Home Work #4

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1 Question 1

We know inertia matrix is symetric so we have:

$$I_{xy} = I_{yx} = 10, \quad I_{xz} = I_{zx} = 0, \quad I_{yz} = I_{zy}$$

$$\mathbf{I} = \begin{bmatrix} 30 & -10 & 0 \\ -10 & 20 & -I_{yz} \\ 0 & -I_{yz} & 30 \end{bmatrix}$$

1.1 part a

We know that:

$$\mathbf{I} \times \boldsymbol{\omega} = \mathbf{h} \tag{1}$$

$$\boldsymbol{\omega} = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}^T_{RPS} = \begin{bmatrix} 10 \times 2\pi & 10 \times 2\pi & 10 \times 2\pi \end{bmatrix}^T_{rad/\sec}, \quad \mathbf{h} = \begin{bmatrix} 200 & 200 & 400 \end{bmatrix}^T_{kg.m^2/s}$$

If we use radian per second instead of revelotion per second $\mathbf{I} \times \boldsymbol{\omega} \neq \mathbf{h}$ would happen.

$$\mathbf{I} \times \boldsymbol{\omega} = \begin{bmatrix} 200 \\ 100 - 10I_{yz} \\ 300 - 10I_{yz} \end{bmatrix} = \begin{bmatrix} 200 \\ 200 \\ 400 \end{bmatrix} \rightarrow I_{yz} = -10 \rightarrow \mathbf{I} = \begin{bmatrix} 30 & -10 & 0 \\ -10 & 20 & 10 \\ 0 & 10 & 30 \end{bmatrix}$$

$$T_{Rotational} = \frac{1}{2}\boldsymbol{\omega}^T \times \mathbf{I} \times \boldsymbol{\omega} = \frac{1}{2} \begin{bmatrix} 10 & 10 & 10 \end{bmatrix} \times \begin{bmatrix} 30 & -10 & 0 \\ -10 & 20 & 10 \\ 0 & 10 & 30 \end{bmatrix} \times \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix} = 4000 \tag{2}$$

1.2 part b

$$T_{Rotational} = \frac{1}{2}\mathrm{I}_\xi \omega^2 \rightarrow \mathrm{I}_\xi = \frac{2T_{Rotational}}{\omega^2} = \frac{2 \times 4000}{300} = 26.67 \tag{3}$$

Rotation matrix calculated via eigen vector of inertia matrix (used MATLAB to calculate).

$$\mathbf{A} = \mathrm{eig}(\mathbf{I}) = \begin{bmatrix} -0.4082 & -0.7071 & -0.5774 \\ -0.8165 & -0.0000 & 0.5774 \\ 0.4082 & -0.7071 & 0.5774 \end{bmatrix} \tag{4}$$

$$\mathbf{I}' = \mathbf{A}^T \times \mathbf{I} \times \mathbf{A} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 40 \end{bmatrix}, \quad \boldsymbol{\omega}' = \mathbf{A}^T \boldsymbol{\omega} = \begin{bmatrix} -8.1650 \\ -14.1421 \\ 5.7735 \end{bmatrix}$$

$$h = |\mathbf{I} \times \boldsymbol{\omega}| = 489.8979$$

We know that above matrix shows the direction cosines of the principal axes with the primary body axes. Used MATLAB function (dcm2angle) to calculate euler angles between two corinate system.

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 45.00^\circ \\ 35.26^\circ \\ -120.00^\circ \end{bmatrix}$$

1.3 part c

Ellipsoid of inertia calculated as folow:

$$\frac{X^2}{\left(\sqrt{\frac{1}{\mathrm{I}_x}}\right)^2} + \frac{Y^2}{\left(\sqrt{\frac{1}{\mathrm{I}_y}}\right)^2} + \frac{Z^2}{\left(\sqrt{\frac{1}{\mathrm{I}_z}}\right)^2} = \frac{X^2}{\left(\sqrt{\frac{1}{10}}\right)^2} + \frac{Y^2}{\left(\sqrt{\frac{1}{30}}\right)^2} + \frac{Z^2}{\left(\sqrt{\frac{1}{40}}\right)^2} = 1 \tag{5}$$

