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Linear Quadratic Integral Differential Game applied to the Real-time Control of a Quadrotor Experimental setup

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Abstract—The accurate attitude control of a quadrotor is necessary, especially when facing disturbance. In this study, a linear quadratic with integral action based on the differential game theory is implemented on a quadrotor experimental setup. A continuous state-space model of the setup is derived using the linearization of nonlinear equations of motion, and its parameters are identified with the experimental results. Then, the attitude control commands of the quadrotor are derived based on two players; one finds the best attitude control command, and the other creates the disturbance by mini-maximizing a quadratic criterion, defined as the sum of outputs plus the weighted control effort and disturbance. The performance of the proposed structure is investigated in level flight and compared to the linear quadratic regulator controller. Results demonstrate that the proposed approach has an excellent performance in dissipating the disturbances.

Index Terms—Linear Quadratic Differential Game, Quadrotor, Real-time, 3DoF Experimental setup, Optimal Control, Robust Control

I. Introduction

A quadrotor is a type of helicopter with four rotors that plays a significant role in today's society, including research, military, imaging, recreation, and agriculture. The performance of the quadrotor relies on the control system, including attitude, altitude, and position subsystems. In the attitude control of the quadrotor, it is vital to maintain the attitude outputs at the desired level using control commands such as the rotational speed of the rotors, when the disturbances occur suddenly. Therefore, much research is being conducted on the automatic control of the attitude's quadrotor in facing the disturbance.

In [1], a Proportional Integral Derivative (PID) controller is used to regulate the quadrotor attitude. However, the control objectives have not been effectively achieved with this controller when the disturbance occurs. To solve this problem the model-based approaches [2] are utilized for controller design. These controllers work based on information from the quadrotor's attitude model and disturbance to produce the best control command.

Various model-based controllers can be found within the literature, the most well-known of which are intelligent control, nonlinear control, robust control, and optimal control to reduce the disturbance effect in the attitude control and provide a faster control algorithm in facing the modeling error. In the intelligent controller category, the artificial intelligence computing approaches like fuzzy logic [3], iterative learning [4], machine learning [5], reinforcement learning [6], and evolutionary computation [7] have been utilized to regulate the quadrotor's attitude.

Moreover, nonlinear control methods such as Feedback Linearization (FBL) [8] and Sliding Mode Control (SMC) [9] have been applied to control the roll, pitch, and yaw angles of the quadrotor. In the optimal controller category, a Linear Quadratic Regulator (LQR) [10] and Linear Quadratic Gaussian (LQG) [11] have been implemented on the quadrotor based on the minimization of a quadratic criterion, including regulation performance and control effort to provide optimally controlled feedback gains.

Linear Quadratic Regulator Differential Game (LQR-DG) control approach [12], [14] is a class of optimal and robust controller methods that controls the outputs of a system based on its linear model and mini-maximization of a cost function. This approach has been utilized to stabilize and control various nonlinear and complex systems such as a ship controller [13]. Moreover, in the LQR-DG control method, the control commands are analytically generated based on a pursuit-evasion of two players, one tracks the best control command, and the other creates the disturbance. This is one of the distinctive features of the LQR-DG controller and an important difference from other optimal control methods.

In this study, an LQR controller method based on the differential game theory, with an integral action called Linear Quadratic Integral Regulator Differential Game (LQIR-DG) controller, is proposed to generate the most efficient control command for an experimental setup of the quadrotor when facing the disturbance. Since the LQIR-DG is affected by an accurate model of the system, first, the dynamic of the three-degree-of-freedom setup of the quadrotor is modeled. Then, the linear state-space form of the quadrotor model is extracted using the linearization of the nonlinear equations of motion to utilize in the proposed control problem. Moreover, the model's parameters are identified and verified against the experimental values. Next, the LQIR-DG technique is applied to the experimental setup of the quadrotor to reduce the effect of disturbance. The performance of the suggested controller is examined when the disturbance occurs. The results show the successful performance of the LQIR-DG scheme in reducing the disturbance.

