

Home Work #2

Ali BaniAsad 401209244

December 9, 2022

1 Question 1

The space shuttle weighs approximately 12.5 tons, whose thrusters can simultaneously produce a total thrust of 53400 Newtons for orbital maneuvers. Assuming that the shuttle is initially in a 300 Km (altitude) circular Earth orbit, it is desired to use a single impulse to transfer the shuttle to a 250x300 Km elliptical orbit.

1.1 part a

$$h = \sqrt{2\mu} \sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6678$$

For first circular orbit $r_a = r_p$.

$$h = 51593 \rightarrow v = 7.7258_{km/sec}$$

Second orbit (elliptical):

$$r_p = 6628, \quad r_a = 6678$$

$$h = 51496 \rightarrow v_a = 7.7113_{km/sec}$$

$$\Delta v = v_a - v = 0.0145_{km/sec}$$

1.2 part b

$$T = \frac{m\Delta v}{\Delta t} \rightarrow \Delta t = \frac{m\Delta v}{T} = 0.0034_{sec}$$

1.3 part c

Assuming the velocity is the mean velocity of circular and elliptical velocity.

$$v_{mean} = \frac{v_{circular} + v_{elliptical}}{2} = 7.7186_{km/sec}$$

$$\begin{aligned} \text{distance} &= v_{mean} \times \Delta t \pm \Delta v \Delta t = 7.7186_{km/sec} \times 0.0034_{sec} \pm 0.0145_{km/sec} \times 0.0034_{sec} \\ &= 0.0268_{km} \pm 4.9415 \times 10^{-5}_{km} \end{aligned}$$

1.4 part d

Period of the first Orbit:

$$\tau_1 = 2\pi \sqrt{\frac{\mu}{r^3}} = 864.3726_{sec} \rightarrow \frac{\Delta t}{\tau_1} = 3.9347 \times 10^{-6}$$

Period of the second Orbit:

$$\tau_2 = 2\pi \sqrt{\frac{\mu}{a^3}} = 859.5233_{sec} \rightarrow \frac{\Delta t}{\tau_1} = 3.9569 \times 10^{-6}$$

1.5 part e

Assume:

$$C_1 = C_2 = 2\pi r = 4.1959e + 04 \rightarrow \frac{\Delta d}{C_1} = 1.1777 \times 10^{-9}$$

2 Question 2

We know that $r_1 = r_2$, $\theta_1 = 90^\circ$, $\theta_2 = 0$, and $e_1 = e_2 = e$.

$$r_1 = \frac{h_1^2}{\mu} \frac{1}{1 + e_1 \cos(\theta_1)} = r_2 = \frac{h_2^2}{\mu} \frac{1}{1 + e_1 \cos(\theta_2)} \xrightarrow[\substack{\theta_1=90^\circ, \theta_2=0 \\ e_1=e_2=e}]{\theta_1=90^\circ, \theta_2=0} \frac{h_1^2}{1 + e} = h_2^2 \rightarrow h_2 = \frac{h_1}{\sqrt{1 + e}}$$

3 Question 3

$$r_p = 8000, r_a = 13000e = \frac{r_a - r_p}{r_a + r_p}, a = \frac{r_a + r_p}{2}, \tau = 2\pi \sqrt{\frac{a^3}{\mu}}$$

Spacecraft 1:

$$r_0 = \frac{h^2}{\mu} \frac{1}{1 + e_1 \cos(\theta_0)}, r_2 = \frac{h^2}{\mu} \frac{1}{1 + e_1 \cos(\theta_2)}$$

$$\mathbf{r}_0 = [r_0 \cos(\theta_0) \quad r_0 \sin(\theta_0) \quad 0], \mathbf{r}_2 = [r_2 \cos(\theta_2) \quad r_2 \sin(\theta_2) \quad 0]$$

$$\mathbf{v}_0 = \frac{\mu}{h} [-\sin(\theta_0) \quad e + \cos(\theta_0) \quad 0], \mathbf{v}_2 = \frac{\mu}{h} [-\sin(\theta_2) \quad e + \cos(\theta_2) \quad 0]$$

