

Bachelor Thesis Presentation

Control of a Three Degree of Freedom Quadcopter Stand Using
a Linear Quadratic Integral Regulator Based on the Differential
Game Theory

By:
Ali BaniAsad

Supervisor:
Dr.Hadi Nobahari

Quadcopter



Three Degree of Freedom
Quadcopter Stand

Game Theory



John Forbes Nash Jr.

LQDG Controller

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_2\mathbf{u}_2(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}_1\mathbf{u}_1(t) + \mathbf{D}_2\mathbf{u}_2(t)$$

$$\mathbf{u}_i(t) = -\mathbf{R}_{ii}^{-1}\mathbf{B}_i^T\mathbf{K}_i(t)\mathbf{x}(t), \quad i = 1, 2$$

$$J_i(\mathbf{u}_1, \mathbf{u}_2) = \int_0^T \left(\mathbf{x}^T(t)\mathbf{Q}_i\mathbf{x}(t) + \mathbf{u}_i^T(t)\mathbf{R}_{ii}\mathbf{u}_i(t) + \mathbf{u}_j^T(t)\mathbf{R}_{ij}\mathbf{u}_j(t) \right) dt$$

$$\dot{\mathbf{K}}_1(t) = -A^T\mathbf{K}_1(t) - \mathbf{K}_1(t)A - \mathbf{Q}_1 + \mathbf{K}_1(t)\mathbf{S}_1(t)\mathbf{K}_1(t) + \mathbf{K}_1(t)\mathbf{S}_2(t)\mathbf{K}_2(t)$$

$$\dot{\mathbf{K}}_2(t) = -A^T\mathbf{K}_2(t) - \mathbf{K}_2(t)A - \mathbf{Q}_2 + \mathbf{K}_2(t)\mathbf{S}_2(t)\mathbf{K}_2(t) + \mathbf{K}_2(t)\mathbf{S}_1(t)\mathbf{K}_1(t)$$

$$\mathbf{S}_i := \mathbf{B}_i\mathbf{R}_{ii}^{-1}\mathbf{B}_i^T$$

LQIDG Controller

Augmented Space State

$$x_a = \begin{bmatrix} x_d - x \\ \int (y_d - y) \end{bmatrix} \quad A_a = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$\dot{x}_a(t) = A_a x_a(t) + B_{a_1} u_{a_1}(t) + B_{a_2} u_{a_2}(t)$$

$$y(t) = C_a x_a(t) + D_{a_1} u_{a_1}(t) + D_{a_2} u_{a_2}(t)$$

LQIDG Controller

$$\boldsymbol{u}_i(t) = -\boldsymbol{R}_{ii}^{-1}\boldsymbol{B}_{a_i}^{\top}\boldsymbol{K}_{a_i}(t)\boldsymbol{x}_a(t), \quad i = 1, 2$$

$$J_i(\boldsymbol{u}_1, \boldsymbol{u}_2) = \int_0^T \left(\boldsymbol{x}_a^{\top}(t) \boldsymbol{Q}_i \boldsymbol{x}_a(t) + \boldsymbol{u}_i^{\top}(t) \boldsymbol{R}_{ii} \boldsymbol{u}_i(t) + \boldsymbol{u}_j^{\top}(t) \boldsymbol{R}_{ij} \boldsymbol{u}_j(t) \right) dt$$

$$\dot{\boldsymbol{K}}_{a_1}(t) = -\boldsymbol{A}^{\top}\boldsymbol{K}_{a_1}(t) - \boldsymbol{K}_{a_1}(t)\boldsymbol{A} - \boldsymbol{Q}_{a_1} + \boldsymbol{K}_{a_1}(t)\boldsymbol{S}_{a_1}(t)\boldsymbol{K}_{a_1}(t) + \boldsymbol{K}_{a_1}(t)\boldsymbol{S}_{a_2}(t)\boldsymbol{K}_{a_2}(t)$$

$$\dot{\boldsymbol{K}}_{a_2}(t) = -\boldsymbol{A}^{\top}\boldsymbol{K}_{a_2}(t) - \boldsymbol{K}_{a_2}(t)\boldsymbol{A} - \boldsymbol{Q}_{a_2} + \boldsymbol{K}_{a_2}(t)\boldsymbol{S}_{a_2}(t)\boldsymbol{K}_{a_2}(t) + \boldsymbol{K}_{a_2}(t)\boldsymbol{S}_{a_1}(t)\boldsymbol{K}_{a_1}(t)$$

$$\boldsymbol{S}_{a_i} := \boldsymbol{B}_{a_i}\boldsymbol{R}_{ii}^{-1}\boldsymbol{B}_{a_i}^{\top}$$

Mathematical Model

$$f = \begin{bmatrix} x_4 + x_5 \sin(x_1) \tan(x_2) + x_6 \cos(x_1) \tan(x_2) \\ x_5 \cos(x_1) - x_6 \sin(x_1) \\ (x_5 \sin(x_1) + x_6 \cos(x_1)) \sec(x_2) \\ A_1 \cos(x_2) \sin(x_1) + A_2 x_5 x_6 + A_3 \sigma_1 + A_4 x_5 \sigma_4 - \frac{x_4}{|x_4|} A_5 + A_6 \cos(x_1) \\ B_1 \sin(x_2) + B_2 x_4 x_6 + B_3 \sigma_2 + B_4 x_4 \sigma_4 - \frac{x_5}{|x_5|} B_5 + B_6 \cos(x_2) \\ C_1 x_4 x_5 + C_2 \sigma_3 - \frac{x_6}{|x_6|} C_3 \end{bmatrix}$$

$$\sigma_1 = \omega_2^2 - \omega_4^2, \quad \sigma_2 = \omega_1^2 - \omega_3^2, \quad \sigma_3 = \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2, \quad \sigma_4 = \omega_1 - \omega_2 + \omega_3 - \omega_4$$

Mathematical Model

$$A_1 = \frac{h_{cg} g m_{tot}}{m_{tot} h_{cg}^2 + J_{11}}$$

$$A_4 = \frac{J_R}{m_{tot} h_{cg}^2 + J_{11}}$$

$$B_1 = \frac{h_{cg} g m_{tot}}{m_{tot} h_{cg}^2 + J_{22}}$$

$$B_4 = \frac{-J_R}{m_{tot} h_{cg}^2 + J_{22}}$$

$$A_2 = \frac{2m_{tot} h_{cg}^2 + J_{22} - J_{33}}{m_{tot} h_{cg}^2 + J_{11}}$$

$$A_5 = \frac{m_1 g \mu r_x}{m_{tot} h_{cg}^2 + J_{11}}$$

$$B_2 = \frac{-2m_{tot} h_{cg}^2 - J_{11} + J_{33}}{m_{tot} h_{cg}^2 + J_{22}}$$

$$B_5 = \frac{m_2 g \mu r_y}{m_{tot} h_{cg}^2 + J_{22}}$$

$$C_1 = \frac{J_{11} - J_{22}}{J_{33}} \quad C_2 = \frac{d}{J_{33}} \quad C_3 = \frac{m_3 g \mu r_z}{J_{33}}$$

$$A_3 = \frac{bd_{cg}}{m_{tot} h_{cg}^2 + J_{11}}$$

$$A_6 = \frac{m_{tot} x_{cg}}{m_{tot} h_{cg}^2 + J_{11}}$$

$$B_3 = \frac{bd_{cg}}{m_{tot} h_{cg}^2 + J_{22}}$$

$$B_6 = \frac{m_{tot} y_{cg}}{m_{tot} h_{cg}^2 + J_{22}}$$

Roll Space State

$$\mathbf{A}_{roll} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ A_1 \cos(x_1) & 0 \end{bmatrix}$$

$$\mathbf{B}_{roll} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} \\ \frac{\partial f_4}{\partial u_1} \end{bmatrix} = \begin{bmatrix} 0 \\ A_3 \end{bmatrix}$$

Pitch Space State

$$\mathbf{A}_{pitch} = \begin{bmatrix} \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_5} \\ \frac{\partial f_5}{\partial x_2} & \frac{\partial f_5}{\partial x_5} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ B_1 \cos(x_1) & 0 \end{bmatrix}$$

$$\mathbf{B}_{pitch} = \begin{bmatrix} \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_5}{\partial u_2} \end{bmatrix} = \begin{bmatrix} 0 \\ B_3 \end{bmatrix}$$

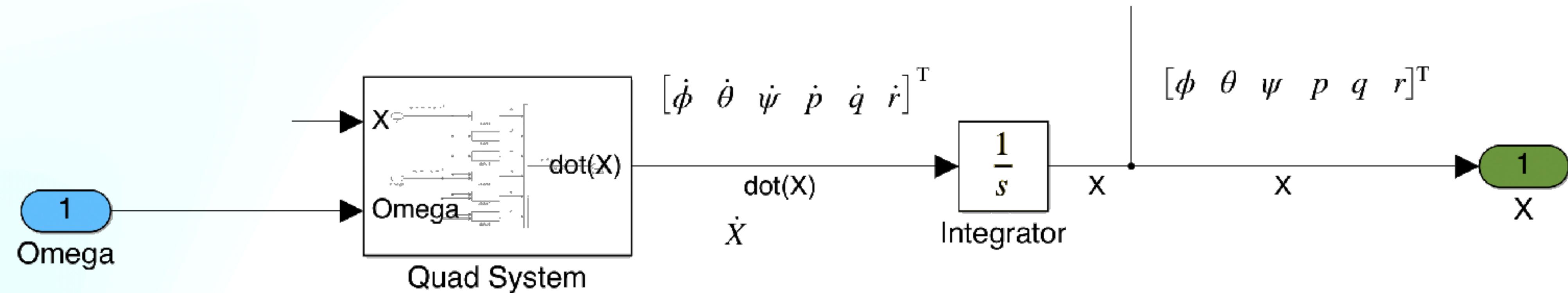
Yaw Space State

$$A_{yaw} = \begin{bmatrix} \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_6} \\ \frac{\partial f_6}{\partial x_3} & \frac{\partial f_6}{\partial x_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

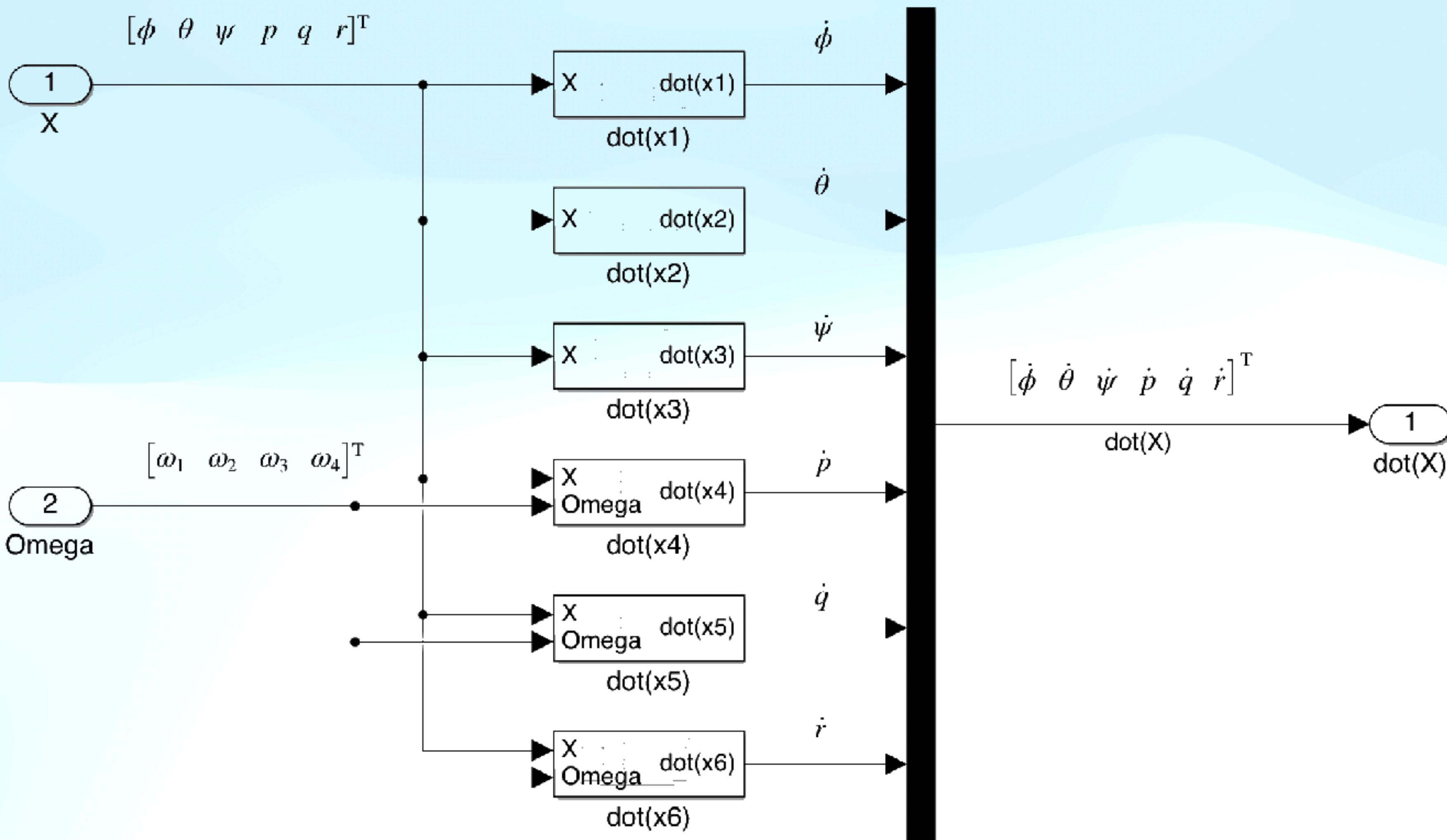
$$B_{yaw} = \begin{bmatrix} \frac{\partial f_3}{\partial u_3} \\ \frac{\partial f_6}{\partial u_3} \end{bmatrix} = \begin{bmatrix} 0 \\ C_2 \end{bmatrix}$$