In the remainder of this study, the problem is defined in section II. The dynamics model for the experimental setup of the quadrotor is derived in detail, in section III. In section IV, The LQIR-DG architecture is denoted. Finally, in sections V and VI, Numerical results and conclusion are provided, respectively.

II. Problem Statement

Here, a nonlinear dynamic is presented for the setup of the quadrotor, as illustrated in Fig.1. The quadrotor is free to rotate about its roll, pitch, and yaw axes. The Euler angles and angular velocities along three orthogonal axes are measured simultaneously using the Attitude and Heading Reference Systems (AHRS). These noisy measurements are utilized in the LQIR-DG for control of the Euler angles. The block diagram of the controller structure is illustrated in Fig.2.

Fig. 1. 3DoF setup of the quadrotor

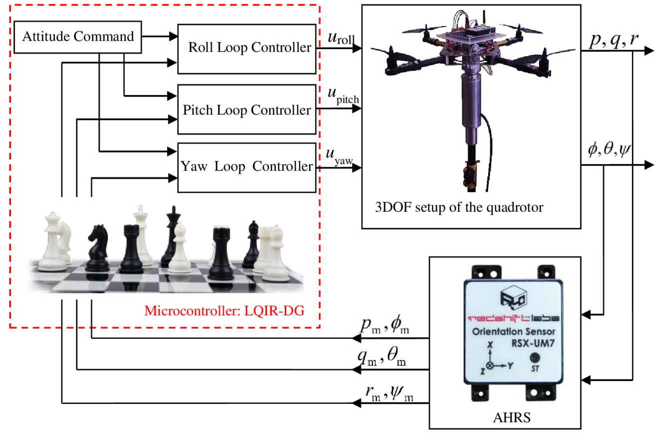


Fig. 2. Block diagram of the LQIR-DG controller structure

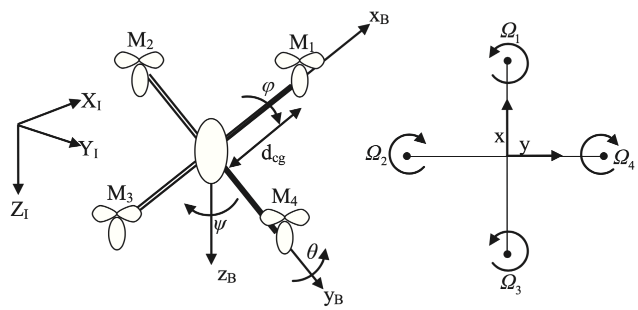


Fig. 3. Configuration of the quadrotor

III. Modeling of the Quadrotor Setup

Here, the model of the three-degree-of-freedom setup of the quadrotor is presented in details. For this purpose, first, the configuration of the quadrotor is denoted. Then, the nonlinear model of the attitude dynamics is derived to denote the state-space form. Finally, the nonlinear model is linearized to utilize for the control purposes.

A. Configuration of the Quadrotor

Fig.3 denotes the quadrotor schematic. Each rotor has an angular velocity, Ω_i, rotating about the z_B axis in the body coordinate system. Rotors 1 and 3 rotate counterclockwise, while rotors 2 and 4 rotate clockwise, to cancel yawing moment.

B. Dynamic Model

The quadrotor kinetic model, derived using the Newton-Euler method, is stated as [15], [16]

$$\dot{p} = \frac{\mathrm{I_{yy}} - \mathrm{I_{zz}}}{\mathrm{I_{xx}}} qr + q\frac{\mathrm{I_{rotor}}}{\mathrm{I_{xx}}}\Omega_r + \frac{u_{\mathrm{roll}}}{\mathrm{I_{xx}}} + \frac{d_{\mathrm{roll}}}{\mathrm{I_{xx}}} \tag{1}$$

$$\dot{q} = \frac{\mathrm{I_{zz}} - \mathrm{I_{zz}}}{\mathrm{I_{yy}}} rp + p\frac{\mathrm{I_{rotor}}}{\mathrm{I_{xx}}}\Omega_r + \frac{u_{\mathrm{pitch}}}{\mathrm{I_{yy}}} + \frac{d_{\mathrm{pitch}}}{\mathrm{I_{yy}}} \tag{2}$$