Figure 1: elipsoid of inertia

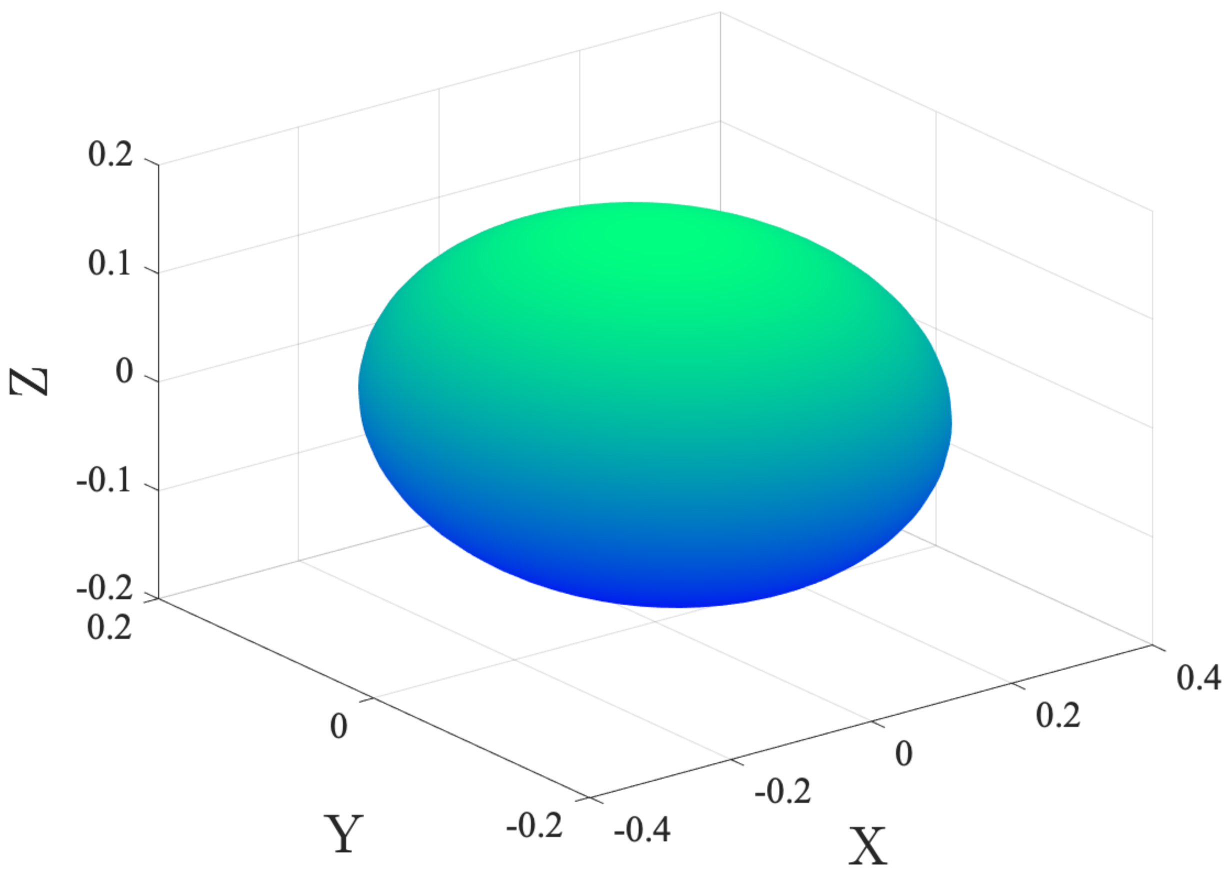


Figure 2: elipsoid of inertia in zx plane

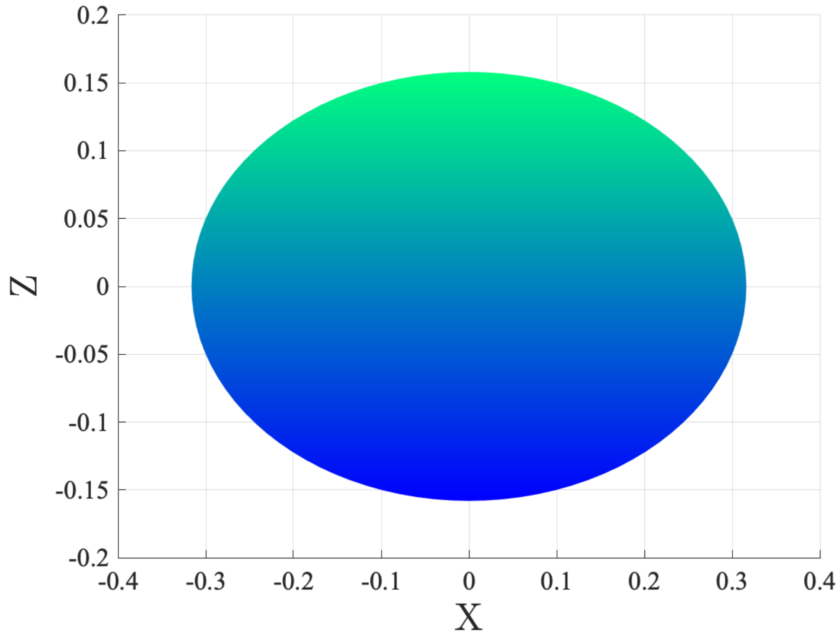


