Control of a Three Degree of Freedom Quadcopter Stand Using Linear Quadratic Integral Based on the Differential Game Theory

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***Abstract*—In this paper, a quadcopter stand with three degrees of freedom was controlled using game theory-based control. The first player tracks a desired input, and the second player creates a disturbance in the tracking of the first player to cause an error in the tracking. The ???? move is chosen using the Nash equilibrium, which presupposes that the other player made the worst move. In addition to being resistant to input interruptions, this method may also be resilient to modeling system uncertainty. This method evaluated the performance through simulation in the Simulink environment and implementation on a three-degree-of-freedom stand.**

***Index Terms*—Quadcopter, Differential Game, Game Theory, Nash Equilibrium, Three Degree of Freedom Stand, Model Base Design, Linear Quadratic Regulator**

I. Introduction

Quadcopter is a type of helicopter with four rotors.

II. Differential Game

Differential games are a series of problems that arise while examining and simulating dynamic systems in game theory. Differential equations simulate how a state variable or set of state variables changes over time.

A. An introduction to the differential game

It is considered that two players are involved in this research. The space states of a continuous linear system are shown below.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B_1}\mathbf{u_1}(t) + \mathbf{B_2}\mathbf{u_2}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D_1}\mathbf{u_1}(t) + \mathbf{D_2}\mathbf{u_2}(t)\end{aligned} \tag{1}$$

Where $\mathbf{x}$ is the vector of the state variables, $\dot{\mathbf{x}}$ is the time derivative of the state vector, $\mathbf{u_1}$ is the first player (controller) input vector, $\mathbf{u_2}$ is the second player (disturbance) input vector, $\mathbf{y}$ is the output vector, $\mathbf{A}$ is the state matrix, $\mathbf{B_1}$ is the first player input matrix, $\mathbf{B_2}$ is the second player input matrix, $\mathbf{C}$ is the output matrix, $\mathbf{D_1}$ is first player the output matrix and $\mathbf{D_2}$ is second player the output matrix. Equation (1) demonstrates how both participants have an impact on the system's dynamics. The second player may progress toward the goal as a result of the first player's exertion, or vice versa. This paper considers the case that players do not cooperate in order to realize their goals. The situation where players do not work together (non-cooperative) to achieve their objectives is examined in the paper. In this case, every player knows at time $t \in [0, T]$ just the initial state $\mathbf{x_0}$ and the model structure. For the game (1,2), we will use the set of Nash equilibria. Formal Nash equilibrium is defined as follows. An admissible set of actions $(\boldsymbol{u_1}^*, \boldsymbol{u_2}^*)$ is a Nash equilibrium for the game (1,2); if for all admissible $(\boldsymbol{u_1}, \boldsymbol{u_2})$, the following inequalities hold:

$$J_1(\mathbf{u_1}^*, \mathbf{u_2}^*) \leq J_1(\mathbf{u_1}, \mathbf{u_2}^*)\ ,\ J_2(\mathbf{u_1}^*, \mathbf{u_2}^*) \leq J_2(\mathbf{u_1}^*, \mathbf{u_2}) \tag{2}$$

B. LQDG controller

For the system described in equation (1), LQDG optimum control effort calculates from equation (3).

$$\mathbf{u_i}(t) = -\mathbf{R_{ii}}^{-1}\mathbf{B_i}^{\mathrm{T}}\mathbf{P_i}(t)\mathbf{x}(t) = -\mathbf{k_i}(t)\mathbf{x}(t), \quad i = 1, 2 \tag{3}$$

In equation (3), $\mathbf{k_i}$ is the optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function (equation 4) of player number i.

$$\begin{aligned}J_i(\mathbf{u_1}, \mathbf{u_2}) = \int_0^T \Big(&\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q_i}\mathbf{x}(t) + \mathbf{u_i}^{\mathrm{T}}(t)\mathbf{R_{ii}}\mathbf{u_i}(t) + \\ &\mathbf{u_j}^{\mathrm{T}}(t)\mathbf{R_{ij}}\mathbf{u_j}(t)\Big)\mathrm{d}t\end{aligned} \tag{4}$$