$$\dot{r} = \frac{\mathrm{I_{xx}} - \mathrm{I_{yy}}}{\mathrm{I_{zz}}} pq + \frac{u_{\mathrm{yaw}}}{\mathrm{I_{zz}}} + \frac{d_{\mathrm{yaw}}}{\mathrm{I_{zz}}} \tag{3}$$

where (p, q, r) are the angular velocities. d_{roll}, d_{pitch}, and d_{yaw} are the disturbances, generated in x_B, y_B and z_B, respectively. Moreover, $\mathrm{I_{xx}}$, $\mathrm{I_{yy}}$, and $\mathrm{I_{zz}}$ are the principal moment of inertia and $\mathrm{I_{rotor}}$ is a rotor inertia about its axis. The relation between the angular body rates and the Euler angles rates is obtained as

$$\dot{\phi} = p + (q\sin(\phi) + r\cos(\phi))\tan(\theta) \tag{4}$$

$$\dot{\theta} = q\cos(\phi) - r\sin(\phi) \tag{5}$$

$$\dot{\psi} = (q\sin(\phi) + r\cos(\phi))/\cos(\theta) \tag{6}$$

where (ϕ, θ, ψ) are roll, pitch, and yaw angles. Moreover, Ω_r, called the overall residual rotor angular velocity, is computed as

$$\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \tag{7}$$

C. Control Commands

The control inputs u_{roll}, u_{pitch}, and u_{yaw} are roll, pitch, and yaw moments, obtained from the rotors, defined as

$$u_{\text{roll}} = \mathrm{b}\,\mathrm{d}_{\text{cg}}(\Omega_2^2 - \Omega_4^2) \tag{8}$$

$$u_{\text{pitch}} = \mathrm{b}\,\mathrm{d}_{\text{cg}}(\Omega_1^2 - \Omega_3^2) \tag{9}$$

$$u_{\text{yaw}} = \mathrm{d}(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \tag{10}$$

Also, d and b are, respectively, drag and thrust coefficients. d_{cg} is the distance of rotors from the gravity center. Hence, the angular velocity commands are obtained as

$$\Omega_{c,1}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2\mathrm{b}\,\mathrm{d}_{\text{cg}}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \tag{11}$$

$$\Omega_{c,2}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2\mathrm{b}\,\mathrm{d}_{\text{cg}}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \tag{12}$$

$$\Omega_{c,3}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2\mathrm{b}\,\mathrm{d}_{\text{cg}}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \tag{13}$$

$$\Omega_{c,4}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2\mathrm{b}\,\mathrm{d}_{\text{cg}}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \tag{14}$$

where Ω_{mean} is the nominal of the rotor angular velocities.

D. State-Space Form

Here, the state-space model is presented for control purposes. By defining $x_1 = p$, $x_2 = q$, $x_3 = r$, $x_4 = \phi$, $x_5 = \theta$, and $x_6 = \psi$; the model of in state-space form are denoted as

$$\dot{x}_1 = \frac{\mathrm{I}_{\text{yy}} - \mathrm{I}_{\text{zz}}}{\mathrm{I}_{\text{xx}}} x_2 x_3 + x_2 \frac{\mathrm{I}_{\text{rotor}}}{\mathrm{I}_{\text{xx}}} \Omega_r + \frac{u_{\text{roll}}}{\mathrm{I}_{\text{xx}}} + \frac{d_{\text{roll}}}{\mathrm{I}_{\text{xx}}} \tag{15}$$

$$\dot{x}_2 = \frac{\mathrm{I}_{\text{zz}} - \mathrm{I}_{\text{zz}}}{\mathrm{I}_{\text{yy}}} x_1 x_3 - x_1 \frac{\mathrm{I}_{\text{rotor}}}{\mathrm{I}_{\text{xx}}} \Omega_r + \frac{u_{\text{pitch}}}{\mathrm{I}_{\text{yy}}} + \frac{d_{\text{pitch}}}{\mathrm{I}_{\text{yy}}} \tag{16}$$

