Home Work #1

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1 Question 1

1.1 a

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$$\boldsymbol{C} = \dot{\boldsymbol{r}} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\boldsymbol{C}}{\mu} = \boldsymbol{e} \rightarrow \boldsymbol{e} = \frac{\boldsymbol{C}}{\mu} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$$

$$\boldsymbol{h}.\boldsymbol{e} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix} . \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} = 0$$

1.2 b, c

$$r = \frac{P}{1 + e\cos(\theta)} \xrightarrow{P=\frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right)$$

Beacuse $\boldsymbol{r}.\boldsymbol{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and $\boldsymbol{h} =$ constant, then v and θ calculated as below:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \ DU/TU$$

$$\theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right) = 2.7862_{rad}$$

2 Question 2

2.1 a

$$h = rv_{\perp} = 48846$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -26.4974 \rightarrow \text{elliptic}$$

$$\varepsilon = -\frac{1}{2}\frac{\mu^2}{h^2}(1-e^2) \rightarrow e = \sqrt{1 + \frac{2\varepsilon h^2}{\mu^2}} = 0.4519$$

2.2 b

$$P = \frac{h^2}{\mu} = 5985.8$$

$$r = \frac{P}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left(\frac{P-r}{re}\right) = 1.8909_{rad}$$

3 Question 3

$$R_e = 6378_{km}$$

$$r_p = R_e + 500$$

$$r_a = R_e + 5000$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.2465$$

$$a = \frac{r_a + r_p}{2} = 9128$$

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 8679.1_{\text{sec}}$$

3.1 a

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1-e^2)\frac{a}{R_e}\sin(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 2.5021 & 1.0022 \end{bmatrix} rad$$

$$E = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan(\theta/2)\right)$$

$$M_e = E - e\sin(E)$$

$$M_1 = 2.1587, \quad M_2 = 0.6275$$

$$t = \frac{M}{2\pi}\tau$$

$$\Delta t = |t_1 - t_2| = 2981.8 - 866.7 = 2115.0\,\text{sec}$$

3.2 b

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1-e^2)\frac{a}{R_e}\cos(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 0.4251 & 2.2506 \end{bmatrix} rad$$

$$M_1 = 0.2521, \quad M_2 = 1.8202$$

$$\Delta t = |t_1 - t_2| = 2514.3 - 348.2 = 2166.13\,\text{sec}$$

4 Question 4

4.1 a

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6570$$

For first circular orbit $r_a = r_p$.

$$h = 51174 \rightarrow v = 7.7891_{km/\,\text{sec}}$$

Second orbit (elliptical):

$$r_p = 6570, \quad r_a = 42160$$

$$h = 67316 \rightarrow v_a = 10.2460_{km/\,\text{sec}}, \quad v_p = 1.5967_{km/\,\text{sec}}$$

Third orbit (circular):

$$h = 129634 \rightarrow v = 3.0748_{km/\,\text{sec}}$$

Total delta change in velocity:

$$\Delta v_{\text{total}} = \Delta v_1 + \Delta v_2 = |10.2460 - 7.7891| + |1.5967 - 3.0748| = 3.9350_{km/\,\text{sec}}$$

4.2 b

Time is half of the period.

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 37850_{\text{sec}} \rightarrow t = 18925_{\text{sec}}$$

5 Quastion 5

First, change foot to km.

$$\boldsymbol{r} = \begin{bmatrix} 1.2756 & 1.9135 & 3.1891 \end{bmatrix} km$$

$$\boldsymbol{v} = \begin{bmatrix} 7.9053 & 0 & 15.8106 \end{bmatrix} km/\sec$$

5.1 a

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 146.0963 \rightarrow \text{hyperbolic}$$

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{bmatrix} 302531.3 & 50421.9 & -151265.7 \end{bmatrix}$$

5.2 b

$$P = \frac{h^2}{\mu} = 293399$$

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r} = \begin{bmatrix} -2.3244 & 14.5133 & 0.1889 \end{bmatrix}, \quad e = 14.6995$$

$$r = \frac{P}{1 + e\cos(\theta)} = \frac{293399}{1 + 14.6995\cos(\theta)}$$

6 short project

6.1 a

Use algorithm 4.2 of Curtis's book for calculation.

$$v_r = \frac{\boldsymbol{r}\boldsymbol{v}}{r} = 0.0102 \ km/\sec \rightarrow \text{away from perigee}$$

$$\boldsymbol{h} = \boldsymbol{r}\boldsymbol{v} = \begin{bmatrix} 11368 & -31882 & 39764 \end{bmatrix}$$

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r}$$

Others Parameters have been calculated in short_project/short_project.m

Table 1: The position and velocity components and magnitude with time for $\Delta t = 100\,\text{sec}$.

