Linear Quadratic Integral Differential Game applied to the Real-time Control of a 3DoF Experimental setup of a Quadrotor

\*Note: Sub-titles are not captured in Xplore and should not be used

line 1: 1st Given Name Surname   
line 2: *dept. name of organization   
(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCID

line 1: 4th Given Name Surname  
line 2: *dept. name of organization*  
*(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCIDline 1: 2nd Given Name Surname  
line 2: *dept. name of organization   
(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCID

line 1: 5th Given Name Surname  
line 2: *dept. name of organization   
(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCIDline 1: 3rd Given Name Surname  
line 2: *dept. name of organization   
(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCID

line 1: 6th Given Name Surname  
line 2: *dept. name of organization   
(of Affiliation)*  
line 3: *name of organization   
(of Affiliation)*line 4: City, Country  
line 5: email address or ORCID

*Abstract*—In this study, a linear quadratic with integral action based on the differential game theory is implemented real-time for a three-degree-of-freedom experimental setup of a quadrotor. For this purpose, two players are considered for each of roll, pitch, and yaw channels of the quadrotor. First player searches the best control command for each channels of the setup of a quadrotor based on the minimization a quadratic criterion; when the worst disturbances are produced by the second player. Performance of the proposed controller is evaluated in level flight and compared to the LQR controller.

Keywords—Linear Quadratic Differential Game, Quadrotor, Real-time, 3DoF Experimental setup, Optimal Control, Robust Control

# Introduction

A quadcopter is a type of helicopter with four rotors. Quadcopters have extensive applications due to their excellent maneuverability and the possibility of hover flight with high balance. In recent years, companies, universities, and research centers have attracted more to this type of UAV. In this way, the facilities and the flight of these UAVs are continuously improving. Quadcopters are widely used in research, military, imaging, recreation, and agriculture. Mathematical models are used in game theory to examine how rational, intelligent beings cooperate or compete. Game theory can be applied to pursuit and evasion as one of its broad applications. There can be two [1] or more players [2] involved in the pursuit-evasion. Pursuit-evasion can occur indoors as well [3]. In some cases, machine learning and differential games pursuit-evade [4]. Players may play different roles in differential games, such as protecting some targets [5]. The differential game’s ability to examine the actions of two or more players makes it powerful. Player cooperation can be used through swarm platooning [6]. Multi-agent [7] and self-driving automobiles [8] motion planning are two other applications of player cooperation.

Due to the widespread use of quadrotors, their control has become an important issue. In order to control quadrotors, neural networks [9] and machine learning [10] methods have been used. Two uses for quadrotor control include swarm flying [12] and motion planning [11]. In [13], Kyuman Lee, Daegyun Choi, and Donghoon Kim worked on Motion Planning for Quadcopters in Three-Dimensional Dynamic Environments with Potential Fields-Aided. To avoid collisions with obstacles, the controller should control the quadrotor to prevent collisions [14].

# MATHEMATICAL MODELING

In this section, a nonlinear dynamic is presented for an experimental setup of a quadrotor, as shown in Fig.1. The quadrotor is free to rotate about its roll, pitch, and yaw axes. The Euler angle angles and angular velocities along three orthogonal axes are measured simultaneously using Attitude and Heading Reference Systems (AHRS). LQDG utilizes these noisy measurements for real-time control of the Euler angles. The block diagram of the control purpose is shown in Fig.2.

