تمرین بخش ج پروژه: (الهی سبحانی - علی مرجانی)

$$|2,2\rangle = |1,+1\rangle \otimes |1,+1\rangle = |++++\rangle$$

$$J_-|2,2\rangle = (J_{-12} \otimes \mathbb{1} + \mathbb{1} \otimes J_{-34})(|1,+1\rangle \otimes |1,+1\rangle)$$

$$\hbar \underbrace{\sqrt{(2+2)(2-2+1)}}_{2}\,|2,1\rangle = \underbrace{J_-|1,+1\rangle}_{\hbar\sqrt{2}|1,0\rangle} \otimes |1,+1\rangle + |1,+1\rangle \otimes \underbrace{J_-|1,+1\rangle}_{\hbar\sqrt{2}|1,0\rangle}$$

$$\Rightarrow \hbar 2\,|2,1\rangle = \hbar\cancel{\sqrt{2}}\left\{\frac{1}{\cancel{\sqrt{2}}}(|+-\rangle + |-+\rangle)\otimes|++\rangle + |++\rangle \otimes \frac{1}{\cancel{\sqrt{2}}}(|+-\rangle \otimes |-+\rangle)\right\}$$

$$\Rightarrow |2,1\rangle = \frac{1}{2}\left\{|+-++\rangle + |-+++\rangle + |+++-\rangle + |++-+\rangle\right\} = \frac{\sqrt{2}}{2}\left(|1,0\rangle \otimes |1,+1\rangle + |1,+1\rangle \otimes |1,0\rangle\right)$$

$$J_-|2,1\rangle = (J_{-12} \otimes \mathbb{1} + \mathbb{1} \otimes J_{-34})(|1,0\rangle \otimes |1,+1\rangle + |1,+1\rangle \otimes |1,0\rangle)$$

$$\hbar \underbrace{\sqrt{(2+1)(2-1+1)}}_{\sqrt{6}}\,|2,0\rangle = \frac{\cancel{\sqrt{2}}}{\cancel{2}}\left\{\underbrace{J_-|1,0\rangle}_{\hbar\cancel{\sqrt{2}}|1,-1\rangle} \otimes |1,+1\rangle + |1,0\rangle \otimes \underbrace{J_-|1,+1\rangle}_{\hbar\cancel{\sqrt{2}}|1,0\rangle} + \underbrace{J_-|1,+1\rangle}_{\hbar\cancel{\sqrt{2}}|1,0\rangle} \otimes |1,0\rangle + |1,+1\rangle \otimes \underbrace{J_-|1,0\rangle}_{\hbar\cancel{\sqrt{2}}|1,-1\rangle}\right\}$$

$$\Rightarrow \hbar\sqrt{6}\,|2,0\rangle = \hbar\left\{\underbrace{|1,-1\rangle \otimes |1,+1\rangle}_{|--\rangle \otimes |++\rangle} + \underbrace{|1,0\rangle \otimes |1,0\rangle}_{*} + |1,0\rangle \otimes |1,0\rangle + |1,+1\rangle \otimes |1,-1\rangle\right\}$$

$$*:\ |1,0\rangle \otimes |1,0\rangle = \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle) = \frac{1}{2}\left(|+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle\right)$$

$$\Rightarrow \hbar\sqrt{6}\,|2,0\rangle = \hbar\left\{|--++\rangle + \frac{1}{2}\left(|+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle\right) + \frac{1}{2}\left(|+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle\right) + |++--\rangle\right\}$$

$$\Rightarrow \cancel{\hbar}\sqrt{6}\,|2,0\rangle = \cancel{\hbar}\left\{|--++\rangle + |+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle + |++--\rangle\right\}$$

$$|2,0\rangle = \frac{1}{\sqrt{6}}\left\{|--++\rangle + |+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle + |++--\rangle\right\}$$

$$= \frac{1}{\sqrt{6}}\left\{|1,-1\rangle\otimes|1,+1\rangle + |1,0\rangle\otimes|1,0\rangle + |1,0\rangle\otimes|1,0\rangle + |1,+1\rangle\otimes|1,-1\rangle\right\}$$

