Home Work #1

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1 Question 1

1.1 a

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix}$$

$$\boldsymbol{C} = \dot{\boldsymbol{r}} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

In Astronomical/Canonical Units: $\mu = 1$

$$\frac{\boldsymbol{C}}{\mu} = \boldsymbol{e} \rightarrow \boldsymbol{e} = \frac{\boldsymbol{C}}{\mu} = \begin{bmatrix} -1 & -1 & 0 \end{bmatrix}$$

$$\boldsymbol{h}.\boldsymbol{e} = \begin{bmatrix} 0 & 0 & -\sqrt{2} \end{bmatrix} . \begin{bmatrix} -1 & -1 & 0 \end{bmatrix} = 0$$

1.2 b, c

$$r = \frac{P}{1 + e\cos(\theta)} \xrightarrow{P=\frac{h^2}{\mu}} r = \frac{h^2}{\mu} \frac{1}{1 + e\cos(\theta)} \rightarrow \theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right)$$

Beacuse $\boldsymbol{r}.\boldsymbol{v} > 0$, θ is in the range $0 \leq \theta \leq \pi$

$$\rightarrow \theta = \pi/2$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 0 = \text{constant}$$

1.3 d

In $r = 32DU$, $\varepsilon = 0$ and $\boldsymbol{h} =$ constant, then v and θ calculated as below:

$$\varepsilon = 0 \rightarrow v = \sqrt{\frac{2\mu}{r}} = 0.25 \ DU/TU$$

$$\theta = \arccos\left((\frac{h^2}{\mu r} - 1)/e \right) = 2.7862_{rad}$$

2 Question 2

2.1 a

$$h = rv_{\perp} = 48846$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -26.4974 \rightarrow \text{elliptic}$$

$$\varepsilon = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2) \rightarrow e = \sqrt{1 + \frac{2\varepsilon h^2}{\mu^2}} = 0.4519$$

2.2 b

$$P = \frac{h^2}{\mu} = 5985.8$$

$$r = \frac{P}{1 + e\cos(\theta)} \rightarrow \theta = \cos^{-1}\left(\frac{P - r}{re}\right) = 1.8909_{rad}$$

3 Question 3

$$R_e = 6378_{km}$$

$$r_p = R_e + 500$$

$$r_a = R_e + 5000$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.2465$$

$$a = \frac{r_a + r_p}{2} = 9128$$

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 8679.1_{\text{sec}}$$

3.1 a

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1 - e^2)\frac{a}{R_e}\sin(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 2.5021 & 1.0022 \end{bmatrix} rad$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan(\theta/2)\right)$$

$$M_e = E - e\sin(E)$$

$$M_1 = 2.1587, \quad M_2 = 0.6275$$

$$t = \frac{M}{2\pi}\tau$$

$$\Delta t = |t_1 - t_2| = 2981.8 - 866.7 = 2115.0\,\text{sec}$$

3.2 b

Solving the below equations with Matlab script in Q3/Q3.m:

$$e\cos(\theta) - (1-e^2)\frac{a}{R_e}\cos(\theta) + 1 = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} 0.4251 & 2.2506 \end{bmatrix} rad$$

$$M_1 = 0.2521, \quad M_2 = 1.8202$$

$$\Delta t = |t_1 - t_2| = 2514.3 - 348.2 = 2166.13\,\text{sec}$$

4 Question 4

4.1 a

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6570$$

For first circular orbit $r_a = r_p$.

$$h = 51174 \rightarrow v = 7.7891_{km/\,\text{sec}}$$

Second orbit (elliptical):

$$r_p = 6570, \quad r_a = 42160$$

$$h = 67316 \rightarrow v_a = 10.2460_{km/\,\text{sec}}, \quad v_p = 1.5967_{km/\,\text{sec}}$$

Third orbit (circular):

$$h = 129634 \rightarrow v = 3.0748_{km/\,\text{sec}}$$

Total delta change in velocity:

$$\Delta v_{\text{total}} = \Delta v_1 + \Delta v_2 = |10.2460 - 7.7891| + |1.5967 - 3.0748| = 3.9350_{km/\,\text{sec}}$$

4.2 b

Time is half of the period.

