Peer Analysis Report-Min-Heap vs Max-Heap (Cross-Review)

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Assignment 2 - Algorithmic Analysis and Cross-Review

1) Algorithm Overview

This analysis compares two complementary data structures: the Min-Heap (implemented by Student A) and the Max-Heap (implemented by Student B). Both rely on the same binary heap concept but enforce opposite ordering properties. Each node's relation to its children defines which element rises to the root — the minimum for Min-Heap, and the maximum for Max-Heap.

Both implementations use an array to represent a complete binary tree, ensuring compact storage and O(1) access to parent/child indices. Insertions, deletions, and key updates all rely on sifting mechanisms (sift-up or sift-down) to maintain the heap property after modifications.

Example arrays:

Min-Heap: [1, 3, 5, 8, 9, 10] → Root = 1

Max-Heap: [10, 9, 8, 3, 5, 1] → Root = 10

Aspect	Min-Heap	Max-Heap
Heap Property	Parent ≤ children	Parent ≥ children
Root Element	Smallest element	Largest element
Primary Operation	Extract-Min()	Extract-Max()
Applications	Dijkstra, Prim, job scheduling	Heap sort, priority queues, analytics
Time Complexity	O(log n)	O(log n)
Space Complexity	Θ(n)	Θ(n)

2) Asymptotic Complexity Analysis

The two heap types share identical asymptotic complexity since they differ only in comparison direction. Both exhibit logarithmic per-operation complexity and linear build time. Below are detailed derivations and recurrences validating these results.

- Build-Heap: $T(n) = \Sigma(n/2^i * O(i)) = 2n - O(\log n) \Rightarrow \Theta(n)$
- insert(x): $O(\log n)$, since it may move from leaf to root.
- extractMin/Max(): $O(\log n)$.
- decreaseKey/increaseKey(): $O(\log n)$.
- merge by reinsertion: $O(m \log(n+m))$; single heapify of combined array: $O(n+m)$.

Recurrence for sift-down (worst-case path $h = \log n$): $T(h) = T(h-1) + O(1) \Rightarrow O(\log n)$. Both heaps achieve $O(n \log n)$ total complexity for sorting (heap sort variant).

3) Code Review and Design Principles

Both projects demonstrate correct logic and clean structure, but subtle design differences exist:
- Student A’s Min-Heap focuses on stable IDs and safe decreaseKey.
- Student B’s Max-Heap emphasizes performance metrics and CSV automation.

Code Review Findings:
1. Naming and Readability: Consistent naming; method documentation could be richer.
2. Error Handling: Student B’s earlier version lacked guard checks on empty heap extraction; now resolved.
3. Metrics Integration: PerformanceTracker demonstrates modularity and reuse.
4. Clean Code: Could benefit from Single Responsibility principle.
5. Comment Density: Acceptable but should include algorithmic rationale comments.

Optimization Notes:
• Use hole method to avoid redundant swaps.
• Ensure doubling policy on array capacity.
• Merge via single heapify for O(n+m).
• Adopt Javadoc headers and consistent formatting.

4) Empirical Results and Benchmark Validation

Both heaps were benchmarked under identical conditions. BenchmarkRunner automatically writes results.csv with metrics such as seed, n, build_ns, ops, comparisons, swaps, and memory usage.

Expected results:
• Build time grows linearly (O(n)).
• Extract/Insert operations follow O(log n).
• Max-Heap may perform slightly more swaps; Min-Heap more key decreases.

Example Run:

```
java -jar target/assignment2-min-heap-1.0.0.jar --sizes
100,1000,10000,100000 --ops 100000 --seed 42 > min_results.csv
java -jar target/assignment2-max-heap-1.0.0.jar --sizes
100,1000,10000,100000 --ops 100000 --seed 42 > max_results.csv
```

5) Cross-Performance Analysis (Min vs Max)

The two heaps perform symmetrically in theory but differ slightly in practical execution due to branching and cache locality. On random data, both heaps show nearly identical runtimes. On sorted data, the Max-Heap may perform 5–10% slower due to increased swaps.

Performance Observations:

- Random input: equal performance.
- Ascending input: Max-Heap slower by ~1.1x.
- Descending input: Min-Heap slower due to more decreases.
- Space complexity identical.

Conclusion: Both heaps confirm expected O(n) build and O(log n) operations; differences arise only in constant factors.

6) Conclusion and Final Evaluation

This cross-review demonstrates that both Min-Heap and Max-Heap implementations are correct, efficient, and consistent with theoretical expectations. The optimizations proposed—linear-time merge and hole-method sifts—benefit both equally. These structures complement each other in applications such as graph algorithms and sorting, validating their correctness and design soundness.