Week 04

- Motivations
- Neural Networks
- Applications

Motivation

Non-linear classification and use more than 5000 features, so sigmoid function g will be complicated.

⇒ Complex

Computer vision problem:

- A car is a matrix for a computer.
- For a car, look and store specific pixels.
- Give the algorithm a list of car images so it learns what is a car.
- A pixel is feature so the calculation will be complex.

Neural network:

- Algorithm trying to mimic the brain
- 80'

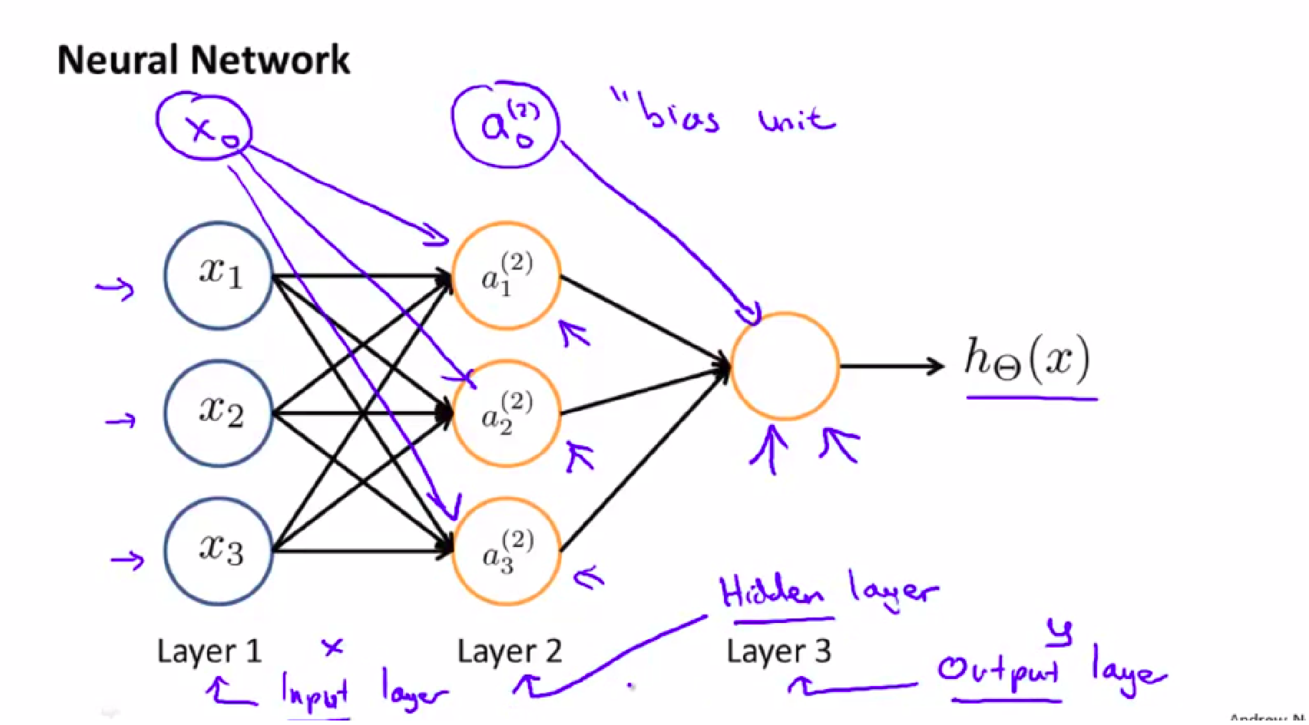
Neural networks

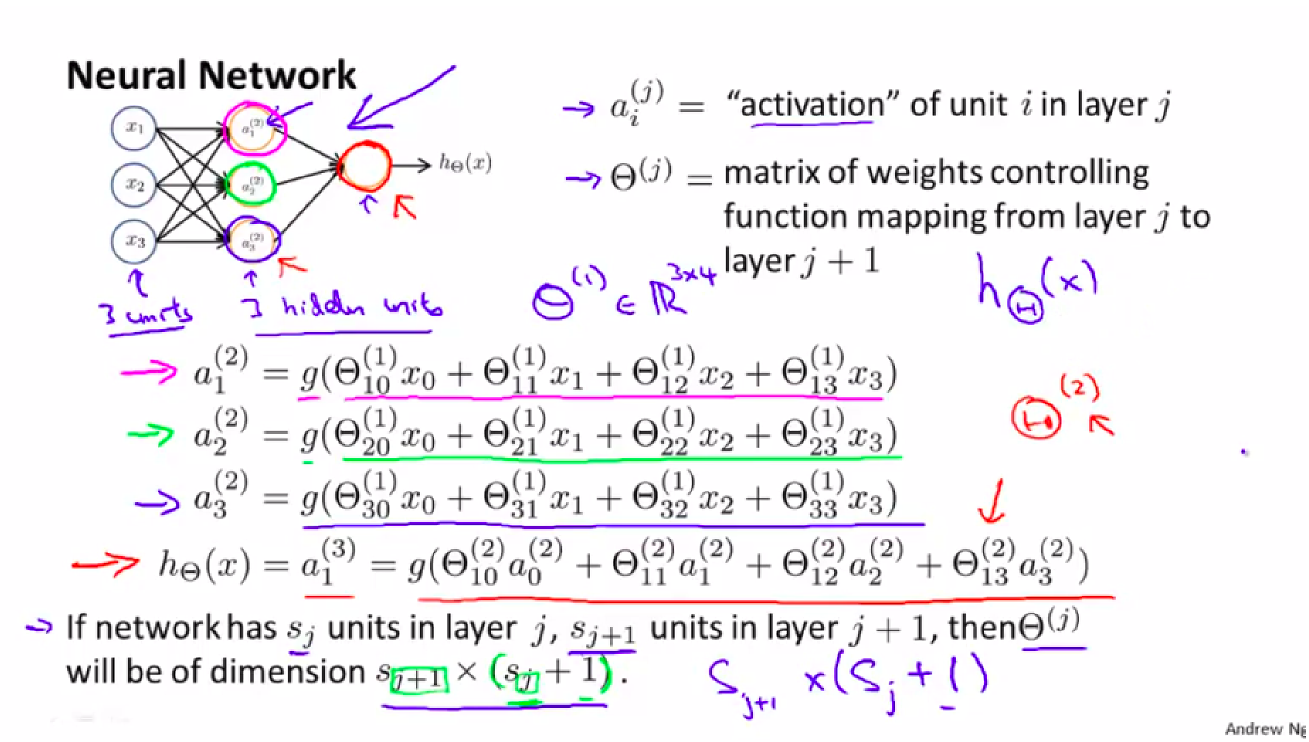
Neuron model: Logistic unit

$x_0, x_1, x_2, x_3 \Rightarrow [wires] \Rightarrow h_\theta(x); x_0 = 1$ bias unit

$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$ $x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

sigmoid (logistic) activation function $g(z) = \frac{1}{1+e^{-z}}$





$\Theta^{(j)}$ = matrix of weights controlling function mapping from layer j to layer $j+1$

$a_i^{(j)}$ = "activation" of unit i in layer j

This is saying that we compute our activation nodes by using a 3×4 matrix of parameters.

$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$

$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$

