Home Work #1

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1 Question 1

1.1 a

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$$\boldsymbol{C} = \dot{\boldsymbol{r}} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\boldsymbol{C}}{\mu} = \boldsymbol{e} \rightarrow \boldsymbol{e} = \frac{\boldsymbol{C}}{\mu} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$$

$$\boldsymbol{h}.\boldsymbol{e} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix} . \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} = 0$$

1.2 b, c

$$r = \frac{P}{1 + e\cos(\theta)} \xrightarrow{P = \frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right)$$

Beacuse $\boldsymbol{r}.\boldsymbol{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and $\boldsymbol{h} = $ constant, then v and θ calculated as below:

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \; DU/TU$$

$$\theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right) = 2.7862_{rad}$$

2 Question 4

2.1 a

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6570$$

For first circular orbit $r_a = r_p$.

$$h = 51174 \rightarrow v = 7.7891_{km/s}$$

Second orbit (elliptical):

$$r_p = 6570, \quad r_a = 42160$$

$$h = 67316 \rightarrow v_a = 10.2460_{km/s}, \quad v_p = 1.5967_{km/s}$$

Third orbit (circular):

$$h = 129634 \rightarrow v = 3.0748_{km/s}$$

Toltal delta change in velocity:

$$\Delta v_{\text{total}} = \Delta v_1 + \Delta v_2 = |10.2460 - 7.7891| + |1.5967 - 3.0748| = 3.9350_{km/s}$$

2.2 b

Time is half of the period.

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 37850_{\text{sec}} \rightarrow t = 18925_{\text{sec}}$$

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