Control of a Three Degree of Freedom Quadcopter Stand Using Linear Quadratic Integral Based on the Differential Game Theory

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***Abstract*—In this paper, a quadcopter stand with three degrees of freedom was controlled using game theory-based control. The first player tracks a desired input, and the second player creates a disturbance in the tracking of the first player to cause an error in the tracking. The move is chosen using the Nash equilibrium, which presupposes that the other player made the worst move.. In addition to being resistant to input interruptions, this method may also be resilient to modeling system uncertainty. This method evaluated the performance through simulation in the Simulink environment and implementation on a three-degree-of-freedom stand.**

***Index Terms*—Quadcopter, Differential Game, Game Theory, Nash Equilibrium, Three Degree of Freedom Stand, Model Base Design, Linear Quadratic Regulator**

I. Introduction

Quadcopter is a type of helicopter with four rotors.

II. Differential Game

Differential games are a series of problems that arise while examining and simulating dynamic systems in game theory. Differential equations simulate how a state variable or set of state variables changes over time.

A. An introduction to the differential game

It is considered that two players are involved in this paper. The space states of a continuous linear system are shown below.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B_1}\mathbf{u_1}(t) + \mathbf{B_2}\mathbf{u_2}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D_1}\mathbf{u_1}(t) + \mathbf{D_2}\mathbf{u_2}(t)\end{aligned} \tag{1}$$

Where $\mathbf{x}$ is the vector of the state variables, $\dot{\mathbf{x}}$ is the time derivative of the state vector, $\mathbf{u_1}$ is the firts player (controller) input vector, $\mathbf{u_2}$ is the second player (disturbance) input vector, $\mathbf{y}$ is the output vector, $\mathbf{A}$ is the state matrix, $\mathbf{B_1}$ is firts player the input matrix, $\mathbf{B_2}$ is the second player input matrix, $\mathbf{C}$ is the output matrix, $\mathbf{D_1}$ is first player the output matrix and $\mathbf{D_2}$ is second player the output matrix. Equation (1) demonstrates how both participants have an impact on the system's dynamics. The second player may progress toward the goal as a result of the first player's exertion, or vice versa. In this paper we consider the case that players do not cooperate in order to realize their goals. The situation where players do not work together (non-cooperative) to achieve their objectives is examined in the paper. In this case, every player knows at time $t \in [0, T]$ just the initial state $\mathbf{x_0}$ and the model structure. This scenario can be interpreted as the players simultaneously determining their actions, next submitting their actions. For the game (1,2), we will use the set of Nash equilibria. Formal Nash equilibrium is defined as follows. An admissible set of actions $(\boldsymbol{u_1}^*, \boldsymbol{u_2}^*)$ is a Nash equilibrium for the game (1,2); if for all admissible $(\boldsymbol{u_1}, \boldsymbol{u_2})$, the following inequalities hold:

$$J_1(\mathbf{u_1}^*, \mathbf{u_2}^*) \leq J_1(\mathbf{u_1}, \mathbf{u_2}^*)\ ,\ J_2(\mathbf{u_1}^*, \mathbf{u_2}^*) \leq J_2(\mathbf{u_1}^*, \mathbf{u_2}) \tag{2}$$

Here admissibility is meant in the sense that $\mathbf{u_i}(.)$ belongs to some restricted set, where this set depends on the information players have on the game, the set of strategies the players like to use to control the system, and the system (2) must have a unique solution.

B. LQDG controller

For the system described in equation (1), LQDG optimum control effort calculates from equation (3).

$$\mathbf{u_i}(t) = -\mathbf{R_{ii}}^{-1}\mathbf{B_i}^{\mathrm{T}}\mathbf{P_i}(t)\mathbf{x}(t) = -\mathbf{k_i}(t)\mathbf{x}(t), \quad i = 1, 2 \tag{3}$$

In equation 3, $\mathbf{k_i}$ is optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function (equation 4) of player number i.

$$J_i(\mathbf{u_1}, \mathbf{u_2}) = \int_0^T \bigg(\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q_i}\mathbf{x}(t) + \mathbf{u_i}^{\mathrm{T}}(t)\mathbf{R_{ii}}\mathbf{u_i}(t) + \mathbf{u_j}^{\mathrm{T}}(t)\mathbf{R_{ij}}\mathbf{u_j}(t)\bigg)\mathrm{d}t \tag{4}$$

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$\mathbf{P_i}$ is found by solving the continuous time couple Riccati differential equation:

$$\begin{aligned}\dot{\mathbf{P}}_{\mathbf{1}}(t) &= -\mathbf{A}^{\mathrm{T}}\mathbf{P_1}(t) - \mathbf{P_1}(t)\mathbf{A} - \mathbf{Q_1} + \mathbf{P_1}(t)\mathbf{S_1}(t)\mathbf{P_1}(t) + \\ &\quad \mathbf{P_1}(t)\mathbf{S_2}(t)\mathbf{P_2}(t) \\ \dot{\mathbf{P}}_{\mathbf{2}}(t) &= -\mathbf{A}^{\mathrm{T}}\mathbf{P_2}(t) - \mathbf{P_2}(t)\mathbf{A} - \mathbf{Q_2} + \mathbf{P_2}(t)\mathbf{S_2}(t)\mathbf{P_2}(t) + \\ &\quad \mathbf{P_2}(t)\mathbf{S_1}(t)\mathbf{P_1}(t)\end{aligned} \tag{5}$$

Using the shorthand notation $\boldsymbol{S_i} := \boldsymbol{B_i}\boldsymbol{R_{ii}}^{-1}\boldsymbol{B_i}^{\mathrm{T}}$.