$$\dot{x}_3 = \frac{\mathrm{I}_{\text{xx}} - \mathrm{I}_{\text{yy}}}{\mathrm{I}_{\text{zz}}} x_1 x_2 + \frac{u_{\text{yaw}}}{\mathrm{I}_{\text{zz}}} + \frac{d_{\text{yaw}}}{\mathrm{I}_{\text{zz}}} \tag{17}$$

$$\dot{x}_4 = x_1 + (x_2 \sin(x_4) + x_3 \cos(x_4)) \tan(x_5) \tag{18}$$

$$\dot{x}_5 = x_2 \cos(x_4) - x_3 \sin(x_4) \tag{19}$$

$$\dot{x}_6 = (x_2 \sin(x_4) + x_3 \cos(x_4)) / \cos(x_5) \tag{20}$$

The measurement model is written as

$$\mathbf{z} = \begin{bmatrix} p_m & q_m & r_m & \phi_m & \theta_m & \psi_m \end{bmatrix}^{\mathrm{T}} \tag{21}$$

E. Linear Model

The continuous-time linear model is utilized to drive the control commands on the quadrotor. The linear state-space model is denoted as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B_d}\mathbf{d}(t) \tag{22}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{B_d}$ are the system, input and disturbance matrices, respectively. Moreover, $\mathbf{d}$ is the disturbance. The measurements equation is stated as

$$\mathbf{z}(t) = \mathbf{x}(t) \tag{23}$$

According to Eqs.(15)-(20), the linear dynamic model around the equilibrium points ($\mathbf{x}_e = 0$ and $\mathbf{u}_e = 0$) of the quadrotor setup is denoted as

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{x}}_{\text{roll}} \\ \dot{\mathbf{x}}_{\text{pitch}} \\ \dot{\mathbf{x}}_{\text{yaw}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\text{roll}} \\ \mathbf{x}_{\text{pitch}} \\ \mathbf{x}_{\text{yaw}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\text{roll}} \\ \mathbf{u}_{\text{pitch}} \\ \mathbf{u}_{\text{yaw}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{d}_{\text{roll}} \\ \mathbf{d}_{\text{pitch}} \\ \mathbf{d}_{\text{yaw}} \end{bmatrix} \tag{24}$$

where $\mathbf{x}_{\text{roll}} = \begin{bmatrix} p & \phi \end{bmatrix}^{\mathrm{T}}$, $\mathbf{x}_{\text{pitch}} = \begin{bmatrix} q & \theta \end{bmatrix}^{\mathrm{T}}$, and $\mathbf{x}_{\text{yaw}} = \begin{bmatrix} r & \psi \end{bmatrix}^{\mathrm{T}}$.

Moreover, the state and input matrices are presented as

$$\mathbf{A}_{\text{roll}} = \mathbf{A}_{\text{pitch}} = \mathbf{A}_{\text{yaw}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{25}$$

$$\mathbf{B}_{\text{roll}} = \begin{bmatrix} \frac{1}{\mathrm{I}_{\text{xx}}} \\ 0 \end{bmatrix};\ \mathbf{B}_{\text{pitch}} = \begin{bmatrix} \frac{1}{\mathrm{I}_{\text{yy}}} \\ 0 \end{bmatrix};\ \mathbf{B}_{\text{yaw}} = \begin{bmatrix} \frac{1}{\mathrm{I}_{\text{zz}}} \\ 0 \end{bmatrix} \tag{26}$$

IV. Formulation of the Controller Design

In the LQIR-DG controller structure, an integral action is added to the LQR-DG controller to cancel the steady-state errors for reference tracking. For this purpose, first, the augmented state space of the linear quadrotor model is defined to utilize in the controller architecture. Then, the LQR-DG controller design procedure is presented to produce the best control commands for the experimental setup of the quadrotor.