$$|2,-2\rangle = |1,-1\rangle\otimes|1,-1\rangle = |----\rangle$$

$$J_+|2,-2\rangle = (J_{+12}\otimes\mathbb{1} + \mathbb{1}\otimes J_{+12})(|1,-1\rangle\otimes|1,-1\rangle) = (J_+|1,-1\rangle\otimes|1,-1\rangle + |1,-1\rangle\otimes J_+|1,-1\rangle)$$

$$\hbar\sqrt{\underbrace{(2+2)(2-2+1)}_{2}}\;|2,-1\rangle = \hbar\sqrt{\underbrace{(1+1)(1-1+1)}_{\sqrt{2}}}\;|1,0\rangle\otimes|1,-1\rangle + |1,-1\rangle\otimes\hbar\sqrt{\underbrace{(1+1)(1-1+1)}_{\sqrt{2}}}\;|1,0\rangle$$

$$\Rightarrow \hbar 2|2,-1\rangle = \hbar\cancel{\sqrt{2}}\left\{\frac{1}{\cancel{\sqrt{2}}}(|+-\rangle + |-+\rangle)\otimes|--\rangle + |--\rangle\otimes\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle)\right\}$$

$$\Rightarrow |2,-1\rangle = \frac{1}{2}\left\{|+---\rangle + |-+--\rangle + |--+-\rangle + |---+\rangle\right\} = \frac{\sqrt{2}}{2}\left\{|1,0\rangle\otimes|1,-1\rangle + |1,-1\rangle\otimes|1,0\rangle\right\}$$

* we find this kets

this ket for: two particle : 1×0 ($\leftarrow$ triplet, $\rightarrow$ singlet)

two particle: 1×0

$1\otimes 1 = 2\oplus 1\oplus 0$

$1\otimes 0 = 1$

$0\otimes 1 = 1$

$0\otimes 0 = 0$

$$\Rightarrow \begin{cases} S_{tot}=2 \Rightarrow 5 \text{ state} \\ S_{tot}=1 \Rightarrow 9 \text{ state} \\ S_{tot}=0 \Rightarrow 2 \text{ state}\end{cases}$$

Now we calculate $1\otimes 0$:

$$|1,+1\rangle = |1,+1\rangle\otimes|0,0\rangle = \frac{1}{\sqrt{2}}(|+++-\rangle - |++-+\rangle)$$

$$|1,-1\rangle = |1,-1\rangle\otimes|0,0\rangle = \frac{1}{\sqrt{2}}(|--+-\rangle - |---+\rangle)$$

$$|1,0\rangle = |1,0\rangle\otimes|0,0\rangle = \frac{1}{2}(|+-+-\rangle + |-++-\rangle - |+--+\rangle - |-+-+\rangle)$$

Now we calculate $0\otimes 1$:

$$|1,+1\rangle = |0,0\rangle\otimes|1,+1\rangle = \frac{1}{\sqrt{2}}(|+-++\rangle - |-+++\rangle)$$

$$|1,-1\rangle = |0,0\rangle\otimes|1,-1\rangle = \frac{1}{\sqrt{2}}(|+---\rangle - |-+--\rangle)$$

$$|1,0\rangle = |0,0\rangle\otimes|1,0\rangle = \frac{1}{2}(|+-+-\rangle - |-++-\rangle + |+--+\rangle - |-+-+\rangle)$$

for $0\otimes 0$ we have:

$$|0,0\rangle = |0,0\rangle\otimes|0,0\rangle = \frac{1}{2}\left(|+-+-\rangle - |-++-\rangle - |+--+\rangle + |-+-+\rangle\right)$$

we have this Hamiltonian for finding eighenvalues:

$$H=\sum_{i} \vec{S}_i\cdot\vec{S}_j = \vec{S}_1\cdot\vec{S}_2 + \vec{S}_2\cdot\vec{S}_3 + \vec{S}_3\cdot\vec{S}_4 + \vec{S}_4\cdot\vec{S}_1$$

