Sharif university of technology
Aerospace department
Flight dynamics

HW series 4

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سوال اول

الف)

$$b = 17_{ft},\ S = 37.8_{ft^2}, W(Gross\ weight\ at\ takeoff) = 960_{lb}$$

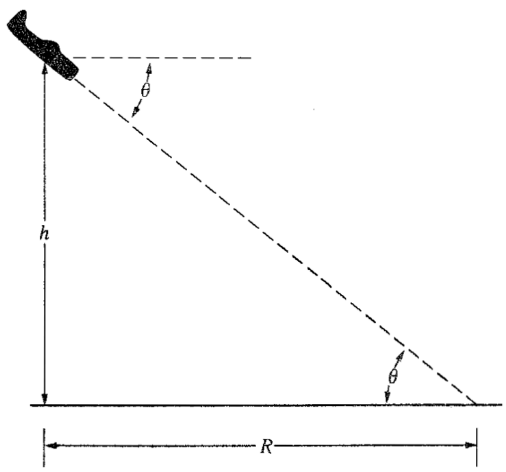
$$T_{\max_{SL}} = 202_{lb}, K = 0.062,\ C_{D_0} = 0.02$$

Maximum thrust at sea level is 202_{lb}.

a)

$$(\frac{C_L}{C_D})_{\max} = (\frac{L}{D})_{\max} = \sqrt{\frac{1}{4C_{D_0}K}} \rightarrow (\frac{L}{D})_{\max} = 14.1990$$

Figure 1



$$\tan\theta = \frac{1}{(L/D)}$$

$$\rightarrow \tan\theta_{min} = \frac{1}{(L/D)_{\max}}\ I$$

$$\rightarrow \tan\theta_{min} = \frac{1}{14.1990}$$

$$\theta_{\min} = 0.0703_{rad} = 4.0279^{\circ}$$

b)

$$\tan\theta = \frac{h}{R} \rightarrow R = \frac{h}{\tan\theta} \rightarrow R_{\max} = \frac{h}{\tan\theta_{\min}} \rightarrow_I R_{\max} = h \times (L/D)_{\max}$$

$$\rightarrow = 10000 \times 14.1990 = R_{\max} = 141990_{ft}$$

c)

$$V_{(L/D)_{\max}} = (\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D_0}}}\frac{W}{S})^{1/2}$$

In 10000_{ft}, $\rho_\infty = 17.56_{lbs/m^3}$:

$$\rightarrow V_{(L/D)_{\max}} = V_{\theta_{\min}} = 225.7206_{ft/s}$$

In sea level $\rho_\infty = 23.77_{lbs/m^3}$:

$$\rightarrow V_{(L/D)_{\max}} = V_{\theta_{\min}} = 193.9751_{ft/s}$$

ب)

$$\frac{T_{\max}}{T_{\max_{SL}}} = \frac{\rho}{\rho_0} \rightarrow T_{max} = 149.1768_{lb}$$

Maximum thrust in 10000_{ft} altitude is 149.1768_{lb}.

$$Z \equiv 1 + \sqrt{1 + \frac{3}{(L/D)^2_{\max}(T/W)^2}}$$

$$(R/C)_{\max} = [\frac{(T/W)(W/S)Z}{3\rho C_{D,0}}]^{1/2}[\frac{T}{W} - \frac{Z}{6}\frac{T}{W} - \frac{6KC_{D,0}}{(T/W)Z}] = [\frac{(T/S)Z}{3\rho C_{D,0}}]^{1/2}[\frac{T}{W} - \frac{Z}{6}\frac{T}{W} - \frac{6KC_{D,0}}{(T/W)Z}]$$

$$\frac{T_{\max_0}}{T_{\max_{SL}}} = \frac{\rho_0}{\rho_{SL}}$$

Figure draw in MATLAB:

Figure 1

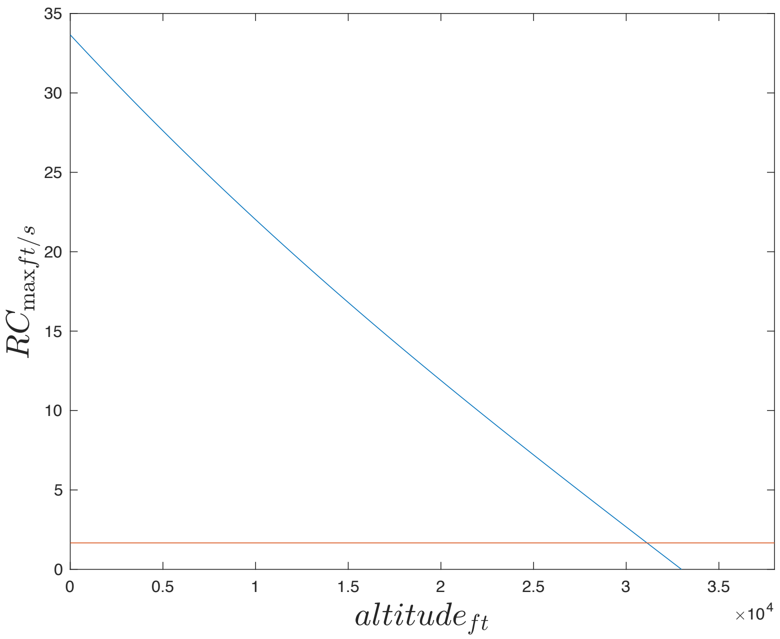
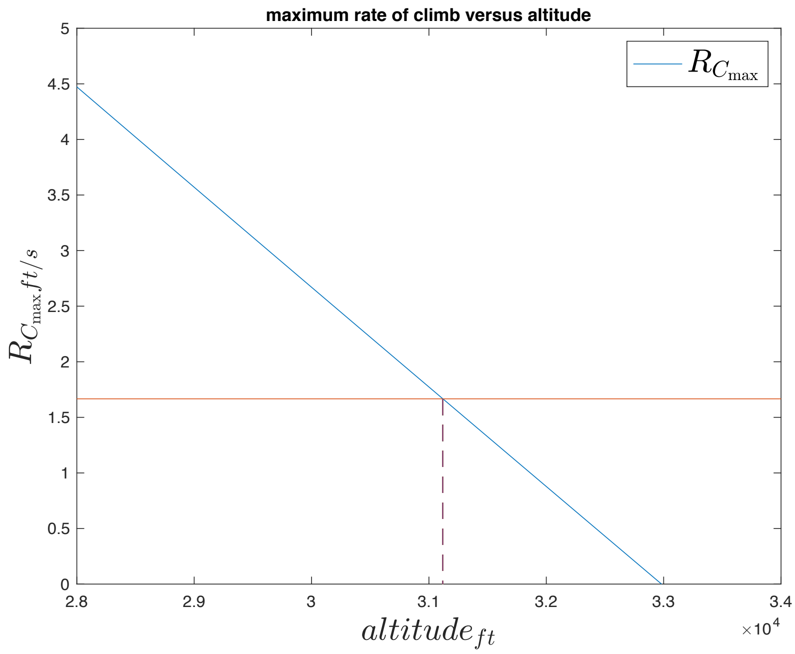


Figure 3



From the figure service ceiling altitude is about 32000_{ft}.

پ)

$$(R/C)_{\max} = [\frac{(T/W)(W/S)Z}{3\rho_{\infty}C_{D,0}}]^{1/2}[\frac{T}{W} - \frac{Z}{6}\frac{T}{W} - \frac{6KC_{D,0}}{(T/W)Z}] = 100/60 ft/s$$

From MATLAB solve we have:

service ceiling altitude is 31118_{ft} with 0.001_{ft} error.