A. Augmented State Space Formulation

To add the integral action to the controller structure, the augmented states are defined as follows:

$$\mathbf{x}_{\mathbf{a_i}} = \begin{bmatrix} \mathbf{x_i} & \int \mathbf{x_i} \end{bmatrix}^{\mathrm{T}} \tag{27}$$

where i = roll, pitch, and yaw. Then, the quadrotor dynamics model, denoted by Eq.(22), is denoted in the augmented state-space model as

$$\dot{\mathbf{x}}_{\boldsymbol{a}}(t) = \mathbf{A_a}\mathbf{x_a}(t) + \mathbf{B_a}\mathbf{u}(t) + \mathbf{B_{d_a}}\mathbf{d}(t) \tag{28}$$

where matrices $\mathbf{A_a}$ and $\mathbf{B_a}$ are defined as follows:

$$\mathbf{A_a} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{29}$$

$$\mathbf{B_a} = \mathbf{B_{d_a}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \tag{30}$$

In the above equation $\mathbf{I}$ denotes the identity matrix.

B. LQIR-DG Controller Method

The LQIR-DG controller is an optimal and robust method based on the differential game theory. This controller consists of two essential players: one finds the best control command, and the other creates the worst disturbance. For this purpose, the first player tries to minimize a cost function; while the second player is assumed to maximize it. Therefore, the quadratic cost function equation is denoted using min-max operators as follows:

$$\begin{aligned}&\min_u \max_d J(\mathbf{x_{a_i}}, u_i, d_i) = J(\mathbf{x_{a_i}}, u_i^*, d_i^*) = \\ &\min_u \max_d \int_0^{\mathrm{t_f}} \left(\mathbf{x_{a_i}^T Q_i x_{a_i}} + u_i^{\mathrm{T}} R u_i - d_i^{\mathrm{T}} R_d d_i \right) \mathrm{d}t\end{aligned} \tag{31}$$

where R and R_d are symmetric nonnegative definite matrices and $\mathbf{Q_i}$ is a symmetric positive definite matrix. Moreover, $\mathrm{t_f}$ is the final time. To solve this problem, connections between the general optimal problem and the LQIR problem are considered [12] and consequently the optimum control effort is computed for the each control loop as follows:

$$u_i(t) = -\mathbf{K}_{\boldsymbol{i}}(t)\mathbf{x_{a_i}}(t) \tag{32}$$

$$d_i(t) = \mathbf{K}_{\boldsymbol{i}}(t)\mathbf{x_{a_i}}(t) \tag{33}$$

where $\mathbf{K_i}$ and $\mathbf{K_{d_i}}$ are a time varying gain, given by

$$\mathbf{K_i} = R^{-1}\mathbf{B}^{\mathbf{T}}_{\boldsymbol{a_i}}\mathbf{P}_{\boldsymbol{a_i}}(t) \tag{34}$$

$$\mathbf{K_{d_i}} = R_d^{-1}\mathbf{B}^{\mathbf{T}}_{\boldsymbol{a_{d_i}}}\mathbf{P}_{\boldsymbol{a_{d_i}}}(t) \tag{35}$$

where $\mathbf{P}_{\boldsymbol{a_i}}(t)$ and $\mathbf{P}_{\boldsymbol{a_{d_i}}}(t)$ satisfy

$$\begin{aligned}\dot{\mathbf{P}}_{\boldsymbol{a_i}}(t) = -\mathbf{A_a^T}\mathbf{P_{a_i}}(t) - \mathbf{P_{a_i}}(t)\mathbf{A_a} - \mathbf{Q_i} + \\ \mathbf{P_{a_i}}(t)\mathbf{S_{a_i}}(t)\mathbf{P_{a_i}}(t) + \mathbf{P_{a_i}}(t)\mathbf{S_{a_{d_i}}}(t)\mathbf{P_{a_{d_i}}}(t)\end{aligned} \tag{36}$$

$$\begin{aligned}\dot{\mathbf{P}}_{\boldsymbol{a_{d_i}}}(t) = -\mathbf{A_a^T}\mathbf{P_{a_{d_i}}}(t) - \mathbf{P_{a_{d_i}}}(t)\mathbf{A_a} - \mathbf{Q_i} + \\ \mathbf{P_{a_{d_i}}}(t)\mathbf{S_{a_{d_i}}}(t)\mathbf{P_{a_{d_i}}}(t) + \mathbf{P_{a_{d_i}}}(t)\mathbf{S_{a_i}}(t)\mathbf{P_{a_i}}(t)\end{aligned} \tag{37}$$