Simulation

Stand Simulation in MATLAB Simulink



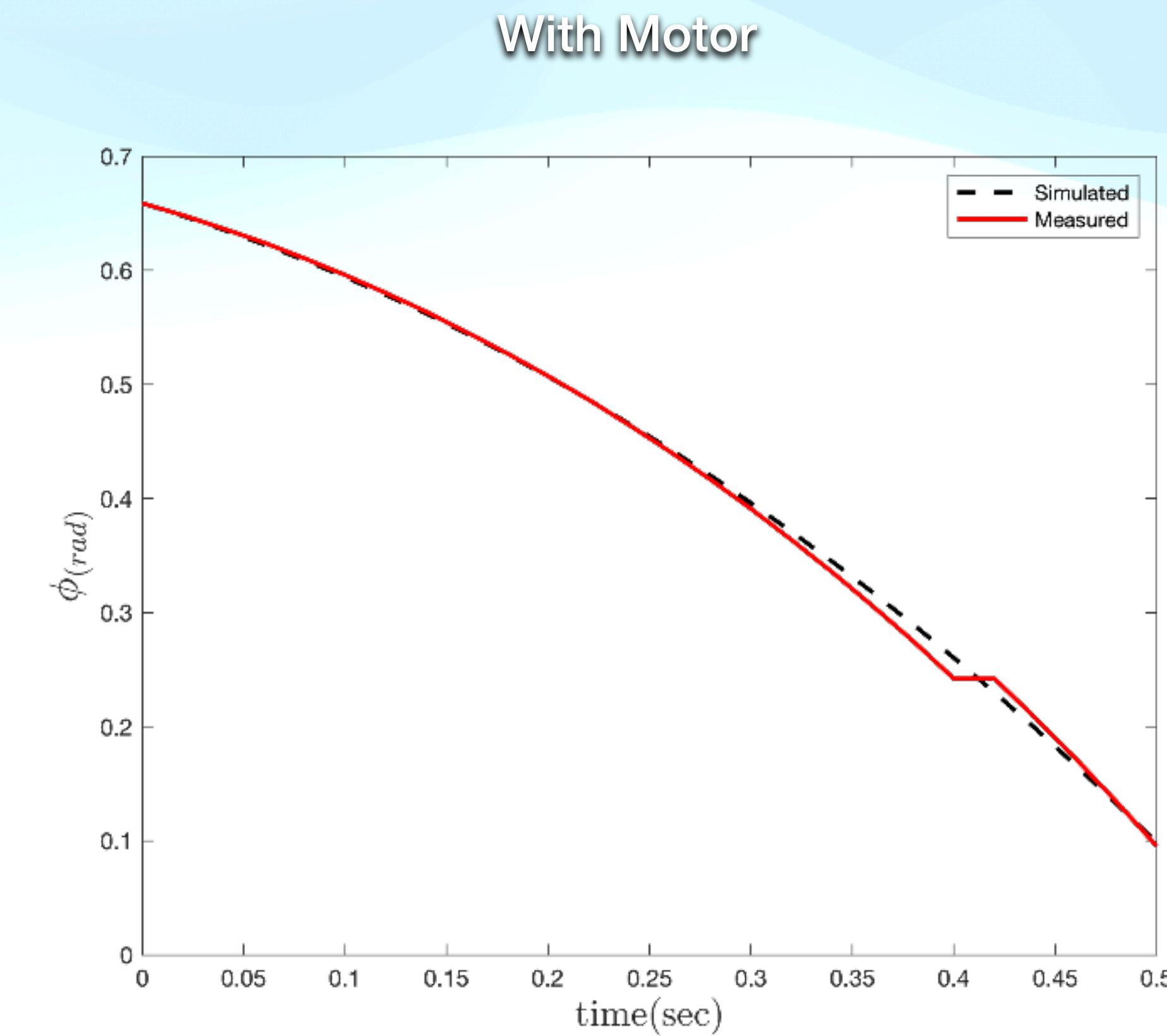
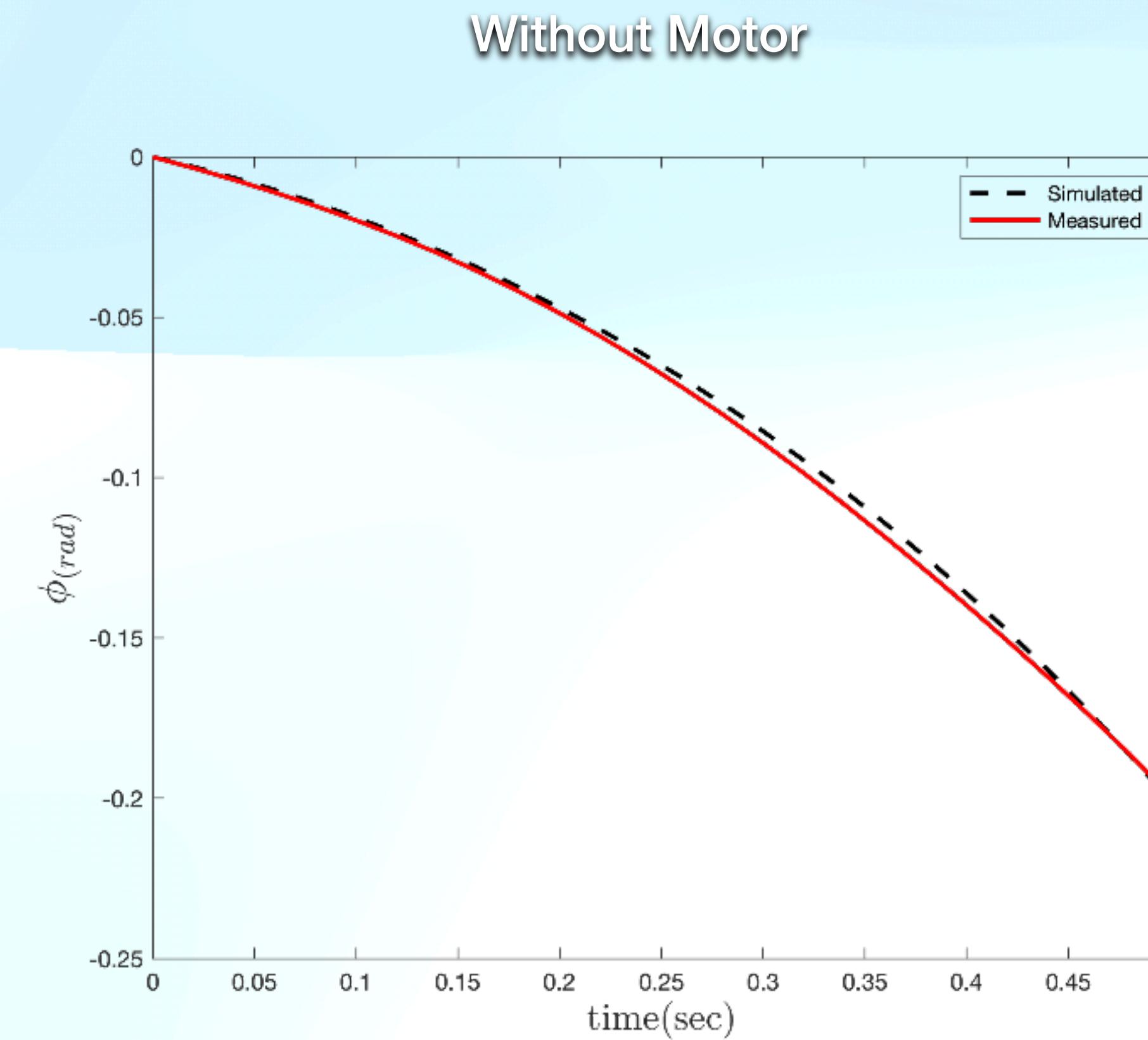
Simulation



