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# A Linear Quadratic Integral Differential Game Approach for Attitude Control of an Experimental Platform of a Quadrotor

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## Abstract

In this study, a linear quadratic integral differential game approach is applied to regulate and track the attitude angle of an experimental setup of the quadrotor using two players. One produces commands for each channel of the quadrotor and another creates the worst disturbance by minimizing a quadratic criterion with integral action. For this purpose, first, the attitude dynamics of the platform are modeled and its parameter is identified based on the Nonlinear Least Squares Trust-Region Reflective Least Squares method. The performance of the proposed controller is evaluated for regulation and tracking problems. The ability of the controller is examined in the disturbance rejection. Moreover, the influence of uncertainty modeling is studied on the obtained results. Then, the performance of the proposed controller is compared with a classic PID, Linear Quadratic Regulator, and Linear Quadratic Integral Regulator. The result demonstrates the effectiveness of the Game Theory on the Linear Quadratic Regulator approach when the input disturbance occurs.

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**Keywords:**

Linear quadratic Gaussian controller, Differential game theory, Quadrotor, Continuous state-space model, three-degree-of-freedom experimental platform, Attitude Control Optimization, Robust disturbance rejection.

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## 1. Introduction

The investigation, strategic operations, optical sensing, entertainment, and farming are all used by quadrotors in today's society [1]. Various subsystems of the quadrotor control system are responsible for the quadrotor's performance, including attitude, altitude, and position. Maintaining the desired attitude outputs is essential for quadrotor attitude control, particularly in the presence of sudden disturbances. This can be achieved by controlling the rotor rotational speeds[2]. Consequently, there is a growing body of research focusing on the development of automatic control strategies for quadrotors to effectively manage disturbances and maintain desired attitudes. The quadrotor attitude is controlled by a Proportional Integral Derivative (PID) controller in [3, 4]. During disturbances, however, this controller has not effectively achieved the control objectives. Model-based approaches to controller design are utilized to resolve this issue [5, 6]. Quadrotor attitude models and disturbances determine the direction of the best control commands using these controllers.

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The literature has proposed numerous models-based controllers to provide a faster control algorithm when dealing with modeling errors and reducing disturbances. An intelligent controller, a robust controller, a nonlinear controller, and an optimal controller are some of these types of controllers.

Various control approaches based on intelligent logic, such as machine learning [7], evolutionary computation [8], iterative learning [9], reinforcement learning [10], and fuzzy logic [11], have been extensively employed for quadrotor attitude regulation. Numerous nonlinear control techniques have been proposed to regulate orientation angles of quadrotors, including Synergetic Control [12], Sliding Mode Control (SMC) [13], and Feedback Linearization (FBL) [14]. Robust control methods such as  $H_\infty$  [15, 16] and  $\mu$ -synthesis [17] have also been utilized to stabilize the attitudes of quadrotors under conditions of extreme uncertainty and worst-case scenarios. Quadrotors have also been controlled using optimal controllers, such as the Linear Quadratic Gaussian (LQG) [18] and the Linear Quadratic Regulator (LQR) [19]. Optimizing feedback gains is achieved by utilizing both regulation and control effort to minimize a quadratic criterion. The Linear Quadratic Regulator Differential Game (LQR-DG) is a robust and optimal control strategy that has been extensively utilized in controlling various nonlinear and complex systems, including quadrotors. This approach employs a linear system model to control its outputs while minimizing a cost function through minimaximization. The LQR-DG approach has been demonstrated to offer excellent regulation performance and control effort while being robust to external disturbances. For instance, it has been successfully applied to control a ship controller, showcasing its effectiveness and versatility in handling complex control problems [20, 21]. By means of an analytical pursuit-evasion process, the LQR-DG control approach generates precise and optimized control commands. This unique feature sets the LQR-DG controller apart from other conventional optimal control techniques and enables one player to track the most appropriate control command while the other creates disturbances.

In this paper, an LQIR method, called LQIR-DG controller is suggested to produce the optimal and robust control command, i.e. rotational velocity command using the game theory approach. Since the LQIR-DG controller is affected by an exact model of the plant, the quadrotor's experimental platform is modeled and its parameters are identified based on experimental data. For control purposes, its linear state-space form is extracted using linearization of the nonlinear UAV. Then, the LQIR-DG technique is applied in real-time to the experimental platform. The performance of the LQIR-DG method is evaluated in the presence of the disturbance and modeling error for regulation and tracking purposes. A comparison is also performed between the results of classical PID, LQR, and LQIR and the proposed approach in real-time mode. The results show the proposed control structure is effective in the control of the quadrotor platform.

The following sections of this research paper provide a comprehensive analysis of the proposed control approach. Section 2 defines detailed derivations of the dynamics model and quadrotor's experimental platform, respectively. Section 4 explicates the proposed LQIG-DG controller architecture. The effectiveness of the controller is assessed in section 5, which is followed by the conclusion in section 6.

## 2. Problem Formulation

The experimental quadrotor platform rotates freely with rotational velocity about its roll, pitch, and yaw axes, as shown in Figure 1. The Euler angles and their derivatives are measured using an Attitude Heading Reference System (AHRS), which is utilized in the structure of the LQIR-DG controller to stabilize the quadrotor platform. The graphical representation of the proposed controller structure is depicted in Figure 2.



Figure 1: 3DoF setup of the quadrotor.

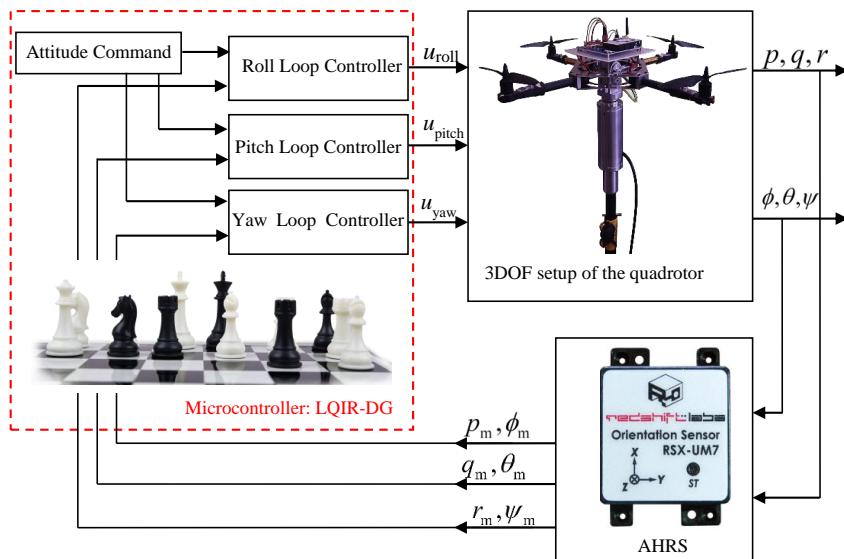


Figure 2: Structure of the LQIG-DG Controller Illustrated in a Block Diagram.

### 3. Dynamic Model of the Quadrotor Platform

In this section, first, a nonlinear model for the quadrotor platform is derived. Then, a state-space model and a linear model are developed for control purposes to be utilized in a controller strategy. Finally, a nonlinear identification method is applied to identify the parameters of the quadrotor.

