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Robust Trajectory Tracking in Satellite Time-Varying Formation Flying

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***Abstract*—The robust time-varying formation control problem for a group of satellites is addressed. By the static state feedback control strategy and the disturbance estimation theory, a formation flying controller is proposed for the satellite group to form desired time-varying formation patterns and trajectories, and achieve the satellite attitude consensus. The dynamics of each satellite is subject to nonlinearities, parametric perturbations, and external disturbances. Robustness analysis shows that the trajectory and attitude tracking errors of the global closed-loop control system can converge into a given neighborhood of the origin in a finite time. The numerical simulation results validate the effectiveness and advantages of the proposed formation flying controller.**

***Index Terms*—Formation control, nonlinear system, robust control, satellite, uncertain system.**

I. Introduction

SATELLITE formation flying has received increasing interest in recent years. Compared to a monolithic satellite, multiple satellites formation flying has various advantages, such as large aperture size, high fault tolerance, and low cost. Using the information exchange between satellites, satellite formation can complete multiple space missions, such as resource exploration, space environment monitoring, meteorological observation, and 3-D imaging, as illustrated in [1] and [2]. However, each satellite model is highly nonlinear and coupled. Furthermore, each satellite dynamics can be easily influenced by parameter perturbations and external disturbances. Therefore, the control system design problem for satellite formation flying is challenging.

In past decades, scholars have developed three typical formation control methods, namely: 1) leader–follower method; 2) virtual structure approach; and 3) behavior-based strategy (see [3]–[7]). These methods possess corresponding advantages and weaknesses. The leader–follower method is easy to implement, but it is overly dependent on the leaders and is not robust against the leader failures. The virtual structure approach has good robustness and high formation precision, but it depends on a comparatively large amount of communications and calculations, which limits its applicability. The behavior-based strategy has a high degree of intelligence, but it is difficult to analyze the formation features. Furthermore, the consensus-based strategy, which can unify the three approaches mentioned above under a general framework, has attracted increasing interest to address the formation control problem. In [8], it was shown that the above mentioned three strategies can be considered as a special case of the consensus-based control approach and their shortcomings can be overcome to a certain extent.

The translational formation control problem has been studied for satellites with three degrees of freedom (3DOF) in [2] and [9]–[11]. In [9], the consensus problem of multiagent systems with general linear dynamics was studied, and the proposed control scheme was applied to solve a spacecraft formation flying problem. But the dynamics was simplified and the effects of disturbances were not further analyzed. An adaptive control method based on the monotonic adaptive gain and the dominant design was discussed in [10] for satellite formation flying. But the formation flying problem in [10] was mainly considered for two satellites, and the formation of more satellites were not discussed fully. A linear time-periodic distributed control scheme was proposed in [11], but the satellite dynamics was described by means of a linear time-periodic approximation, and the communication among satellites was undirected. The relative translational tracking problem under thrust misalignment was studied in [2], and an adaptive control method by combining a backstepping algorithm was developed. However, in [2] and [9]–[11], the rotational dynamics was not further studied in the formation controller design for multiple satellites. Recently, the attitude control problem, as an important issue in satellite formation flying, has attracted much attention, because the rotational dynamics can influence the translational dynamics seriously. A decentralized coordinated rotational control method was presented in [12], which could deal with the nonlinearity of the satellite dynamics. But

Manuscript received February 28, 2019; accepted December 9, 2019. This work was supported in part by the National Natural Science Foundation of China under Grant 61873012, Grant 61503012, and Grant 61633007, in part by the Fundamental Research Funds for the Central Universities under Grant YWF-18-BJ-Y-81 and Grant YWF-17-BJ-Y-86, and in part by the Office of Naval Research under Grant N00014-17-1-2239. This article was recommended by Associate Editor Z. Ju. *(Corresponding author: Hao Liu.)*

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Digital Object Identifier 10.1109/TCYB.2019.2960363

the influence of disturbances and uncertainties on the global control system were ignored in [12]. In [6], a robust attitude rotational control problem for a team of satellites in the virtual structure was discussed and subject to model uncertainties, but the translational formation control problem was not further studied. Moreover, the formation flying control problems for multiple satellites with 6DOF rotational and translational dynamics were addressed in [13] and [14]. A unified synchronization framework was presented in [13], to achieve precision formation flying spacecraft, but a simplified circular reference orbit was used in the relative translational dynamics. In [14], a nonlinear adaptive control law was presented to solve the control problem of the electromagnetic satellite formation, but the dynamical performance cannot be specified for the adaptive-based control method. Therefore, the time-varying formation control problem for a team of satellites considering complex dynamics models is still open.

In this article, the robust time-varying formation control problem is addressed for a team of satellites. Based on the static state feedback control method as illustrated in [15] and [16] and the disturbance estimation theory as shown in [17]–[19], a formation flying controller was proposed for the satellite group to form the desired time-varying formation patterns and track the desired formation trajectories, while achieving the satellite attitude consensus. Compared to the previous research of satellite formation flying control, the main contributions of this article are shown as follows.

First, each satellite model is highly nonlinear and coupled with 3 translational DOF and 3 rotational DOF. Second, the influence of nonlinearities, parametric perturbations, and external time-varying disturbances on the closed-loop control system are analyzed. It is proven that the trajectory and attitude tracking errors of the global closed-loop control system can converge into a given neighborhood of the origin ultimately, subject to uncertainties. The simulation results show that the proposed robust controller can restrain the influence of the uncertainties on the formation flying control system, compared to a leader-following controller. In addition, the communication among satellites is described by a directed graph, and the resulting control protocol is distributed in nature. The desired time-varying formation patterns can be achieved via the proposed robust controller based on the static state feedback control method.

This article is organized as follows. Section II formulates the graph theory, the satellite model, and the satellite formation flying problem. Section III gives the controller design. Robust analysis and the numerical simulation results are shown in Sections IV and V, respectively. Section VI concludes the whole work of this article.

II. Problem Formulation

A. Graph Theory

In order to describe the information transmission between finite homogeneous satellites, a directed graph $G = \{V, E, W\}$ is adopted in this article, where the satellites are numbered from 1 to n. Let $\Phi = \{1, 2, \ldots, n\}$. Let $V = \{v_1, v_2, \ldots, v_n\}$ denote a finite set of nodes, and $E \in \{(v_i, v_j) : v_i, v_j \in V\}$ a set of edges where an edge $(v_i, v_j) \in E$ denotes that satellite j can obtain information from its neighbor satellite i. Node v_i represents satellite i. An adjacency matrix $W = [w_{ij}] \in \mathbb{R}^{n\times n}$ is defined with each non-negative element w_{ij}, where $w_{ii} = 0$ ($i \in \Phi$). The in-degree of node $d_i(v_i) = \sum_{j=1}^{n} w_{ij}$ and $D = \mathrm{diag}\{d_i(v_i)\} \in \mathbb{R}^{n\times n}$ is the in-degree matrix. The Laplacian matrix of the graph is defined as $L = D - W$. If there is a directed path from a node to every other node, the graph has a spanning tree and the node is called the root of the tree.

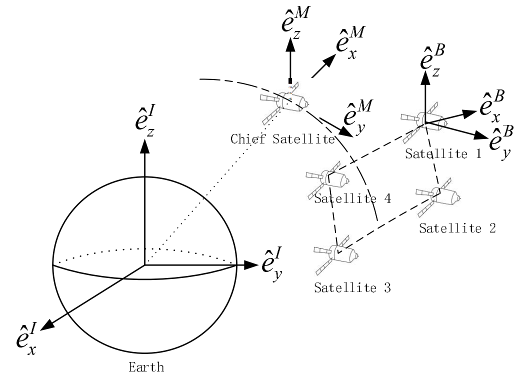

Fig. 1. Schematic of satellite formation.

