Home Work #2

Ali BaniAsad 401209244

December 10, 2022

1 Question 1

The space shuttle weighs approximately 12.5 tons, whose thrusters can simultaneously produce a total thrust of 53400 Newtons for orbital maneuvers. Assuming that the shuttle is initially in a 300 Km (altitude) circular Earth orbit, it is desired to use a single impulse to transfer the shuttle to a 250x300 Km elliptical orbit.

1.1 part a

$$h = \sqrt{2\mu}\sqrt{\frac{r_a r_p}{r_a + r_p}}$$

$$v = \frac{h}{r}$$

First orbit (circular):

$$r = 6678$$

For first circular orbit $r_a = r_p$.

$$h = 51593 \rightarrow v = 7.7258_{km/\sec}$$

Second orbit (elliptical):

$$r_p = 6628, \quad r_a = 6678$$

$$h = 51496 \rightarrow v_a = 7.7113_{km/\sec}$$

$$\Delta v = v_a - v = 0.0145_{km/\sec}$$

1.2 part b

$$T = \frac{m\Delta v}{\Delta t} \rightarrow \Delta t = \frac{m\Delta v}{T} = 0.0034_{\sec}$$

1.3 part c

Assuming the velocity is the mean velocity of circular and elliptical velocity.

$$v_{mean} = \frac{v_{circular} + v_{elliptical}}{2} = 7.7186_{km/\sec}$$

$$\begin{aligned} \text{distance} &= v_{mean} \times \Delta t \pm \Delta v \Delta t = 7.7186_{km/\sec} \times 0.0034_{\text{sec}} \pm 0.0145_{km/\sec} \times 0.0034_{\text{sec}} \\ &= 0.0268_{km} \pm 4.9415 \times 10^{-5}_{km} \end{aligned}$$

1.4 part d

Period of the first Orbit:

$$\tau_1 = 2\pi\sqrt{\frac{\mu}{r^3}} = 864.3726_{\text{sec}} \rightarrow \frac{\Delta t}{\tau_1} = 3.9347 \times 10^{-6}$$

Period of the second Orbit:

$$\tau_2 = 2\pi\sqrt{\frac{\mu}{a^3}} = 859.5233_{\text{sec}} \rightarrow \frac{\Delta t}{\tau_1} = 3.9569 \times 10^{-6}$$

1.5 part e

Assume:

$$C_1 = C_2 = 2\pi r = 4.1959e + 04 \rightarrow \frac{\Delta d}{C_1} = 1.1777 \times 10^{-9}$$

2 Question 2

We know that $r_1 = r_2$, $\theta_1 = 90^\circ$, $\theta_2 = 0$, and $e_1 = e_2 = e$.

$$r_1 = \frac{h_1^2}{\mu}\frac{1}{1 + e_1\cos(\theta_1)} = r_2 = \frac{h_2^2}{\mu}\frac{1}{1 + e_1\cos(\theta_2)} \xrightarrow[e_1=e_2=e]{\theta_1=90^\circ,\ \theta_2=0} \frac{h_1^2}{1+e} = h_2^2 \rightarrow h_2 = \frac{h_1}{\sqrt{1+e}}$$

3 Question 3

$$r_p = 8000,\ r_a = 13000 e = \frac{r_a - r_p}{r_a + r_p},\ a = \frac{r_a + r_p}{2},\ \tau = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Spacecraft 1:

$$r_0 = \frac{h^2}{\mu}\frac{1}{1 + e_1\cos(\theta_0)},\ r_2 == \frac{h^2}{\mu}\frac{1}{1 + e_1\cos(\theta_2)}$$

$$\boldsymbol{r}_0 = \begin{bmatrix} r_0\cos(\theta_0) & r_0\sin(\theta_0) & 0 \end{bmatrix},\ \boldsymbol{r}_2 = \begin{bmatrix} r_2\cos(\theta_2) & r_2\sin(\theta_2) & 0 \end{bmatrix}$$

$$\boldsymbol{v}_0 = \frac{\mu}{h}\begin{bmatrix} -\sin(\theta_0) & e + \cos(\theta_0) & 0 \end{bmatrix},\ \boldsymbol{v}_2 = \frac{\mu}{h}\begin{bmatrix} -\sin(\theta_2) & e + \cos(\theta_2) & 0 \end{bmatrix}$$

Spacecraft 2:

$$E = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan(\theta/2)\right),\ M_e = E - e\sin(E),\ M = \frac{2\pi}{\tau}t \xrightarrow[\theta_1=\pi/6]{\theta_2=\pi/2} t_1 = 542.785_{\text{sec}},\ t_2 = 1873.138_{\text{sec}}$$

Furthermore, I Used lambert's formulas to solve the problem, taking r0, r2, and delta t to lambert's formulas as input for the function.

$$\boldsymbol{v}_{0l} = \begin{bmatrix}-2.7240 & 9.4690 & 0\end{bmatrix}\frac{km}{sec},\ \boldsymbol{v}_{2l} = \begin{bmatrix}-7.8503 & 4.3427 & 0\end{bmatrix}\frac{km}{sec}$$

$$\Delta\boldsymbol{v}_0 = \boldsymbol{v}_{0l} - \boldsymbol{v}_0 = \begin{bmatrix}-2.7240 & 1.6149 & 0\end{bmatrix}\frac{km}{sec},\ \Delta\boldsymbol{v}_2 = \boldsymbol{v}_{2l} - \boldsymbol{v}_2 = \begin{bmatrix}-1.5066 & 2.8323 & 0\end{bmatrix}\frac{km}{sec}$$

