{sec} \rightarrow t = 18925_{sec}$$

5 Question 5

First, change foot to km.

$$\mathbf{r} = [1.2756 \quad 1.9135 \quad 3.1891] km$$

$$\mathbf{v} = [7.9053 \quad 0 \quad 15.8106] km/sec$$

5.1 a

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 146.0963 \rightarrow \text{hyperbolic}$$

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} = [302531.3 \quad 50421.9 \quad -151265.7]$$

5.2 b

$$P = \frac{h^2}{\mu} = 293399$$

$$e = \frac{\mathbf{v} \times \mathbf{h}}{\mu} - \frac{\mathbf{r}}{r} = \begin{bmatrix} -2.3244 & 14.5133 & 0.1889 \end{bmatrix}, \quad e = 14.6995$$

$$r = \frac{P}{1 + e \cos(\theta)} = \frac{293399}{1 + 14.6995 \cos(\theta)}$$

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