Notations: Denote $0_{a\times b} \in \mathbb{R}^{a\times b}$ as a zero matrix, $I_n \in \mathbb{R}^{n\times n}$ an identity matrix, and $c_{n,j} \in \mathbb{R}^{n\times 1}$ a column vector with all 0 except for 1 on the jth element. $\otimes$ represents the Kronecker product, and s the Laplace operator. Defining a norm as $\|B(s)\|_1 = \|b(t)\|_1 = \max_i[\sum_j \int_0^\infty |b_{ij}(t)|dt]$, $\|v\|_\infty = \max_i \sup_{t\geq 0}|v_i(t)|$, where $b(t) = [b_{ij}(t)]_{m\times n} = L^{-1}(B(s))$, $v \in \mathbb{R}^{n\times\Gamma}$, and L indicates the Laplace transform. For a vector $\pi = [\pi_1 \quad \pi_2 \quad \pi_3]^T \in \mathbb{R}^{3\times 1}$, define the skew-symmetric matrix $S(\cdot)$ as

$$S(\pi) = \begin{bmatrix} 0 & -\pi_3 & \pi_2 \\ \pi_3 & 0 & -\pi_1 \\ -\pi_2 & \pi_1 & 0 \end{bmatrix}.$$

B. Translational Dynamics and Rotational Dynamics

As shown in Fig. 1, denote $\hat{E}_I = \{\hat{e}_x^I, \hat{e}_y^I, \hat{e}_z^I\}$ as the inertial system attached to the Earth, $\hat{E}_M = \{\hat{e}_x^M, \hat{e}_y^M, \hat{e}_z^M\}$ the chief satellite-fixed frame, and $\hat{E}_{Bi} = \{\hat{e}_{xi}^B, \hat{e}_{yi}^B, \hat{e}_{zi}^B\}$ the body-fixed frame attached to satellite i. Let $p_i(t) = [p_{xi}(t) \quad p_{yi}(t) \quad p_{zi}(t)]^T \in \mathbb{R}^{3\times 1}$ represent the relative 3DOF translational position in $\hat{E}_M$ and $u_{pi}(t) = [u_{xi}(t) \quad u_{yi}(t) \quad u_{zi}(t)]^T \in \mathbb{R}^{3\times 1}$ the relative control acceleration vector. As shown in [20], for satellite i, the relative translational dynamics can be written as

$$\begin{aligned}\ddot{p}_{xi}(t) &= \dot{c}_{faL}^2 p_{xi}(t) + \ddot{c}_{faL}^2 p_{yi}(t) + 2\dot{c}_{faL}\dot{p}_{yi}(t) - \mu_g h_{pi} p_{xi}(t) \\ &\quad + \mu_g\left(1 - h_{pi} r_{0_L}^3\right)\Big/ r_{0_L}^2 + u_{xi}(t) + d_{xi}(t) \\ \ddot{p}_{yi}(t) &= -\ddot{c}_{faL} p_{xi}(t) + \dot{c}_{faL}^2 p_{yi}(t) - 2\dot{c}_{faL}\dot{p}_{xi}(t) \\ &\quad - \mu_g h_{pi} p_{yi}(t) + u_{yi}(t) + d_{yi}(t) \\ \ddot{p}_{zi}(t) &= -\mu_g h_{pi} p_{zi}(t) + u_{zi}(t) + d_{zi}(t)\end{aligned} \tag{1}$$

where μ_g indicates the geocentric gravitational constant, r_{0_L} the chief satellite's orbit radius, $d_{pi}(t) =$

$[\,d_{xi}(t) \quad d_{yi}(t) \quad d_{zi}\,(t)]^T \in \mathbb{R}^{3\times 1}$ the external disturbance, and $h_{pi} = ((r_{0_L} + p_{xi})^2 + p_{yi}^2 + p_{zi}^2)^{-3/2}$. The derivatives of the true anomaly c_{faL} of the chief satellite in (1) can be obtained by

$$\begin{aligned}\dot{c}_{faL} &= c_p\Big[1 + 2c_{c1}e + (1+5c_{c2})e^2/2 + O\Big(e^3\Big)\Big]\\ \ddot{c}_{faL} &= -2c_p\Big[c_{nL}c_{s1}e + (c_{nL}c_{s2} + 3c_{nL}c_{c2}c_{s1})e^2 + O\Big(e^3\Big)\Big]\\ \dot{c}_{faL}^2 &= c_p^2\Big[1 + 4c_{c1}e + (3+7c_{c2})e^2/2 + O\Big(e^3\Big)\Big] \end{aligned}\tag{2}$$

where a is the orbital semilong axis, e is the eccentricity, $c_p = \sqrt{\mu_g a(1-e^2)}/a^2$, $c_{c1} = \cos c_{fML}$, $c_{s1} = \sin c_{fML}$, $c_{c2} = \cos(2c_{fML})$, $c_{s2} = \sin(2c_{fML})$, $c_{nL} = \sqrt{\mu_g/a^3}$ means the average orbital angular velocity, and $c_{fML} = c_{nL}t$ indicates the mean anomaly when the initial mean anomaly is 0°. The function $O(\cdot)$ represents the infinitesimal of higher order. The satellite model given in (1) and (2) is applicable for the circular orbits and elliptical orbits with different eccentricities.

MRPs are used to describe the attitude of rigid bodies in [21]. In this article, a parameter vector $\sigma_i(t)$ satisfies $\sigma_i(t) = [\sigma_{1i}(t), \sigma_{2i}(t), \sigma_{3i}(t)]^T = \vec{n}_i \tan(\theta_i/4) \in \mathbb{R}^{3\times 1}$ for satellite i, where θ_i is the rotation angle about the Euler axis $\vec{n}_i$. Then, as shown in [21], the rotational dynamics is given by

$$\dot{\sigma}_i(t) = H(\sigma_i)\omega_i(t) \tag{3}$$

$$J_i\dot{\omega}_i(t) = -S(\omega_i)J_i\omega_i(t) + \tau_i(t) + d_{\tau i}(t) \tag{4}$$

where $H(\sigma_i)$ is an invertible matrix and $H(\sigma_i) = ((1-\sigma_i{}^T\sigma_i)I_3 + 2\sigma_i\sigma_i{}^T + 2S(\sigma_i))/4$. Let $J_i = \mathrm{diag}\{J_{xi}, J_{yi}, J_{zi}\} \in \mathbb{R}^{3\times 3}$ represent the inertia matrix of each satellite, $\omega_i(t) = [\,\omega_{xi}(t) \quad \omega_{yi}(t) \quad \omega_{zi}(t)]^T \in \mathbb{R}^{3\times 1}$ the angular velocity vector, $\tau_i(t) = [\,\tau_{xi}(t) \quad \tau_{yi}(t) \quad \tau_{zi}(t)\,]^T \in \mathbb{R}^{3\times 1}$ the internal control torque, and $d_{\tau i}(t) = [\,d_{\tau xi}(t) \quad d_{\tau yi}(t) \quad d_{\tau zi}(t)\,]^T \in \mathbb{R}^{3\times 1}$ the external disturbance. Defining $u_{\tau i}(t)$ as the torque control input of satellite i, and $u_{\tau i}(t) = H^{-1}(\sigma_i)\tau_i(t)$. By combining (3) and (4), one can obtain the following Lagrangian expression with respect to MRPs as:

$$M(\sigma_i)\ddot{\sigma}_i(t) + C(\sigma_i,\dot{\sigma}_i)\dot{\sigma}_i(t) = u_{\tau i}(t) + H(\sigma_i)^{-1}d_{\tau i}(t) \tag{5}$$

where $M(\sigma_i) = H^{-1}(\sigma_i)J_iH^{-1}(\sigma_i)$ and $C(\sigma_i,\dot{\sigma}_i) = -H^{-1}(\sigma_i)(J_iH^{-1}(\sigma_i)\dot{H}(\sigma_i)H^{-1}(\sigma_i) + S(J_i\omega_i)H^{-1}(\sigma_i))$.

Remark 1: The 6DOF satellite model described in (1) and (5) with 3 relative translational DOF and 3 rotational DOF involves highly nonlinear and coupled dynamics.