Simulation

Parameter Estimation

Roll

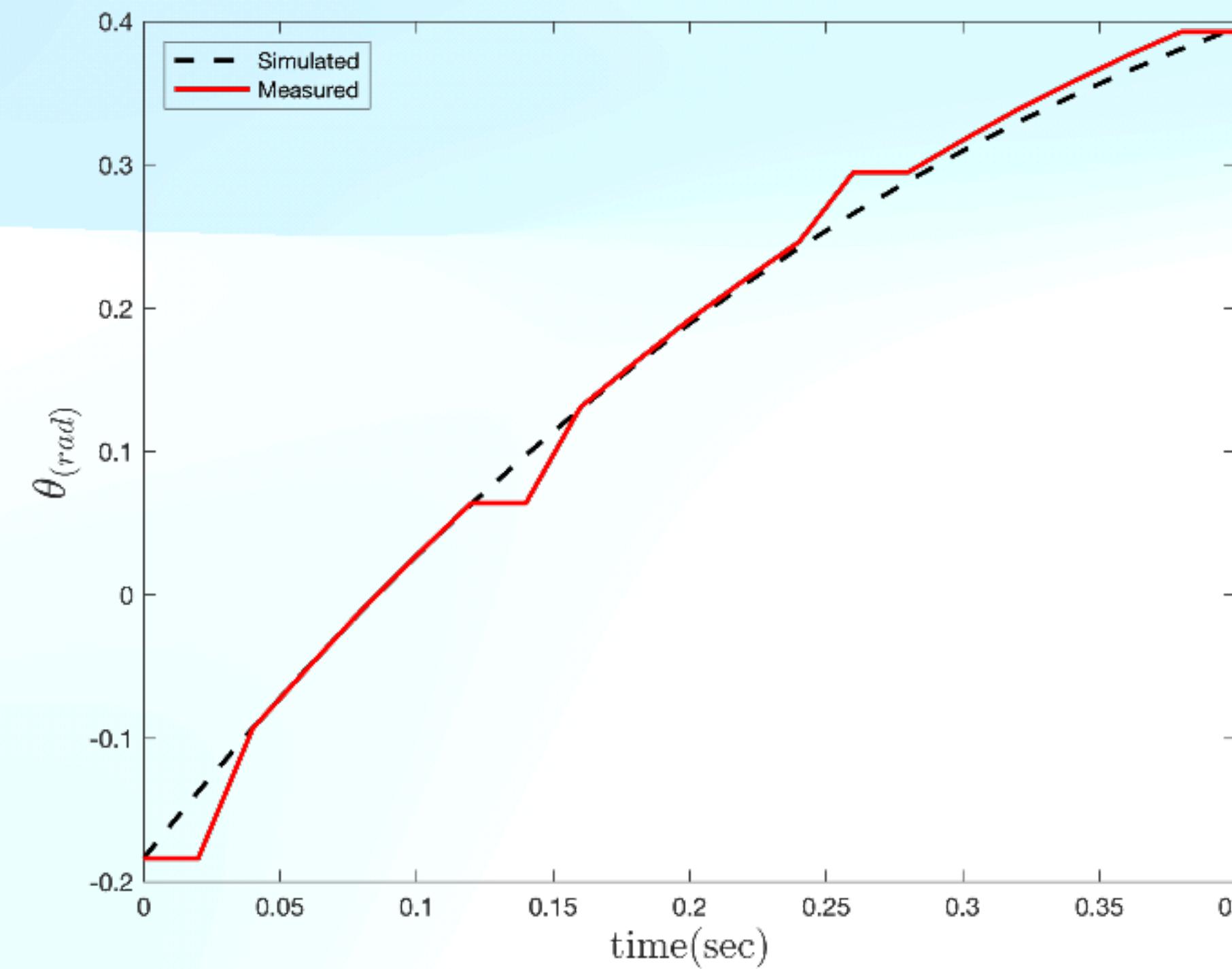


Simulation

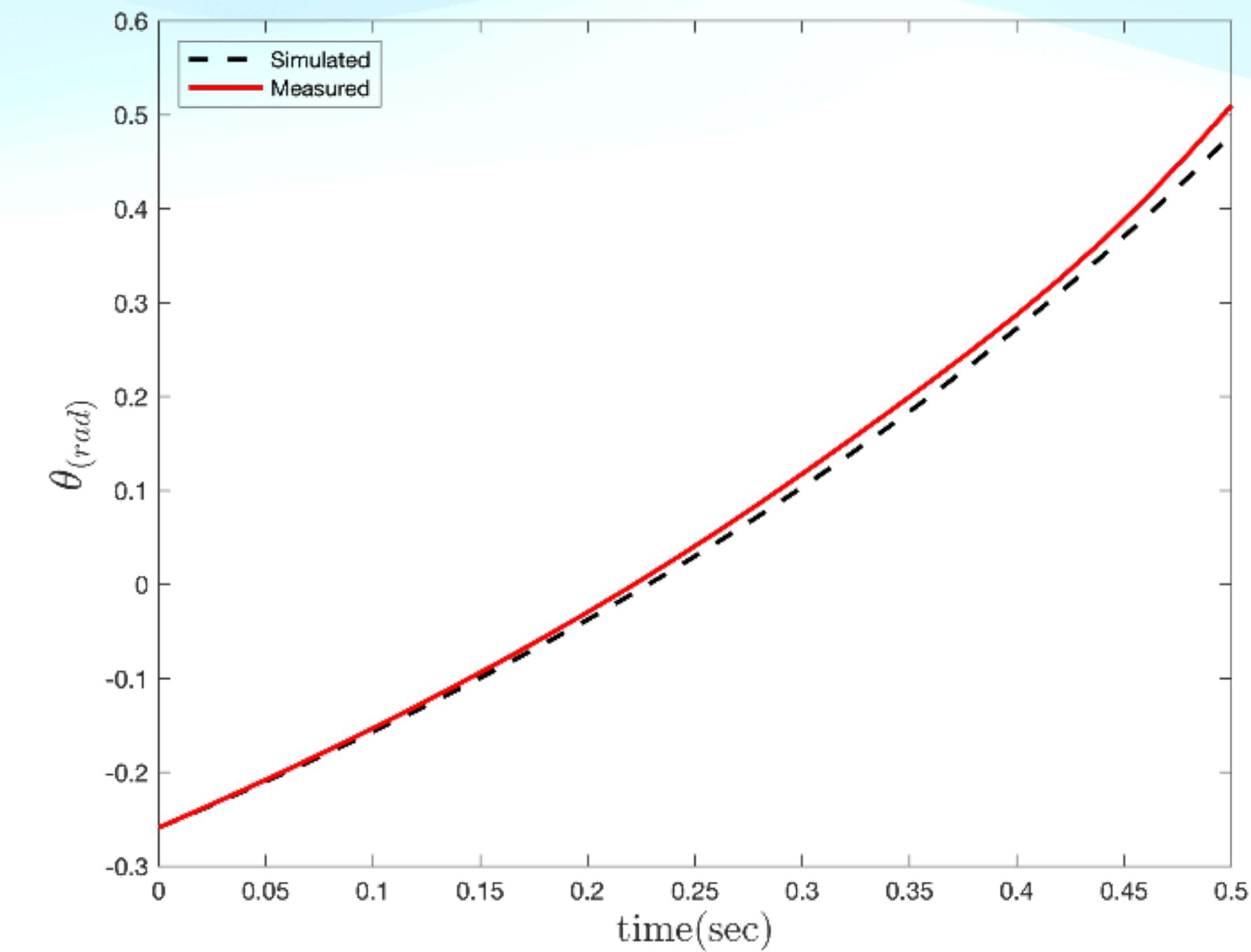
Parameter Estimation

Pitch

Without Motor



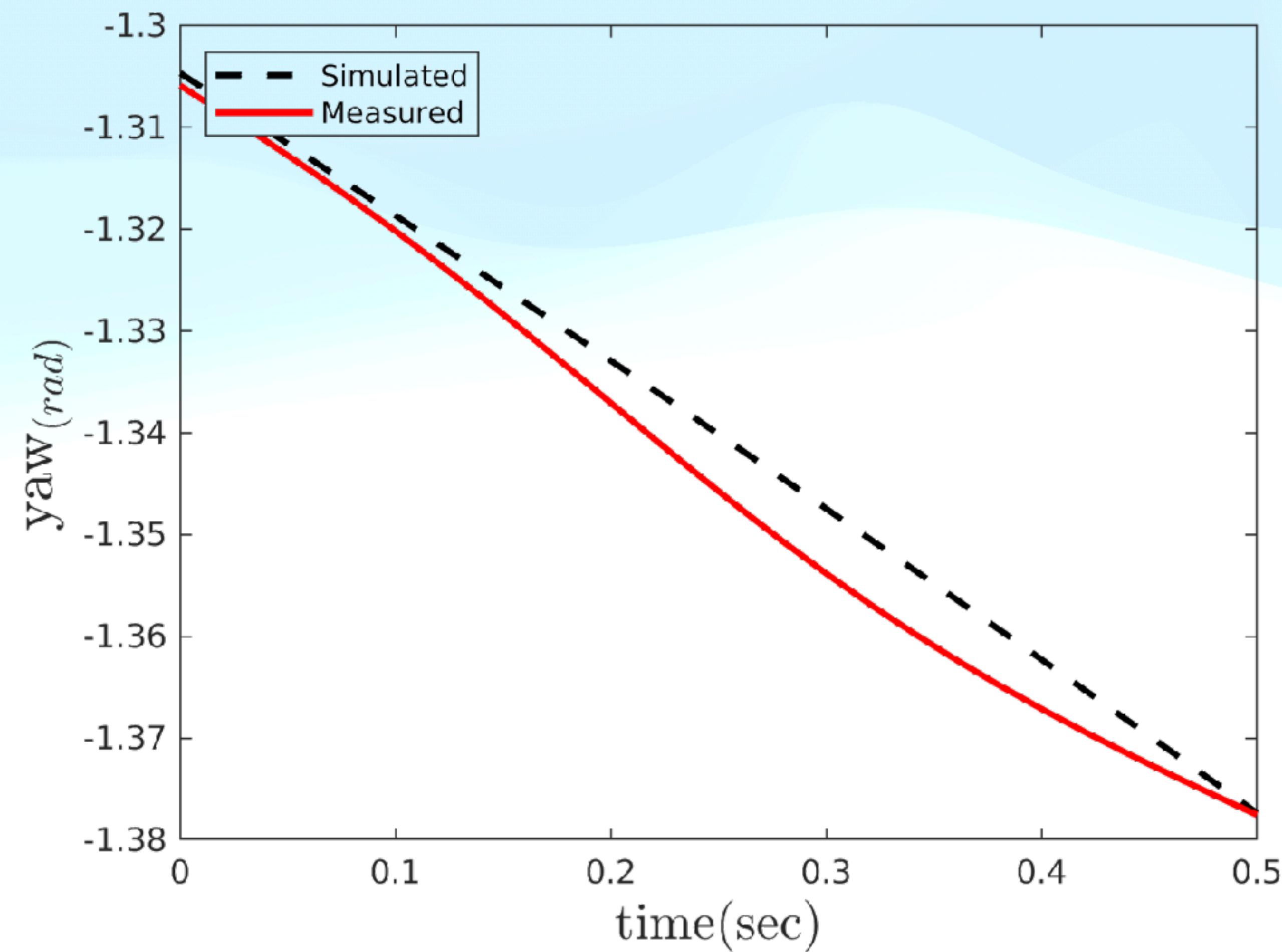
With Motor



Simulation

Parameter Estimation

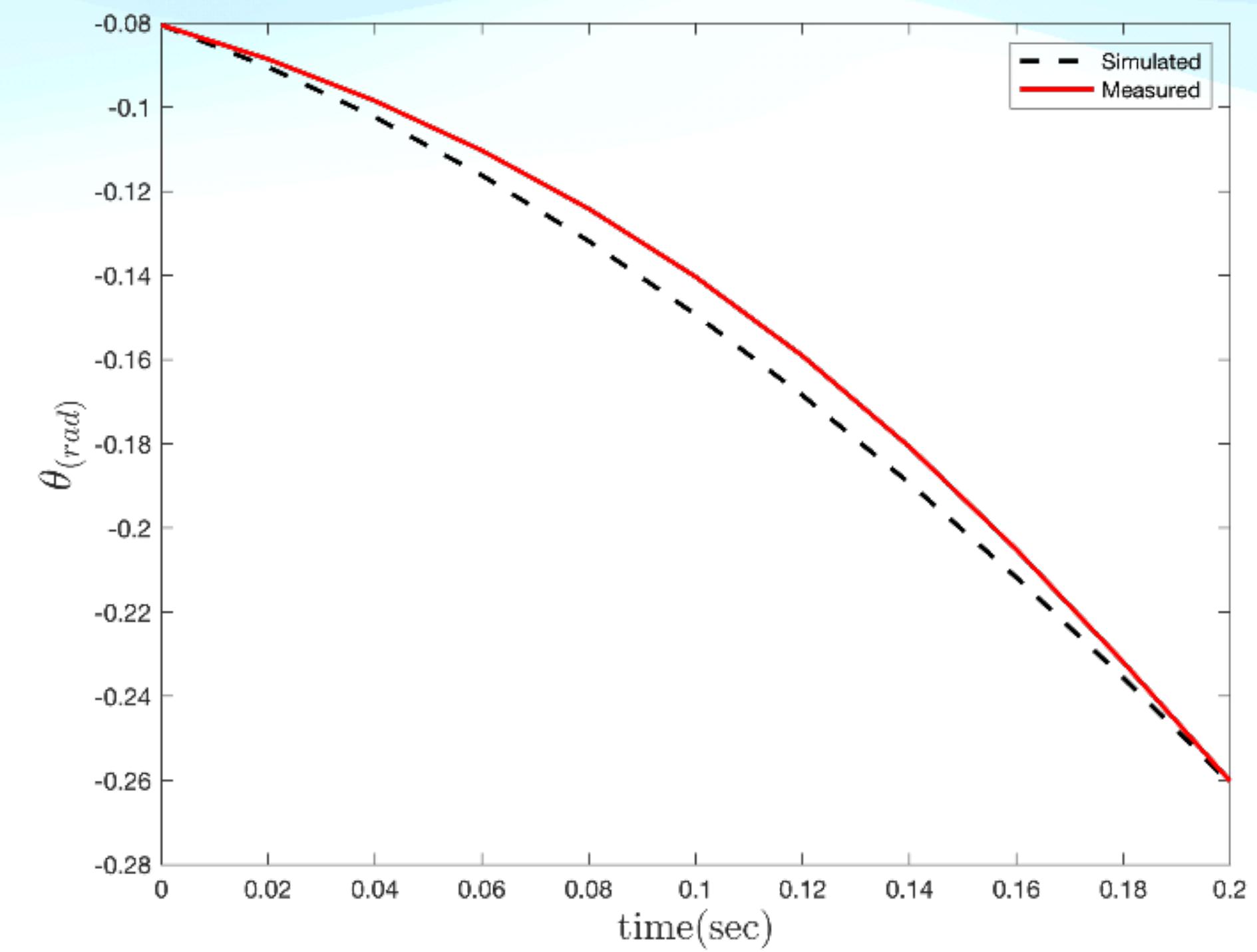
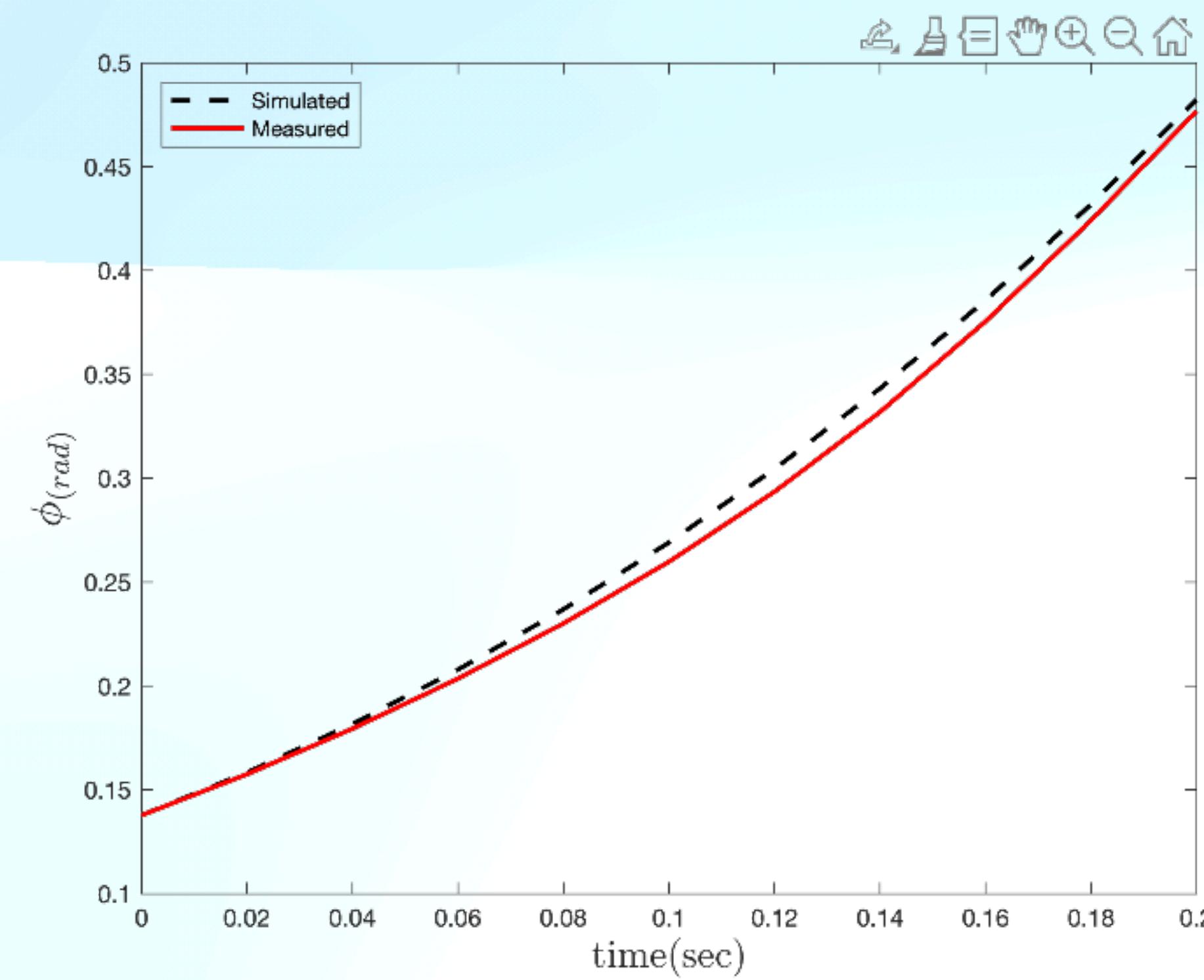
Yaw



Simulation

Parameter Estimation

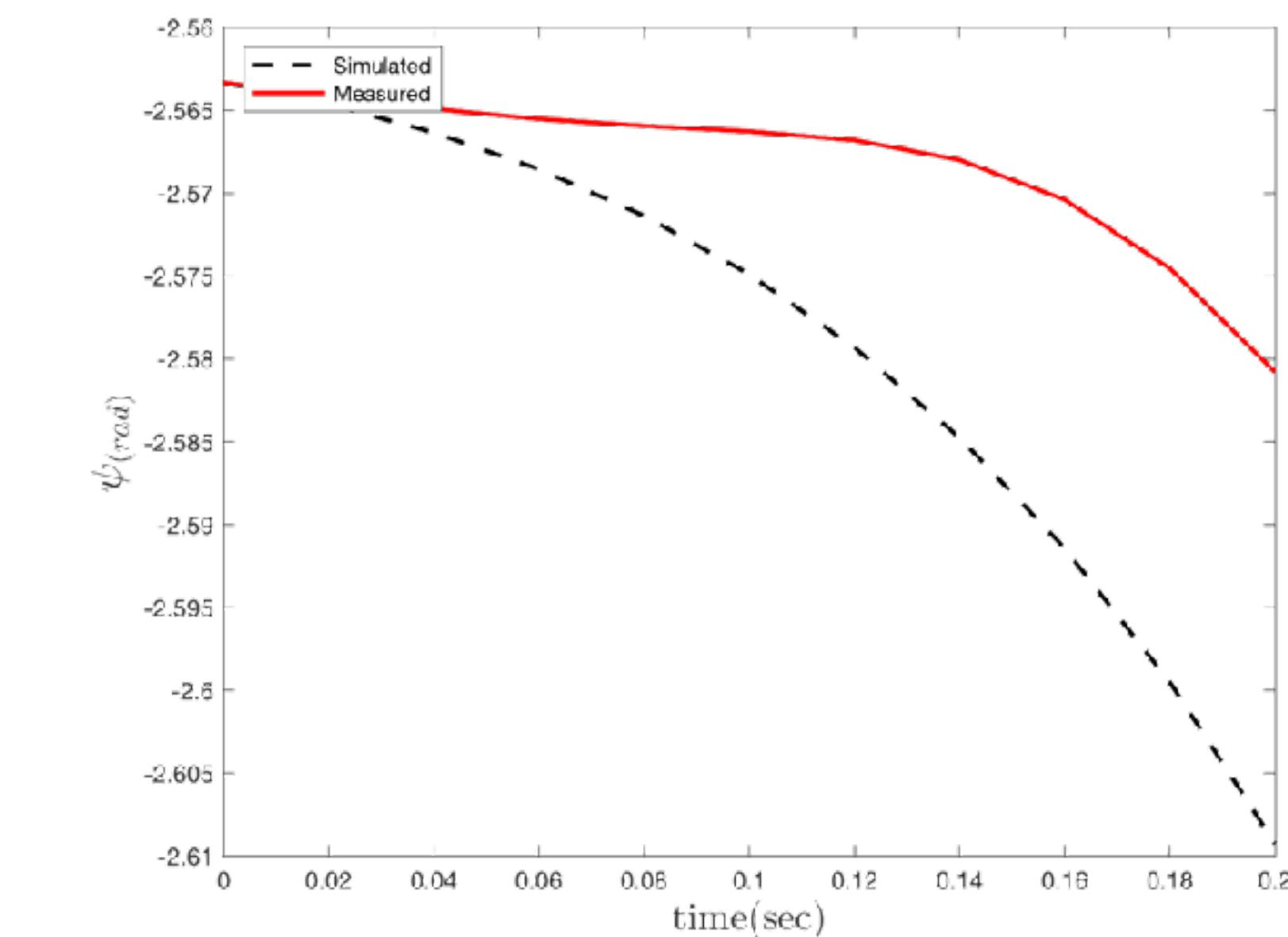
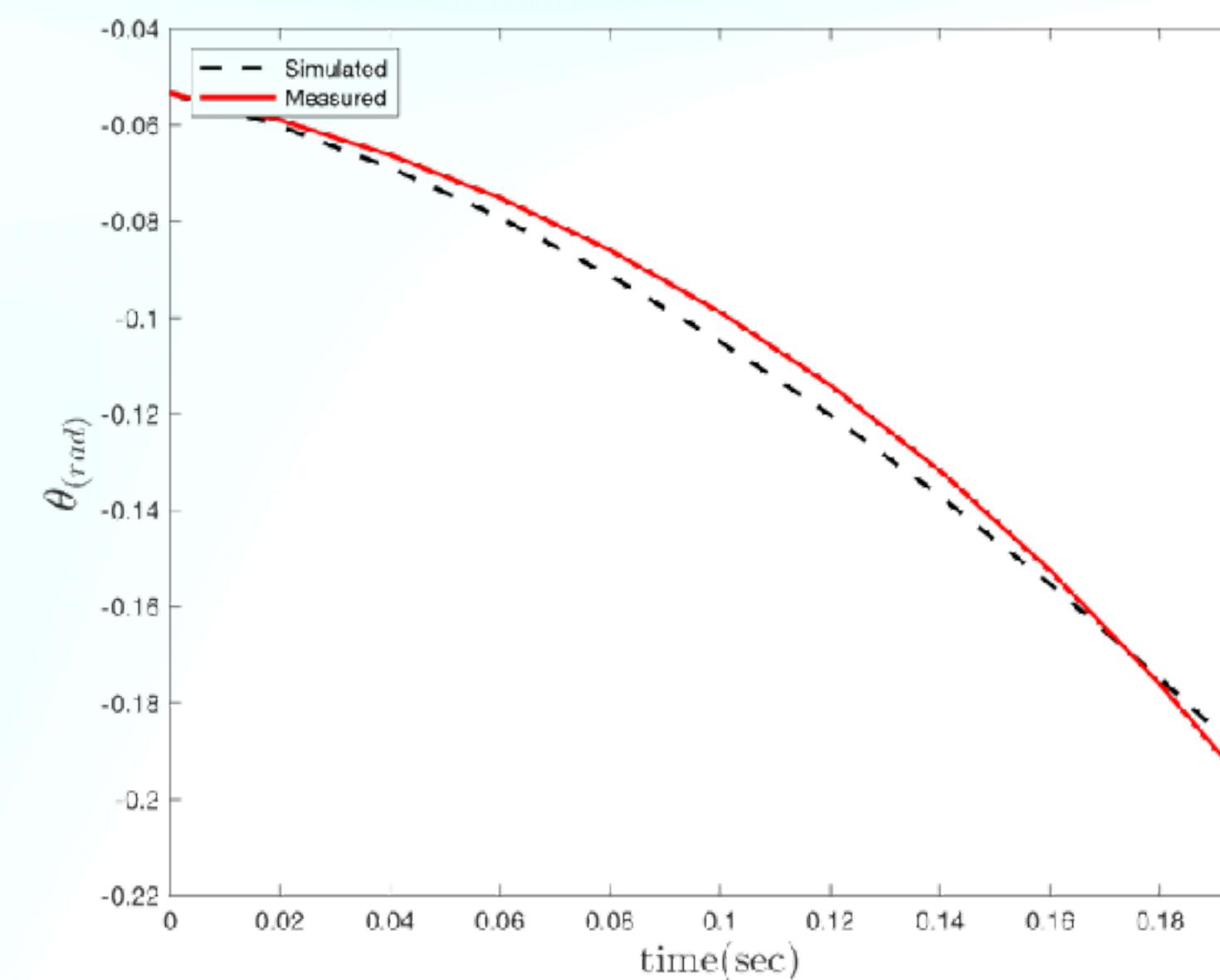
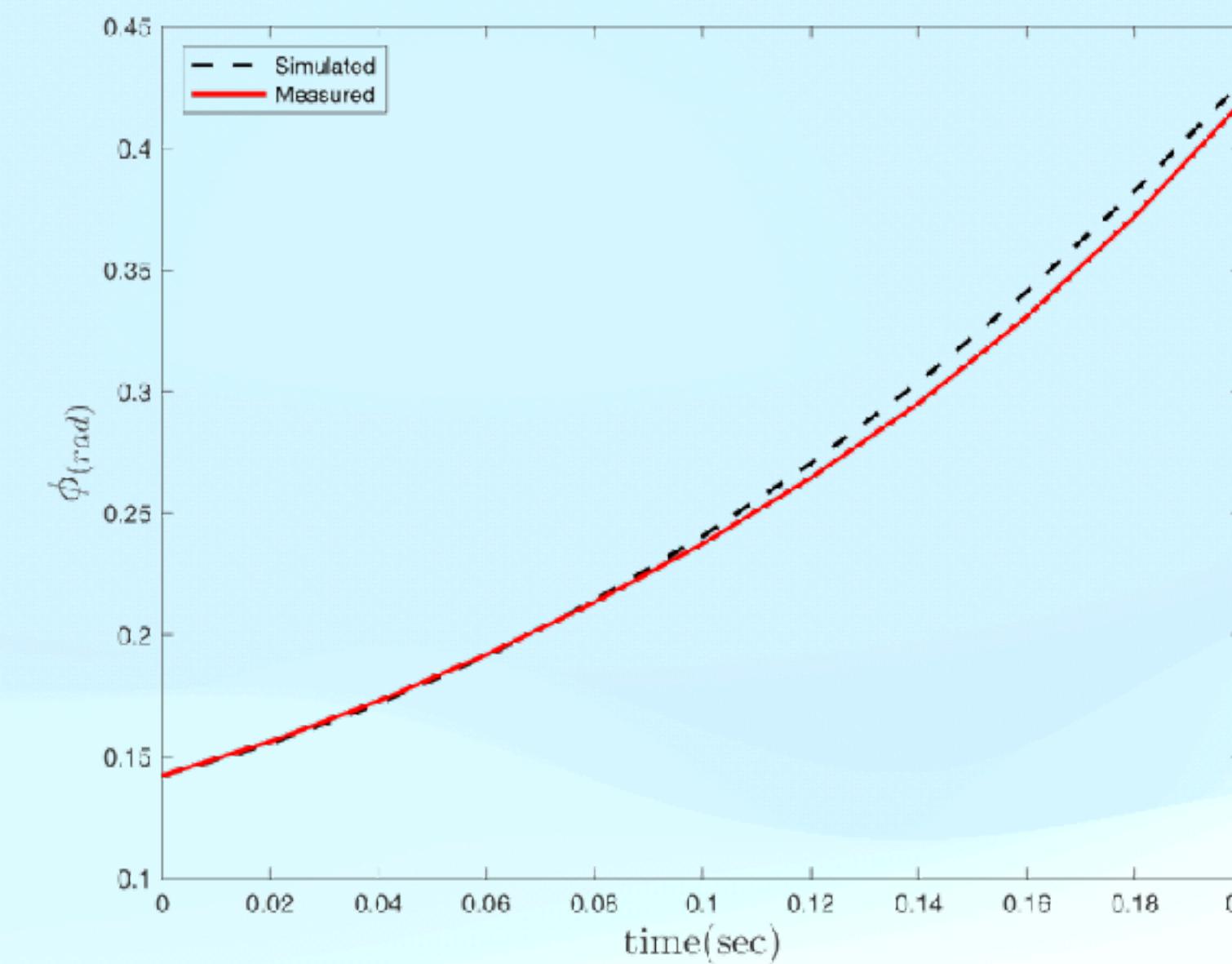
Roll-Pitch