ρ_0 is density in sea level.

$$\frac{T_{\max}}{T_{\max_{SL}}} = \frac{\rho}{\rho_0} \rightarrow T_{max} = T_{\max_{SL}}\frac{\rho}{\rho_0}$$

$$\rightarrow (R/C)_{\max} = [\frac{(T_{\max_{SL}}\frac{\rho}{\rho_0}/S)Z}{3\rho C_{D,0}}]^{1/2}[\frac{T_{\max_{SL}}\frac{\rho}{\rho_0}}{W} - \frac{Z}{6}\frac{T_{\max_{SL}}\frac{\rho}{\rho_0}}{W} - \frac{6KC_{D,0}}{(T_{\max_{SL}}\frac{\rho}{\rho_0}/W)Z}]$$

$$\rightarrow (R/C)_{\max} = f(\rho)$$

بر اساس کد MATLAB ضمیمه شده و حل معادله بالا داریم:

$$\rho = 0.000891153_{lbs/ft^3} \rightarrow h = 29940_{ft}$$

ت)

بر اساس محاسبات عددی برای curve fitting داریم:

$$(R/C)_{\max} = a + bh$$

$$S_h = \sum_{h=0}^{h_2} h,\ S_{(R/C)_{\max}} = \sum_{h=0}^{h_2}(R/C)_{(h)_{\max}},\ S_{h(R/C)_{\max}} = \sum_{h=0}^{h_2} h(R/C)_{(h)_{\max}}, S_{hh} = \sum_{h=0}^{h_2} h^2$$

در معادلات پایین n برابر با تعداد داده ها است.

$$a = \frac{S_{hh}S_{(R/C)_{\max}} - S_hS_{h(R/C)_{\max}}}{nS_{hh} - S_h^2} \rightarrow a = 32.856171$$

$$b = \frac{nS_{h(R/C)_{\max}} - S_hS_{(R/C)_{\max}}}{S_{hh} - S_h^2} \rightarrow b = -0.00102132$$

$$t_{\min} = \int_0^{h_2}\frac{dh}{a+bh} = \frac{1}{b}(\ln(a+bh_2) - \ln(a))$$

$$h_2 = 10000_{ft}$$

$$\rightarrow t_{\min} = 364.5187_s = 6.0753_{min} = 0.1013_{hr}$$

ث)

$Fuel\ capacity = 55_{gal}$

$Weight\ empty = 550_{lb}$

$c_t = 1.3_{h^{-1}} = 3.6111 \times 10^{-4}_{s^{-1}}$

In 10000_{ft} altitude ρ_∞ is $17.56 \times 10^{-4}_{lbs/ft^3}$.

$$\frac{C_L^{1/2}}{C_D} = \frac{3}{4}(\frac{1}{3KC_{D_0}^3})^{1/4} \rightarrow \frac{C_L^{1/2}}{C_D} = 21.4739$$

$W_0 = 960_{lb},\ W_1 = 550_{lb}$

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2}) \rightarrow R_{\max} = 4.9180 \times 10^6_{ft}$$

$R_{\max} = 931.45_{mi}$

$$V_{\infty(\max\ \ range)} = V_{(\frac{C_L^{1/2}}{C_D})_{\max}} = (\frac{2}{\rho_\infty}\sqrt{\frac{3K}{C_{D_0}}}\frac{W}{S})^{1/2}$$

$$\rightarrow V_{\infty(\max\ \ range)} = 297.0650_{ft/s} = 202.544_{mi/h}$$

ج)

$$E = \frac{1}{c_t}\frac{L}{D}\ln(\frac{W_0}{W_1})$$

$$(\frac{L}{D})_{\max} = 14.1990$$

$$\rightarrow E_{\max} = \frac{1}{c_t}(\frac{L}{D})_{\max}\ln(\frac{W_0}{W_1}) \rightarrow E_{max} = 21902.0721_s = 365.0345_{min} = 6.0839_{hr}$$

چ)

At 10000_{ft} altitude.

تمامی مثال های قبل بر فرض اتموسفر بدون باد بود. برای حالت tailwind داریم.

$$V_{tw} = 40_{mi/h} = 58.67_{ft/s}$$

$$V_g = V_\infty + V_{tw}$$

$$R = R_{no\ wind} + V_{tw}E = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2}) + V_{tw}\frac{1}{c_t}(\frac{C_L}{C_D})\ln(\frac{W_0}{W_1})$$

$$\alpha = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}(W_0^{1/2} - W_1^{1/2})$$

$$\beta = V_{tw}\frac{1}{c_t}\ln(\frac{W_0}{W_1})$$

α and β are constant.

$$C_D = C_{D_0} + KC_L^2$$

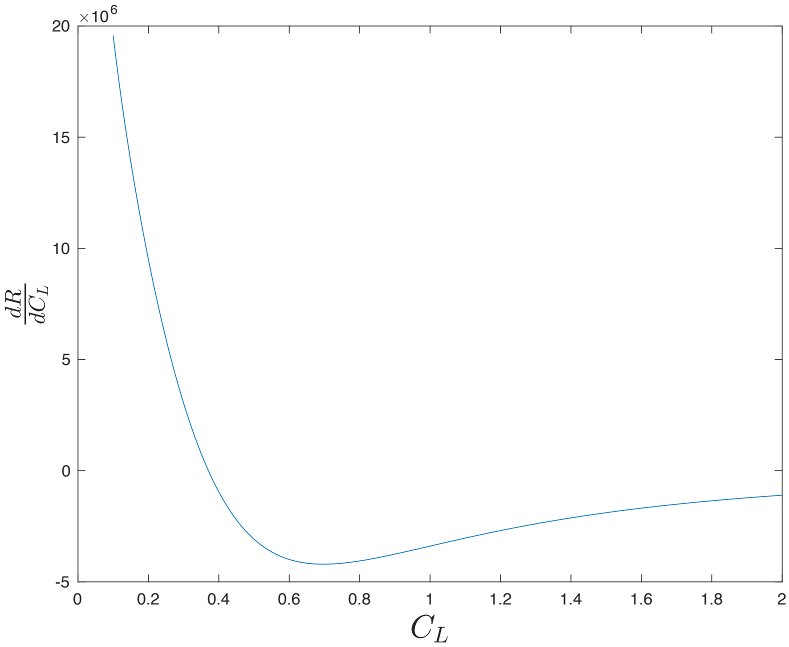
$$R = \alpha\frac{C_L^{1/2}}{C_{D_0} + KC_L^2} + \beta\frac{C_L}{C_{D_0} + KC_L^2} \rightarrow R = f(C_L)$$

$$\frac{dR}{dC_L} = \alpha\frac{\frac{1}{2}C_L^{-1/2}(C_{D_0} + KC_L^2) - C_L^{1/2}2KC_L}{(C_{D_0} + KC_L^2)^2} + \beta\frac{C_{D_0} + KC_L^2 - 2KC_LC_L}{(C_{D_0} + KC_L^2)^2}$$

$$\frac{dR}{dC_L} = \alpha\frac{\frac{1}{2}C_L^{-1/2}C_{D_0} - \frac{3}{2}KC_L^{3/2}}{(C_{D_0} + KC_L^2)^2} + \beta\frac{C_{D_0} - KC_L^2}{(C_{D_0} + KC_L^2)^2}$$

For maximum R $\rightarrow \frac{dR}{dC_L} = 0.$

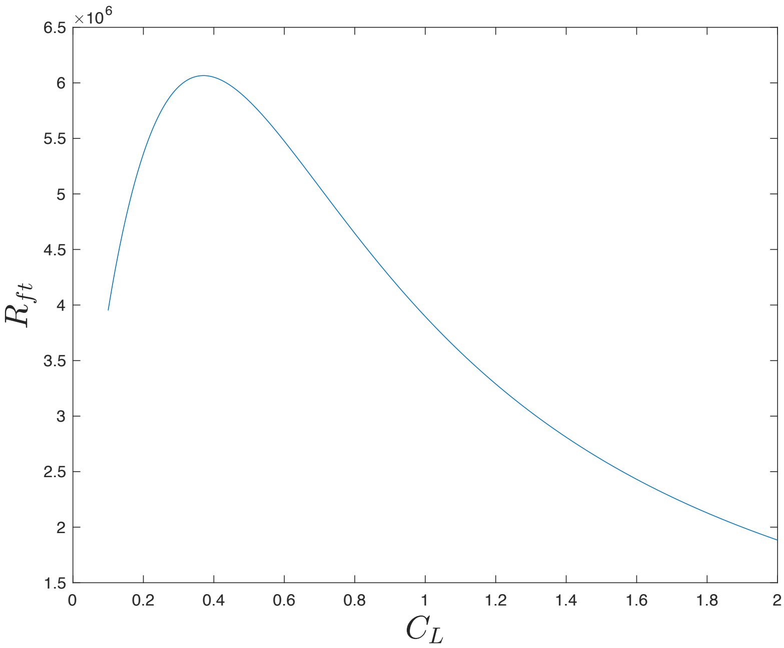
Figure 4



بر اساس نمودار مشتق بالا یک جواب برای سوال داریم.