* $\vec{S}_1\cdot\vec{S}_2$ and $\vec{S}_3\vec{S}_4 = \frac{1}{2}(S_{12}^2 - S_1^2 - S_2^2$ or $\frac{1}{2}(S_{34}^2 - S_3^2 - S_4^2)$

proof: $S_{12}^2 = (S_1+S_2)^2 = S_1^2 + S_2^2 + 2S_1\cdot S_2$

$$\Rightarrow S_1\cdot S_2 = \frac{1}{2}(S_{12}^2 - S_1^2 - S_2^2)$$

* $\vec{S}_2\cdot\vec{S}_3$ and $\vec{S}_4\cdot\vec{S}_1 = S_{2z}S_{3z} + \frac{1}{2}\{(S_2^+S_3^- + S_2^-S_3^+)\}$ or $S_{4z}S_{1z} + \frac{1}{2}\{S_4^+S_1^- + S_4^-S_1^+\}$

$$\Rightarrow H = \frac{1}{2}(S_{12}^2 - S_1^2 - S_2^2) + S_{2z}S_{3z} + \frac{1}{2}\{S_2^+S_3^- + S_2^-S_3^+\} + \frac{1}{2}(S_{34}^2 - S_3^2 - S_4^2)$$
$$+ S_{4z}S_{1z} + \frac{1}{2}\{S_4^+S_1^- + S_4^-S_1^+\}$$

برای یافتن کت‌های مربوط به حالت $\underbrace{1\otimes 1}_{(\text{triplet}\otimes\text{triplet})}$ که $S_{tot}=1$ را دارند از روش زیر استفاده می‌کنیم:

$$|\chi^{(m)}\rangle = |1,m\rangle = C_1\left(|1,m_1\rangle_{12}\otimes|1,m_2\rangle_{34}\right) + C_2\left(|1,m_2\rangle_{12}\otimes|1,m_1\rangle_{34}\right)$$

* $m_1=0$ & $m_2=1$

حال برای یافتن کت $|1,+1\rangle$ داریم:

$$|\chi^{(+1)}\rangle = C_1\left(|1,0\rangle\otimes|1,+1\rangle\right) + C_2\left(|1,+1\rangle\otimes|1,0\rangle\right)$$

باید برای یافتن ضرایب C_1 و C_2 دو شرط را برقرار کنیم بهنجارش و تعامد:

$$\langle\chi^{(+1)}|\chi^{(+1)}\rangle = 1 \Rightarrow |C_1|^2 + |C_2|^2 = 1 \qquad (\mathrm{I})$$

$$\langle 2,1|\chi^{(+1)}\rangle = 0 \Rightarrow \frac{1}{\sqrt{2}}\left(\langle 1,0|\otimes\langle 1,+1| + \langle 1,+1|\otimes\langle 1,0|\right) C_1\left(|1,0\rangle\otimes|1,+1\rangle\right)$$
$$+ C_2\left(|1,+1\rangle\otimes|1,0\rangle\right) = 0$$

$$\Rightarrow \frac{1}{\sqrt{2}}(C_1 + C_2) = 0 \Rightarrow C_1 = -C_2 \qquad (\mathrm{II})$$

با استفاده از نتایج I و II پاسخ که داریم:

$C_1 = \pm\frac{1}{\sqrt{2}}$ and $C_2 = \mp\frac{1}{\sqrt{2}}$

در نتیجه داریم:

$$|1,+1\rangle = \frac{1}{\sqrt{2}}\left\{\left(|1,0\rangle \otimes |1,+1\rangle\right) - \left(|1,+1\rangle \otimes |1,0\rangle\right)\right\} = \frac{1}{2}\left\{|+-++\rangle + |-+++\rangle - |+++-\rangle - |++-+\rangle\right\}$$

حال با زدن J_- روی کت $|1,+1\rangle$ داریم:

$$J_-|1,+1\rangle = \left(J_{-12} \otimes \mathbb{1} + \mathbb{1} \otimes J_{-34}\right) \frac{1}{\sqrt{2}}\left\{|1,0\rangle \otimes |1,+1\rangle - |1,+1\rangle \otimes |1,0\rangle\right\}$$