where $\mathbf{S_{a_i}} = \mathbf{B_{a_i}} R^{-1}\mathbf{B}^{\mathbf{T}}_{\boldsymbol{a_i}}$ and $\mathbf{S_{a_{d_i}}} = \mathbf{B}_{\boldsymbol{a_{d_i}}} R_d^{-1}\mathbf{B}^{\mathbf{T}}_{\boldsymbol{a_{d_i}}}$. In this study, the steady-state values of the above equations ($\mathbf{P}$ as $\mathrm{t_f} \to \infty$) are utilized to generate a feedback control law.

V. Result and Discussion

Here, the results of the LQIR-DG controller method are devoted to the control loops of the roll, pitch, and yaw of the experimental setup of the quadrotor. First, the controller parameters are tuned using the results of numerical simulations. Moreover, the performance of the LQIR-DG controller is compared to an LQR control strategy. The quadrotor parameters are shown in table I. Moreover, the parameters of LQIR-DG controller weight are denoted in table II.

TABLE I
The Parameter of the Quadrotor

Parameter	Value	Unit
$\mathrm{I_{xx}}$	0.02839	$\mathrm{kg.m^2}$
$\mathrm{I_{yy}}$	0.03066	$\mathrm{kg.m^2}$
$\mathrm{I_{zz}}$	0.0439	$\mathrm{kg.m^2}$
$\mathrm{I_{rotor}}$	4.4398×10^{-5}	$\mathrm{kg.m^2}$
b	3.13×10^{-5}	$\mathrm{N.\sec^2/rad^2}$
d	3.2×10^{-6}	$\mathrm{N.m.\sec^2/rad^2}$
Ω_{mean}	3000	rpm
$\mathrm{d_{cg}}$	0.2	m

TABLE II
The Parameters of the LQIR-DG Controller

Control Loop	Weight	Value
Roll	$\mathbf{Q}_{\mathrm{roll}}$	diag([7.91, 0.01, 631.85, 214.28])
Pitch	$\mathbf{Q}_{\mathrm{pitch}}$	diag([9853.09, 0.12, 0.01, 873.93])
Yaw	$\mathbf{Q}_{\mathrm{yaw}}$	diag([1.81e−4, 4.5e−4, 3e−6, 1.7e−5])
-	R	1
-	R_d	1.2577

A. Performance of the LQIR-DG Controller

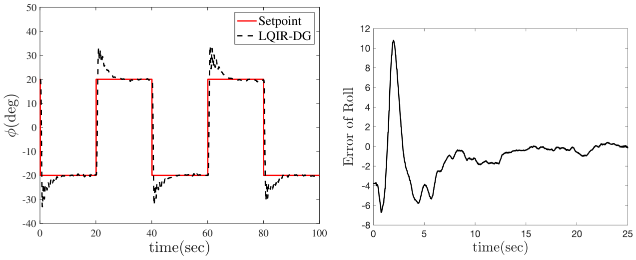
Here, the performance of the LQIR-DG controller is evaluated. The desired and actual outputs, including the roll, pitch, and yaw angles, are compared in Fig.4. The desired scenario of the simulator is considered a level flight. These figures show that the attitude outputs of the quadrotor converge to the desired values in less than three seconds. Moreover, Fig.5 show the angular velocity command of the quadrotor, respectively. These results illustrate that the LQIR-DG approach appropriately controls the attitude of the experimental setup of the quadrotor.

B. Comparison with LQR

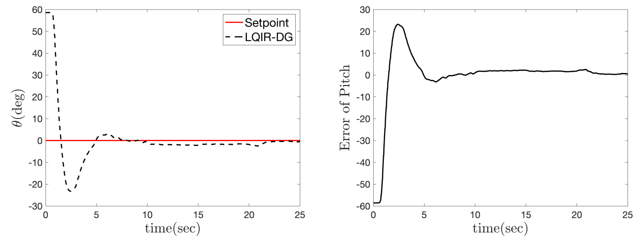
Here, the LQIR-DG controller performance is compared with famous control strategies such as the LQR controller method. Fig.6 compares the quadrotor's desired and actual pitch angle in the presence of these controllers. This result indicates that the LQIR-DG controller can provide high tracking performance, such as good transient response and high rapid convergence relative to the LQR controller for pitch angle control of the quadrotor setup.

