

# Linear Quadratic Integral Differential Game applied to the Real-time Control of a 3DoF Experimental setup of a Quadrotor

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**Abstract**—The accurate attitude control of a quadrotor is necessary, especially when facing disturbance. In this study, a linear quadratic with integral action based on the differential game theory is implemented on a three-degree-of-freedom experimental setup of a quadrotor. For this purpose, first, a continuous state-space model of the quadrotor is derived based on the linearization of the nonlinear equations of motion, and the parameters of the state-space model structure are identified with the experimental results. Then, the attitude control commands of the quadrotor are derived based on two players; one finds the best attitude control command, and the other creates the disturbance by mini-maximizing a quadratic criterion, defined as the sum of outputs plus the weighted control effort and disturbance. The performance of the proposed controller is evaluated in level flight and compared to the linear quadratic regulator controller. Results demonstrate that the proposed approach has an excellent performance in dissipating the disturbances.

**Index Terms**—Linear Quadratic Differential Game, Quadrotor, Real-time, 3DoF Experimental setup, Optimal Control, Robust Control

## I. INTRODUCTION

A quadrotor is a type of helicopter with four rotors that plays a significant role in today's society, including research, military, imaging, recreation, and agriculture. The performance of the quadrotor relies on the control system, including attitude, altitude, and position subsystems. In the attitude control of the quadrotor, it is necessary to maintain the attitude outputs, including roll, pitch, and yaw angles of the quadrotor, at the desired level using control commands such as the rotational speed of the rotors, when the disturbances occur suddenly. Therefore, much research is being conducted on the automatic control of the attitudes' quadrotor in facing the disturbance.

In [1], a Proportional Integral Derivative (PID) controller is used to control the attitude of the quadrotor. However, the control objectives have not been effectively achieved with this controllers when the disturbance occurs. To solve this problem the model-based approaches [2] are utilized for controller design. These controllers work based on information from the attitude model of the quadrotor and the disturbance to produce the best control command.

Various model-based controllers can be found within the literature, the most well-known of which are intelligent control, the nonlinear control, robust control, and optimal control to reduce the disturbance effect in the attitude control and provide a faster control algorithm in facing the modeling error. In the intelligent controller category, the artificial intelligence computing approaches like fuzzy logic [3], iterative learning [4], machine learning [5], reinforcement learning [6], and evolutionary computation [7] have been utilized to control the attitude of the quadrotor.

Moreover, nonlinear control methods such as Sliding Mode Control (SMC) [8] and Feedback Linearization (FBL) [9] have been applied to control the roll, pitch, and yaw angles of the quadrotor. In the optimal controller category, a Linear Quadratic Regulator (LQR) [10] and Linear Quadratic Gaussian (LQG) [11] have been implemented on the quadrotor based on the minimization of a quadratic criterion, including regulation performance and control effort to provide optimally controlled feedback gains.

Linear Quadratic Regulator Differential Game (LQR-DG) control approach [12], [14] is a class of optimal and robust controller methods that controls the outputs of a system based on its linear model and mini-maximization of a cost function. This approach has been utilized to stabilize and control various nonlinear and complex systems such as a ship controller [13]. Moreover, in the LQR-DG control method, the control commands are analytically generated based on a pursuit-evasion of two players, one tracks the best control command, and the other creates the disturbance. This is one of the distinctive features of the LQR-DG controller and an important difference from other optimal control methods.

In this study, an LQR controller method based on the differential game theory, with an integral action called Linear Quadratic Integral Regulator Differential Game (LQIR-DG) controller, is proposed to generate the most efficient control command for an experimental setup of the quadrotor when facing the disturbance. Since the LQIR-DG is affected by an accurate model of the system, first, the dynamic of the three-degree-of-freedom setup of the quadrotor is modeled. Then,

the linear state-space form the quadrotor model is extracted using the linearization of the nonlinear equations of motion to utilize in the proposed control problem. Moreover, the model's parameters are identified and verified against the experimental values. Next, the LQIR-DG technique is applied to the experimental setup of the quadrotor to reduce the effect of disturbance. The performance of the proposed controller is examined in the presence of the disturbance. The results show the successful performance of the LQIR-DG scheme in reducing the disturbance.