Simulation

Parameter Estimation

3 Degree Of Freedom



Controller Design

LQR

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

$$\mathbf{u}_i(t) = -\mathbf{K}_{LQR}\mathbf{x}(t)$$

$$J(\mathbf{u}) = \int_0^T (\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)) dt$$

$$\mathbf{K}_{LQR} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$$

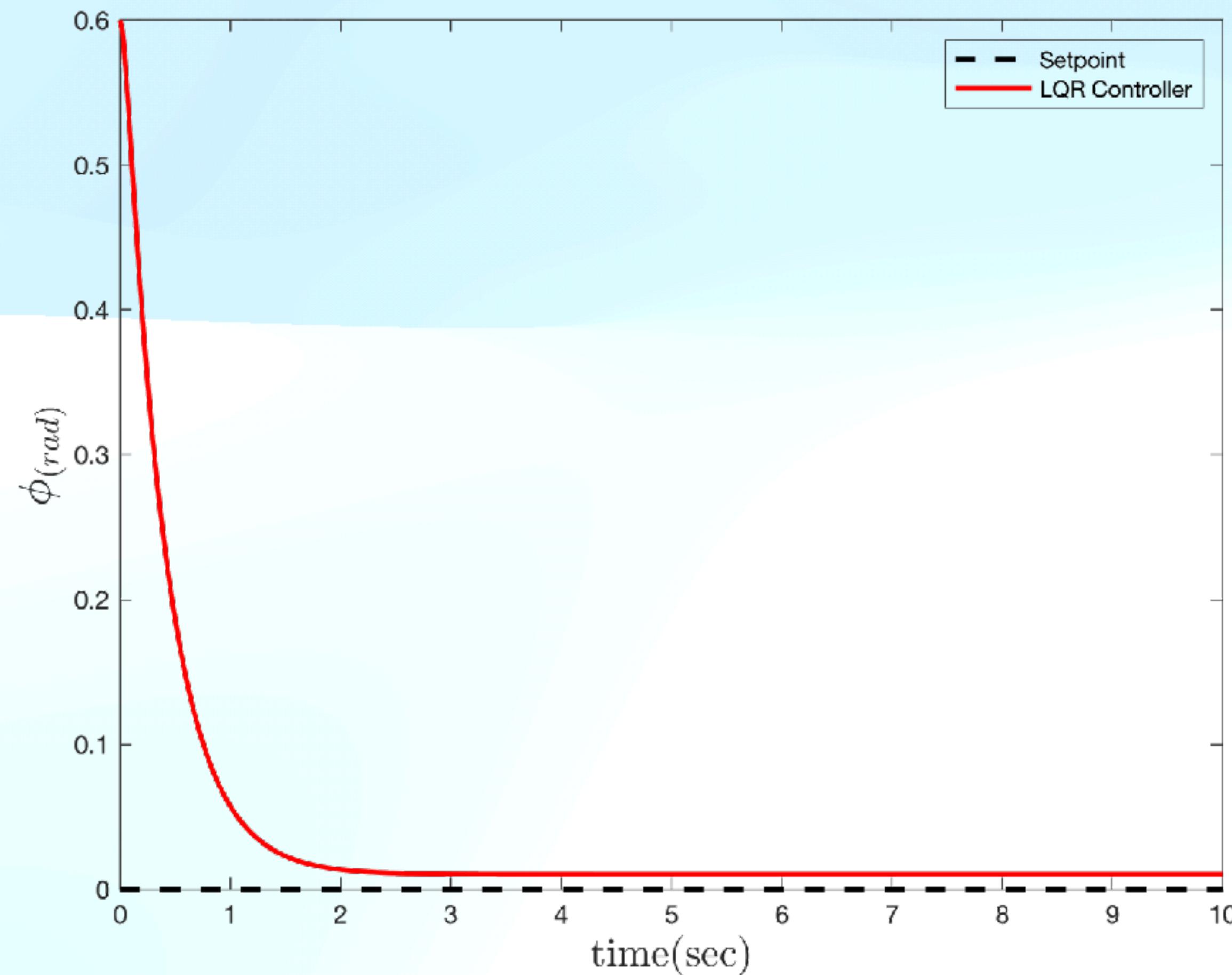
$$\dot{\mathbf{P}}(t) = \mathbf{A}^T\mathbf{P}(t) + \mathbf{P}(t)\mathbf{A} - \mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t) + \mathbf{Q}$$

Weighting matrices optimized using TCACS.

