Linear Quadratic Integral Differential Game applied to the Real-time Control of a 3DoF Experimental setup of a Quadrotor

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*Abstract*—In this study, a linear quadratic with integral action based on the differential game theory is implemented real-time for a three-degree-of-freedom experimental setup of a quadrotor. For this purpose, two players are considered for each of roll, pitch, and yaw channels of the quadrotor. First player searches the best control command for each channels of the setup of a quadrotor based on the minimization a quadratic criterion; when the worst disturbances are produced by the second player. Performance of the proposed controller is evaluated in level flight and compared to the LQR controller.

Keywords—Linear Quadratic Differential Game, Quadrotor, Real-time, 3DoF Experimental setup, Optimal Control, Robust Control

# Introduction

A quadcopter is a type of helicopter with four rotors. Quadcopters have extensive applications due to their excellent maneuverability and the possibility of hover flight with high balance. In recent years, companies, universities, and research centers have attracted more to this type of UAV. In this way, the facilities and the flight of these UAVs are continuously improving. Quadcopters are widely used in research, military, imaging, recreation, and agriculture. Mathematical models are used in game theory to examine how rational, intelligent beings cooperate or compete. Game theory can be applied to pursuit and evasion as one of its broad applications. There can be two [1] or more players [2] involved in the pursuit-evasion. Pursuit-evasion can occur indoors as well [3]. In some cases, machine learning and differential games pursuit-evade [4]. Players may play different roles in differential games, such as protecting some targets [5]. The differential game’s ability to examine the actions of two or more players makes it powerful. Player cooperation can be used through swarm platooning [6]. Multi-agent [7] and self-driving automobiles [8] motion planning are two other applications of player cooperation.

Due to the widespread use of quadrotors, their control has become an important issue. In order to control quadrotors, neural networks [9] and machine learning [10] methods have been used. Two uses for quadrotor control include swarm flying [12] and motion planning [11]. In [13], Kyuman Lee, Daegyun Choi, and Donghoon Kim worked on Motion Planning for Quadcopters in Three-Dimensional Dynamic Environments with Potential Fields-Aided. To avoid collisions with obstacles, the controller should control the quadrotor to prevent collisions [14].

# MATHEMATICAL MODELING

In this section, a nonlinear dynamic is presented for an experimental setup of a quadrotor, as shown in Fig.1. The quadrotor is free to rotate about its roll, pitch, and yaw axes. The Euler angle angles and angular velocities along three orthogonal axes are measured simultaneously using Attitude and Heading Reference Systems (AHRS). LQDG utilizes these noisy measurements for real-time control of the Euler angles. The block diagram of the control purpose is shown in Fig.2.

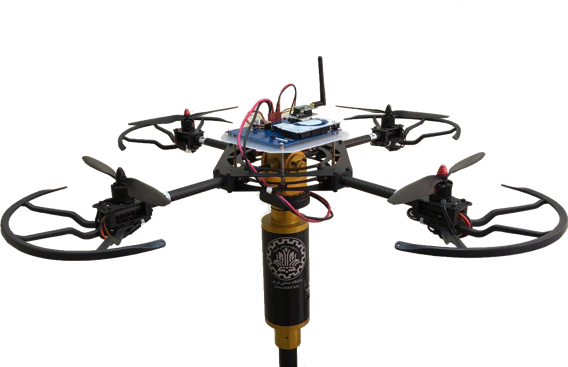


Figure 1 3DoF experimental setup of a quadrotor

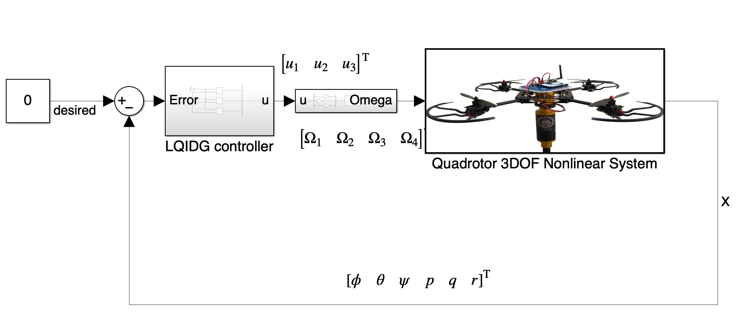


Figure 2 block diagram of the control purpose

## block diagram of the control purpose

The schematic of a quadrotor is given in Fig.3. Each rotor is considered a rigid disk is rotating about the axis ZB in the body fixed frame with an angular velocity Ωi. Rotors 1 and 3 rotate in the same direction, i.e., counterclockwise, while rotors 2 and 4 rotate in the opposite direction, i.e., clockwise, to cancel yawing moment of the quadrotor.

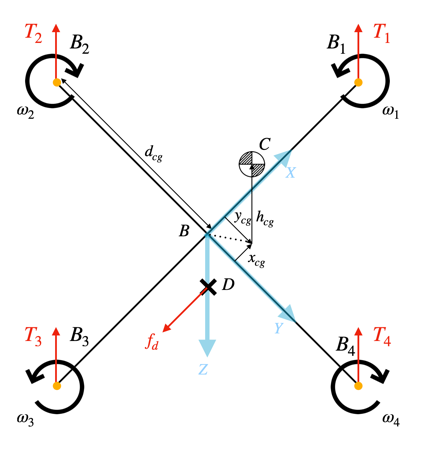


Figure 3 Configuration of the quadrotor

## Modeling of Quadrotor

The dynamic model of the quadrotor, obtained from the Newton-Euler method, is stated as follows [15], [16]:

where droll, dpitch, and dyaw are roll, pitch, and yaw moments, generated by the disturbance and (p,q,r) are the angular velocities, and (φ,θ,ψ) are roll, pitch, and yaw angles. The relation between Euler angles rates and the angular body rates are obtained as follows:

where Ixx, Iyy, and Izz are the principal moment of inertia and Irotor is the inertia of a rotor about its axis. Moreover, Ωr, called the overall residual propeller angular speed, is computed as:

The control inputs uroll , upitch , and uyaw are roll, pitch, and

yaw moments, generated by the propellers, defined as:

Also, b and d are thrust and drag coefficients, respectively, and dcg is the horizontal distance of each rotor from the center of gravity, as shown in Fig.3. Therefore, the angular velocity commands are obtained as:

where Ωmean is the average angular velocity of the rotors. Here, the state-space model of the experimental setup of the quadrotor is presented for the control purpose. by defining x1 =p,x2 =q,x3 =r,x4 =φ,x5 =θ,andx6 =ψ; the model of the experimental setup in state-space form are expressed as:

The measurement model is written as:

The continuous-time linear model is utilized to drive the control commands on the quadrotor. The linear state-space model is denoted as:

where d is the unknown input. A, B, and Bd are the system input and unknown input matrices, respectively. Moreover, the measurements equation is stated as:

where C is the output matrix. Also, D and D are the

feedforward matrices due to known and unknown inputs, respectively. According to eq ? - ?, the linear dynamic model around the equilibrium points (xe = 0 and u = 0) of the quadrotor setup is denoted as:

where bold X subscript roll =

.Also d= x ,is the........Moreover, the state and input matrices are derived as:

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*a**b* 

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