C. LQIDG controller

The absence of an integrator in the LQDG controller may result in steady-state errors due to disturbances or modeling errors. The LQIDG controller is based on the LQDG controller to eliminate this error.

The LQIDG controller adds the integral of the difference between the system output and the desired value to the state vector. Therefore, The augmented space states of a continuous linear system are shown below.

$$\mathbf{x_a} = \begin{bmatrix} \mathbf{x_d} - \mathbf{x} \\ \int (\mathbf{y_d} - \mathbf{y}) \end{bmatrix} \tag{6}$$

Where $\mathbf{x_a}$ is the vector of augmented state variables, $\mathbf{x_d}$ is the vector of the desired state variables, and $\mathbf{y_d}$ is the desired output vector. As a result, the state vector and the output vector are equal.

$$\mathbf{y} = \mathbf{x} \tag{7}$$

Following is a representation of the system dynamics in the augmented state space.

$$\dot{\mathbf{x}}_{\boldsymbol{a}}(t) = \mathbf{A_a}\mathbf{x_a}(t) + \mathbf{B_{a_1}}\mathbf{u_{a_1}}(t) + \mathbf{B_{a_2}}\mathbf{u_{a_2}}(t) \tag{8}$$

Where matrices $\mathbf{A_a}$ and $\mathbf{B_a}$ are defined as follows:

$$\mathbf{A_a} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{C} & 0 \end{bmatrix}, \quad \mathbf{B_a} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \tag{9}$$

By introducing a new space state for the system, the remaining design phases of the LQIDG controller are comparable to those of the LQDG controller. LQIDG optimum control effort calculates from equation (10).

$$\mathbf{u_i}(t) = -\mathbf{R_{ii}}^{-1}\mathbf{B_{a_i}}^{\mathrm{T}}\mathbf{P_{a_i}}(t)\mathbf{x_a}(t) = -\mathbf{K_{a_i}}(t)\mathbf{x_a}(t),\ i = 1,2 \tag{10}$$

In equation 10, $\mathbf{K_{a_i}}$ is optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function (equation 11) of player number i.

$$\begin{aligned}J_i(\boldsymbol{u_1}, \boldsymbol{u_2}) = \int_0^T \Big(&\boldsymbol{x_a}^{\mathrm{T}}(t)\boldsymbol{Q_i}\boldsymbol{x_a}(t) + \boldsymbol{u_i}^{\mathrm{T}}(t)\boldsymbol{R_{ii}}\boldsymbol{u_i}(t) + \\ &\boldsymbol{u_j}^{\mathrm{T}}(t)\boldsymbol{R_{ij}}\boldsymbol{u_j}(t)\Big)\mathrm{d}t\end{aligned} \tag{11}$$

$\dot{\mathbf{P}}_{\boldsymbol{a_i}}$ is found by solving the continuous time couple Riccati differential equation:

$$\begin{aligned}\dot{\mathbf{P}}_{\boldsymbol{a_1}}(t) &= -\mathbf{A_a}^{\mathrm{T}}\mathbf{P_{a_1}}(t) - \mathbf{P_{a_1}}(t)\mathbf{A_a} - \mathbf{Q_1} + \\ &\quad \mathbf{P_{a_1}}(t)\mathbf{S_{a_1}}(t)\mathbf{P_{a_1}}(t) + \mathbf{P_{a_1}}(t)\mathbf{S_{a_2}}(t)\mathbf{P_{a_2}}(t) \\ \dot{\mathbf{P}}_{\boldsymbol{a_2}}(t) &= -\mathbf{A_a}^{\mathrm{T}}\mathbf{P_{a_2}}(t) - \mathbf{P_{a_2}}(t)\mathbf{A_a} - \mathbf{Q_2} + \\ &\quad \mathbf{P_{a_2}}(t)\mathbf{S_{a_2}}(t)\mathbf{P_{a_2}}(t) + \mathbf{P_{a_2}}(t)\mathbf{S_{a_1}}(t)\mathbf{P_{a_1}}(t)\end{aligned} \tag{12}$$

Using the shorthand notation $\boldsymbol{S_{a_i}} := \boldsymbol{B_{a_i}}\boldsymbol{R_{ii}}^{-1}\boldsymbol{B_{a_i}}^{\mathrm{T}}$.

III. Mathematical modeling