C. Problem Description

The objectives for the satellite group in this article are shown as follows: to achieve the desired time-varying formation patterns and track the formation center tracks the desired time-varying formation trajectories, while to align the satellite attitudes. Let $\xi_{pij}(t)$ and $\xi_{\sigma ij}(t)$ $(i,j \in \Phi)$ denote the desired position and attitude deviation between satellites i and j, respectively. The desired trajectory of the virtual leader (formation center) is defined as $p^r(t) \in \mathbb{R}^{3\times 1}$, and the desired attitude of the virtual leader as $\sigma^r(t) \in \mathbb{R}^{3\times 1}$. The second derivatives of the trajectory and attitude references are assumed to satisfy $\ddot{p}^r(t) = 0$ and $\ddot{\sigma}^r(t) = 0$. In fact, let $\xi_{pij}(t) = \xi_{pi}(t) - \xi_{pj}(t)$ and $\xi_{\sigma ij}(t) = \xi_{\sigma i}(t) - \xi_{\sigma j}(t)$, where $\xi_{pi}(t)$ and $\xi_{\sigma i}(t)$ can be considered as the desired position and attitude deviation between the virtual leader and satellite i, and $\sum_{i=1}^{n} \xi_{pi}(t) = 0_{3\times 1}$. $\xi_{pij}(t)$ is time varying and determines the formation pattern of the satellite group. $\xi_{\sigma i}(t) = 0$ and $\xi_{\sigma ij}(t)$ are set to be $0_{3\times 1}$ to guarantee that the satellites can achieve the same attitudes.

Let the superscript N represents the nominal parameter and the superscript Δ represents the parameter uncertainty satisfying, for example, $J_i = J_i^N + J_i^\Delta$. Based on the following feedback linearization control method as shown in [22]:

$$u_{\tau i}(t) = C^N(\sigma_i,\dot{\sigma}_i)\dot{\sigma}_i(t) + M^N(\sigma_i)u_{\sigma i}(t) \tag{6}$$

the satellite motion (1) and (5) can be rewritten by the following equations:

$$\begin{aligned}\ddot{p}_i(t) &= C_{p1i}^N\dot{p}_i(t) + C_{p2i}^N p_i(t) + u_{pi}(t) + \Delta_{p_i}(t)\\ \ddot{\sigma}(t) &= u_{\sigma i}(t) + \Delta_{\sigma i}(t),\ i\in\Phi\end{aligned}\tag{7}$$

where $u_{\sigma i}(t)$ is the introduced virtual attitude control input to be designed, and

$$C_{p1i}^N = \begin{bmatrix} 0 & 2\dot{c}_{faL}^N & 0\\ -2\dot{c}_{faL}^N & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$C_{p2i}^N = \begin{bmatrix} \Big(\dot{c}_{faL}^2\Big)^N + 2\mu_g/r_{0_L}^3 & \ddot{c}_{faL}^N & 0\\ -\ddot{c}_{faL}^N & \Big(\dot{c}_{faL}^2\Big)^N - \mu_g/r_{0_L}^3 & 0\\ 0 & 0 & -\mu_g/r_{0_L}^3\end{bmatrix}$$

$\Delta_{pi}(t) \in \mathbb{R}^{3\times 1}$ and $\Delta_{\sigma i}(t) \in \mathbb{R}^{3\times 1}$ in (7) are named equivalent disturbances, involving nonlinear terms, parametric perturbations, and external disturbances, and satisfy

$$\begin{aligned}\Delta_{pi}(t) &= C_{p1i}^\Delta\dot{p}_i(t) + C_{p2i}^\Delta p_i(t) + C_{p3i}(t) + d_{pi}(t)\\ \Delta_{\sigma i}(t) &= M^{-1}(\sigma_i)\Big(H^{-1}(\sigma_i)d_{\tau i}(t) - C^\Delta(\sigma_i,\dot{\sigma}_i)\dot{\sigma}_i(t)\Big)\\ &\quad - M^{-1}(\sigma_i)M^\Delta(\sigma_i)u_{\sigma i}(t)\end{aligned}\tag{8}$$

where

$$C_{p1i}^\Delta = \begin{bmatrix} 0 & 2\dot{c}_{faL}^\Delta & 0\\ -2\dot{c}_{faL}^\Delta & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$C_{p2i}^\Delta = \begin{bmatrix} \Big(\dot{c}_{faL}^2\Big)^\Delta & \ddot{c}_{faL}^\Delta & 0\\ -\ddot{c}_{faL}^\Delta & \Big(\dot{c}_{faL}^2\Big)^\Delta & 0\\ 0 & 0 & 0\end{bmatrix}$$

$C^\Delta(\sigma_i,\dot{\sigma}_i) = -H^{-1}(\sigma_i)(J_i^\Delta H^{-1}(\sigma_i)\dot{H}(\sigma_i)H^{-1}(\sigma_i) + S(J_i^\Delta\omega_i)H^{-1}(\sigma_i))$, $M^\Delta(\sigma_i) = H^{-1}(\sigma_i)J_i^\Delta H^{-1}(\sigma_i)$, $C_{p3i} = [\,\mu_g(1-h_{pi}r_{0_L}^3)/r_{0_L}^2 \quad 0 \quad 0]^T$. The parameter perturbations and external disturbances are assumed to be bounded and satisfy $\|M^{-1}(\sigma_i)M^\Delta(\sigma_i)\|_1 < 1$.

Remark 2: By ignoring the equivalent disturbances $\Delta_{pi}(t)$ and $\Delta_{\sigma i}(t)$, the satellite model (7) can be regarded as the nominal model. Actually, the real model of the satellite includes the nominal model and equivalent disturbances.

III. Formation Controller Design

In this section, a formation flying controller is proposed for the team of satellites, which consists of a translational controller to form the desired time-varying formation patterns and track the desired formation trajectories, and a rotational controller to achieve the satellite attitude consensus.

A. Translational Controller

The relative control acceleration input $u_{pi}(t)$ for satellite i consists of two parts: 1) the nominal part $u_{pi}^F(t) \in \mathbb{R}^{3\times 1}$ based on the static state feedback control approach designed for the nominal model and 2) the disturbance estimating part $u_{pi}^R(t) \in \mathbb{R}^{3\times 1}$ constructed to restrain the effects of the equivalent disturbance $\Delta_{pi}(t)$, as

$$u_{pi}(t) = u_{pi}^F(t) + u_{pi}^R(t), i \in \Phi. \tag{9}$$

By ignoring the equivalent disturbance $\Delta_{pi}(t)$, one can design $u_{pi}^F(t)$ based on [15] as

$$\begin{aligned} u_{pi}^F(t) = &-\eta_{Fp}\left(K_p L\left(p_i(t) - \xi_{pi}(t)\right) + K_{\dot p} L\left(\dot p_i(t) - \dot\xi_{pi}(t)\right)\right) \\ &- \eta_{Fp}\beta_{li}K_p\left(p_i(t) - \xi_{pi}(t) - p^r(t)\right) \\ &- \eta_{Fp}\beta_{li}K_{\dot p}\left(\dot p_i(t) - \dot\xi_{pi}(t) - \dot p^r(t)\right) \end{aligned} \tag{10}$$

where $\eta_{Fp} \in \mathbb{R}^{1\times 1}$ is a scalar coupling gain, and $K_p, K_{\dot p} \in \mathbb{R}^{3\times 3}$ are diagonal nominal controller parameter matrices. β_{li} represents the link weight between the virtual leader and satellite i, and when $\beta_{li} = 1$, satellite i can get the information from the virtual leader; otherwise, $\beta_{li} = 0$. Let $K_{\bar p} = [\, K_p \quad K_{\dot p}\,]$, and

$$A_z = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}, B_z = \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}.$$

