Linear Quadratic Integral Differential Game applied to the Real-time Control of a 3DoF Experimental setup of a Quadrotor

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*Abstract*—In this study, a linear quadratic with integral action based on the differential game theory is implemented real-time for a three-degree-of-freedom experimental setup of a quadrotor. For this purpose, two players are considered for each of roll, pitch, and yaw channels of the quadrotor. First player searches the best control command for each channels of the setup of a quadrotor based on the minimization a quadratic criterion; when the worst disturbances are produced by the second player. Performance of the proposed controller is evaluated in level flight and compared to the LQR controller.

Keywords—Linear Quadratic Differential Game, Quadrotor, Real-time, 3DoF Experimental setup, Optimal Control, Robust Control

# Introduction

A quadcopter is a type of helicopter with four rotors. Quadcopters have extensive applications due to their excellent maneuverability and the possibility of hover flight with high balance. In recent years, companies, universities, and research centers have attracted more to this type of UAV. In this way, the facilities and the flight of these UAVs are continuously improving. Quadcopters are widely used in research, military, imaging, recreation, and agriculture. Mathematical models are used in game theory to examine how rational, intelligent beings cooperate or compete. Game theory can be applied to pursuit and evasion as one of its broad applications. There can be two [1] or more players [2] involved in the pursuit-evasion. Pursuit-evasion can occur indoors as well [3]. In some cases, machine learning and differential games pursuit-evade [4]. Players may play different roles in differential games, such as protecting some targets [5]. The differential game’s ability to examine the actions of two or more players makes it powerful. Player cooperation can be used through swarm platooning [6]. Multi-agent [7] and self-driving automobiles [8] motion planning are two other applications of player cooperation.

Due to the widespread use of quadrotors, their control has become an important issue. In order to control quadrotors, neural networks [9] and machine learning [10] methods have been used. Two uses for quadrotor control include swarm flying [12] and motion planning [11]. In [13], Kyuman Lee, Daegyun Choi, and Donghoon Kim worked on Motion Planning for Quadcopters in Three-Dimensional Dynamic Environments with Potential Fields-Aided. To avoid collisions with obstacles, the controller should control the quadrotor to prevent collisions [14].

# MATHEMATICAL MODELING

In this section, a nonlinear dynamic is presented for an experimental setup of a quadrotor, as shown in Fig.1. The quadrotor is free to rotate about its roll, pitch, and yaw axes. The Euler angle angles and angular velocities along three orthogonal axes are measured simultaneously using Attitude and Heading Reference Systems (AHRS). LQDG utilizes these noisy measurements for real-time control of the Euler angles. The block diagram of the control purpose is shown in Fig.2.

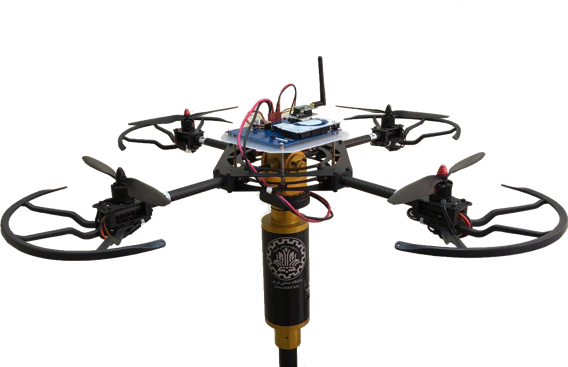


Figure 1 3DoF experimental setup of a quadrotor

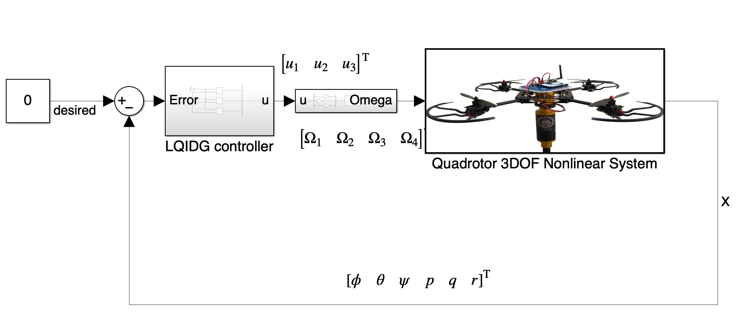


Figure 2 block diagram of the control purpose

## block diagram of the control purpose

The schematic of a quadrotor is given in Fig.3. Each rotor is considered a rigid disk is rotating about the axis ZB in the body fixed frame with an angular velocity Ωi. Rotors 1 and 3 rotate in the same direction, i.e., counterclockwise, while rotors 2 and 4 rotate in the opposite direction, i.e., clockwise, to cancel yawing moment of the quadrotor.

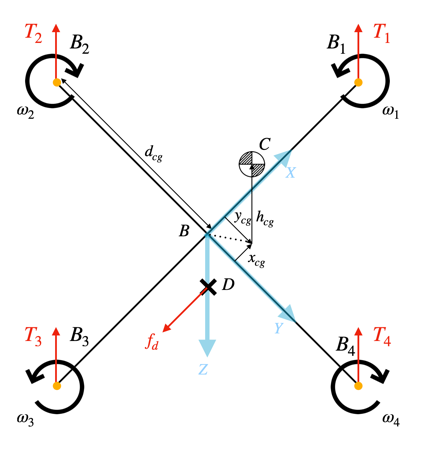


Figure 3 Configuration of the quadrotor

## Modeling of Quadrotor

The dynamic model of the quadrotor, obtained from the Newton-Euler method, is stated as follows [15], [16]:



where, d subscript roll, d subscript pitch, and d subscript yaw are roll, pitch, and yaw moments, generated by the disturbance and left parenthesis p comma q comma r right parenthesis are the angular velocities, and left parenthesis phi comma theta comma psi right parenthesis spaceare roll, pitch, and yaw angles. The relation between Euler angles rates and the angular body rates are obtained as follows:



where I subscript x x end subscript comma space I subscript y y end subscript, and I subscript z z end subscriptare the principal moment of inertia and I subscript rotor is the inertia of a rotor about its axis. Moreover, capital omega subscript r, called the overall residual propeller angular speed, is computed as:



The control inputs uroll , upitch , and uyaw are roll, pitch, and yaw moments, generated by the propellers, defined as:



Also, b and d are thrust and drag coefficients, respectively, and d subscript c g end subscriptis the horizontal distance of each rotor from the center of gravity, as shown in Fig.3. Therefore, the angular velocity commands are obtained as:



where capital omega subscript meanis the average angular velocity of the rotors. Here, the state-space model of the experimental setup of the quadrotor is presented for the control purpose. by defining x subscript 1 equals p comma space x subscript 2 equals q comma space x subscript 3 equals r comma space x subscript 4 equals ϕ comma space x subscript 5 equals theta, and x subscript 6 equals psi; the model of the experimental setup in state-space form are expressed as:



The measurement model is written as:



The continuous-time linear model is utilized to drive the control commands on the quadrotor. The linear state-space model is denoted as:



where d is the unknown input. A, B, and bold B subscript bold d are the system input and unknown input matrices, respectively. Moreover, the measurements equation is stated as:



where C is the output matrix. Also, D and bold D subscript bold d are the

feedforward matrices due to known and unknown inputs, respectively. According to eq ? - ?, the linear dynamic model around the equilibrium points open parentheses bold x subscript e bold space bold equals bold space 0 bold space and space bold u bold space equals space 0 close parenthesesof the quadrotor setup is denoted as:



where Error converting from MathML to accessible text. , Error converting from MathML to accessible text., and Error converting from MathML to accessible text.

.Also d= x ,is the........Moreover, the state and input matrices are derived as:





# DIFFERENTIAL GAME

Differential games are a series of problems that arise while examining and simulating dynamic systems in game theory. Differential equations simulate how a state variable or set of state variables changes over time.

## Differential Game Usage in a Quadrotor Control Loop

This paper describes the state of two players in different loop control of a quadrotor. Three groups of players are identified: two players for roll loop control, two players for pitch loop control, and two players for yaw loop control. The space state of roll, pitch, and yaw are defined below.



Where x is the vector of the state variables, x ̇ is the time derivative of the state vector, u is the controller input vector, ud is the disturbance input vector, y is the output vector, A is the state matrix, B is the controller input matrix, Bd is the disturbance input matrix, C is the output matrix, D is controller the output matrix and Dd is disturbance the output matrix. Equation (23) demonstrates how both participants have an impact on the quadrotor’s dynamics. The second player may progress toward the goal as a result of the first player’s exertion, or vice versa. This paper considers the case that players do not cooperate in order to realize their goals. In this case, every player knows at time t ∈ [0, T ] just the initial state x0 and the model structure. For the game between two players in each loop control, the set of Nash equilibria is used. Formal Nash equilibrium is defined as follows. An admissible set of actions (u ∗ , u ∗ ) is a Nash equilibrium for the game between two player in each loop control; if for all admissible (ui , uid ), the following inequalities hold:



## LQDG controller

For the each control loop described in equation (23), LQDG optimum control effort calculates from equation (25).



In equation (25), Ki is the optimal feedback gain. Assuming that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function equation (26) of controller player for each control loop of quadrotor.



Here the matrices Qi and Ri are assumed to be symmetric and Ri positive definite. Pi is found by solving the continuous time couple Riccati differential equation:

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Using the shorthand notation Si := Bi Ri −1 Bi T .

## LQIDG controller

The absence of an integrator in the LQDG controller may result in steady-state errors due to disturbances or modeling errors. The LQIDG controller is based on the LQDG controller to eliminate this error.

The LQIDG controller adds the integral of the difference between the system output and the desired value to the state vector. Therefore, The augmented space states of a continuous linear system are shown below.



Where xa is the vector of augmented state variables, xd is the vector of the desired state variables, and yd is the desired output vector. As a result, the state vector and the output vector are equal.



The following represents the system dynamics in the augmented state space.



Where matrices Aa and Ba are defined as follows:



By introducing a new space state for the system, the remaining design phases of the LQIDG controller are comparable to those of the LQDG controller. LQIDG optimum control effort calculates from equation (32).



In equation (32), Kai is the optimal feedback gain. Assum- ing that the other players will make their worst move, this gain is calculated to minimize the quadratic cost function, equation (33), of player number i.



is found by solving the continuous time couple Riccati differential equation:

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Using the shorthand notation SR −1B T.

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*a**b* 

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2. J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.
3. I. S. Jacobs and C. P. Bean, “Fine particles, thin films and exchange anisotropy,” in Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.
4. K. Elissa, “Title of paper if known,” unpublished.
5. R. Nicole, “Title of paper with only first word capitalized,” J. Name Stand. Abbrev., in press.
6. Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, “Electron spectroscopy studies on magneto-optical media and plastic substrate interface,” IEEE Transl. J. Magn. Japan, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].
7. M. Young, The Technical Writer’s Handbook. Mill Valley, CA: University Science, 1989.

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