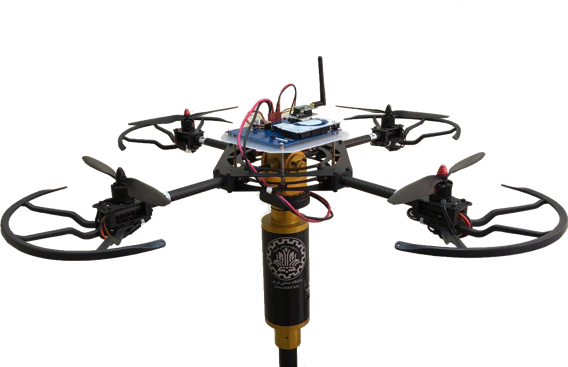


Figure 3DoF experimental setup of a quadrotor

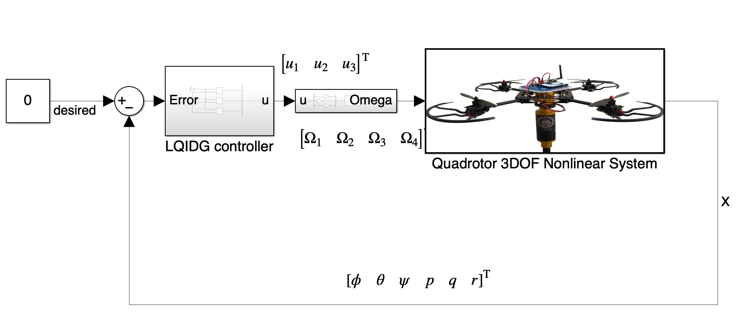


Figure block diagram of the control purpose

## block diagram of the control purpose

The schematic of a quadrotor is given in Fig.3. Each rotor is considered a rigid disk is rotating about the axis ZB in the body fixed frame with an angular velocity Ωi. Rotors 1 and 3 rotate in the same direction, i.e., counterclockwise, while rotors 2 and 4 rotate in the opposite direction, i.e., clockwise, to cancel yawing moment of the quadrotor.

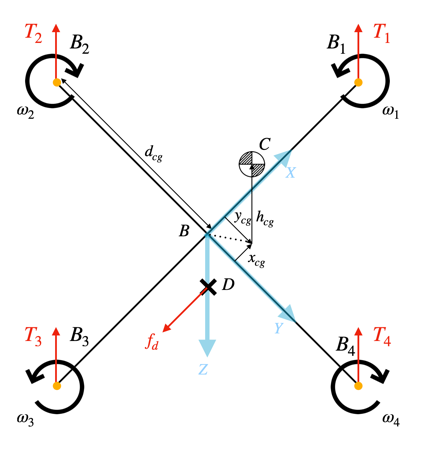


Figure Configuration of the quadrotor

## Modeling of Quadrotor

The dynamic model of the quadrotor, obtained from the Newton-Euler method, is stated as follows [15], [16]:



where, d subscript roll, d subscript pitch, and d subscript yaw are roll, pitch, and yaw moments, generated by the disturbance and left parenthesis p comma q comma r right parenthesis are the angular velocities, and left parenthesis phi comma theta comma psi right parenthesis spaceare roll, pitch, and yaw angles. The relation between Euler angles rates and the angular body rates are obtained as follows:



where I subscript x x end subscript comma space I subscript y y end subscript, and I subscript z z end subscriptare the principal moment of inertia and I subscript rotor is the inertia of a rotor about its axis. Moreover, capital omega subscript r, called the overall residual propeller angular speed, is computed as:



The control inputs uroll , upitch , and uyaw are roll, pitch, and yaw moments, generated by the propellers, defined as:



Also, b and d are thrust and drag coefficients, respectively, and d subscript c g end subscriptis the horizontal distance of each rotor from the center of gravity, as shown in Fig.3. Therefore, the angular velocity commands are obtained as:



where capital omega subscript meanis the average angular velocity of the rotors. Here, the state-space model of the experimental setup of the quadrotor is presented for the control purpose. by defining x subscript 1 equals p comma space x subscript 2 equals q comma space x subscript 3 equals r comma space x subscript 4 equals ϕ comma space x subscript 5 equals theta, and x subscript 6 equals psi; the model of the experimental setup in state-space form are expressed as:



The measurement model is written as:



The continuous-time linear model is utilized to drive the control commands on the quadrotor. The linear state-space model is denoted as:



where d is the unknown input. A, B, and bold B subscript bold d are the system input and unknown input matrices, respectively. Moreover, the measurements equation is stated as:



where C is the output matrix. Also, D and bold D subscript bold d are the

feedforward matrices due to known and unknown inputs, respectively. According to eq ? - ?, the linear dynamic model around the equilibrium points open parentheses bold x subscript e bold space bold equals bold space 0 bold space and space bold u bold space equals space 0 close parenthesesof the quadrotor setup is denoted as:



where Error converting from MathML to accessible text. , Error converting from MathML to accessible text., and Error converting from MathML to accessible text.