Consider the design symmetric and positive-definite matrices $M_p = M_p^T \in \mathbb{R}^{6\times 6}$ and $\Pi_p = \Pi_p^T \in \mathbb{R}^{3\times 3}$. The translational controller parameter matrix $K_{\bar p}$ by the static state feedback control approach can be given as

$$K_{\bar p} = \Pi_p^{-1} B_z^T Q_p \tag{11}$$

where Q_p is the positive-definite solution to the following Riccati equation:

$$A_z^T Q_p + Q_p A_z + M_p - Q_p B_z \Pi_p^{-1} B_z^T Q_p = 0.$$

Moreover, u_{pi}^R is constructed based on the disturbance estimation filter $F_{pi}(s)$ as

$$u_{pi}^R(s) = -F_{pi}(s)\Delta_{pi}(s), i \in \Phi \tag{12}$$

where $F_{pi}(s) = \mathrm{diag}\{F_{p1,i}(s), F_{p2,i}(s), F_{p3,i}(s)\}$. $F_{pj,i}(s) = f_{pj,i}^2/(s + f_{pj,i})^2$ $(j = 1, 2, 3)$ and the positive-disturbance estimator parameters $(j = 1, 2, 3)$ need to be determined. Let $f_{pi} = \mathrm{diag}\{f_{p1,i}, f_{p2,i}, f_{p3,i}\}$. By selecting larger $f_{pj,i}$, $F_{pj,i}(s)$ could have a wider frequency band, under which $F_{pj,i}(s)$ closes to a unit matrix. In this case, u_{pi}^R approximates to $-\Delta_{pi}$. Therefore, by choosing appropriate disturbance estimator parameters, the influence of equivalent disturbance on the closed-loop control system can be restrained. However, Δ_{pi} cannot be measured directly. It can be obtained from (7) that

$$\Delta_{pi}(t) = \ddot p_i(t) - C_{p1i}^N \dot p_i(t) - C_{p2i}^N p_i(t) - u_{pi}(t). \tag{13}$$

By substituting (13) to (12), one can obtain that

$$u_{pi}^R(s) = -f_{pj,i}^2 p_i(s) + f_{pj,i}^2 \delta_{2i}^p(s)$$

where

$$\delta_{2i}^p(s) = \frac{1}{s + f_{pj,i}}\left(\left(2f_{pj,i} + C_{p1i}^N\right)p_i(s) + \frac{-\left(f_{pj,i}^2 + C_{p1i}^N f_{pj,i} - C_{p2i}^N\right)p_i(s) + u_{pi}(s)}{s + f_{pj,i}}\right).$$

Defining

$$\delta_{1i}^p(s) = \frac{-\left(f_{pj,i}^2 + C_{p1i}^N f_{pj,i} - C_{p2i}^N\right)p_i(s) + u_{pi}(s)}{s + f_{pj,i}}$$

and

$$\delta_{2i}^p(s) = \frac{1}{s + f_{pj,i}}\left(\left(2f_{pj,i} + C_{p1i}^N\right)p_i(s) + \delta_{1i}^p(s)\right)$$

therefore, one can realize the disturbance estimating part $u_{\sigma i}^R(t)$ with filter states $\delta_{1i}^p(t)$ and $\delta_{2i}^p(t)$ via the following state-space form as:

$$\begin{aligned} \dot\delta_{1i}^p(t) &= -f_{pi}\delta_{1i}^p(t) - \left(f_{pi}^2 + C_{p1}^N f_{pi} - C_{p2}^N\right)p_i(t) + u_{pi}(t) \\ \dot\delta_{2i}^p(t) &= -f_{pi}\delta_{2i}^p(t) + \left(2f_{pi} + C_{p1}^N\right)p_i(t) + \delta_{1i}^p(t) \\ u_{pi}^R(t) &= -f_{pi}^2 p_i(t) + f_{pi}^2\delta_{2i}^p(t), i \in \Phi. \end{aligned} \tag{14}$$

The nominal model can be obtained by ignoring the equivalent disturbances $\Delta_{pi}(t)$ and $\Delta_{\sigma i}(t)$ $(i \in \Phi)$ in (7). The real model can be regarded as the nominal model added with the equivalent disturbances $\Delta_{pi}(t)$ and $\Delta_{\sigma i}(t)$. For satellite i, the relative control acceleration input $u_{pi}(t)$ includes two parts, e.g., the nominal control input $u_{pi}^F(t)$ and the disturbance estimating part $u_{pi}^R(t)$. The nominal controller is designed based on the linear quadratic regulator control method to achieve the desired formation control for the nominal translational system and the disturbance estimating part to restrain the influences of $\Delta_{pi}(t)$ on the real translational system.

B. Rotational Controller

Similar to the translational controller, the virtual attitude control input $u_{\sigma i}(t)$ for satellite i is designed as

$$u_{\sigma i}(t) = u_{\sigma i}^F(t) + u_{\sigma i}^R(t), i \in \Phi \tag{15}$$

where $u_{\sigma i}^F(t) \in \mathbb{R}^{3\times 1}$ is the nominal control part based on the static state feedback control approach and $u_{\sigma i}^R(t) \in \mathbb{R}^{3\times 1}$ is the disturbance estimating part. The nominal part $u_{\sigma i}^F(t)$ is designed similar to $u_{pi}^F(t)$ as

$$\begin{aligned} u_{\sigma i}^F(t) = &-\eta_{F\sigma}\left(K_\sigma L(\sigma_i(t) - \xi_{\sigma i}(t)) + K_{\dot\sigma} L\left(\dot\sigma_i(t) - \dot\xi_{\sigma i}(t)\right)\right) \\ &- \eta_{F\sigma}\beta_{li}K_\sigma\left(\sigma_i(t) - \xi_{\sigma i}(t) - \sigma^r(t)\right) \\ &- \eta_{F\sigma}\beta_{li}K_{\dot\sigma}\left(\dot\sigma_i(t) - \dot\xi_{\sigma i}(t) - \dot\sigma^r(t)\right) \end{aligned} \tag{16}$$

where $\eta_{F\sigma} \in \mathbb{R}^{1\times 1}$ is a scalar coupling gain, and $K_\sigma, K_{\dot\sigma} \in \mathbb{R}^{3\times 3}$ are diagonal nominal controller parameter matrices. Let $K_{\bar\sigma} = [\, K_\sigma \quad K_{\dot\sigma} \quad c]$. Define symmetric and positive-definite matrices $M_\sigma = M_\sigma^T \in \mathbb{R}^{6\times 6}$ and $\Pi_\sigma = \Pi_\sigma^T \in \mathbb{R}^{3\times 3}$. The attitude controller parameter matrix $K_{\bar{\sigma}}$ can be obtained by

$$K_{\bar{\sigma}} = \Pi_\sigma^{-1} B_z^T Q_\sigma \tag{17}$$

where Q_σ is the positive-definite solution to the associated Riccati equation

$$A_z^T Q_\sigma + Q_\sigma A_z + M_\sigma - Q_\sigma B_z \Pi_\sigma^{-1} B_z^T Q_\sigma = 0.$$

Moreover, designing the disturbance estimating part $u_{\sigma i}^R$ as

$$u_{\sigma i}^R(s) = -F_{\sigma i}(s)\Delta_{\sigma i}(s) \tag{18}$$

where $F_{\sigma i}(s) = \operatorname{diag}\{F_{\sigma 1,i}(s), F_{\sigma 2,i}(s), F_{\sigma 3,i}(s)\}$. Let $F_{\sigma j,i}(s) = f_{\sigma j,i}^2/(s+f_{\sigma j,i})^2$ $(j = 1, 2, 3)$ and $f_{\sigma i} = \operatorname{diag}\{f_{\sigma 1,i}, f_{\sigma 2,i}, f_{\sigma 3,i}\}$. $u_{\sigma i}^R(t)$ can be realized in a similar way as

$$\begin{aligned}
\dot{\delta}_{i1}^\sigma(t) &= -f_{\sigma i}\delta_{i1}^\sigma(t) - f_{\sigma i}^2\sigma_i(t) + u_{\sigma\, i}(t)\\
\dot{\delta}_{i2}^\sigma(t) &= -f_{\sigma i}\delta_{i2}^\sigma(t) + 2f_{\sigma i}\sigma_i(t) + \delta_{i1}^\sigma(t)\\
u_{\sigma i}^R(t) &= -f_{\sigma i}^2\sigma_i(t) + f_{\sigma i}^2\delta_{i2}^\sigma(t), \; i \in \Phi.
\end{aligned} \tag{19}$$

Remark 3: The formation controller designed in this section is distributed, because only the information of satellite i and its neighbors is needed in the controller design.

