Passives in Power Electronics: Magnetic Component Design and Simulation

Prof. W. G. Hurley

Power Electronics Research Centre

National University of Ireland, Galway

Dr. Werner Wölfle

Traco Power Solutions

Wexford, Ireland

ECPE Copenhagen
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Outline

- **Magnetics Basics**
  - Laws of electromagnetism
  - Losses in magnetic components
  - Ferromagnetic materials
- **Inductor Design**
  - Basic equations
  - Design methodology
  - Buck converter inductor design: gapped core
  - Dual windings inductor design methodology
  - Flyback converter inductor design: dual windings
  - ZCS-QRC converter inductor design

NUI Galway
OÉ Gaillimh

Outline

- **Transformer Design**
  - Basic equations
  - Design methodology
  - Push-pull converter transformer design
  - Transformer insulation

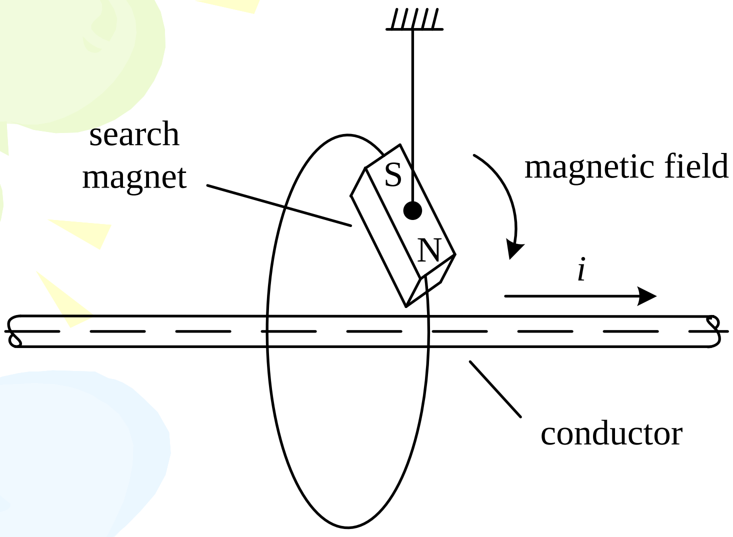
- **High Frequency Effects in the Winding**
  - Skin and Proximity effect
  - Optimize the thickness of the windings: pushpull
  - Litz wire
  - Interleaving the windings
  - Fringing effect
  - Leakage inductance in transformer windings

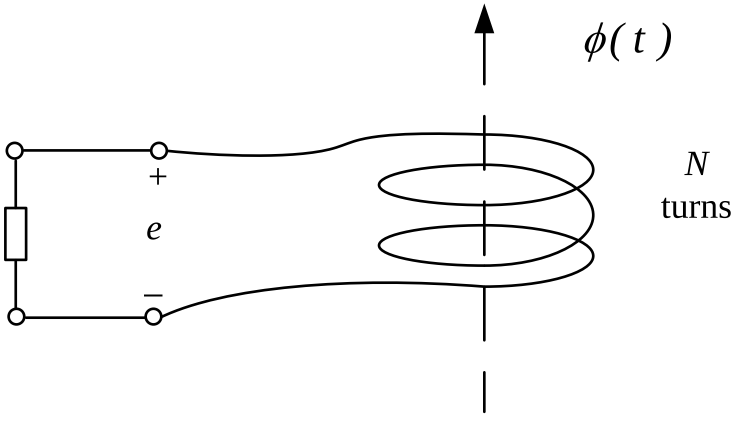
Outline

- **High Frequency Effects in the Core**
  - Eddy current in the core
  - Core losses calculation (GSE, iGSE)
  - Core losses in the transformer: push-pull
- **Conclusions**

Review of Fundamentals

Laws of Electromagnetism





Ampere's Law

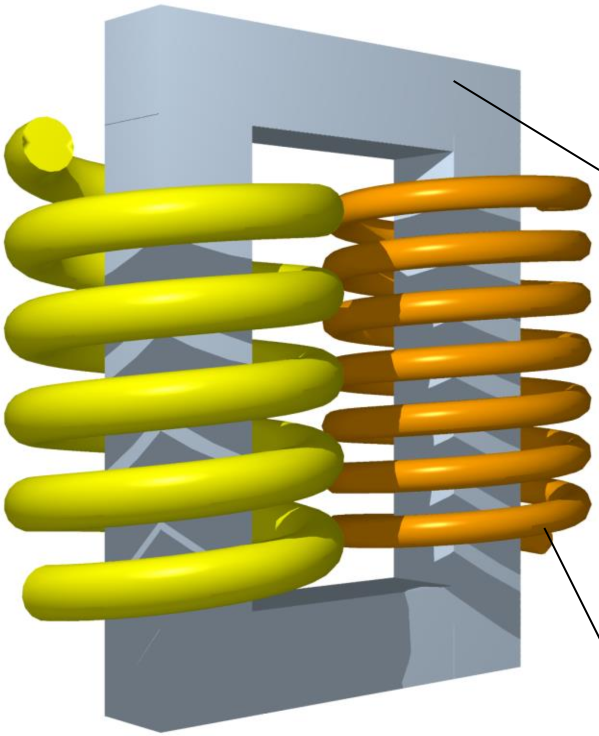
Faraday's Law

$$\sum H \cdot l = Ni$$

$$e = -N \frac{d\phi}{dt}$$

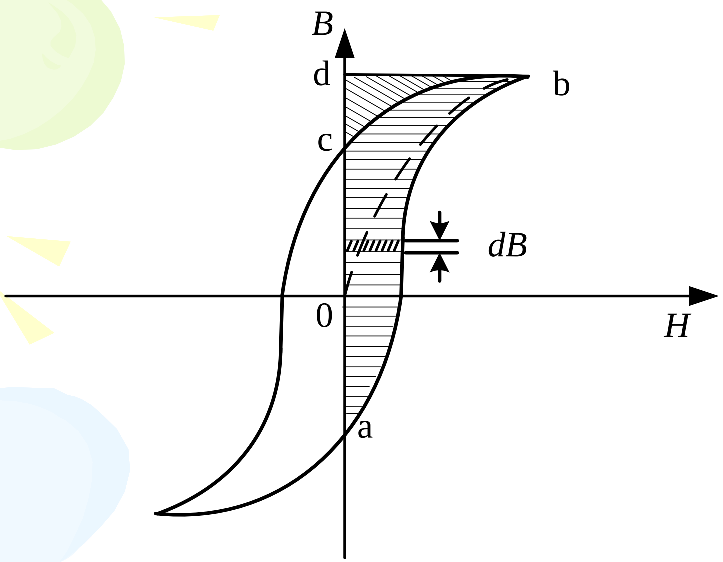
$$\mathbf{B} = \mu \mathbf{H} \qquad \mu = \mu_r \mu_0$$

Losses in Magnetic Components

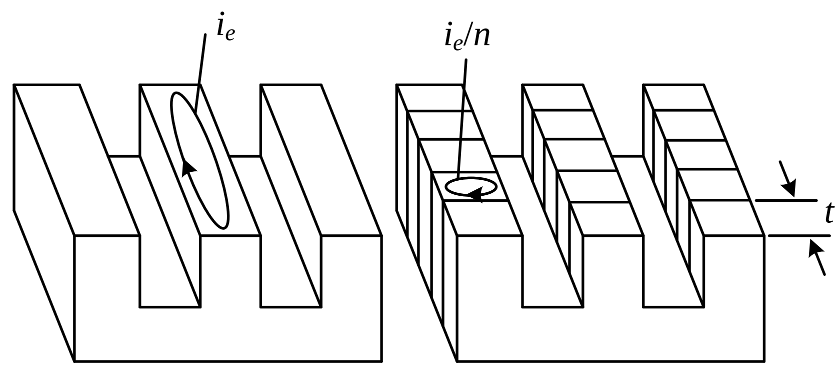


- Core losses
  - Hysteresis loss
  - Eddy current loss
- Copper losses
  - Skin effect loss
  - Proximity effect loss

Core Loss



Hysteresis loss in a ferromagnetic material

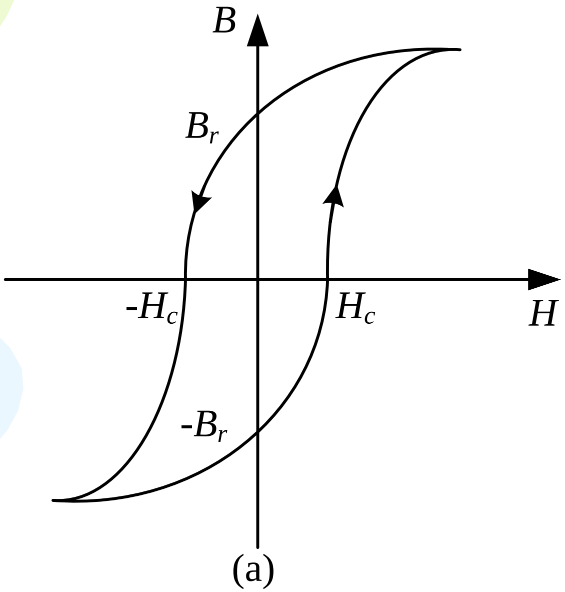


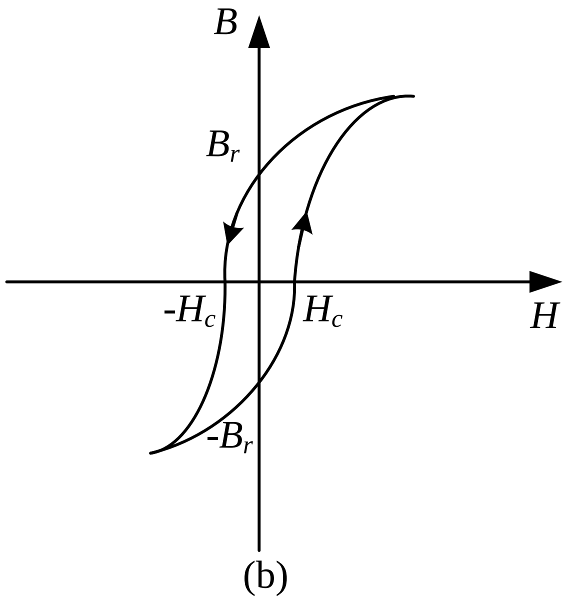
Eddy current loss in a ferromagnetic material

$$P_{fe} = K_c f^{\alpha} \hat{B}^{\beta}$$

- Hysteresis loss is the area inside the B-H loop
- Eddy current loss is reduced by laminations in steel
- Eddy current loss is reduced by higher resistivity in ferrites

Ferromagnetic Materials





(a) Hard magnetic materials

(b) Soft magnetic materials

Ferromagnetic Materials

Soft magnetic materials are classified as:

- Ferrites
- Laminated iron alloys
- Powered iron
- Amorphous alloys
- Nanocrystalline materials

Soft Magnetic Materials

The magnetic and operating properties of some soft magnetic materials

Materials	**Ferrites**	**Nanocrystalline**	**Amorphous**	**Si Iron**	**Ni-Fe (Permalloy)**	**Powdered iron**
Model	TDK P40	VIROPERM 500F	METGLAS 2605	AK Oriented M-4	MAGNETICS PERMALLOY 80	MICROMET -ALS 35μ
Permeability, μ_i	1500-4000	15000	10,000-150, 000	5,000-10,000	20,000	3-550
B_{peak}, T	0.45-0.81	1.2	1.56	2.0	0.82	0.6-1.3
ρ, μΩm	6.5×10^6	1.15	1.3	0.51	0.57	10^6
Curie temp. T_c, ° C	215	600	399	746	460	665
P_{loss}	60 mW/ cm^3 at 0.1T/50kHz	588 mW/cm^3 at 0.3T/100kHz	72 m W/cm^3 at 0.2T/25kHz	2.295-30.6mW/cm^3 at 1.5T/50Hz	192.28mW/cm^3 at 0.2T/5kHz	126-315mW/cm^3 at 0.1T/10kHz

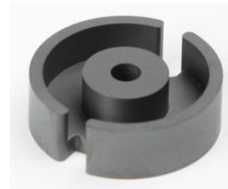
Magnetic Materials Applications

Materials	Shapes	Applications
Ferrites	Toroid Planar EI EE IU	Inductors (with air-gape) for switched mode power supplies; Transformers for switch mode power supplies; RF transformers and inductors; Pulse and signal transformers Noise filters; Magnetic Sensor applications Differential filter inductors; Common mode inductors
Nanocrystalline	U	Common mode choke; HF absorption filter; Signal transformers
Amorphous	Tape wound toroid	Current transformer for low frequency Current and magnetic Sensors for low frequency Low frequency transformer; Shielding for devices and buildings
Si Iron	Tape wound toroid Strip wound core Laminated stack	Mains transformer 50/60Hz Military/Aerospace Transformer 400Hz DC Energy Storage Inductor
Ni-Fe (Permalloy)	Toroid	Flyback Inductors/Transformers; Buck/Boost Inductors/Transformers Power Factor Correction (PFC) inductors; Resonant Circuits Inductors with high AC currents; In-Line Noise Filter Inductors; High Q Filters
Powdered iron	Toroid	Inductor applications for low frequency (<50kHz); Inductor applications for Very low AC current (DC Inductors); In Line Filter Inductors;

Core Shapes


Toroid core

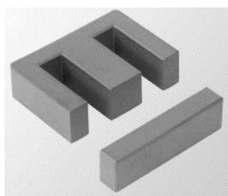

PQ core


Pot core


RS/DS core


RM core


EE core


EI core


ER core


EFD core

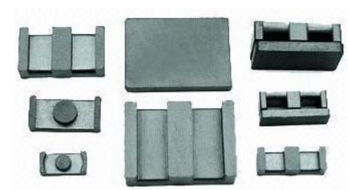

ETD core


EP core

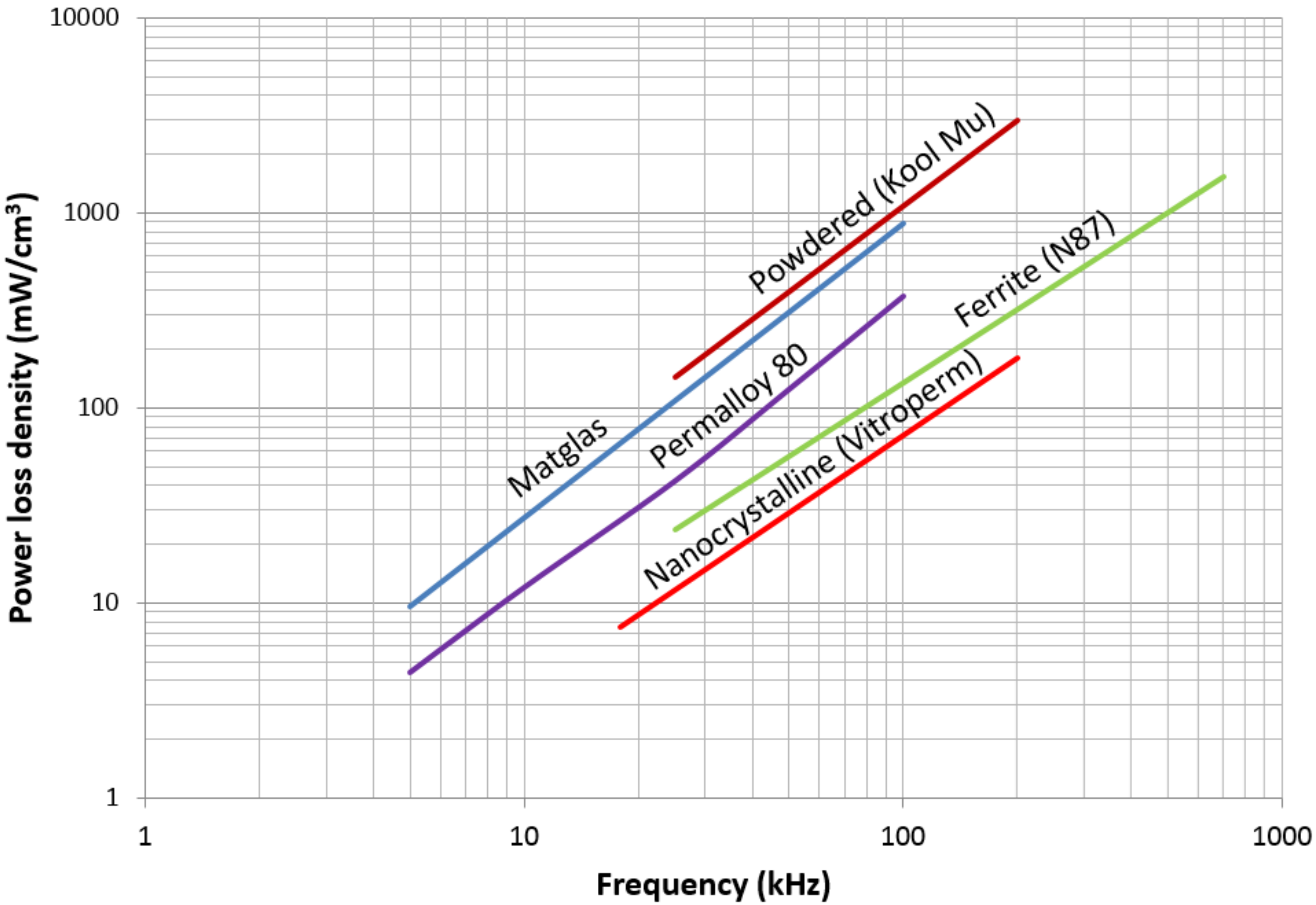

UR core


U core


C core

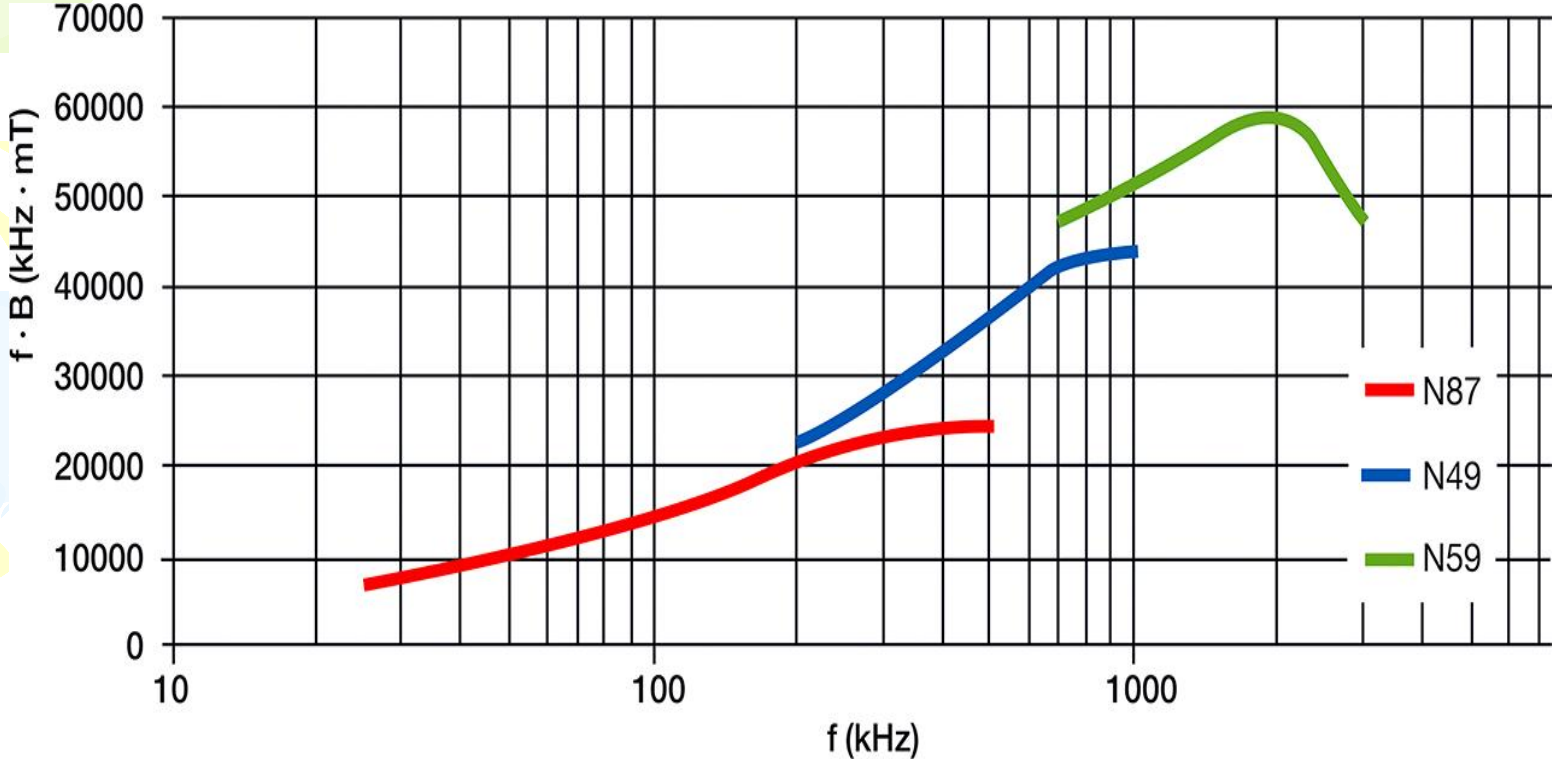

Planar core

Core Loss Density vs Frequency



B=0.1 T

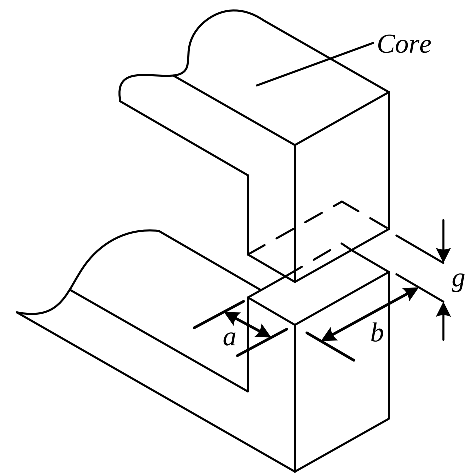
Performance Factor



Effective Permeability

$$\mu_{\text{eff}} = \frac{1}{\dfrac{1}{\mu_r} + \dfrac{1}{l_c / g}} \approx \frac{l_c}{g}$$

$$\mu_r \gg 1$$



l_c is the magnetic core length

g is the gap length

Inductor Design

Inductor Design Equations

Inductance

$$L = \frac{\mu_{\mathrm{eff}} \mu_0 N^2 A_c}{l_c}$$

Maximum Flux density

$$B_{\max} = \mu_{\mathrm{eff}} \mu_0 H_{\max} = \frac{\mu_{\mathrm{eff}} \mu_0 N \hat{I}}{l_c}$$

Copper loss

$$P_{cu} = \rho_w \frac{l_w}{A_w} I_{rms}^2 = \rho_w \frac{N\,\mathrm{MLT}\left(K_i \hat{I}\right)^2}{A_w} = \rho_w \frac{N^2\,\mathrm{MLT}\left(K_i \hat{I}\right)^2}{N A_w}$$

Core loss

$$P_{fe} = K_c f^{\alpha} \left(\frac{\Delta B}{2}\right)^{\beta} \qquad P_{fe} = \gamma P_{cu}$$

μ_{eff}: effective relative permeability
N: turns
A_c: cross section of the core
l_c: length of the core
$\hat{I}$: the peak value of current
ρ_w: the resistivity of the conductor
MLT: the mean length of a turn
A_w: cross section of the conductor
ΔB: peak to peak flux density ripple

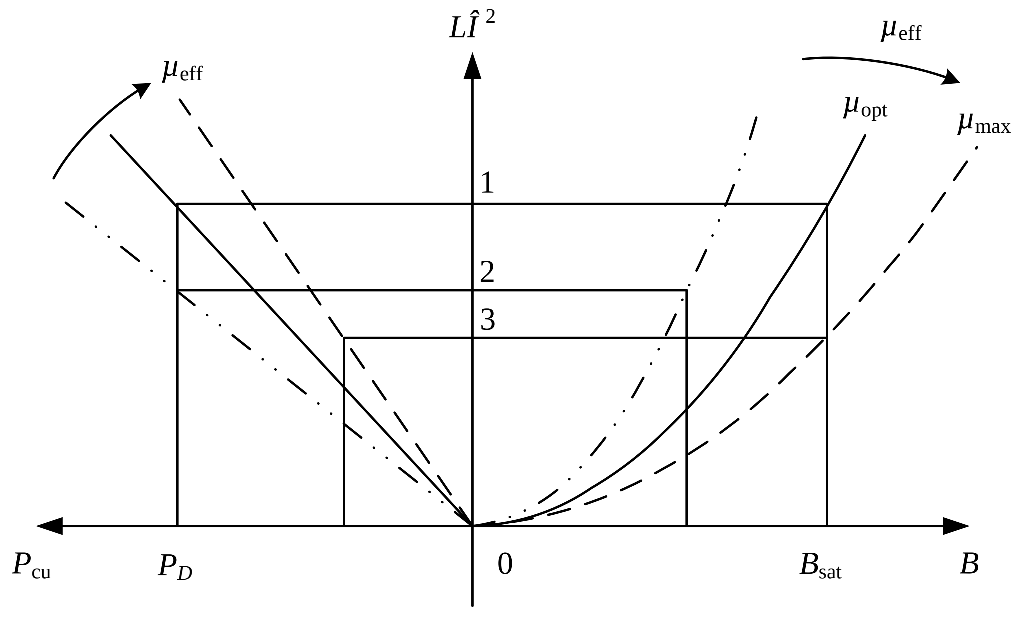
Optimum Effective Permeability

Stored energy

$$\tfrac{1}{2} L\hat{I}^2 = \tfrac{1}{2}\frac{A_c l_c}{\mu_{\text{eff}}\mu_0} B_{\max}^2 \qquad \tfrac{1}{2} L\hat{I}^2 = \tfrac{1}{2}\frac{\mu_{\text{eff}}\mu_0 A_c N A_w}{\rho_w \,\text{MLT}\, K_i^2 l_c} P_{\text{cu}}$$

Design results

$$\mu_{\text{opt}} = \frac{B_{\text{sat}} l_c K_i}{\mu_0 \sqrt{\dfrac{P_{\text{cumax}} N A_w}{\rho_w \text{MLT}}}}$$



Stored energy as a function of flux density and dissipation

Thermal Equations

Temperature rise

$$\Delta T = R_\theta Q = \frac{1}{h_c A_t} Q$$

Q: the total power loss
R_θ: the thermal resistance
h_c: the coefficient of heat transfer
A_t: surface area
V_c: the volume of the core
MLT: the mean length of a turn
A_w: cross section of the conductor
ΔB: peak to peak flux density ripple

Thermal resistance

$$R_\theta \approx \frac{C}{\sqrt{V_c}}$$

C=0.06, V_c in the range 0.4×10^{-6} m^3 to 100×10^{-6} m^3

Empirical formula

$$h_c = 1.42 \left[\frac{\Delta T}{H} \right]^{0.25}$$

- The typical value of h_c = 10 W/m$^{2\circ}$C is often used for cores encountered in switching power supplies.