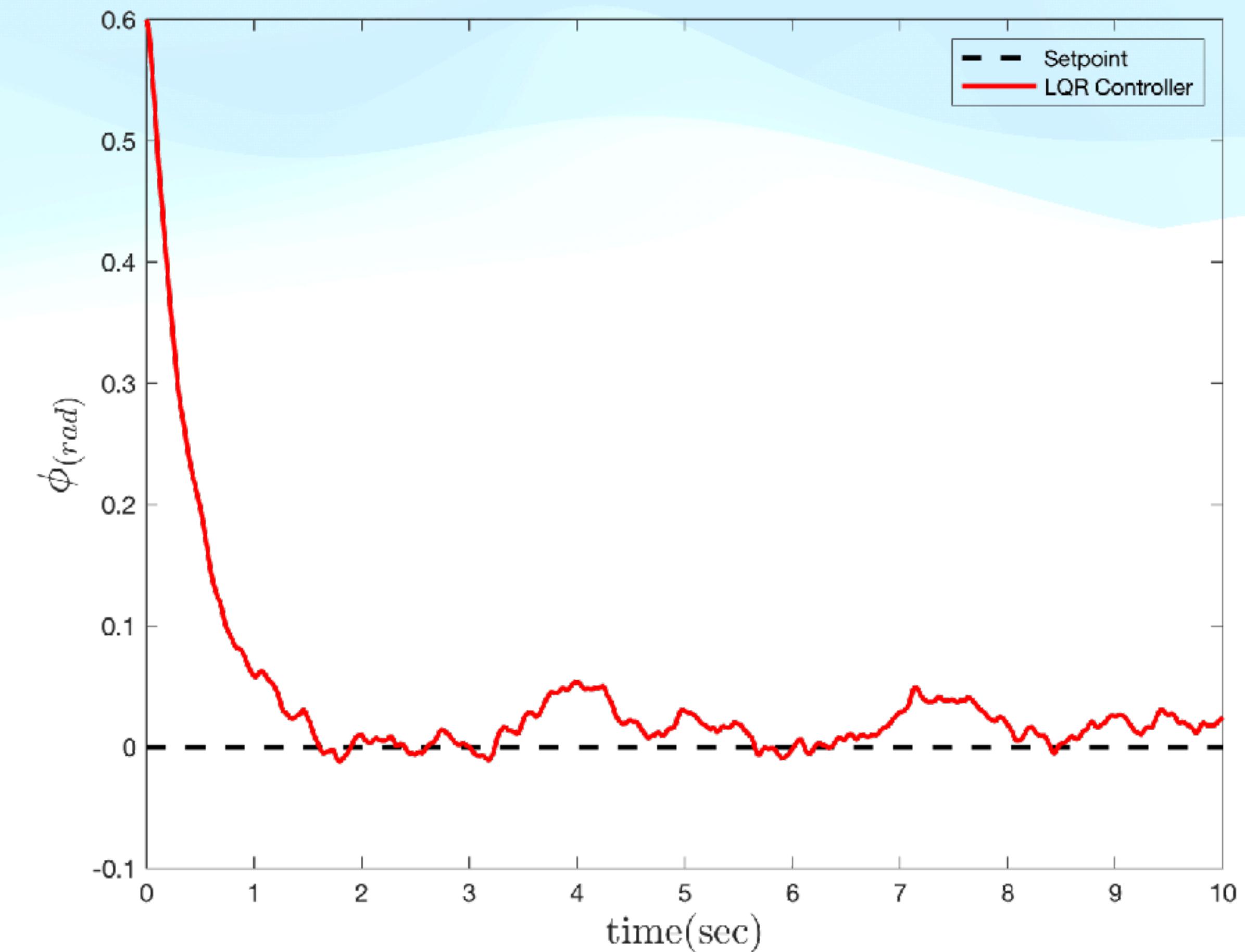
Controller Design

LQR Controller for Roll

Without noise



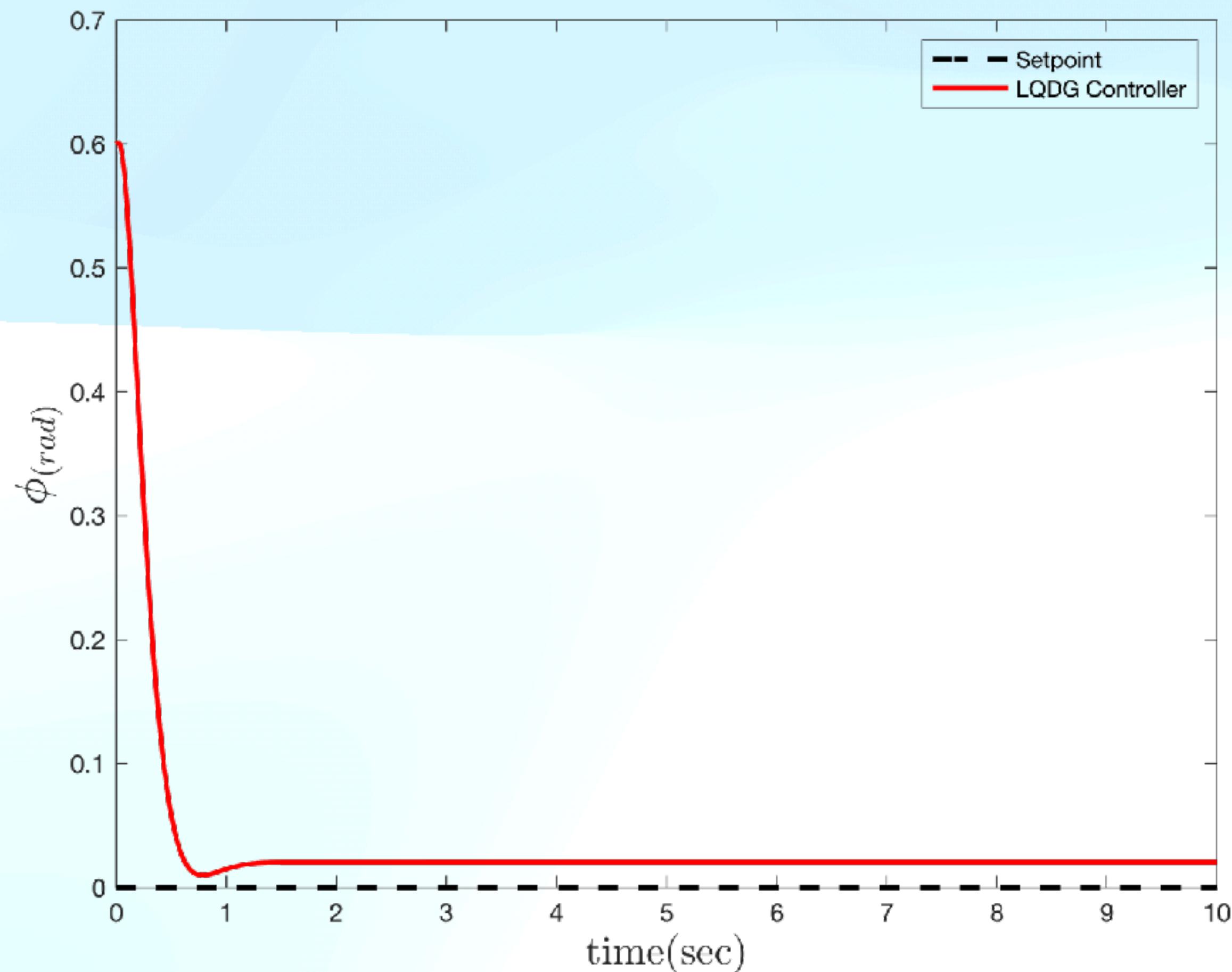
With noise



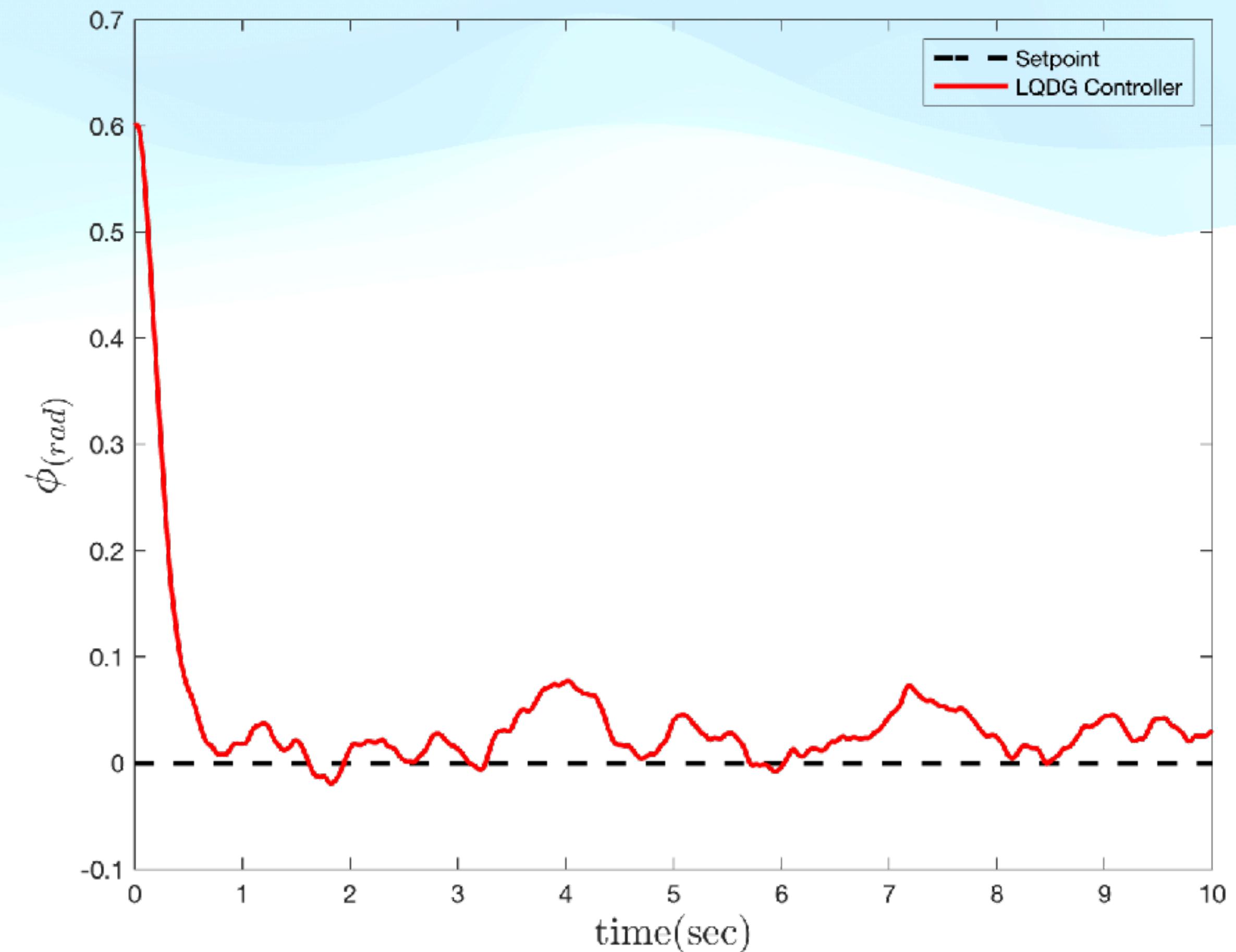
Controller Design

LQDG Controller Roll

Without noise



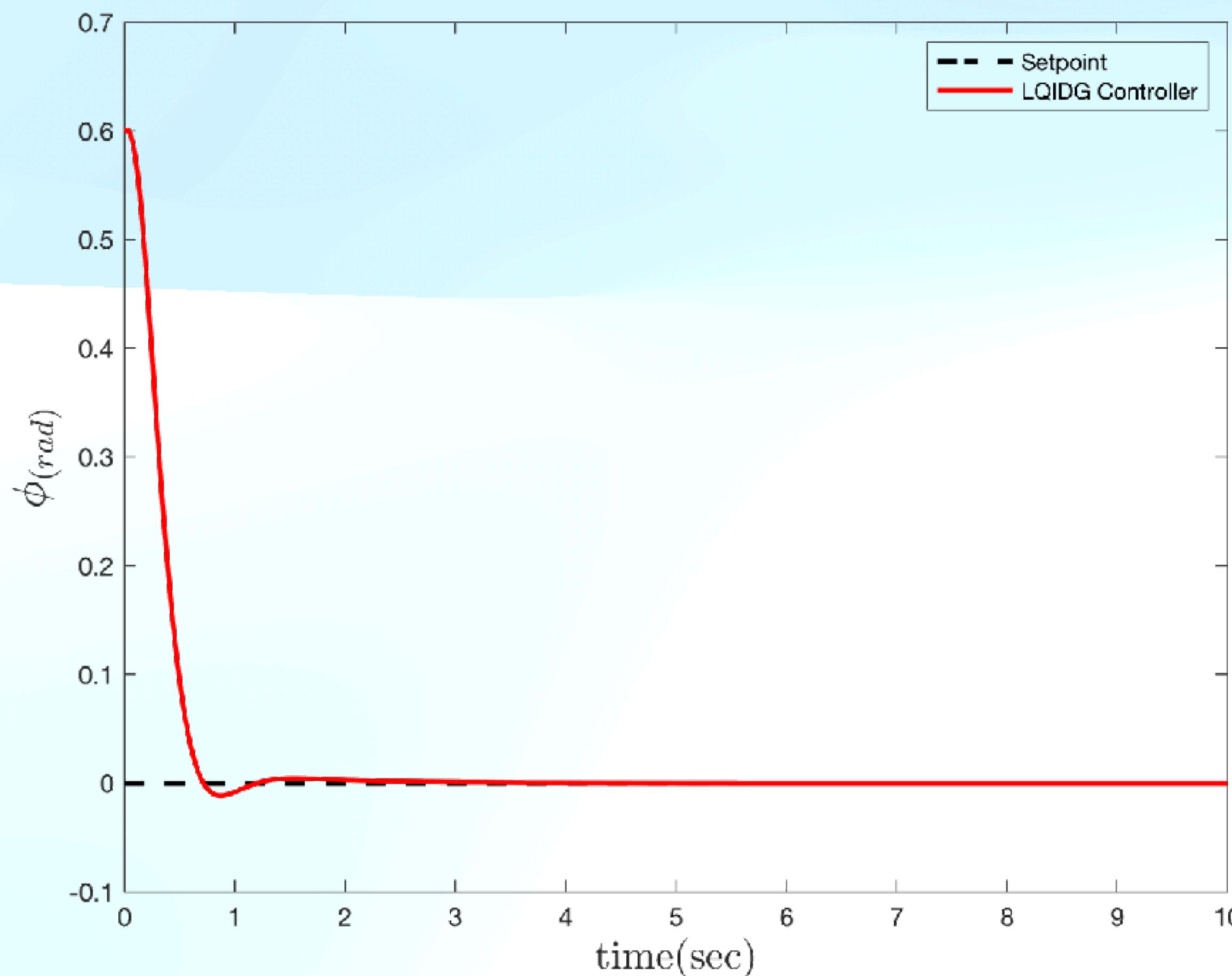
With noise



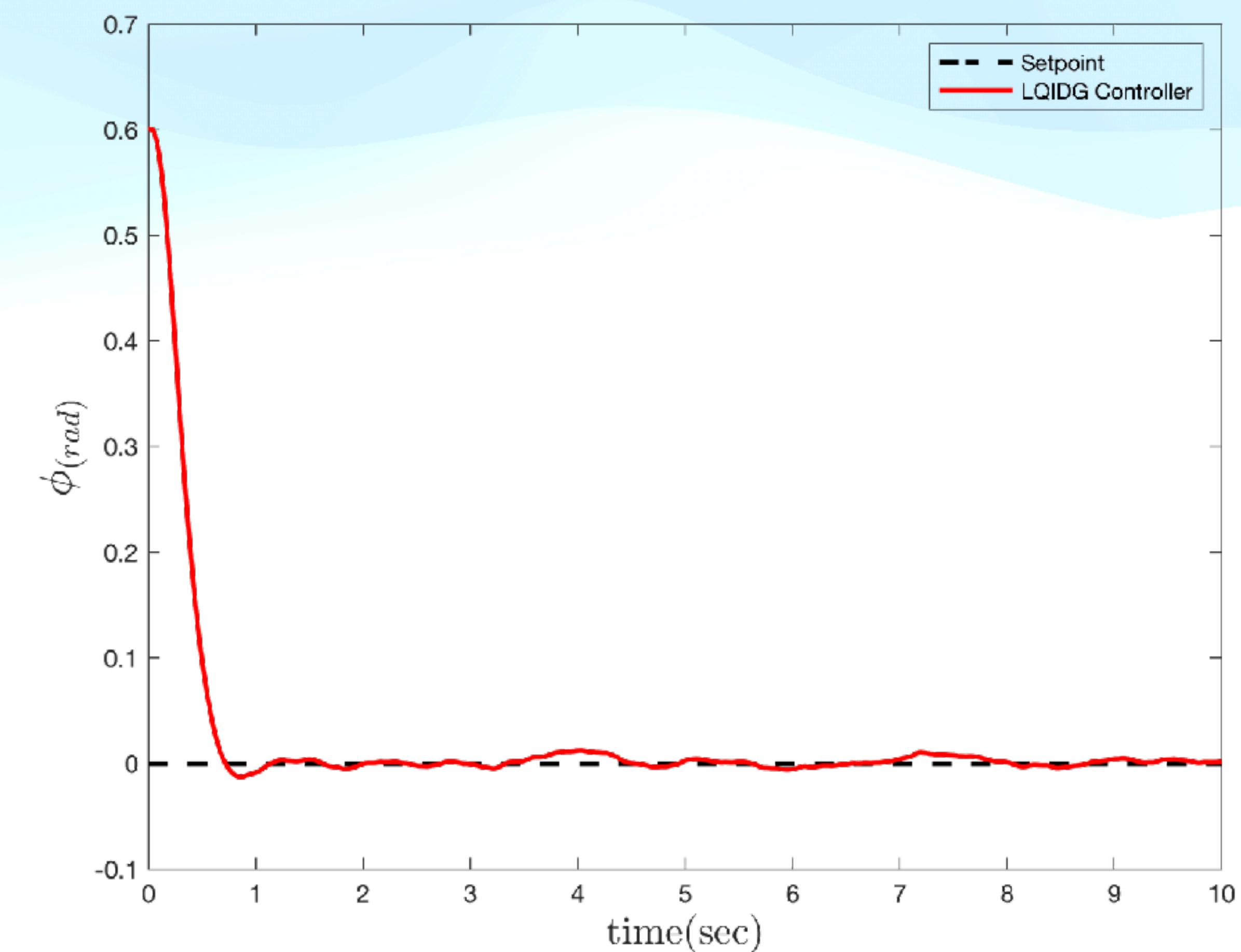
Controller Design

LQIDG Controller for Roll