IV. Main Results

Define the trajectory tracking error of satellite i as $e_{pi}(t) = [e_{pj,i}(t)] = p_i(t) - \xi_{pi}(t) - p^r(t) \in \mathbb{R}^{3\times 1}$ and the attitude tracking error $e_{\sigma i}(t) = [e_{\sigma j,i}(t)] = \sigma_i(t) - \xi_{\sigma i}(t) - \sigma^r(t) \in \mathbb{R}^{3\times 1}$. Let $z_{pi}(t) = [\, e_{pi}^T(t) \quad \dot{e}_{pi}^T(t) \,]^T = [z_{pj,i}(t)] \in \mathbb{R}^{6\times 1}$ and $z_{\sigma i}(t) = [\, e_{\sigma}{}_i^T(t) \quad \dot{e}_{\sigma i}^T(t) \,]^T = [z_{\sigma j,i}(t)] \in \mathbb{R}^{6\times 1}$. Then, from (7), (9), (10), (15), and (16), the error system model for satellite i can be obtained as

$$\begin{aligned}
\dot{z}_{pi}(t) &= A_z z_{pi}(t) - \eta_{Fp} B_z\big(K_p L e_{pi}(t) + K_{\dot{p}} L \dot{e}_{pi}(t)\big)\\
&\quad - \eta_F \beta_{li} B_z\big(K_p e_{pi}(t) + K_{\dot{p}} \dot{e}_{pi}(t)\big)\\
&\quad + B_z\Big(u_{pi}^R(t) + \Delta_{pi}(t)\Big)\\
\dot{z}_{\sigma i}(t) &= A_z z_{\sigma i}(t) - \eta_{F\sigma} B_z(K_\sigma L e_{\sigma i}(t) + K_{\dot{\sigma}} L \dot{e}_{\sigma i}(t))\\
&\quad - \eta_F \beta_{li} B_z(K_\sigma e_{\sigma i}(t) + K_{\dot{\sigma}} \dot{e}_{\sigma i}(t))\\
&\quad + B_z\big(u_{\sigma i}^R(t) + \Delta_{\sigma i}(t)\big).
\end{aligned} \tag{20}$$

Now, one can obtain the global closed-loop control system as

$$\begin{aligned}
\dot{z}_p(t) &= A_{\bar{p}} z_p(t) + B_{\bar{z}} \tilde{\Delta}_p(t)\\
\dot{z}_\sigma(t) &= A_{\bar{\sigma}} z_\sigma(t) + B_{\bar{z}} \tilde{\Delta}_\sigma(t)
\end{aligned} \tag{21}$$

where $z_p(t) \in \mathbb{R}^{6n\times 1}$, $z_\sigma(t) \in \mathbb{R}^{6n\times 1}$, $A_{\bar{p}} = I_n \otimes A_z - \eta_{Fp}(L + B_L) \otimes B_z K_{\bar{p}}$, $A_{\bar{\sigma}} = I_n \otimes A_z - \eta_{F\sigma}(L + B_L) \otimes B_z K_{\bar{\sigma}}$, $B_{\bar{z}} = I_n \otimes B_z$, $\tilde{\Delta}_p(t) = u_{pi}^R(t) + \Delta_{pi}(t)$, $\tilde{\Delta}_\sigma(t) = u_{\sigma i}^R(t) + \Delta_{\sigma i}(t)$, and $B_L = \operatorname{diag}\{\beta_{li}\} \in \mathbb{R}^n$. From [16, Th. 1], $A_{\bar{p}}$ and $A_{\bar{\sigma}}$ are asymptotically stable by using the linear quadratic regulator-based optimal design, if $\eta_{Fp} \geq \lambda_p^{\min}/2$ and $\eta_{F\sigma} \geq \lambda_p^{\min}/2$ when the directed graph G has a spanning tree and the information of the virtual leader can be transmitted to the root, where $\lambda_p^{\min} = \min_{i\in\Phi}\operatorname{Re}(\lambda_{pi})$ and $\lambda_{pi}(i \in \Phi)$ indicate the eigenvalues of $(L + B_L)$.

Theorem 1: Consider the satellite motion described in (7), the proposed formation flying control scheme in Section III, the scalar coupling gains η_{Fp} and $\eta_{F\sigma}$, and the nominal controller parameter matrices $K_{\bar{p}}$ and $K_{\bar{\sigma}}$ determined by (11) and (17). If the directed graph G has a spanning tree, the information of the virtual leader can be transmitted to the root, and the initial states $z_p(0)$ and $z_\sigma(0)$ are bounded, then, for a given positive constant ε_e, there exist finite positive constants T^* and f^* such that for any $f_{pj,i} \geq f^*$ and $f_{\sigma j,i} \geq f^*$ $(i \in \Phi)$, all states in the global closed-loop system are bounded, and the tracking errors $z_{pi}(t)$ and $z_{\sigma i}(t)$ satisfy $\max_j|z_{pj,i}(t)| \leq \varepsilon_{ep}$ and $\max_i|z_{\sigma i}(t)| \leq \varepsilon_{e\sigma}\quad \forall t \geq T^*$.

Proof: By substituting (12) and (18) into (21), one can have that

$$\begin{aligned}
\dot{z}_p(t) &= A_{\bar{p}} z_p(t) + L^{-1}\big(B_{\bar{z}}\big(I_{3n} - F_p(s)\big)\Delta_p(s)\big)\\
\dot{z}_\sigma(t) &= A_{\bar{\sigma}} z_\sigma(t) + L^{-1}(B_{\bar{z}}(I_{3n} - F_\sigma(s))\Delta_\sigma(s))
\end{aligned} \tag{22}$$

where $\Delta_p(s) = [\Delta_{pi}(s)] \in \mathbb{R}^{3n\times 1}$, $\Delta_\sigma(s) = [\Delta_{\sigma i}(s)] \in \mathbb{R}^{3n\times 1}$, $F_p(s) = \operatorname{diag}\{F_{pi}(s)\} \in \mathbb{R}^{3n\times 3n}$, and $F_\sigma(s) = \operatorname{diag}\{F_{\sigma i}(s)\} \in \mathbb{R}^{3n\times 3n}$. The matrices $A_{\bar{p}}$ and $A_{\bar{\sigma}}$ are asymptotically stable and from (22), one can obtain that

$$\begin{aligned}
\|z_p\|_\infty &\leq \|\zeta_{p(0)}\|_\infty + \eta_p\|\Delta_p\|_\infty\\
\|z_\sigma\|_\infty &\leq \|\zeta_{\sigma(0)}\|_\infty + \eta_\sigma\|\Delta_\sigma\|_\infty
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
\max_j|z_{pj}| &\leq \max_j\Big|c_{6n,j}^T e^{A_{\bar{p}}t} z_p(0)\Big| + \eta_p\|\Delta_p\|_\infty\\
\max_j|z_{\sigma j}| &\leq \max_j\Big|c_{6n,j}^T e^{A_{\bar{\sigma}}t} z_\sigma(0)\Big| + \eta_\sigma\|\Delta_\sigma\|_\infty
\end{aligned} \tag{24}$$

where $\zeta_{p(0)} = \max_j \sup_{t\geq 0}|c_{6n,j}^T e^{A_{\bar{p}}t} z_p(0)|$, $\zeta_{\varsigma(0)} = \max_j \sup_{t\geq 0}|c_{6n,j}^T e^{A_{\bar{\sigma}}t} z_\sigma(0)|$, and

$$\begin{aligned}
\eta_p &= \Big\|\big(sI_{6n} - A_{\bar{p}}\big)^{-1} B_{\bar{z}}\big(I_{3n} - F_p(s)\big)\Big\|_1\\
\eta_\sigma &= \Big\|(sI_{6n} - A_{\bar{\sigma}})^{-1} B_{\bar{z}}(I_{3n} - F_\sigma(s))\Big\|_1.
\end{aligned} \tag{25}$$

By selecting the disturbance estimator parameters f_{pj} and $f_{\sigma j}$ properly, η_p and η_σ can be as small as desired. According to [16], because $A_{\bar{p}}$ and $A_{\bar{\sigma}}$ are asymptotically stable, there exists a positive constant $\underline{f^*}$ such that for any $f_p^{\min} \geq \underline{f^*}$ and $f_\sigma^{\min} \geq \underline{f^*}$ $(i \in \Phi)$, one can obtain finite positive constants χ_p and χ_σ independent of f_{pj} and $f_{\sigma j}$, and satisfy the following inequalities:

$$\chi_p \geq \eta_p f_p^{\min},\ \chi_\sigma \geq \eta_\sigma f_\sigma^{\min}$$

where $f_p^{\min} = \min\{f_{pj}\}$ and $f_\sigma^{\min} = \min\{f_{\sigma j}\}$.