Time (s)	**Position**			**Pos magnitude**	**Velocity**			**magnitude**
65.57	1600.00	5310.00	3800.00	6722.80	-7.35	0.46	2.47	7.77
165.57	856.12	5321.11	4021.57	6724.60	-7.51	-0.24	1.96	7.77
265.57	101.01	5262.55	4190.48	6727.90	-7.57	-0.93	1.42	7.76
365.57	-655.39	5135.19	4304.60	6732.70	-7.54	-1.61	0.86	7.76
465.57	-1403.24	4940.84	4362.57	6738.91	-7.40	-2.27	0.30	7.75
565.57	-2132.91	4682.10	4363.72	6746.36	-7.17	-2.90	-0.27	7.74
665.57	-2834.92	4362.59	4308.23	6754.99	-6.85	-3.48	-0.83	7.73
765.57	-3500.22	3986.83	4197.15	6764.79	-6.44	-4.02	-1.38	7.72
865.57	-4120.41	3559.88	4032.13	6775.59	-5.95	-4.51	-1.91	7.71
965.57	-4687.82	3087.35	3815.48	6787.14	-5.39	-4.93	-2.42	7.69
1065.57	-5195.49	2575.36	3550.11	6799.18	-4.76	-5.29	-2.89	7.68
1165.57	-5637.30	2030.58	3239.63	6811.58	-4.07	-5.59	-3.32	7.67
1265.57	-6008.06	1460.29	2888.38	6824.36	-3.34	-5.81	-3.70	7.65
1365.57	-6303.53	871.83	2501.03	6837.38	-2.57	-5.95	-4.04	7.64
1465.57	-6520.44	272.58	2082.58	6850.37	-1.77	-6.02	-4.32	7.62
1565.57	-6656.50	-330.08	1638.28	6863.08	-0.95	-6.02	-4.55	7.61
1665.57	-6710.39	-928.79	1173.66	6875.28	-0.13	-5.94	-4.73	7.59
1765.57	-6681.99	-1516.15	694.61	6886.96	0.69	-5.79	-4.84	7.58
1865.57	-6572.19	-2084.95	207.18	6898.08	1.50	-5.57	-4.90	7.57
1965.57	-6382.70	-2628.35	-282.68	6908.48	2.29	-5.29	-4.89	7.56
2065.57	-6116.11	-3139.98	-769.09	6917.93	3.04	-4.94	-4.83	7.55
2165.57	-5775.82	-3613.83	-1246.29	6926.26	3.76	-4.53	-4.71	7.54
2265.57	-5366.09	-4044.38	-1708.63	6933.35	4.43	-4.07	-4.53	7.53
2365.57	-4892.26	-4426.58	-2150.52	6939.28	5.04	-3.57	-4.30	7.52
2465.57	-4360.25	-4755.99	-2566.72	6944.01	5.59	-3.02	-4.02	7.52
2565.57	-3776.47	-5028.79	-2952.34	6947.43	6.08	-2.43	-3.69	7.51
2665.57	-3147.81	-5241.84	-3302.87	6949.43	6.49	-1.82	-3.32	7.51
2765.57	-2481.63	-5392.61	-3614.20	6949.91	6.82	-1.19	-2.90	7.51
2865.57	-1785.89	-5479.34	-3882.63	6948.91	7.08	-0.54	-2.46	7.51
2965.57	-1069.05	-5501.04	-4104.96	6946.59	7.25	0.11	-1.98	7.51
3065.57	-339.66	-5457.43	-4278.50	6942.94	7.33	0.76	-1.48	7.52
3165.57	393.70	-5348.88	-4401.13	6937.97	7.33	1.41	-0.97	7.52
3265.57	1122.40	-5176.49	-4471.22	6931.64	7.23	2.04	-0.43	7.53
3365.57	1837.79	-4942.03	-4487.76	6923.96	7.06	2.65	0.10	7.54
3465.57	2531.12	-4648.26	-4450.42	6915.13	6.79	3.22	0.64	7.55
3565.57	3193.84	-4298.58	-4359.52	6905.35	6.45	3.76	1.17	7.56
3665.57	3817.85	-3897.02	-4215.94	6894.70	6.02	4.26	1.70	7.57
3765.57	4395.43	-3448.11	-4021.14	6883.23	5.52	4.71	2.20	7.58
3865.57	4919.32	-2956.99	-3777.14	6870.97	4.95	5.10	2.68	7.59
3965.57	5382.78	-2429.41	-3486.64	6858.06	4.31	5.44	3.13	7.61
4065.57	5779.73	-1871.92	-3153.13	6844.82	3.62	5.70	3.54	7.62
4165.57	6104.87	-1291.31	-2780.57	6831.43	2.88	5.90	3.91	7.64
4265.57	6353.76	-694.67	-2373.35	6818.04	2.09	6.02	4.23	7.65
4365.57	6522.78	-89.28	-1936.29	6804.69	1.28	6.07	4.50	7.67
4465.57	6609.19	517.33	-1474.62	6791.44	0.45	6.05	4.72	7.68
4565.57	6611.49	1117.39	-994.18	6778.55	-0.40	5.94	4.88	7.70