Current Density in the Windings

Window utilization factor

$$k_u = \frac{W_c}{W_a}$$

Total conducting area

$$W_c = NA_w = k_u W_a$$

Optimum value of the effective relative permeability

$$\mu_{\text{opt}} = \frac{B_{\text{sat}} l_c K_i}{\mu_0 \sqrt{\dfrac{P_{\text{cu max}} k_u W_a}{\rho_w \text{MLT}}}}$$

k_u:	window utilization factor
A_w:	cross section of the conductor
W_c:	electrical conduction area
W_a:	window winding area of core
A_t:	surface area of wound transformer
V_w:	volume of winding
K_i:	current waveform factor

Current density

$$J_o = \frac{I_{\text{rms}}}{A_w}$$

Combine

$$P_{\text{cu}} = \rho_w \frac{N^2 \text{MLT} \left(J_o A_w\right)^2}{NA_w} = \rho_w V_w k_u J_o^2$$

Total power loss

$$Q = P_{\text{cu}} + P_{\text{fe}} = (1+\gamma)\left[\rho_w V_w k_u J_0^2\right] = h_c A_t \Delta T$$

Core Window Area Product

(3.25)

Core area by window area product

$$A_p = \left[\frac{\sqrt{1+\gamma}\,K_i L \hat{I}^2}{B_{max} K_\theta \sqrt{k_u \Delta T}} \right]^{8/7}$$

$$J_o = \sqrt{\frac{1}{1+\gamma} \frac{h_c A_t \Delta T}{\rho_w V_w k_u}}$$

$$= \sqrt{\frac{h_c k_t}{\rho_w k_w}} \sqrt{\frac{1}{1+\gamma} \frac{\Delta T}{k_u} \frac{1}{A_p^{1/4}}} = K_\theta \sqrt{\frac{1}{1+\gamma} \frac{\Delta T}{k_u} \frac{1}{A_p^{1/4}}}$$

Dimensional analysis

$$V_w = k_w A_p^{3/4}$$

$$V_c = k_c A_p^{3/4}$$

$$A_t = k_t A_p^{1/2}$$

Here

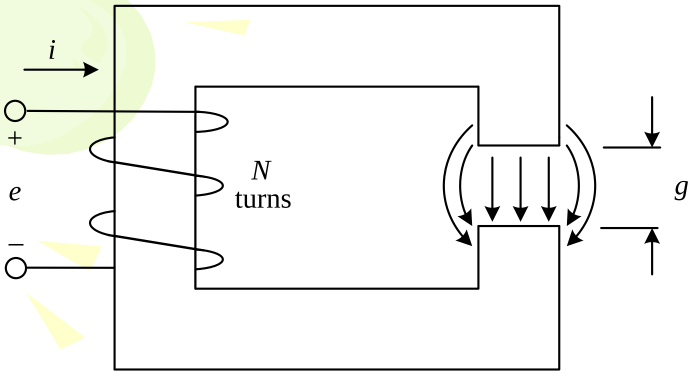
$A_p = A_c W_a$ *Core cross-sectional area by the Window winding area*

γ is the ratio P_{fe}/P_{cu} , often taken as 0 in inductors with low current ripple

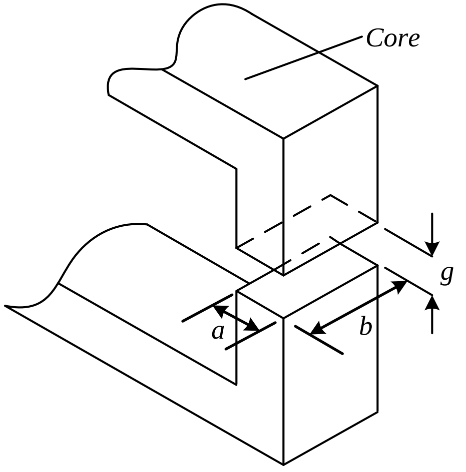
K_θ is a constant 48,200 with SI units, derived from dimensional analysis

k_u is the window utilisation factor

Fringing Effect in Gap



Fringing effect in an air-gap

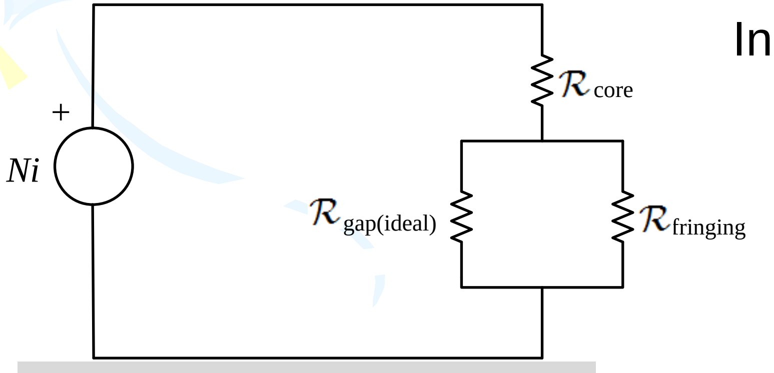


Increase gap dimensions (*a*+*g*) and (*b*+*g*)

Inductance increased by

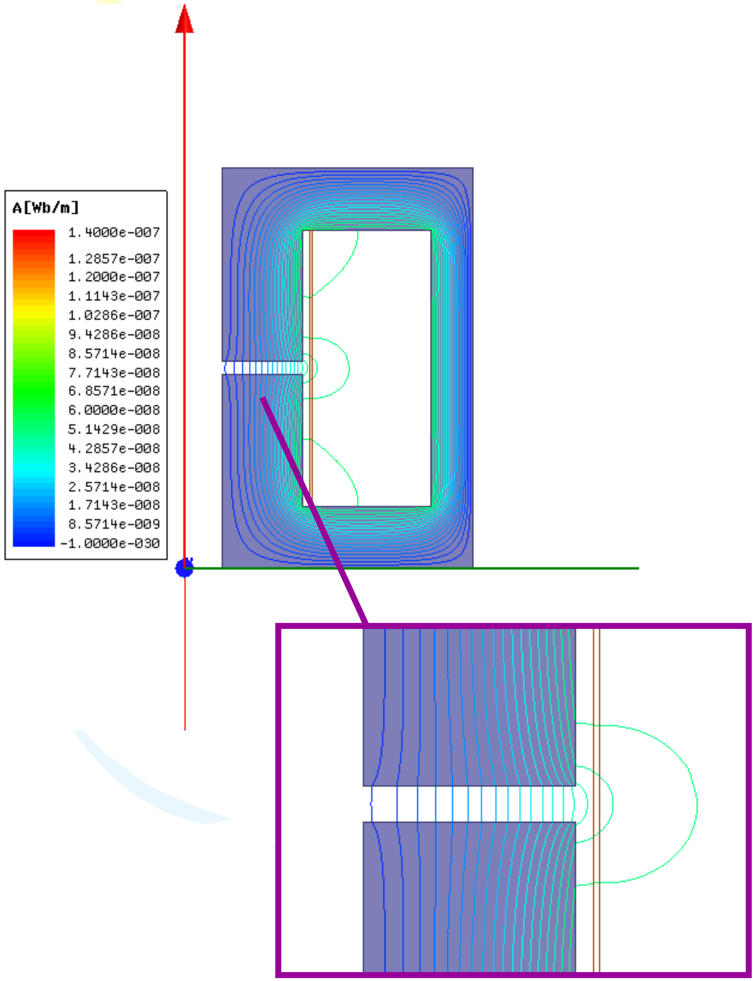
$$L' \approx L\left(1+\frac{a+b}{ab}g\right)=L\left(1+\frac{2g}{a}\right) \quad \text{for a=b}$$

$$L=\frac{\mu_0 N^2 A_c}{\left(g+\frac{l_c}{\mu_c}\right)} \approx \frac{\mu_0 N^2 A_c}{g}$$

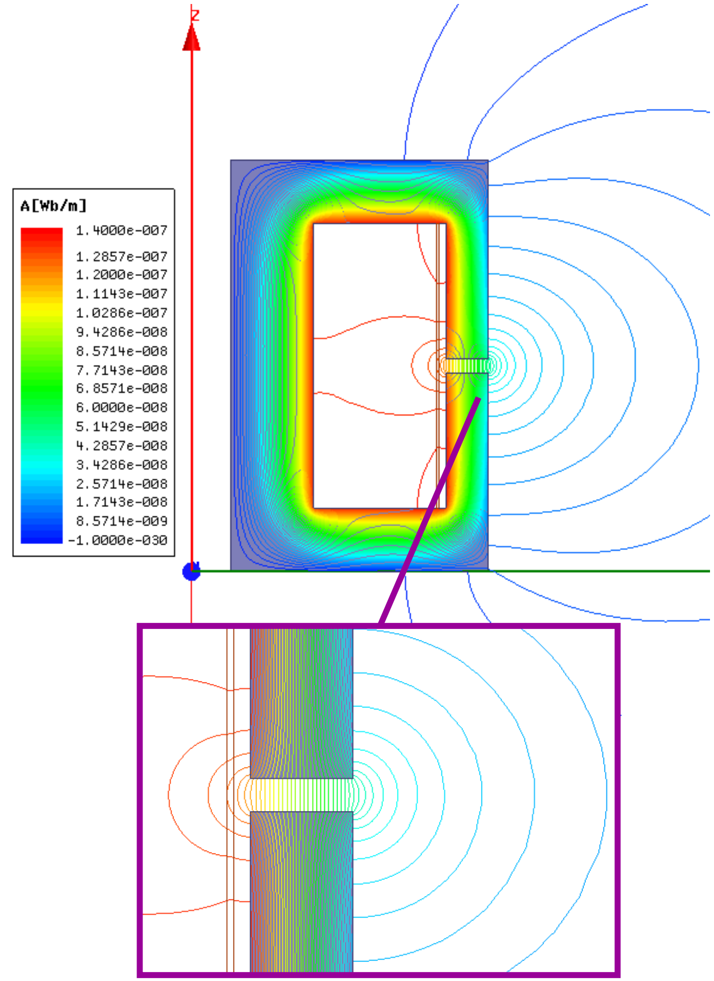


Equivalent magnetic circuit for fringing effects in an air-gap

Fringing (Flux)

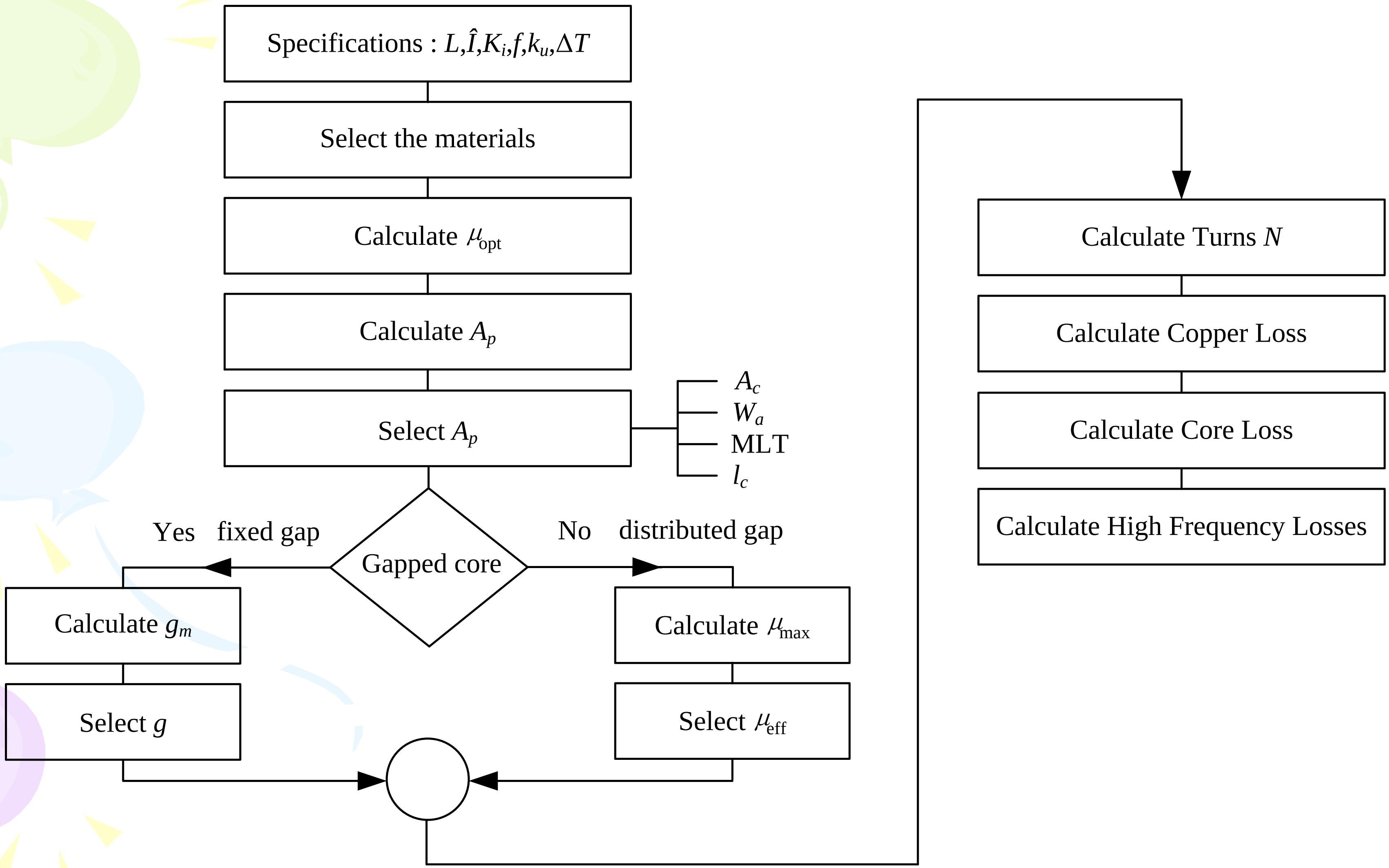


Gap in the centre leg



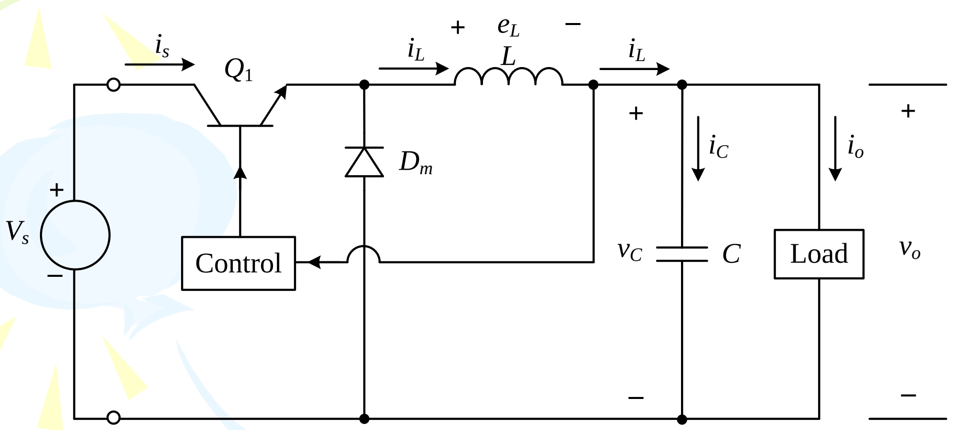
Gap in the outer leg

Design Methodology

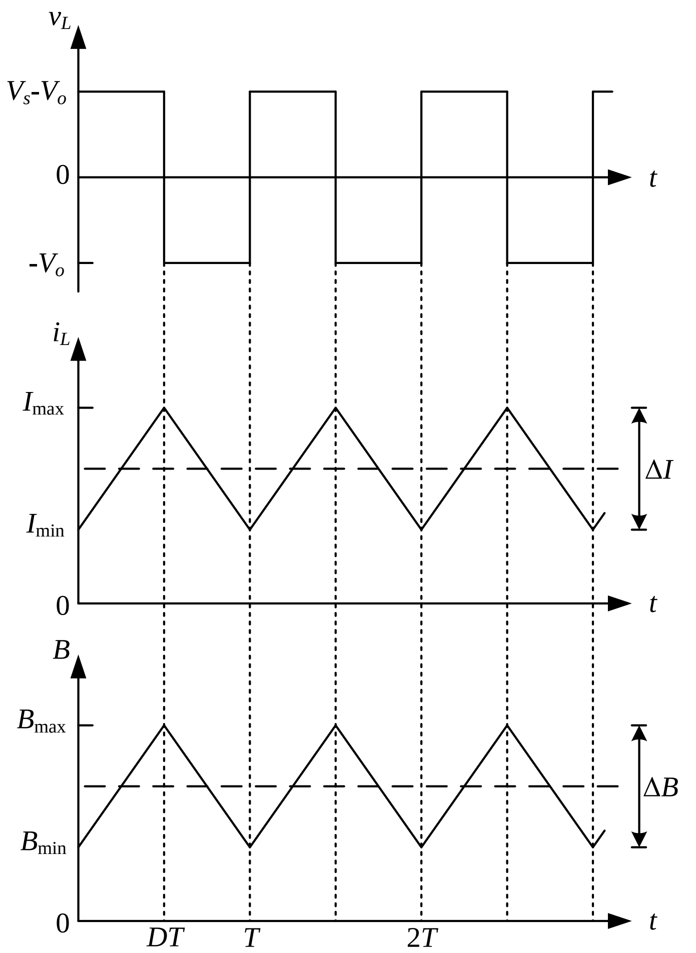


Buck Converter Inductor Design

Circuit



Waveforms



Buck Converter: Specifications

Design specifications for inductor

Input voltage	12 V
Output voltage	6 V
Inductance	34 µH
DC Current	20 A
Frequency, *f*	80 kHz
Temperature Rise, ΔT	15ºC
Ambient Temperature, T_a	70ºC
Window utilization factor	0.8

Core data: EPCOS N87 Mn-Zn

K_c	16.9
α	1.25
β	2.35
B_{sat}	0.4T

Core loss

$$P_{fe} = K_c f^{\alpha} B_{\max}^{\beta}$$

Buck Converter: Core size

Calculations:

(1) Current ripple

$$\Delta I_L = \frac{(V_s - V_o)DT}{L} = \frac{(12-6)(0.5)}{(34\times10^{-6})(80\times10^{3})} = 1.1\,\text{A}$$

(2) Peak current

$$\hat{I} = I_{dc} + \frac{\Delta I}{2} = 20.0 + \frac{1.1}{2} = 20.55\,\text{A}$$

$$L\hat{I}^2 = (34\times10^{-6})(20.55)^2 = 0.0144\,\text{J}$$

(3) A_p

$$A_p = \left[\frac{\sqrt{1+\gamma}\,K_i L\hat{I}^2}{B_{max} K_\theta \sqrt{k_u \Delta T}}\right]^{8/7} = \left[\frac{\sqrt{1+0}\;\;0.0144}{(0.25)(48.2\times10^{3})\sqrt{(0.8)(15)}}\right]^{8/7} \times 10^8 = 4.12\,\text{cm}^4$$

ETD49 Core Data

A_c	2.09 cm^2
l_c	11.4 cm
W_a	2.69 cm^2
A_p	5.62 cm^4
V_c	24.1 cm^3
k_f	1.0
k_u	0.8
K_i	1.0
MLT	8.6 cm
ρ_{20}	1.72 μΩ-cm
α_{20}	0.00393

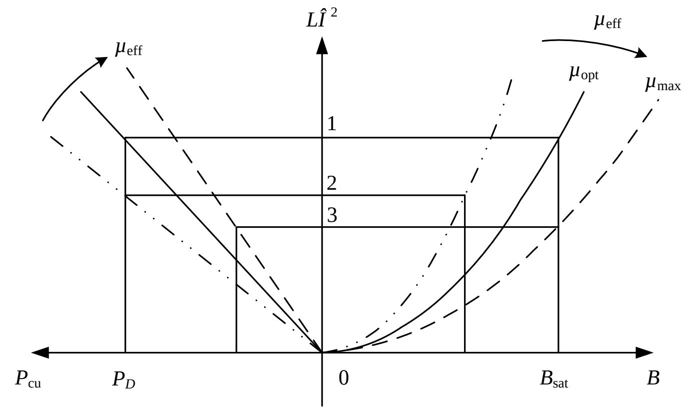
Thermal resistance: $R_\theta = 11°C/W$

Buck Converter: Gap

Calculations:

(4) Maximum dissipation

$$P_D = \frac{\Delta T}{R_\theta} = \frac{15}{11} = 1.36\,\mathrm{W}$$



(5) Optimum value of the effective permeability

$$\mu_{opt} = \frac{B_{max} l_c K_i}{\mu_0 \sqrt{\dfrac{P_{cu\,max} k_u W_a}{\rho_w \mathrm{MLT}}}} = \frac{(0.25)(11.4\times10^{-2})(1.0)}{(4\pi\times10^{-7})\sqrt{\dfrac{(1.36)(0.8)(2.69\times10^{-4})}{(1.72\times10^{-8})(8.6\times10^{-2})}}} = 51$$

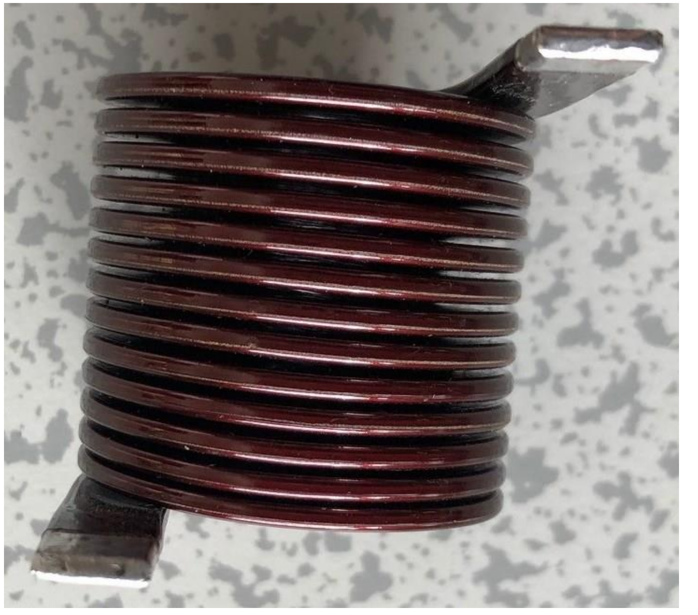
(6) Maximum gap length

$$g_{max} = \frac{l_c}{\mu_{min}} = \frac{11.4\times10^{-2}}{51}\times10^{3} = 2.24\,\mathrm{mm}$$

Select g=2 mm, A_L=188 nH/turn

Buck Converter: Wire size

<u>Calculations:</u>



(7) Turns

$$N = \sqrt{\frac{L}{A_L}} = \sqrt{\frac{(34\times10^{-6})}{(188\times10^{-9})}} = 13.5\ \text{turns}$$

Select *N*=13 turns

(8) Wire size

$$J_o = K_\theta \sqrt{\frac{1}{1+\gamma}\frac{\Delta T}{k_u}\frac{1}{A_p^{1/4}}} = 48.2\times10^3 \sqrt{\frac{1}{1+0}\frac{15}{0.8}\frac{1}{(5.62\times10^{-8})^{1/4}}} = 1.68\ \text{A/mm}^2$$

$$A_w = I_{rms} / J = 20/1.68 = 11.9\ \text{mm}^2$$

An 8 mm × 2 mm wire meets this specification, with a dc resistance of 10.75 × 10^{-6} Ω/cm @20ºC

Window utilisation factor 13x8x2/269=0.77 √

Buck Converter: Losses

Loss calculations:

(9) Copper loss

$$T_{\max} = 70 + 15 = 85\ °\mathrm{C}$$

$$R_{\mathrm{dc}} = (13)(8.6)(10.75\times10^{-6})[1+(0.00393)(85-20)]\times10^{3} = 1.51\ \mathrm{m}\Omega$$

$$P_{\mathrm{cu}} = R_{\mathrm{dc}} I_{\mathrm{rms}}^{2} = (1.51\times10^{-3})(20.0)^{2} = 0.604\ \mathrm{W}$$

(10) Core loss

$$\Delta B = \frac{(V_i - V_o)DT}{NA_c} = \frac{(12-6)(0.5)}{(13)(2.09\times10^{-4})(80\times10^{3})} = 0.014\,\mathrm{T}$$

$$P_{\mathrm{fe}} = V_c K_c f^{\alpha} B_{\max}^{\ \beta} = (23.8\times10^{-6})(16.9)(80\,000)^{1.25}(0.014/2)^{2.35} = 0.005\,\mathrm{W}$$

(11) Total loss

$$P_{\mathrm{total}} = P_{\mathrm{cu}} + P_{\mathrm{fe}} = 0.609\ \mathrm{W}$$

Dual Windings Inductor

- The total current is divided in the ratio *m*:(1 - *m*)
- The areas are distributed in the ratio *n*:(1 - *n*)
- *mI* flows in an area nW_c and current *(1 - m)I* flows in an area (1 - *n*)W_c

Total copper loss

$$P_{cu} = \rho_w \frac{l_w}{W_a}\left[\frac{m^2}{n} + \frac{(1-m)^2}{1-n}\right] I_{rms}^2 \xrightarrow{\text{normalization}} P_{cu} = P_o\left[\frac{m^2}{n} + \frac{(1-m)^2}{1-n}\right]$$

The minimum loss occurs when *m* = *n.*

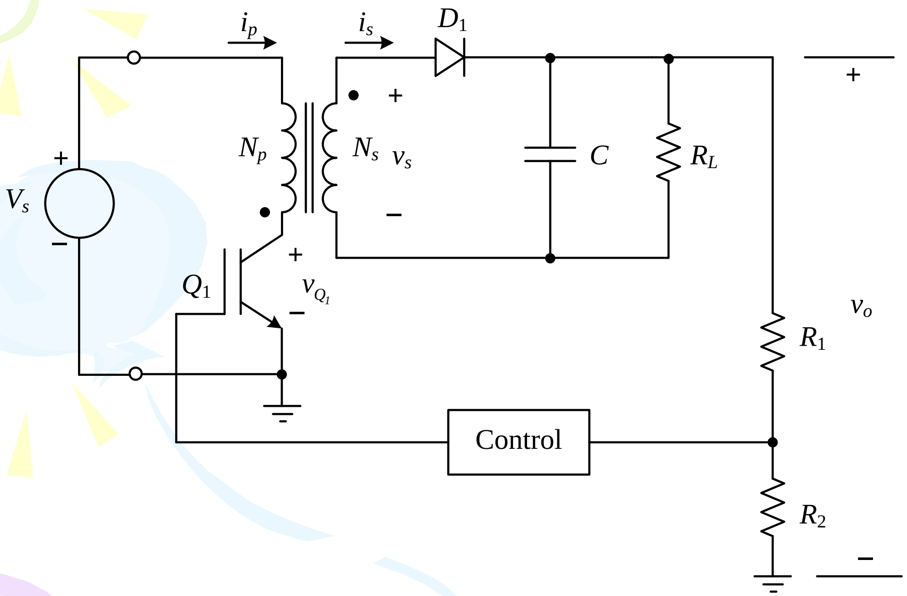
Current density

$$\frac{mI}{nW_a} = \frac{(1-m)I}{(1-n)W_a} = \frac{I}{W_a} = J$$

- The optimum distribution of current in the available area is to have the same current density in each winding which leads to minimum power losses
- In an ideal transformer the winding areas are equal

Flyback Converter Inductor Design

Circuit



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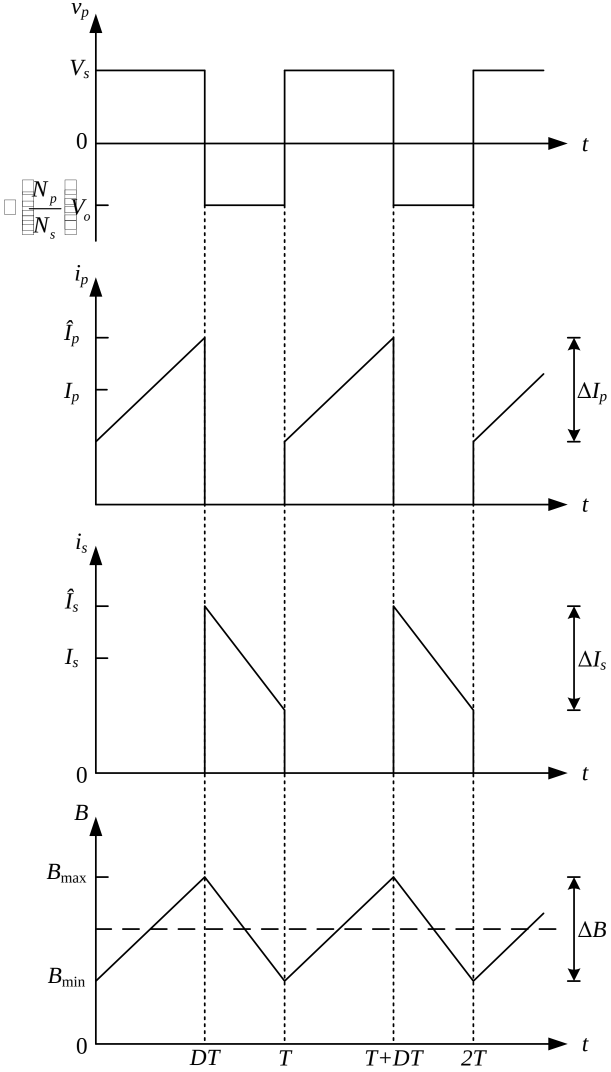


Flyback Converter: Specifications

Design specifications for inductor

Input voltage	325 V
Output voltage	24 V
Inductance	700 µH
DC Current	10 A
Frequency, *f*	70 kHz
Temperature Rise, ΔT	30ºC
Ambient Temperature, T_a	60ºC
Window utilization factor	0.25

Core data: EPCOS N87 Mn-Zn

K_c	16.9
α	1.25
β	2.35
B_{sat}	0.4T

Flyback Converter: Turns ratio

Calculations:

(1) Duty cycle

The turns ratio is chosen to ensure that the switch stress is minimized.
Set D=0.314

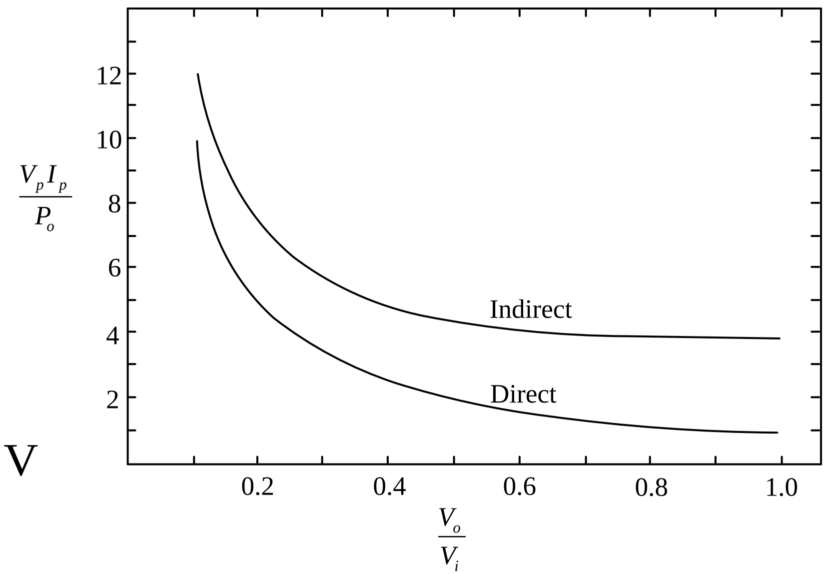
(2) Turns ratio

$$a = \frac{V_i}{V_o}\frac{D}{1-D} = \frac{325.3}{24}\frac{0.314}{1-0.314} = 6.2$$

$$\text{Rectified voltage} = \sqrt{2}(240) + 10\% = 372\,\text{V}$$

$$\text{Blocking voltgage of switch} = 6.2(24)+372=520\,\text{V}$$

$$\text{Rated Blocking Voltage} = 600\,\text{V}; \text{Margin} = 80\,\text{V}$$



Switch stress factors for direct and indirect converters [1]

[1] J. G. Kassakian, M. F. Schlecht, and G. C. Verghese, Principles of Power Electronics (Addison-Wesley Series in Electrical Engineering). Reading, MA: Prentice Hall, 1991.