نمودار برد نسبت به C_L را نیز رسم می کنیم.

Figure 5



بر اساس حل عددی با متلب و بیشترین برد در حالت tailwind به صورت زیر است.

$$R_{\max with\ tailwind} = 6065508.6646_{ft} = 1148.770_{mi}$$

At $C_L = 0.3705$

ه)

$$S = 960_{ft^2},\ C_{D_0} = 0.015,\ K = 0.08,\ W = 73000_{lb}$$

$$(T_A)_{V=0} = 13850_{lb}$$

$$T_R = \frac{1}{2}\rho_\infty V_\infty^2 (C_{D_0} + KC_L^2)$$

$$C_L = \frac{2W}{\rho_\infty V_\infty^2 S}$$

$$T_R = \frac{1}{2}\rho_\infty V_\infty^2 (C_{D_0} + K(\frac{2W}{\rho_\infty V_\infty^2 S})^2)$$

At sea level:

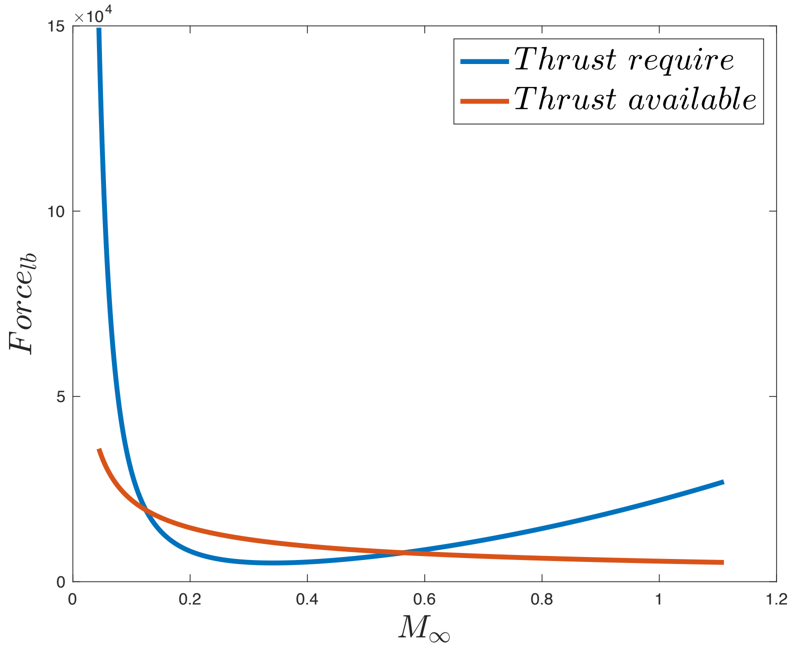
$$\frac{T_A}{(T_A)_{V=0}} = 0.4M_\infty^{-0.6}$$

سوال را با فرض ثابت بودن سرعت صوت حل می کنیم.

$$a_{Sonic\ Speed} = 1126_{ft/s}$$

$$M_\infty = \frac{V_\infty}{a}$$

Figure 6



بر اساس نمودار و حل MATLAB داریم:

$$V_{\max} = 858_{ft}$$

$$M_{\max} = 0.7620$$

At 30000_{ft} altitude:

$$\frac{T_A}{(T_A)_{V=0}} = 0.222 M_\infty^{-0.6}$$

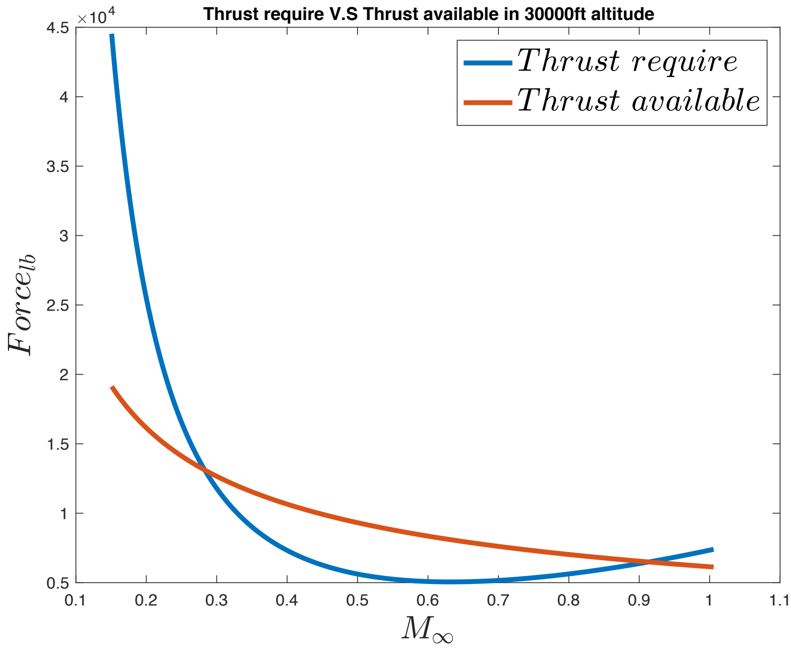
سوال را با فرض ثابت بودن سرعت صوت حل می کنیم.

$$a_{Sonic\ Speed} = 994.42_{ft/s}$$

$$M_\infty = \frac{V_\infty}{a}$$

بر اساس نمودار و حل MATLAB داریم:

Figure 7



$$V_{\max} = 823_{ft}$$

$$M_{\max} = 0.8276$$

سوال دوم

الف)

$$T_{\max} = 23500\sigma_N,\ C_{D_0} = 0.015,\ C_{L_{\max}} = 1.8$$

$$e = 0.9,\ b = 17_m = 55.775_{ft},\ S = 32_{m^2} = 344.445_{ft}, W = 50_{KN}$$

$$Z \equiv 1 + \sqrt{1 + \frac{3}{(L/D)^2_{\max}(T/W)^2}}$$

$$(R/C)_{\max} = [\frac{(T/W)(W/S)Z}{3\rho C_{D,0}}]^{1/2}[\frac{T}{W} - \frac{Z}{6}\frac{T}{W} - \frac{6KC_{D,0}}{(T/W)Z}]$$

$$AR = \frac{b^2}{S} = 9.0312$$

$$K_1 = \frac{1}{3}K_3,\ K_2 = 0,\ K_3 = \frac{1}{\pi eAR}$$

$$K \equiv K_1 + K_2 + K_3$$

$$(\frac{C_L}{C_D})_{\max} = (\frac{L}{D})_{\max} = \sqrt{\frac{1}{4C_{D_0}K}} \rightarrow (\frac{L}{D})_{\max} = 17.8659$$

نمودار Rate of climb نسبه ارتفاع به صورت زیر است(figure 8).

بر اساس رابطه زیر داریم:

$$t_{\min} = \int_0^{h_2} \frac{dh}{R/C_{\max}}$$

حداقل زمان برای رسیدن ارتفاع Service Ceiling برابر با مساحت داخل نمودار(figure 2) است.

در روش گرافیکی به صورت عددی از آن انتگرال می گیرم و مساحت را حساب می کنیم.

برای محاسبه مساحت از روش ذوزنقه ای با گام یک فوت (0.3048_m) استفاده می کنیم.