Figure 3: elipsoid of inertia zy plane

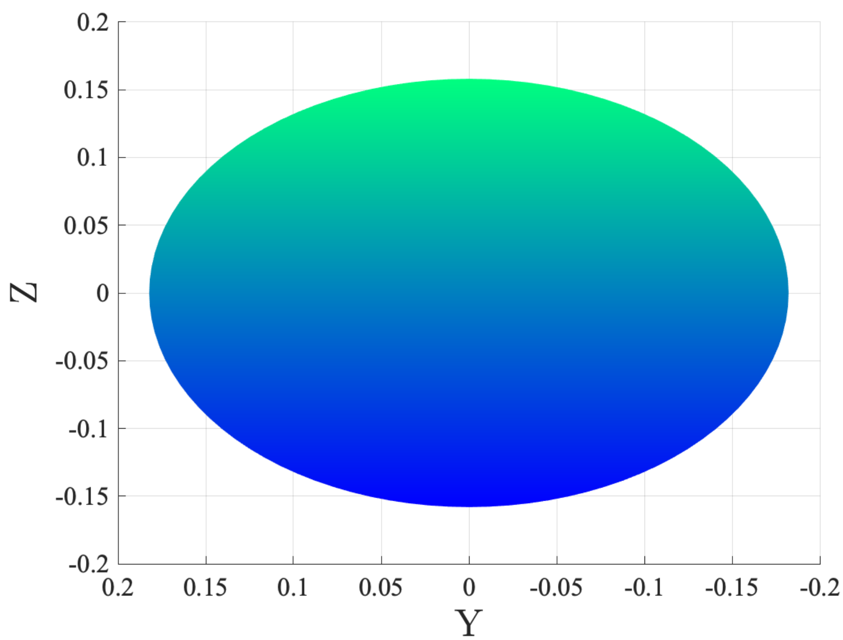
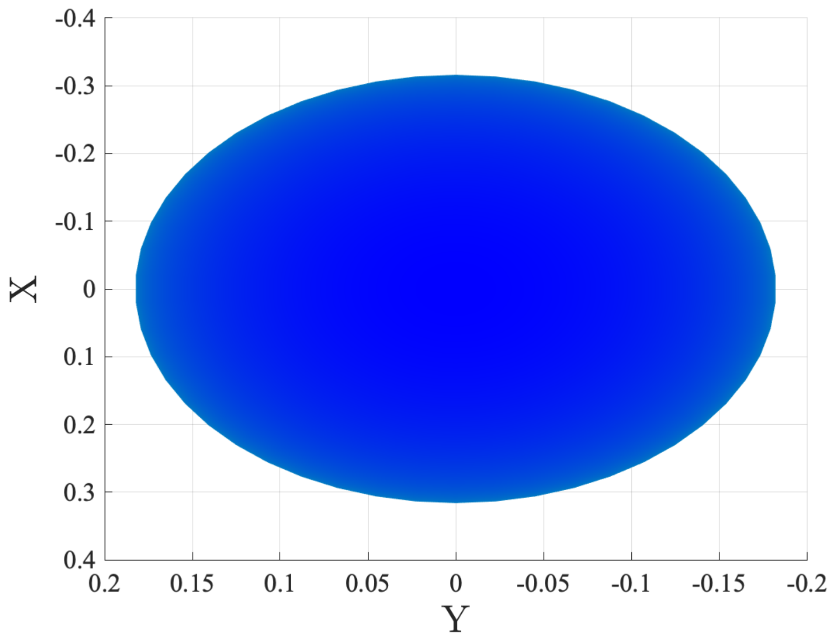