This paper is organized as follows: The problem architecture is defined in section II. In section III, the dynamics model for the experimental setup of the quadrotor is derived in detail. In section IV, The LQIR-DG architecture is denoted. Numerical results are provided in section V. Finally, a conclusion is made in section VI.

## II. PROBLEM STATEMENT

In this section, a nonlinear dynamic is presented for the experimental setup of the quadrotor, as shown in Fig.1. The quadrotor is free to rotate about its roll, pitch, and yaw axes. The Euler angle angles and angular velocities along three orthogonal axes are measured simultaneously using the Attitude and Heading Reference Systems (AHRS). LQIR-DG utilizes these noisy measurements for real-time control of the Euler angles. The block diagram of the proposed controller structure is shown in Fig.2.

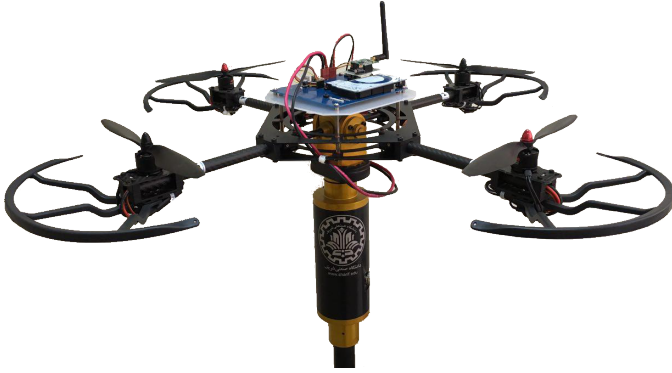


Fig. 1. 3DoF experimental setup of a quadrotor

## III. MODELING OF AN EXPERIMENTAL SETUP OF A QUADROTOR

Here, the model of the three-degree-of-freedom experimental setup of the quadrotor is presented in details. For this purpose, first, the configuration of the quadrotor is denoted. Then, the nonlinear model of the attitude dynamics is derived to denote the state-space form. Finally, the nonlinear model is linearized to utilize in the control purposes.

### A. Configuration of the Quadrotor

The schematic of the quadrotor is given in Fig.3. Each rotor is considered a rigid disk is rotating about the axis  $z_B$  in the body fixed frame with an angular velocity  $\Omega_i$ . Rotors 1 and

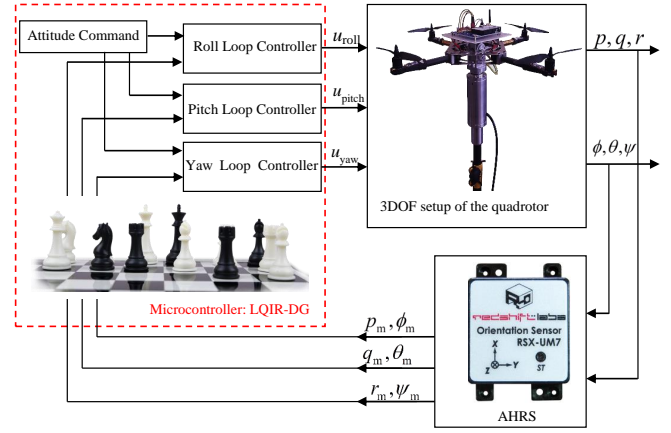


Fig. 2. Block diagram of the LQIR-DG controller structure

3 rotate in the same direction, i.e., counterclockwise, while rotors 2 and 4 rotate in the opposite direction, i.e., clockwise, to cancel yawing moment of the quadrotor.