VI. Conclusion

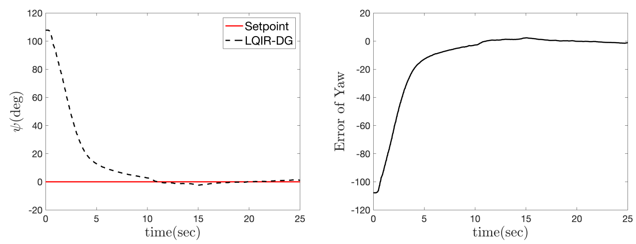
In this study, a linear quadratic with integral action based on the differential game theory, called LQIR-DG, was implemented for level attitude control in an experimental setup of a quadrotor. To implement the proposed controller structure, first, an accurate model of the quadrotor was linearized in the state-space form, and then the model parameters were estimated. Next, two players were considered for each of the quadrotor's roll, pitch, and yaw channels. The first player found the



(a)



(b)



(c)

Fig. 4. Performance of the LQIR-DG controller (a) roll angle (b) pitch angle (c) yaw angle

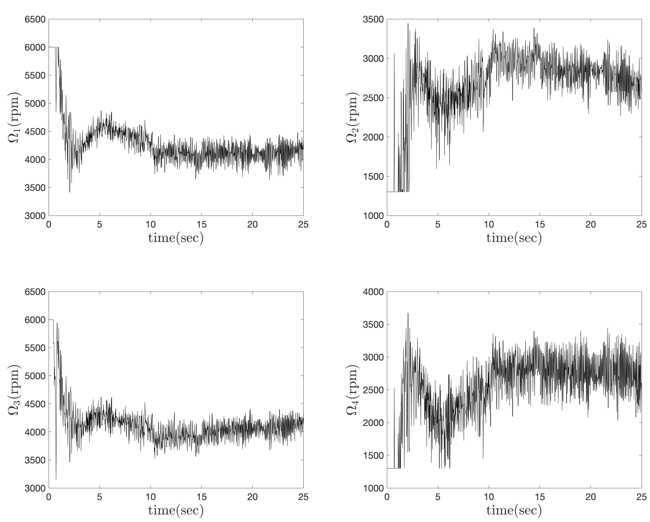


Fig. 5. Time history of angular velocity commands

best control command for each channel of the setup of a quadrotor based on the mini-maximization of a quadratic criterion; when the second player produced the worst disturbances. Finally, the performance of the proposed

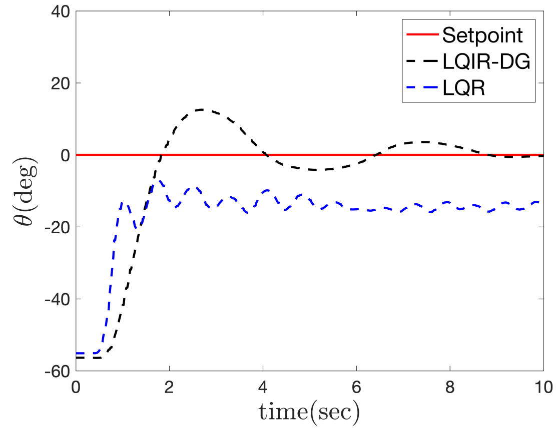


Fig. 6. Comparison of the LQIR-DG to the LQR in control of the pitch angle of the experimental setup

controller was investigated in level flight and compared to the LQR controller. The implementation results verify the successful performance of the LQIR-DG method in the level flight of the attitude control for the actual plant.

References

[1] H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati and S. Smailzadeh, "Attitude Control of a Quadrotor with Optimized PID Controller," Intelligent Control and Automation, Vol. 4 No. 3, 2013, pp. 335-342. doi: 10.4236/ica.2013.43039.