Figure 4: elipsoid of inertia in xy plane



Part I

part d

Angular momentum and rotational kinetic energy elipsoid calculated as folow:

$$\frac{\omega_x^2}{\left(\frac{h}{\mathrm{I}_x}\right)^2} + \frac{\omega_y^2}{\left(\frac{h}{\mathrm{I}_y}\right)^2} + \frac{\omega_z^2}{\left(\frac{h}{\mathrm{I}_z}\right)^2} = \frac{\omega_x^2}{\left(\frac{490}{10}\right)^2} + \frac{\omega_y^2}{\left(\frac{490}{30}\right)^2} + \frac{\omega_z^2}{\left(\frac{490}{40}\right)^2} = 1 \tag{6}$$

$$\frac{\omega_x^2}{\left(\sqrt{\frac{2T}{\mathrm{I}_x}}\right)^2} + \frac{\omega_y^2}{\left(\sqrt{\frac{2T}{\mathrm{I}_y}}\right)^2} + \frac{\omega_z^2}{\left(\sqrt{\frac{2T}{\mathrm{I}_z}}\right)^2} = \frac{\omega_x^2}{\left(\sqrt{\frac{2\times 4000}{10}}\right)^2} + \frac{\omega_y^2}{\left(\sqrt{\frac{2\times 4000}{30}}\right)^2} + \frac{\omega_z^2}{\left(\sqrt{\frac{2\times 4000}{40}}\right)^2} = 1 \tag{7}$$

Ellipsoid parameter calculated before, now, use them to draw the elipsoid.

Figure 5: Angular momentum and rotational kinetic energy elipsoid of inertia

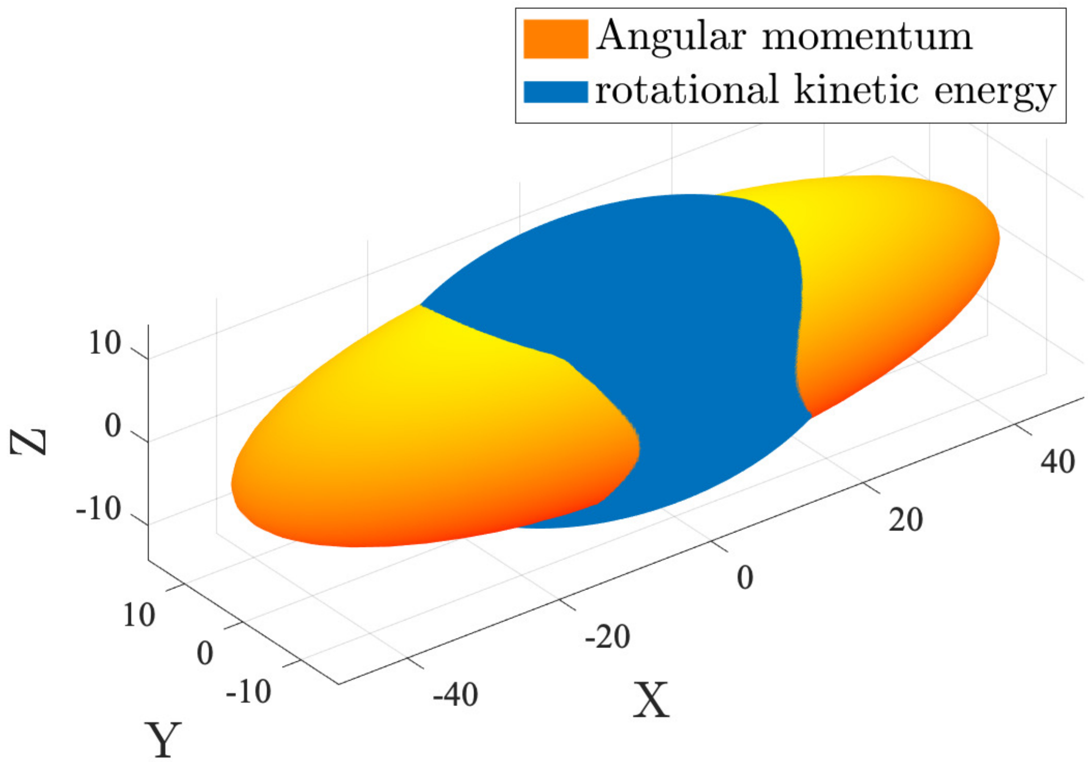


Figure 6: Angular momentum and rotational kinetic energy elipsoid of inertia in zx plane

