Übungsblatt 6

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Aufgabe 18

$$
\begin{aligned}
Var(\bar{X}) = & \\
& = Var(\frac{1}{n} \cdot (X_1 + \cdots + X_n)) \\
& = \frac{1}{n^2}(Var(X_1 + \cdots + X_n)) \\
& = \frac{1}{n^2}(Var(X_1) + Var(X_2) + \cdots + Var(X_n)
\end{aligned}
$$

$$
\begin{aligned}
E(\bar{X}) = E(\frac{1}{n}(X_1 + X_2 + \cdots + X_n)) \\
= \frac{1}{n}\dot{E}(X_1 + \cdots + X_n) \\
= \frac{1}{n}\dot{(}E(X_1) + \cdots + E(X_n))
\end{aligned}
$$

a)

$Var(\bar{X}) = \frac{1}{12n}$ $E(\bar{X}) = \frac{1}{2}$

b)

$Var(\bar{X}) = \frac{1}{\lambda^2 n}$ $E(\bar{X}) = \lambda$

c)

$Var(\bar{X}) = \frac{\sigma^2}{n}$ $E(\bar{X}) = \mu$

Aufgabe 19

$$
\begin{aligned}
X_i - \bar{X} &= X_i - \mu + \mu - \bar{X} \\
(X_i - \bar{X})^2 &= ((X_i - \mu + \mu - \bar{X}))^2 \\
&= (X_i - \mu)^2 + 2(X_i - \mu)(\mu - \bar{X}) + (\mu - \bar{X})^2
\end{aligned}
$$

$$
\begin{aligned}
\sum (X_i - \bar{X})^2 &= \sum (X_i - \mu)^2 + \sum 2 \cdot (X_i - \mu)(\mu - \bar{X}) + \sum (\mu - \bar{X})^2 \\
&= \sum (X_i - \mu)^2 + n\dot{\sum}(\mu - \bar{X})^2 + 2\sum \cdot (X_i - \mu)(\mu - \bar{X}) \\
&= \underbrace{\sum (X_i - \mu)^2}_{:=a} + n(\mu - \bar{X})^2 + 2(\mu - \bar{X}) \sum (X_i - \mu) \\
&= a + 2(\mu - \bar{X})(n \cdot \bar{X} - n\mu) \\
&= a - 2n(-\mu + \bar{X})(\bar{X} - \mu) \\
&= a - 2n(\bar{X} - \mu)(\bar{X} - \mu) \\
&= a - 2n(\bar{X} - \mu)^2 \Rightarrow \sum (X_i - \mu)^2 - n(\mu - \bar{X})^2
\end{aligned}
$$

$$
\begin{aligned}
E(s_n^2) &= E[\frac{1}{n-1} \sum (X_i - \bar{X})^2] \\
&= \frac{1}{n-1} E[\sum (X_i - \bar{X})^2] \\
&= \frac{1}{n-1} [\sum \underbrace{E((X_i - \mu)^2)}_{\sigma^2} - nE(\mu - \underbrace{\bar{X}^2}_{\frac{\sigma^2}{n}})] \\
&= \frac{1}{n-1} [n \cdot \sigma^2 - n \cdot \frac{\sigma^2}{n}] \\
&= \sigma^2 = Var
\end{aligned}
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Aufgabe 20

a)

$$
\begin{aligned}
P(X \leq 69) &= P(Z \leq \frac{69-70}{2}) = P(Z \leq -\frac{1}{2}) \\
&= 0.5 - P(0 \leq Z \leq \frac{1}{2}) \\
&= 0.5 - 0.1915 \\
&= 0.3085 \\
P(x \geq 73) &= P(Z \geq \frac{73-70}{2}) = P(Z \geq 1.5) \\
&= 0.5 - P(0 \leq Z \leq 1.5) \\
&= 0.0668
\end{aligned}
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b)

$$
\begin{aligned}
P(X_\mu \leq 70) \leq 0.1 &\iff P(Z \leq \frac{X_\mu - 70}{\sigma}) \leq 0.1 \\
&\iff P(Z \leq \frac{X_\mu - 70}{2}) \leq 0.1 \\
&\iff [0.5 - P(0 \leq Z \leq \frac{X_\mu - 70}{2})] \leq 0.1 \\
&\frac{X_\mu - 70}{2} = 1.29 \\
&X_\mu = 72.58
\end{aligned}
$$