Figure 8

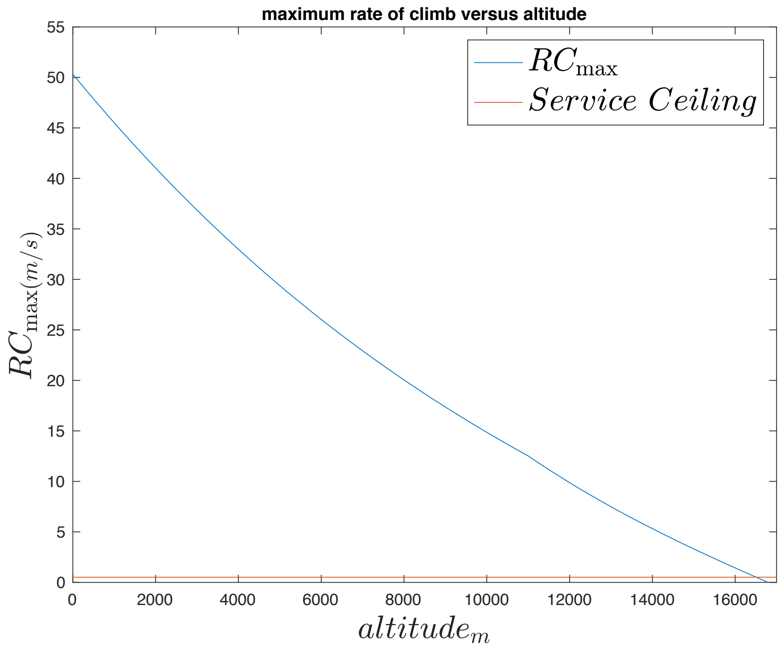
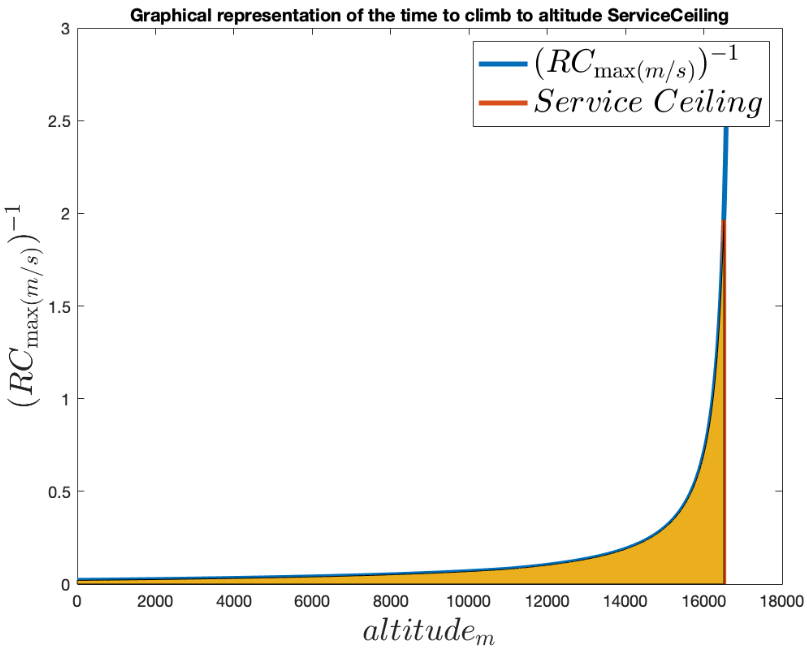


Figure 9



$t_{\min} = 2066.1_s = 34.4350_{min} = 0.5739_{hr}$

راه حل تحلیلی مانند سوال قبل a و b را محاسبه می کنیم.

$(R/C)_{\max} = a + bh$

$S_h = \sum_{h=0}^{h_2} h,\ S_{(R/C)_{\max}} = \sum_{h=0}^{h_2} (R/C)_{(h)_{\max}},\ S_{h(R/C)_{\max}} = \sum_{h=0}^{h_2} h(R/C)_{(h)_{\max}}, S_{hh} = \sum_{h=0}^{h_2} h^2$

در معادلات پایین n برابر با تعداد داده ها است.

$a = \dfrac{S_{hh}S_{(R/C)_{\max}} - S_h S_{h(R/C)_{\max}}}{nS_{hh} - S_h^2} \rightarrow a = 24.6668$

$b = \dfrac{nS_{h(R/C)_{\max}} - S_h S_{(R/C)_{\max}}}{S_{hh} - S_h^2} \rightarrow b = -0.0010$

$h_2 = 54169_{ft} = 16511_m$

$t_{\min} = \int_0^{h_2} \dfrac{dh}{a + bh} = t_{\min} = \int_0^{16511} \dfrac{dh}{a + bh} = \dfrac{1}{b}(\ln(a + bh_2) - \ln(a))$

$t_{\min} = 1145.0_s = 19.0833_{min} = 0.3181_{hr}$

ب)

از حل معادله زیر داریم:

$T = 23500\sigma_N$

$(R/C)_{\max} = [\dfrac{(T/W)(W/S)Z}{3\rho C_{D,0}}]^{1/2}[\dfrac{T}{W} - \dfrac{Z}{6}\dfrac{T}{W} - \dfrac{6KC_{D,0}}{(T/W)Z}]$

بیشینه ارتفاع زمانی است که $(R/C)_{\max}$ برابر با صفر باشد.

از حل معادله در MATLAB داریم:

$\rho = 0.145882075362854155 7_{kg/m^3}$

و بر اساس برنامه ساخته شده در سری یک ارتفاع را بدست می آوریم.

$h_{max} = 55108_{ft} = 16797_m$

برای حالت اول $\theta_{\min}$ در $(\frac{L}{D})_{\max}$ رخ می دهد. بنابراین سرعت را از همین طریق بدست می آوریم.

$$V_{(L/D)_{\max}} = (\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D_0}}}\frac{W}{S})^{1/2} = 88_{m/s}$$

از حل معادله در MATLAB داریم:

$$\rho = 0.7529017262163628_{kg/m^3}$$

و بر اساس برنامه ساخته شده در سری یک ارتفاع را بدست می آوریم.

$$h = 15719_{ft} = 4791.2_m$$

$$\Delta h = 16797 - 4791.2 = 12006_m$$

$$\tan\theta_{\min} = \frac{1}{(\frac{L}{D})_{\max}} \to \theta_{\min} = 0.0559_{rad}$$

$$R = \Delta h \times (L/D)_{\max} \to R = 214494.422_m$$

برای حالت دوم $\theta_{\min_{sink\ rate}}$ در $(\frac{L^{3/2}}{D})_{\max}$ رخ می دهد. بنابراین سرعت را از همین طریق بدست می آوریم.

$$V_{(L^{3/2}/D)_{\max}} = (\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D_0}}}\frac{W}{S}\cos\theta_{\min_{sink\ rate}})^{1/2} = 88_{m/s}$$

$$\theta \approx 0 \to \cos\theta \approx 1$$

از حل معادله در MATLAB داریم:

$$\rho = 0.4346880143_{kg/m^3}$$

و بر اساس برنامه ساخته شده در سری یک ارتفاع را بدست می آوریم.

$$h = 31426_{ft} = 9578.6_m$$

$$\Delta h = 16797 - 9578.6 = 7218.4_m$$

$$(\frac{C_L^{3/2}}{C_D})_{\max} = \frac{1}{4}(\frac{3}{kC_{D_0}^{1/3}})^{3/4} \rightarrow (\frac{C_L^{3/2}}{C_D})_{\max} = 14.9077$$

$$C_L = \sqrt{\frac{3C_{D_0}}{K}} \rightarrow C_L = 0.9283 \rightarrow C_D = 0.0600 \rightarrow \frac{L}{D} = 15.4723$$

$$\tan\theta_{\min_{sink\ rate}} = \frac{1}{(\frac{L}{D})} \rightarrow \theta_{\min_{sink\ rate}} = 0.0645_{rad}$$

$$R = \Delta h \times (L/D) \rightarrow R = 111685.25_m$$

$$\theta_{\min} = 0.0559_{rad} < \theta_{\min_{sink\ rate}} = 0.0645_{rad}$$

پ)

$$V_{V_{\theta_{\min}}} = V_{(L/D)_{\max}} \sin\theta = (\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D_0}}}\frac{W}{S})^{1/2}\sin\theta$$

$$V_{V_{\theta_{\min\ sink\ rate}}} = \sqrt{\frac{2}{\rho(C_L^{1.5}/C_D)^2_{\max}}\frac{W}{S}}$$