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 37850_{\text{sec}} \rightarrow t = 18925_{\text{sec}}$$

5 Quastion 5

First, change foot to km.

$$\boldsymbol{r} = \begin{bmatrix} 1.2756 & 1.9135 & 3.1891 \end{bmatrix} km$$

$$\boldsymbol{v} = \begin{bmatrix} 7.9053 & 0 & 15.8106 \end{bmatrix} km/\sec$$

5.1 a

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = 146.0963 \rightarrow \text{hyperbolic}$$

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} = \begin{bmatrix} 302531.3 & 50421.9 & -151265.7 \end{bmatrix}$$

5.2 b

$$P = \frac{h^2}{\mu} = 293399$$

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r} = \begin{bmatrix} -2.3244 & 14.5133 & 0.1889 \end{bmatrix}, \quad e = 14.6995$$

$$r = \frac{P}{1 + e\cos(\theta)} = \frac{293399}{1 + 14.6995\cos(\theta)}$$

6 short project

6.1 a

Use algorithm 4.2 of Curtis's book for calculation.

$$v_r = \frac{\boldsymbol{r}\boldsymbol{v}}{r} = 0.0102 \ km/\sec \rightarrow \text{away from perigee}$$

$$\boldsymbol{h} = \boldsymbol{r}\boldsymbol{v} = \begin{bmatrix} 11368 & -31882 & 39764 \end{bmatrix}$$

$$\boldsymbol{e} = \frac{\boldsymbol{v} \times \boldsymbol{h}}{\mu} - \frac{\boldsymbol{r}}{r}$$

Others Parameters have been calculated in short_project/short_project.m

Table 1: The position and velocity components and magnitude with time for $\Delta t = 100\,\text{sec}$.

Time (s)	**Position**			**Pos magnitude**	**Velocity**			**magnitude**
65.57	1600.00	5310.00	3800.00	6722.80	-7.35	0.46	2.47	7.77
165.57	856.12	5321.11	4021.57	6724.60	-7.51	-0.24	1.96	7.77
265.57	101.01	5262.55	4190.48	6727.90	-7.57	-0.93	1.42	7.76
365.57	-655.39	5135.19	4304.60	6732.70	-7.54	-1.61	0.86	7.76
465.57	-1403.24	4940.84	4362.57	6738.91	-7.40	-2.27	0.30	7.75
565.57	-2132.91	4682.10	4363.72	6746.36	-7.17	-2.90	-0.27	7.74
665.57	-2834.92	4362.59	4308.23	6754.99	-6.85	-3.48	-0.83	7.73
765.57	-3500.22	3986.83	4197.15	6764.79	-6.44	-4.02	-1.38	7.72
865.57	-4120.41	3559.88	4032.13	6775.59	-5.95	-4.51	-1.91	7.71
965.57	-4687.82	3087.35	3815.48	6787.14	-5.39	-4.93	-2.42	7.69
1065.57	-5195.49	2575.36	3550.11	6799.18	-4.76	-5.29	-2.89	7.68
1165.57	-5637.30	2030.58	3239.63	6811.58	-4.07	-5.59	-3.32	7.67