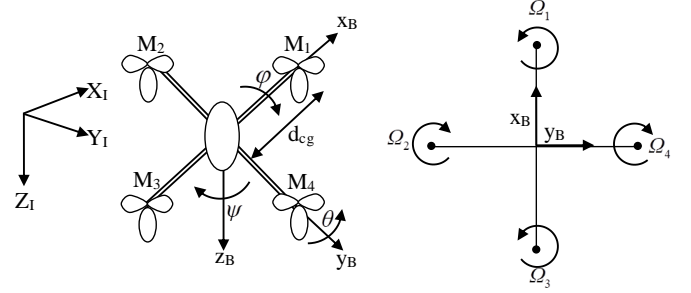


Fig. 3. Configuration of the quadrotor

### B. Dynamic Model

The dynamic model of the quadrotor, obtained from the Newton-Euler method, is stated as follows [15], [16]:

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr + q \frac{I_{rotor}}{I_{xx}} \Omega_r + \frac{u_{roll}}{I_{xx}} + \frac{d_{roll}}{I_{xx}} \quad (1)$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} rp + p \frac{I_{rotor}}{I_{xx}} \Omega_r + \frac{u_{pitch}}{I_{yy}} + \frac{d_{pitch}}{I_{yy}} \quad (2)$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{u_{yaw}}{I_{zz}} + \frac{d_{yaw}}{I_{zz}} \quad (3)$$

where  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$  are the principal moment of inertia and  $I_{rotor}$  is the inertia of a rotor about its axis. Moreover,  $d_{roll}$ ,  $d_{pitch}$ , and  $d_{yaw}$  are the disturbances, generated in  $x_B$ ,  $y_B$  and  $z_B$ , respectively.  $(p, q, r)$  are the angular velocities, and  $(\phi, \theta, \psi)$  are roll, pitch, and yaw angles. The relation between the Euler angles rates and the angular body rates are obtained as follows:

$$\dot{\phi} = p + q \sin(\phi) \cos(\theta) + r \cos(\phi) \tan(\theta) \quad (4)$$

$$\dot{\theta} = q \cos(\phi) - r \sin(\phi) \quad (5)$$

$$\dot{\psi} = (q \sin(\phi) + r \cos(\phi)) \sec(\theta) \quad (6)$$

Moreover,  $\Omega_r$ , called the overall residual propeller angular speed, is computed as

$$\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \quad (7)$$

### C. Control Commands

The control inputs  $u_{\text{roll}}$ ,  $u_{\text{pitch}}$ , and  $u_{\text{yaw}}$  are roll, pitch, and yaw moments, generated by the propellers, defined as

$$u_{\text{roll}} = b d_{\text{cg}} (\Omega_2^2 - \Omega_4^2) \quad (8)$$

$$u_{\text{pitch}} = b d_{\text{cg}} (\Omega_1^2 - \Omega_3^2) \quad (9)$$

$$u_{\text{yaw}} = d (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (10)$$

Also,  $b$  and  $d$  are thrust and drag coefficients, respectively, and  $d_{\text{cg}}$  is the horizontal distance of each rotor from the center of gravity. Therefore, the angular velocity commands are obtained as

$$\Omega_{c,1}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2b d_{\text{cg}}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \quad (11)$$

$$\Omega_{c,2}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2b d_{\text{cg}}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \quad (12)$$

$$\Omega_{c,3}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2b d_{\text{cg}}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \quad (13)$$

$$\Omega_{c,4}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2b d_{\text{cg}}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \quad (14)$$

where  $\Omega_{\text{mean}}$  is the average angular velocity of the rotors.

### D. State-Space Form

Here, the state-space model of the experimental setup of the quadrotor is presented for the control purposes. by defining  $x_1 = p$ ,  $x_2 = q$ ,  $x_3 = r$ ,  $x_4 = \phi$ ,  $x_5 = \theta$ , and  $x_6 = \psi$ ; the model of the experimental setup in state-space form are expressed as

$$\dot{x}_1 = \frac{I_{yy} - I_{zz}}{I_{xx}} x_2 x_3 + x_2 \frac{I_{\text{rotor}}}{I_{xx}} \Omega_r + \frac{u_{\text{roll}}}{I_{xx}} + \frac{d_{\text{roll}}}{I_{xx}} \quad (15)$$

$$\dot{x}_2 = \frac{I_{zz} - I_{xx}}{I_{yy}} x_1 x_3 - x_1 \frac{I_{\text{rotor}}}{I_{yy}} \Omega_r + \frac{u_{\text{pitch}}}{I_{yy}} + \frac{d_{\text{pitch}}}{I_{yy}} \quad (16)$$