.Also d= x ,is the........Moreover, the state and input matrices are derived as:





# Prepare Your Paper Before Styling

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections A-D below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not use hard tabs, and limit use of hard returns to only one return at the end of a paragraph. Do not add any kind of pagination anywhere in the paper. Do not number text heads-the template will do that for you.

## Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, sc, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

## Units

* Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive”.
* Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
* Do not mix complete spellings and abbreviations of units: “Wb/m2” or “webers per square meter”, not “webers/m2”. Spell out units when they appear in text: “. . . a few henries”, not “. . . a few H”.

Identify applicable funding agency here. If none, delete this text box.

* Use a zero before decimal points: “0.25”, not “.25”. Use “cm3”, not “cc”. (*bullet list*)

## Equations

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled.

Number equations consecutively. Equation numbers, within parentheses, are to position flush right, as in (1), using a right tab stop. To make your equations more compact, you may use the solidus ( / ), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

*a**b* 

Note that the equation is centered using a center tab stop. Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(1)”, not “Eq. (1)” or “equation (1)”, except at the beginning of a sentence: “Equation (1) is . . .”

## Some Common Mistakes

* The word “data” is plural, not singular.
* The subscript for the permeability of vacuum **0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.
* In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
* A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
* Do not use the word “essentially” to mean “approximately” or “effectively”.
* In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
* Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
* Do not confuse “imply” and “infer”.
* The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
* There is no period after the “et” in the Latin abbreviation “et al.”.
* The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [7].

# Using the Template

After the text edit has been completed, the paper is ready for the template. Duplicate the template file by using the Save As command, and use the naming convention prescribed by your conference for the name of your paper. In this newly created file, highlight all of the contents and import your prepared text file. You are now ready to style your paper; use the scroll down window on the left of the MS Word Formatting toolbar.

## Authors and Affiliations

**The template is designed for, but not limited to, six authors.** A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

### For papers with more than six authors: Add author names horizontally, moving to a third row if needed for more than 8 authors.

### For papers with less than six authors: To change the default, adjust the template as follows.

#### Selection: Highlight all author and affiliation lines.

#### Change number of columns: Select the Columns icon from the MS Word Standard toolbar and then select the correct number of columns from the selection palette.

#### Deletion: Delete the author and affiliation lines for the extra authors.

## Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced. Styles named “Heading 1”, “Heading 2”, “Heading 3”, and “Heading 4” are prescribed.

## Figures and Tables

#### Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. 1”, even at the beginning of a sentence.

1. Table Type Styles

| Table Head | Table Column Head | | |
| --- | --- | --- | --- |
| Table column subhead | Subhead | Subhead |
| copy | More table copya |  |  |

1. Sample of a Table footnote. (*Table footnote*)
2. Example of a figure caption. (*figure caption*)

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

##### Acknowledgment *(Heading 5)*

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

##### References

The template will number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first ...”

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors’ names; do not use “et al.”. Papers that have not been published, even if they have been submitted for publication, should be cited as “unpublished” [4]. Papers that have been accepted for publication should be cited as “in press” [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

1. G. Eason, B. Noble, and I. N. Sneddon, “On certain integrals of Lipschitz-Hankel type involving products of Bessel functions,” Phil. Trans. Roy. Soc. London, vol. A247, pp. 529–551, April 1955. *(references)*
2. J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.
3. I. S. Jacobs and C. P. Bean, “Fine particles, thin films and exchange anisotropy,” in Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.
4. K. Elissa, “Title of paper if known,” unpublished.
5. R. Nicole, “Title of paper with only first word capitalized,” J. Name Stand. Abbrev., in press.
6. Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, “Electron spectroscopy studies on magneto-optical media and plastic substrate interface,” IEEE Transl. J. Magn. Japan, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].
7. M. Young, The Technical Writer’s Handbook. Mill Valley, CA: University Science, 1989.

**IEEE conference templates contain guidance text for composing and formatting conference papers. Please ensure that all template text is removed from your conference paper prior to submission to the conference. Failure to remove template text from your paper may result in your paper not being published.**