Homework 5: Due May 3

Entanglement and Tensor Networks, Spring 2016, Prof. White

1. **TEBD Code Using ITensor**

In this problem you will take a partial TEBD code based on ITensor and fill in some missing pieces to make it produce the ground state of the Heisenberg chain and measure its energy. Download HW5_tebd.cc and HW5_Makefile from the class homepage.

(a) Download ITensor from http://itensor.org/ and make sure you have the version 2.0.5 or later. (If you cloned ITensor using git, doing "git pull" in the ITensor folder and recompiling should be sufficient.) Edit the Makefile `LIBRARY_DIR` variable to point to where you installed ITensor. Make sure you can compile and run the code before continuing.

(b) Implement the `applyGate` function (line 60), which should apply the ITensor `gate` to the ITensor `phi`. First use the "`*`" operator to contract the two ITensors. It's a good idea to print the result using `Print(phi);` to see what prime levels the indices now have. Next restore the prime levels of `phi`'s indices to their original values. Finally, normalize `phi` by dividing by the number `norm(phi)`. One indication things are now working is that the printout of the bond dimension should now report values greater than 1.

(c) Add code to the loops over `b` (lines 101 and 113) which measures the energy of the Hamiltonian term on that bond $\langle\Psi|\hat{H}_b|\Psi\rangle = \langle\Psi|\vec{S}_b \cdot \vec{S}_{b+1}|\Psi\rangle$. You can obtain the operator $\hat{H}_b = \vec{S}_b \cdot \vec{S}_{b+1}$ from the vector `H` by writing

```
auto Hb = H.at(b);
```

Compute the expectation value of this operator by contracting it on both sides with `phi`. You will need to increase the prime levels of the site indices of one copy of `phi` so that they match the corresponding indices of `Hb`. (Recall that the site indices have an index type of `Site`.) Call `.real()` on the resulting scalar ITensor to extract its value as a real number which you can add to the total energy "`Energy`".

(d) Use your code to get the most accurate energy you can for two chains of lengths N_1 and $N_2 > N_1$. Subtract the *total* energies of the shorter chain from the longer chain and divide by $(N_2 - N_1)$ to obtain an estimate of the energy per site of the infinite system. How close can you get to the exact result $E/N = (\frac{1}{4} - \ln 2)$?

2. In Julia, using either an exact diagonalization routine, or your TEBD code, split a 16 site ground state into the product of two 8-site tensors. Consider the Hamiltonian in split form H_L, H_R, $S^z_8 S^z_9$, etc., where H_L lives on sites 1-8, H_R on sites 9-16. Draw the tensor diagrams showing how to form $\langle\psi|H|\psi\rangle$ using the split wavefunction and operators. Implement the diagrams in Julia and evaluate the energy, and check that you got the right answer with another energy evaluation.