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Table 1 – continued from previous page

Time (s)	Position			magnitude	Velocity			magnitude
4665.57	6529.30	1703.04	-501.12	6766.33	-1.24	5.76	4.97	7.71
4765.57	6363.28	2266.70	-1.73	6754.95	-2.08	5.50	5.01	7.72
4865.57	6115.09	2801.05	497.64	6744.47	-2.89	5.17	4.97	7.73
4965.57	5787.40	3299.02	990.58	6734.89	-3.66	4.78	4.88	7.74
5065.57	5384.01	3753.97	1470.66	6726.26	-4.39	4.31	4.71	7.75
5165.57	4910.19	4159.74	1931.45	6718.92	-5.07	3.79	4.49	7.76
5265.57	4372.08	4510.86	2366.80	6713.02	-5.68	3.22	4.21	7.77
5365.57	3776.56	4802.60	2770.96	6708.62	-6.22	2.61	3.87	7.77
5465.57	3131.21	5030.98	3138.55	6705.65	-6.68	1.95	3.48	7.78
5565.57	2444.39	5192.81	3464.65	6704.03	-7.05	1.28	3.04	7.78
5665.57	1725.27	5286.00	3744.95	6703.95	-7.32	0.58	2.56	7.78

6.2 b

Figure 1: 3D trajectory

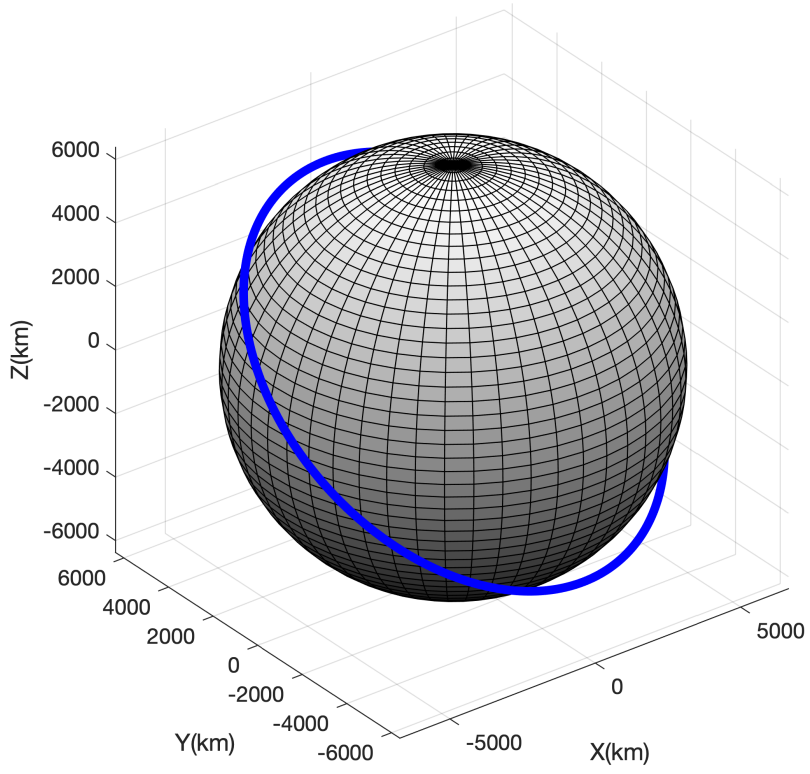


Figure 2: 3D trajectory in zx axis

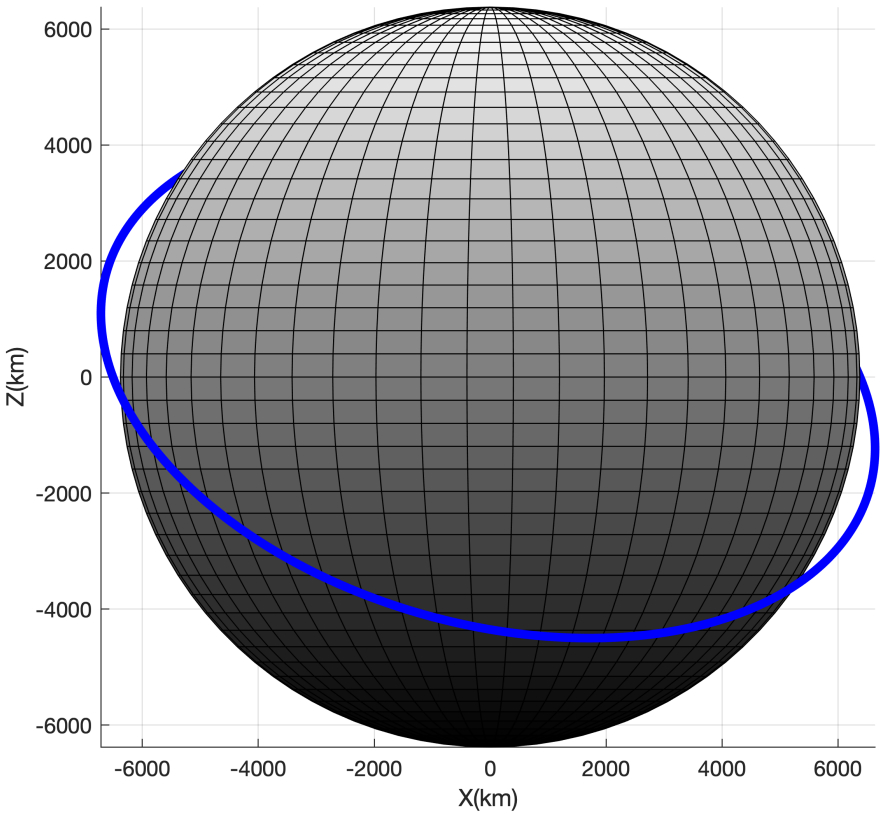


Figure 3: 3D trajectory in zy axis

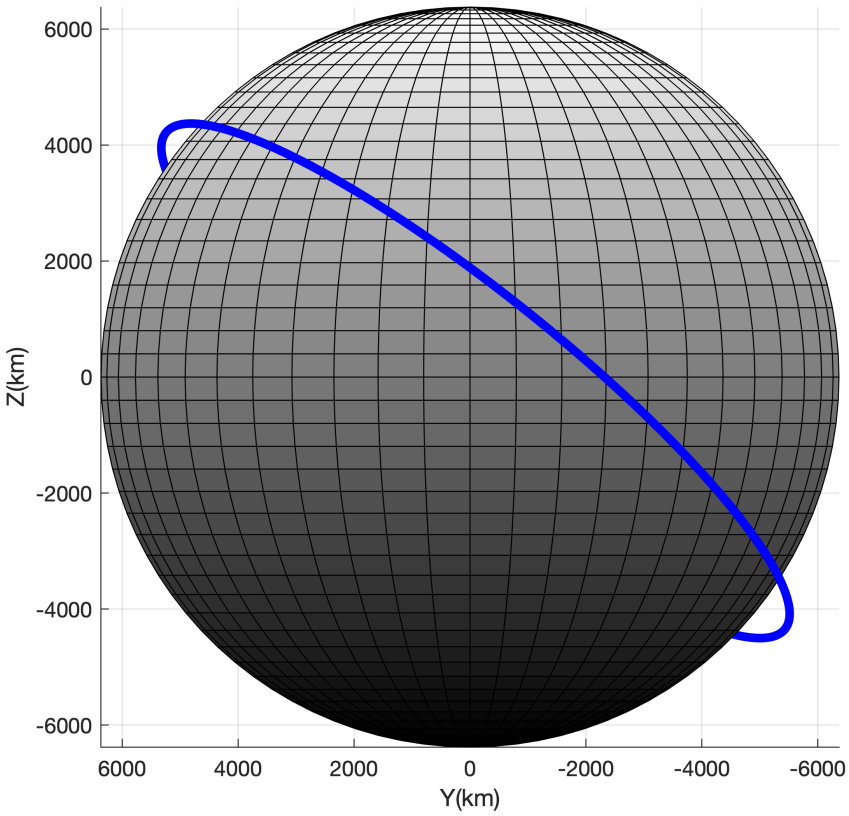
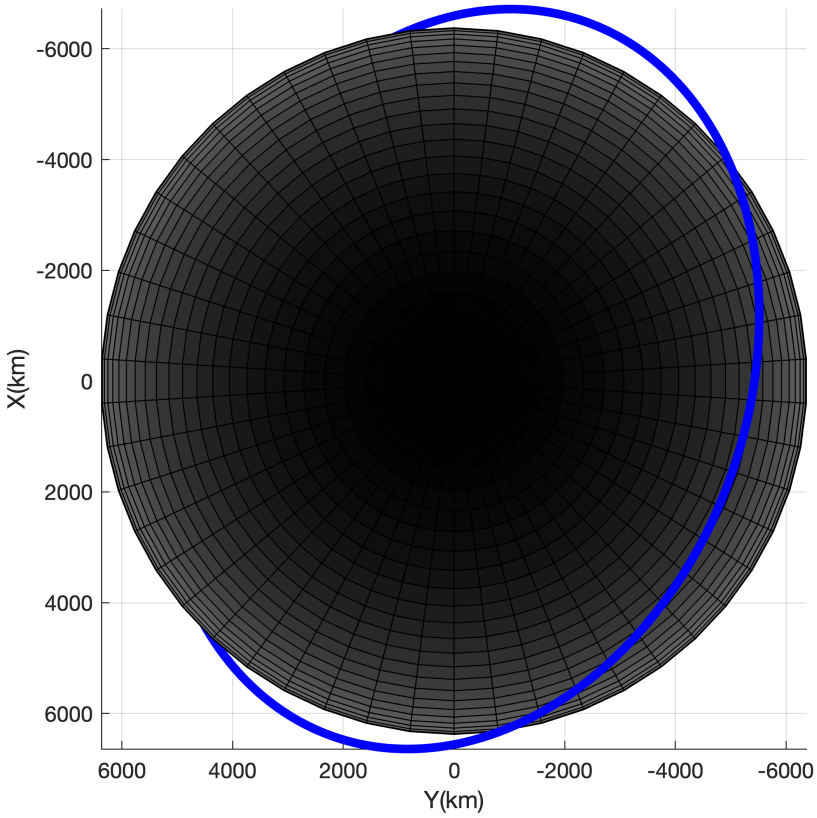


