Advanced Mathematics I

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1. Illustrate the use of the Gauss reduction in obtaining the general solution of each of the following sets of equations:

$I)2x_1 + x_3 = 4,$
$\quad x_1 - 2x_2 + 2x_3 = 7,$
$\quad 3x_1 + 2x_2 = 1$

$II)2x_1 - x_2 = 6,$
$\quad - x_1 + 3x_2 - 2x_3 = 1,$
$\quad - 2x_2 + 4x_3 - 3x_4 = -2,$
$\quad -3x_3 + 5x_4 = 1$

2. If A and B are $n \times n$ matrices, when is it true that:

$$(A + B)(A - B) = A^2 - B^2$$

Give an example in which this relation does not hold.

3. Detemine those values λ for which the following set of equations may possess a nontrivial solution:

$$3x_1 + x_2 - \lambda x_3 = 0,$$
$$4x_1 - 2x_2 - 3x_3 = 0,$$
$$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$$

For each permissible value of λ ,determine the most general solution.

4. Let A and B represent diagonal matrices of order n.
  a) Prove that A B is also a diagonal matrix.
  b) Prove that BA =AB.

5. a)Show that the set
$$2x_1 - 2x_2 + x_3 = \lambda x_1,$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2,$$
$$-x_1 + 2x_2 = \lambda x_3$$

can possess a nontrivial solution only if $\lambda = 1$ or $\lambda = -3$

  b) Obtain the general solution in each case.

6. a) If $a_{ij} = r_i s_j$, prove that $A = [a_{ij}]$ is of rank one or zero.

  b) If $A = [a_{ij}]$ is of rank one, prove that a_{ij} can be writen as $r_i s_j$.

7. If $D = [d_i \delta_{ij}]$ is a nonsingular diagonal matrix, prove that its invers e is given by: (δ_{ij} is a function of two variables, The function is 1 if the variables are equal, and 0 otherwise)

$$D^{-1} = \left[\frac{1}{d_i} \delta_{ij}\right].$$

Good Luck