$$\dot{x}_3 = \frac{I_{xx} - I_{yy}}{I_{zz}} x_1 x_2 + \frac{u_{\text{yaw}}}{I_{zz}} + \frac{d_{\text{yaw}}}{I_{zz}} \quad (17)$$

$$\dot{x}_4 = x_1 + x_2 \sin(x_4) \cos(x_5) + x_3 \cos(x_4) \tan(x_5) \quad (18)$$

$$\dot{x}_5 = x_2 \cos(x_4) - x_3 \sin(x_4) \quad (19)$$

$$\dot{x}_6 = (x_2 \sin(x_4) + x_3 \cos(x_4)) \sec(x_5) \quad (20)$$

The measurement model is written as

$$\begin{aligned} \mathbf{z} &= [p_m \ q_m \ r_m \ \phi_m \ \theta_m \ \psi_m]^T \\ &= [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T \end{aligned} \quad (21)$$

### E. Linear Model

The continuous-time linear model is utilized to drive the control commands on the quadrotor. The linear state-space model is denoted as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{B}_d\mathbf{d}(t) \quad (22)$$

where  $\mathbf{d}$  is the disturbance.  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{B}_d$  are the system input and disturbance matrices, respectively. Moreover, the measurements equation is stated as

$$\mathbf{z}(t) = \mathbf{x}(t) \quad (23)$$

According to Eqs.(15)-(20), the linear dynamic model around the equilibrium points ( $\mathbf{x}_e = 0$  and  $\mathbf{u}_e = 0$ ) of the quadrotor setup is denoted as

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} \dot{\mathbf{x}}_{\text{roll}} \\ \dot{\mathbf{x}}_{\text{pitch}} \\ \dot{\mathbf{x}}_{\text{yaw}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\text{roll}} \\ \mathbf{x}_{\text{pitch}} \\ \mathbf{x}_{\text{yaw}} \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{B}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{\text{roll}} \\ \mathbf{u}_{\text{pitch}} \\ \mathbf{u}_{\text{yaw}} \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{B}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{yaw}} \end{bmatrix} \begin{bmatrix} \mathbf{d}_{\text{roll}} \\ \mathbf{d}_{\text{pitch}} \\ \mathbf{d}_{\text{yaw}} \end{bmatrix} \end{aligned} \quad (24)$$

where  $\mathbf{x}_{\text{roll}} = [p \ \phi]^T$ ,  $\mathbf{x}_{\text{pitch}} = [q \ \theta]^T$ , and  $\mathbf{x}_{\text{yaw}} = [r \ \psi]^T$ .

Moreover, the state and input matrices are presented as

$$\mathbf{A}_{\text{roll}} = \mathbf{A}_{\text{pitch}} = \mathbf{A}_{\text{yaw}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad (25)$$

$$\mathbf{B}_{\text{roll}} = \begin{bmatrix} \frac{1}{I_{xx}} \\ 0 \end{bmatrix}; \mathbf{B}_{\text{pitch}} = \begin{bmatrix} \frac{1}{I_{yy}} \\ 0 \end{bmatrix}; \mathbf{B}_{\text{yaw}} = \begin{bmatrix} \frac{1}{I_{zz}} \\ 0 \end{bmatrix} \quad (26)$$

## IV. FORMULATION OF THE CONTROLLER DESIGN

In the LQIR-DG controller structure, an integral action is added to the LQR-DG controller to cancel the steady-state errors for reference tracking. For this purpose, first, the augmented state space of the linear quadrotor model is defined to utilize in the controller architecture. Then, the LQR-DG controller design procedure is presented to generate the best control commands for the experimental setup of the quadrotor.