Here the matrices $\mathbf{Q_i}$ and $\mathbf{R_{ii}}$ are assumed to be symmetric and $\mathbf{R_{ii}}$ positive definite. $\mathbf{P_i}$ is found by solving the continuous time couple Riccati differential equation:

$$\begin{aligned}\dot{\mathbf{P}}_\mathbf{1}(t) = &-\mathbf{A}^{\mathrm{T}}\mathbf{P_1}(t) - \mathbf{P_1}(t)\mathbf{A} - \mathbf{Q_1} + \mathbf{P_1}(t)\mathbf{S_1}(t)\mathbf{P_1}(t) + \\ &\mathbf{P_1}(t)\mathbf{S_2}(t)\mathbf{P_2}(t) \\ \dot{\mathbf{P}}_\mathbf{2}(t) = &-\mathbf{A}^{\mathrm{T}}\mathbf{P_2}(t) - \mathbf{P_2}(t)\mathbf{A} - \mathbf{Q_2} + \mathbf{P_2}(t)\mathbf{S_2}(t)\mathbf{P_2}(t) + \\ &\mathbf{P_2}(t)\mathbf{S_1}(t)\mathbf{P_1}(t)\end{aligned} \tag{5}$$

Using the shorthand notation $\boldsymbol{S_i} := \boldsymbol{B_i}\boldsymbol{R_{ii}}^{-1}\boldsymbol{B_i}^{\mathrm{T}}$.

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C. LQIDG controller

The absence of an integrator in the LQDG controller may result in steady-state errors due to disturbances or modeling errors. The LQIDG controller is based on the LQDG controller to eliminate this error.

The LQIDG controller adds the integral of the difference between the system output and the desired value to the state vector. Therefore, The augmented space states of a continuous linear system are shown below.

$$\mathbf{x_a} = \begin{bmatrix} \mathbf{x_d} - \mathbf{x} \\ \int (\mathbf{y_d} - \mathbf{y}) \end{bmatrix} \tag{6}$$

Where $\mathbf{x_a}$ is the vector of augmented state variables, $\mathbf{x_d}$ is the vector of the desired state variables, and $\mathbf{y_d}$ is the desired output vector. As a result, the state vector and the output vector are equal.

$$\mathbf{y} = \mathbf{x} \tag{7}$$

The following represents the system dynamics in the augmented state space.

$$\dot{\mathbf{x}}_{\boldsymbol{a}}(t) = \mathbf{A_a}\mathbf{x_a}(t) + \mathbf{B_{a_1}}\mathbf{u_{a_1}}(t) + \mathbf{B_{a_2}}\mathbf{u_{a_2}}(t) \tag{8}$$

Where matrices $\mathbf{A_a}$ and $\mathbf{B_a}$ are defined as follows:

$$\mathbf{A_a} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{C} & 0 \end{bmatrix}, \quad \mathbf{B_a} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \tag{9}$$

By introducing a new space state for the system, the remaining design phases of the LQIDG controller are comparable to those of the LQDG controller. LQIDG optimum control effort calculates from equation (10).

$$\mathbf{u_i}(t) = -\mathbf{R_{ii}}^{-1}\mathbf{B_{a_i}}^{\mathrm{T}}\mathbf{P_{a_i}}(t)\mathbf{x_a}(t) = -\mathbf{K_{a_i}}(t)\mathbf{x_a}(t), \ i = 1, 2 \tag{10}$$

In equation (10), $\mathbf{K_{a_i}}$ is the optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function, equation 11, of player number i.

$$\begin{aligned} J_i(\mathbf{u_1}, \mathbf{u_2}) = \int_0^T \Big(& \mathbf{x_a}^{\mathrm{T}}(t)\mathbf{Q_i}\mathbf{x_a}(t) + \mathbf{u_i}^{\mathrm{T}}(t)\mathbf{R_{ii}}\mathbf{u_i}(t) + \\ & \mathbf{u_j}^{\mathrm{T}}(t)\mathbf{R_{ij}}\mathbf{u_j}(t)\Big)\mathrm{d}t \end{aligned} \tag{11}$$