The robust characteristics of translational tracking are first analyzed. Let $\varepsilon_{\Delta p} = \max_i \varepsilon_{\Delta pi}$. Since the parameter perturbations and external disturbances are bounded, every element in the matrix C_{p1i}^Δ and C_{p2i}^Δ are bounded, and every element of vector C_{p3i} is bounded, there exists a positive constant $\varepsilon_{\Delta p}$ such that $\Delta_p(t)$ in (8) satisfies

$$\|\Delta_p\|_\infty \leq \varepsilon_{\Delta p}. \tag{26}$$

Because the initial state $z_p(0)$ is bounded and $A_{\bar{p}}$ is asymptotically stable, then from (23) and (26), one can obtain that $z_p(t)$ is bounded if $f_p^{\min} \geq \underline{f^*}$. It follows that:

$$\|z_p\|_\infty \leq \varepsilon_{ep}. \tag{27}$$

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The states of the robust filter $\delta^p_{1i}(t)$ and $\delta^p_{2i}(t)$ are bounded, and the bounds depend on the disturbance estimator parameters. Therefore, all sates in the translational dynamics are bounded. From (8), (21), (26), and (27), one can obtain that

$$\max_j \left| z_p(t) \right| \leq \max_j \left| c^T_{6n,j} \zeta_{p(0)} \right| + \chi_p \varepsilon_{\Delta p} / f^{\min}_p.$$

Moreover, the attitude tracking performance can be analyzed in a similar way. One can obtain the following inequality for the equivalent disturbance $\Delta_{\sigma i}(t)$ in (8) as:

$$\|\Delta_{\sigma i}\|_\infty \leq \left\| M^{-1}(\sigma_i) M^{\Delta}(\sigma_i) \right\|_1 \|u_{\sigma i}\|_\infty + \chi_{d\sigma i} \tag{28}$$

where $\chi_{d\sigma i}$ is a positive constant. From (15), (16), and (18), there exists a positive constant $\chi_{\sigma z i}$ depending on $f_{pj,i}$ such that

$$\|u_{\sigma i}\|_\infty \leq \chi_{\sigma z i} \|z_{\sigma i}\|_\infty + \|\Delta_{\sigma i}\|_\infty. \tag{29}$$

Combining (28) and (29) yields that

$$\|\Delta_\sigma\|_\infty \leq \chi_{\Delta\sigma z} \|z_\sigma\|_\infty + \chi_{\Delta\sigma d} \tag{30}$$

where $\chi_{\Delta\sigma z}$ and $\chi_{\Delta\sigma d}$ are positive constants determined by $f_{pj,i}$. If the positive constant $f^{\min}_\sigma$ satisfies $f^{\min}_\sigma \geq \underline{f^*}$ and $f^{\min}_\sigma > \chi_{\Delta z} \chi_\sigma$, one can obtain the following inequality by substituting (30) to (23) as:

$$\left(1 - \chi_{\Delta\sigma z} \chi_\sigma / f^{\min}_p\right) \|\Delta_\sigma\|_\infty \leq \chi_{\Delta\sigma z} \left\| \zeta_{\sigma(0)} \right\|_\infty + \chi_{\Delta\sigma d}. \tag{31}$$

For the asymptotically stable $A_{\bar{\sigma}}$ and the bounded initial state $z_\sigma(0)$, $\zeta_{\sigma(0)}$ is bounded. Therefore, from (23) and (31), one can obtain that $\Delta_\sigma(t)$ and $z_\sigma(t)$ are bounded as

$$\|\Delta_\sigma\|_\infty \leq \varepsilon_{\Delta\sigma}, \|z_\sigma\|_\infty \leq \varepsilon_{e\sigma} \tag{32}$$

where $\varepsilon_{\Delta\sigma}$ and $\varepsilon_{e\sigma}$ are positive constants. Similarly, from (18), (21), and (32), one has that

$$\max_j |z_\sigma(t)| \leq \max_j \left| c^T_{6n,j} \zeta_{\sigma(0)} \right| + \chi_\sigma \varepsilon_{\Delta\sigma} / f^{\min}_\sigma.$$

Now, for a given positive constant ε_e, there exist finite positive constants T^* and f^* such that for any $f_{pj,i} \geq f^*$ and $f_{\sigma j,i} \geq f^*$ $(i \in \Phi)$, all sates in the global closed-loop system are bounded, and the tracking errors $z_{pi}(t)$ and $z_{\sigma i}(t)$ satisfy $\max_j |z_{pj,i}(t)| \leq \varepsilon_{ep}$ and $\max_i |z_{\sigma i}(t)| \leq \varepsilon_{e\sigma}$ $\quad \forall t \geq T^*$. ■

Remark 4: It should be pointed out that the theoretical values of the disturbance estimator parameters f_{pi} and $f_{\sigma i}$ $(i \in \Phi)$ obtained from Theorem 1 are conservative. It means that the real values of the disturbance estimator parameters may be much smaller than the theoretical ones. In practical applications, a unidirectional adjustment method can be given to determine the disturbance estimator parameters as follows. First, the initial parameters f_{pi} and $f_{\sigma i}$ $(i \in \Phi)$ are set with small positive constants. Then, increase the disturbance estimator parameters until the specified tracking performance of the closed-loop system can be achieved.

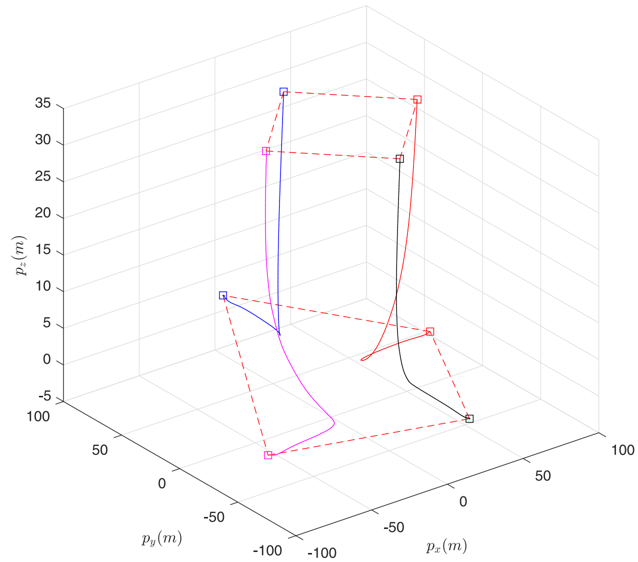

Fig. 2. 3-D trajectories by the proposed controller.