Table 1

altitude(ft)	Vertical velocity in minimum theta(m/s)	Vertical velocity in minimum sink rate(m/s)
55108	11.1695	9.8175
52352	10.4537	9.1883
49597	9.7840	8.5997
46841	9.157	8.0486
44086	8.5703	7.5330
41331	8.0213	7.0504
38575	7.5072	6.5985
35820	7.0351	6.1836
33064	6.6731	5.8654
30309	6.3379	5.5707
27554	6.0268	5.2974
24798	5.7376	5.0432
22043	5.4685	4.8066
19287	5.2174	4.5859
16532	4.9830	4.3798
13777	4.7637	4.1871
11021	4.5583	4.0066
8266	4.3657	3.8373
5510	4.1847	3.6782
2755	4.0147	3.5287
0	3.8546	3.3881

Table 2

altitude(m)	Vertical velocity in minimum theta(m/s)	Vertical velocity in minimum sink rate(m/s)
16797	11.1695	9.8175
15957	10.4537	9.1883
15117	9.7840	8.5997
14277	9.157	8.0486
13437	8.5703	7.5330
12598	8.0213	7.0504
11758	7.5072	6.5985
10918	7.0351	6.1836
10078	6.6731	5.8654
9238.2	6.3379	5.5707
8398.5	6.0268	5.2974
7558.4	5.7376	5.0432
6718.7	5.4685	4.8066
5878.7	5.2174	4.5859
5039	4.9830	4.3798
4199.2	4.7637	4.1871
3359.2	4.5583	4.0066
2519.5	4.3657	3.8373
1679.4	4.1847	3.6782
839.7	4.0147	3.5287
0	3.8546	3.3881

Figure 10

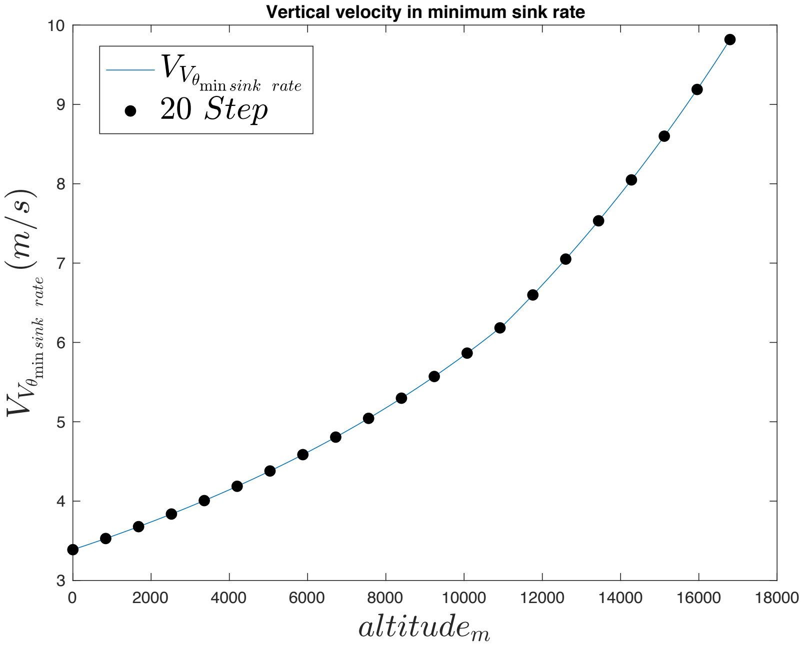


Figure 11

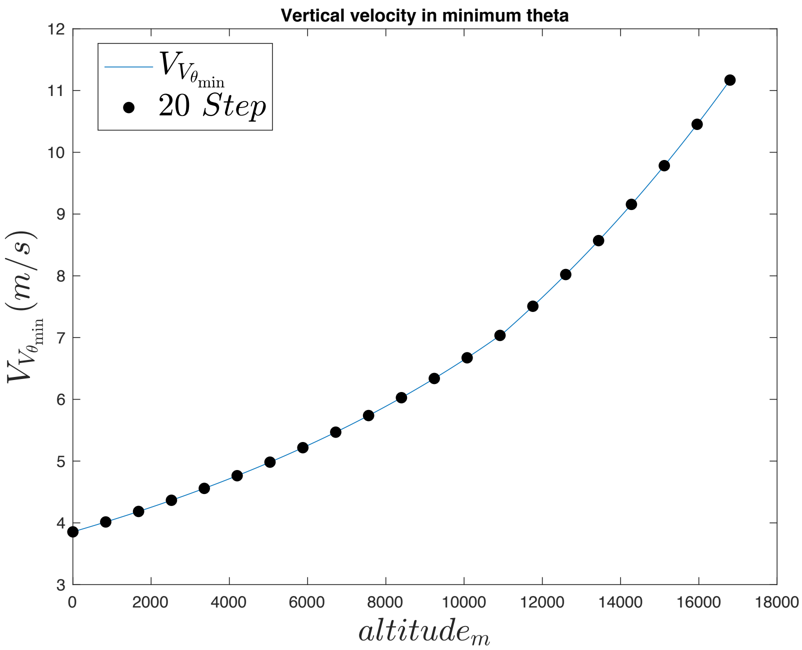
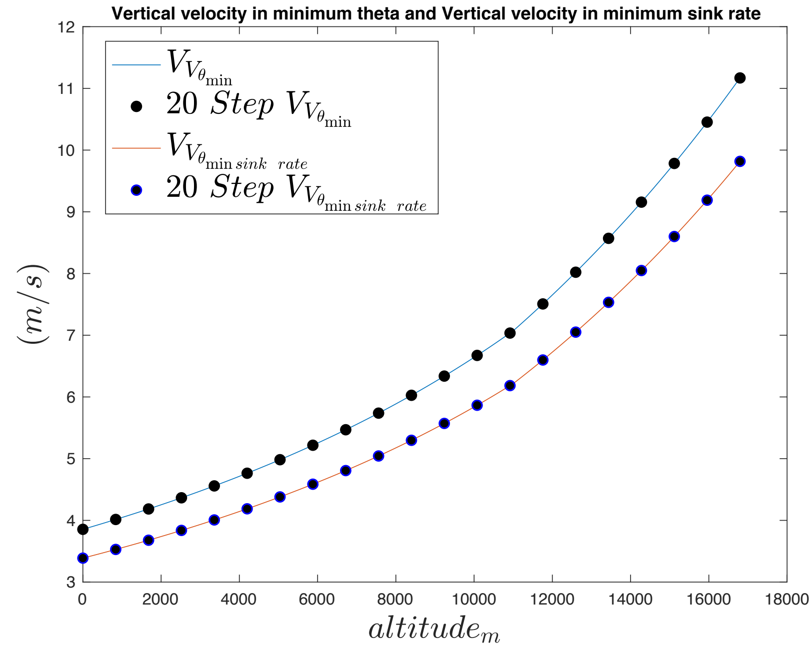


Figure 12



ت)

$$V = \frac{ds}{dt} \rightarrow dt = \frac{ds}{V}$$

بنابراین زمان برابر با مساحت زیر نمودار(figure 13,14) است.