### A. Augmented State Space Formulation

To add the integral action to the controller structure, the augmented states are defined as follows:

$$\mathbf{x}_{a_i} = \begin{bmatrix} \mathbf{x}_i \\ \int \mathbf{x}_i \end{bmatrix}^T \quad (27)$$

where  $i$  = roll, pitch, and yaw. Then, the model of the quadrotor dynamics, denoted by Eq.(22), is represented in the augmented state-space model as

$$\dot{\mathbf{x}}_{\mathbf{a}}(t) = \mathbf{A}_{\mathbf{a}}\mathbf{x}_{\mathbf{a}}(t) + \mathbf{B}_{\mathbf{a}}\mathbf{u}(t) + \mathbf{B}_{\mathbf{d}_{\mathbf{a}}}\mathbf{d}(t) \quad (28)$$

where matrices  $\mathbf{A}_{\mathbf{a}}$  and  $\mathbf{B}_{\mathbf{a}}$  are defined as follows:

$$\mathbf{A}_{\mathbf{a}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (29)$$

$$\mathbf{B}_{\mathbf{a}} = \mathbf{B}_{\mathbf{d}_{\mathbf{a}}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \quad (30)$$

In the above equation  $\mathbf{I}$  denotes the identity matrix.

### B. LQIR-DG Controller Method

The LQIR-DG controller is an optimal and robust method based on the differential game theory. This controller consists of two essential players: one finds the best control command, and the other creates the worst disturbance. For this purpose, the first player tries to minimize a cost function; while the second is assumed to maximize it. Therefore, the quadratic cost function equation is denoted using min-max operators as follows:

$$\begin{aligned} \min_u \max_d J(\mathbf{x}_{\mathbf{a}_i}, u_i, d_i) &= J(\mathbf{x}_{\mathbf{a}_i}, u_i^*, d_i^*) = \\ \min_u \max_d \int_0^{t_f} &\left( \mathbf{x}_{\mathbf{a}_i}^T \mathbf{Q}_i \mathbf{x}_{\mathbf{a}_i} + u_i^T R u_i - d_i^T R_d d_i \right) dt \end{aligned} \quad (31)$$

where  $t_f$  is the final time. Moreover,  $R$  and  $R_d$  are symmetric nonnegative definite matrices and  $\mathbf{Q}_i$  is a symmetric positive definite matrix. To solve this problem, connections between the general optimal problem and the LQIR problem are considered [12] and consequently the optimum control effort is computed for the each control loop as follows:

$$u_i(t) = -\mathbf{K}_i(t)\mathbf{x}_{\mathbf{a}_i}(t) \quad (32)$$

$$d_i(t) = \mathbf{K}_{d_i}(t)\mathbf{x}_{\mathbf{a}_i}(t) \quad (33)$$

where  $\mathbf{K}_i$  and  $\mathbf{K}_{d_i}$  are a time varying gain, given by

$$\mathbf{K}_i = R^{-1} \mathbf{B}_{\mathbf{a}_i}^T \mathbf{P}_{\mathbf{a}_i}(t) \quad (34)$$

$$\mathbf{K}_{d_i} = R_d^{-1} \mathbf{B}_{\mathbf{a}_{d_i}}^T \mathbf{P}_{\mathbf{a}_{d_i}}(t) \quad (35)$$

where  $\mathbf{P}_{\mathbf{a}_i}(t)$  and  $\mathbf{P}_{\mathbf{a}_{d_i}}(t)$  satisfy

$$\begin{aligned} \dot{\mathbf{P}}_{\mathbf{a}_i}(t) &= -\mathbf{A}_{\mathbf{a}_i}^T \mathbf{P}_{\mathbf{a}_i}(t) - \mathbf{P}_{\mathbf{a}_i}(t) \mathbf{A}_{\mathbf{a}_i} - \mathbf{Q}_i + \\ &\mathbf{P}_{\mathbf{a}_i}(t) \mathbf{S}_{\mathbf{a}_i}(t) \mathbf{P}_{\mathbf{a}_i}(t) + \mathbf{P}_{\mathbf{a}_i}(t) \mathbf{S}_{\mathbf{a}_{d_i}}(t) \mathbf{P}_{\mathbf{a}_{d_i}}(t) \end{aligned} \quad (36)$$

$$\begin{aligned} \dot{\mathbf{P}}_{\mathbf{a}_{d_i}}(t) &= -\mathbf{A}_{\mathbf{a}_{d_i}}^T \mathbf{P}_{\mathbf{a}_{d_i}}(t) - \mathbf{P}_{\mathbf{a}_{d_i}}(t) \mathbf{A}_{\mathbf{a}_{d_i}} - \mathbf{Q}_i + \\ &\mathbf{P}_{\mathbf{a}_{d_i}}(t) \mathbf{S}_{\mathbf{a}_{d_i}}(t) \mathbf{P}_{\mathbf{a}_{d_i}}(t) + \mathbf{P}_{\mathbf{a}_{d_i}}(t) \mathbf{S}_{\mathbf{a}_i}(t) \mathbf{P}_{\mathbf{a}_i}(t) \end{aligned} \quad (37)$$