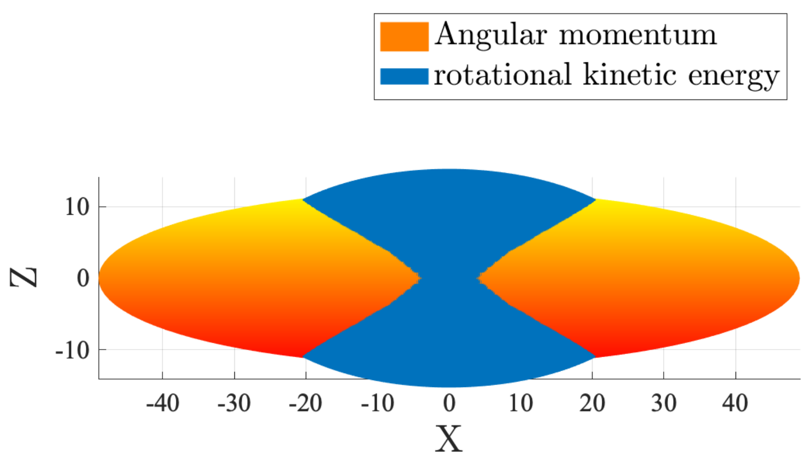


Figure 7: Angular momentum and rotational kinetic energy elipsoid of inertia zy plane

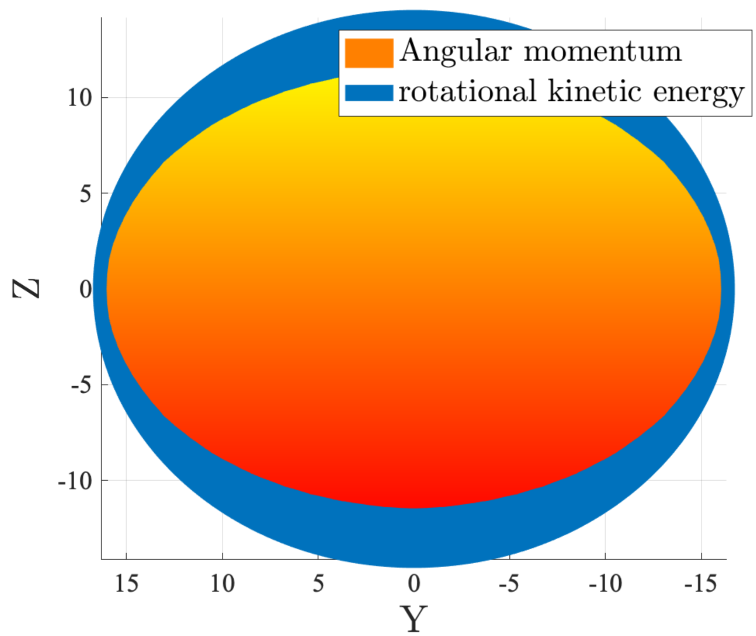
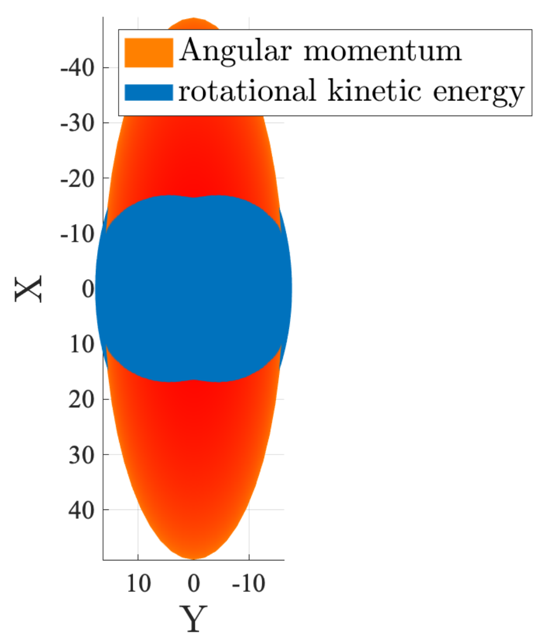


Figure 8: Angular momentum and rotational kinetic energy elipsoid of inertia in xy plane



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