

- Consider an spacecraft at time  $t_0$  in canonical units (for which  $\mu = 1$ ), whose position and velocity are given within the context of 2BP:  $\vec{r} = [0 \ 2 \ 0]^T DU$ ;  $\vec{v} = \frac{1}{\sqrt{2}} [1 \ 1 \ 0]^T \frac{DU}{TU}$ 
  - Calculate the vectors  $\vec{h}$  and  $\vec{e}$ , then verify that  $\vec{h} \cdot \vec{e} = 0$ .
  - Write the orbit polar equation  $r(\theta) = ?$ .
  - What is the value of the true anomaly  $\theta$ , at  $t_0$ ?
  - Determine the speed and the true anomaly of the spacecraft when  $r = 32 DU$ .
- Radar tracking data for an Earth satellite indicates an altitude of  $600 \text{ km}$ ,  $r\dot{\theta} = 7 \text{ km/s}$ , and  $\dot{r} = 3.5 \text{ km/s}$ . Determine the orbit eccentricity  $e$  and the satellite true anomaly  $\theta$ , at the instant of observation.  
 $(R_E = 6378 \text{ km}, \mu_E = 398600 \text{ km}^3 \cdot \text{s}^{-2})$ .

- Let a satellite be in a  $500 \times 5000 \text{ Km}$  orbit around the Earth.

3-1 Assume the apse line to be parallel to the line from the Earth to the Sun (as shown in the course presentation example).

Find the time that the satellite is in the Earth's shadow if the Perigee is toward the Sun.

**Hint:** This is a continuation of example 8 part b.

3-2 Assume the Sun is aligned with the Earth's North Pole as shown, and determine the time that the satellite will be within the Earth shadow in this case.

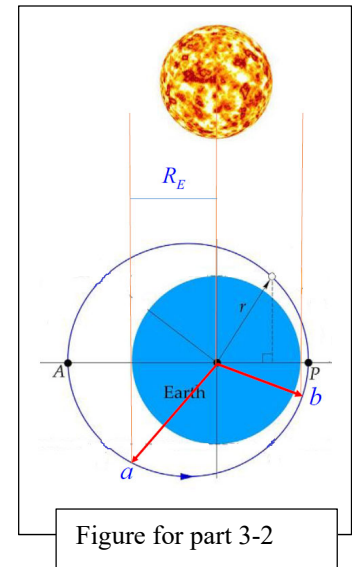


Figure for part 3-2

- It is desired to place a communications satellite into a geosynchronous orbit from a low-Earth circular parking orbit. What is the required  $\Delta V_{total}$  for this transfer and how long will it take?

$$(r_{orbit 1} = 6570 \text{ km}, r_{orbit 2} = 42160 \text{ km})$$

- The inertial position and velocity of an Earth orbiting satellite is given by :

$$\vec{r}_{ECI} = (4.1852I + 6.2778J + 10.463K) \times 10^7 \text{ ft}; \quad \vec{v}_{ECI} = (2.5936I + 5.1872K) \times 10^4 \text{ ft/sec}$$

3-1 Determine the satellite specific energy  $\mathcal{E}$ , and specific angular momentum,  $h$ .

3-2 Compute the satellite flight-path angle,  $\gamma$  at the current location as well as its planar (2D) orbital equation.

- Suggested Practice problems from Curtis : (not to be turned in!)**

2-3, 2-5, 2-6, 2-7, 2-8, 2-9, 2-10.

3-3, 3-4, 3-5, 3-6, 3-7, 3-8, 3-11, 3-13.

### Short Project one: (Mandatory)

Consider the **TBP** governing equation of motion in **3D**. As mentioned, this equation governs the behavior of satellites or space vehicles in the Earth Centered Inertial Frame (ECI).

*TBP Governing Equation of Motion*

$$\ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r} ; \mu_{Earth} = 398,600 \text{ km}^3/\text{s}^2$$

$$\vec{r}(0) = 1600\mathbf{I} + 5310\mathbf{J} + 3800\mathbf{K} \text{ (km)}$$

$$\dot{\vec{r}}(0) = -7.350\mathbf{I} + 0.4600\mathbf{J} + 2.470\mathbf{K} \text{ (km/s)}$$

Perform the following tasks:

- Use the initial condition of an arbitrary satellite given below to propagate its orbit for at least one complete period (the orbit is closed). You can **tabulate** the position and velocity components and magnitude with time for  $\Delta t = 100 \text{ Sec}$ .
- Show (plot) the resulting satellite 3D trajectory for one complete period.
- Show the satellite ground track or its geographical position on the Earth surface. This can be simply shown as a graph of the satellite latitude versus its longitude for one period. See the sample given below.