#### 3.1. Quadrotor Configuration

Figure 3 shows the quadrotor schematic. It depicts four rotors rotating around the  $z_B$  axis in the coordinate system of the body. The rotors have a rotational velocity of  $\Omega_r$ . The quadrotor platform has 3 degrees of freedom,

including roll, pitch, and yaw motions, which are described by roll ( $\phi$ ), pitch ( $\theta$ ), and yaw ( $\psi$ ) angles, respectively. The counterclockwise rotation of Rotors 1 and 3 generates a moment that counteracts the yawing moment, while the clockwise rotation of Rotors 2 and 4 produces a moment that also counteracts the yawing moment.

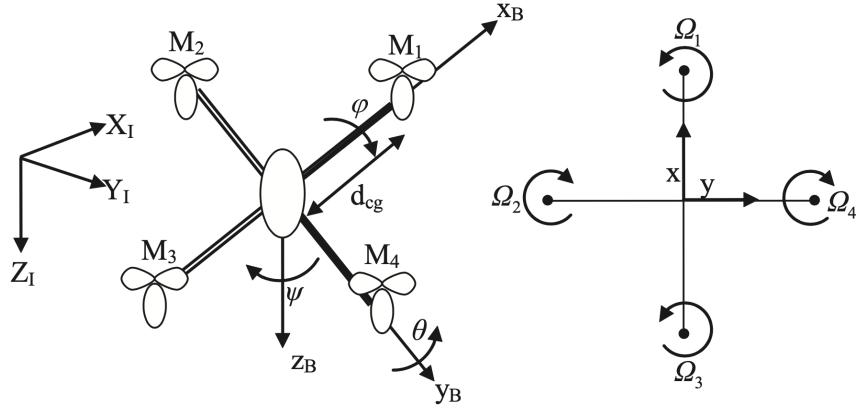


Figure 3: Quadrotor Configuration.

### 3.2. Dynamic Modeling of the Quadrotor Platform

Here, according to Newton-Euler, the dynamic model of the quadrotor platform is derived as follows [22, 23]:

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}} qr + q \frac{I_{rotor}}{I_{xx}} \Omega_r + \frac{u_{roll}}{I_{xx}} + \frac{d_{roll}}{I_{xx}} \quad (1)$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}} rp + p \frac{I_{rotor}}{I_{xx}} \Omega_r + \frac{u_{pitch}}{I_{yy}} + \frac{d_{pitch}}{I_{yy}} \quad (2)$$

$$\dot{r} = \frac{I_{xx} - I_{yy}}{I_{zz}} pq + \frac{u_{yaw}}{I_{zz}} + \frac{d_{yaw}}{I_{zz}} \quad (3)$$

The ( $p, q, r$ ) represent the rotational variables, and  $d_{roll}$ ,  $d_{pitch}$ , and  $d_{yaw}$  denote the disturbances produced in the  $x_B$ ,  $y_B$ , and  $z_B$  axes, respectively. Additionally,  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$  are the principal moments of inertia, and  $I_{rotor}$  is the rotor inertia about its axis. Euler angle rates are also determined from angular body rates as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (4)$$

The residual rotor velocity, denoted by  $\Omega_r$ , is calculated as follows:

$$\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \quad (5)$$

### 3.3. Input of the Dynamic Model

The control inputs  $u_{roll}$ ,  $u_{pitch}$ , and  $u_{yaw}$  are to the moments generated by the quadrotor's rotors along the roll, pitch, and yaw axes, respectively, defined as follows:

$$u_{roll} = b d_{cg} (\Omega_2^2 - \Omega_4^2) \quad (6)$$

$$u_{pitch} = b d_{cg} (\Omega_1^2 - \Omega_3^2) \quad (7)$$

$$u_{yaw} = d (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (8)$$

where  $d_{cg}$ ,  $d$ , and  $b$  represent the distance between the rotors and the gravity center, drag factor, and thrust factor, respectively. The rotational velocity commands are computed as follows:

$$\Omega_{c,1}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2b d_{cg}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \quad (9)$$

$$\Omega_{c,2}^2 = \Omega_{\text{mean}}^2 + \frac{1}{2b d_{cg}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \quad (10)$$

$$\Omega_{c,3}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2b d_{cg}} u_{\text{pitch}} + \frac{1}{4d} u_{\text{yaw}} \quad (11)$$

$$\Omega_{c,4}^2 = \Omega_{\text{mean}}^2 - \frac{1}{2b d_{cg}} u_{\text{roll}} - \frac{1}{4d} u_{\text{yaw}} \quad (12)$$

In the above equation,  $\Omega_{\text{mean}}$  is nominal rotational velocities of the rotors.

### 3.4. State-Space Formulation

Here, by defining  $x_1 = p$ ,  $x_2 = q$ ,  $x_3 = r$ ,  $x_4 = \phi$ ,  $x_5 = \theta$ , and  $x_6 = \psi$ , the formulation of the quadrotor platform is presented as follows:

$$\dot{x}_1 = \Gamma_1 x_2 x_3 + \Gamma_2 x_2 \Omega_r + \Gamma_3 u_{\text{roll}} + \Gamma_3 d_{\text{roll}} \quad (13)$$

$$\dot{x}_2 = \Gamma_4 x_1 x_3 - \Gamma_5 x_1 \Omega_r + \Gamma_6 u_{\text{pitch}} + \Gamma_6 d_{\text{pitch}} \quad (14)$$

$$\dot{x}_3 = \Gamma_7 x_1 x_2 + \Gamma_8 u_{\text{yaw}} + \Gamma_8 d_{\text{yaw}} \quad (15)$$

$$\dot{x}_4 = x_1 + (x_2 \sin(x_4) + x_3 \cos(x_4)) \tan(x_5) \quad (16)$$

$$\dot{x}_5 = x_2 \cos(x_4) - x_3 \sin(x_4) \quad (17)$$

$$\dot{x}_6 = (x_2 \sin(x_4) + x_3 \cos(x_4)) / \cos(x_5) \quad (18)$$

where  $\Gamma_i (i = 1, \dots, 8)$  is defined as:

$$\begin{aligned} \Gamma_1 &= \frac{I_{yy} - I_{zz}}{I_{xx}}, & \Gamma_2 &= \frac{I_{\text{rotor}}}{I_{xx}}, & \Gamma_3 &= \frac{1}{I_{xx}} \\ \Gamma_4 &= \frac{I_{zz} - I_{xx}}{I_{yy}}, & \Gamma_5 &= \frac{I_{\text{rotor}}}{I_{xx}}, & \Gamma_6 &= \frac{1}{I_{yy}} \\ \Gamma_7 &= \frac{I_{xx} - I_{yy}}{I_{zz}}, & \Gamma_8 &= \frac{1}{I_{zz}} \end{aligned} \quad (19)$$

Moreover, the measurement vector, obtained from the AHRS sensor is presented as follows:

$$\mathbf{z} = [p \ q \ r \ \phi \ \theta \ \psi]^T + \nu \quad (20)$$

where  $\nu$  is a Gaussian white noise. In the above equation, the superscripts T indicate the transpose notation.