Figure 13

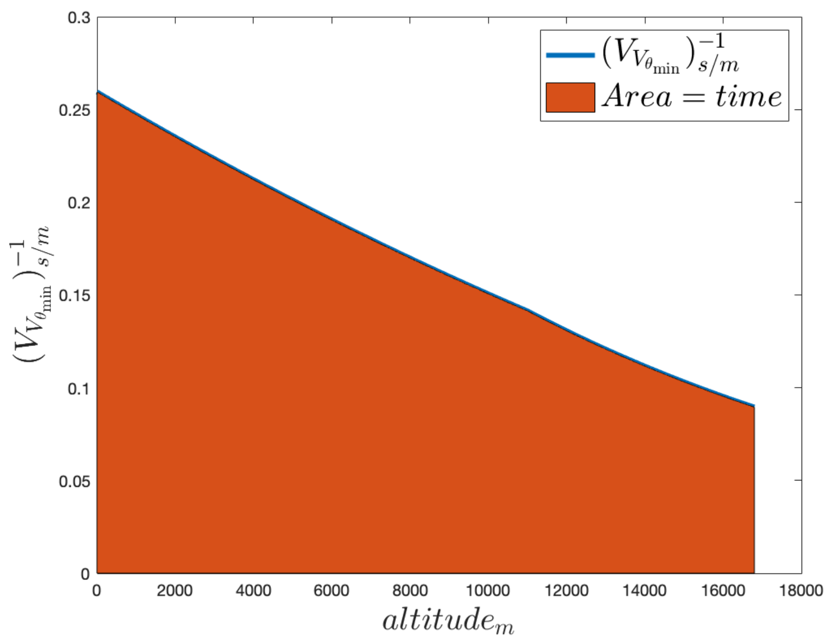
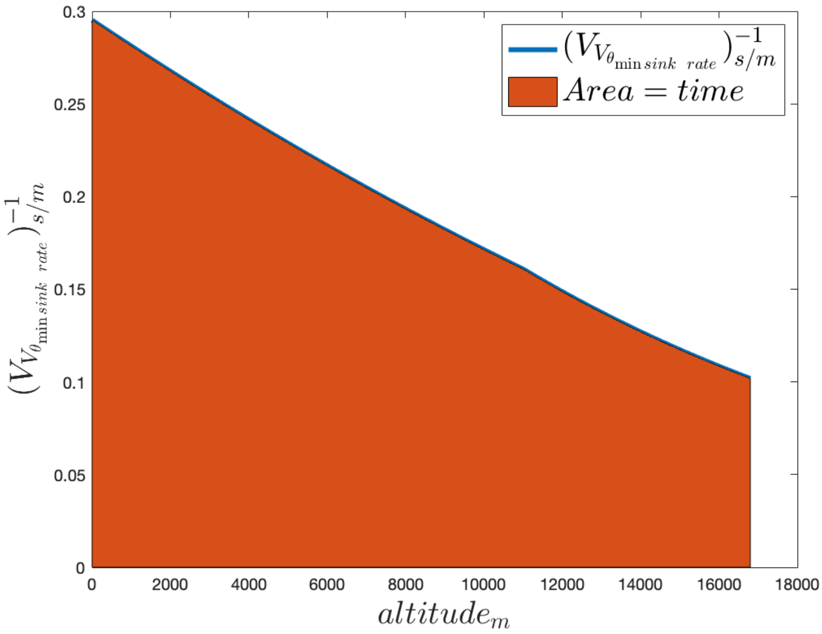


Figure 14



مساحت را با کد MATLAB بدست آوردیم.

$$t_{\theta_{\min}} = 2828.0_s = 47.1333_{min} = 0.7856_{hr}$$

$$t_{\min\ \ sink\ rate} = 3217.4_s = 53.6233_{min} = 0.8937_{hr}$$

از طرفی چون نمدار بالا خیلی شبیه خطی هست از روش تحلیلی و تقریبی برای حل آن استفاده می کنیم(که چون وقت کم است انجام نمی دهیم).

سوال سوم

$$W_0 = 22000 \times 9.8 = 215600_N,\ S = 80_{m^2},\ c_t = 6.4 \times 10^{-5}_{N/N.s}$$

$$\sigma = 0.5,\ C_D = 0.025 + 0.06C_L^2 \rightarrow C_{D_0} = 0.025,\ K = 0.06$$

الف)

$$R_{\max} = 3800_{km}$$

a) altitude and attack angle is constant

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2})$$

$$\sigma = \frac{\rho_\infty}{\rho_{\infty_0}} = 0.5 \rightarrow \rho_\infty = 0.6125_{kg/m^3}$$

$$(\frac{C_L^{1/2}}{C_D})_{\max} = \frac{3}{4}(\frac{1}{3KC_{D_0}^3})^{1/4} \rightarrow (\frac{C_L^{1/2}}{C_D})_{\max} = 18.3142$$

$$R_{\max} = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}(\frac{C_L^{1/2}}{C_D})_{\max}(W_0^{1/2} - W_1^{1/2})$$

$$W_1 = (W_0^{1/2} - \frac{R_{\max}}{\frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}(\frac{C_L^{1/2}}{C_D})_{\max}})^2 \rightarrow W_0 = 186160.19_N$$

$$fuel = W_0 - W_1 = 29440_N$$

b) V_∞ and attack angle is constant

$$R = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})\sqrt{\frac{2W_0}{\rho_\infty S}} \ln \frac{W_0}{W_1}$$

$$R_{\max} = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})_{\max}\sqrt{\frac{2W_0}{\rho_\infty S}} \ln \frac{W_0}{W_1}$$

$$R_{\max} = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})_{\max}\sqrt{\frac{2W_0}{\rho_\infty S}} \ln \frac{W_0}{W1} \rightarrow \frac{R_{\max}}{\frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})_{\max}\sqrt{\frac{2W_0}{\rho_\infty S}}} = \ln \frac{W_0}{W1}$$

$$\rightarrow W_1 = \frac{W_0}{\exp(\frac{R_{\max}}{\frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})_{\max}\sqrt{\frac{2W_0}{\rho_\infty S}}})}$$

$$W_1 = 187141.86_N$$

$$fuel = W_0 - W_1 = 28458_N$$

$$fuel_{(\rho_\infty,\alpha)const} > fuel_{(V_\infty,\alpha)const}$$

ب)

همان معادلات بالا را برای $R_{\max}/2$ حل می کنیم و چون معادلات خطی نیستند انتظار جواب خطی و نصف مقدار بدست آمده را نداریم.

a) altitude and attack angle is constant

$$R_{\max}/2 = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}(\frac{C_L^{1/2}}{C_D})_{\max}(W_0^{1/2} - W_1^{1/2}) \rightarrow W_0 = 200610.07_N$$

$$fuel = W_0 - W_1 = 14990_N$$

$$fuel_{R_{\max}/2} = 14990_N$$

$$fuel_{R_{\max}}/2 = 14720_N$$

$$fuel_{R_{\max}/2} > fuel_{R_{\max}}/2$$

b) V_∞ and attack angle is constant

$$R_{\max}/2 = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})_{\max}\sqrt{\frac{2W_0}{\rho_\infty S}}\ln\frac{W_0}{W_1} \rightarrow W_1 = 200867.58_N$$

$$fuel_{R_{\max}/2} = 14732_N$$

$$fuel_{R_{\max}}/2 = 14229_N$$

$$fuel_{R_{\max}/2} > fuel_{R_{\max}}/2$$

بر اساس معلادلات با در هر دو حالت مصرف سوخت در نیمه اول مسیر بیشتر بود به این دلیل که در نیمه اول مسیر هواپیما سوخت بیشتری نسبت به نیمه دوم حمل می کند که برای حمل این مقدار سوخت باید سوخت بیشتری مصرف کند اما در نیمه دوم چون وزن کمتر است (به علت مصرف سوخت) سوخت کمتری لازم است پس مسافت بیشتری را به ازای یه مقدار مشخص سوخت می رود در نتیجه نیمه دوم راه را با سوخت کمتری طی می کند.

پ)

a) altitude and attack angle is constant

$$E = \frac{1}{c_t}(\frac{C_L}{C_D})\ln\frac{W_0}{W1}$$

$E_{\max} = 8_{hr} = 28800_s$

$(\frac{C_L}{C_D})_{\max} = \sqrt{\frac{1}{4C_{D_0}K}} \rightarrow (\frac{C_L}{C_D})_{\max} = 12.9099$

$E_{\max} = \frac{1}{c_t}(\frac{C_L}{C_D})_{\max} \ln \frac{W_0}{W1} \rightarrow \frac{E_{\max}}{\frac{1}{c_t}(\frac{C_L}{C_D})_{\max}} = \ln \frac{W_0}{W1} \rightarrow W_1 = \frac{W_0}{\exp \frac{E_{\max}}{\frac{1}{c_t}(\frac{C_L}{C_D})_{\max}}}$

$W1 = 186914.47_N$

$fuel = W_0 - W_1 = 28686_N$

b) V_∞ and attack angle is constant

$E = \frac{1}{c_t}(\frac{C_L}{C_D}) \ln \frac{W_0}{W1}$

$E_{\max} = 8_{hr} = 28800_s$

$(\frac{C_L}{C_D})_{\max} = \sqrt{\frac{1}{4C_{D_0}K}} \rightarrow (\frac{C_L}{C_D})_{\max} = 12.9099$

$E_{\max} = \frac{1}{c_t}(\frac{C_L}{C_D})_{\max} \ln \frac{W_0}{W1} \rightarrow \frac{E_{\max}}{\frac{1}{c_t}(\frac{C_L}{C_D})_{\max}} = \ln \frac{W_0}{W1} \rightarrow W_1 = \frac{W_0}{\exp \frac{E_{\max}}{\frac{1}{c_t}(\frac{C_L}{C_D})_{\max}}}$

$W1 = 186914.47_N$

$fuel = W_0 - W_1 = 28686_N$

محاسبات و فرضیات دو حالت ثابت است پس مصرف سوخت نیز برابر است.

