

Control of a Three Degree of Freedom Quadcopter Stand Using Linear Quadratic Integral Based on the Differential Game Theory

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Abstract—In this paper, a quadcopter stand with three degrees of freedom was controlled using game theory-based control. The first player tracks a desired input, and the second player creates a disturbance in the tracking of the first player to cause an error in the tracking. The move is chosen using the Nash equilibrium, which presupposes that the other player made the worst move.. In addition to being resistant to input interruptions, this method may also be resilient to modeling system uncertainty. This method evaluated the performance through simulation in the Simulink environment and implementation on a three-degree-of-freedom stand.

Index Terms—Quadcopter, Differential Game, Game Theory, Nash Equilibrium, Three Degree of Freedom Stand, Model Base Design, Linear Quadratic Regulator

I. INTRODUCTION

Quadcopter is a type of helicopter with four rotors.

II. DIFFERENTIAL GAME

Differential games are a series of problems that arise while examining and simulating dynamic systems in game theory. Differential equations simulate how a state variable or set of state variables changes over time.

A. An introduction to the differential game

It is considered that two players are involved in this paper. The space states of a continuous linear system are shown below.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_2\mathbf{u}_2(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}_1\mathbf{u}_1(t) + \mathbf{D}_2\mathbf{u}_2(t)\end{aligned}\quad (1)$$

where \mathbf{x} is the vector of all the state variables, $\dot{\mathbf{x}}$ is the time derivative of the state vector, \mathbf{u}_1 is the first player (controller) input vector, \mathbf{u}_2 is the second player (disturbance) input vector, \mathbf{y} is the output vector, \mathbf{A} is the state matrix, \mathbf{B}_1 is first player the input matrix, \mathbf{B}_2 is the second player input matrix, \mathbf{C} is the output matrix, \mathbf{D}_1 is first player the output matrix and \mathbf{D}_2 is second player the output matrix. Equation (1) demonstrates how both participants have an impact on the system's dynamics. The second player may progress toward

the goal as a result of the first player's exertion, or vice versa. In this paper we consider the case that players do not cooperate in order to realize their goals. The situation where players do not work together (non-cooperative) to achieve their objectives is examined in the paper. In this case, every player knows at time $t \in [0, T]$ just the initial state \mathbf{x}_0 and the model structure. This scenario can be interpreted as the players simultaneously determining their actions, next submitting their actions. For the game (1,2), we will use the set of Nash equilibria. Formal Nash equilibrium is defined as follows. An admissible set of actions $(\mathbf{u}_1^*, \mathbf{u}_2^*)$ is a Nash equilibrium for the game (1,2); if for all admissible $(\mathbf{u}_1, \mathbf{u}_2)$, the following inequalities hold:

$$J_1(\mathbf{u}_1^*, \mathbf{u}_2^*) \leq J_1(\mathbf{u}_1, \mathbf{u}_2^*), \quad J_2(\mathbf{u}_1^*, \mathbf{u}_2^*) \leq J_2(\mathbf{u}_1^*, \mathbf{u}_2) \quad (2)$$

Here admissibility is meant in the sense that $\mathbf{u}_i(\cdot)$ belongs to some restricted set, where this set depends on the information players have on the game, the set of strategies the players like to use to control the system, and the system (2) must have a unique solution.

B. LQDG controller

For the system described in equation (1), LQDG optimum control effort calculates from equation (3).

$$\mathbf{u}_i(t) = -\mathbf{R}_{ii}^{-1}\mathbf{B}_i^T\mathbf{P}_i(t)\mathbf{x}(t) = -\mathbf{k}_i(t)\mathbf{x}(t), \quad i = 1, 2 \quad (3)$$

In equation 3, \mathbf{k}_i is optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function (equation 4) of player number i .

$$J_i(\mathbf{u}_1, \mathbf{u}_2) = \int_0^T \left(\mathbf{x}^T(t)\mathbf{Q}_i\mathbf{x}(t) + \mathbf{u}_i^T(t)\mathbf{R}_{ii}\mathbf{u}_i(t) + \mathbf{u}_j^T(t)\mathbf{R}_{ij}\mathbf{u}_j(t) \right) dt \quad (4)$$

\mathbf{P}_i is found by solving the continuous time couple Riccati differential equation:

$$\begin{aligned}\dot{\mathbf{P}}_1(t) &= -\mathbf{A}^T \mathbf{P}_1(t) - \mathbf{P}_1(t) \mathbf{A} - \mathbf{Q}_1 + \mathbf{P}_1(t) \mathbf{S}_1(t) \mathbf{P}_1(t) + \\ &\quad \mathbf{P}_1(t) \mathbf{S}_2(t) \mathbf{P}_2(t) \\ \dot{\mathbf{P}}_2(t) &= -\mathbf{A}^T \mathbf{P}_2(t) - \mathbf{P}_2(t) \mathbf{A} - \mathbf{Q}_2 + \mathbf{P}_2(t) \mathbf{S}_2(t) \mathbf{P}_2(t) + \\ &\quad \mathbf{P}_2(t) \mathbf{S}_1(t) \mathbf{P}_1(t)\end{aligned}\tag{5}$$

III. MATHEMATICAL MODELING

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections III-A–III-E below for more information on proofreading, spelling and grammar. Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads— \LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive”.
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Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{6}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(6)”, not “Eq. (6)” or “equation (6)”, except at the beginning of a sentence: “Equation (6) is . . .”

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Please use “soft” (e.g., `\eqref{Eq}`) cross references instead of “hard” references (e.g., (1)). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don’t use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in \LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you’ve discovered a new method of counting.

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- The subscript for the permeability of vacuum μ_0 , and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
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- Do not use the word “essentially” to mean “approximately” or “effectively”.
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- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
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- There is no period after the “et” in the Latin abbreviation “et al.”.
- The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [7].

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Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

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Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an

TABLE I
TABLE TYPE STYLES

Table Head	Table Column Head		
	Table column subhead	Subhead	Subhead
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^aSample of a Table footnote.



Fig. 1. Example of a figure caption.

example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

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Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first ...”

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