Flyback Converter: Primary current

Calculations:

(3) Peak value of the primary current

$$I_p = \frac{P}{DV_i} = \frac{240}{(0.314)(\sqrt{2})(230)} = 2.351\,\mathrm{A} \qquad \Delta I_p = \frac{V_i DT}{L_p} = \frac{(\sqrt{2})(230)(0.314)}{(700\times 10^{-6})(70\times 10^{3})} = 2.084\,\mathrm{A}$$

$$\hat{I}_p = I_p + \frac{\Delta I_p}{2} = 2.351 + \frac{2.084}{2} = 3.393\,\mathrm{A}$$

(4) Rms value of the primary current

Current Waveform factor

$$y_p = \frac{\Delta I_p}{\hat{I}_p} = \frac{2.084}{3.393} = 0.614$$

$$K_{ip} = \sqrt{D\left(1 - y_p + \frac{y_p^2}{3}\right)} = \sqrt{(0.314)\left(1 - 0.614 + \frac{(0.614)^2}{3}\right)} = 0.4$$

$$I_{prms} = K_{ip}\hat{I}_p = (0.4)(3.393) = 1.357\,\mathrm{A}$$

Flyback Converter: Secondary current

Calculations:

(5) Peak value of the secondary current

$$I_s = \frac{P}{(1-D)V_o} = \frac{240}{(1-0.314)(24)} = 14.577\,\text{A} \qquad \Delta I_s = \frac{N_p}{N_s}\Delta I_p = (6.2)(2.084) = 12.92\,\text{A}$$

$$\hat{I}_s = I_s + \frac{\Delta I_s}{2} = 14.577 + \frac{12.92}{2} = 21.037\,\text{A}$$

(6) Rms value of the secondary current

Current Waveform factor

$$y_s = \frac{\Delta I_s}{\hat{I}_s} = \frac{12.92}{21.037} = 0.614$$

$$K_{is} = \sqrt{(1-D)\left(1 - y_s + \frac{y_s^2}{3}\right)} = \sqrt{(1-0.314)\left(1 - 0.614 + \frac{(0.614)^2}{3}\right)} = 0.592$$

$$I_{srms} = K_{is}\hat{I}_s = (0.592)(21.037) = 12.454\,\text{A}$$

Flyback Converter: Core size

Calculations:

(7) Window utilization factor

Select the total window utilisation factor as 0.235, because of the voltage insulation requirement

$$k_{up} = k_u \frac{1}{1+\dfrac{I_{srms}}{aI_{prms}}} = (0.235)\frac{1}{1+\dfrac{12.454}{(6.2)(1.359)}} = (0.235)(0.4035) = 0.0948$$

(8) A_p

$$L\hat{I}_p^2 = (700\times10^{-6})(3.393)^2 = 0.0081\text{J}$$

Assume γ = 2

$$A_p = \left[\frac{\sqrt{1+\gamma}K_{ip}L_p\hat{I}_p^2}{B_{max}K_\theta\left(k_{up}/\sqrt{k_u}\right)\sqrt{\Delta T}}\right]^{8/7} = \left[\frac{\sqrt{1+2}(0.4)(0.0081)}{(0.2)(48.2\times10^3)(0.0948/\sqrt{0.235})(\sqrt{30})}\right]^{8/7}\times10^8 = 6.89\text{ cm}^4$$

E55/28/21 Core Data

A_c	3.51 cm^2
l_c	12.4 cm
W_a	2.77 cm^2
A_p	9.72 cm^4
V_c	43.5 cm^3
k_f	1.0
k_u	0.235
MLT	11.3 cm
ρ_{20}	1.72 μΩ-cm
α_{20}	0.00393

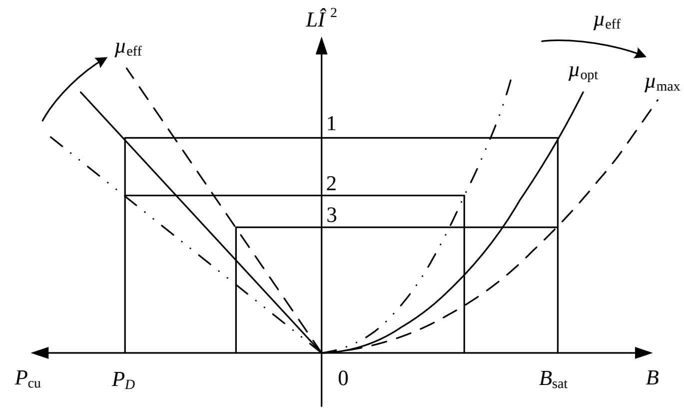
Thermal resistance: $R_\theta = 10°C / W$

Flyback: Effective permeability

Calculations:

(8) Maximum dissipation

$$P_D = \frac{\Delta T}{R_\theta} = \frac{30}{10} = 3.0\,\mathrm{W}$$



Since $\gamma = 2$, the total copper loss in 1.0 W.

$$P_{cu_p\max} = \frac{k_{up}}{k} P_{cu} = \frac{0.0948}{0.235} 1.0 = 0.403\ \mathrm{W}$$

(9) Optimum value of the effective permeability

$$\mu_{opt} = \frac{B_{\max} l_c K_{ip}}{\mu_0 \sqrt{\frac{P_{cu_p\max} k_{up} W_a}{\rho_w \mathrm{MLT}}}} = \frac{(0.2)(12.4\times10^{-2})(0.4)}{(4\pi\times10^{-7})\sqrt{\frac{(0.403)(0.155)(2.77\times10^{-4})}{(1.72\times10^{-8})(11.3\times10^{-2})}}} = 83.5$$

Flyback Converter: Gap and turns

Calculations:

(10) Maximum gap length

$$g_{max} = \frac{l_c}{\mu_{min}} = \frac{(12.4\times10^{-2})}{83.5}10^{3} = 1.48\,\mathrm{mm}$$

Select *g*=1 mm, A_L=496 nH/turn

(11) Turns

$$N = \sqrt{\frac{L}{A_L}} = \sqrt{\frac{(700\times10^{-6})}{(496\times10^{-9})}} = 37.6\,Turns$$

Select primary *N*=38 turns, the secondary turns is 38/6.2 = 6 turns.

Flyback Converter: Wire size

Calculations:

(12) Wire size

$$J_o = K_t \frac{\sqrt{\Delta T}}{\sqrt{k_u(1+\gamma)\sqrt[8]{A_p}}} = 48.2 \times 10^3 \frac{\sqrt{30}}{\sqrt{(0.235)(1+2)\sqrt[8]{(9.72 \times 10^{-8})}}} \times 10^{-4} = 236.6\,\mathrm{A/cm^2}$$

Primary windings:

$$A_{wp} = I_{prms} / J_o = 1.357 / 236.6 = 0.00574\,\mathrm{cm^2}$$

This corresponds to four 0.428 mm diameter copper wires in parallel. A 0.5 mm diameter wire has a dc resistance of 871×10^{-6} Ω/cm @ 20°C.

Secondary windings:

$$A_{ws} = I_{srms} / J_o = 12.454 / 236.6 = 0.0526\,\mathrm{cm^2}$$

This corresponds to a copper foil 25.4×0.2 mm, with a dc resistance of 33.86×10^{-6} Ω/cm @ 20°C.

Skin depth at 70 kHz = 0.25mm

Flyback Converter: Losses

Loss calculations:

(13) Copper loss

Primary windings

$$T_{max} = 60 + 30 = 90\ °C$$

$$R_{dc} = (38)(11.3)((871/4)\times10^{-6})[1+(0.00393)(90-20)]\times10^{3} = 119.2\ \text{m}\Omega$$

$$P_{cu} = R_{dc} I_{rms}^{2} = (119.2\times10^{-3})(1.357)^{2} = 0.220\ \text{W}$$

Secondary windings

$$R_{dc} = (6)(11.3)(33.86\times10^{-6})[1+(0.00393)(90-20)]\times10^{3} = 2.927\ \text{m}\Omega$$

$$P_{cu} = R_{dc} I_{rms}^{2} = (2.927\times10^{-3})(12.454)^{2} = 0.454\ \text{W}$$

The total copper loss is 0.674 W.

(14) Core loss

$$\Delta B = \frac{V_i DT}{N_p A_c} = \frac{325.3\times0.314}{38\times3.51\times10^{-4}\times70\times10^{3}} = 0.109\,\text{T}$$

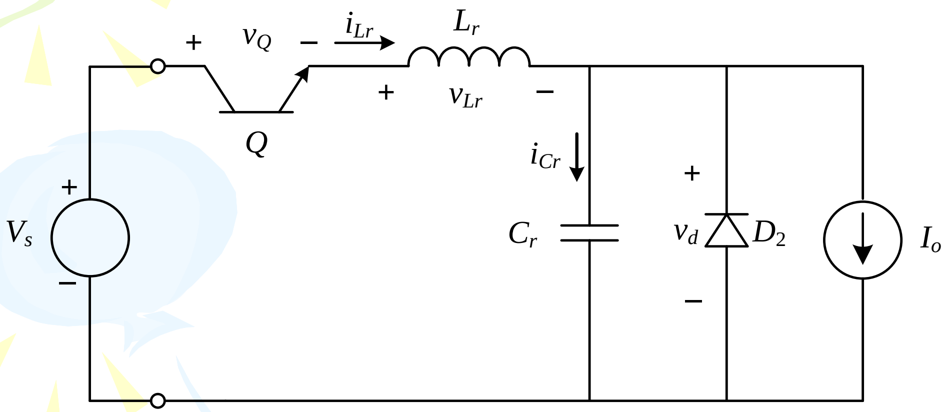
$$P_{fe} = V_c K_c f^{\alpha} B_{max}^{\beta} = (43.9\times10^{-6})(16.9)(70\ 000)^{1.25}(0.109/2)^{2.35} = 0.898\,\text{W}$$

(15) Total loss

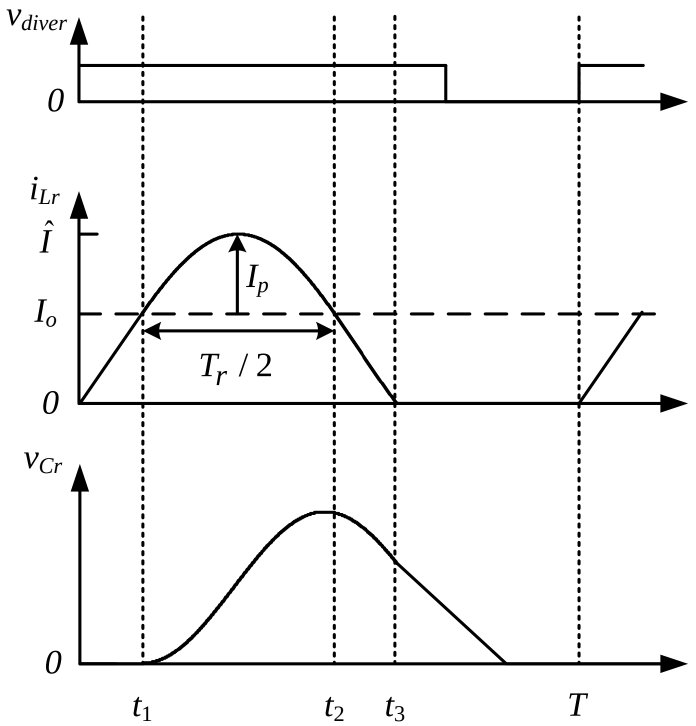
$$P_{total} = 0.674 + 0.898 = 1.572\ \text{W}$$

ZCS-QRC Converter Inductor Design

Circuit



Waveforms



ZCS-QRC Converter: Specifications

Design specifications for inductor

Input voltage	48 V
Inductance	85 µH
Capacitor	0.033 µF
DC Current	0.8 A
Frequency, f	85 kHz
Temperature Rise, ΔT	45ºC
Ambient Temperature, T_a	65ºC
Window utilization factor	0.2

Core data: EPCOS Mn-Zn

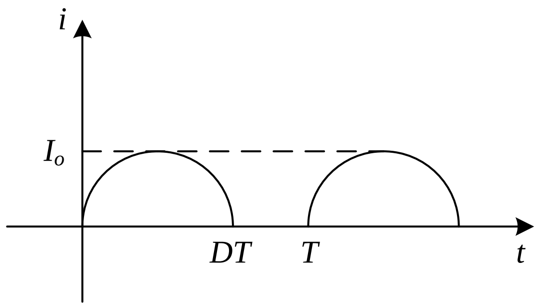
K_c	16.9
α	1.25
β	2.35
B_{sat}	0.4T

ZCS-QRC Converter: Current and Core

Calculations:

(1) Peak current

$$I_o = \frac{V_s}{Z_c} = \frac{V_s}{\sqrt{\frac{L_r}{C_r}}} = 0.94\,\mathrm{A} \quad \hat{I} = I_o + I_p = 0.8 + 0.94 = 1.74\,\mathrm{A}$$



(2) Rms value of the current

$$T = \frac{1}{f} = 1.176\times10^{-5}\,\mathrm{s}; \quad T_r = 2\pi\sqrt{L_r C_r} = 1.058\times10^{-5}\,\mathrm{s}$$

$$t_1 = \frac{I_o L_r}{V_s} = 1.433\times10^{-6}\,\mathrm{s}; \quad t_3 - t_2 \approx t_1; \quad t_3 = t_1 + \frac{T_r}{2}; \quad t_3 = 8.159\times10^{-6}\,\mathrm{s}$$

$$D = \frac{t_3}{T} = 0.7; \quad I_{rms} = \sqrt{\frac{D}{2}}\hat{I} = 1.03\,\mathrm{A}; \quad K_i = \frac{I_{rms}}{\hat{I}} = 0.592$$

(3) A_p

Select $\hat{B}$ as 0.2T to avoid saturation and control core losses

$$A_p = \left[\frac{\sqrt{1+\gamma}K_i L\hat{I}^2}{B_{max}K_t\sqrt{k_u \Delta T}}\right]^{8/7} = 0.03258\,\mathrm{cm}^4$$

E16/8/5 (EF 16) Core Data

A_c	0.201 cm^2
l_c	3.8 cm
W_a	0.233 cm^2
A_p	0.0448 cm^4
V_c	0.756 cm^3
k_f	1.0
k_u	0.45
MLT	3.4 cm
ρ_{20}	1.72 $\mu\Omega$-cm
α_{20}	0.00393

ZCS-QRC Converter: Gap

Calculations:

(4) Maximum dissipation

$$R_\theta = \frac{0.06}{\sqrt{V_c}} = 69.007\ ^{O}\mathrm{C/W} \qquad P_D = \frac{\Delta T}{\mathrm{R}_\theta} = 0.652\,\mathrm{W}$$

(5) Optimum value of the effective permeability

$$\mu_{\mathrm{opt}} = \frac{B_{\max} l_c K_i}{\mu_0 \sqrt{\dfrac{P_{\mathrm{cu\,max}} k_u W_a}{\rho_w \mathrm{MLT}}}} = \frac{(0.2)(3.8\times10^{-2})(0.592)}{(4\pi\times10^{-7})\sqrt{\dfrac{(0.3)(0.45)(0.233\times10^{-4})}{(1.72\times10^{-8})(3.4\times10^{-2})}}} = 49.209$$

(6) Maximum gap length

$$g_{\max} = \frac{l_c}{\mu_{\min}} = \frac{3.8\times10^{-2}}{49.209}\times10^{3} = 7.641\mathrm{mm}$$

Select g=0.5 mm, A_L=69 nH/turn, μ_{eff}=102

ZCS-QRC Converter: Wire size

Calculations:

(7) Turns

$$N = \sqrt{\frac{L_r}{A_L}} = \sqrt{\frac{(85\times10^{-6})}{(69\times10^{-9})}} = 35.1\ \text{turns}$$

Select *N*=35 turns

(8) Wire size

$$J_o = K_\theta \frac{\sqrt{\Delta T}}{\sqrt{k_u(1+\gamma)}\sqrt[8]{A_p}} = 524\ \text{A/cm}^2$$

$$A_w = I_{\text{rms}} / J = 1.03/524 = 0.01966\ \text{cm}^2$$

A multi strand wire at least 30 strands of 0.1 mm wire can meet this specification, with a dc resistance of 7.25×10^{-4} Ω/cm

Skin depth at 85 kHz = 0.23 mm

ZCS-QRC Converter: Losses

Loss calculations:

(9) Copper loss

$$T_{max} = 65 + 45 = 110\ °C$$

$$R_{dc} = (35)(3.4)(7.25\times10^{-4})[1+(0.00393)(110-20)]\times10^{3} = 0.117\ \Omega$$

$$P_{cu} = R_{dc} I_{rms}^{2} = (0.117)(1.03)^{2} = 0.124\ \mathrm{W}$$

(10) Core loss

$$\Delta B = \Delta I \cdot N \cdot \frac{\mu_o \cdot \mu_{eff}}{l_c} = 0.208\,\mathrm{T}$$

$$P_{fe} = V_c K_c f^{\alpha} B_{max}{}^{\beta} = 0.09\,\mathrm{W}$$

(11) Total loss

$$P_{total} = P_{cu} + P_{fe} = 0.214\ \mathrm{W}$$

Transformer Design

NUI Galway OÉ Gaillimh

Basic Equations

Voltage equation

$$V_{rms} = K_v f N\hat{B}A_c$$

K_v =4.44 for a sinewave
=4.00 for a squarewave

Power equation

$$\sum VA = K_v f\hat{B} \cdot \sum N_i I_i \cdot A_c$$

$$I_i = J_o A_{wi}$$

Window utilisation factor

$$k_u = \frac{\sum_{i=1}^{n} N_i A_{wi}}{W_a}$$

$$\sum VA = K_v f\hat{B} J_o k_u W_a A_c$$

$$A_p = W_a \times A_c$$

$$Window\, area \times cross-sectional\, area$$

$$\sum VA = K_v f\hat{B} k_u A_p J_o$$

V_{rms}: the rms value of the applied voltage
K_v: the voltage waveform factor
f: the frequency of the applied voltage
J_o: the current density in each winding
$\hat{B}$: the maximum flux density in the core
I_i: the current in winding i
N_i: the number of turns in winding i
A_{wi}: the conductor area in winding i
k_u: the window utilisation factor

Transformer Losses

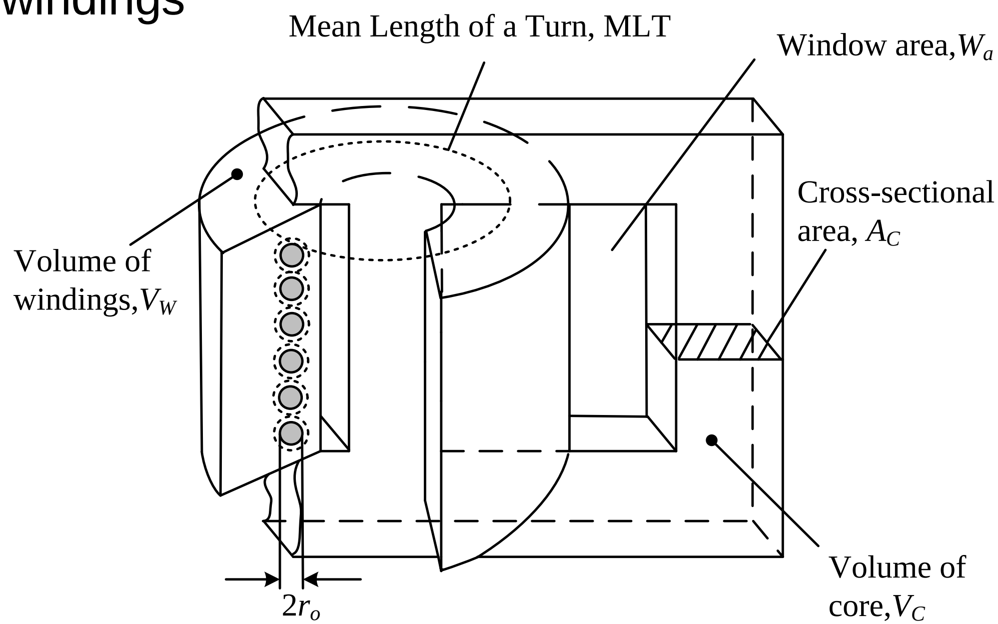
Winding losses

Total resistive losses $$P_{cu} = \sum RI^2 = \rho_w \sum_{i=1}^{n} \frac{N_i MLT (J_o A_{wi})^2}{A_{wi}}$$

$$k_u = \frac{\sum_{i=1}^{n} N_i A_{wi}}{W_a}$$ is window utilization factor

$$V_w = MLT \times W_a$$ is volume of the windings

$$P_{cu} = \rho_w V_w k_u J_o^2$$



Typical layout of a transformer

Core losses

$$P_{fe} = V_c K_c f^{\alpha} \hat{B}^{\beta}$$

Heat loss by convection

$$P_{total} = P_{cu} + P_{fe} = h_c A_t \Delta T$$

Dimensional Analysis

(14)

(15)

$$V_w = k_w A_p^{3/4}$$

$$V_c = k_c A_p^{3/4}$$

$$A_t = k_t A_p^{1/2}$$

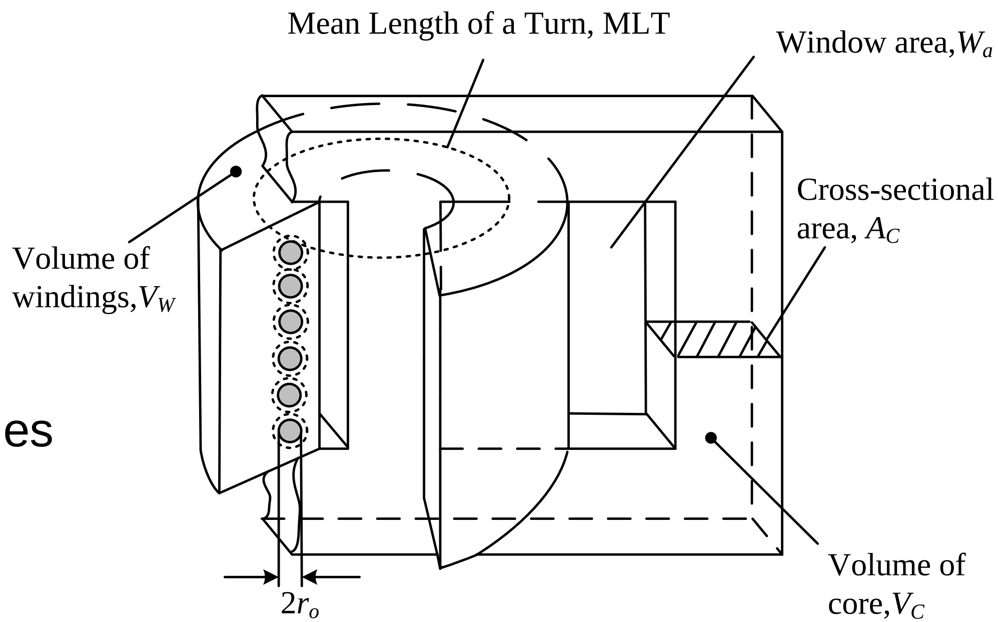
k_w=10, k_c =5.6 and k_t=40 are typical values

$$P_{cu} = \rho_w k_w A_p^{3/4} k_u J_o^2$$

$$P_{fe} = \mathrm{k}_c A_p^{3/4} K_c f^{\alpha} \hat{B}^{\beta}$$

$$P_{total} = \mathrm{h}_c k_t A_p^{1/2} \Delta T$$

$$\sum VA = K_v f \hat{B} k_u A_p J_o$$



Typical layout of a transformer

Losses Optimization

Winding losses

$$P_{cu} = \rho_w V_w k_u \left[\frac{\sum \mathrm{VA}}{K_v f \hat{B} k_f k_u A_p} \right]^2 = \frac{a}{f^2 \hat{B}^2}$$

Core losses

$$P_{fe} = V_c K_c f^{\alpha} \hat{B}^{\beta} = b f^{\alpha} \hat{B}^{\beta}$$

Total losses

$$P = \frac{a}{f^2 \hat{B}^2} + b f^{\alpha} \hat{B}^{\beta}$$

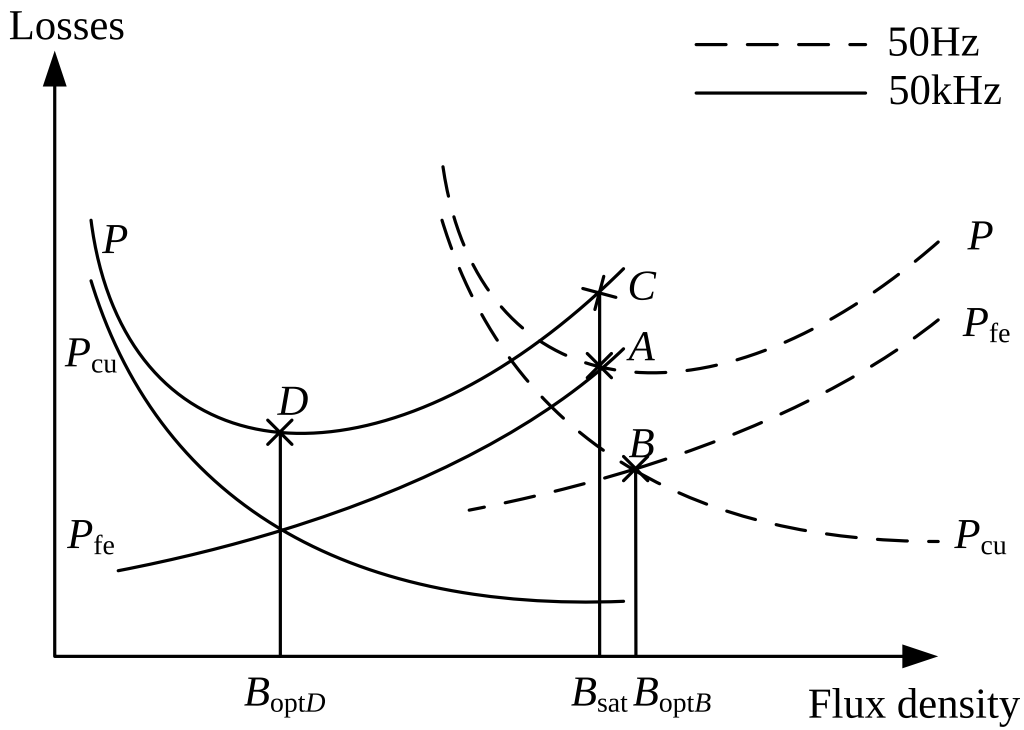
At a given operation frequency,

$$\frac{\partial P}{\partial \hat{B}} = -\frac{2a}{f^2 \hat{B}^3} + \beta b f^{\alpha} \hat{B}^{\beta - 1} = 0$$

The minimum losses occur when

$$P_{cu} = \frac{\beta}{2} P_{fe} \qquad P_{total} = \frac{\beta + 2}{\beta} P_{cu}$$

Losses Optimization



Winding, core and total losses at different frequencies

- The first step in the design is to establish whether the optimum flux density given by the optimization criterion is greater or less than the saturation flux density.