ت)

I) with $10_{m/s}$ tail wind

$R = R_{no\ wind} + V_{tw}E$

$V_{tw} = 10_{m/s}$

a) altitude and attack angle is constant

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2}) + V_{tw}\frac{1}{c_t}(\frac{C_L}{C_D})\ln\frac{W_0}{W1}$$

$$\alpha = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}(W_0^{1/2} - W_1^{1/2})$$

$$\beta = V_{tw}\frac{1}{c_t}\ln(\frac{W_0}{W_1})$$

α and β are constant.

$$C_D = C_{D_0} + KC_L^2$$

$$R = \alpha\frac{C_L^{1/2}}{C_{D_0} + KC_L^2} + \beta\frac{C_L}{C_{D_0} + KC_L^2} \rightarrow R = f(C_L)$$

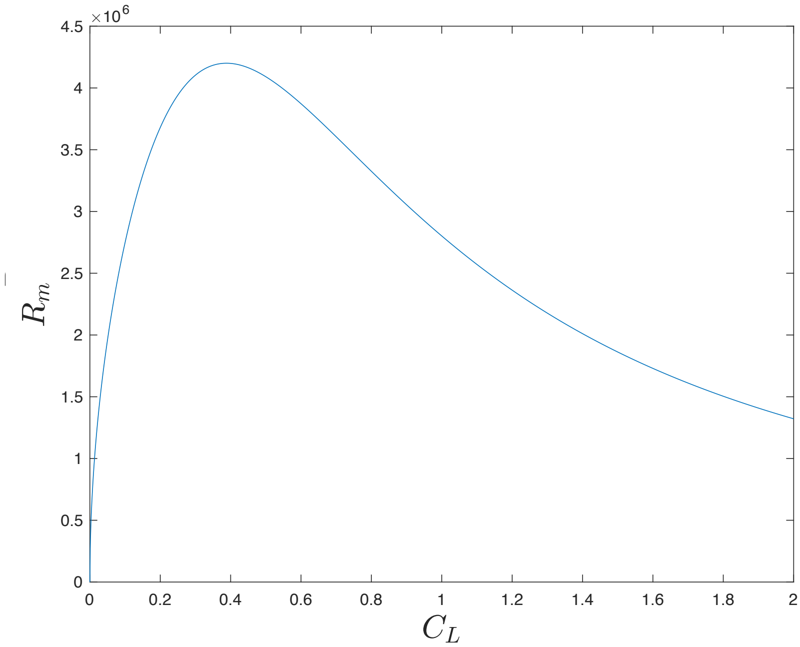
$$\frac{dR}{dC_L} = \alpha\frac{\frac{1}{2}C_L^{-1/2}(C_{D_0} + KC_L^2) - C_L^{1/2}2KC_L}{(C_{D_0} + KC_L^2)^2} + \beta\frac{C_{D_0} + KC_L^2 - 2KC_LC_L}{(C_{D_0} + KC_L^2)^2}$$

$$\frac{dR}{dC_L} = \alpha\frac{\frac{1}{2}C_L^{-1/2}C_{D_0} - \frac{3}{2}KC_L^{3/2}}{(C_{D_0} + KC_L^2)^2} + \beta\frac{C_{D_0} - KC_L^2}{(C_{D_0} + KC_L^2)^2}$$

For maximum R $\rightarrow \frac{dR}{dC_L} = 0.$

در MATLAB برد را بر حسب C_L را بدست می آوریم و بیشترین بر را از روی برد بدست آمده می خوانیم.

Figure 16



$R_{\max} = 4059197.87_m$

در این حالت با وجود tailwind 259200_m به برد اضافه می شود.

b) V_∞ and attack angle is constant

$W_1 = 187141.86_N$

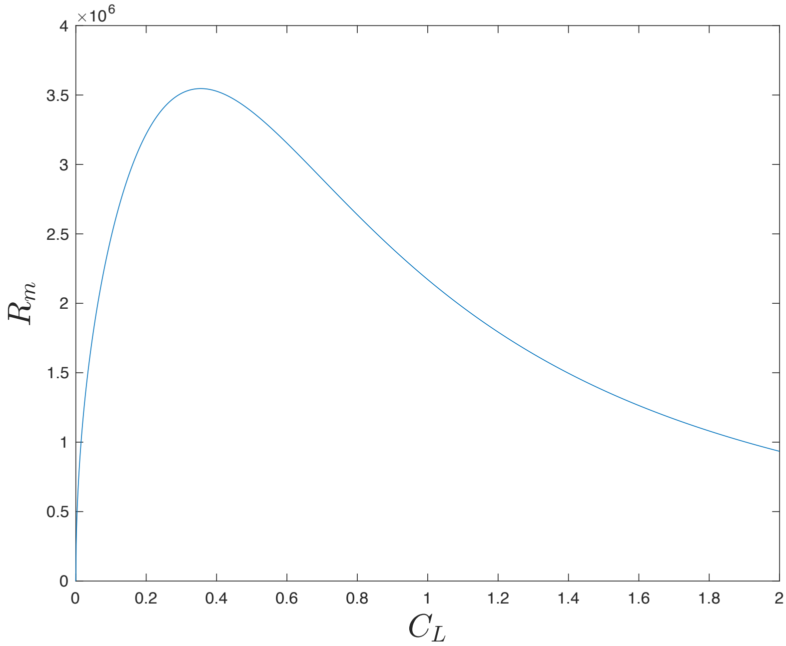
$R = R_{no\ wind} + V_{tw}E$

$$R = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})\sqrt{\frac{2W_0}{\rho_\infty S}}\ln\frac{W_0}{W_1} + V_{tw}\frac{1}{c_t}(\frac{C_L}{C_D})\ln\frac{W_0}{W1}$$

$C_D = C_{D_0} + KC_L^2$

مانند مساله قبل برد برای C_L های متفاوت بدست می آوریم و بیشترین مقدار آن را می خوانیم.

Figure 17



$R_{\max} = 4200289.23_m$

$$R_{\max_{(\rho_\infty,\alpha)const}} < R_{\max_{(V_\infty,\alpha)const}}$$

II) with $10_{m/s}$ head wind

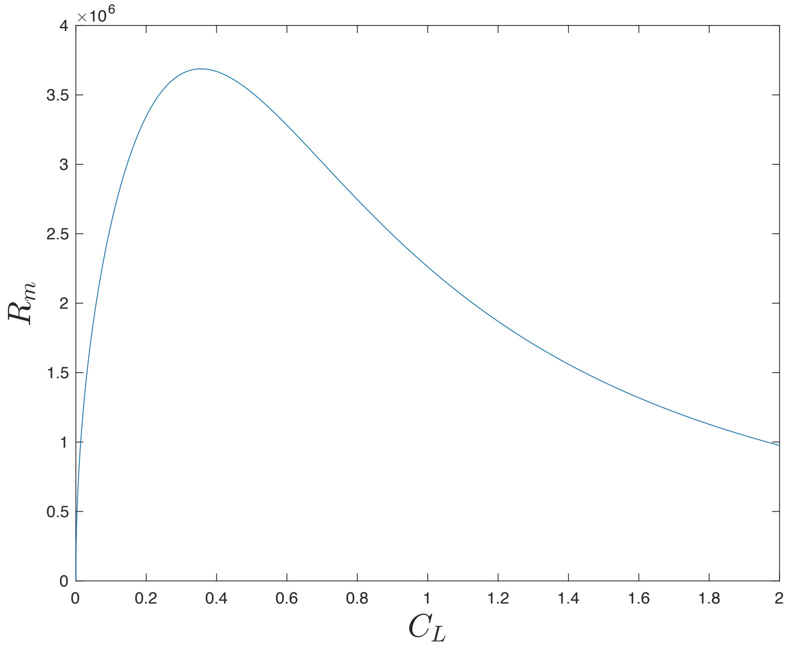
$R = R_{no\ wind} - V_{hw}E$

$V_{hw} = 10_{m/s}$

a) altitude and attack angle is constant

$$R = \frac{2}{c_t}\sqrt{\frac{2}{\rho_\infty S}}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2}) - V_{tw}\frac{1}{c_t}(\frac{C_L}{C_D})\ln\frac{W_0}{W1}$$