این دو ترم با یکدیگر خط می‌خورند

$$\Rightarrow \cancel{\hbar\sqrt{2}}\,|1,0\rangle = \frac{1}{\sqrt{2}}\left\{\cancel{\hbar\sqrt{2}}\,|1,-1\rangle \otimes |1,+1\rangle + \cancel{|1,0\rangle \otimes \hbar\sqrt{2}\,|1,0\rangle} - \cancel{\hbar\sqrt{2}\,|1,0\rangle \otimes |1,0\rangle} - |1,+1\rangle \otimes \cancel{\hbar\sqrt{2}}\,|1,-1\rangle\right\}$$

$$\Rightarrow |1,0\rangle = \frac{1}{\sqrt{2}}\left\{|1,-1\rangle \otimes |1,+1\rangle - |1,+1\rangle \otimes |1,-1\rangle\right\} = \frac{1}{\sqrt{2}}\left\{|--++\rangle - |++--\rangle\right\}$$

حال با زدن J_- روی کت $|1,0\rangle$ داریم:

$$J_-|1,0\rangle = \left(J_{-12} \otimes \mathbb{1} + \mathbb{1} \otimes J_{-34}\right) \frac{1}{\sqrt{2}}\left\{|1,-1\rangle \otimes |1,+1\rangle - |1,+1\rangle \otimes |1,-1\rangle\right\}$$

$$\Rightarrow \cancel{\hbar\sqrt{2}}\,|1,-1\rangle = \frac{1}{\sqrt{2}}\left\{-\cancel{\hbar\sqrt{2}}\,|1,0\rangle \otimes |1,-1\rangle + |1,-1\rangle \otimes \cancel{\hbar\sqrt{2}}\,|1,0\rangle\right\}$$

$$\Rightarrow |1,-1\rangle = \frac{1}{\sqrt{2}}\left\{|1,-1\rangle \otimes |1,0\rangle - |1,0\rangle \otimes |1,-1\rangle\right\} = \frac{1}{2}\left\{|--+-\rangle + |---+\rangle - |+---\rangle - |-+--\rangle\right\}$$

حال برای یافتن کت $|0,0\rangle$ در حالت $1 \otimes 1$ داریم:

$$|0,0\rangle = \sum_{m_1,m_2} C^{0,0}_{1m_1;1m_2}\, |1,m_1\rangle_{12} \otimes |1,m_2\rangle_{34}$$

$$= C_1\left(|1,+1\rangle \otimes |1,-1\rangle\right) + C_2\left(|1,0\rangle \otimes |1,0\rangle\right) + C_3\left(|1,-1\rangle \otimes |1,+1\rangle\right)$$

حال از یک شرط بهنجارش و دو شرط تعامد کت‌ها، ضرایب C_1 و C_2 و C_3 شان مشخص می‌شود.

$$\langle 0,0|0,0\rangle = |C_1|^2 + |C_2|^2 + |C_3|^2 = 1 \quad (\mathrm{I})$$

$$\langle 1,0|0,0\rangle = \frac{1}{\sqrt{2}}\left\{\langle 1,-1| \otimes \langle 1,+1| - \langle 1,+1| \otimes \langle 1,-1|\right\}|0,0\rangle = \frac{1}{\sqrt{2}}\left(C_3 - C_1\right) = 0 \quad (\mathrm{II})$$

$$\langle 2,0|0,0\rangle = \frac{1}{\sqrt{6}}\left\{\langle 1,-1| \otimes \langle 1,+1| + 2\langle 1,0| \otimes \langle 1,0| + \langle 1,+1| \otimes \langle 1,-1|\right\}|0,0\rangle = \frac{1}{\sqrt{6}}\left(C_3 + 2C_2 + C_1\right) = 0 \quad (\mathrm{III})$$