Losses Optimization

Core size

$$A_p = \left[\frac{\rho_w k_w}{h k_t} \frac{\beta+2}{\beta} \frac{1}{k_u \Delta T} \right]^{4/7} \left[\frac{\sum \mathrm{VA}}{K_v f \hat{B}_o K_\theta} \right]^{8/7}$$

Current density

$$J_o = \sqrt{\frac{\beta}{\beta+2} \frac{h k_t}{\rho k_w} \frac{\Delta T}{k_u} \frac{1}{A_p^{1/4}}}$$

Optimum flux density

$$\left(f_o B_o \right)^{7\beta-2} f_o^{7(\alpha-\beta)} = \frac{2^7 \beta}{(\beta+2)^8} \frac{[h k_t \Delta T]^8}{[\rho_w k_w][k_c K_c]^7} \left[\frac{K_v{}^2 k_u}{\sum \mathrm{VA}^2} \right]$$

$$P_{\mathrm{cu}} = \rho_w k_w A_p^{3/4} k_u J_o^2$$

$$P_{\mathrm{fe}} = \mathrm{k}_c A_p^{3/4} K_c f^{\alpha} \hat{B}^{\beta}$$

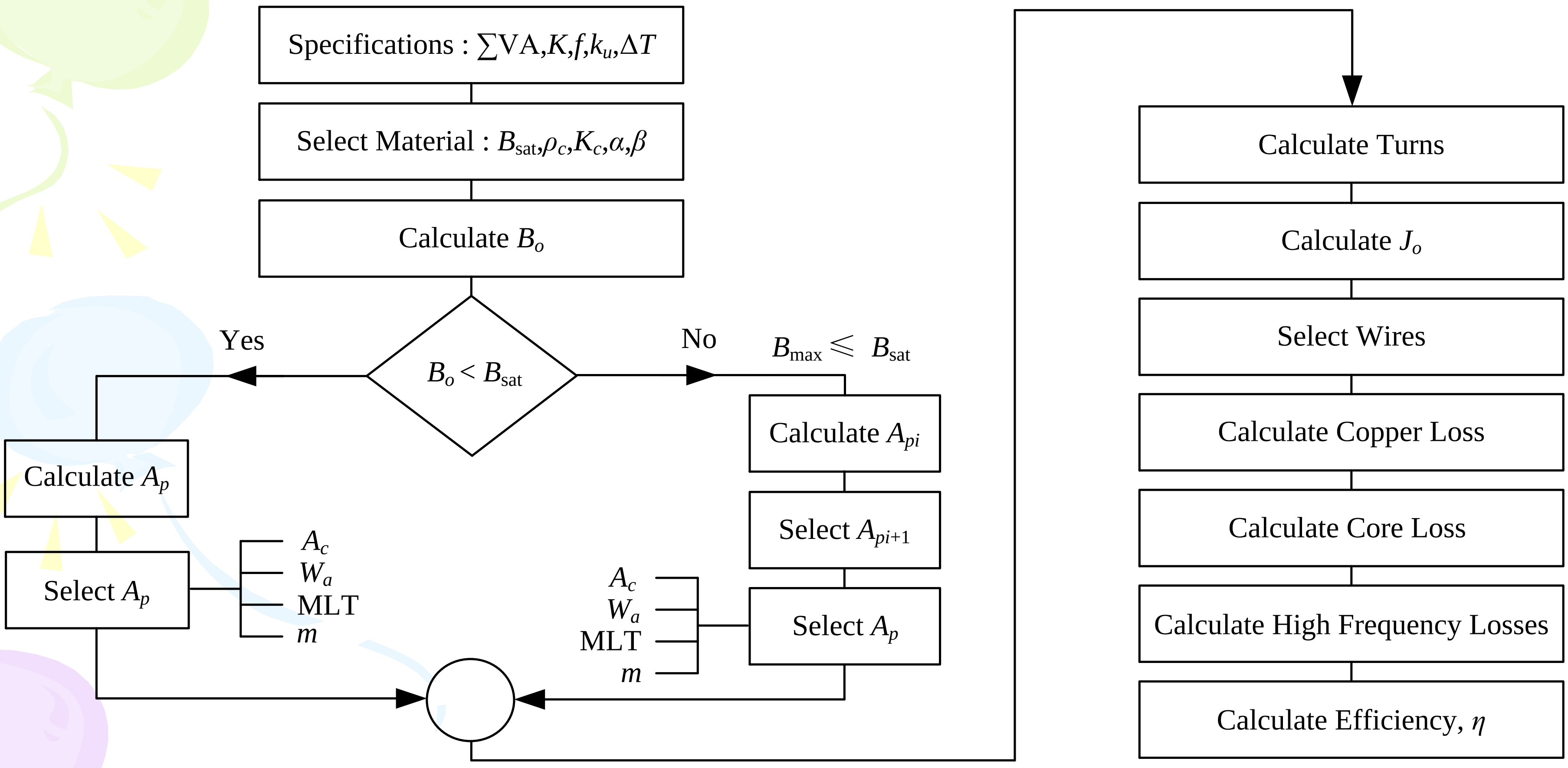
$$P_{\mathrm{total}} = \mathrm{h}_c k_t A_p^{1/2} \Delta T$$

$$\sum VA = K_v f \hat{B} k_u A_p J_o$$

$$P_{\mathrm{total}} = \frac{\beta+2}{\beta} P_{\mathrm{cu}}$$

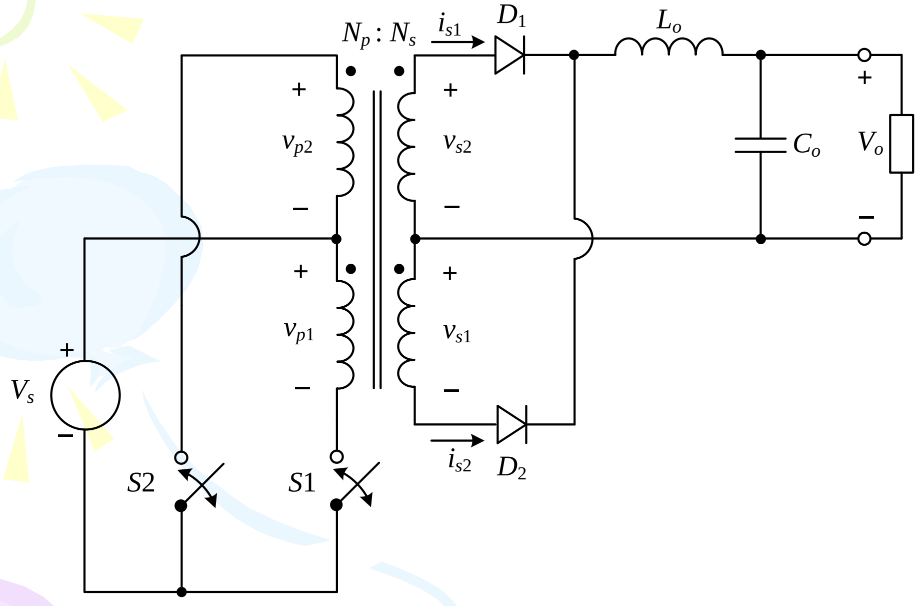
$$P_{\mathrm{total}} = \frac{\beta+2}{2} P_{\mathrm{fe}}$$

Design Methodology

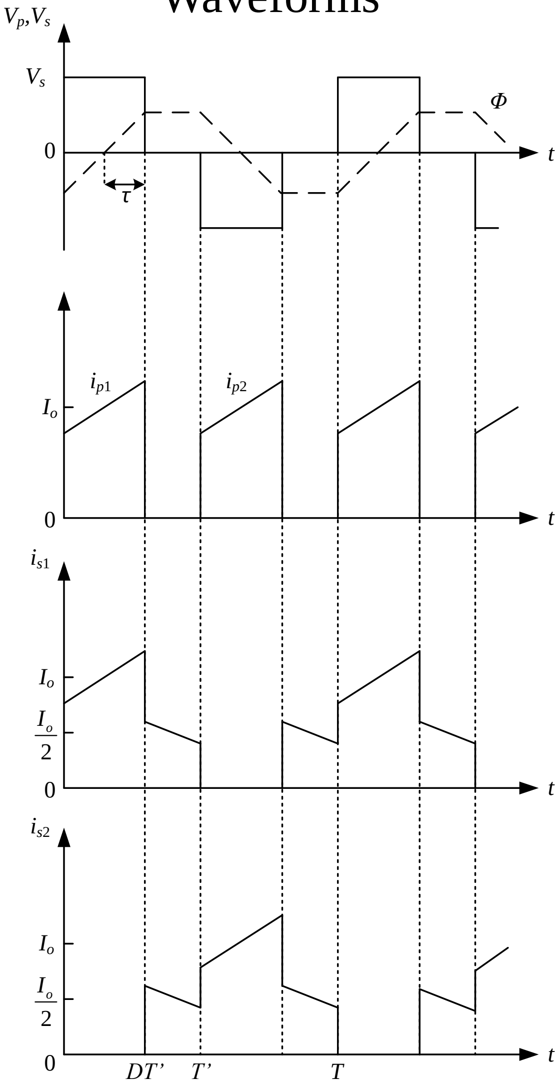


Push-pull Converter Transformer

Circuit



Waveforms



Push-pull Converter: Specifications

Design specifications

Input	36 $\rightarrow$ 72 V
Output	24 V, 300 W
Frequency, f	50 kHz
Temperature Rise, ΔT	35 ºC
Ambient Temperature, T_a	45 ºC

Core data: EPCOS N67 Mn-Zn

K_c	9.12
α	1.24
β	2.0
B_{sat}	0.4 T

Core loss

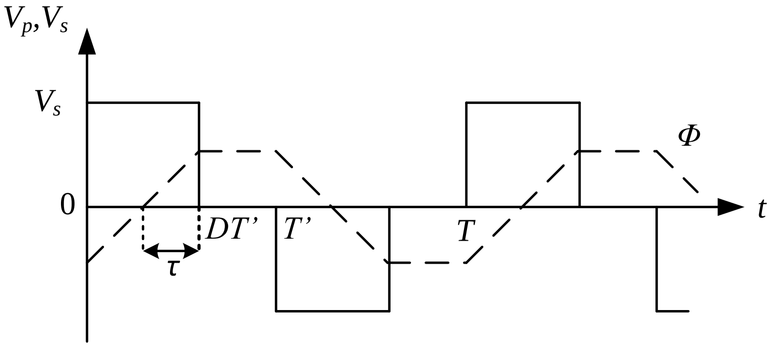
$$P_{fe} = K_c f^{\alpha} B_m^{\beta}$$

Push-pull Converter: Voltage factor

Calculations:

(1) Voltage waveform factor K_v

$$D = \frac{24}{36} = 0.67$$



Push-pull converter voltage and flux waveforms

$$V_s = N_p \frac{d\phi}{dt} = N_p A_c \frac{dB}{dt} = N_p A_c \frac{B_{max}}{DT'/2} = N_p A_c \frac{4B_{max}}{DT} = \frac{4}{D} f N_p A_c B_{max}$$

$$V_{rms} = \sqrt{D} V_s = \frac{4}{\sqrt{D}} f N_p B_{max} A_c = K_v \, f N_p B_{max} A_c$$

$$K_v = \frac{4}{\sqrt{D}} = 4.88$$

Push-pull Converter: Power factor

Calculations:

(2) Power factor k_{pp}, k_{ps}

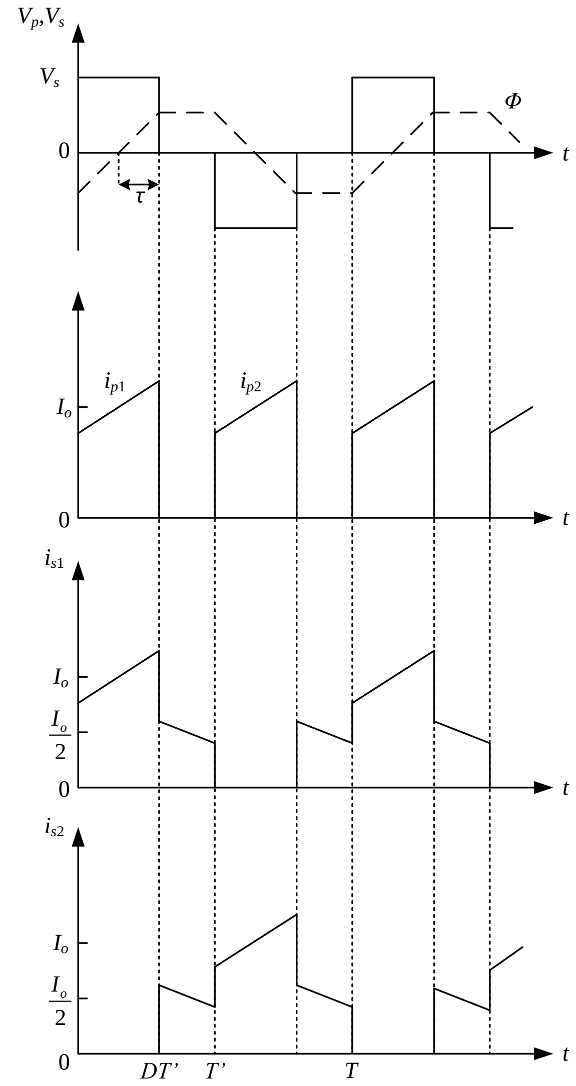
$$V_{prms} = \sqrt{D}V_s;\ I_{prms} = \sqrt{(D/2)}I_s;$$

$$V_{srms} = \sqrt{D}V_s = \frac{V_o}{\sqrt{D}};\ I_{srms} = \frac{I_o}{2}\sqrt{(1+D)}$$

$$V_{srms}I_{srms} = \frac{1}{2}\frac{\sqrt{1+D}}{\sqrt{D}}V_oI_o = \frac{\sqrt{1+D}}{\sqrt{D}}\frac{P_o}{2}$$

$$< p >= \int_0^{DT'} v(t)i(t)dt = \frac{1}{T}V_sI_sDT' = \frac{D}{2}V_sI_s$$

$$k_{pp} = \frac{< p >}{V_{prms}I_{prms}} = \frac{1}{\sqrt{2}};\quad k_{ps} = \frac{< p >}{V_{srms}I_{srms}} = \sqrt{\frac{D}{1+D}}$$



Push-pull Converter: Core size

Calculations:

(3) *VA* ratings of the windings

$$\sum VA = \left(\frac{1}{k_{pp}} \left(\frac{P_o}{2} + \frac{P_o}{2} \right) + \frac{1}{k_{ps}} \left(\frac{P_o}{2} + \frac{P_o}{2} \right) \right) = \left(\sqrt{2} + \sqrt{\frac{1+D}{D}} \right) P_o$$

$$= \left(\sqrt{2} + \sqrt{\frac{1+0.67}{0.67}} \right) (300) = 898.6 \text{ VA}$$

(4) Optimum A_p

$$\hat{B}_o = \left\{ \frac{2^7 \times 2.0}{(2.0+2)^8} \frac{\left[(10)(40)(35)\right]^8}{\left[(1.72\times 10^{-8})(10)\right]\left[(5.6)(9.12)\right]^7} \left[\frac{4.899^2 (0.4)}{898.6^2} \right] \right\}^{\frac{1}{7\times 2.07-2}} \bullet 50000^{\frac{-(7\times 1.24-2)}{7\times 2.0-2}}$$

$$= 0.127 \, \text{T}$$

The optimum flux density is less than B_{sat}

$$A_p = \left[\frac{(1.72\times 10^{-8})(10)}{(10)(40)} \frac{2.0+2}{2.0} \frac{1}{0.4\times 35} \right]^{4/7} \left[\frac{898.6}{(4.899)(50000)(0.127)} \right]^{8/7} = 2.54\,\text{cm}^4$$

ETD44 Core Data

A_c	1.73 cm^2
W_a	2.78 cm^2
A_p	4.81 cm^4
V_c	17.70 cm^3
k_f	1.0
k_u	0.4
MLT	7.77 cm
ρ_{20}	1.72 $\mu\Omega$-cm
α_{20}	0.00393

Push-pull Converter: Turns

Calculations:

(5) Turns

Primary turns:

$$N_p = \frac{V_p}{K_v B_{\max} A_c f} = \frac{\sqrt{0.67}(36)}{(4.88)(0.126)(1.73\times 10^{-4})(50\ 000)} = 5.5 \text{ turns}$$

Rounded up to 6 turns

Secondary turns:

We assumed a 1:1 turns ratio so the number of secondary turns is 6.

Push-pull Converter: Wire size

Calculations:

(6) Wire size

$$J_o = \sqrt{\frac{\beta}{\beta+2}\frac{hk_t}{\rho_w k_w}\frac{\Delta T}{k_u}\frac{1}{A_p^{1/4}}} = \sqrt{\frac{2.0}{2.0+2}\frac{(10)(40)}{(1.72\times10^{-8})(10)}\frac{35}{0.4}\frac{1}{4.81^{1/4}}} = 2.62\,\text{A/mm}^2$$

Primary windings:

$$I_p = \frac{P_o/2}{k_{pp}V_p} = \frac{300/2}{(0.707)(29.394)} = 7.217\ \text{A}$$

$$A_w = I_p / J_o = 7.217/2.62 = 2.753\,\text{mm}^2$$

Skin depth at 50 kHz = 0.295 mm

Standard 0.1×30 mm copper foil with a dc resistance of 5.8 mΩ/m @ 20ºC meets this requirement.

Secondary windings:

$$I_s = \frac{I_o}{2}\sqrt{1+D} = \frac{12.5}{2}\sqrt{1+0.67} = 8.07\ \text{A}$$

$$A_w = I_s / J_o = 8.069/2.62 = 3.078\,\text{mm}^2$$

Again, standard 0.1×30 mm copper foil meets this requirement.

Push-pull Converter: Losses

Loss calculations:

(7) Copper loss

$$T_{max} = 45 + 35 = 80\ °\mathrm{C}$$

Each primary windings

$$R_{dc} = (6)(7.77\times10^{-2})(5.80\times10^{-3})[1+(0.00393)(80-20)]\times10^{3} = 3.34\ \mathrm{m\Omega}$$

$$P_{cu} = R_{dc} I_{rms}^{2} = (3.34\times10^{-3})(7.217)^{2} = 0.174\ \mathrm{W}$$

Each secondary windings

$$R_{dc} = (6)(7.77\times10^{-2})(5.80\times10^{-3})[1+(0.00393)(75-20)]\times10^{3} = 3.29\ \mathrm{m\Omega}$$

$$P_{cu} = R_{dc} I_{rms}^{2} = (3.34\times10^{-3})(8.07)^{2} = 0.218\ \mathrm{W}$$

The total copper loss is 0.784 W.

(8) Core loss

$$B_{max} = \frac{\sqrt{D}V_{dc}}{K_v f N_p A_c} = \frac{\sqrt{0.67}(36)}{(4.88)(50000)(6)(1.73\times10^{-4})} = 0.116\ \mathrm{T}$$

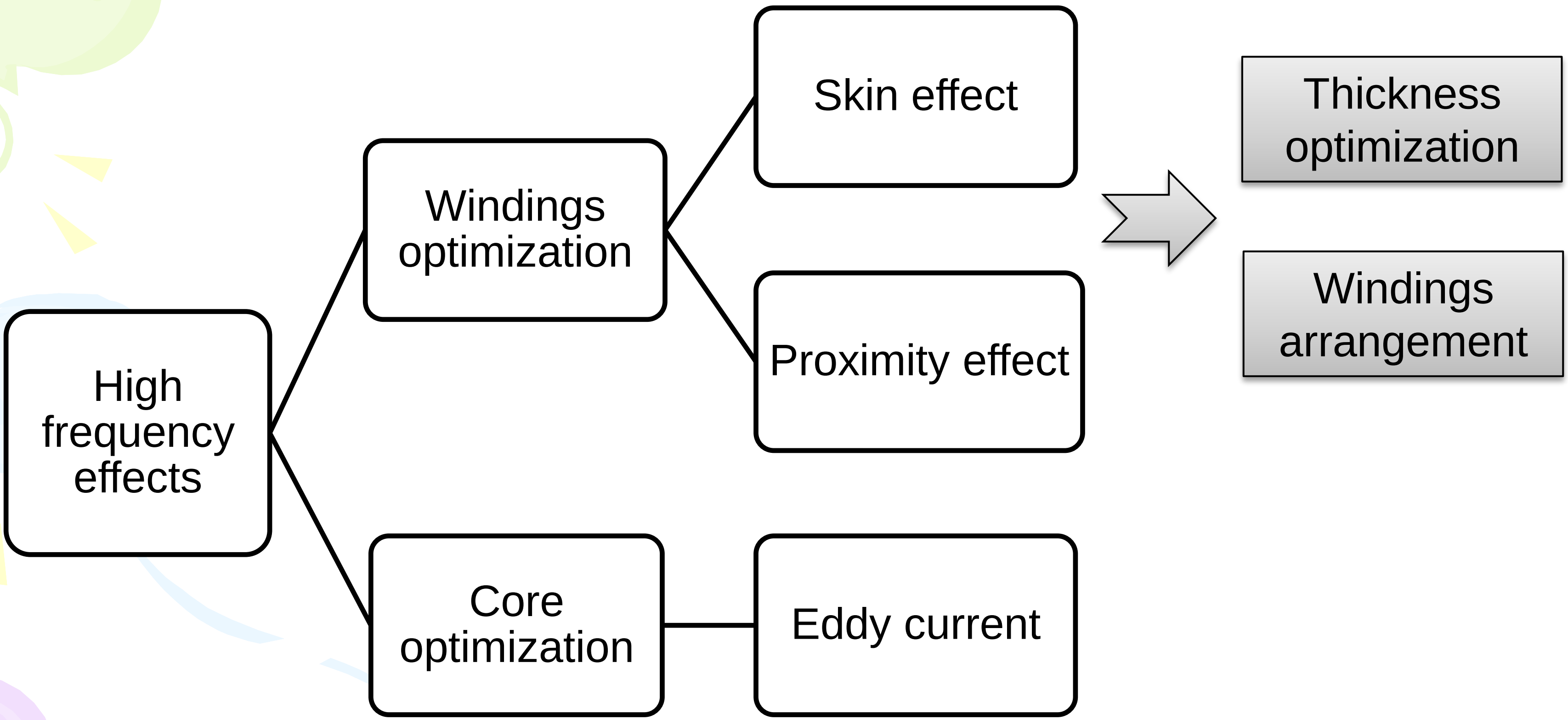
$$P_{fe} = V_c K_c f^{\alpha} B_{max}^{\beta} = (17.7\times10^{-6})(9.12)(50000)^{1.24}(0.116)^{2.0} = 1.448\ \mathrm{W}$$

(9) Total loss

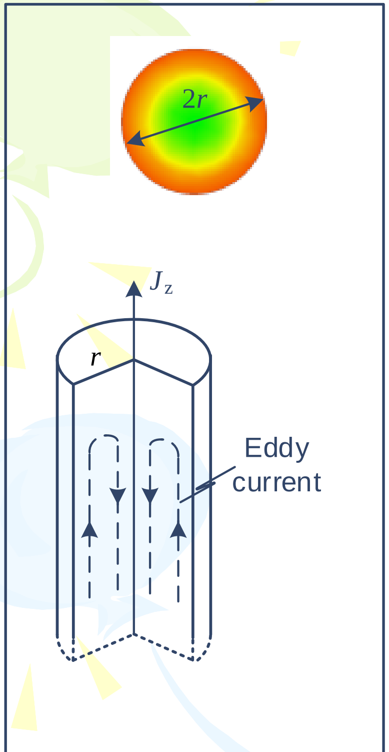
$$P_{total} = 0.784 + 1.448 = 2.231\ \mathrm{W}$$

High Frequency Effects in the Windings

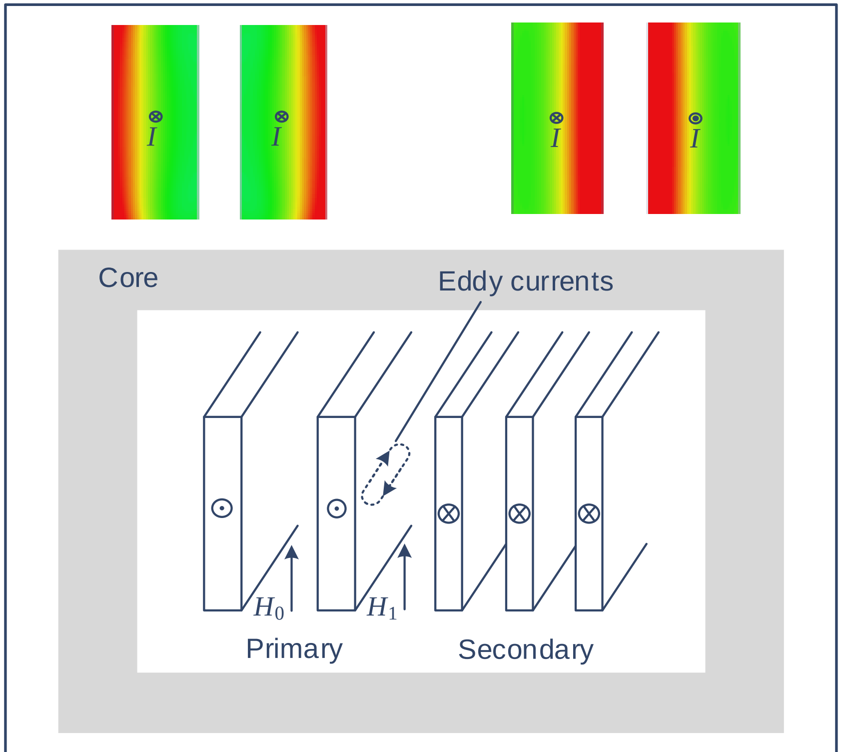
High Frequency Effects



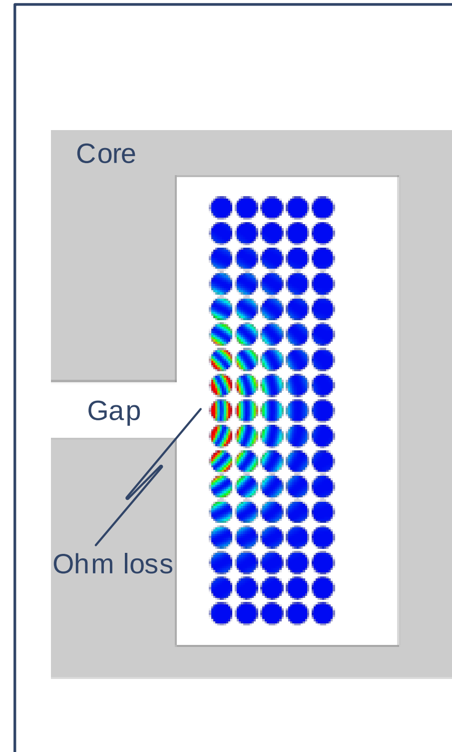
Design Issues for High Frequency



Skin effect



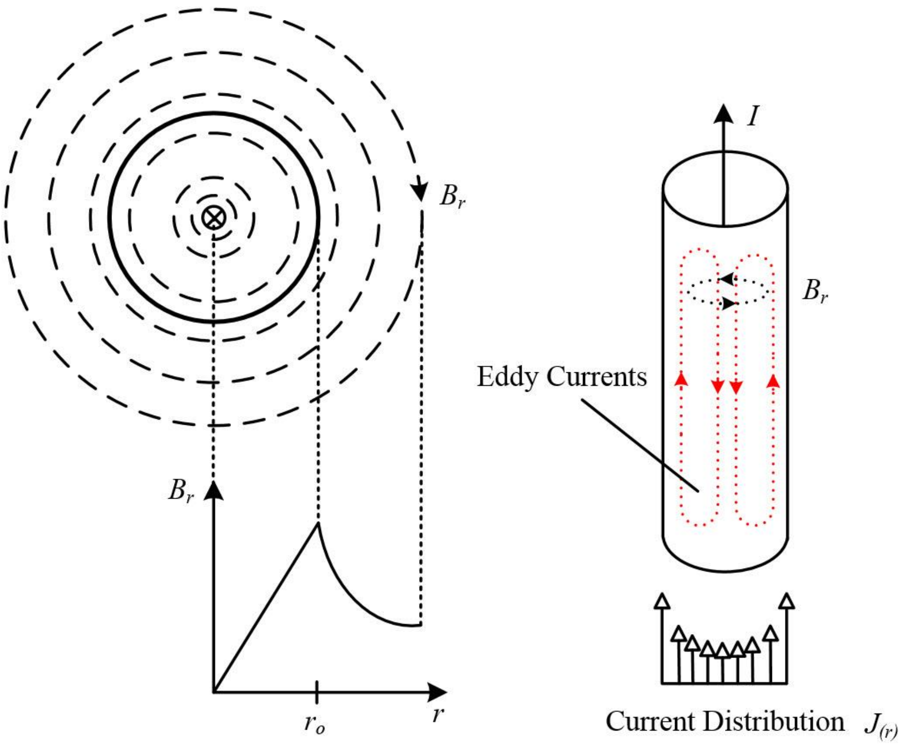
Proximity effect



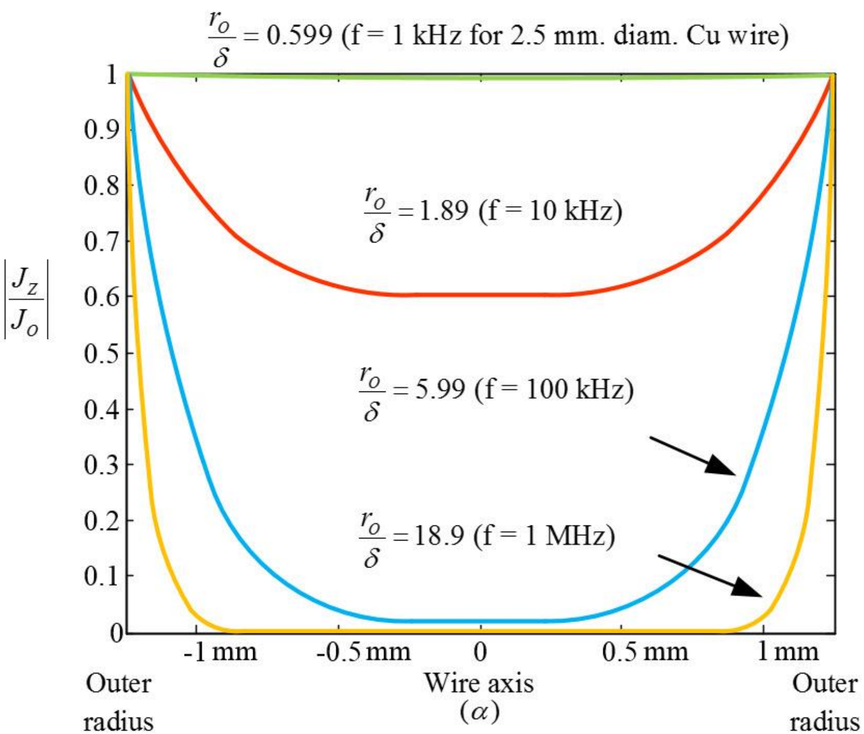
Fringing effect

- High frequency winding loss
- Core loss: Steinmetz equation, iGSE.
- Parasitic parameters: leakage inductance, stray capacitance