1265.57	-6008.06	1460.29	2888.38	6824.36	-3.34	-5.81	-3.70	7.65
1365.57	-6303.53	871.83	2501.03	6837.38	-2.57	-5.95	-4.04	7.64
1465.57	-6520.44	272.58	2082.58	6850.37	-1.77	-6.02	-4.32	7.62
1565.57	-6656.50	-330.08	1638.28	6863.08	-0.95	-6.02	-4.55	7.61
1665.57	-6710.39	-928.79	1173.66	6875.28	-0.13	-5.94	-4.73	7.59
1765.57	-6681.99	-1516.15	694.61	6886.96	0.69	-5.79	-4.84	7.58
1865.57	-6572.19	-2084.95	207.18	6898.08	1.50	-5.57	-4.90	7.57
1965.57	-6382.70	-2628.35	-282.68	6908.48	2.29	-5.29	-4.89	7.56
2065.57	-6116.11	-3139.98	-769.09	6917.93	3.04	-4.94	-4.83	7.55
2165.57	-5775.82	-3613.83	-1246.29	6926.26	3.76	-4.53	-4.71	7.54
2265.57	-5366.09	-4044.38	-1708.63	6933.35	4.43	-4.07	-4.53	7.53
2365.57	-4892.26	-4426.58	-2150.52	6939.28	5.04	-3.57	-4.30	7.52
2465.57	-4360.25	-4755.99	-2566.72	6944.01	5.59	-3.02	-4.02	7.52
2565.57	-3776.47	-5028.79	-2952.34	6947.43	6.08	-2.43	-3.69	7.51
2665.57	-3147.81	-5241.84	-3302.87	6949.43	6.49	-1.82	-3.32	7.51
2765.57	-2481.63	-5392.61	-3614.20	6949.91	6.82	-1.19	-2.90	7.51
2865.57	-1785.89	-5479.34	-3882.63	6948.91	7.08	-0.54	-2.46	7.51
2965.57	-1069.05	-5501.04	-4104.96	6946.59	7.25	0.11	-1.98	7.51
3065.57	-339.66	-5457.43	-4278.50	6942.94	7.33	0.76	-1.48	7.52
3165.57	393.70	-5348.88	-4401.13	6937.97	7.33	1.41	-0.97	7.52
3265.57	1122.40	-5176.49	-4471.22	6931.64	7.23	2.04	-0.43	7.53
3365.57	1837.79	-4942.03	-4487.76	6923.96	7.06	2.65	0.10	7.54
3465.57	2531.12	-4648.26	-4450.42	6915.13	6.79	3.22	0.64	7.55
3565.57	3193.84	-4298.58	-4359.52	6905.35	6.45	3.76	1.17	7.56
3665.57	3817.85	-3897.02	-4215.94	6894.70	6.02	4.26	1.70	7.57
3765.57	4395.43	-3448.11	-4021.14	6883.23	5.52	4.71	2.20	7.58
3865.57	4919.32	-2956.99	-3777.14	6870.97	4.95	5.10	2.68	7.59
3965.57	5382.78	-2429.41	-3486.64	6858.06	4.31	5.44	3.13	7.61
4065.57	5779.73	-1871.92	-3153.13	6844.82	3.62	5.70	3.54	7.62
4165.57	6104.87	-1291.31	-2780.57	6831.43	2.88	5.90	3.91	7.64
4265.57	6353.76	-694.67	-2373.35	6818.04	2.09	6.02	4.23	7.65
4365.57	6522.78	-89.28	-1936.29	6804.69	1.28	6.07	4.50	7.67
4465.57	6609.19	517.33	-1474.62	6791.44	0.45	6.05	4.72	7.68
4565.57	6611.49	1117.39	-994.18	6778.55	-0.40	5.94	4.88	7.70