where  $\mathbf{S}_{\mathbf{a}_i} = \mathbf{B}_{\mathbf{a}_i} R^{-1} \mathbf{B}_{\mathbf{a}_i}^T$  and  $\mathbf{S}_{\mathbf{a}_{d_i}} = \mathbf{B}_{\mathbf{a}_{d_i}} R_d^{-1} \mathbf{B}_{\mathbf{a}_{d_i}}^T$ . In this study, the steady-state values of the above equations ( $\mathbf{P}$  as  $t_f \rightarrow \infty$ ) are utilized to generate a feedback control law.

## V. RESULT AND DISCUSSION

Here, the results of the LQIR-DG controller method are devoted to the control loops of the roll, pitch, and yaw of the experimental setup of the quadrotor. First, the controller parameters are tuned using the results of numerical simulations. Moreover, the performance of the LQIR-DG controller is compared to an LQR control strategy. The parameters of the quadrotor are shown in table I. Moreover, the parameters of LQIR-DG controller weight are denoted in table II.

TABLE I  
THE PARAMETER OF THE QUADROTOR

Parameter	Value	Unit
$I_{xx}$	0.02839	$kg.m^2$
$I_{yy}$	0.03066	$kg.m^2$
$I_{zz}$	0.0439	$kg.m^2$
$I_{rotor}$	$4.4398 \times 10^{-5}$	$kg.m^2$
$b$	$3.13 \times 10^{-5}$	1
$d$	$3.2 \times 10^{-6}$	1
$\Omega_{mean}$	3000	rpm
$d_{cg}$	0.2	$m$

TABLE II  
THE PARAMETERS OF THE LQIR-DG CONTROLLER

Control Loop	Weight	Value
Roll	$\mathbf{Q}_{roll}$	diag([7.91, 0.01, 631.85, 214.28])
Pitch	$\mathbf{Q}_{pitch}$	diag([9853.09, 0.12, 0.01, 873.93])
Yaw	$\mathbf{Q}_{yaw}$	diag([1.81e-4, 4.5e-4, 3e-6, 1.7e-5])
-	$R$	1
-	$R_d$	1.2577

### A. Performance of the LQIR-DG Controller

The performance of the LQIR-DG controller is evaluated in Figure 8. The desired and actual outputs, including the roll, pitch, and yaw angles, are compared in Fig.4. The desired scenario of the simulator is considered as a level flight. These figures show that the attitude outputs of the quadrotor converge to the desired values in less than three seconds. Moreover, Fig.5 show the angular velocity command of the quadrotor, respectively. These results illustrate that the LQIR-DG approach appropriately controls the attitude of the experimental setup of the quadrotor.

### B. Comparison with LQR

Here, the LQIR-DG controller performance is compared with famous control strategy such as LQR controller method. Fig.6 compares the quadrotor's desired and actual pitch angle in the presence of these controllers. This results indicates that the LQIR-DG controller can provide high tracking performance, such as good transient response and high rapid convergence relative to LQR controller for pitch angle control of the experimental setup of the quadrotor.