Skin Effect

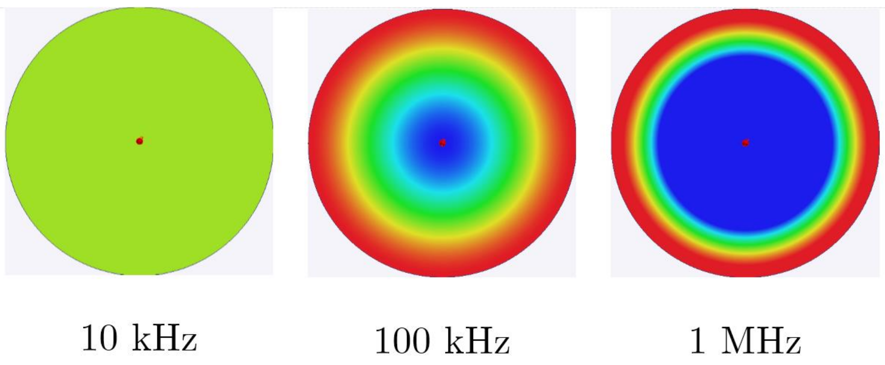


Eddy currents in a circular conductor

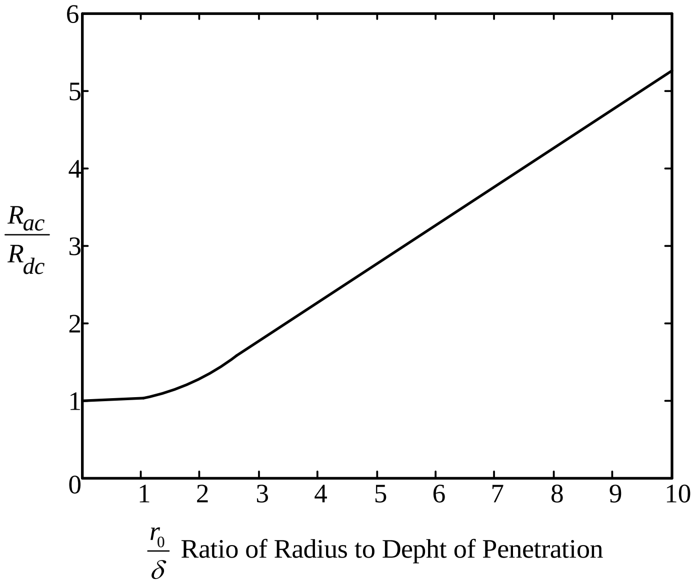


Current distribution in a circular conductor

Skin Effect Factor



Current distribution in a circular conductor



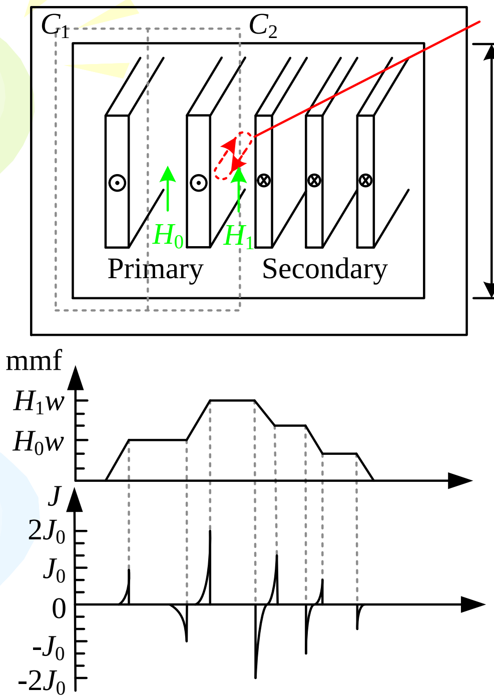
R_{ac}/R_{dc} due to the skin effect

$$\frac{R_{ac}}{R_{dc}} = 1 + \frac{(r_o/\delta)^4}{48 + 0.8(r_o/\delta)^4} \qquad r_o/\delta < 2$$

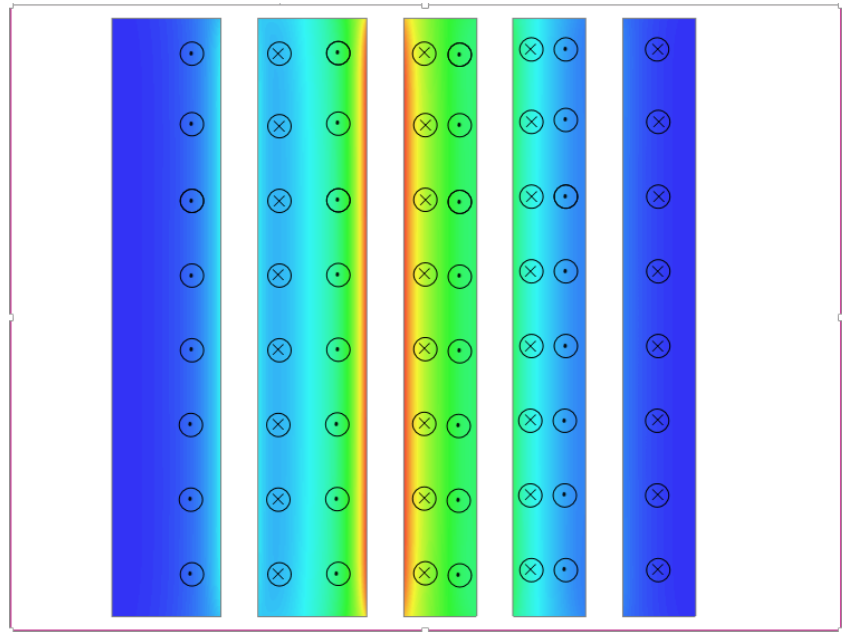
$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

- The ac resistance is proportional to the square root of frequency at very high frequencies.

Proximity Effect



Transformer cross-section with current density distribution

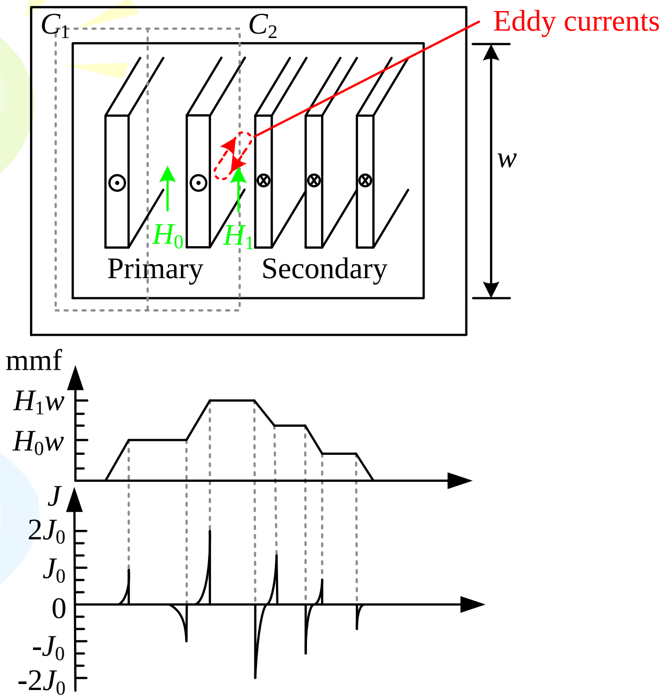


Proximity effect factor for sinusoidal excitation

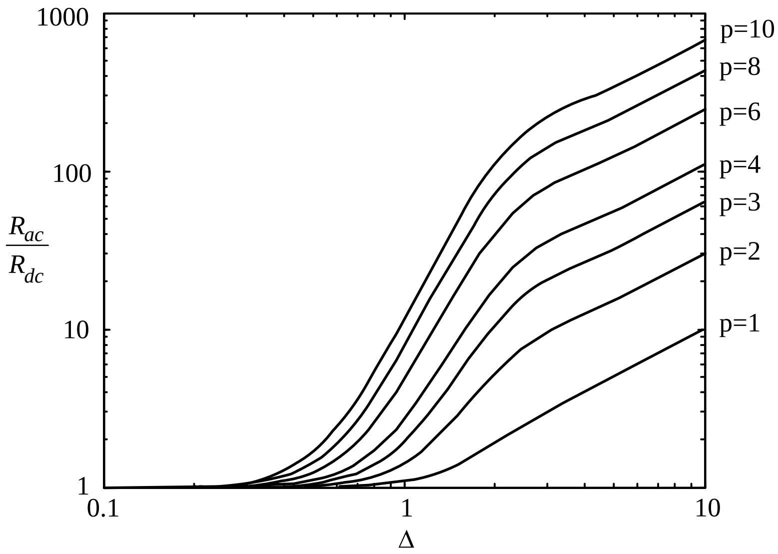
- As the number of layers increase there is a substantial increase in the ac resistance for a given layer thickness and frequency.

P. L. Dowell, 'Effects of eddy currents in transformer windings,' Proceedings of the Institution of Electrical Engineers, vol. 113(8), pp. 1387-1394, 1966.

Proximity Effect Factor



Transformer cross-section with current density distribution



R_{ac}/R_{dc} due to the proximity effect

$$\frac{R_{ac}}{R_{dc}} = \phi_{prox}(\Delta) = 1 + \frac{5p^2 - 1}{45}\Delta^4 \text{ where } \Delta = \frac{d}{\delta}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

- As the number of layers *p* increase there is a substantial increase in the ac resistance for a given layer thickness *d* and frequency.

Porosity Factor

A round conductor of diameter *D* is equivalent to a square conductor of side length

$$d = \sqrt{\frac{\pi}{4}} D$$

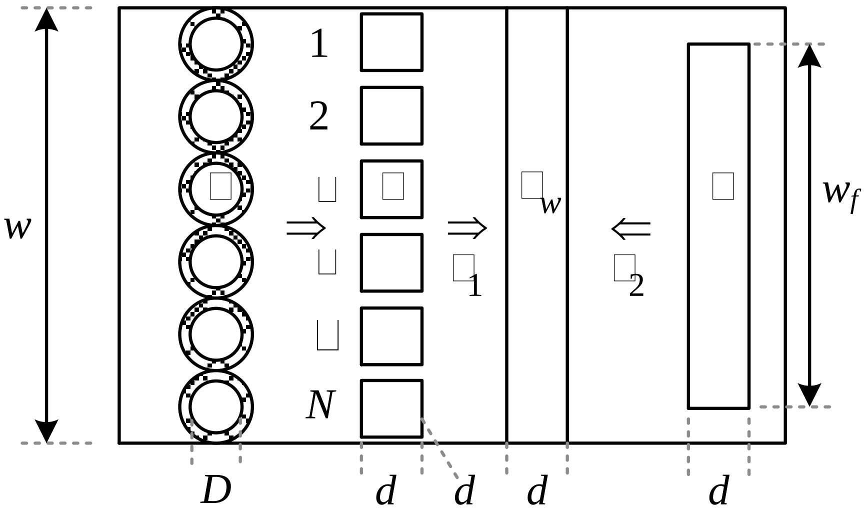
The porosity factor

$$\eta = \frac{Nd}{w}$$

The effective conductivity

$$\sigma_w = \eta \sigma$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_w}}$$



Porosity factor for foils and round conductors

The porosity factor adjusts the conductivity value for a foil with the same area as the round conductors.

Proximity Effect: arbitrary waveform

An arbitrary periodic current waveform may be represented by its Fourier series

$$i(t) = I_{dc} + \sum_{n=1}^{\infty} \hat{I}_n \cos(n\omega t + \varphi_n)$$

The total power loss due to all the harmonics

$$P = R_{dc} I_{dc}^2 + \sum_{n=1}^{\infty} R_{dc} \phi_{prox}(\Delta_n) I_{n,rms}^2$$

so

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \Delta_n = \frac{d}{\delta_n} = \sqrt{n}\frac{d}{\delta_o} = \sqrt{n}\Delta$$

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{dc}^2 + \sum_{n=1}^{\infty} \phi_{prox}(\Delta_n) I_{n,rms}^2}{I_{rms}^2} \qquad \phi_{prox}(\Delta_n) = 1 + \frac{5p^2-1}{45}\Delta_n^4 = 1 + \frac{5p^2-1}{45} n^2 \Delta^4$$

$$= \frac{I_{dc}^2 + \sum_{n=1}^{\infty} I_{n,rms}^2 + \frac{5p^2-1}{45}\Delta^4 \sum_{n=1}^{\infty} n^2 I_{n,rms}^2}{I_{rms}^2}$$

Proximity Effect: arbitrary waveform

$$i(t) = I_{dc} + \sum_{n=1}^{\infty} \hat{I}_n \cos(n\omega t + \varphi_n)$$

$$\frac{di(t)}{dt} = I' = -\omega \sum_{n=1}^{\infty} n\hat{I}_n \sin(n\omega t + \varphi_n)$$

$$I_{rms}^2 = I_{dc}^2 + \sum_{n=1}^{\infty} I_{n,rms}^2$$

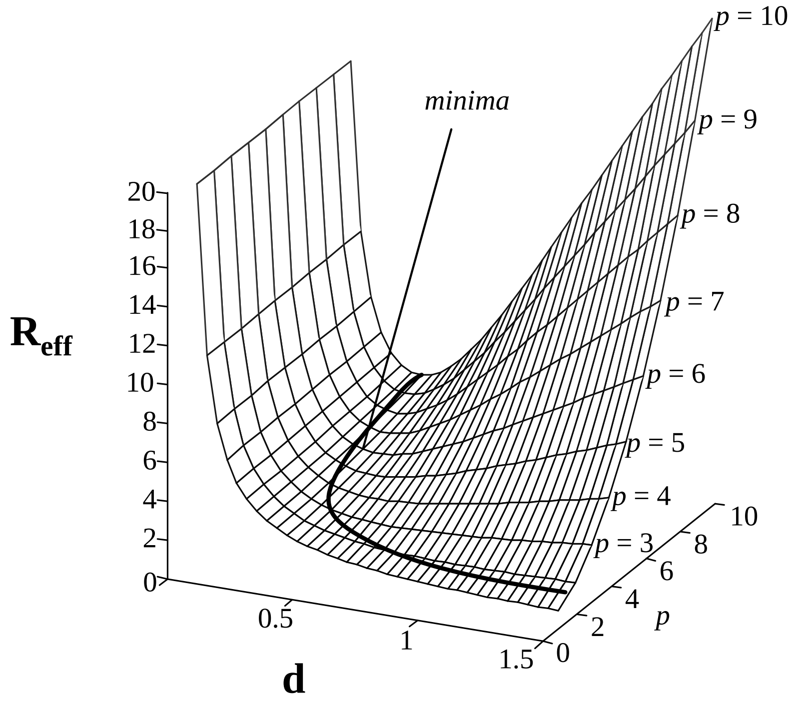
$$I'^2_{rms} = \omega^2 \sum_{n=1}^{\infty} n^2 I_{n,rms}^2$$

$$\frac{R_{eff}}{R_{dc}} = \frac{I_{dc}^2 + \sum_{n=1}^{\infty} I_{n,rms}^2 + \frac{5p^2-1}{45}\Delta^4 \sum_{n=1}^{\infty} n^2 I_{n,rms}^2}{I_{rms}^2}$$

$$= \frac{I_{rms}^2 + \frac{5p^2-1}{45}\Delta^4 \left[\frac{I'_{rms}}{\omega}\right]^2}{I_{rms}^2}$$

$$= 1 + \frac{5p^2-1}{45}\Delta^4 \left[\frac{I'_{rms}}{\omega I_{rms}}\right]^2$$

$$\Delta = \frac{d}{\delta_o}$$



Optimum Thickness

The optimum value of *Δ*

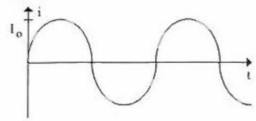
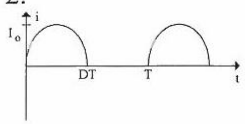
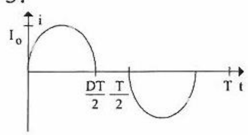
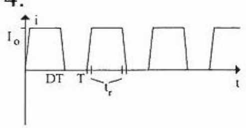
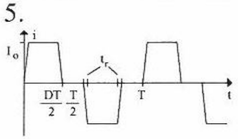
$$\Delta_{opt} = \sqrt{\frac{15}{5p^2-1}}\sqrt{\frac{\omega I_{\mathrm{rms}}}{\left[\frac{di}{dt}\right]_{\mathrm{rms}}}}$$

Finally

$$\frac{R_{eff}}{R_{dc}} = 1 + \frac{1}{3}\left(\frac{\Delta}{\Delta_{\mathrm{opt}}}\right)^4$$

$$\left(\frac{R_{eff}}{R_{dc}}\right)_{\mathrm{opt}} = \frac{4}{3}$$

Table of Waveforms

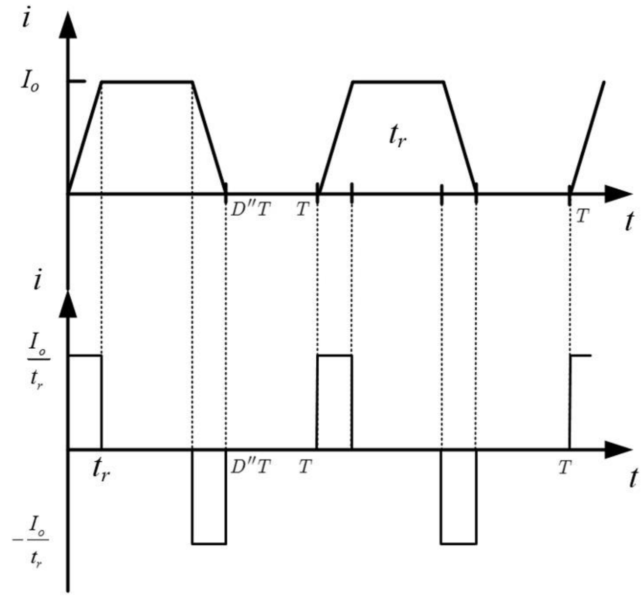
Current Waveform	I_{rms} and I_{rms}'	Fourier series, $i(t)$	Δ_{opt}
1. 	$I_{rms} = \dfrac{I_o}{\sqrt{2}}$ $I_{rms}' = \dfrac{2\pi}{T\sqrt{2}} I_o$	$I_o \sin(\omega t)$	$\Delta_{opt} = \sqrt[4]{\dfrac{1}{\Psi}}$
2. 	$I_{rms} = I_o\sqrt{\dfrac{D}{2}}$ $I_{rms}' = I_o \dfrac{\pi}{DT}\sqrt{\dfrac{D}{2}}$	$\dfrac{2DI_o}{\pi} + \displaystyle\sum_{n=1}^{\infty} \dfrac{4DI_o}{\pi}\left\{\dfrac{\cos(n\pi D)}{\left(1-4n^2D^2\right)}\right\} \blacklozenge$ $\times\cos(n\omega t)$	$\Delta_{opt} = \sqrt[4]{\dfrac{4D^2}{\Psi}}$
3. 	$I_{rms} = I_o\sqrt{\dfrac{D}{2}}$ $I_{rms}' = I_o \dfrac{2\pi}{DT}\sqrt{\dfrac{D}{2}}$	$\displaystyle\sum_{n=1,\text{ odd}}^{\infty} \dfrac{4DI_o}{\pi}\left\{\dfrac{\cos\left(n\pi D/2\right)}{\left(1-n^2D^2\right)}\right\} \blacklozenge$ $\times\cos(n\omega t)$	$\Delta_{opt} = \sqrt[4]{\dfrac{D^2}{\Psi}}$
4. 	$I_{rms} = I_o\sqrt{D - \dfrac{4t_r}{3T}}$ $I_{rms}' = I_o\sqrt{\dfrac{2}{t_r T}}$	$I_o\left(D - \dfrac{t_r}{T}\right) + \displaystyle\sum_{n=1}^{\infty} \dfrac{2I_o}{n\pi}\sin\left[n\pi\left(D - \dfrac{t_r}{T}\right)\right]$ $\times\text{sinc}\left(n\pi\dfrac{t_r}{T}\right)\cos(n\omega t)$	$\Delta_{opt} = \sqrt[4]{\dfrac{\left[D - \dfrac{4t_r}{3T}\right]2\pi^2\dfrac{t_r}{T}}{\Psi}}$
5. 	$I_{rms} = I_o\sqrt{D - \dfrac{8t_r}{3T}}$ $I_{rms}' = I_o\sqrt{\dfrac{4}{t_r T}}$	$\displaystyle\sum_{n=1,\text{ odd}}^{\infty} \dfrac{4I_o}{n\pi}\sin\left[n\pi\left(\dfrac{D}{2} - \dfrac{t_r}{T}\right)\right]$ $\times\text{sinc}\left(n\pi\dfrac{t_r}{T}\right)\cos(n\omega t)$	$\Delta_{opt} = \sqrt[4]{\dfrac{\left[D - \dfrac{8t_r}{3T}\right]\pi^2\dfrac{t_r}{T}}{\Psi}}$

♦ In waveform 2 for $n = k = 1/2D \in \mathbf{N}$ (the set of natural numbers), and in waveform 3 for $n = k = 1/D \in \mathbf{N}$, the {expression in curly brackets} is replaced by $\pi/4$.

$\Psi = (5p^2 - 1)/15$, p = No. of layers, sinc $(x) = \sin(x)/x$

Optimum Winding Thickness: Pushpull

$$\Delta_{opt} = \sqrt{\frac{15}{5p^2-1}}\sqrt{\frac{\omega I_{\mathrm{rms}}}{I'_{\mathrm{rms}}}}$$



Skin depth

$$\delta_0 = \frac{66}{\sqrt{f}} = \frac{66}{\sqrt{(50\times 10^3)}} = 0.295\,mm$$

Optimum layer *Δ*

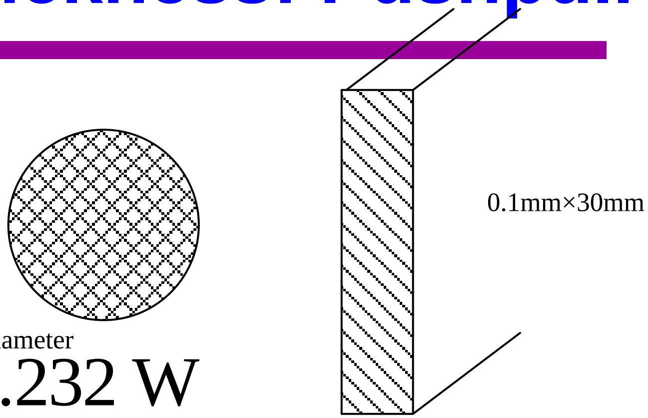
$$\Delta_{\mathrm{opt}} = \sqrt[4]{\frac{\left[D-\frac{4t_r}{3T}\right]2\pi^2\frac{t_r}{T}}{(5p^2-1)15}} = \sqrt[4]{\frac{\left[0.67-\frac{(4)(0.025)}{3}\right]2\pi^2(0.025)}{[(5)(6)^2-1]/15}} = 0.402$$

Optimum layer thickness

$$d_{\mathrm{opt}} = \Delta_{\mathrm{opt}}\delta_o = (0.402)(0.295) = 0.1\,\mathrm{mm}$$

Optimum Winding Thickness: Pushpull

Effective ac resistance: foil $R_{\text{eff}} = \frac{4}{3} R_{\text{dc}} = 4.455\,\text{m}\Omega$



$$P_{\text{cu}} = R_{\text{eff}} I_{\text{rms}}^2 = 4.455 \times 10^{-3} \times (7.217)^2 = 0.232\ \text{W}$$

Round versus foil conductor

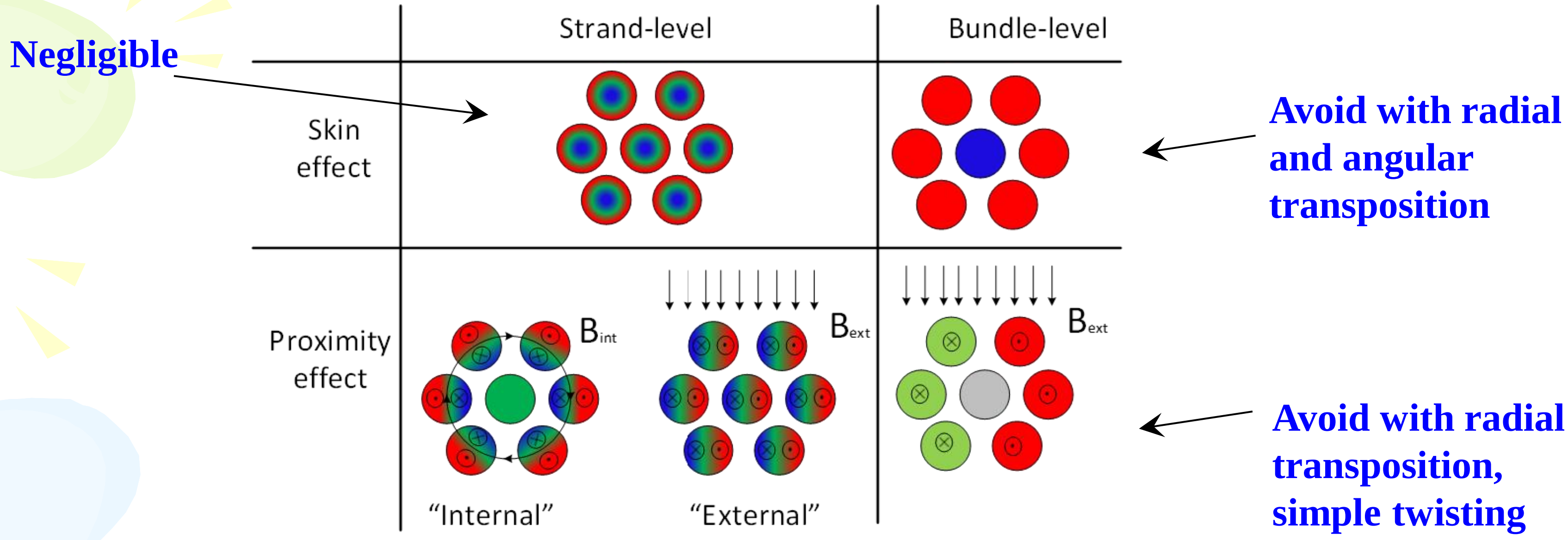
AC resistance of round conductor

$$R_{ac} = \left[0.25 + 0.5\frac{r_o}{\delta} + \frac{3}{32}\frac{\delta}{r_o}\right] R_{dc} = \left[0.25 + 0.5\frac{1}{0.295} + \frac{3}{32}\frac{0.295}{1}\right] 3.134 = 6.179\,\text{m}\Omega$$

$$P_{\text{cu}} = R_{\text{ac}} I_{\text{rms}}^2 = 6.179 \times 10^{-3} \times (7.217)^2 = 0.322\ \text{W}$$