Figure 18



$R_{\max} = 3546586.6_m$

b) V_∞ and attack angle is constant

$W_1 = 187141.86_N$

$R = R_{no\ wind} V_{hw} E$

$$R = \frac{1}{c_t}(\frac{C_L^{1/2}}{C_D})\sqrt{\frac{2W_0}{\rho_\infty S}}\ln\frac{W_0}{W_1} - V_{hw}\frac{1}{c_t}(\frac{C_L}{C_D})\ln\frac{W_0}{W1}$$

$R_{\max} = 3687652.7_m$

$$R_{\max_{(\rho_\infty,\alpha) const}} < R_{\max_{(V_\infty,\alpha) const}}$$

سوال چهارم

$W_0 = 50_{KN},\ W_1 = 40_{KN},\ S = 30^2,\ P = 840_{KW}$
$\eta = 0.85,\ C_{D_0} = 0.025,\ K = 0.05,\ c = 4.5_{(N/KW.h)} = 1.25 \times 10^{-6}_{N/W.s}$
In 3_{Km} altitude $\rho_\infty = 1.007_{kg/m^3}$.

الف)

I) maximum rate

a) altitude and attack angle is constant

$$(\frac{C_L}{C_D})_{\max} = \sqrt{\frac{1}{4C_{D_0}K}} \rightarrow (\frac{C_L}{C_D})_{\max} = 14.1421$$

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W1} \rightarrow R_{\max} = \frac{\eta}{c}(\frac{C_L}{C_D})_{\max}\ln\frac{W_0}{W1} \rightarrow R_{\max} = 2145893.9_m$$

b) altitude and velocity is constant

$$\xi = \frac{\Delta W}{W_0} = 0.2$$

$$R = \frac{2\eta}{c}\frac{C_L}{C_D}\tan^{-1}(\frac{\xi}{2\sqrt{1-\xi}}) \rightarrow R_{\max} = \frac{2\eta}{c}(\frac{C_L}{C_D})_{\max}\tan^{-1}(\frac{\xi}{2\sqrt{1-\xi}})$$

$$R_{\max} = 2141455.6_m$$

$$R_{\max_{(h,\alpha)const}} > R_{\max_{(h,V)const}}$$

II) maximum endurance

a) altitude and attack angle is constant

$$E = \frac{\eta}{c}\sqrt{2\rho S}(\frac{C_L^{1.5}}{C_D})_{\max}(W_1^{-1/2} - W_0^{-1/2}) \rightarrow E_{\max} = \frac{\eta}{c}\sqrt{2\rho_\infty S}\frac{C_L^{1.5}}{C_D}(W_1^{-1/2} - W_0^{-1/2})$$

$$(\frac{C_L^{3/2}}{C_D})_{\max} = \frac{1}{4}(\frac{3}{kC_{D_0}^{1/3}})^{3/4} \rightarrow (\frac{C_L^{3/2}}{C_D})_{\max} = 13.5540$$

$$E_{\max} = 37817.237_s = 630.2873_{min} = 10.5048_{hr}$$

b) altitude and velocity is constant

$$E = \frac{2\eta}{c}\frac{2}{3^{3/4}}3^{1/4}\frac{C_L^{1.5}}{C_D}\sqrt{\frac{\rho_\infty S}{2W_0}}\tan^{-1}(\frac{\sqrt{3}\xi}{4-3\xi})$$

$$\rightarrow E_{\max} = \frac{2\eta}{c}\frac{2}{3^{3/4}}3^{1/4}(\frac{C_L^{1.5}}{C_D})_{\max}\sqrt{\frac{\rho_\infty S}{2W_0}}\tan^{-1}(\frac{\sqrt{3}\xi}{4-3\xi})$$

$$E_{\max} = 37563.6_s = 626.06_{min} = 10.4343_{hr}$$

$$E_{\max_{(h,\alpha)const}} > E_{\max_{(h,V)const}}$$

ب)

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W1} \rightarrow R_{\max}/2 = \frac{\eta}{c}(\frac{C_L}{C_D})_{\max}\ln\frac{W_0}{W1'} \rightarrow W_1' = 44721.359_N$$

$$C_L = \sqrt{\frac{C_{D_0}}{K}} \rightarrow C_L = 0.7071$$

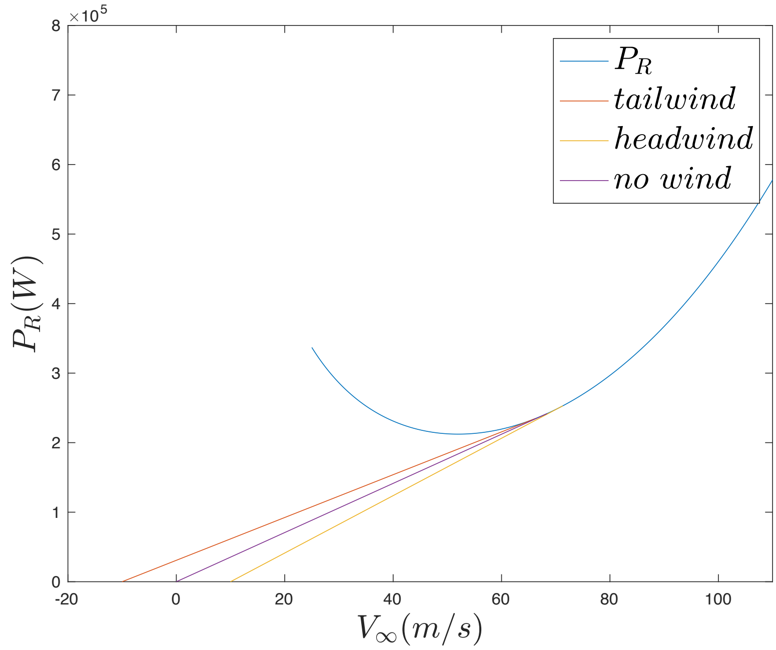
$$V_\infty = \sqrt{\frac{2W_1'}{\rho_\infty C_L}} \rightarrow V_\infty = 354.4176_{m/s}$$

ت)

$$P_R = \sqrt{\frac{2W^3}{\rho_\infty S}}\frac{1}{(C_L^{3/2}/C_D)_{\max}}$$

$$(\frac{C_L^{3/2}}{C_D})_{\max} = 13.5540$$

Figure 19



I) tail wind

بر اساس محاسبات MATLAB داریم

$$V_{\infty_{best\ tailwind}} = 66.2_{m/s} \rightarrow V_g = 56.2_{m/s}$$

$$P_R = 314.55_{HP}$$

$$\frac{HP_R}{V_g} = 5.7$$

II) head wind

$$V_{\infty_{best\ headwind}} = 71.3_{m/s} \rightarrow V_g = 61.3_{m/s}$$

$$P_R = 339.2_{HP}$$

$$\frac{HP_R}{V_g} = 5.53$$

III) no wind

$$V_{\infty_{best}} = 68.40_{m/s} \rightarrow V_g = 68.40_{m/s}$$

$$P_R = 324.3_{HP}$$

$$\frac{HP_R}{V_g} = 4.741$$

سوالات کامپیوتری

سوال اول

ب)

I) $n = 5 \rightarrow time = 312.218_{\mathrm{sec}}$

II) $n = 10 \rightarrow time = 309.97_{sec}$

III) $n = 15 \rightarrow time = 309.55_{\mathrm{sec}}$

IV) $n = 30 \rightarrow time = 309.29_{sec}$

V) $n = 60 \rightarrow time = 309.235_{sec}$

VI) $n = 30000 \rightarrow time = 309.218512_{sec}$