$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

$h_\Theta(x) = a_1^{(3)} = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$

Example: If layer 1 has 2 input nodes and layer 2 has 4 activation nodes. Dimension of $\Theta^{(1)}$ is going to be 4×3 where $s_j = 2$ and $s_{j+1} = 4$, so $s_{j+1} \times (s_j + 1) = 4 \times 3$

Forward Propagation: Vectorized implementation

$a_1^{(2)} = g(z_1^{(2)})$

$a_2^{(2)} = g(z_2^{(2)})$

$a_3^{(2)} = g(z_3^{(2)})$

$$z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$z^{(2)} = \Theta^{(1)} a^{(1)}$

$a^{(2)} = g(z^{(2)})$

Add $a_0^{(2)} = 1$

$z^{(3)} = \Theta^{(2)} a^{(2)}$

$h_\Theta(x) = a^{(3)} = g(z^{(3)})$

Example

$a(2) =?$

$z(2) = \Theta(1)a(1); a(2) = g(z(2))$

Summary

$z^{(j)} = \Theta^{(j-1)} a^{(j-1)}$

$a^{(j)} = g^{(z(j))}$

$h\Theta^{(x)} = a^{(j+1)} = g(z^{(j+1)})$

Applications

- $AND; x_1, x_2 \in 0, 1; y = x_1 \text{ AND } x_2$

  Graph: $\begin{bmatrix} x_0 \\ x_1 \\ x_3 \end{bmatrix} \rightarrow g(z^{(2)}) \rightarrow h_\Theta(x)$

$\Theta^{(1)} = [-30 \quad 20 \quad 20]$

$h_\Theta(x) = g(\Theta^{(1)}_{10} + \Theta^{(1)}_{11} + \Theta^{(1)}_{12}) = g(-30 + 20x_1 + 20x_2)$