Could replace solid wire with stranded Litz wire

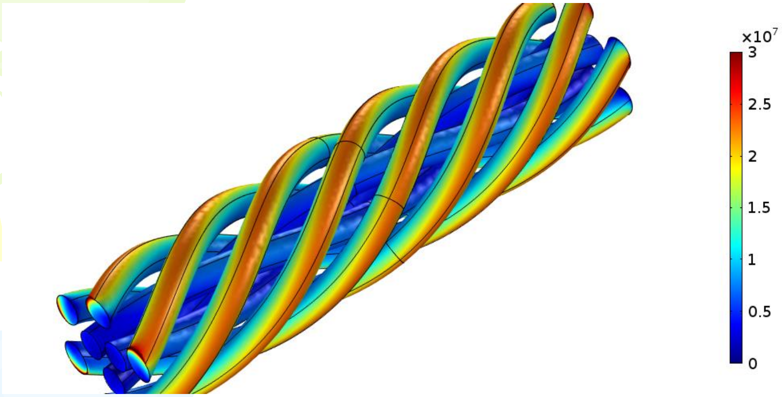
Litz Wire

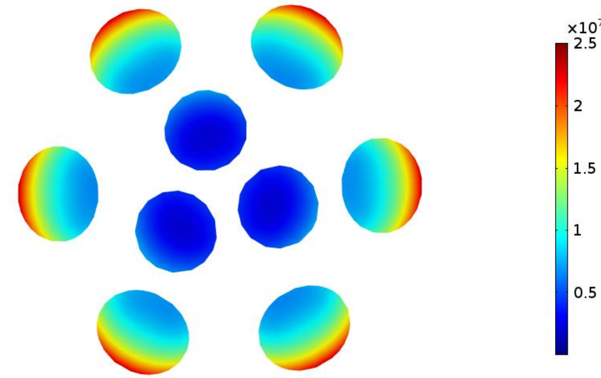


- Litz wire reduces the window utilisation factor, core may be 30% larger for same temperature rise
- Use strands with diameter less than δ/4
- This corresponds to 400 kHz for 50 AWG or 0.025 mm Cu wire
- Proximity effect occurs at strand level when wire is twisted
- Twisting cancels proximity effect at bundle level

Sullivan C. R., Zhang R. Y., "Analytical Model for Effects of Twisting on Litz-wire Losses", IEEE 15th Workshop on Control and Modelling for Power Electronics, (COMPEL), pp. 1-10. 2014

Litz Wire: skin effect

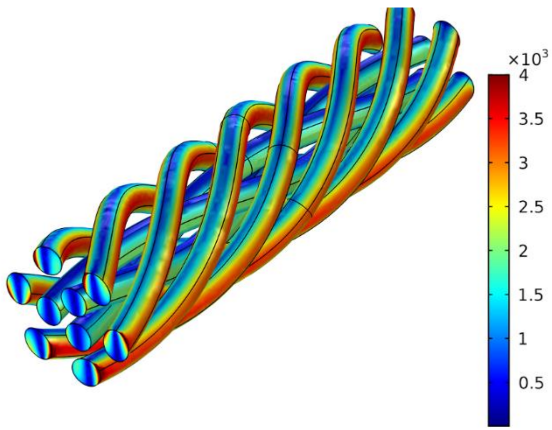


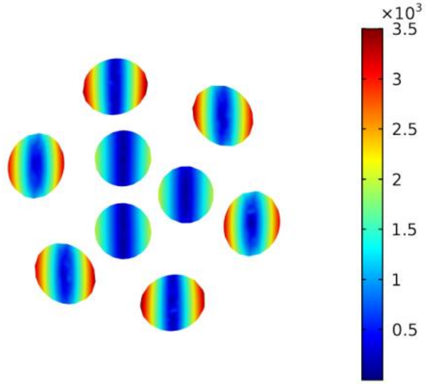


- Skin effect acts like a solid conductor at the bundle level
- Use strands with diameter less than δ/4
- This corresponds to 400 kHz for 50 AWG or 0.025mm Cu wire

Acero J., Lope I., Burdio J.M., Carretero C., Alonso R., “Loss Analysis of Multistranded Twisted Wires by Usinf 3D-FEA Simulation”, IEEE 15th Workshop on Control and Modelling for Power Electronics, (COMPEL), pp. 1-6, 2014

Litz Wire: proximity effect

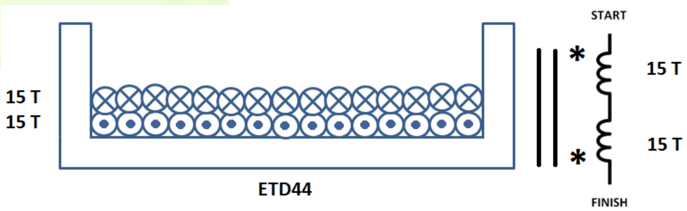


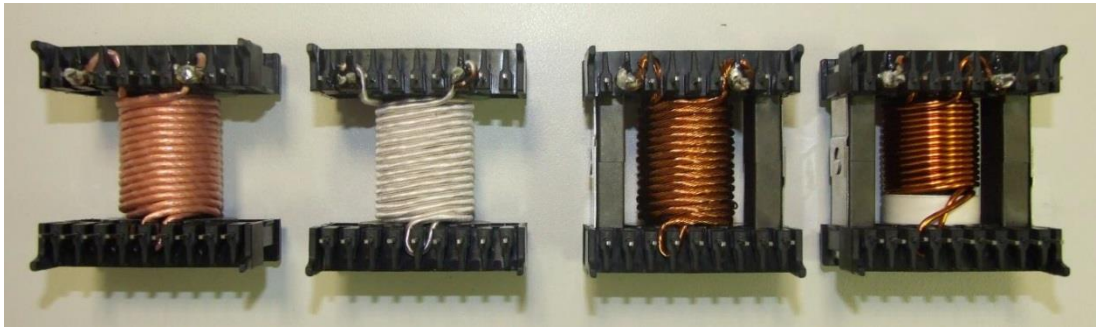


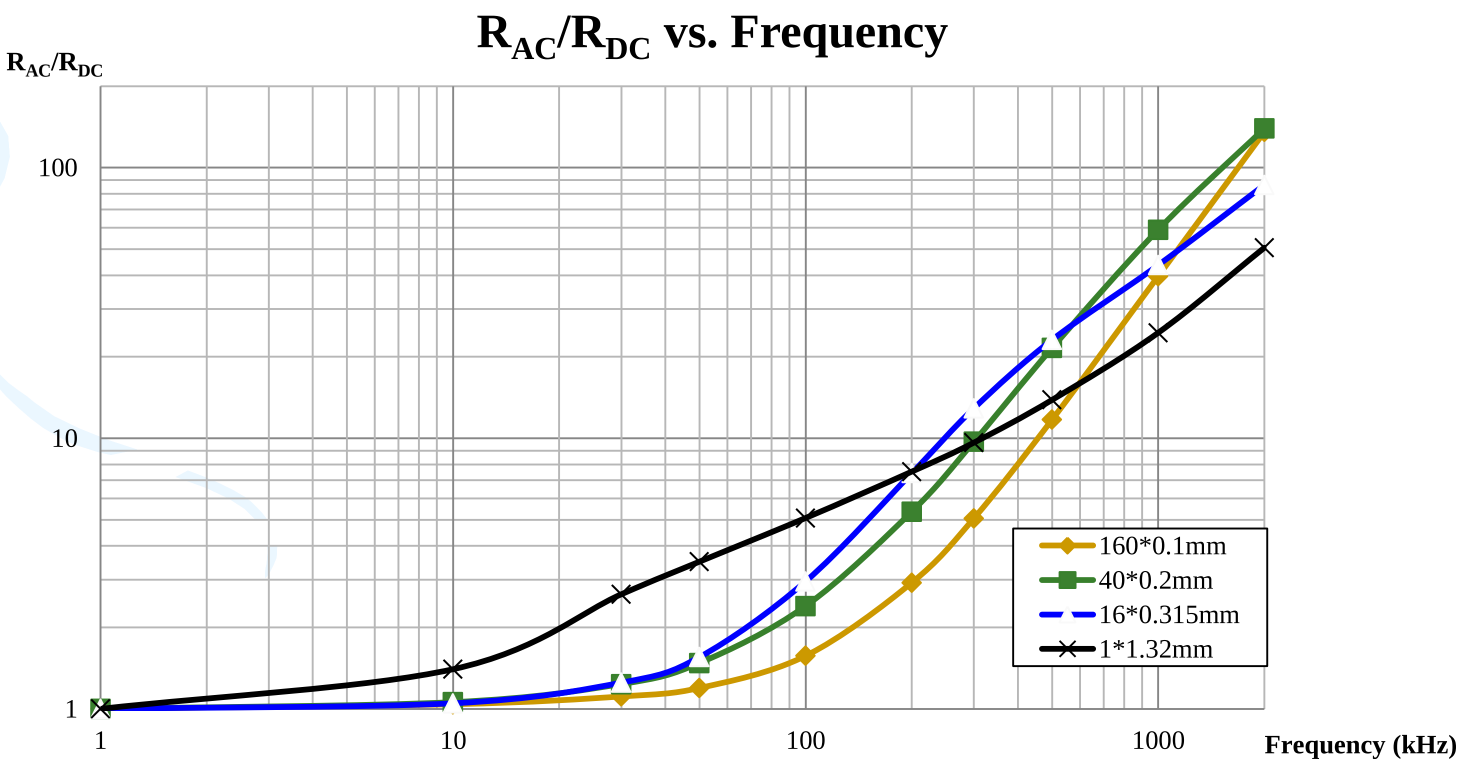
- Proximity effect occurs at strand level when wire is twisted
- Twisting cancels proximity effect at bundle level

Acero J., Lope I., Burdio J.M., Carretero C., Alonso R., "Loss Analysis of Multistranded Twisted Wires by Usinf 3D-FEA Simulation", IEEE 15th Workshop on Control and Modelling for Power Electronics, (COMPEL), pp. 1-6, 2014

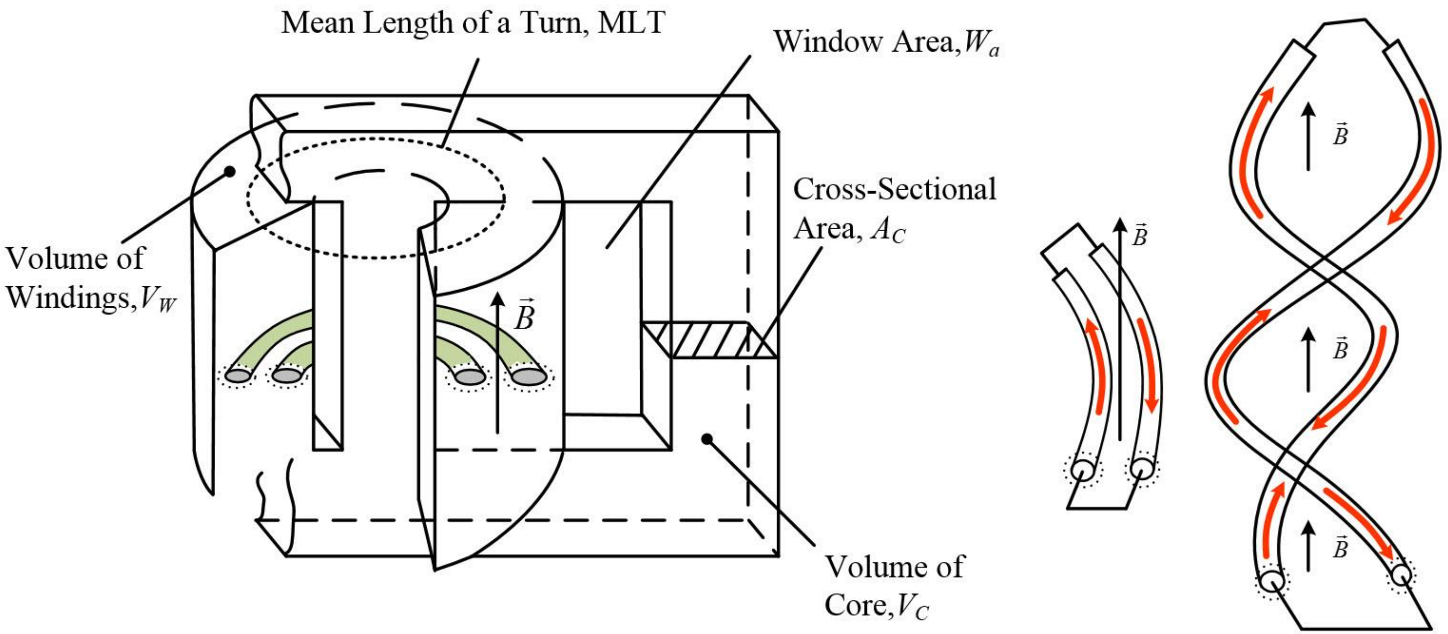
Litz Wire: Comparison Rac/Rdc





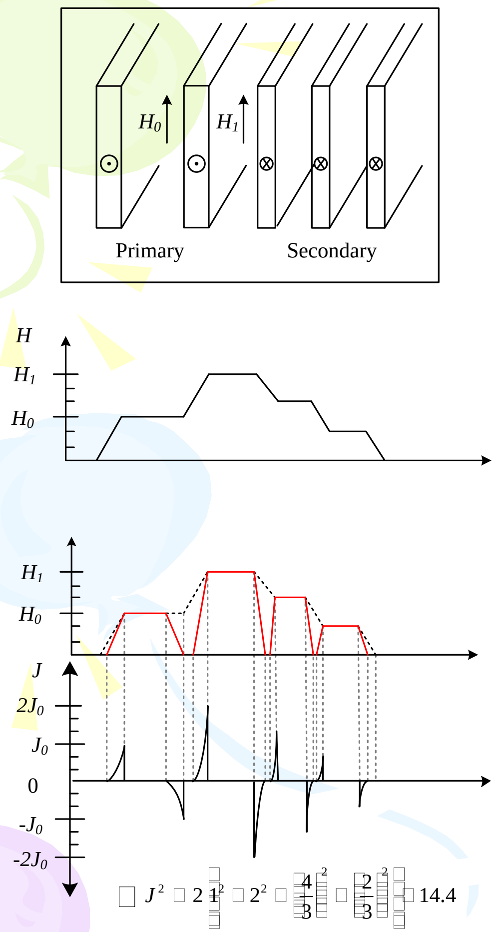


Litz Wire: Affect of twisting

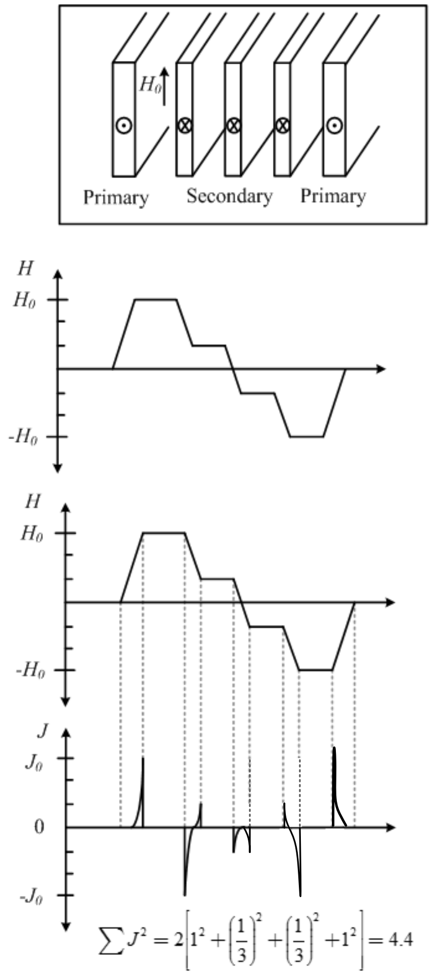


- Twisting cancels emf's due to magnetic field

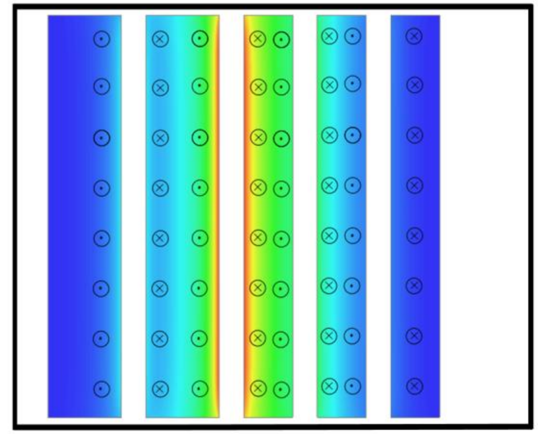
Interleaving the Windings



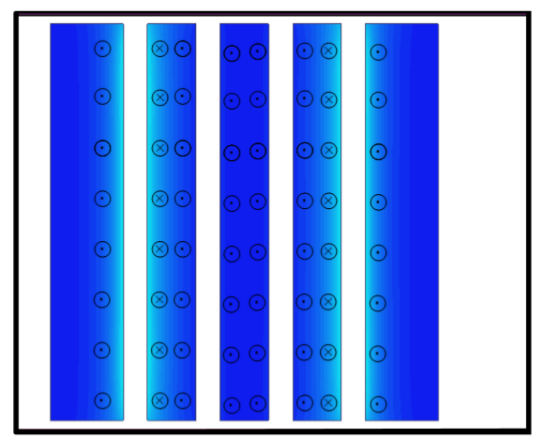
Current density distribution before interleaving



Current density distribution after interleaving

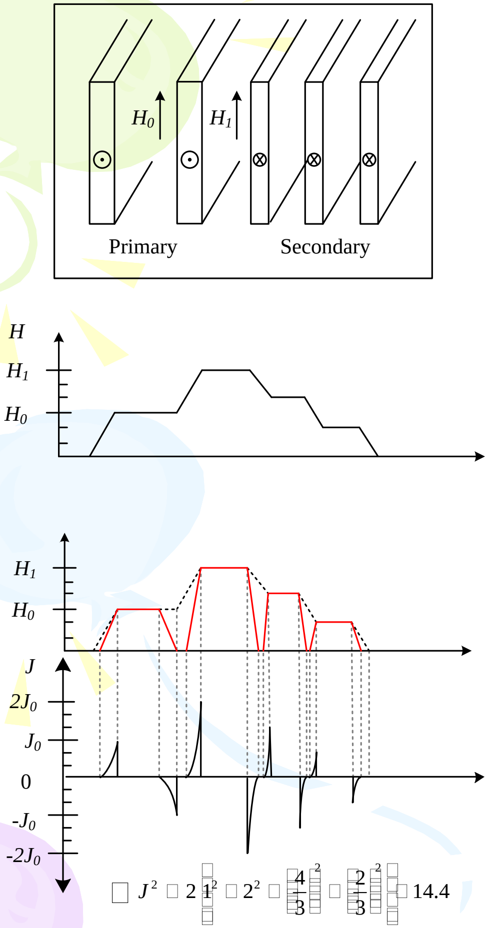


Current density distribution before interleaving in FEA

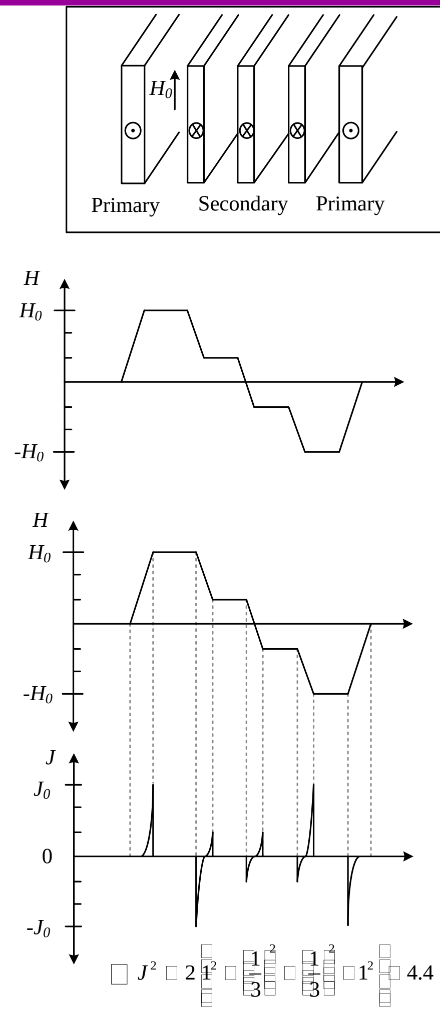


Current density distribution after interleaving in FEA

Interleaving the Windings



Current density distribution before interleaving



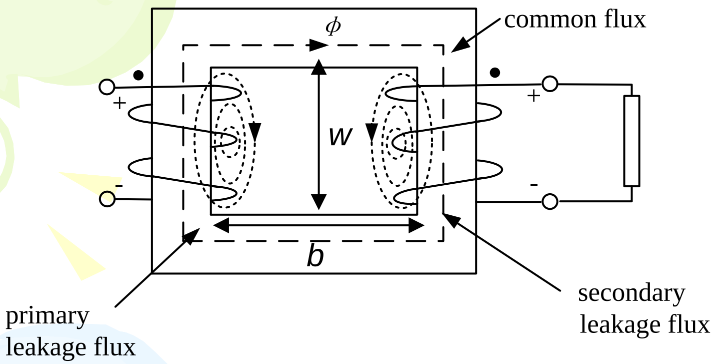
Current density distribution after interleaving

$$W_m = \tfrac{1}{2}\mu_0 \int_{\text{volume}} H^2 dV$$

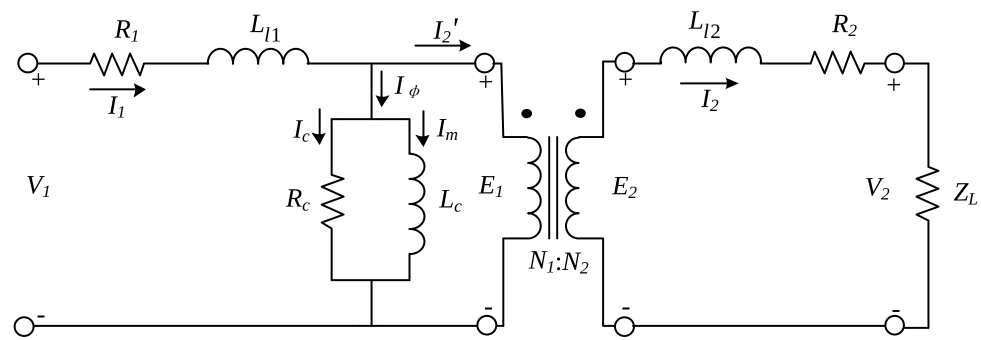
$$W_m = \tfrac{1}{2}\frac{\mu_0 N_p^{\,2}\,\mathrm{MLT}\,b}{3w} I_p^{\,2}$$

- A reduction in losses can be achieved by a factor of more than three in the proximity effect losses.
- **Proximity effect** reduces the leakage inductance.
- **Interleaving** also reduces leakage inductance.

Leakage Inductance



Leakage inductance in a transformer

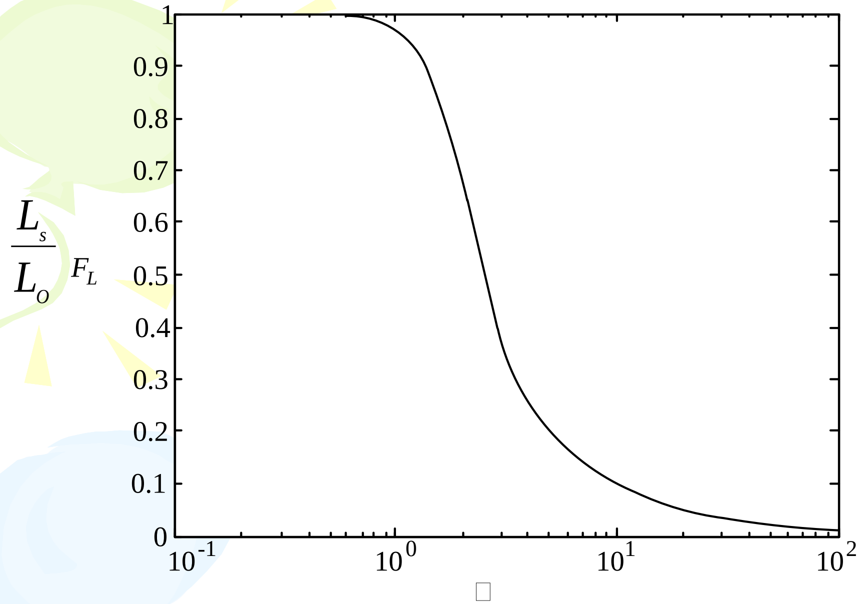


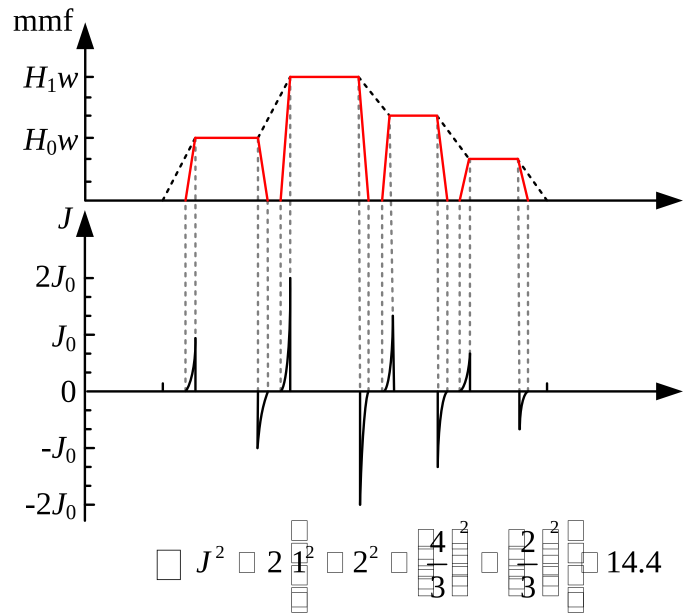
Transformer equivalent circuits

The leakage inductance $$L_{l1} + L_{l2} = \frac{\mu_o N_p^{\ 2} MLT\, b}{3w}$$

- The leakage inductance can be reduced by using less turns; interleaving the windings and reducing the ratio *b/w* which means the winding should be placed in a long narrow window.