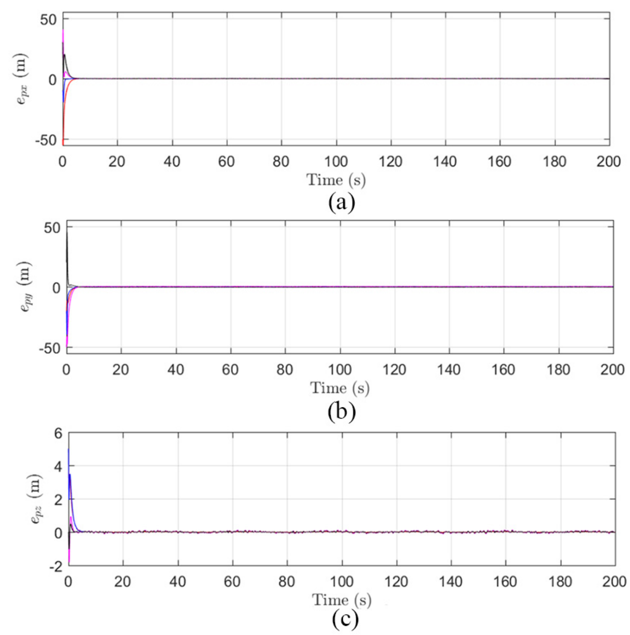

Fig. 3. Trajectory tracking errors by the proposed controller. (a) Tracking error along $\hat{e}^M_x$. (b) Tracking error along $\hat{e}^M_y$. (c) Tracking error along $\hat{e}^M_x$.

V. Simulation Studies

Simulation studies of four satellites are provided to demonstrate the tracking performance of the proposed robust formation control scheme, and $\Phi = \{1, 2, 3, 4\}$. Using the satellite motion model in Section II with $\mu_g = 3.986 \times 10^{14}$ and $J^N_i = \mathrm{diag}\{4.34, 4.33, 3.66\}$ with all parameters in the international units. The initial orbital elements of the chief satellite are shown as: $a = 7.4 \times 10^6$, $e = 0.1$, the inclination $i_L = 30°$, the longitude of the ascending node $\Omega_L = 100°$, the argument of periapsis $\omega_L = 0°$, and the mean anomaly $c_{fML} = 0°$, respectively. The desired trajectory of the formation center is $p^r(t) = [0.15t \quad 0.15t \quad 0.15t]^T$ and the desired attitude of the virtual leader is $\sigma^r(t) = [0.05 \quad 0.05 \quad 0.05]^T$. The

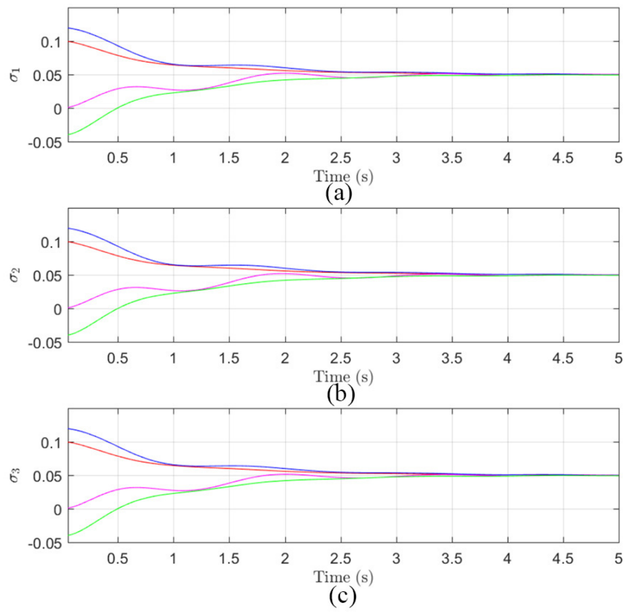

Fig. 4. Transient attitude responses by the proposed controller. (a) σ_1. (b) σ_2. (c) σ_3.

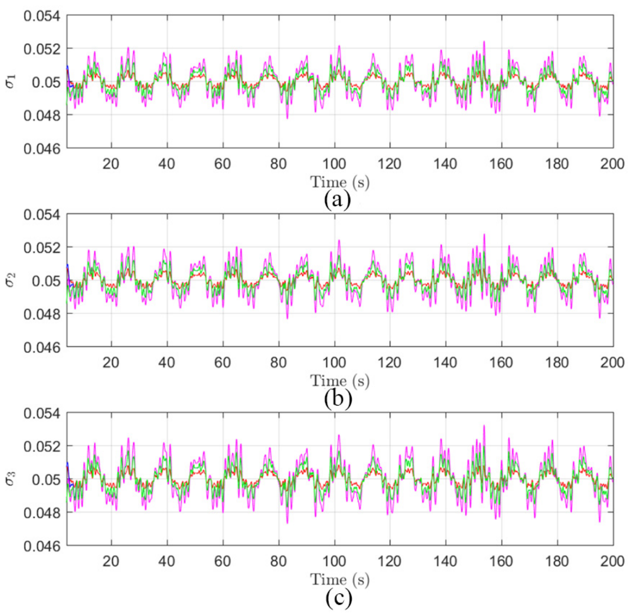

Fig. 5. Steady-state attitude responses by the proposed controller. (a) σ_1. (b) σ_2. (c) σ_3.

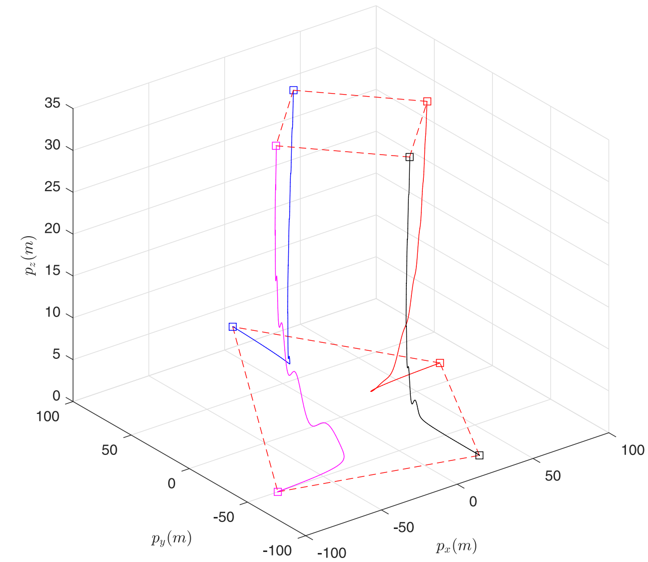

Fig. 6. 3-D trajectories by the leader-following controller.

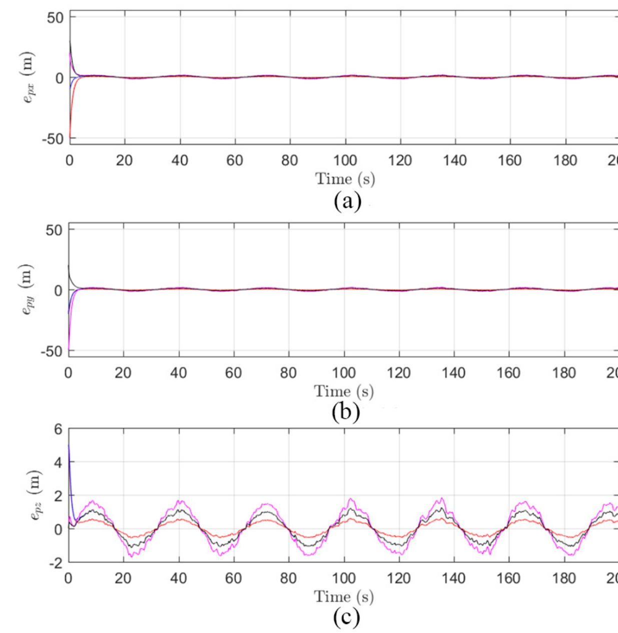

Fig. 7. Trajectory tracking errors by the leader-following controller. (a) Tracking error along $\hat{e}_x^M$. (b) Tracking error along $\hat{e}_y^M$. (c) Tracking error along $\hat{e}_x^M$.

desired trajectory $p^r(t)$ of the formation center is relative to the chief satellite. The formation center is considered as a virtual leader, and the desired trajectory of the formation center is described in the chief satellite-fixed frame $\hat{E}_M$. The chief satellite is moving on the desired orbit in the inertial system $\hat{E}_I$ attached to the Earth, and the formation center is moving away from the chief satellite in $\hat{E}_M$. The four satellites are required to form a time-varying formation pattern as $\xi_{p1}(t) = 50(1 + e^{-t})c_{3,1}$, $\xi_{p2}(t) = 50(1 + e^{-t})c_{3,2}$, $\xi_{p3}(t) = -50(1 + e^{-t})c_{3,1}$, and $\xi_{p4}(t) = -50(1 + e^{-t})c_{3,2}$. The topological structure among satellites is described by the edge set $E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4)\}$ and the adjacency matrix W is given as

$$[w_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

which means the information can flow from satellites 1 to 2, from satellites 2 to 3, and from satellites 1 to 4.

