Homework 3: Due April 19

Entanglement and Tensor Networks, Spring 2016, Prof. White

1. Use your Julia program that diagonalizes a Heisenberg spin chain (using $J = 1$) using a sparse H and Lanczos (but not restricting to the Sz=0 subspace) to get the ground state for $N = 16$. Do the Schmidt decomposion on the ground state (i.e. do the SVD) for a split down the middle. Form the approximate ground state that comes from truncating the Schmidt decomposition, keeping only m states, for m ranging from 1 up to $2^{N/2}$. After you make the approximation to the matrices, you can multiply them back together to get a vector like the original ground state vector. For each m, calculate (1) the truncation error, namely the sum of discarded squared singular values

$$\delta_m = \sum_{j=m+1}^{2^{N/2}} \lambda_j^2;$$

(2) the error in the ground state squared, i.e. vecnorm(psi-psiapprox)^2; (3) the error in the ground state energy from using the approximate ground state, using

$$E = \langle\psi| H |\psi\rangle / \langle\psi| \psi\rangle.$$

Make a plot of ΔE_m versus δ_m; you should find a roughly linear relationship. This plot is widely used in DMRG to extrapolate the energy to $m \to \infty$.

2. Modify the code of problem 1 to apply an edge magnetic term $0.1S^z$ to the first site. Get the ground state, and from it form the approximate reduced density matrix ρ, truncated to m states, for the left half of the system. Using $\langle A\rangle = tr\{\rho A\}$, measure $\langle S_1^z\rangle$. Find the exact value using the original ground state, and plot the error versus δ_m.

3. Take your ground state of problem 2, and find its MPS representation by repeatly SVD'ing it to break it up to site-tensors, with a bond dimension $m = 10$. Within the MPS representation, contracting tensors left to right, calculate the norm of the state. Similarly, measure $\langle S_1^z\rangle$ with this type of contractions.