Without noise

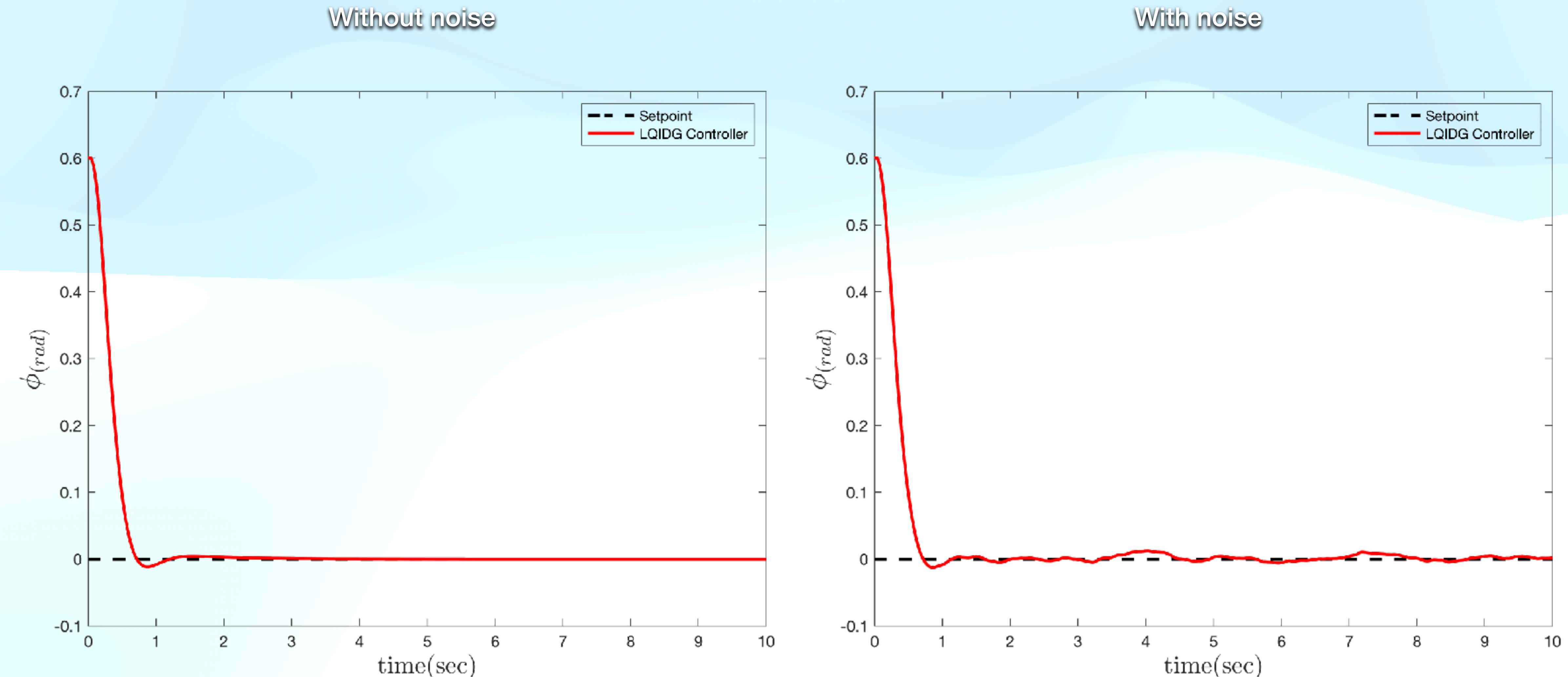


With noise



Controller Design

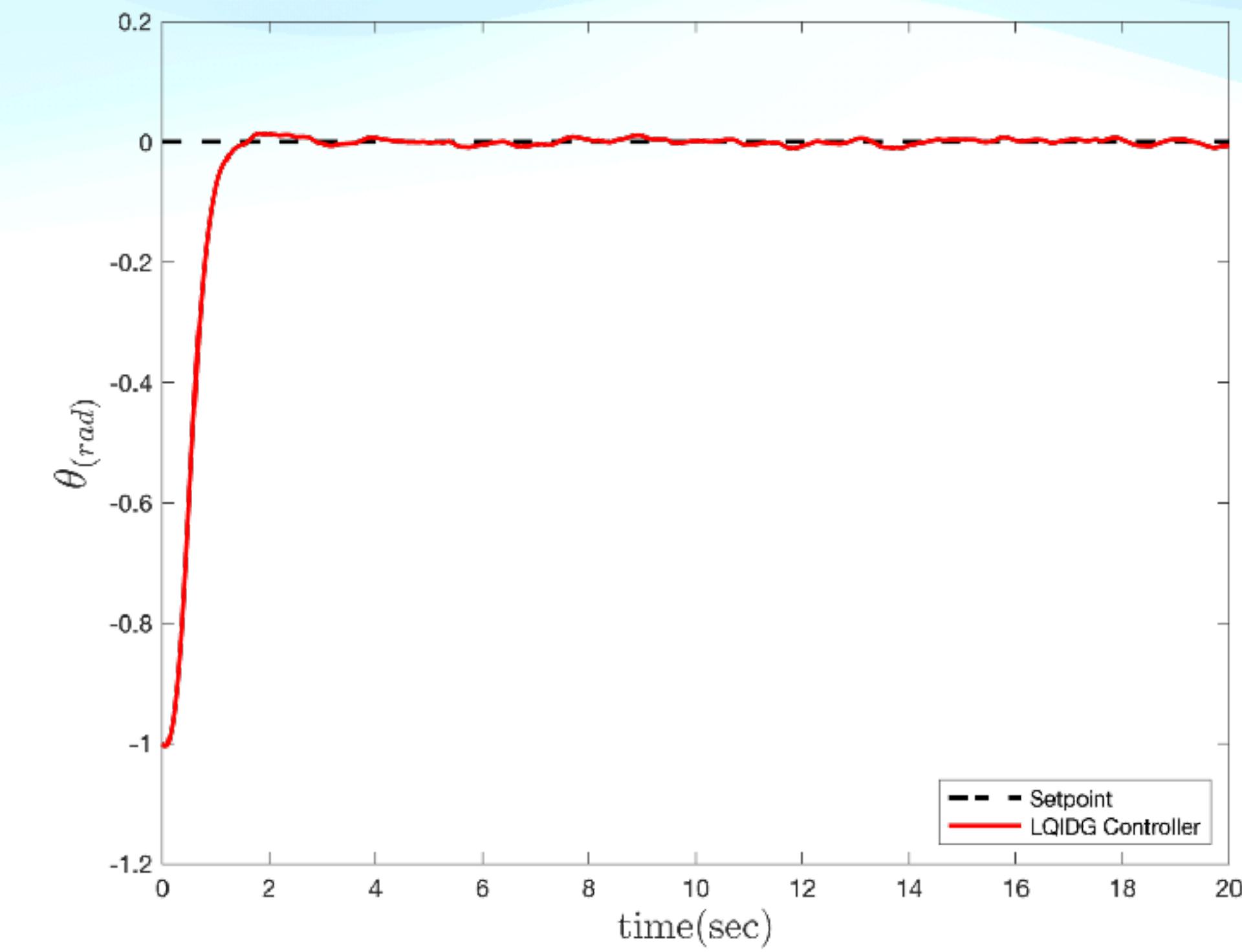
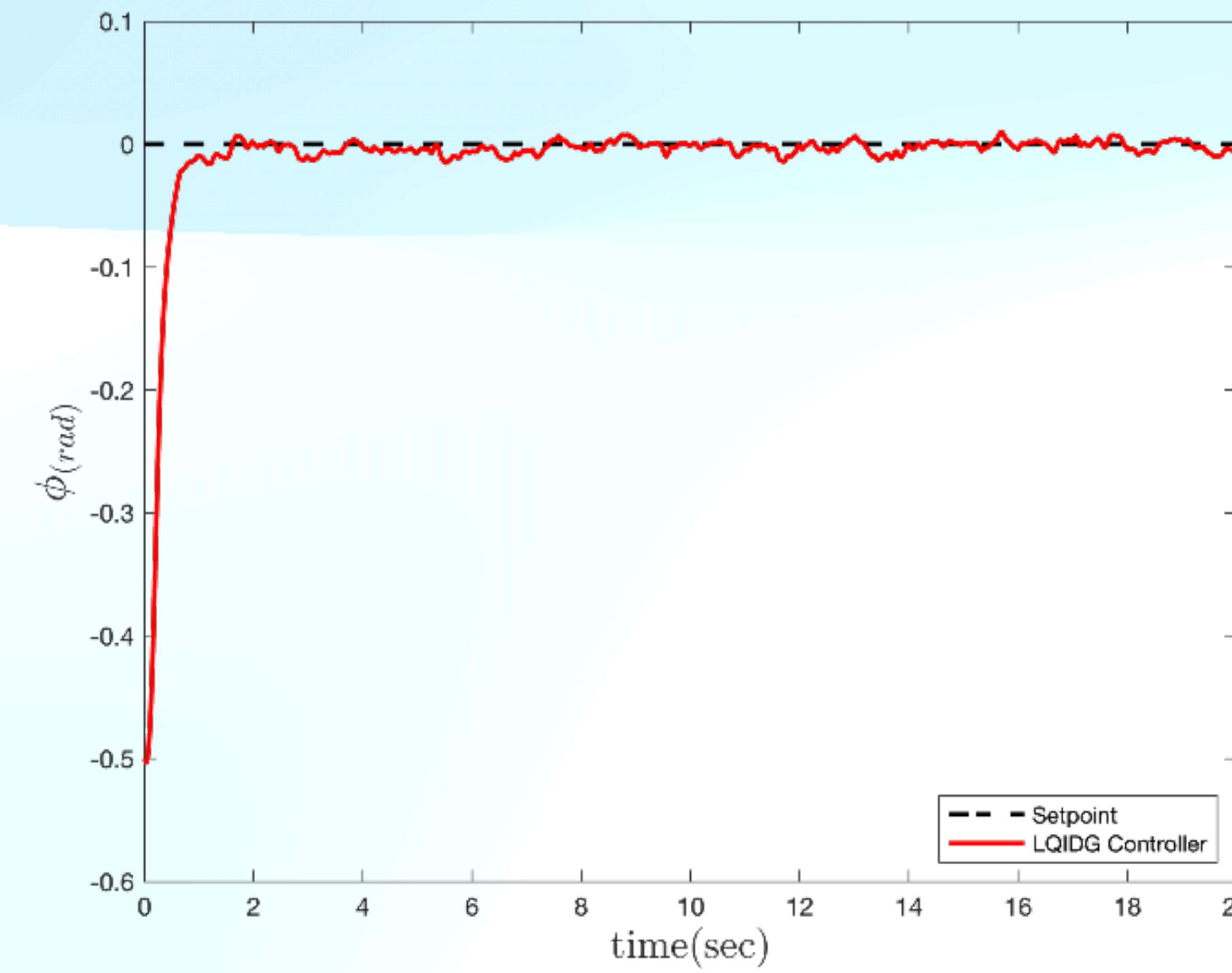
LQIDG Controller for Roll



Controller Design

LQIDG Controller

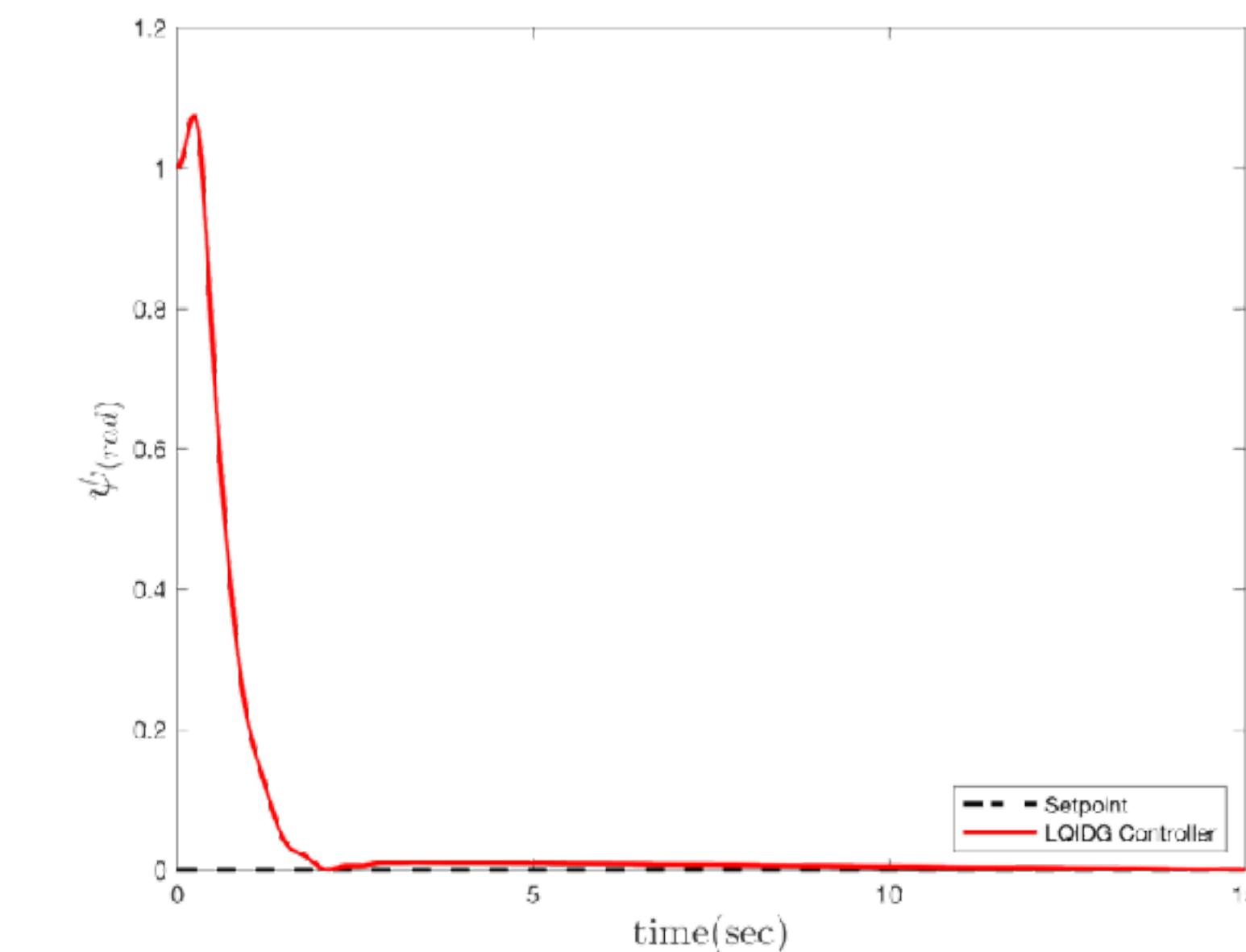
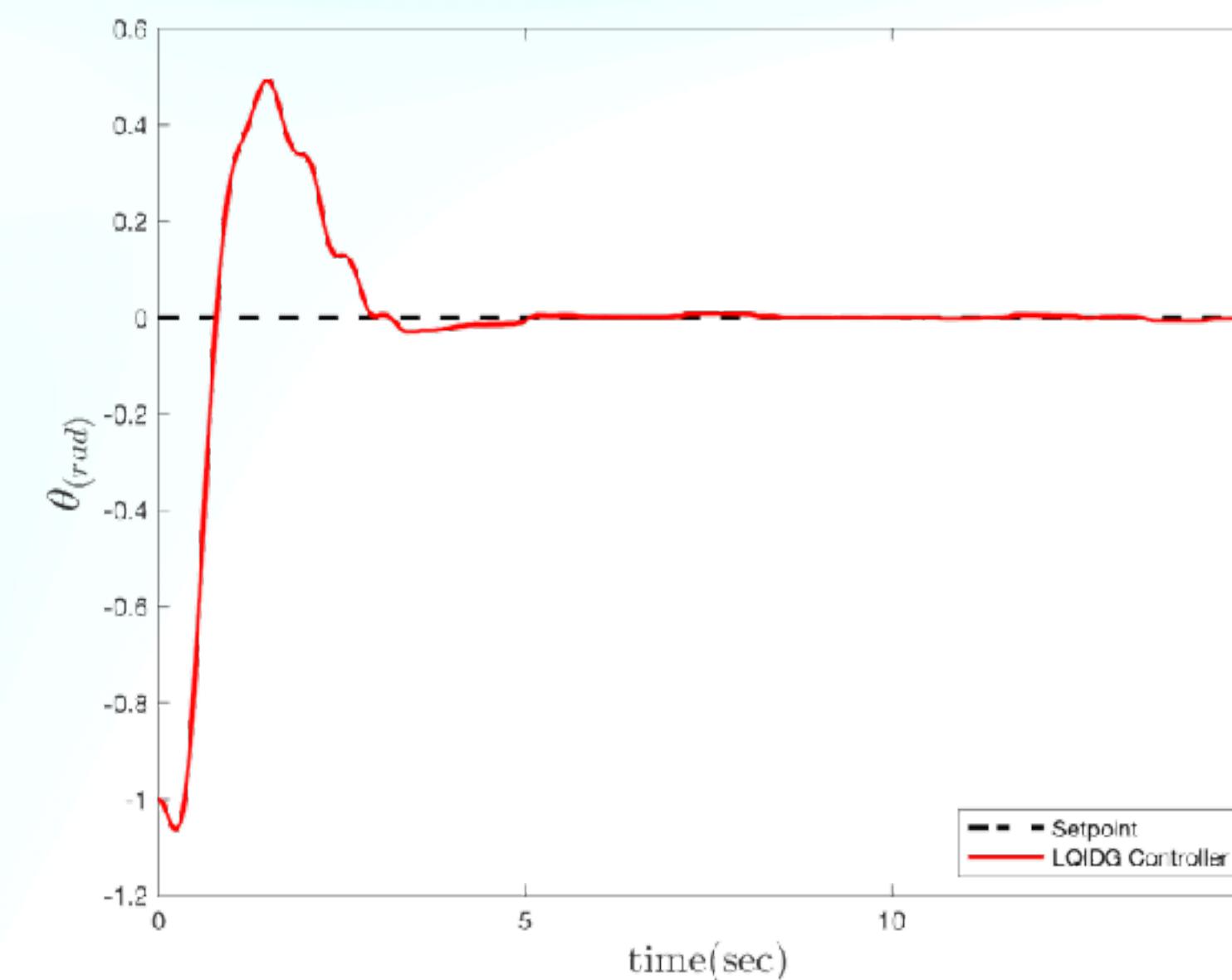
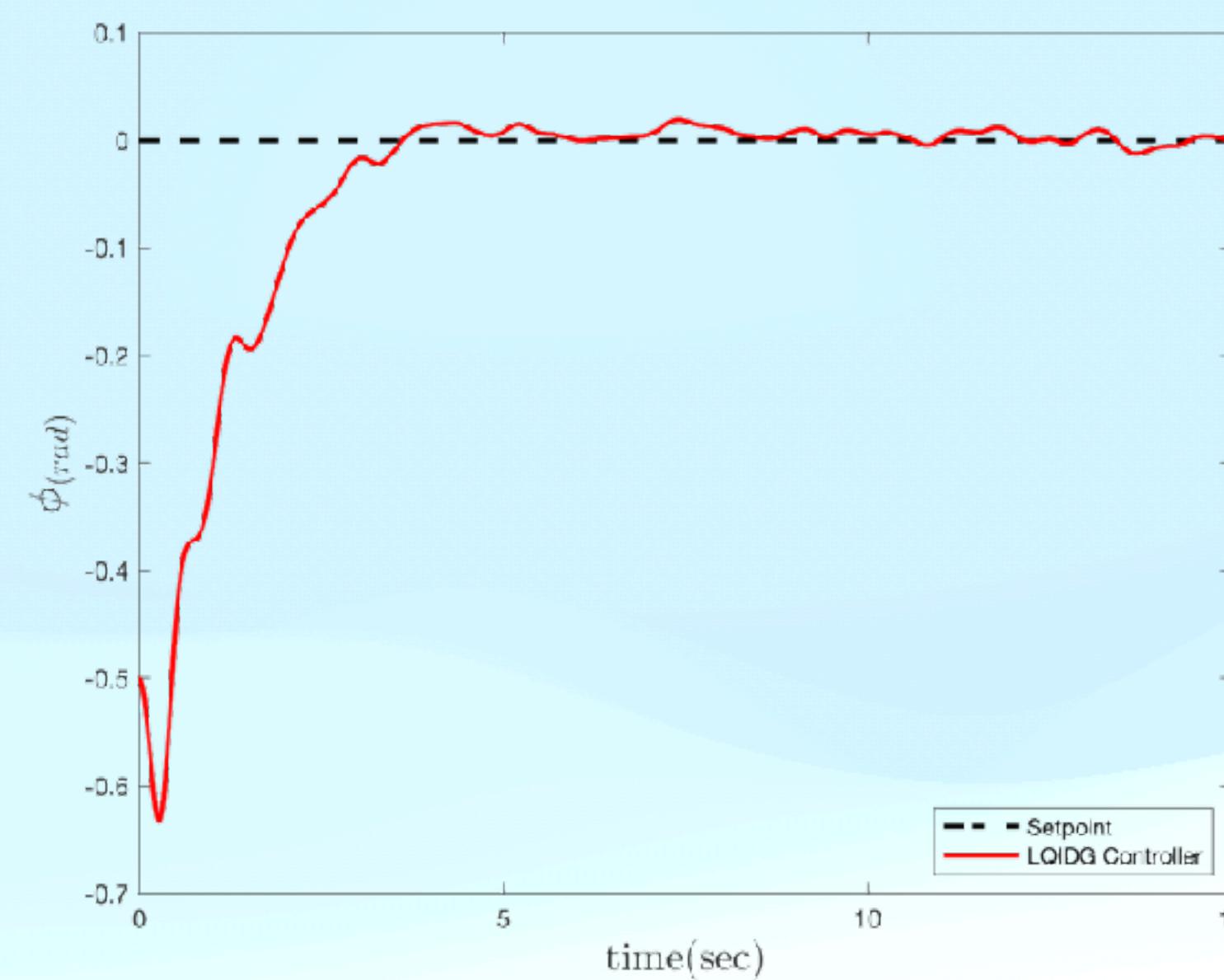
Roll-Pitch with noise