### 3.5. Linear Model

The linear continuous-time model of the quadrotor platform about the equilibrium points ( $\mathbf{x}_e \neq 0$  and  $\mathbf{u}_e \neq 0$ ) is represented as:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{B}_d \mathbf{d}(t) \quad (21)$$

where  $\dot{\mathbf{x}} = [\dot{x}_{\text{roll}} \ \dot{x}_{\text{pitch}} \ \dot{x}_{\text{yaw}}]$ , defined as:

$$\mathbf{x}_{\text{roll}} = \begin{bmatrix} p \\ \phi \end{bmatrix}, \quad \mathbf{x}_{\text{pitch}} = \begin{bmatrix} q \\ \theta \end{bmatrix}, \quad \mathbf{x}_{\text{yaw}} = \begin{bmatrix} r \\ \psi \end{bmatrix} \quad (22)$$

where  $\mathbf{A}$  is the dynamic system matrix, denoted as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{\text{yaw}} \end{bmatrix} \quad (23)$$

Moreover,  $\mathbf{B}$  and  $\mathbf{B}_d$  are the input and disturbance matrices, respectively, and are defined as:

$$\mathbf{B} = \mathbf{B}_d = \begin{bmatrix} \mathbf{B}_{\text{roll}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\text{pitch}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\text{yaw}} \end{bmatrix} \quad (24)$$

In addition, the input matrices and state are shown as:

$$\mathbf{A}_{\text{roll}} = \mathbf{A}_{\text{pitch}} = \mathbf{A}_{\text{yaw}} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad (25)$$

$$\mathbf{B}_{\text{roll}} = \begin{bmatrix} \frac{1}{I_{xx}} \\ 0 \end{bmatrix}; \quad \mathbf{B}_{\text{pitch}} = \begin{bmatrix} \frac{1}{I_{yy}} \\ 0 \end{bmatrix}; \quad \mathbf{B}_{\text{yaw}} = \begin{bmatrix} \frac{1}{I_{zz}} \\ 0 \end{bmatrix} \quad (26)$$

### 3.6. Identification of the Platform Parameters

This section describes the utilization of the Nonlinear Least Squares (NLS) algorithm for estimating the model parameters ( $\Gamma$ ) of the 3DoF experimental platform using experimental data. This technique is based on the Trust-Region Reflective Least Squares (TRRLS) method, which iteratively finds the values of the model parameters by minimizing a cost function, defined as follows:

$$\min_{\Gamma_i} (\| e(\Gamma_i) \|^2) = \min_{\Gamma_i} = \left( \sum_{j=1}^n (\mathbf{z}_j - \hat{\mathbf{z}}_j)(\mathbf{z}_j - \hat{\mathbf{z}}_j)^T \right) \quad (27)$$

where  $\mathbf{z}$  and  $\hat{\mathbf{z}}$  are the experimental and simulated output signals, when the same input signals are applied ones. The structure of the proposed identification approach is illustrated in figure 4

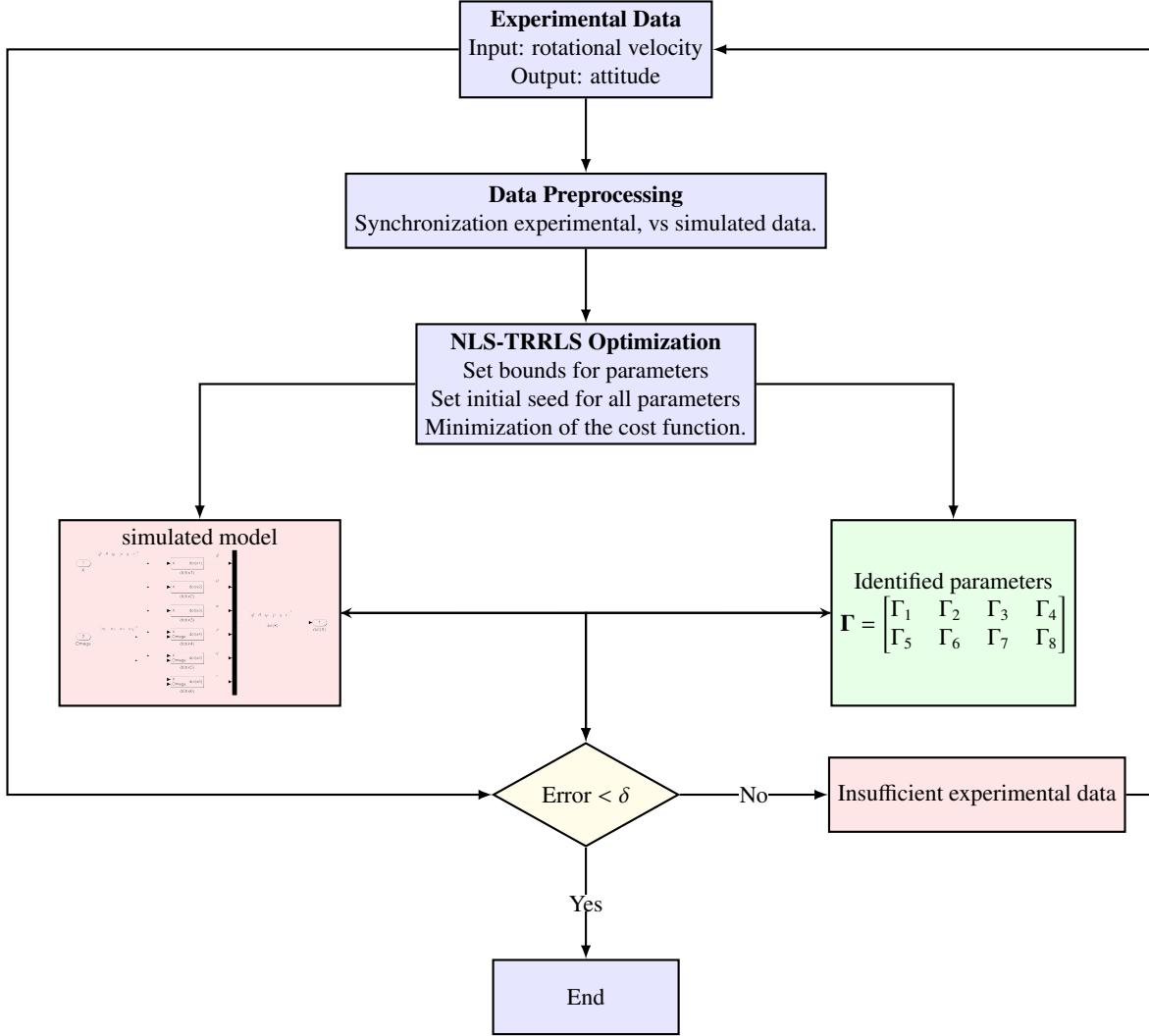


Figure 4: Structure of TRRLS identification approach.

#### 4. Formulation of the LQDG Controller

Here, the LQR-DG controller is augmented with an integral action to eliminate steady-state errors. For this purpose, the augmented states of the quadrotor platform, which include states and their integrals, are selected. The design methodology of the LQR-DG controller is introduced. It generates optimal control signals for the three degrees of freedom platform.

##### 4.1. Augmented State Space Development

To integrate the integrator into the control strategy architecture, the augmented state variables are defined below:

$$\mathbf{x}_{\mathbf{a}_i} = \begin{bmatrix} \mathbf{x}_i \\ \int \mathbf{x}_i \end{bmatrix} \quad (28)$$

The dynamics model of the quadrotor, as expressed by Eq. (21), is reformulated into an augmented state-space model that incorporates the roll ( $\phi$ ), pitch ( $\theta$ ), and yaw ( $\psi$ ) angles. The augmented state-space model provides a

comprehensive description of the quadrotor's dynamic behavior, enabling the design of effective control strategies. The ensuing state-space model that integrates the augmented state variables is mathematically represented and defined in the following manner:

$$\dot{\mathbf{x}}_a(t) = \mathbf{A}_a \mathbf{x}_a(t) + \mathbf{B}_a \mathbf{u}(t) + \mathbf{B}_{d_a} \mathbf{d}(t) \quad (29)$$

$\mathbf{B}_a$  and  $\mathbf{A}_a$ , which are expressed as below:

$$\mathbf{B}_a = \mathbf{B}_{d_a} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \quad (30)$$

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \quad (31)$$

In the aforementioned expression, the notation  $\mathbf{I}$  denotes the identity matrix.