Only satellite 1 can directly receive information from the virtual leader, thereby $\beta_{l1} = 1$ and $\beta_{li} = 0 (i = 2, 3, 4)$. The initial conditions of the four satellites are given as: $p_1(0) = [50 \ \ -20 \ \ 5]^T$, $p_2(0) = [-10 \ \ 80 \ \ 5]^T$, $p_3(0) = [-80 \ \ -50 \ \ 0]^T$,

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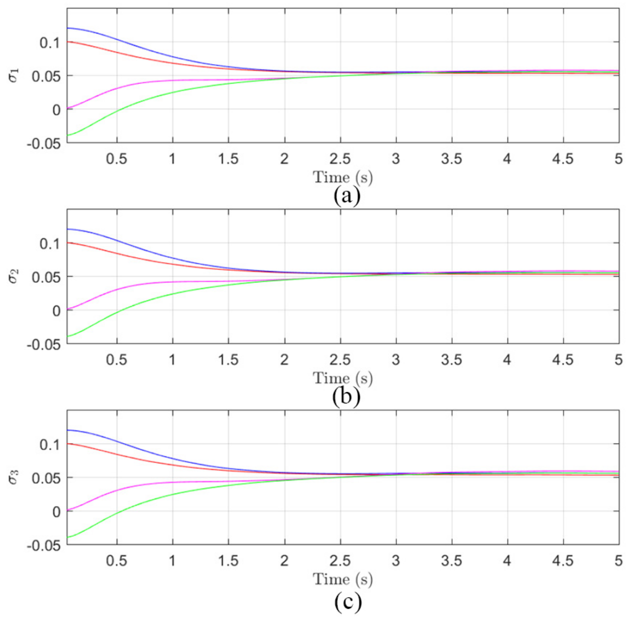

Fig. 8. Transient attitude responses by the leader-following controller. (a) σ_1. (b) σ_2. (c) σ_3.

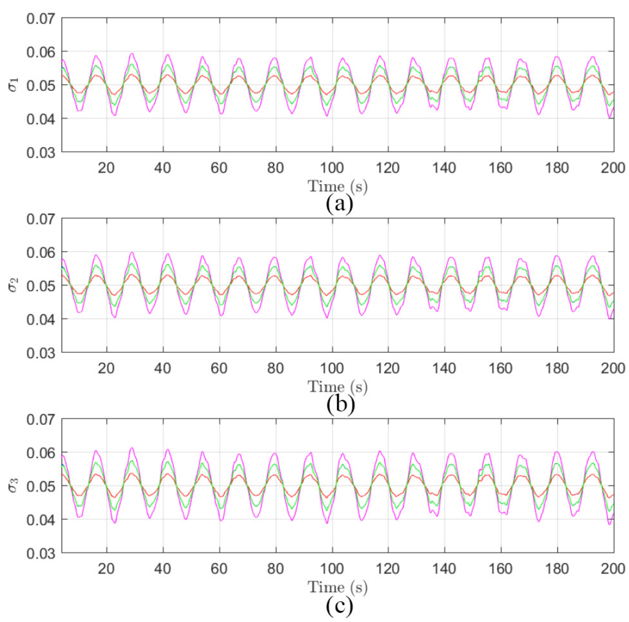

Fig. 9. Steady-state attitude responses by the leader-following controller. (a) σ_1. (b) σ_2. (c) σ_3.

$p_4(0) = [30 \quad -80 \quad 0]^T$, $\dot{p}_1(0) = [0.1 \quad 0.05 \quad 0.1]^T$, $\dot{p}_2(0) = [0 \quad 0.1 \quad 0.05]^T$, $\dot{p}_3(0) = [-0.1 \quad -0.08 \quad 0]^T$, $\dot{p}_4(0) = [0 \quad 0.08 \quad 0.1]^T$, $\sigma_1(0) = [0.1 \quad 0.1 \quad 0.1]^T$, $\sigma_2(0) = [0.12 \quad 0.12 \quad 0.12]^T$, $\sigma_3(0) = [0 \quad 0 \quad 0]^T$, and $\sigma_4(0) = [-0.04 \quad -0.04 \quad -0.04]^T$. Assume each parameter uncertainty is 10% of the nominal parameter, and the external time-varying disturbances on each satellite include sinusoidal signals and random signals. For the translational dynamics, external time-varying disturbances include $d_p(t) = 5[\sin(2t) \quad \sin(2t) \quad \sin(2t)]^T$ and a Gaussian distributed random signal, with mean 0 and variance 1. For the rotational dynamics, external time-varying disturbances include $d_\tau(t) = 0.3[\sin t \quad \sin t \quad \sin t]^T$ and a Gaussian distributed random signal, with mean 0 and variance 0.01. The nominal controller parameter matrices K_p, $K_{\dot{p}}$, and the scalar coupling gain η_{Fp} are chosen according to [16, Th. 1], by the linear quadratic regulator-based optimal design. In this case, the matrix $A_{\tilde{p}}$ is asymptotically stable. Similarly, the nominal controller parameter matrices K_σ, $K_{\dot{\sigma}}$, and the scalar coupling gain $\eta_{F\sigma}$ are determined to guarantee that the matrix $A_{\tilde{\sigma}}$ is asymptotically stable. The disturbance estimator parameters f_{pi} and $f_{\sigma i}$ are selected by the unidirectional adjustment method in Remark 4. The controller parameters are shown in Table I.

TABLE I
CONTROLLER PARAMETERS

Parameter	Value	Parameter	Value
K_P	$diag(10, 10, 10)$	$K_{\dot{P}}$	$diag(10, 10, 10)$
K_σ	$diag(100, 100, 100)$	$K_{\dot{\sigma}}$	$diag(90, 90, 90)$
f_{pi}	$diag(15, 15, 15)$	$f_{\sigma i}$	$diag(10, 10, 10)$
η_{Fp}	1	$\eta_{F\sigma}$	1

Figs. 2–5 depict the 3-D trajectory p_i, the trajectory tracking error e_{pi}, the transient attitude σ_i, and the steady-state σ_i, respectively. The trajectories of satellites 1–4 are represented by the red, blue, pink, and black solid lines, and the formation pattern by the red dotted lines. The trajectory tracking error e_{pi} can converge into the neighborhood of the origin in 10 s bounded by $\varepsilon_{ep} = 0.1$, and the satellite attitude error $e_{\sigma i}$ in 5 s bounded by $\varepsilon_{e\sigma} = 0.002$. Compared to the proposed controller, the 3-D trajectory, the trajectory tracking error, the transient attitude, and the steady-state attitude showed in Figs. 6–9 are obtained via the leader-following controller in [23]. The steady position and attitude tracking errors by the leader-following controller are nearly 2 and 0.01, respectively. The comparison demonstrates that the proposed robust time-varying formation flying control method can improve the tracking performance of the global closed-loop control system and restrain the effects of nonlinearities and uncertainties.

VI. CONCLUSION

In this article, a robust formation flying control scheme for a team of satellites subject to nonlinearities and uncertainties was proposed. With the resulting formation controller based on the consensus theory and the disturbance estimation theory, the satellites can form desired time-varying formation patterns and trajectories, and achieve the satellite attitude consensus. Robustness analysis of the proposed closed-loop control system is given, and the numerical simulation results demonstrate that the proposed robust controller has advantages over the baseline controller.

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