(3) از رابطه II داریم

$$c_1 = c_3$$

با جایگذاری ۱ در رابطه‌ی III داریم:

$$c_3 + 2c_2 + c_3 = 0 \Rightarrow c_3 = -c_2$$

با جایگذاری ۱ این رابطه در رابطه‌ی I داریم:

$$|c_1|^2 + |-c_1|^2 + |c_1|^2 = 3|c_1|^2 = 1$$

$$\Rightarrow c_1 = \pm\frac{1}{\sqrt{3}} \quad \& \quad c_2 = \mp\frac{1}{\sqrt{3}} \quad \& \quad c_3 = \pm\frac{1}{\sqrt{3}}$$

حال که ضرایب را یافتیم کت $|0,0\rangle$ خواهد شد:

$$|0,0\rangle = \frac{1}{\sqrt{3}}\left\{|1,+1\rangle\otimes|1,-1\rangle - |1,0\rangle\otimes|1,0\rangle + |1,-1\rangle\otimes|1,+1\rangle\right\}$$

$$= \frac{1}{\sqrt{3}}\left\{|++--\rangle - \frac{1}{2}\left(|+-+-\rangle + |+--+\rangle + |-++-\rangle + |-+-+\rangle\right) + |--++\rangle\right\}$$

برای زدن هامیلتونی روی کت ها از دو نوتیشن هامیلتونی استفاده کردیم که یک نوتیشن را در برگه‌ی

(2) توضیح دادیم و نوتیشن دوم را نیز اینجا بازگو کنیم:

$$H = \sum_{\substack{i=1\\ j=1}}^{4} S_i \cdot S_j = \sum_{i=1}^{4} S_{z(i)} \cdot S_{z(i+1)} + \sum_{i=1}^{4} \frac{1}{2}\left(S_i^+ S_{(i+1)}^- + S_i^- S_{(i+1)}^+\right)$$

$$H = \left(S_z^1 S_z^2 + S_z^2 S_z^3 + S_z^3 S_z^4 + S_z^4 S_z^1\right)$$
$$+\frac{1}{2}\left\{\left(S_1^- S_2^+ + S_1^+ S_2^-\right) + \left(S_2^- S_3^+ + S_2^+ S_3^-\right) + \left(S_3^- S_4^+ + S_3^+ S_4^-\right) + \left(S_4^- S_1^+ + S_1^+ S_4^-\right)\right\}$$

حال که دو نوتیشن از هامیلتونی را داریم شروع کنیم هامیلتونی را روی کت ها

اثر دهیم تا نتیجه آن را بیابیم:

* روی کت های $T\otimes T$ هامیلتونی را در شکل دوم خود اثر دهیم.

* روی کت های $T\otimes S$ ، $S\otimes T$ ، $S\otimes S$ هامیلتونی را در شکل اول خود که در برگه‌ی 2 گفتیم بسط اثر دهیم.

$$H|1,0\rangle_{(TOT)} = H\left(\frac{1}{\sqrt{2}}\left(|--++\rangle - |++--\rangle\right)\right)$$

$H|1,0\rangle$ Terms:

$S_1^z S_2^z : \left(\frac{1}{4}\right)|--++\rangle - \left(\frac{1}{4}\right)|++--\rangle$; $\frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right) = \frac{1}{2}(0) = 0$

$S_2^z S_3^z : \left(-\frac{1}{4}\right)|--++\rangle - \left(-\frac{1}{4}\right)|++--\rangle$; $\frac{1}{2}\left(S_2^+ S_3^- + S_2^- S_3^+\right) = \frac{1}{2}\left(|-+-+\rangle - |+-+-\rangle\right)$

$S_3^z S_4^z : \left(\frac{1}{4}\right)|--++\rangle - \left(\frac{1}{4}\right)|++--\rangle$; $\frac{1}{2}\left(S_3^+ S_4^- + \frac{1}{2} S_3^- S_4^+\right) = 0$

$S_4^z S_1^z : \left(-\frac{1}{4}\right)|--++\rangle - \left(-\frac{1}{4}\right)|++--\rangle$; $\frac{1}{2}\left(S_4^+ S_1^- + \frac{1}{2} S_4^- S_1^+\right) = \frac{1}{2}\left(|+-+-\rangle - |-+-+\rangle\right)$

$$\Longrightarrow \boxed{H|1,0\rangle = 0|1,0\rangle}$$

$$H|1,-1\rangle_{(TOT)} = H\left(\frac{1}{\sqrt{2}}\left(|--+-\rangle + |---+\rangle - |+---\rangle - |-+--\rangle\right)\right)$$