#### 4.2. LQR-DG Control Scheme with Integral Action

In line with the principles of differential game theory, the LQIR-DG controller is designed to be both robust and optimal. The LQIR-DG scheme involves selecting two fundamental players, one responsible for determining the control command and the other for generating the worst possible disturbance. To achieve the primary objective, the primary player minimizes the following cost function, while the other player must maximize it:

$$\min_u \max_d J(\mathbf{x}_{a_i}, d_i, u_i) = J(\mathbf{x}_{a_i}, u_i^*, d_i^*) = \min_d \max_u \int_0^{t_f} \left( \mathbf{x}_{a_i}^T \mathbf{Q}_i \mathbf{x}_{a_i} + u_i^T R u_i - d_i^T R_d d_i \right) dt \quad (32)$$

where  $\mathbf{Q}_i$ ,  $R_d$ , and  $R$  are weight coefficients of the function. The final time is denoted by  $t_f$ . By solving the above problem, the control command is computed as follows [24]:

$$d_i(t) = \mathbf{K}_{d_i}(t) \mathbf{x}_{a_i}(t) \quad (33)$$

Moreover, the worst disturbance is obtained as:

$$u_i(t) = -\mathbf{K}_i(t) \mathbf{x}_{a_i}(t) \quad (34)$$

Here,  $\mathbf{K}_{d_i}$  and  $\mathbf{K}_i$  are gain values defined as follows:

$$\mathbf{K}_i = R^{-1} \mathbf{B}_{a_i}^T \mathbf{P}_{a_i}(t) \quad (35)$$

$$\mathbf{K}_{d_i} = R_d^{-1} \mathbf{B}_{a_{d_i}}^T \mathbf{P}_{a_{d_i}}(t) \quad (36)$$

where  $\mathbf{P}_{a_i}(t)$  and  $\mathbf{P}_{a_{d_i}}(t)$  satisfy

$$-\mathbf{A}_a^T \mathbf{P}_{a_{d_i}}(t) - \mathbf{Q}_i - \mathbf{P}_{a_{d_i}}(t) \mathbf{A}_a + \mathbf{P}_{a_{d_i}}(t) \mathbf{S}_{a_i}(t) \mathbf{P}_{a_i}(t) + \mathbf{P}_{a_{d_i}}(t) \mathbf{S}_{a_{d_i}}(t) \mathbf{P}_{a_{d_i}}(t) = \mathbf{0} \quad (37)$$

$$-\mathbf{A}_a^T \mathbf{P}_{a_i}(t) - \mathbf{Q}_i - \mathbf{P}_{a_i}(t) \mathbf{A}_a + \mathbf{P}_{a_i}(t) \mathbf{S}_{a_{d_i}}(t) \mathbf{P}_{a_{d_i}}(t) + \mathbf{P}_{a_i}(t) \mathbf{S}_{a_i}(t) \mathbf{P}_{a_i}(t) = \mathbf{0} \quad (38)$$

where

$$\mathbf{S}_{a_i} = \mathbf{B}_{a_i} R^{-1} \mathbf{B}_{a_i}^T, \quad \mathbf{S}_{a_{d_i}} = \mathbf{B}_{a_{d_i}} R_d^{-1} \mathbf{B}_{a_{d_i}}^T$$

## 5. Result and Discussion

Here, the simulation results of parameter identification and LQIDG-Controller for a quadrotor platform are presented. First, the quadrotor parameters are estimated based on the NLS method. Then, the performance of the LQDG controller structure is evaluated. The performance of the quadrotor LQIDG controller is presented in Tables 1 and 2.

Table 1: Quadrotor Parameters

Parameter	Value	Unit
$d_{cg}$	0.2	m
$d$	$3.2 \times 10^{-6}$	N.m.sec <sup>2</sup> /rad <sup>2</sup>
$b$	$3.13 \times 10^{-5}$	N.sec <sup>2</sup> /rad <sup>2</sup>
$I_{xx}$	0.02839	kg.m <sup>2</sup>
$I_{yy}$	0.03066	kg.m <sup>2</sup>
$I_{zz}$	0.0439	kg.m <sup>2</sup>
$I_{rotor}$	$4.4398 \times 10^{-5}$	kg.m <sup>2</sup>
$\Omega_{mean}$	3000	rpm

Table 2: LQIR-DG Controller Parameters

Channel	Weighting Matrix	Matrix Values
Roll	$Q_{roll}$	diag([0.02, 65.96, 83.04, 0.00])
Pitch	$Q_{pitch}$	diag([435.01, 262.60, 262.60, 0.00])
Yaw	$Q_{yaw}$	diag([ $4 \times 10^{-4}$ , 0.00, 0.133, 0])
	$R$	1
	$R_d$	1.2764

### 5.1. Identification of the 3DoF quadrotor platform model

As described in section 3.4, the parameters of the quadrotor platform, denoted by  $\Gamma_i (i = 1, \dots, 8)$ , are identified using the NRS algorithm. The NLS-TRRLS algorithm is implemented in Matlab R2022b®. To increase the accuracy of parameter identification, three scenarios according to Table 3. In the first scenario, depicted in Figure 5, the quadrotor is rotated about only one axis (roll, pitch, or yaw axes) to identify the parameters  $\Gamma_3$ ,  $\Gamma_6$ , and  $\Gamma_8$ . In the second scenario, as illustrated in Figure 6, the parameters  $\Gamma_2$  and  $\Gamma_5$  are estimated by moving the experimental platform to freely rotate around its roll and pitch axes simultaneously. When the stopping condition of the NLS algorithm is met, the optimal values of the quadrotor parameters are computed and presented in Table 4. These results illustrate that the outputs of the simulation results for the quadrotor model are consistent with reality.

Table 3: Scenarios for Identification of Quadrotor Parameters.

<b>Scenario</b>	<b>Description</b>	<b>Initial Condition (deg)</b>			<b>Rotational Velocity Commands</b>			
		$\phi$	$\theta$	$\psi$	$\Omega_1$	$\Omega_2$	$\Omega_3$	$\Omega_4$
I	Roll free	38	-	-	2000	2000	2000	3400
	Pitch free	-	-15	-	3700	2000	2000	2000
	Yaw free	-	-	-75	2000	3300	2000	3300
II	Roll and Pitch free	8	-5	-	1700	3800	2400	1700
III	Roll, Pitch, and Yaw free	8	-3	-146	1700	3800	2400	1700

Table 4: True values of the quadrotor parameters.