Figure 4: 3D trajectory in xy axis



6.3 c

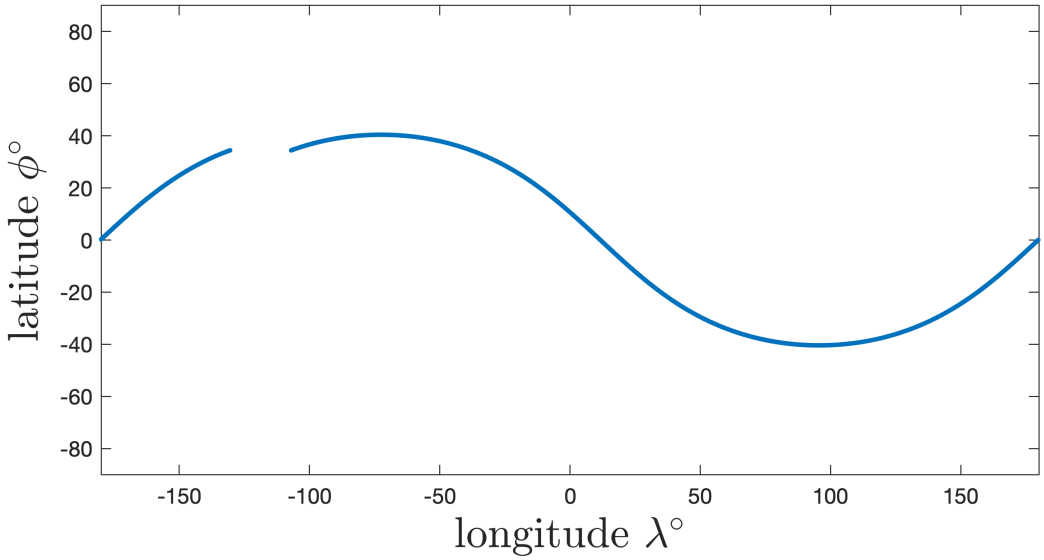
Using below tranfer matrix to tranfer from ECI corrdinate to ECEF corrdinate.

$$\boldsymbol{T}^{ECCF-ECI} = \begin{bmatrix} \cos(\omega_E t) & -\sin(\omega_E t) & 0 \\ \sin(\omega_E t) & \cos(\omega_E t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \arccos(\frac{\boldsymbol{r}(3)}{r})$$

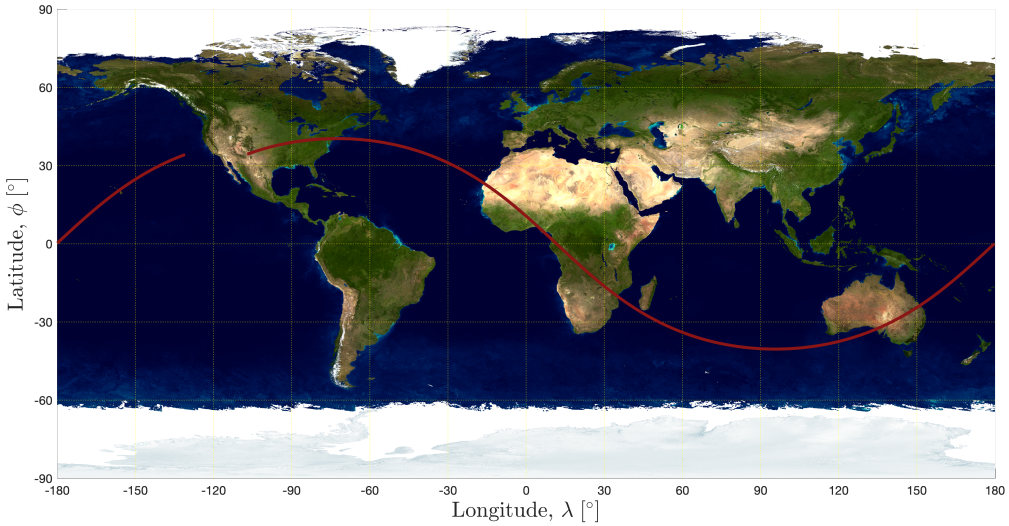
$$\lambda = \begin{cases} \arctan(\frac{\boldsymbol{r}(1)}{r_{xy}}), & \boldsymbol{r}(2) > 0 \\ 2\pi - \arctan(\frac{\boldsymbol{r}(1)}{r_{xy}}), & \boldsymbol{r}(2) \leq 0 \end{cases}$$

Figure 5: Satellite latitude versus its longitude for one period



Below the figure drawn provided by tamaskis, please click here to see the source code. Please use mentioned library to run code or skip part on earth fig.

Figure 6: Satellite latitude versus its longitude for one period



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