$g(z) = \_\_/\overline{\quad}$

x_1	x_2	y
0	0	g(-30) = 0
0	1	g(-10) = 0
1	0	g(-10) = 0
1	1	g(10) = 1

$AND \Rightarrow \Theta^{(1)} = [-30 \quad 20 \quad 20]$

$OR \Rightarrow \Theta^{(1)} = [-10 \quad 20 \quad 20]$

$NOT x_1 \text{ AND } NOT x_2 \Rightarrow \Theta^{(1)} = [10 \quad -20 \quad -20]$

$NOR \Rightarrow \Theta^{(1)} = [10 \quad -20 \quad -20]$

$XNOR$

Graph: $\begin{bmatrix} x_0 \\ x_1 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \end{bmatrix} \rightarrow [a^{(3)}] \rightarrow h_\Theta(x)$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

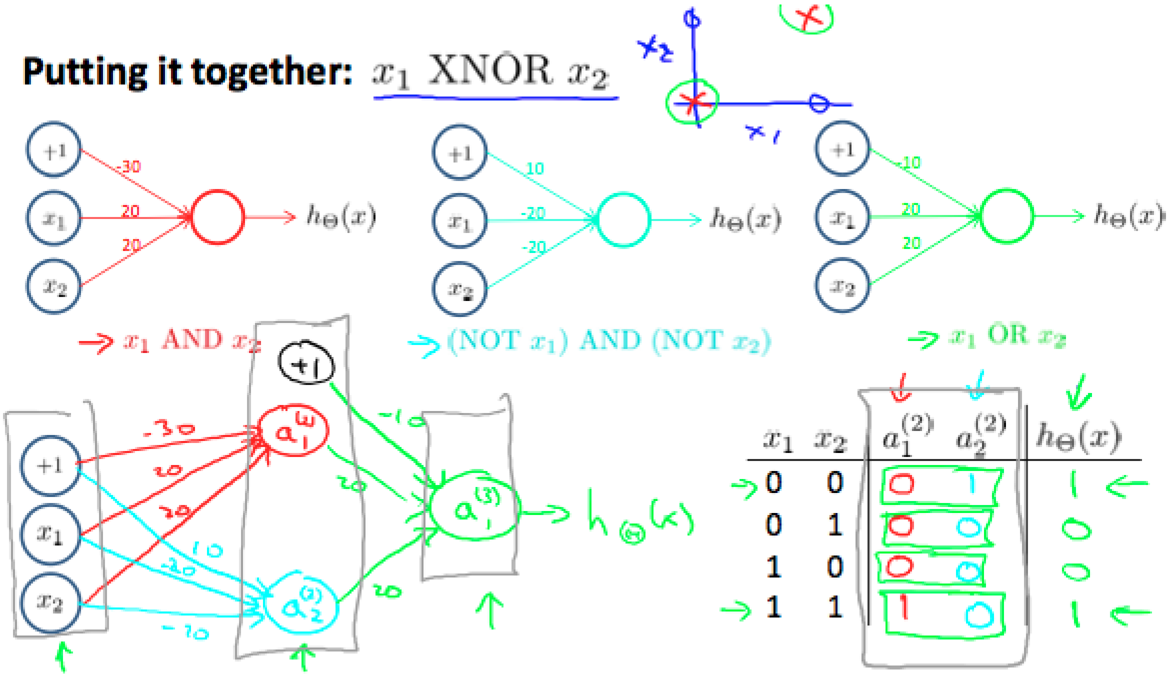
$\Theta^{(2)} = [-10 \quad 20 \quad 20]$

Values for all our nodes:

$a^{(2)} = g(\Theta(1).\, x)$

$a^{(3)} = g(\Theta(2).\, a^{(2)})$

$h_\Theta(x) = a^{(3)}$



Multiclass Classification

To classify data into multiple classes, we let our hypothesis function return a vector of values.

If it is class C1 return $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

We can define our set of resulting classes as y. Each $y^{(i)}$ represents a different image corresponding to either a car, pedestrian, truck, or motorcycle.

The inner layers, each provide us with some new information which leads to our final hypothesis function. The setup looks like:

$$\begin{bmatrix} x_0 \\ .. \\ x_n \end{bmatrix} \rightarrow \begin{bmatrix} a_1^2 \\ a_0^2 \\ .. \end{bmatrix} \rightarrow .. \rightarrow \begin{bmatrix} h_\Theta(x)_1 \\ .. \\ h_\Theta(x)_4 \end{bmatrix}$$