<b>Parameter</b>	<b>Value</b>	<b>Parameter</b>	<b>Value</b>
$\Gamma_1$	-0.9622	$\Gamma_5$	$3.6441 \times 10^{-4}$
$\Gamma_2$	-0.0154	$\Gamma_6$	$7.5395 \times 10^{-5}$
$\Gamma_3$	$5.4716 \times 10^{-5}$	$\Gamma_7$	0.1308
$\Gamma_4$	1.0457	$\Gamma_8$	$4.3753 \times 10^{-5}$

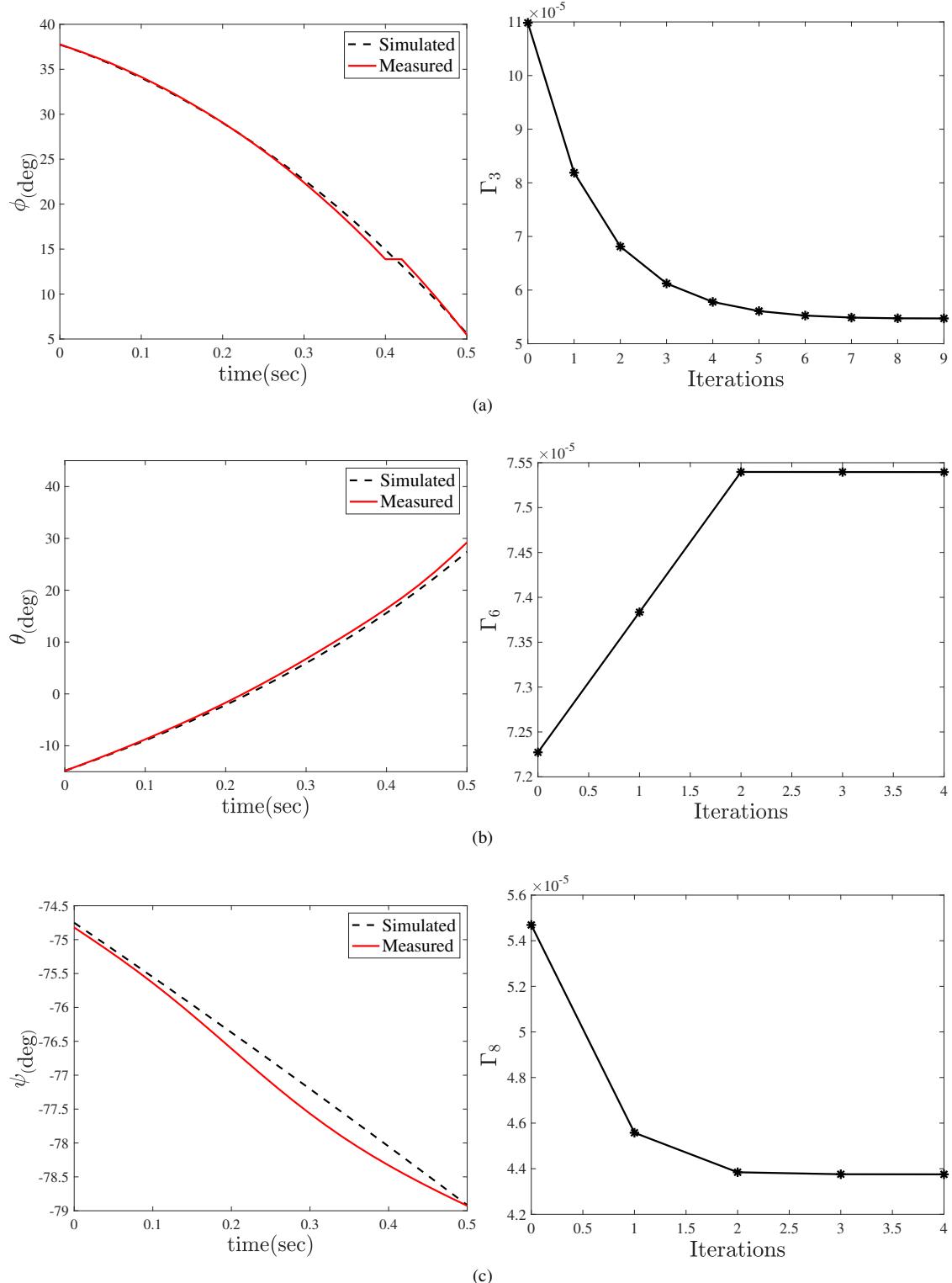


Figure 5: Identification process results when the quadrotor rotates about only one axis: (a) Identification of  $\Gamma_3$  in free roll motion. (b) Identification of  $\Gamma_6$  in free pitch motion. (c) Identification of  $\Gamma_8$  in free yaw motion.

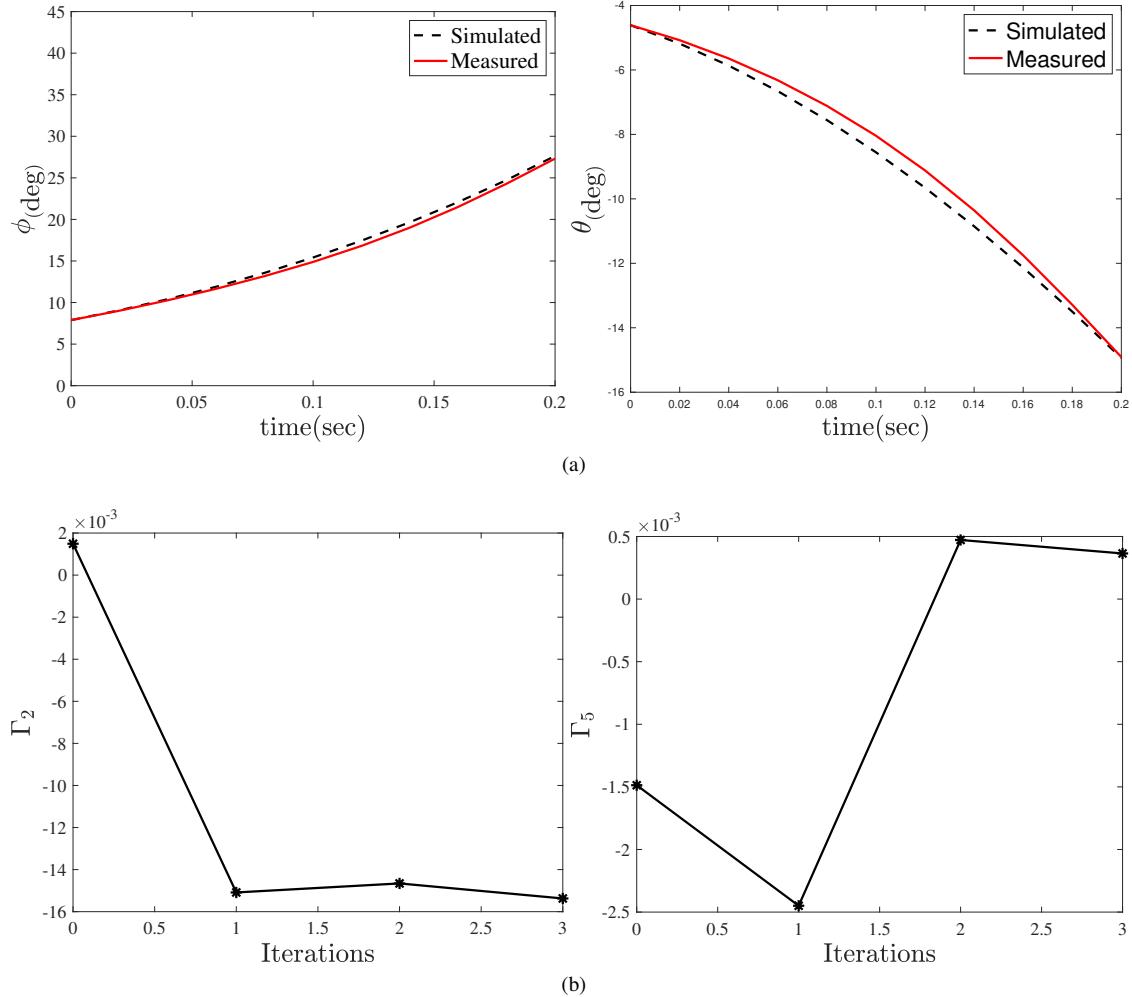


Figure 6: Identification process results when the quadrotor rotates about its roll and pitch axes: (a) Comparison of Simulation and experimental results. (b) Identification of  $\Gamma_2$  and  $\Gamma_5$ .