4 Question 4

$$\boldsymbol{r}_{ap} = \begin{bmatrix}9798 & 5657 & 11314\end{bmatrix},\ \boldsymbol{v} = \begin{bmatrix}-0.1 & 0.2 & 0.3\end{bmatrix}$$

$$a = \frac{r_{ap}}{2}$$

$$\theta = \arccos\left(\frac{\boldsymbol{r}_{ap}.\boldsymbol{v}}{r_{ap}v}\right)$$

There is 2 equations and 2 unknowns, so we can solve it by using MATLAB.

$$\frac{h^2}{\mu} = a(1-e^2),\ v_r = \frac{\mu}{h}e\sin\theta \rightarrow e = 0.0656,\ h = 56348$$

4.1 part a

$$\cancel{r = \frac{h^2}{\mu}\frac{1}{(1+e\cos\theta)} = \frac{h^2}{\mu(1+e\cos\theta)} = 7667.5\frac{km}{\text{sec}}}$$

4.2 part b

$$v_\perp = \frac{h}{r} = 3.5217\frac{km}{\text{sec}}$$

$$\gamma = \arctan\left(\frac{v_r}{v_\perp}\right) = 0.1058_{\text{rad}}$$

5 Question 5

$$\theta_{B_1} = -\pi/2,\ \theta_{B_2} = \pi/2$$

$$r_B = \frac{h^2}{\mu}\frac{1}{(1 + e\cos(\pm\pi/2))} = \frac{h^2}{\mu}$$

$$v_{\perp_{B_1}} = v_{\perp_{B_2}} = \frac{h}{r_B} = \frac{\mu}{h}$$

$$v_{r_{B_1}} = \frac{\mu}{h}e\sin(\theta_{B_1}) = -\frac{\mu e}{h}$$

$$v_{r_{B_2}} = \frac{\mu}{h}e\sin(\theta_{B_2}) = \frac{\mu e}{h}$$

$$\Delta v_B = \sqrt{\left(v_{r_{B_2}} - v_{r_{B_1}}\right)^2 + v_{\perp_{B_1}}^2 + v_{\perp_{B_2}}^2 + -2v_{\perp_{B_1}}v_{\perp_{B_2}}\cos(\pi/2)} = \Delta v_B = \sqrt{4\frac{\mu^2}{h^2}e^2 + 2\frac{\mu^2}{h^2}}$$

$$\Delta v_b = \frac{\sqrt{2}\mu}{h}\sqrt{1 + 2e^2}$$

6 Bonus

For changing the orbit blow equation must be true.

$$r_1 = r_2 \rightarrow \frac{7200}{1 + 0.5\cos(\theta_1)} = \frac{8064}{1 + 0.2\cos(\theta_2)}$$

Now we find θ_1 respect to θ_2.

$$\theta_1 = \arccos\left((5\cos(\theta_2))/7 - 3/14\right)$$

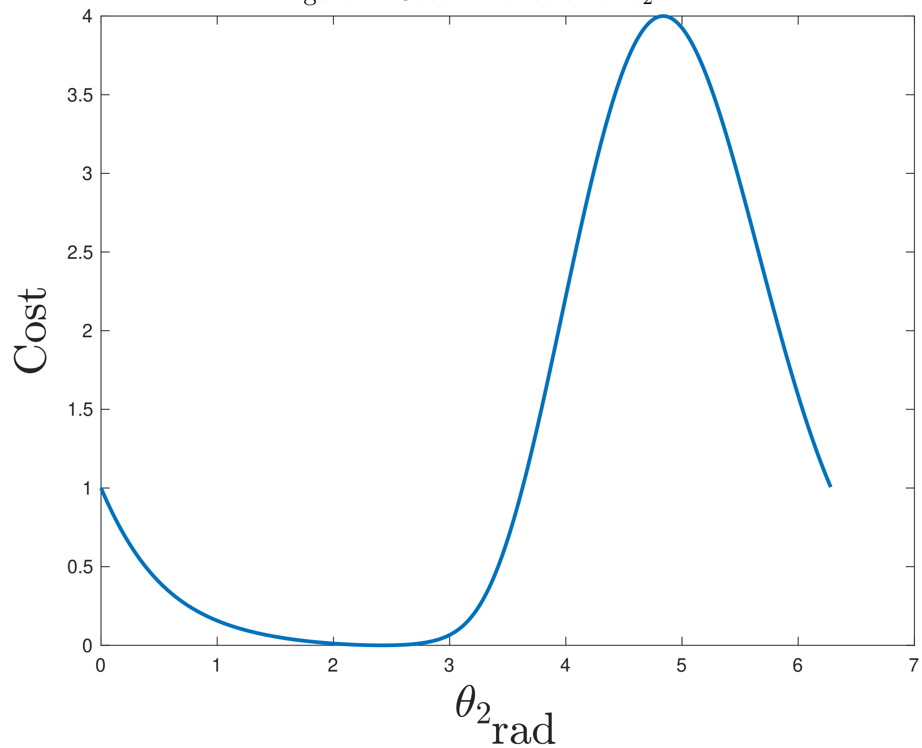
Now we make cost function.

$$\text{Cost} = (\boldsymbol{v}_1 - \boldsymbol{v}_2)^2\ ,\ \boldsymbol{v}_1\begin{bmatrix}-\sin(\theta_1) & \cos(\theta_1)\end{bmatrix},\ \boldsymbol{v}_2\begin{bmatrix}-\sin(\theta_2) & \cos(\theta_2)\end{bmatrix}$$

Now we have a cost only function of θ_2. Then, use fminbnd function of MATLAB to find minimum θ_2.

$$\min\theta_2 = -2.4189_{rad} = -138.6_{deg}$$

Figure 1: Cost in variation of θ_2



Contents

List of Figures