Leakage Inductance at High Frequency



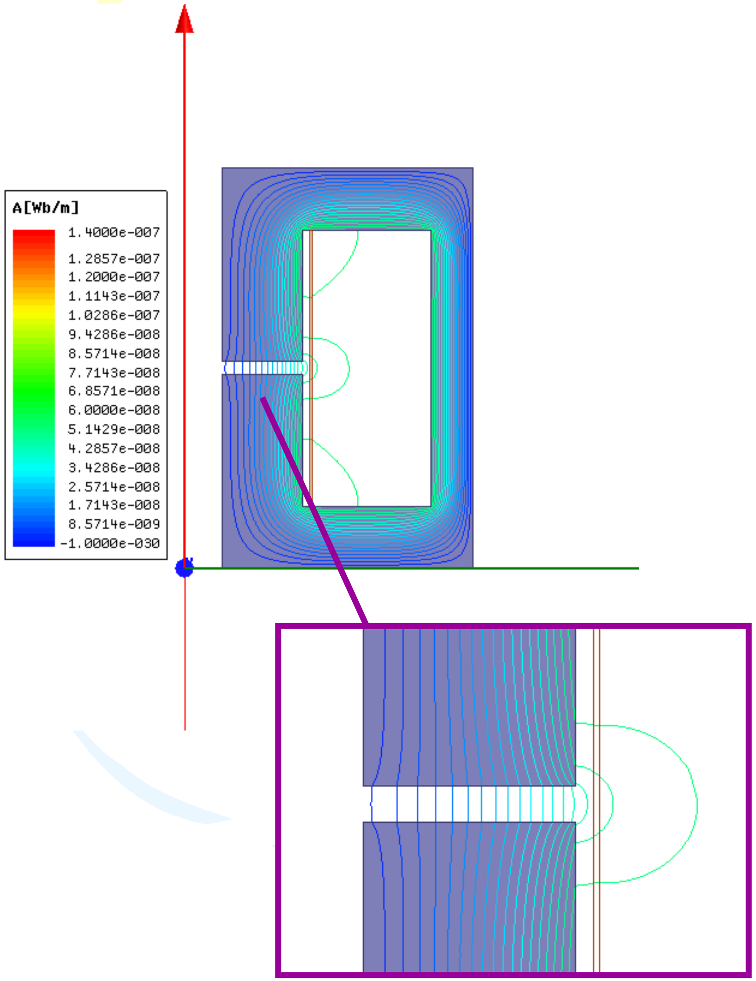


Leakage inductance of the transformer versus the frequency

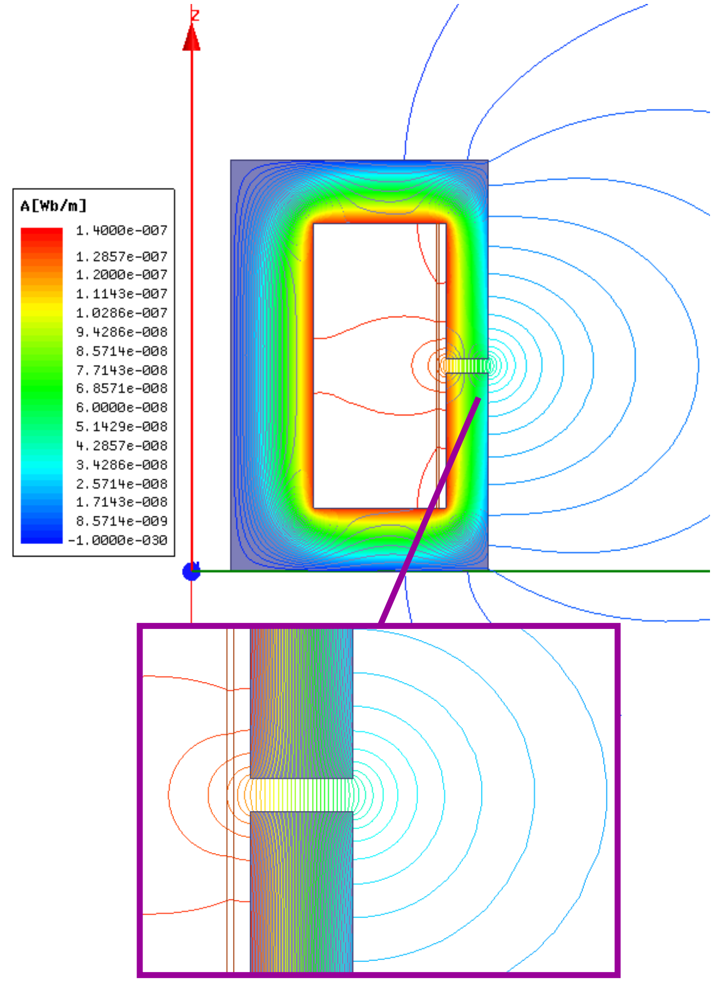
$$\frac{L_s}{L_o} = \mathrm{Im}\left\{\Delta\left[\frac{\sinh 2\Delta + \sin 2\Delta}{\cosh 2\Delta - \cos 2\Delta} + \frac{2(p^2-1)}{3}\frac{\sinh \Delta - \sin \Delta}{\cosh \Delta + \cos \Delta}\right]\right\} \quad \text{where} \quad \Delta = \frac{d}{\delta_0}$$

- The total leakage is directly related to the total volume occupied by the windings and the greater spread of windings along a core (increased *w*) reduces leakage effects.

Fringing (Flux)



Gap in the centre leg

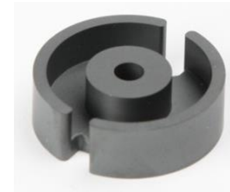


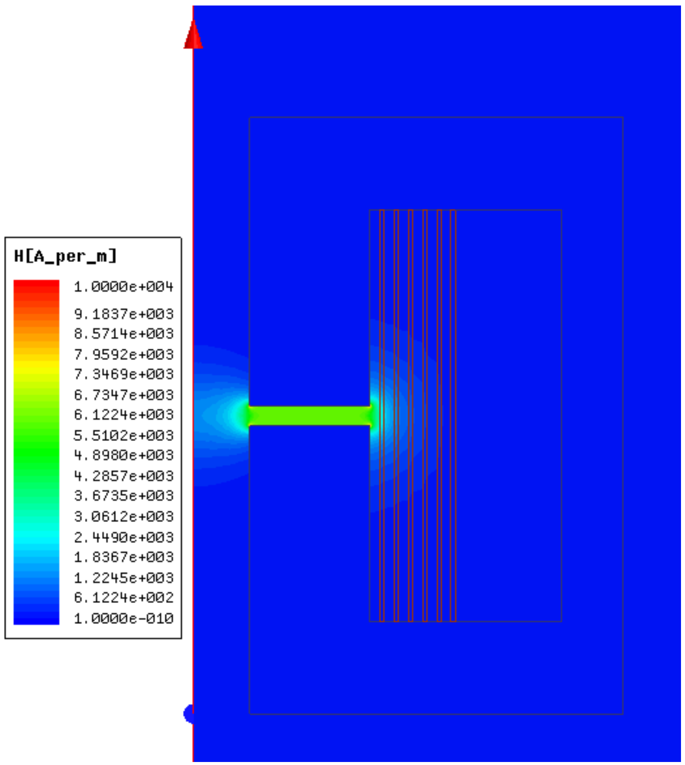
Gap in the outer leg

Fringing (Different Frequencies)

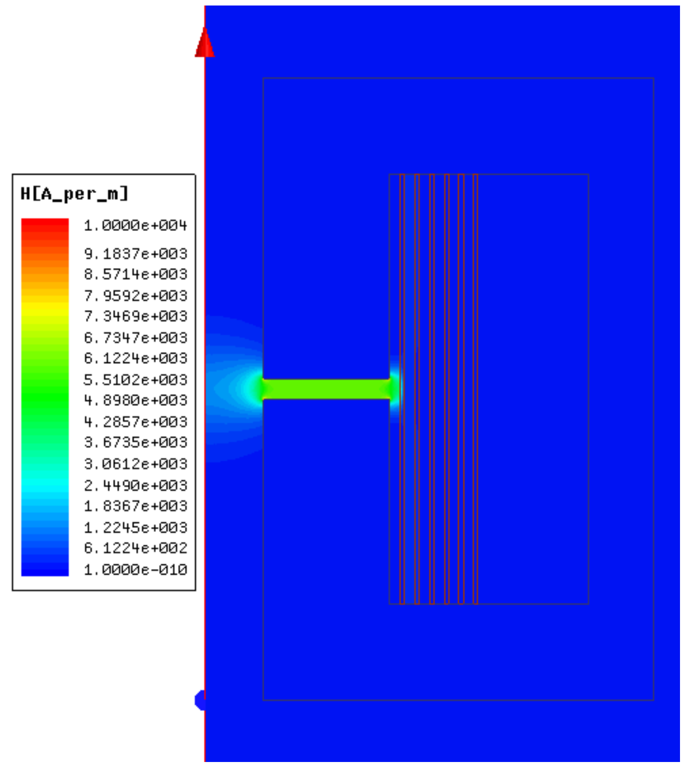
Magnetic Field Intensity

Width of conductor: 0.2mm   Core: Magnetics® port core





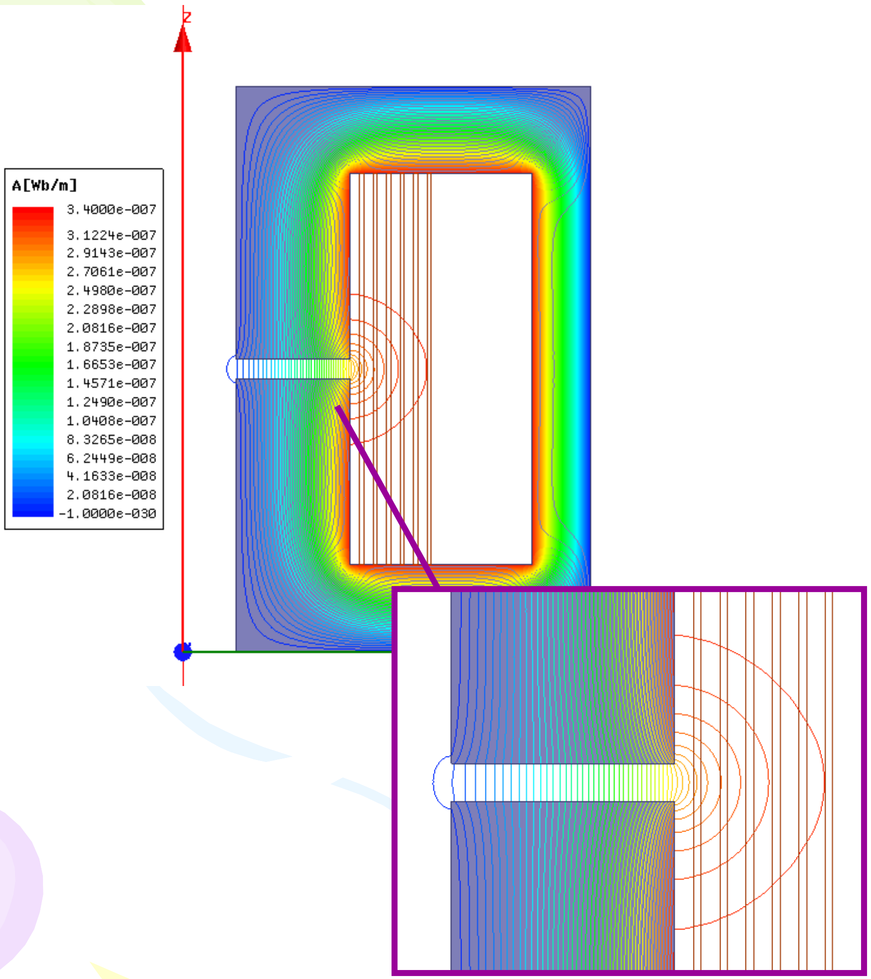
Frequency 1kHz



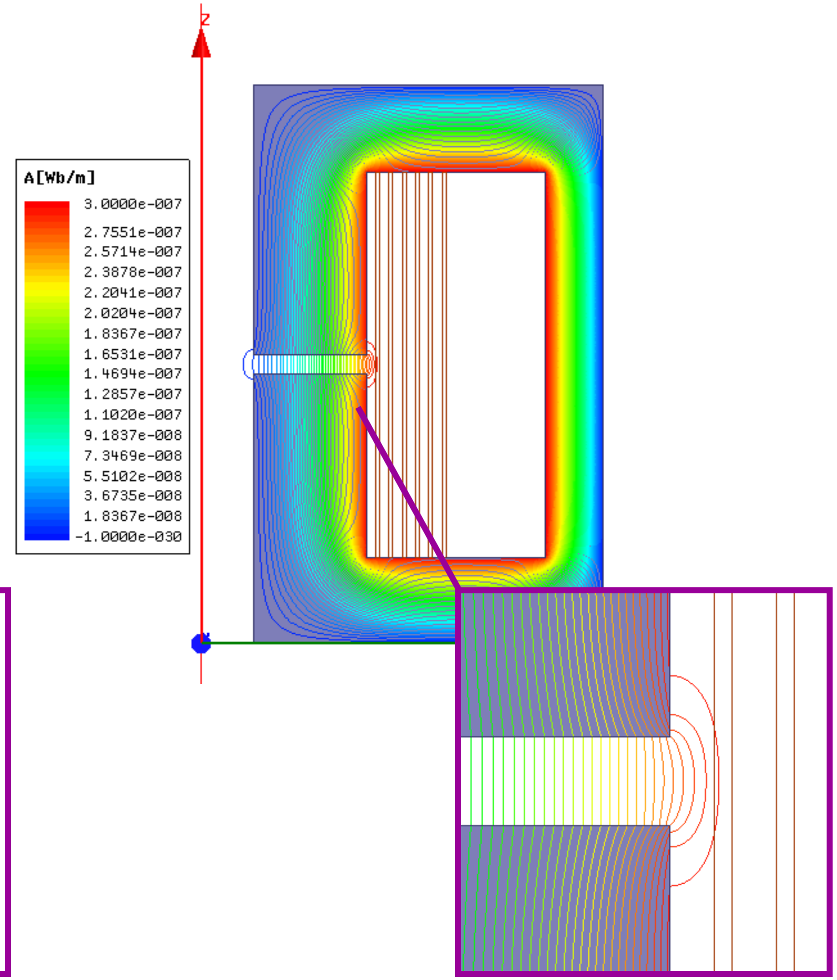
Frequency 100kHz

Fringing (Different Frequencies)

Magnetic Flux



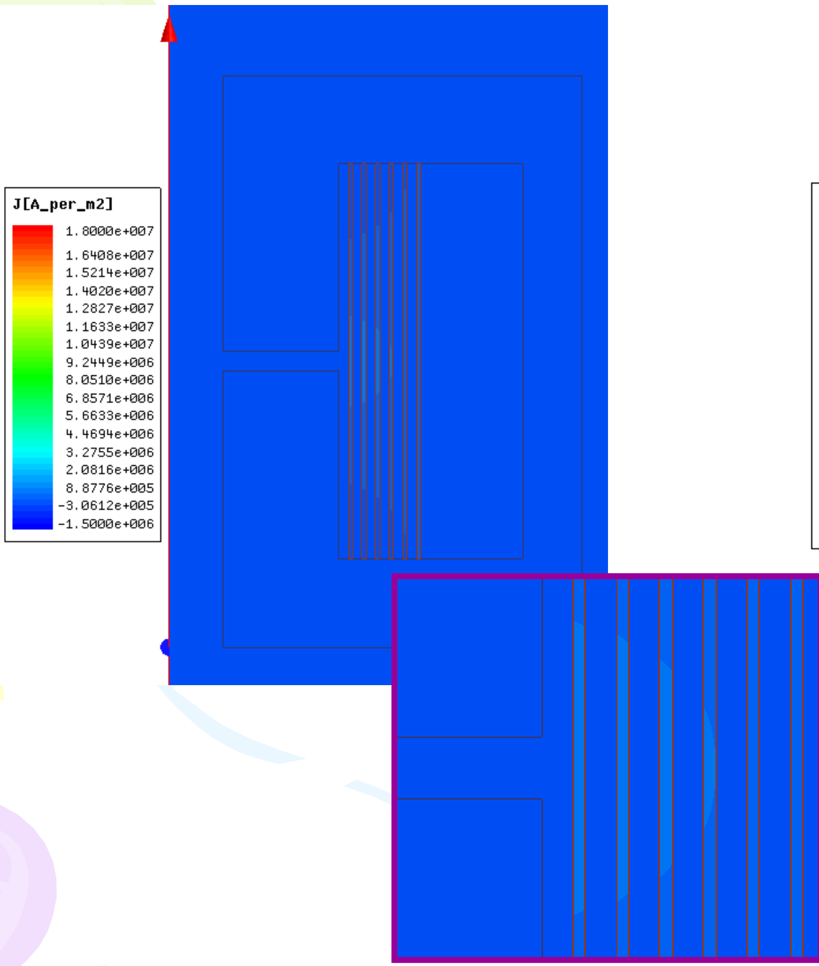
Frequency 1kHz



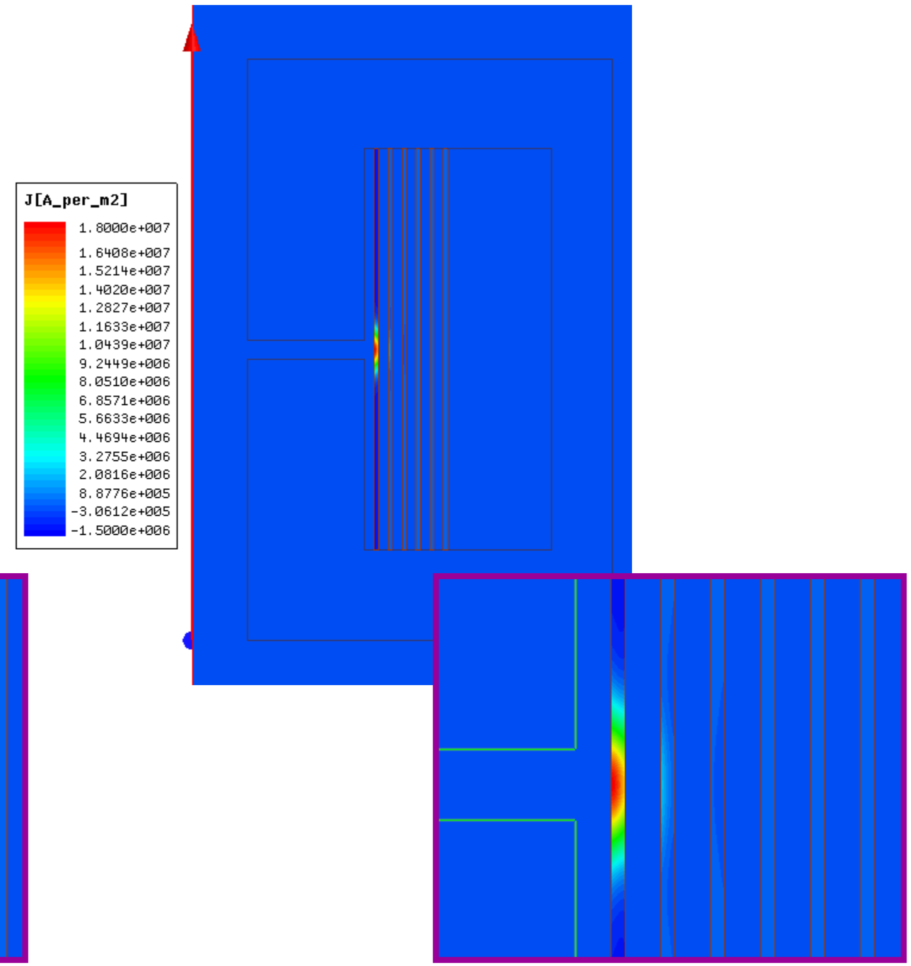
Frequency 100kHz

Fringing (Different Frequencies)

Current Density



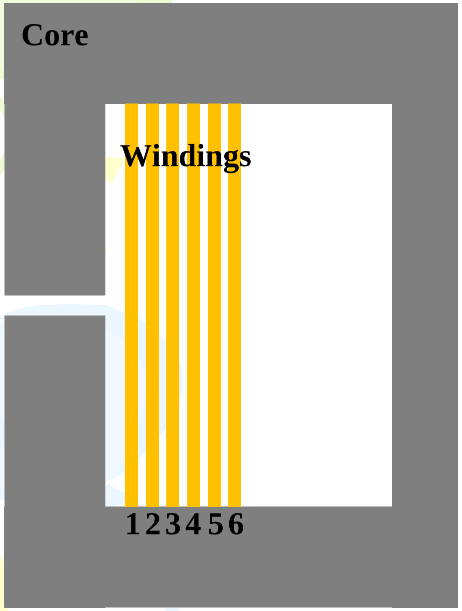
Frequency 1kHz



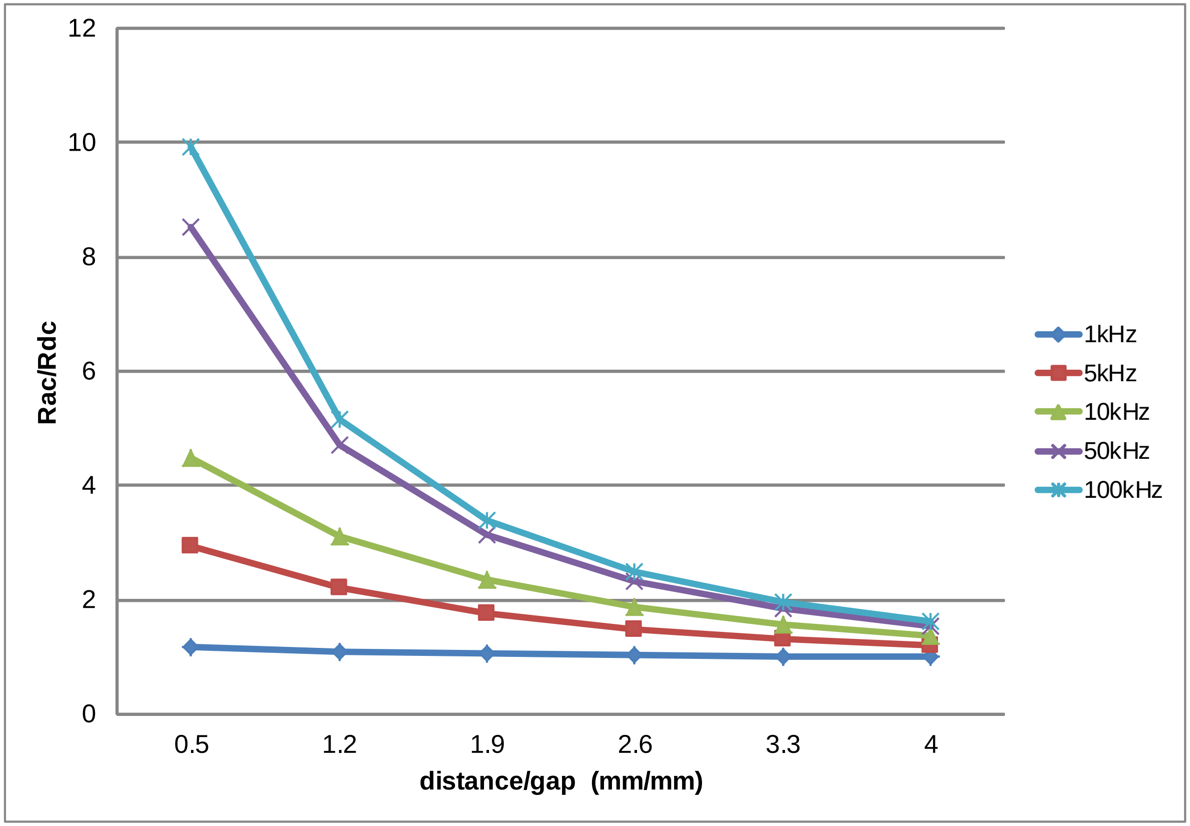
Frequency 100kHz

Winding Resistance

Winding Resistance related to the fringing effect, g=1mm



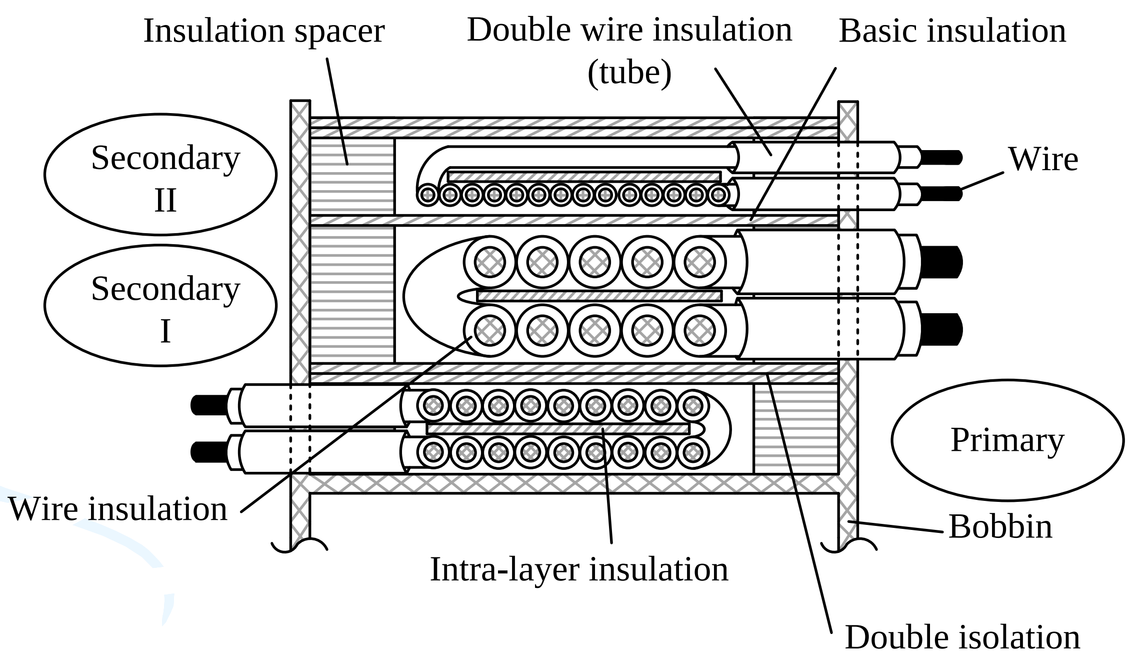
Distance 1: 0.5mm
Distance 2: 1.2mm
Distance 3: 1.9mm
Distance 4: 2.6mm
Distance 5: 3.3mm
Distance 6: 4.0mm



Transformer Insulation

Basic means of providing insulation:

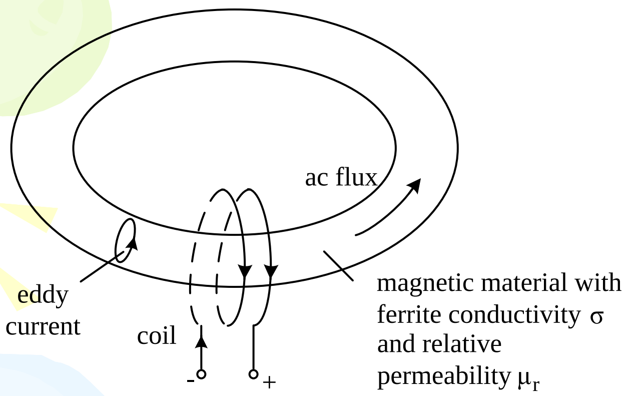
- Insulator
- Creepage
- Clearance



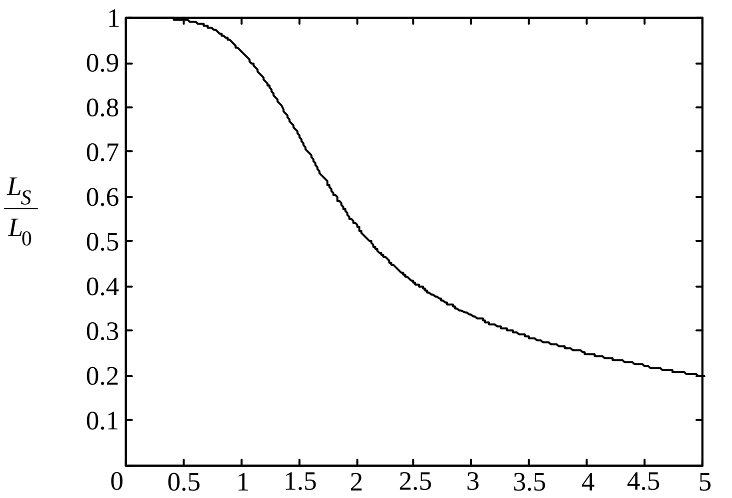
Transformer insulation

High Frequency Effects in the Core

Eddy Current in the Core



Eddy current losses in a toroidal core



Equivalent core inductance versus frequency

The inductance of the toroid under the lower frequency $L_0 = \dfrac{\mu_r \mu_0 N^2 A_c}{\ell_c}$

The inductance terms of the core impendence is

$$L_s = L_0\left(1 - \frac{\Delta^4}{12 + 1.43\Delta^4}\right) \quad \Delta < 2.1$$

$$= L_0\left(\frac{1}{\Delta} + \frac{1}{16\Delta^3} + \frac{1}{16\Delta^4}\right) \quad \Delta > 2.1$$

Eddy Current Core Losses

The equivalent core resistance:

$$R_s = \omega L_0 \frac{\Delta^2}{4} = \pi f^2 \sigma \left(\frac{\mu_r \mu_0 N}{l_c} \right)^2 \frac{l_c \pi b^2}{2}$$

The average power loss in the core due to eddy currents is

$$p = \frac{\pi f^2 \sigma B_{max}^2 \pi b^2}{4}$$

- The core losses may be reduced by increasing the electrical resistivity or reducing the electrical conductivity of the core material.
- The use of a smaller core cross-section to reduce eddy current losses suggests the use of laminations.