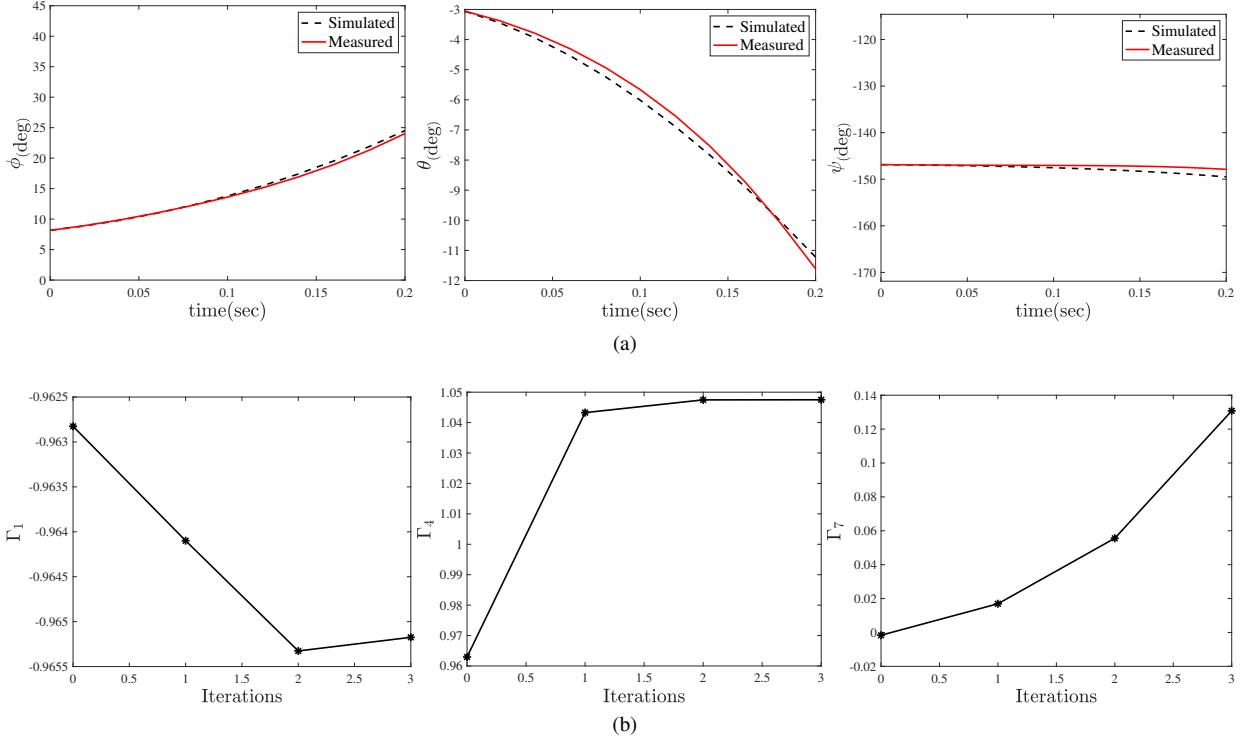


Figure 7: Identification process results when the quadrotor rotates about its roll, pitch, and yaw axes: (a) Comparison of Simulation and experimental results. (b) Identification of  $\Gamma_1$ ,  $\Gamma_4$  and  $\Gamma_7$  parameters.

### 5.2. Evaluation of LQIR-DG Performance

Here, the performance of the LQIR-DG controller algorithm is evaluated for regulation and tracking performance, disturbance rejection, and the impact of model uncertainty. Finally, the performance of the proposed controller is compared to the PID controller and variants of the LQR controller. The parameters of the PID controller are presented in Table 5.

Table 5: Parameters of PID Controller

Channel	$K_p$	$K_i$	$K_d$
Roll	2.0	0.5	1.0
Pitch	1.5	0.4	0.8
Yaw	3.0	0.6	1.2

#### 5.2.1. Performance of the LQIR-DG Controller

Here, the results of the LQIR-DG controller method are presented for the control loops of the Euler angles of the experimental platform in Figures 8 and 9. The performance of the proposed controller is evaluated in Figure 8. Figure 8 (a) compares the desired and output signals, i.e., the Euler angles during regulation. Figure 8 (b) compares the desired square wave input with a frequency of 0.02 Hz and an amplitude of 20 degrees with the output signals in the two-degree-of-freedom coupling mode.

Moreover, Figures 9 (a) and (b) show the rotational velocity command of the quadrotor in the regulation and tracking problems, respectively. These results demonstrate that the roll, pitch, and yaw angles are accurately controlled by the proposed approach.