Controller Design

LQIDG Controller

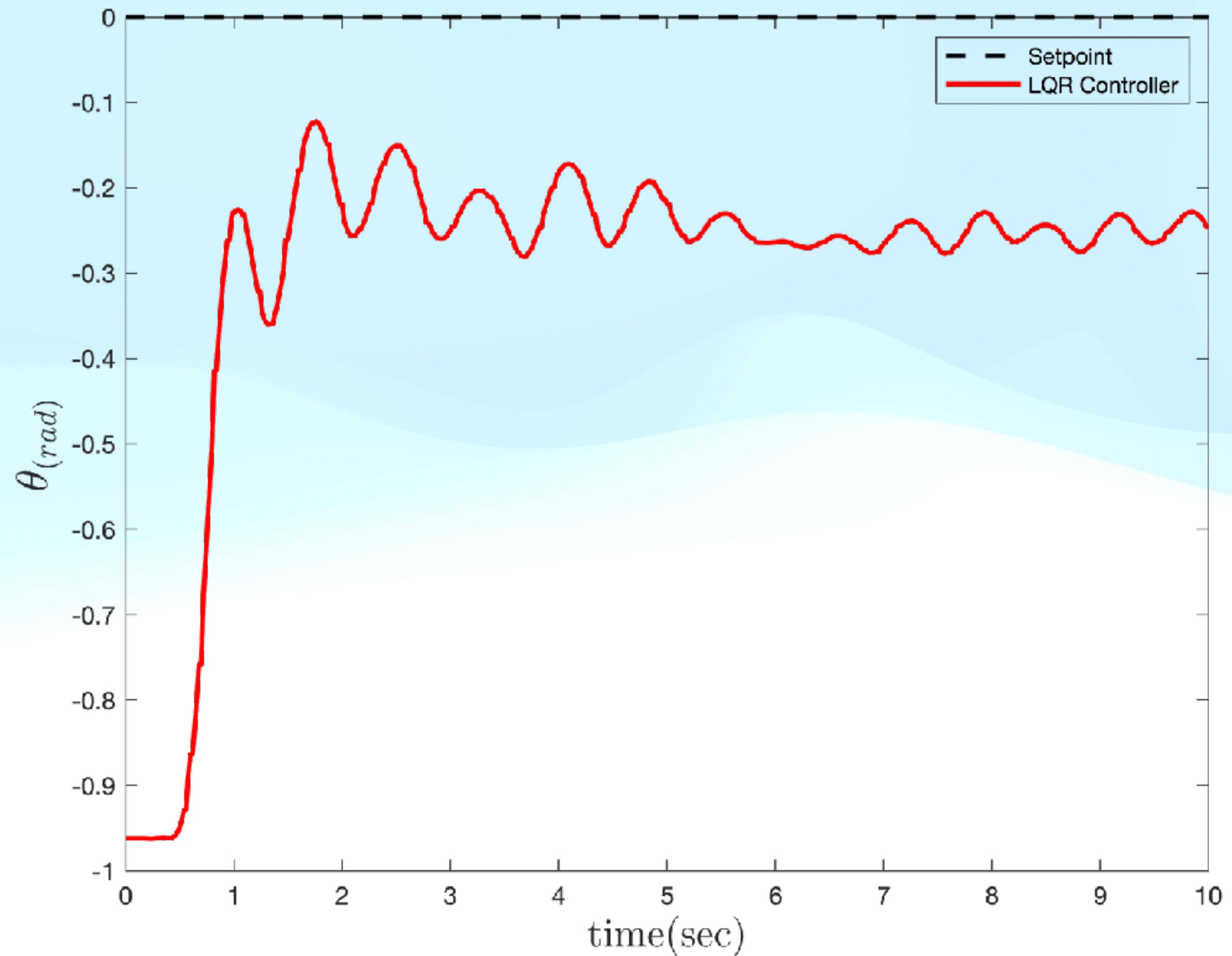
3 Degree Of Freedom
with noise



Implementation

LQR Controller

Pitch

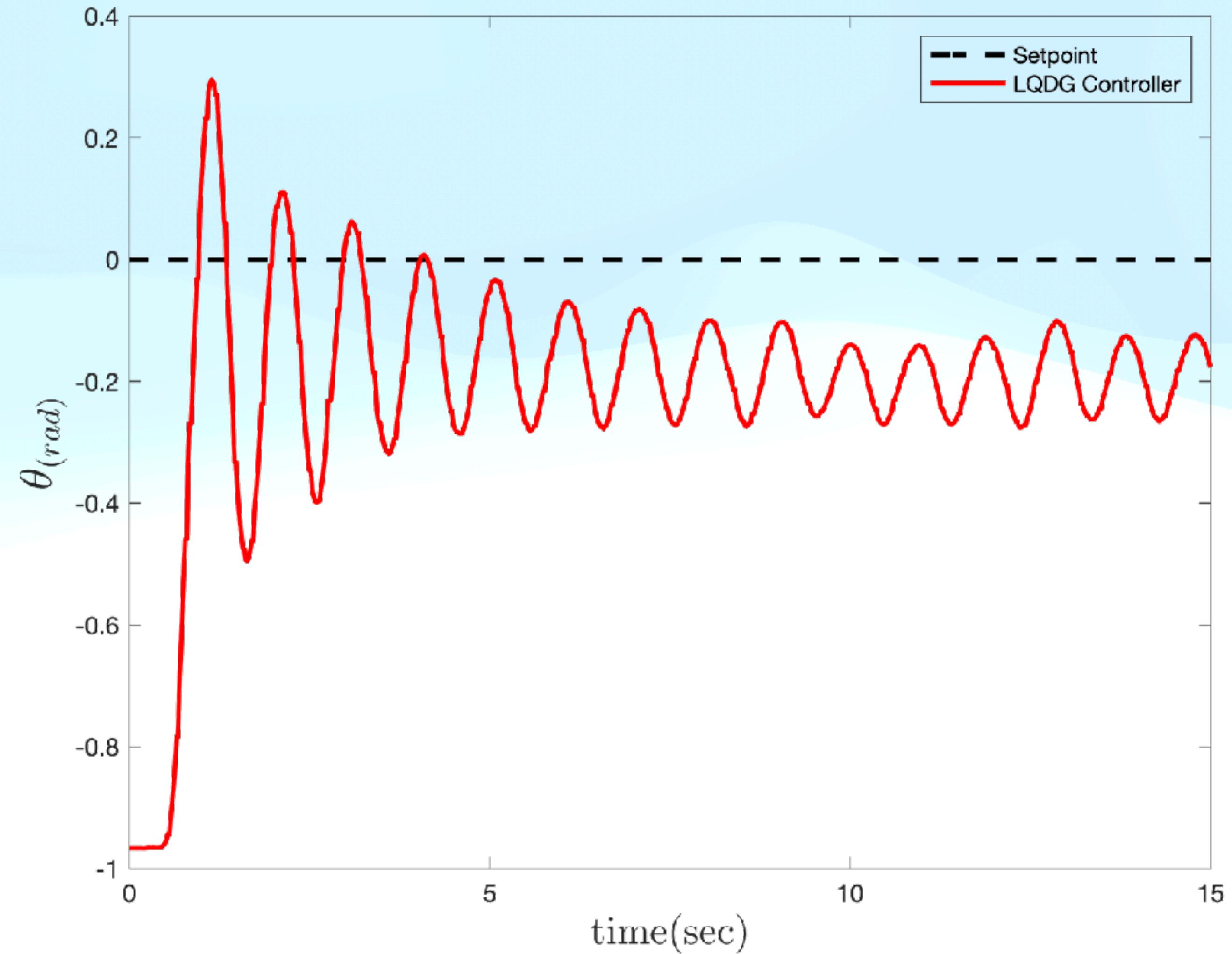


Simulating and implementing have the same weighting matrix.