Core Losses (GSE, iGSE)

Steinmetz equation:

$$P_{\mathrm{fe}} = K_c f^{\alpha} B_{\max}{}^{\beta}$$

The time-average power loss with non-sinusoidal excitation using the iGSE

$$P_v = \frac{1}{T}\int_0^T k_i \left|\frac{dB(t)}{dt}\right|^{\alpha} \left|\Delta B\right|^{\beta-\alpha} dt = k_i \left|\Delta B\right|^{\beta-\alpha} \frac{1}{T}\int_0^T \left|\frac{dB(t)}{dt}\right|^{\alpha} dt$$

$$= k_i \left|\Delta B\right|^{\beta-\alpha} \overline{\left|\frac{dB(t)}{dt}\right|^{\alpha}}$$

where

$$k_i = \frac{K_c}{2^{\beta-1}\pi^{\alpha-1}\int_0^{2\pi}\left|\cos\theta\right|^{\alpha} d\theta}$$

A useful approximation is

$$k_i = \frac{K_c}{2^{\beta-1}\pi^{\alpha-1}\left(1.1044 + \dfrac{6.8244}{\alpha + 1.354}\right)}$$

Push-pull Converter Transformer

Core data: EPCOS N67 Mn-Zn

K_c	9.12
α	1.24
β	2.0
B_{sat}	0.4 T

Core data: ETD44

A_c	1.73 cm^2
W_a	2.78 cm^2
A_p	4.81 cm^4
V_c	17.70 cm^3
k_f	1.0
k_u	0.4
MLT	7.77 cm
ρ_{20}	1.72 μΩ-cm
α_{20}	0.00393

Push-pull Converter Transformer

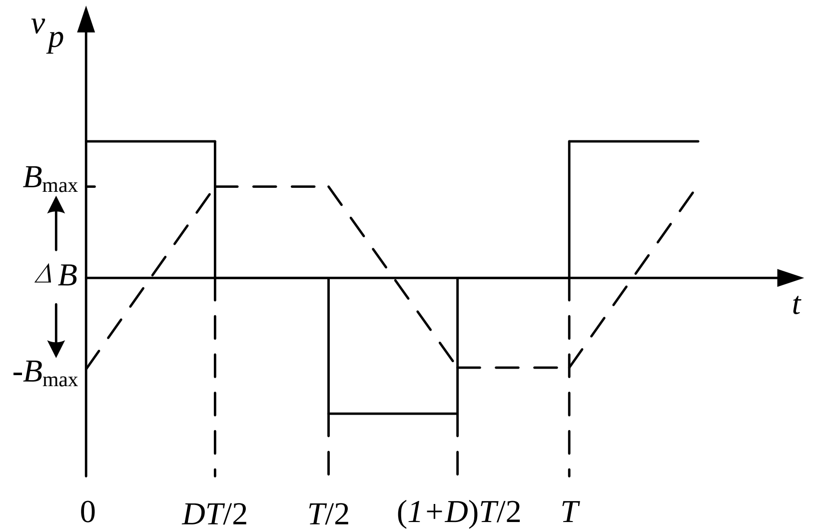
Calculations:

(1) power loss per unit volume

$$P_v = k_i \left|\Delta B\right|^{\beta-\alpha} \frac{1}{T}\left[\int_0^{DT/2}\left|\frac{\Delta B}{DT/2}\right|^{\alpha} dt + \int_{T/2}^{(1+D)T/2}\left|\frac{\Delta B}{DT/2}\right|^{\alpha} dt\right] \approx k_i \left|\Delta B\right|^{\beta-\alpha} \frac{1}{T}\left[\left|2\Delta B\right|^{\alpha}(DT)^{1-\alpha}\right]$$

$$k_i = \frac{K_c}{2^{\beta-1}\pi^{\alpha-1}\left(1.1044+\dfrac{6.8244}{\alpha+1.354}\right)}$$

$$= \frac{9.12}{2^{2.0-1}\pi^{1.24-1}\left(1.1044+\dfrac{6.8244}{1.24+1.354}\right)} = 0.9275$$



Flux waveform for the push-pull converter

Push-pull Converter Transformer

Calculations:

(2) *ΔB*

$$B_{max} = \frac{\sqrt{D}V_{dc}}{K_v f N_p A_c} = \frac{\sqrt{0.67}(36)}{(4.88)(50000)(6)(1.73\times10^{-4})} = 0.116\ \mathrm{T}$$

$$\Delta B = 2B_{max} = 0.232\ \mathrm{T}$$

(3) The core loss per unit volume

$$P_v = k_i \left|\Delta B\right|^{\beta-\alpha} \frac{1}{T}\left[\left|2\Delta B\right|^{\alpha}(DT)^{1-\alpha}\right]$$

$$= (0.9275)(0.232)^{2.0-1.24}(50000)\left[(2\times0.232)^{1.24}\times(0.67/(50000))^{1-1.24}\right]$$

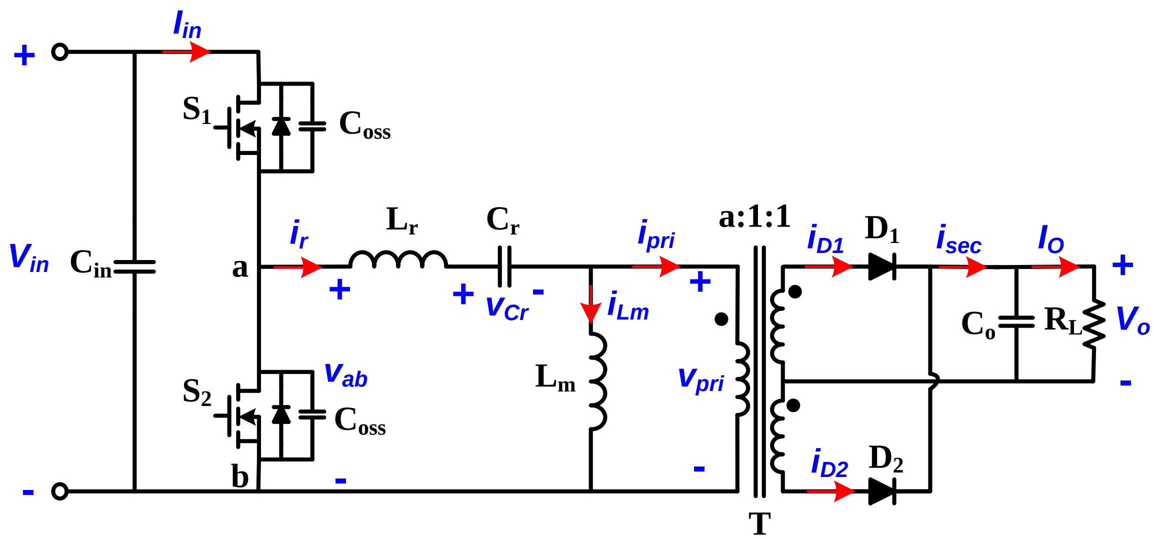
$$= 0.871\times10^{5}\ \mathrm{W/m^3}$$

(4) The total core loss

$$17.71\times10^{-6}\times0.871\times10^{5} = 1.543\ \mathrm{W}$$

(5) The total core loss by GSE = 1.466 W

LLC Resonant Converter



Half-bridge LLC resonant converter with uncontrolled rectifier

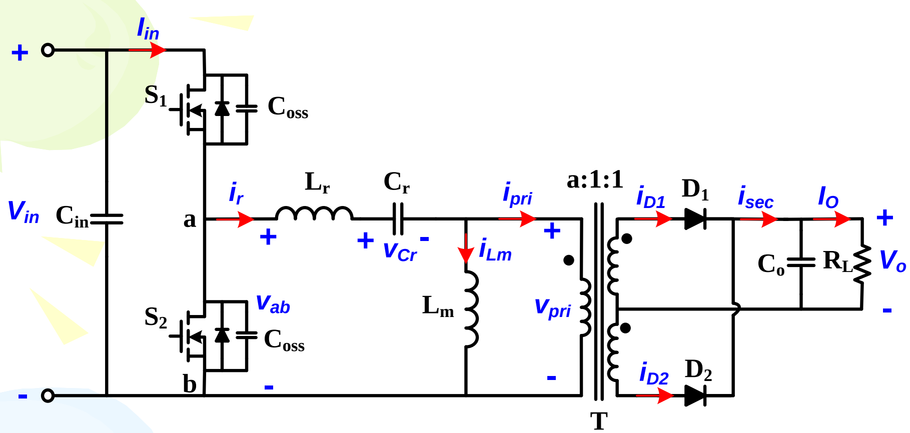
Soft Switching

High Efficiency

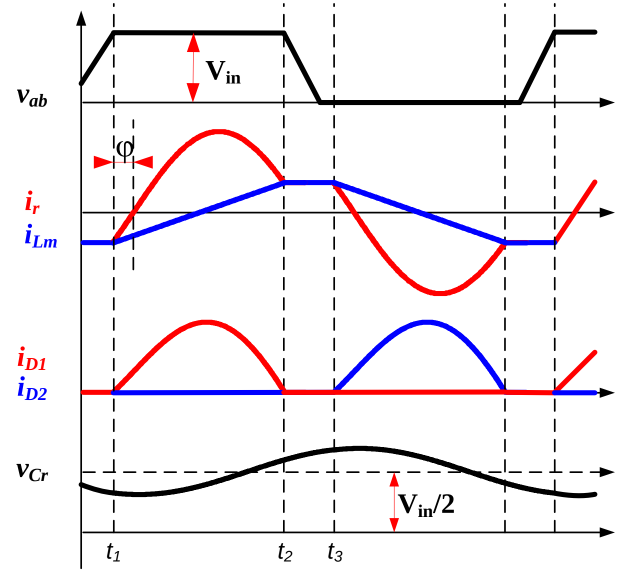
High Frequency

Low Size

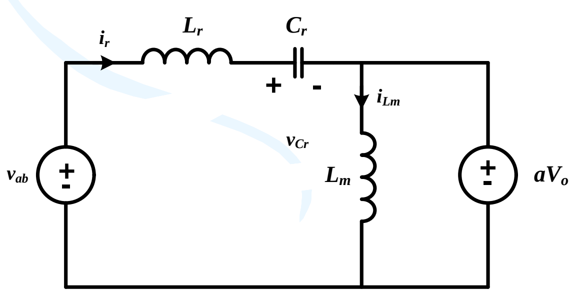
Basic Operation Principle



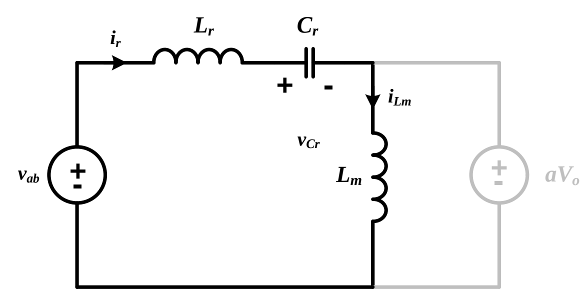
Half-bridge LLC resonant converter with the uncontrolled rectifier



Waveforms of the LLC resonant converter operated at the full load condition

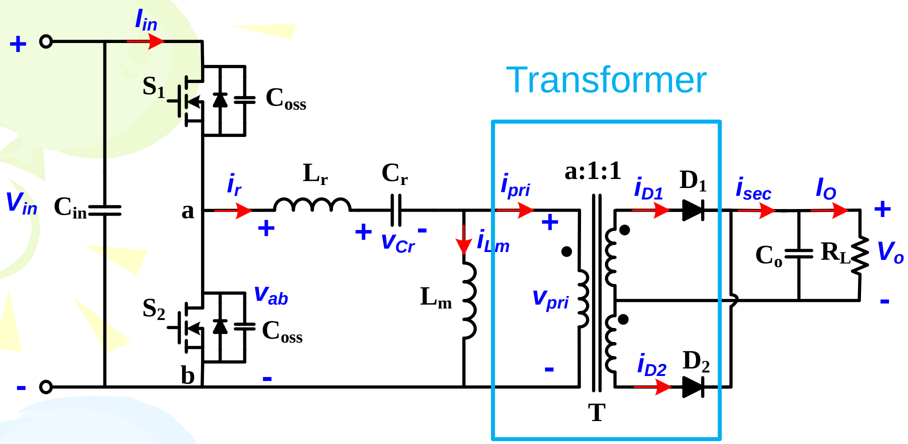


Mode 1: t_1~t_2

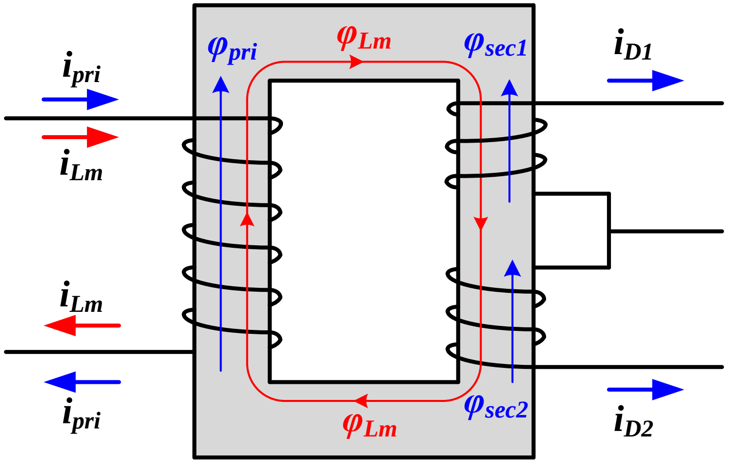


Mode 2: t_2~t_3

LLC Resonant Converter Transformer



Half-bridge LLC resonant converter with the uncontrolled rectifier



Currents cross the transformer and the flux in the magnetic core

- The magnetizing current will set up the common flux in the core.
- The flux set up by the primary current and the secondary current will cancel each other.
- The current density in each winding is the same (see slide 41)

LLC Transformer Design

Design specifications for transformer

Input voltage, V_{in}	400 V
Output voltage, V_o	24V
Output power, P_o	200W
Resonant Capacitance, C_r	0.033 µF
Resonant Inductance, L_r	86 µH
Magnetizing Inductance, L_m	680 µH
Turns ratio, a	8
Frequency, f	90 kHz
Temperature Rise, ΔT	50ºC
Ambient Temperature, T_a	50ºC
Window utilization factor	0.65

Core data: TDK PC40

K_c	0.344
α	1.585
β	2.661
B_{sat}	0.4T

LLC Resonant Converter Transformer Design

Calculations:

(1) Peak value of the current

$$f_r = \frac{1}{2\pi\sqrt{L_r C_r}} = 94.47\ \text{kHz};\quad T_r = \frac{1}{f_r} = 10.58\ \mu\text{s};\quad T_s = \frac{1}{f_s} = 11.11\ \mu\text{s};\quad f_n = \frac{f_s}{f_r} = 0.953$$

$$I_{p\_peak} = \sqrt{\left(\frac{\pi I_o}{2af_n}\right)^2 + \left(\frac{aV_o}{4f_r L_m}\right)^2} = 1.873\ \text{A} \qquad I_{Lm\_peak} = \frac{aV_o}{4L_m f_r} = 0.747\ \text{A} \qquad I_{s\_peak} = 13.909\ \text{A}$$

(2) RMS value of the current

$$I_{p\_rms} = \sqrt{\frac{a^2 V_o^2 T_r^2 (2T_s - T_r)}{32 L_m^2 T_s} + \frac{\pi^2 I_o^2 T_s^2}{8a^2 T_r^2}} = 1.329\ \text{A} \qquad I_{s\_rms} = 6.735\ \text{A}$$

(3) Current waveform factors

$$K_{ip} = \frac{I_{p\_rms}}{I_{p\_peak}} = \frac{1.329}{1.873} = 0.710 \qquad K_{is} = \frac{I_{s\_rms}}{I_{s\_peak}} = \frac{6.735}{13.909} = 0.484$$

Calculations:

(4) Window utilization factor

The optimum distribution of current in the available area is to have the same current density in each winding.

$$\frac{N_p I_{p\_rms}}{W_{cp}} = \frac{N_s I_{s\_rms}}{W_{cs1}} = \frac{N_s I_{s\_rms}}{W_{cs2}}$$

$$\frac{W_{cp}}{W_{cp}+W_{cs1}+W_{cs1}} = \frac{W_{cp}}{W_c} = \frac{N_p I_{p\_rms}}{N_p I_{p\_rms}+2N_s I_{s\_rms}} = \frac{1}{1+2\frac{I_{s\_rms}}{aI_{p\_rms}}}$$

$$\frac{W_{cp}}{W_c} = \frac{W_{cp}}{W_a}\frac{W_a}{W_c} = \frac{k_{up}}{k_u} = \frac{1}{1+2\frac{I_{s\_rms}}{aI_{p\_rms}}} \qquad k_{up} = k_u\frac{1}{1+\frac{2I_{s\_rms}}{aI_{p\_rms}}} = 0.287$$

(5) A_p

$$A_p = \left[\frac{\sqrt{k_u(1+\gamma)}K_{ip}L_m I^2_{Lm\_peak}}{B_{\max}k_{up}\sqrt{\Delta T}}\frac{I_{p\_peak}}{I_{Lm\_peak}}\right]^{8/7} = 0.729\ \text{cm}^4$$

with $B_{\max} = 0.18\text{T}; \quad \gamma = 5$ $\qquad$ γ must be checked later

EER28L Core

A_c	0.814 cm^2
l_c	7.55 cm
W_a	0.924 cm^2
A_p	0.752 cm^4
V_c	6.15 cm^3
k_f	1.0
k_u	0.65
MLT	5.5 cm
ρ_{20}	1.72 $\mu\Omega$-cm
α_{20}	0.00393

LLC Resonant Converter Transformer Design

Calculations:

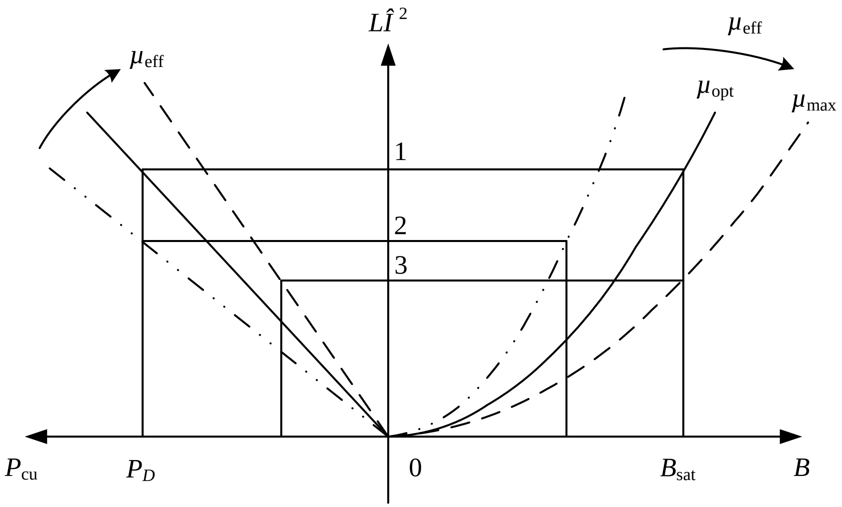
(6) Maximum dissipation

$$R_\theta = \frac{0.06}{\sqrt{V_c}} = \frac{0.06}{\sqrt{6.15\times10^{-6}}} = 24.2\ °\text{C/W}$$

$$P_D = \frac{\Delta T}{R_\theta} = \frac{50}{24.2} = 2.067\,\text{W}$$

$$P_{cu} = \frac{P_D}{1+\gamma} = \frac{2.067}{1+5} = 0.344\,\text{W}$$

$$P_{cu_p} = 0.172\,\text{W}$$



(7) Optimum value of the effective permeability

$$\mu_{\text{opt}} = \frac{B_{\max} l_c K_{ip}}{\mu_0 \sqrt{\dfrac{P_{\text{cu}_p} k_{up} W_a}{\rho_w \text{MLT}}}} \frac{I_{p\_peak}}{I_{Lm\_peak}} = \frac{(0.18)(7.55\times10^{-2})(0.710)}{(4\pi\times10^{-7})\sqrt{\dfrac{(0.172)(0.287)(0.924\times10^{-4})}{(1.72\times10^{-8})(5.5\times10^{-2})}}} \frac{1.872}{0.747} = 277.04$$

LLC Resonant Converter Transformer Design

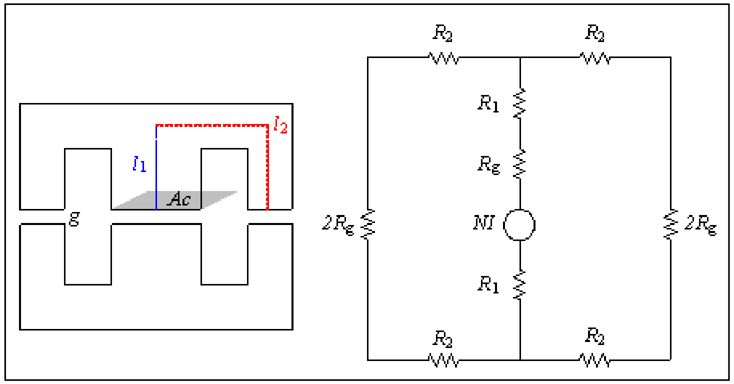
Calculations:

(8) Maximum gap length

The gap is distributed in three legs of the core.

$$R_{total} = R_g + 2R_1 + R_2 + R_g$$

$$= \frac{2g}{\mu_0 A_c} + \frac{2l_1}{\mu_0 \mu_r A_c} + \frac{l_2}{\mu_0 \mu_r \frac{A_c}{2}} = \frac{l_c}{\mu_0 \mu_{eff} A_c}$$



$$\mu_{eff} = \frac{\mu_r l_c}{2g\mu_r + l_c}$$

$$g_{\max} = \frac{l_c}{2}\left(\frac{1}{\mu_{opt}} - \frac{1}{\mu_r}\right) = 0.12 \text{ mm}$$

Select g=0.075 mm

$$\mu_{eff} = \frac{1}{\frac{1}{\mu_r} + \frac{g}{l_c}} = \frac{1}{\frac{1}{2300} + \frac{0.075}{75.5}} = 412.96$$

$$A_L = \frac{\mu_0 \mu_{eff} A_c}{l_c} = 559.5 \text{ nH/Turn}^2$$

LLC Resonant Converter Transformer Design

Calculations:

(9) Turns

$$N_p = \sqrt{\frac{L_m}{A_L}} = \sqrt{\frac{680\times10^{-6}}{559.5\times10^{-9}}} = 34.9 \text{ turns} \qquad N_s = \frac{N_p}{a} = \frac{32}{4} = 8 \text{ turns}$$

Select N_p=43 turns, N_s=8 turns

(10) Wire size

$$J_o = K_\theta \frac{\sqrt{\Delta T}}{\sqrt{k_u(1+\gamma)}\sqrt[8]{A_p}} = 178.84 \text{ A/cm}^2$$

$$A_{p\_w} = \frac{I_{p\_rms}}{J_o} = \frac{1.329}{178.84} = 0.0074 \text{ cm}^2 \qquad A_{s\_w} = \frac{I_{s\_rms}}{J_o} = \frac{6.735}{178.84} = 0.038 \text{ cm}^2$$

Primary windings:

The 3×AWG24 wire meets this specification. The cross-sectional area of the wire is 0.0062cm^2. The dc resistance is 2.81×10^{-4}Ω/cm@20ºC.

Secondary windings:

The 0.15 mm×16 mm copper foil meets this specification. As there are two secondary windings, the equivalent cross-sectional area of the copper is 0.048cm^2. The dc resistance is 3.58×10^{-5}Ω/cm@20ºC.

LLC Resonant Converter Transformer Design

Losses calculations:

(11) Copper loss

$T_{max} = 50 + 50 = 100\ °C$

$$R_{p\_dc} = (32)(5.5)(2.81\times10^{-4})[1+(0.00393)(100-20)] = 0.065\ \Omega$$

$$R_{s\_dc} = (4)(5.5)(3.58\times10^{-5})[1+(0.00393)(100-20)] = 1.035\ \text{m}\Omega$$

$$P_{p\_cu} = R_{p\_dc} I_{p\_rms}^2 = 0.115\ \text{W} \qquad P_{s\_cu} = 2R_{s\_dc} I_{s\_rms}^2 = 0.094\ \text{W}$$

$$P_{cu} = P_{p\_cu} + P_{s\_cu} = 0.115 + 0.094 = 0.209\ \text{W}$$

(12) Core loss

$$\Delta B = \frac{\mu_{eff}\mu_0 N_s I_{Lm\_peak}}{l_c} = \frac{(412.96)(4\pi\times10^{-7})(32)(0.747)}{7.55\times10^{-2}} = 0.164\text{T}$$

$$P_{fe} = V_c K_c f^{\alpha} B_{max}{}^{\beta} = 1.233\text{W}$$

(13) Total loss

Copper loss	0.209 W
Core loss	1.233 W
Total losses	1.442 W<2.067 W

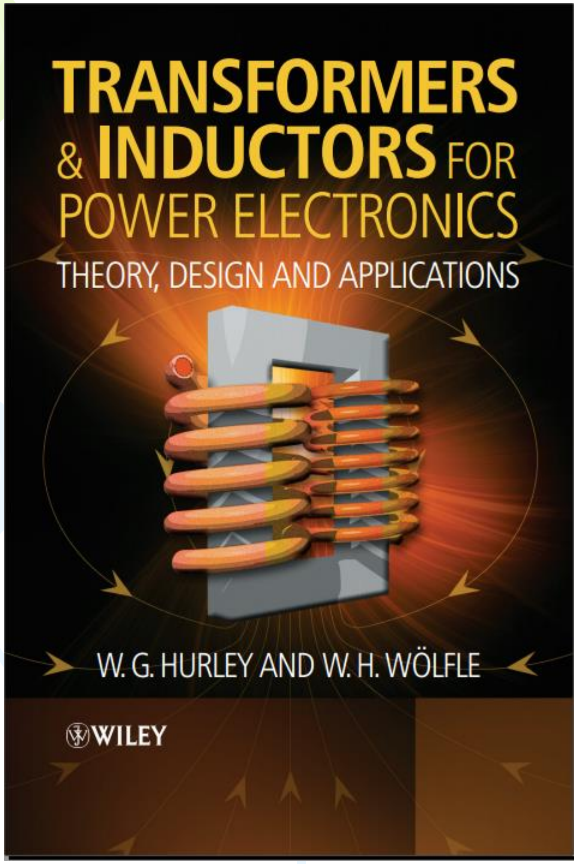
Check γ

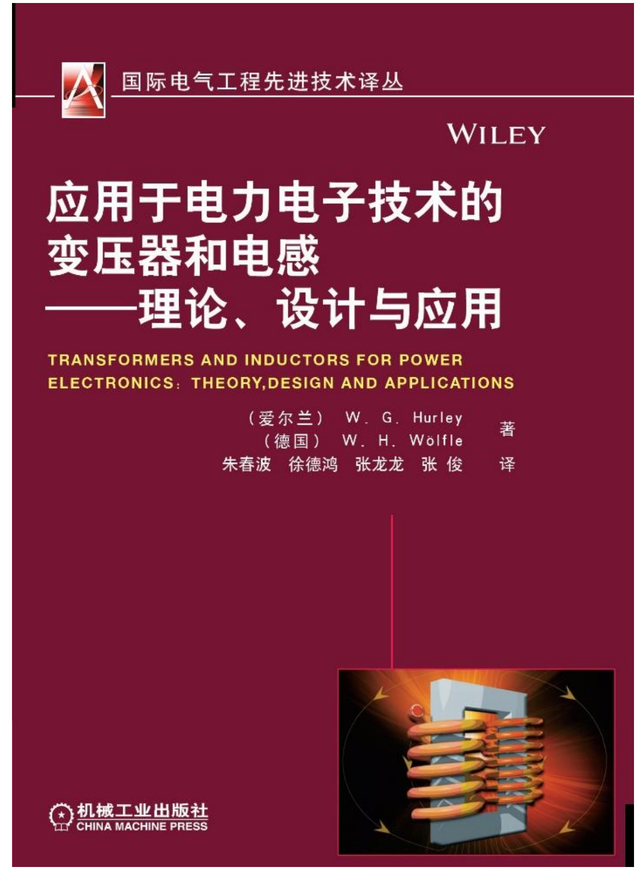
$$\gamma = \frac{P_{fe}}{P_{cu}} = \frac{1.233}{0.209} = 5.91$$

Conclusions

- High frequency effects have a major influence on transformer and inductor design for power electronics.
- Core selection is determined by the main specifications such as voltage, current and temperature rise.
- Winding loss can be reduced by optimising the thickness of the winding layer and interleaving.
- Core loss can be reduced by laminations.
- Interleaving the winding reduces leakage inductance.

Text Book





"With its comprehensive scope and careful organization of topics, covering fundamentals, high-frequency effects, unusual geometries, loss mechanisms, measurements and application examples, this book is a 'must have' reference for the serious power electronics engineer. Hurley and Wölfle have produced a text that is destined to be a classic on all our shelves" Professor John Kassakian of the Massachusetts Institute of Technology

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