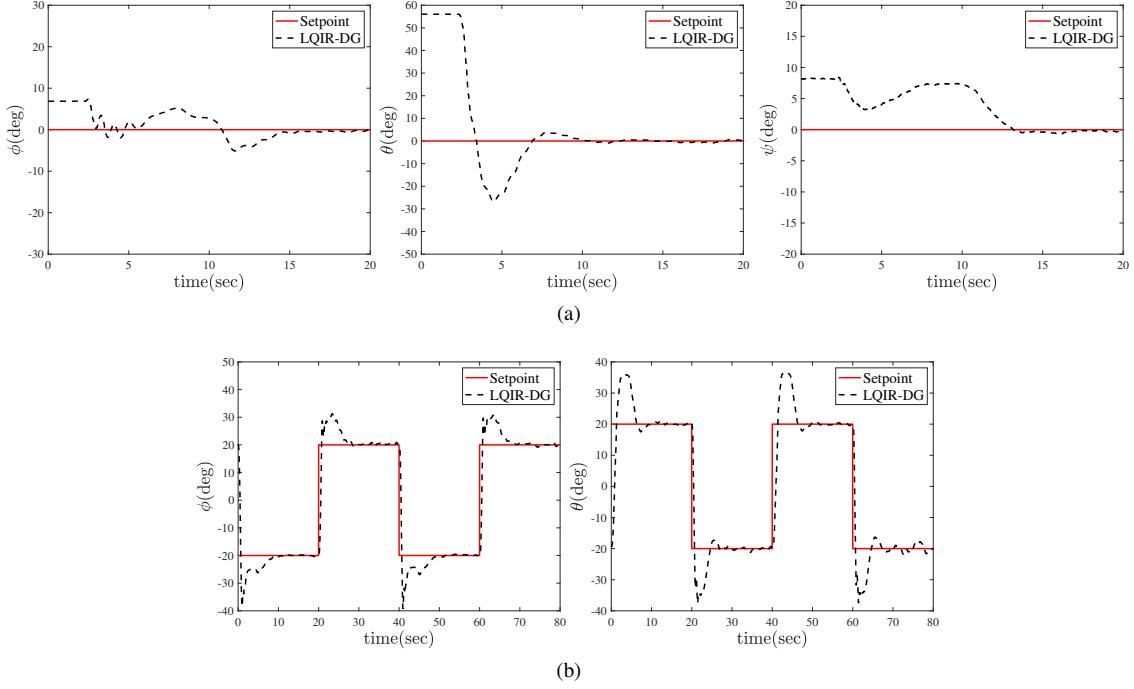


Figure 8: Comparison of Euler angles in (a) regulation (b) Tracking Conditions

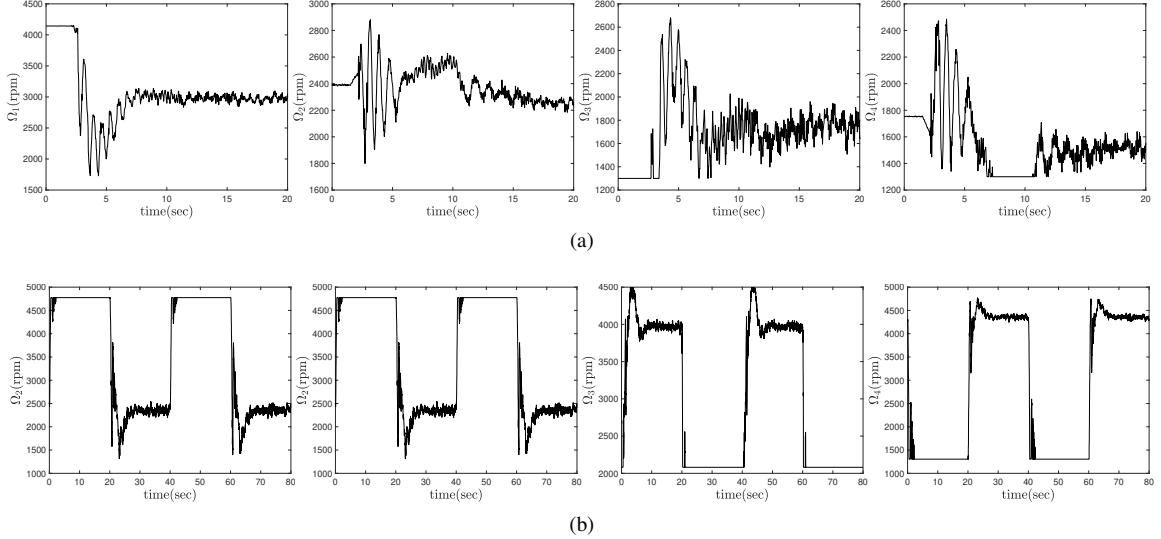


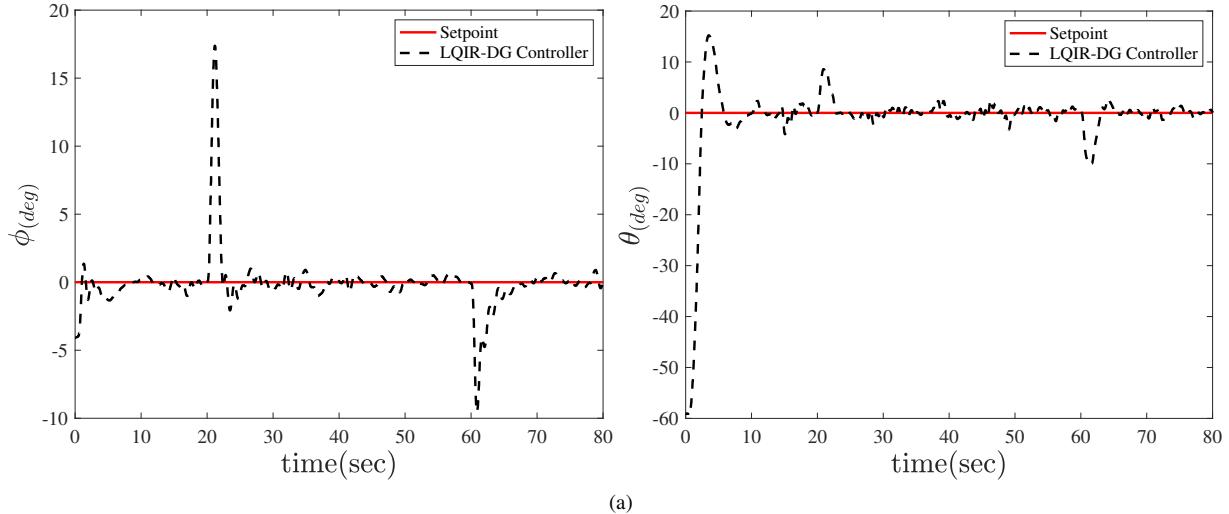
Figure 9: Temporal Evolution of Angular Velocity Commands for the LQIR-DG Controller

### 5.2.2. Performance of the LQIR-DG in Disturbance Rejection

The performance of the proposed controller in the input disturbance is shown in Figure 10. The input disturbance,  $d\Omega_i$ , is considered as a change in the command of the rotational velocity, modeled as:

$$d\Omega_1 = d\Omega_2 = -d\Omega_3 = -d\Omega_4 = \begin{cases} 500_{\text{rpm}} & 20 < t < 60 \\ 0 & \text{otherwise} \end{cases} \quad (39)$$

Figure 10 (a) illustrate the desired and the actual roll and pitch angles in the regulation problem. The results indicate that the LQIR-DG controller can stabilize the quadrotor platform when the input disturbance is present.



(a)

Figure 10: Comparison of the desired and actual roll and pitch angle in the presence of the input disturbance.

### 5.2.3. Investigating the Impact of Modeling Uncertainty

In this section, the performance of the LQIR-DG controller is investigated while considering uncertainty in the 3DoF experimental model. To achieve this, 50 and 100 grams were added to the roll and pitch axes, respectively, as shown in Figure 11. Figure 12 (a) compares the desired and the actual roll angle and Figure 12 (b) shows the desired and the actual pitch angle when the uncertainty of moments of inertia is presented. Moreover, Figure 12 (c) shows the rotational velocity command of the experimental platform in the presence of model uncertainty. The implementation results indicate that the LQIR-DG controller converges to the desired values in the presence of the modeling uncertainty.

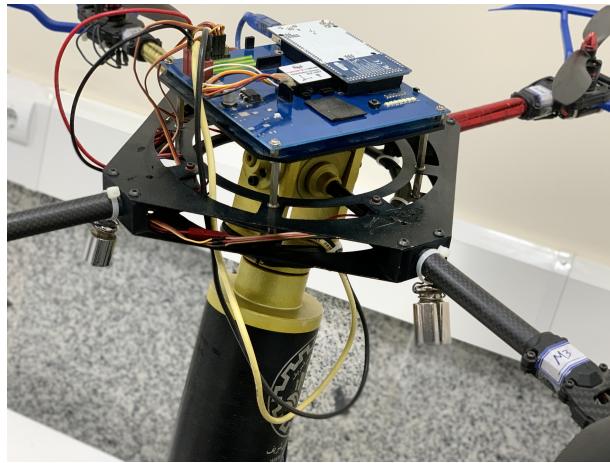


Figure 11: Quadrotor 3DoF Platform with Added Weight.