Implementation

LQDG Controller

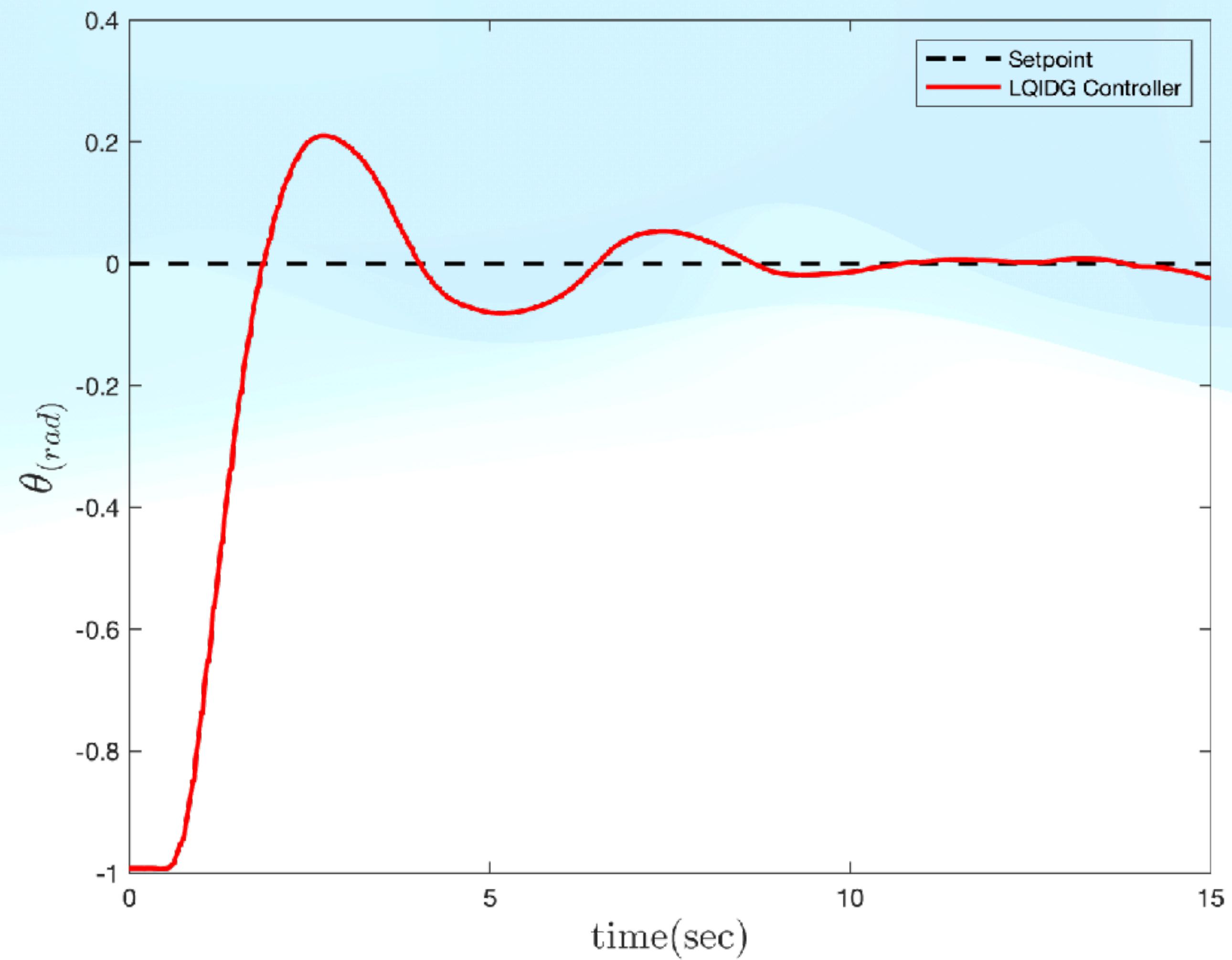
Pitch



Implementation

LQIDG Controller

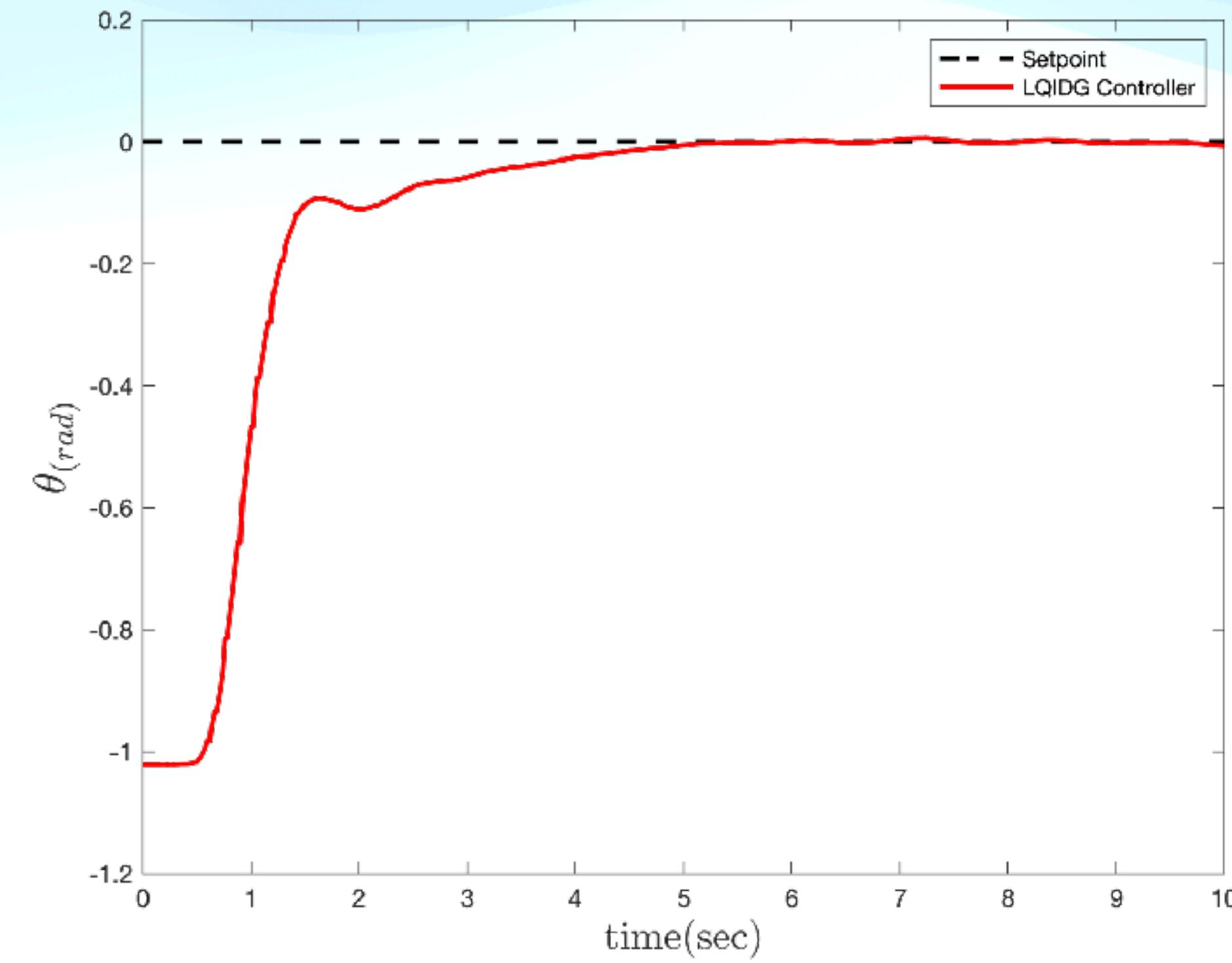
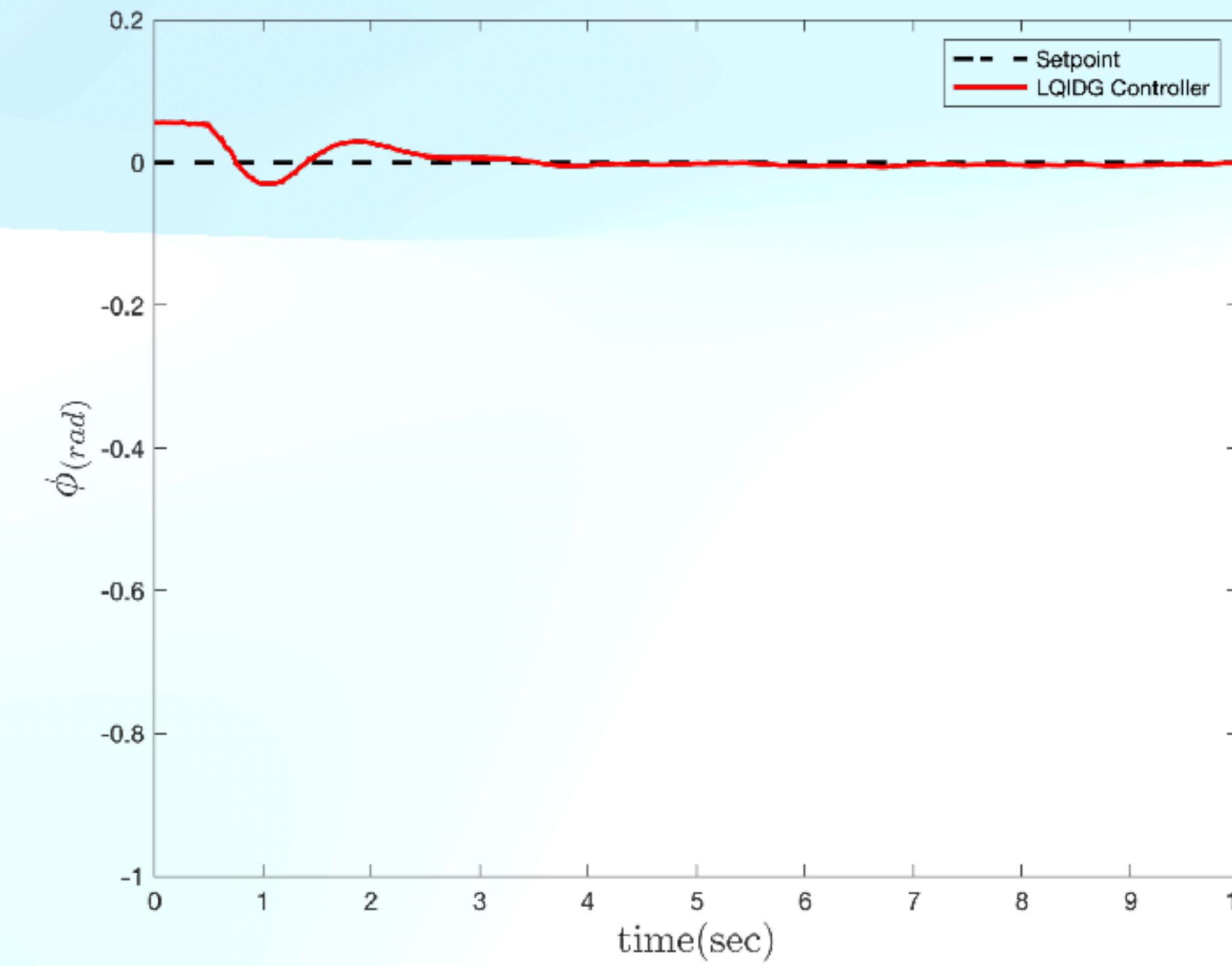
Pitch



Implementation

LQIDG Controller

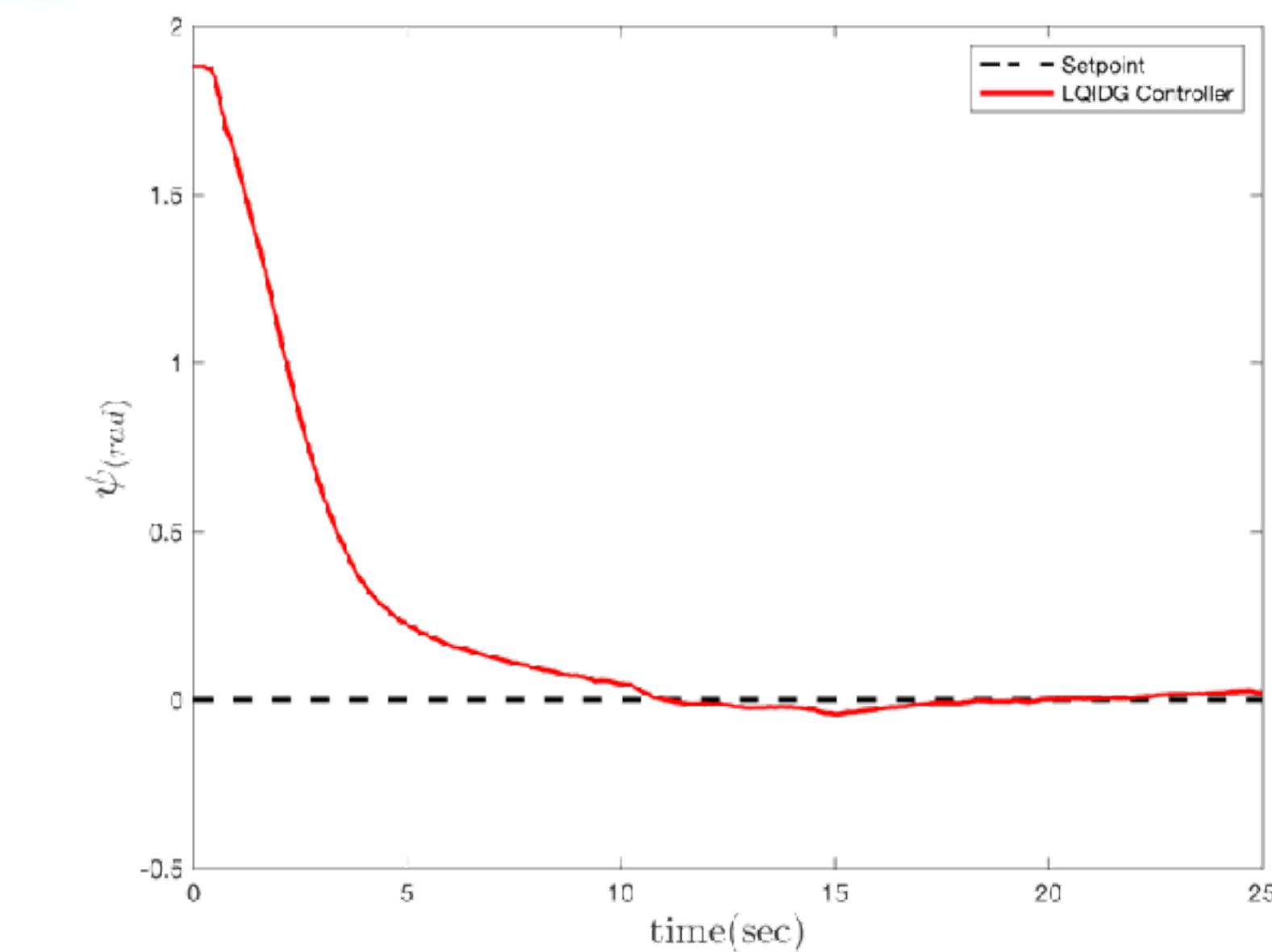
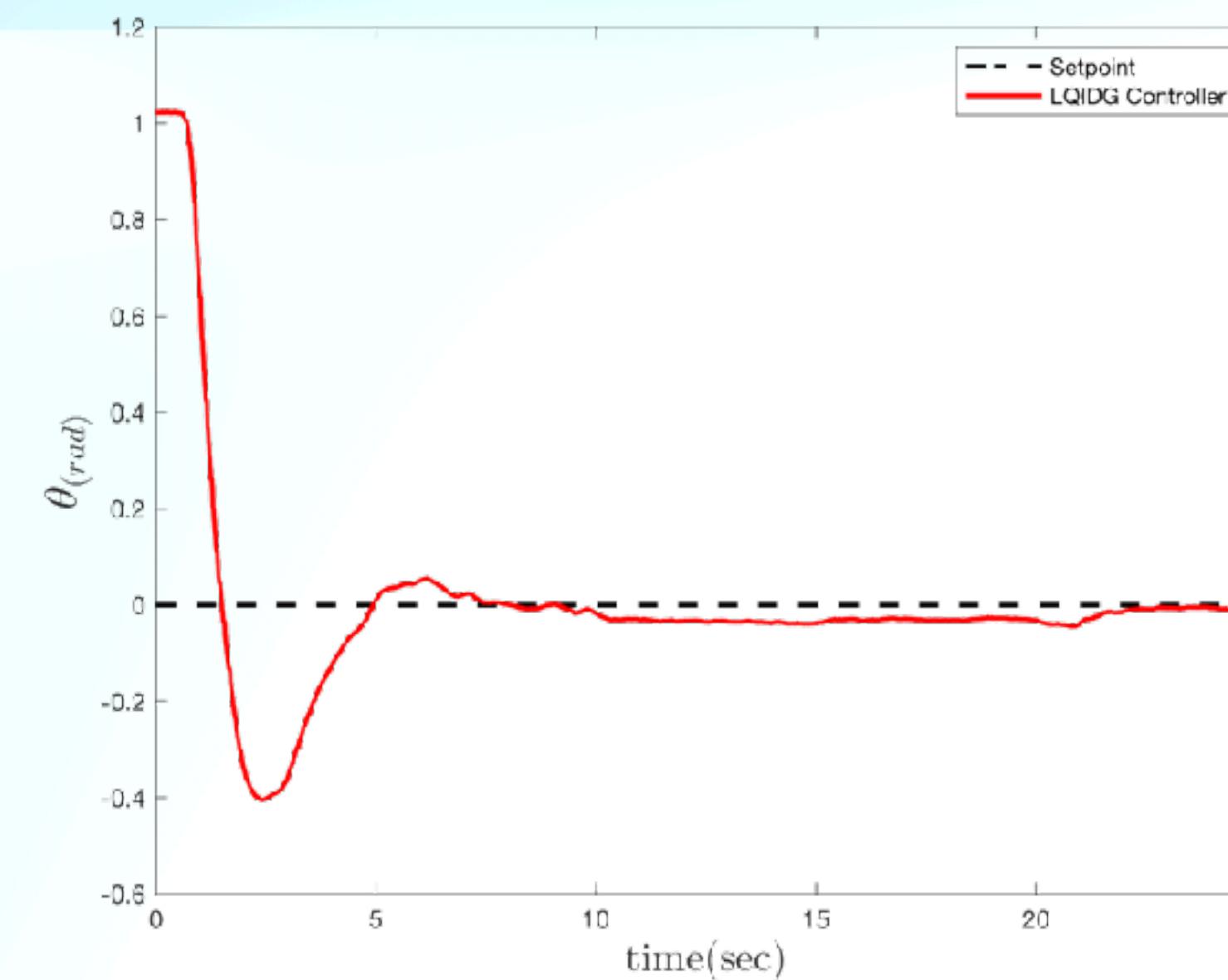
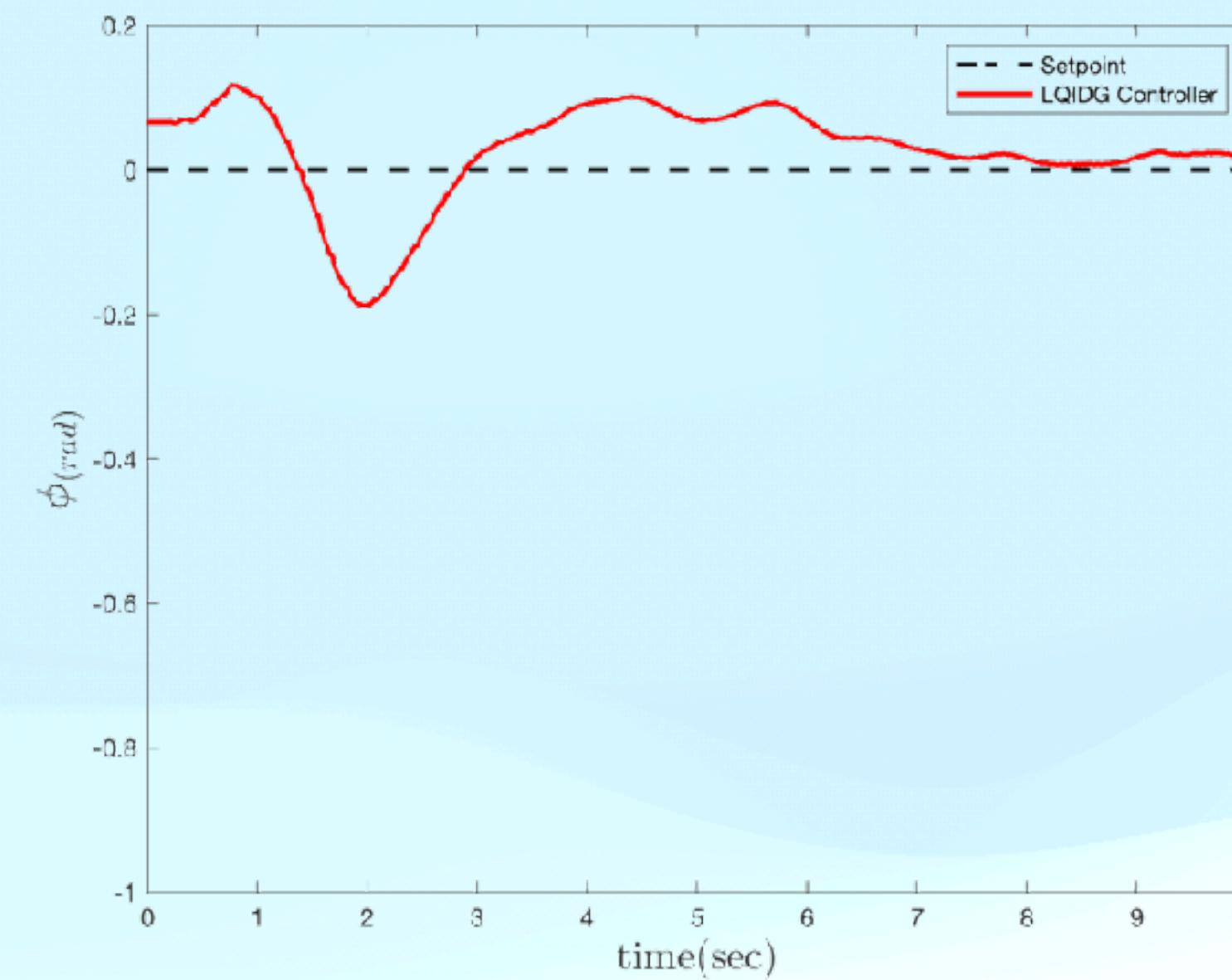
Roll-Pitch with noise



Implementation

LQIDG Controller

3 Degree Of Freedom



Thank You For Your Attention