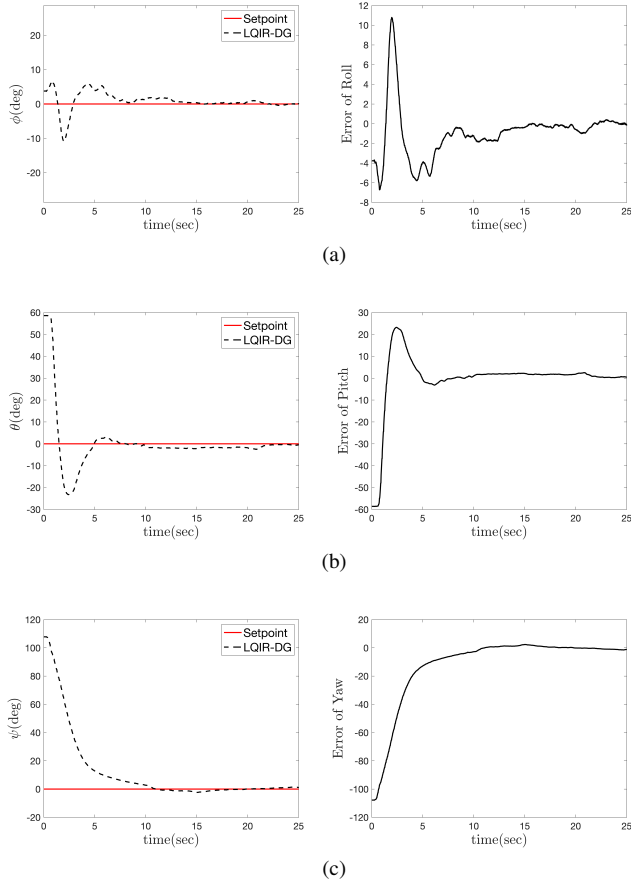


Fig. 4. Performance of the LQIR-GT controller (a) roll angle (b) pitch angle (c) actual yaw

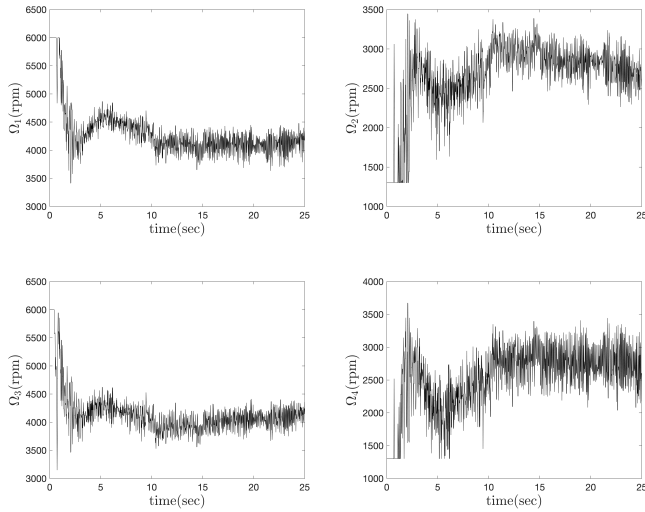


Fig. 5. Time history of angular velocity commands

## VI. CONCLUSION

In this study, a linear quadratic with integral action based on the differential game theory, called LQIR-DG, was implemented for level attitude control in an experimental setup of

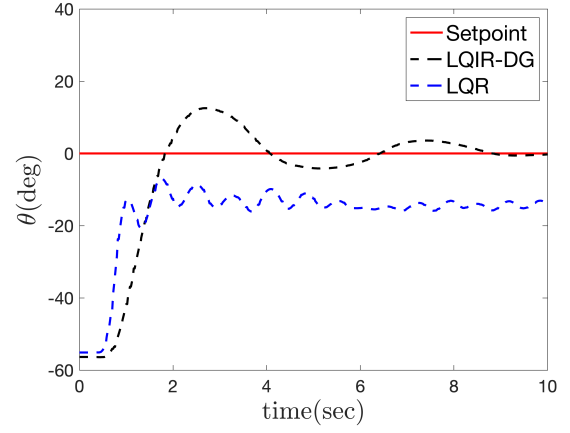


Fig. 6. Comparison of the LQIR-DG to the LQR in control of the pitch angle of the experimental setup

a quadrotor. To implement the proposed controller structure, first, an accurate dynamic model of the quadrotor was linearized in the state-space form, and then the model parameters were estimated. Next, two players were considered for each of the quadrotor's roll, pitch, and yaw channels. The first player found the best control command for each channel of the setup of a quadrotor based on the mini-maximization of a quadratic criterion; when the second player produced the worst disturbances. Finally, the performance of the proposed controller was evaluated in level flight and compared to the LQR controller. The implementation results verify the successful performance of the LQIR-DG method in the level flight of the attitude control for the actual plant.

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