$\dot{\mathbf{P}}_{\boldsymbol{a_i}}$ is found by solving the continuous time couple Riccati differential equation:

$$\begin{aligned} \dot{\mathbf{P}}_{\boldsymbol{a_1}}(t) = & -\mathbf{A_a}^{\mathrm{T}}\mathbf{P_{a_1}}(t) - \mathbf{P_{a_1}}(t)\mathbf{A_a} - \mathbf{Q_1} + \\ & \mathbf{P_{a_1}}(t)\mathbf{S_{a_1}}(t)\mathbf{P_{a_1}}(t) + \mathbf{P_{a_1}}(t)\mathbf{S_{a_2}}(t)\mathbf{P_{a_2}}(t) \\ \dot{\mathbf{P}}_{\boldsymbol{a_2}}(t) = & -\mathbf{A_a}^{\mathrm{T}}\mathbf{P_{a_2}}(t) - \mathbf{P_{a_2}}(t)\mathbf{A_a} - \mathbf{Q_2} + \\ & \mathbf{P_{a_2}}(t)\mathbf{S_{a_2}}(t)\mathbf{P_{a_2}}(t) + \mathbf{P_{a_2}}(t)\mathbf{S_{a_1}}(t)\mathbf{P_{a_1}}(t) \end{aligned} \tag{12}$$

Using the shorthand notation $\boldsymbol{S_{a_i}} := \boldsymbol{B_{a_i}}\boldsymbol{R_{ii}}^{-1}\boldsymbol{B_{a_i}}^{\mathrm{T}}$.

III. Mathematical modeling

$$\mathbf{f} = \begin{bmatrix} x_4 + x_5\sin(x_1)\tan(x_2) + x_6\cos(x_1)\tan(x_2) \\ x_5\cos(x_1) - x_6\sin(x_1) \\ (x_5\sin(x_1) + x_6\cos(x_1))\sec(x_2) \\ A_1\cos(x_2)\sin(x_1) + A_2x_5x_6 + A_3u_1 \\ B_1\sin(x_2) + B_2x_4x_6 + B_3u_2 \\ C_1x_4x_5 + C_2u_3 \end{bmatrix} \tag{13}$$

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^{\mathrm{T}} \tag{14}$$

$$\delta\dot{\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta\mathbf{u} \tag{15}$$

$$\mathbf{x}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{16}$$

$$\mathbf{u}^* = \begin{bmatrix} 0 & 0 & 0 & 4 \times 2000^2 \end{bmatrix}^{\mathrm{T}} \tag{17}$$

$$\mathbf{A} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}^*} \tag{18}$$

$$\mathbf{B} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{\mathbf{u}^*} \tag{19}$$

$$\mathbf{A}_{\text{roll}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ A_1\cos(x_1) & 0 \end{bmatrix} \tag{20}$$

$$\mathbf{B}_{\text{roll}} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \end{bmatrix} = \begin{bmatrix} 0 \\ A_3 \end{bmatrix} \tag{21}$$

$$\mathbf{A}_{\text{pitch}} = \begin{bmatrix} \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_5} \\ \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_5} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ B_1\cos(x_1) & 0 \end{bmatrix} \tag{22}$$

$$\mathbf{B}_{\text{pitch}} = \begin{bmatrix} \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_5}{\partial u_2} \end{bmatrix} = \begin{bmatrix} 0 \\ B_3 \end{bmatrix} \tag{23}$$

$$\mathbf{A}_{\text{yaw}} = \begin{bmatrix} \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{24}$$

$$\mathbf{B}_{\text{yaw}} = \begin{bmatrix} \frac{\partial f_3}{\partial u_3} \\ \frac{\partial f_6}{\partial u_3} \end{bmatrix} = \begin{bmatrix} 0 \\ C_2 \end{bmatrix} \tag{25}$$

$$\begin{aligned} u_1 &= \omega_2^2 - \omega_4^2 \\ u_2 &= \omega_1^2 - \omega_3^2 \\ u_3 &= \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 \\ u_4 &= \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 \end{aligned} \tag{26}$$

$$
\begin{aligned}
\omega_1 &= \sqrt{\frac{u_4 + u_3 + 2u_2}{4}} \\
\omega_2 &= \sqrt{\frac{u_4 - u_3 + 2u_1}{4}} \\
\omega_3 &= \sqrt{\frac{u_4 + u_3 - 2u_2}{4}} \\
\omega_4 &= \sqrt{\frac{u_4 - u_3 - 2u_1}{4}}
\end{aligned} \tag{27}
$$

IV. Simulation