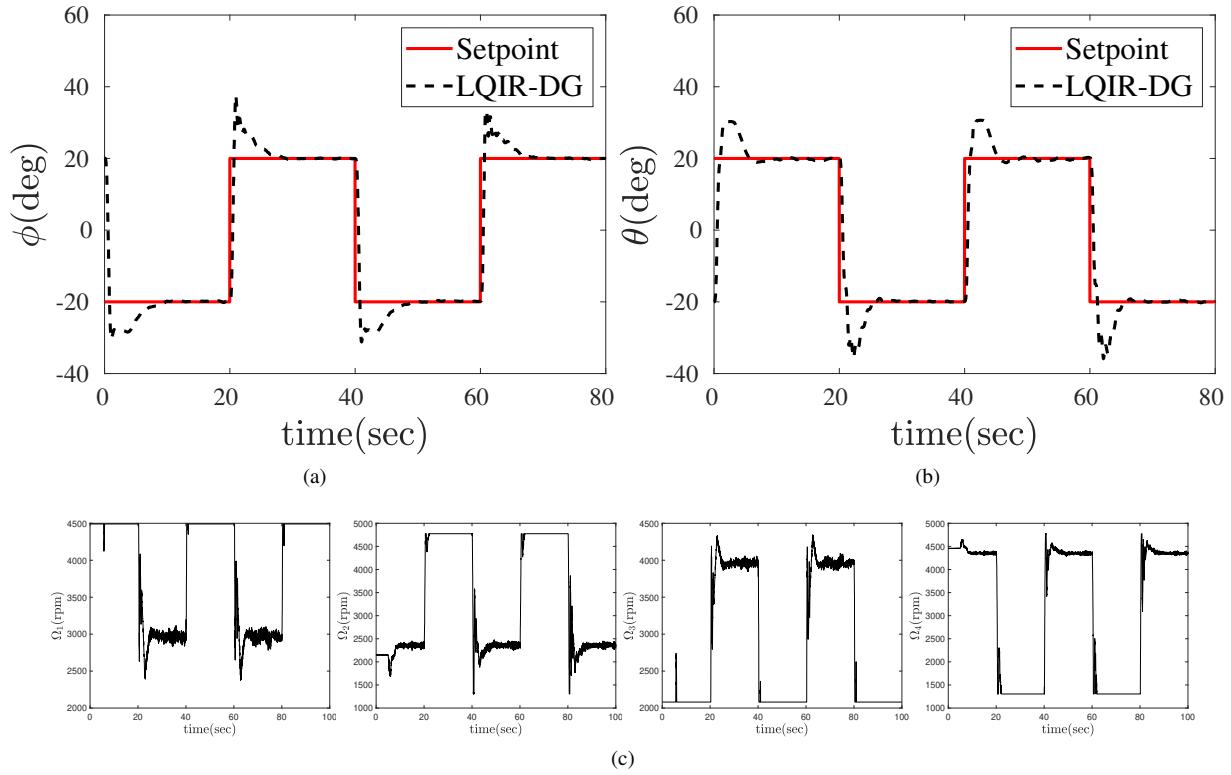


Figure 12: Comparison of the LQIR-DG controller when the uncertainty of moment of inertia is presented

#### 5.2.4. Comparison with the Control Strategies

Here, the LQIR-DG controller performance is compared with the PID controller and variant of the LQR strategies such as the LQR and LQIR in Figure 13. Figure 13 compare the desired and the actual attitude of the quadrotor platform in the presence of these controllers. Moreover, the RSME of the controllers of all the controllers, i.e, boxplot, are compared in Figure 14. The median of RMSE is shown in the crossline in the boxplot.

These results indicate that the LQIR-DG controller is able to provide an excellent transient response and rapid convergence relative to other controllers for attitude control of the quadrotor experimental platform.

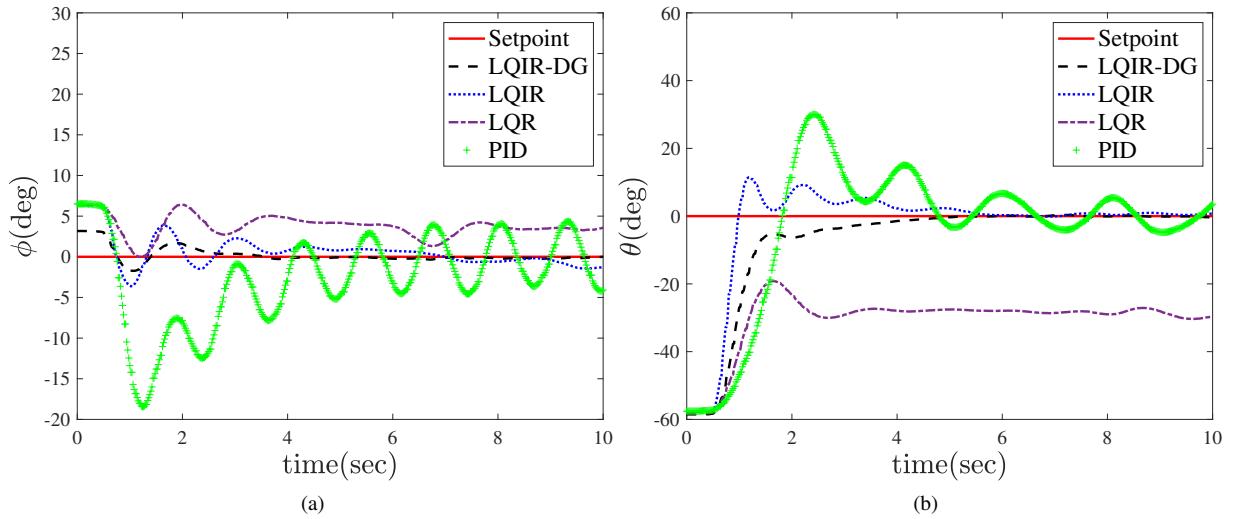


Figure 13: Comparison of LQIR-DG Controller to the LQR, LQIR, and PID in control of the quadrotor outputs: (a) roll angle (b) pitch angle.

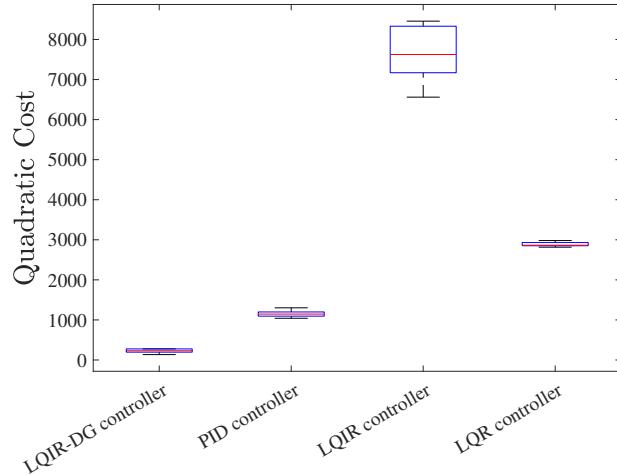


Figure 14: Comparative Analysis of the LQIR-DG Control Strategy versus LQR, LQIR, and PID Controllers using Quadratic Cost Function

## 6. Conclusion

In this paper, the LQIR-DG method, which is an optimal controller based on the game theory, was used in real-time for attitude control of an experimental platform for a quadrotor. To implement the proposed controller, an accurate dynamic model was considered for the quadrotor platform. Then, the model parameters were identified using the NSL method. For evaluation of the proposed method, the purpose of regulation and tracking problem was successfully performed. Moreover, the method's ability to reject the input disturbance and modelling error was investigated in the laboratory experiment. The implementation results illustrated the successful performance of the proposed method in attitude control for the quadratic platform.

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