1) Consider a rigid SC with the given inertia tensor and other information given in the body coordinate system.

$$I=\begin{bmatrix} 30 & -I_{xy} & -I_{xz} \\ -10 & 20 & -I_{yz} \\ 0 & -I_{zy} & 30 \end{bmatrix} Kg.m^2;\ \vec{\omega}=[10,10,10]^T rps\ \ ;\ \vec{h}=[200,200,400]^T Kg.m^2/s$$

1-1 Determine the SC rotational kinetic energy (RKE) and its moment of inertia about the given $\vec{\omega}$, vector.

1-2 Determine the SC Principal MOIs, the associated rotation matrix (between the body and the principle axes), as well as the corresponding Euler angles between the two BCSs.

1-3 Determining and draw the SC ellipsoid of inertia.

1-4 Determine and draw the SC AM and KRE ellipsoids for the given energy.

1-5 Plot/show the intersection between the AM and KRE ellipsoids (Polhode curves).

1-6 **Optional Bonus**: Present mathematical formulas for the Polhode curves.

2) Consider an Earth orbiting satellite is in a circular orbit at an altitude of 500 km. The satellite MOI in in the principal axis is provided as $I_x=8, I_y=10, I_z=14\ \ Kg.m^2$. The satellite **IC** (say at the injection time) is also given as $\vec{q}(0)=[0.3,0.2,0.5,0.7874]^T;\ \ \vec{\omega}(0)=[2,3,-5]^T\times 10^{-3} rps$.

2-1 Determine the initial (injecton) Euler angles.

2-2 Simulate the satellite **torque free motion** using the given IC for 50 seconds, using both the nonlinear AD EOM as well as the linearized EOM (both with GG effect). Compare and plot the Euler angles resulting from the two simulations.

2-3 If the IC is nulled , that is $\psi(0)=\theta(0)=\varphi(0)=0; with\ \ \vec{\omega}(0)=\vec{0}$, simulate the satellite AD behavior for 50 seconds, using the linearized AD EOM with GG effect with an initial torque disturbance of : $M_x=M_y=M_z=8\times 10^{-6}\ N.m$.

3) Show that the characteristic equation for pure passive coupled roll-yaw stability, is as given: continued.

$$T_{Dx}=I_x\ddot{\varphi}+4\omega_0^2\left(I_y-I_z\right)\varphi-\omega_0\left(I_x+I_y-I_z\right)\dot{\psi}$$

$$T_{Dz}=I_z\ddot{\psi}+\omega_0\left(I_z+I_x-I_y\right)\dot{\varphi}+\omega_0^2\left(I_y-I_x\right)\psi$$

$$s^4+\omega_0^2\left[3\sigma_x+\sigma_x\sigma_z+1\right]s^2+4\omega_0^2\sigma_x\sigma_z=0$$

4) Investigate the pure passive GG stability of a satellite with MOI : $I_x=8, I_y=10, I_z=14\ \ Kg.m^2$ and how does it compare with the solution of problem 2-2 above.

5) TBC