Spacecraft 2:

$$E = 2 \arctan \left(\sqrt{\frac{1-e}{1+e}} \tan(\theta/2) \right), \quad M_e = E - e \sin(E), \quad M = \frac{2\pi}{\tau} t \xrightarrow[\theta_1=\pi/6]{\theta_2=\pi/2} t_1 = 542.785_{\text{sec}}, \quad t_2 = 1873.138_{\text{sec}}$$

Furthermore, I Used lambert's formulas to solve the problem, taking r0, r2, and delta t to lambert's formulas as input for the function.

$$\mathbf{v}_{0l} = \begin{bmatrix} -2.7240 & 9.4690 & 0 \end{bmatrix} \frac{km}{sec}, \quad \mathbf{v}_{2l} = \begin{bmatrix} -7.8503 & 4.3427 & 0 \end{bmatrix} \frac{km}{sec}$$

$$\Delta \mathbf{v}_0 = \mathbf{v}_{0l} - \mathbf{v}_0 = \begin{bmatrix} -2.7240 & 1.6149 & 0 \end{bmatrix} \frac{km}{sec}, \quad \Delta \mathbf{v}_2 = \mathbf{v}_{2l} - \mathbf{v}_2 = \begin{bmatrix} -1.5066 & 2.8323 & 0 \end{bmatrix} \frac{km}{sec}$$

4 Question 4

$$\mathbf{r}_{ap} = \begin{bmatrix} 9798 & 5657 & 11314 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -0.1 & 0.2 & 0.3 \end{bmatrix}$$

$$a = \frac{r_{ap}}{2}$$

$$\theta = \arccos \left(\frac{\mathbf{r}_{ap} \cdot \mathbf{v}}{r_{ap} v} \right)$$

There is 2 equations and 2 unknowns, so we can solve it by using MATLAB.

$$\frac{h^2}{\mu} = a(1 - e^2), \quad v_r = \frac{\mu}{h} e \sin \theta \rightarrow e = 0.0656, \quad h = 56348$$

4.1 part a

$$r = \frac{h^2}{\mu} \frac{1}{(1 + e \cos \theta)} = \frac{h^2}{\mu(1 + e \cos \theta)} = 7667.5 \frac{km}{sec}$$

4.2 part b

$$v_{\perp} = \frac{h}{r} = 3.5217 \frac{km}{sec}$$

$$\gamma = \arctan \left(\frac{v_r}{v_{\perp}} \right) = 0.1058_{\text{rad}}$$

5 Question 5

$$\theta_{B_1} = -\pi/2, \theta_{B_2} = \pi/2$$

$$r_B = \frac{h^2}{\mu} \frac{1}{(1 + e \cos(\pm\pi/2))} = \frac{h^2}{\mu}$$

$$v_{\perp_{B_1}} = v_{\perp_{B_2}} = \frac{h}{r_B} = \frac{\mu}{h}$$

$$v_{r_{B_1}} = \frac{\mu}{h} e \sin(\theta_{B_1}) = -\frac{\mu e}{h}$$

$$v_{r_{B_2}} = \frac{\mu}{h} e \sin(\theta_{B_2}) = \frac{\mu e}{h}$$

$$\Delta v_B = \sqrt{(v_{r_{B_2}} - v_{r_{B_1}})^2 + v_{\perp_{B_1}}^2 + v_{\perp_{B_2}}^2 - 2v_{\perp_{B_1}} v_{\perp_{B_2}} \cos(\pi/2)} = \Delta v_B = \sqrt{4\frac{\mu^2}{h^2} e^2 + 2\frac{\mu^2}{h^2}}$$

$$\Delta v_b = \frac{\sqrt{2}\mu}{h} \sqrt{1 + 2e^2}$$

Contents

| | | |
|----------|-------------------|----------|
| 1 | Question 1 | 1 |
| 1.1 | part a | 1 |
| 1.2 | part b | 1 |
| 1.3 | part c | 2 |
| 1.4 | part d | 2 |
| 1.5 | part e | 2 |
| 2 | Question 2 | 2 |
| 3 | Question 3 | 2 |
| 4 | Question 4 | 3 |
| 4.1 | part a | 3 |
| 4.2 | part b | 3 |
| 5 | Question 5 | 4 |

List of Figures

List of Tables