$H|1,-1\rangle$ terms:

$S_1^z S_2^z : \left(\frac{1}{4}\right)|--+-\rangle + \left(\frac{1}{4}\right)|---+\rangle - \left(-\frac{1}{4}\right)|+---\rangle - \left(-\frac{1}{4}\right)|-+--\rangle$

$S_2^z S_3^z : \left(-\frac{1}{4}\right)|--+-\rangle + \left(\frac{1}{4}\right)|---+\rangle - \left(\frac{1}{4}\right)|+---\rangle - \left(-\frac{1}{4}\right)|-+--\rangle$

$S_3^z S_4^z : \left(-\frac{1}{4}\right)|--+-\rangle + \left(-\frac{1}{4}\right)|---+\rangle - \left(\frac{1}{4}\right)|+---\rangle - \left(\frac{1}{4}\right)|-+--\rangle$

$S_4^z S_1^z : \left(\frac{1}{4}\right)|--+-\rangle + \left(-\frac{1}{4}\right)|---+\rangle - \left(-\frac{1}{4}\right)|+---\rangle - \left(\frac{1}{4}\right)|-+--\rangle$

$\frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right) = \frac{1}{2}\left\{-|-+--\rangle - |+---\rangle\right\}$

$\frac{1}{2}\left(S_2^+ S_3^- + S_2^- S_3^+\right) = \frac{1}{2}\left\{|-+--\rangle - |--+-\rangle\right\}$

$\frac{1}{2}\left(S_3^+ S_4^- + S_3^- S_4^+\right) = \frac{1}{2}\left\{|---+\rangle + |--+-\rangle\right\}$

$\frac{1}{2}\left(S_4^+ S_1^- + S_4^- S_1^+\right) = \frac{1}{2}\left\{|+---\rangle - |---+\rangle\right\}$

$$\Longrightarrow \boxed{H|1,-1\rangle = 0|1,-1\rangle}$$

$$H|0,0\rangle = \frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|0,0\rangle$$
(S⊗S)

$$+\left(\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|0,0\rangle$$

$$+\frac{1}{4}\left\{|++--\rangle+|--++\rangle\right\}$$

$$+\frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|0,0\rangle$$

$$+\left(\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|0,0\rangle$$

$$+\frac{1}{4}\left\{|--++\rangle+|++--\rangle\right\}$$

$$=-\frac{3}{8}\underbrace{|+-+-\rangle}_{|\varphi_1\rangle}+\frac{3}{8}\underbrace{|-++-\rangle}_{|\varphi_2\rangle}+\frac{3}{8}\underbrace{|+--+\rangle}_{|\varphi_3\rangle}-\frac{3}{8}\underbrace{|-+-+\rangle}_{|\varphi_4\rangle}$$

$$-\frac{1}{8}\left(|\varphi_1\rangle+|\varphi_2\rangle+|\varphi_3\rangle+|\varphi_4\rangle\right)+\frac{1}{4}|++--\rangle+\frac{1}{4}|--++\rangle$$

$$-\frac{3}{8}|\varphi_1\rangle+\frac{3}{8}|\varphi_2\rangle+\frac{3}{8}|\varphi_3\rangle-\frac{3}{8}|\varphi_4\rangle$$

$$-\frac{1}{8}\left(|\varphi_1\rangle+|\varphi_2\rangle+|\varphi_3\rangle+|\varphi_4\rangle\right)+\frac{1}{4}|--++\rangle+\frac{1}{4}|++--\rangle$$