[2] Bouzid, Y., Zareb, M., Siguerdidjane, H. et al. Boosting a Reference Model-Based Controller Using Active Disturbance Rejection Principle for 3D Trajectory Tracking of Quadrotors: Experimental Validation. J Intell Robot Syst 100, 597–614 (2020). https://doi.org/10.1007/s10846-020-01182-4

[3] K. Liu, R. Wang, S. Dong and X. Wang, "Adaptive Fuzzy Finite-time Attitude Controller Design for Quadrotor UAV with External Disturbances and Uncertain Dynamics," 2022 8th International Conference on Control, Automation and Robotics (ICCAR), 2022, pp. 363-368, doi: 10.1109/ICCAR55106.2022.9782598.

[4] L. V. Nguyen, M. D. Phung, and Q. P. Ha, "Iterative Learning Sliding Mode Control for UAV Trajectory Tracking," Electronics, vol. 10, no. 20, 2021, doi: 10.3390/electronics10202474.

[5] C. Nicol, C. J. B. Macnab and A. Ramirez-Serrano, "Robust neural network control of a quadrotor helicopter," 2008 Canadian Conference on Electrical and Computer Engineering, 2008, pp. 001233-001238, doi: 10.1109/CCECE.2008.4564736.

[6] C.-H. Pi, W.-Y. Ye, and S. Cheng, "Robust Quadrotor Control through Reinforcement Learning with Disturbance Compensation," Applied Sciences, vol. 11, no. 7, 2021, doi: 10.3390/app11073257.

[7] P. Ghiglino, J. L. Forshaw, and V. J. Lappas, "Online PID Self-Tuning using an Evolutionary Swarm Algorithm with Experimental Quadrotor Flight Results," in AIAA Guidance, Navigation, and Control (GNC) Conference, doi: 10.2514/6.2013-5098.

[8] A. Aboudonia, A. El-Badawy, and R. Rashad, "Disturbance observer-based feedback linearization control of an unmanned quadrotor helicopter," Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 230, no. 9, pp. 877–891, 2016, doi: 10.1177/0959651816656951.

[9] H. Wang and M. Chen, "Sliding mode attitude control for a quadrotor micro unmanned aircraft vehicle using disturbance observer," Proceedings of 2014 IEEE Chinese Guidance, Navigation and Control Conference, 2014, pp. 568-573, doi: 10.1109/CGNCC.2014.7007285.

[10] Z. Shulong, A. Honglei, Z. Daibing and S. Lincheng, ”A new feedback linearization LQR control for attitude of quadrotor,” 2014 13th International Conference on Control Automation Robotics & Vision (ICARCV), 2014, pp. 1593-1597, doi: 10.1109/ICARCV.2014.7064553.

[11] E. Barzanooni, K. Salahshoor and A. Khaki-Sedigh, ”Attitude flight control system design of UAV using LQG multivariable control with noise and disturbance,” 2015 3rd RSI International Conference on Robotics and Mechatronics (ICROM), 2015, pp. 188-193, doi: 10.1109/ICRoM.2015.7367782.

[12] Engwerda, J. C. (2006). Linear Quadratic Games: An Overview (CentER Discussion Paper, Vols. 2006–110) [WorkingPaper]. Macroeconomics.

[13] Z. Zwierzewicz, ”On the ship course-keeping control system design by using robust and adaptive control,” 2014 19th International Conference on Methods and Models in Automation and Robotics (MMAR), 2014, pp. 189-194, doi: 10.1109/MMAR.2014.6957349.

[14] Engwerda, J. Min-Max Robust Control in LQ-Differential Games. Dyn Games Appl (2022). https://doi.org/10.1007/s13235-021-00421-z

[15] Bouabdallah, S., 2007. Design and Control of Quadrotors with Application to Autonomous Flying (Ph.D. thesis), University of Pennsylvania, Philadelphia.

[16] S. Bouabdallah and R. Siegwart, ”Full control of a quadrotor,” 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2007, pp. 153-158, doi: 10.1109/IROS.2007.4399042.