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Time (s)	Position			magnitude	Velocity			magnitude
4665.57	6529.30	1703.04	-501.12	6766.33	-1.24	5.76	4.97	7.71
4765.57	6363.28	2266.70	-1.73	6754.95	-2.08	5.50	5.01	7.72
4865.57	6115.09	2801.05	497.64	6744.47	-2.89	5.17	4.97	7.73
4965.57	5787.40	3299.02	990.58	6734.89	-3.66	4.78	4.88	7.74
5065.57	5384.01	3753.97	1470.66	6726.26	-4.39	4.31	4.71	7.75
5165.57	4910.19	4159.74	1931.45	6718.92	-5.07	3.79	4.49	7.76
5265.57	4372.08	4510.86	2366.80	6713.02	-5.68	3.22	4.21	7.77
5365.57	3776.56	4802.60	2770.96	6708.62	-6.22	2.61	3.87	7.77
5465.57	3131.21	5030.98	3138.55	6705.65	-6.68	1.95	3.48	7.78
5565.57	2444.39	5192.81	3464.65	6704.03	-7.05	1.28	3.04	7.78
5665.57	1725.27	5286.00	3744.95	6703.95	-7.32	0.58	2.56	7.78

6.2 part b

Figure 1: 3D trajectory

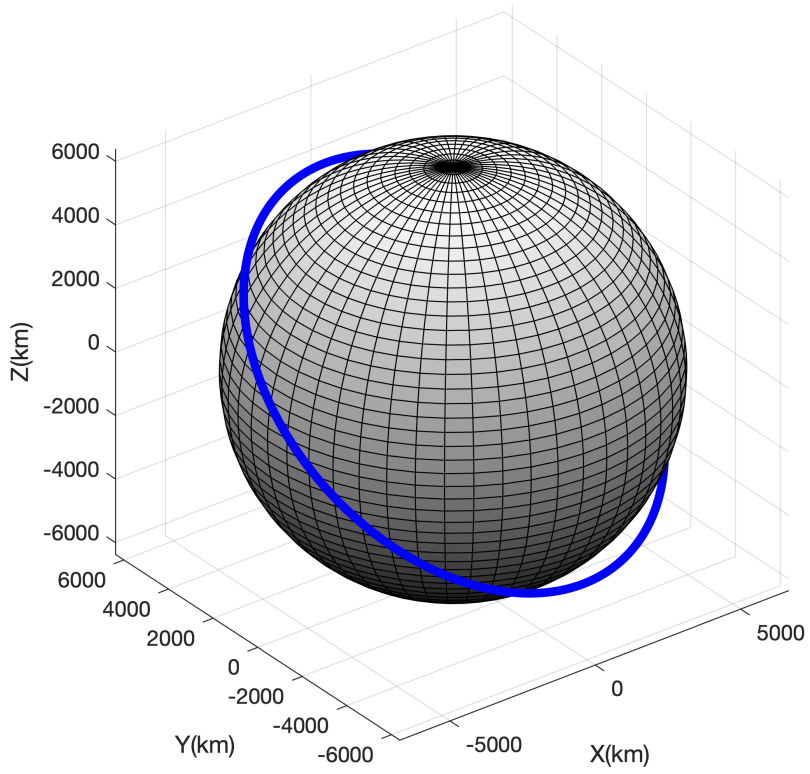


Figure 2: 3D trajectory in zx axis

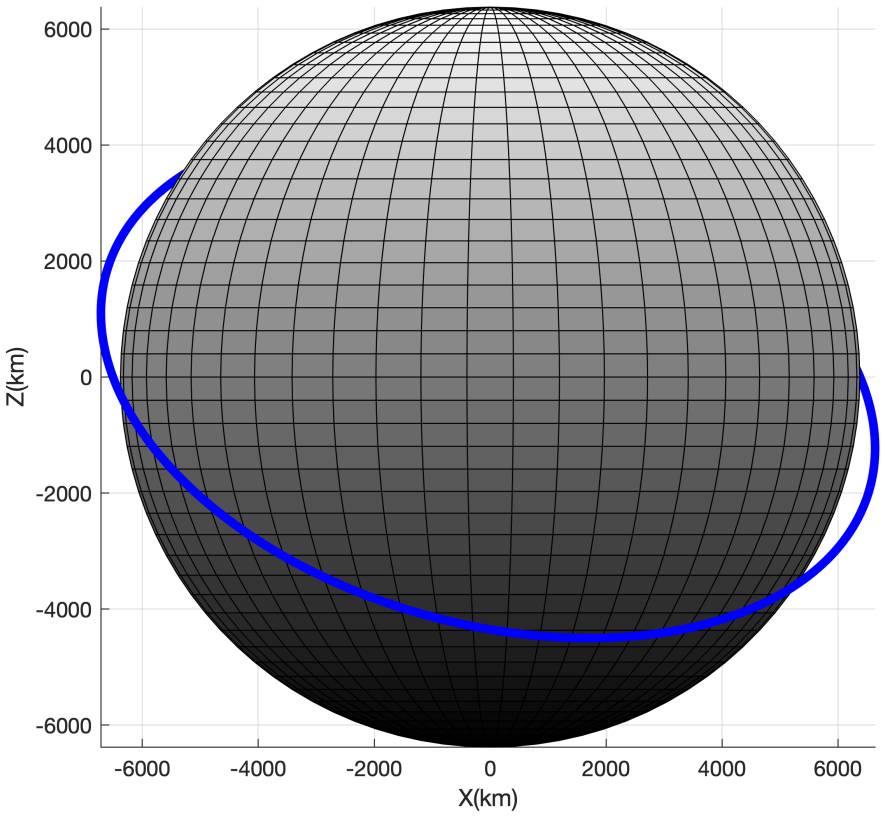


Figure 3: 3D trajectory in zy axis

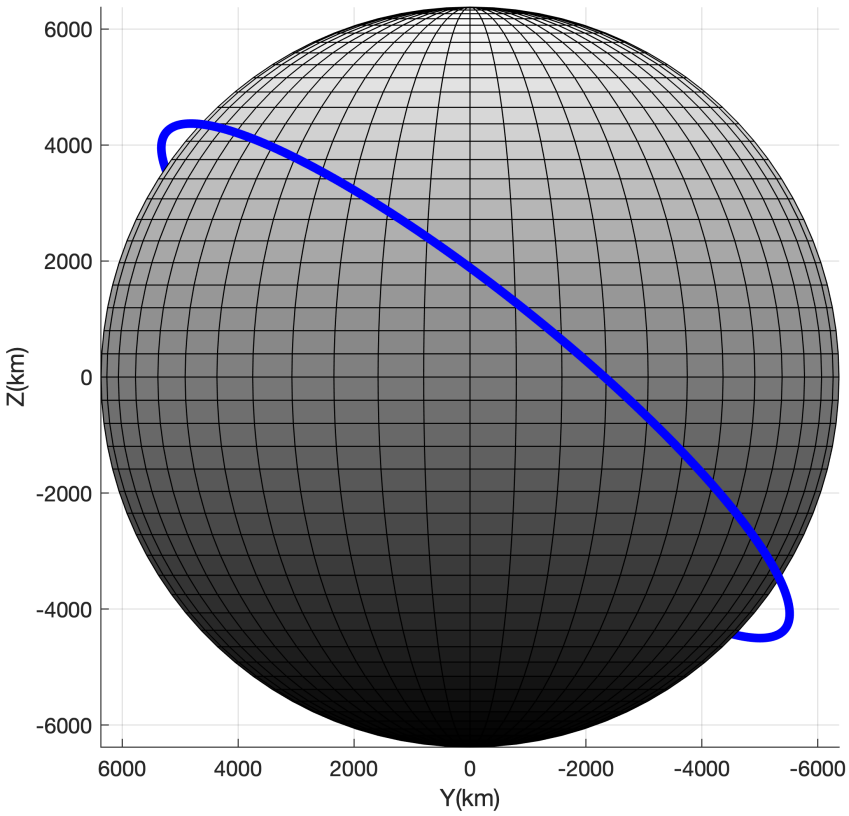
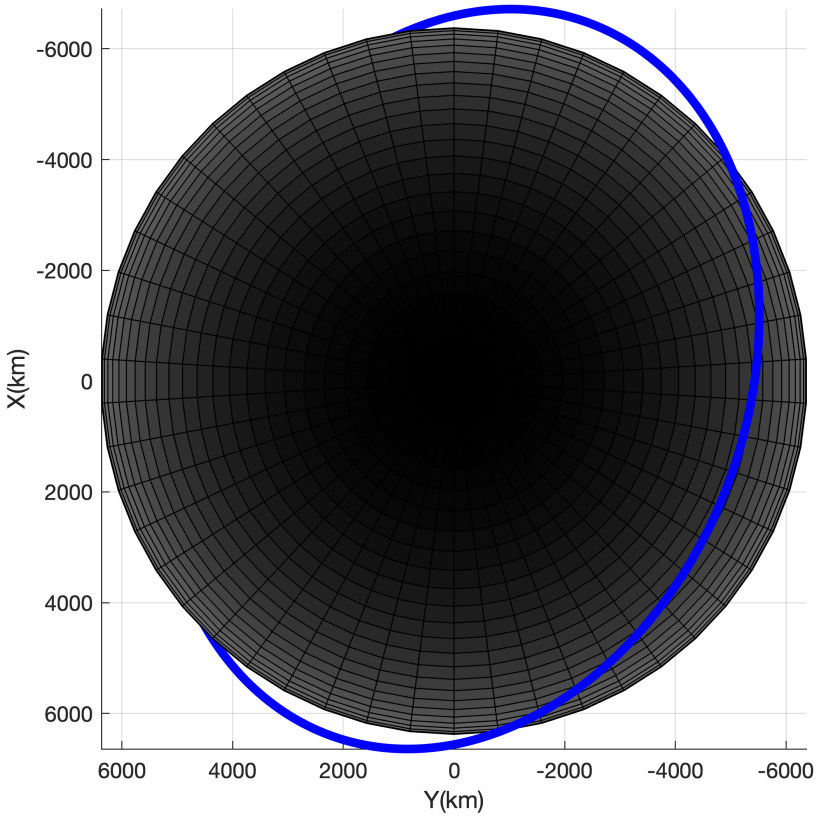


Figure 4: 3D trajectory in xy axis



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