(نتیجه) $H|0,0\rangle = -|+-+-\rangle+\frac{1}{2}|+--+\rangle+\frac{1}{2}|-++-\rangle-|-+-+\rangle+\frac{1}{2}|++--\rangle+\frac{1}{2}|--++\rangle$

$$H|1,0\rangle = H\left(|0,0\rangle\otimes|1,0\rangle\right)=\frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|1,0\rangle$$
(S⊗T)

$$+\left(\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|1,0\rangle$$

$$+\frac{1}{4}\left\{|++--\rangle-|--++\rangle\right\}$$

$$+\frac{1}{2}\left(2-\frac{3}{4}-\frac{3}{4}\right)|1,0\rangle$$

$$+\left(\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|1,0\rangle$$

$$+\frac{1}{4}\left\{|--++\rangle-|++--\rangle\right\}$$

$$=-\frac{3}{8}\left(\underbrace{|+-+-\rangle}_{|\varphi_1\rangle}-\underbrace{|-++-\rangle}_{|\varphi_2\rangle}+\underbrace{|+--+\rangle}_{|\varphi_3\rangle}-\underbrace{|-+-+\rangle}_{|\varphi_4\rangle}\right)$$

$$+\left(+\frac{1}{8}\right)\left(-|\varphi_1\rangle-|\varphi_2\rangle+|\varphi_3\rangle+|\varphi_4\rangle\right)+\frac{1}{4}\left\{|++--\rangle-|--++\rangle\right\}$$

$$+\frac{1}{8}\left(|\varphi_1\rangle-|\varphi_2\rangle+|\varphi_3\rangle-|\varphi_4\rangle\right)$$

$$+\left(\frac{1}{8}\right)\left(-|\varphi_1\rangle - |\varphi_2\rangle + |\varphi_3\rangle + |\varphi_4\rangle\right) + \frac{1}{4}\left\{|--++\rangle - |++--\rangle\right\}$$

درنتیجه: $H|1,0\rangle = -\frac{1}{2}|+-+-\rangle + \frac{1}{2}|-+-+\rangle$

$$H|1,-1\rangle = \underset{(S\otimes T)}{H}\left(|0,0\rangle \otimes |1,-1\rangle\right) = \frac{1}{2}\left(0 - \frac{3}{4} - \frac{3}{4}\right)|1,-1\rangle$$

$$+\left((-\tfrac{1}{2})(-\tfrac{1}{2}) + (\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,-1\rangle$$

$$-\frac{1}{2\sqrt{2}}|--+-\rangle$$

$$+\left((\tfrac{1}{2})(-\tfrac{1}{2}) + (-\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,-1\rangle$$

$$+\frac{1}{2\sqrt{2}}|---+\rangle$$

$$+\frac{1}{2}\left(2 - \frac{3}{4} - \frac{3}{4}\right)|1,-1\rangle$$

$$= -\frac{3}{4\sqrt{2}}\left(|+---\rangle - |-+--\rangle\right)$$

$$+\frac{1}{4\sqrt{2}}\left(|+---\rangle + |-+--\rangle\right) - \frac{1}{2\sqrt{2}}|--+-\rangle$$

$$+\frac{1}{4\sqrt{2}}\left(|+---\rangle - |-+--\rangle\right)$$

$$+\frac{1}{4\sqrt{2}}\left(-|+---\rangle - |-+--\rangle\right) + \frac{1}{2\sqrt{2}}|---+\rangle$$

درنتیجه: $H|1,-1\rangle = -\frac{1}{\sqrt{8}}|+---\rangle + \frac{1}{\sqrt{8}}|-+--\rangle - \frac{1}{\sqrt{8}}|--+-\rangle + \frac{1}{\sqrt{8}}|---+\rangle$

$$H|1,+1\rangle = \underset{(S\otimes T)}{H}\left(|0,0\rangle \otimes |1,+1\rangle\right) = \frac{1}{2}\left(0 - \frac{3}{4} - \frac{3}{4}\right)|1,+1\rangle$$

$$+\left((-\tfrac{1}{2})(+\tfrac{1}{2}) + (\tfrac{1}{2})(\tfrac{1}{2})\right)|1,+1\rangle$$

$$+\frac{1}{2\sqrt{2}}\left\{|++-+\rangle\right\}$$

$$+\frac{1}{2}\left(2 - \frac{3}{4} - \frac{3}{4}\right)|1,+1\rangle$$

$$+\left((\tfrac{1}{2})(\tfrac{1}{2}) + (\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,+1\rangle$$

$$+\frac{1}{2\sqrt{2}}\left\{-|+++-\rangle\right\}$$

$$= -\frac{3}{4\sqrt{2}}\left(|+-++\rangle - |-+++\rangle\right)$$

$$+\frac{1}{4\sqrt{2}}\left(-|+-++\rangle - |-+++\rangle\right) + \frac{1}{2\sqrt{2}}|++-+\rangle$$

$$+\frac{1}{4\sqrt{2}}\left(|+-++\rangle - |-+++\rangle\right)$$

$$+\frac{1}{4\sqrt{2}}\left(|+-++\rangle + |-+++\rangle\right) - \frac{1}{2\sqrt{2}}|+++-\rangle$$

درنتیجه: $H|1,+1\rangle = -\frac{1}{\sqrt{8}}|+-++\rangle + \frac{1}{\sqrt{8}}|-+++\rangle + \frac{1}{\sqrt{8}}|++-+\rangle - \frac{1}{\sqrt{8}}|+++-\rangle$

$$\underset{(T\otimes S)}{H|1,0\rangle} = H(|1,0\rangle \otimes |0,0\rangle) = \frac{1}{2}\left(2-\frac{3}{4}-\frac{3}{4}\right)|1,0\rangle$$

$$+\left((-\tfrac{1}{2})(\tfrac{1}{2}) + (\tfrac{1}{2})(+\tfrac{1}{2}) + (-\tfrac{1}{2})(-\tfrac{1}{2}) + (\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,0\rangle$$

$$+\frac{1}{2}\left\{\frac{1}{2}|++--\rangle - \frac{1}{2}|--++\rangle\right\}$$

$$+\frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|1,0\rangle$$

$$+\left((-\tfrac{1}{2})(+\tfrac{1}{2}) + (-\tfrac{1}{2})(-\tfrac{1}{2}) + (\tfrac{1}{2})(\tfrac{1}{2}) + (\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,0\rangle$$

$$+\frac{1}{2}\left\{\frac{1}{2}|--++\rangle - |++--\rangle\right\}$$

$$= \frac{1}{8}\left(\underbrace{|+-+-\rangle}_{|\varphi_1\rangle} + \underbrace{|-++-\rangle}_{|\varphi_2\rangle} - \underbrace{|+--+\rangle}_{|\varphi_3\rangle} - \underbrace{|-+-+\rangle}_{|\varphi_4\rangle}\right)$$

$$+\frac{1}{8}\left(-|\varphi_1\rangle + |\varphi_2\rangle - |\varphi_3\rangle + |\varphi_4\rangle\right)$$

$$+\frac{1}{4}\left\{|++--\rangle - |--++\rangle\right\}$$

$$-\frac{3}{8}\left(|\varphi_1\rangle + |\varphi_2\rangle - |\varphi_3\rangle - |\varphi_4\rangle\right)$$

$$+\frac{1}{8}\left(-|\varphi_1\rangle + |\varphi_2\rangle - |\varphi_3\rangle + |\varphi_4\rangle\right)$$

$$+\frac{1}{4}\left\{|--++\rangle - |++--\rangle\right\}$$

درنتیجه: $H|1,0\rangle = -\frac{1}{2}|+-+-\rangle + \frac{1}{2}|-+-+\rangle$

$$\underset{(T\otimes S)}{H|1,-1\rangle} = H(|1,-1\rangle \otimes |0,0\rangle) = \frac{1}{2}\left(2-\frac{3}{4}-\frac{3}{4}\right)|1,-1\rangle$$

$$+\left((\tfrac{1}{2})(-\tfrac{1}{2}) + (-\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,-1\rangle$$

$$+\frac{1}{2}\left\{\frac{1}{\sqrt{2}}|-+--\rangle\right\}$$

$$+\frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|1,-1\rangle$$

$$+\left((-\tfrac{1}{2})(-\tfrac{1}{2}) + (\tfrac{1}{2})(-\tfrac{1}{2})\right)|1,-1\rangle$$

$$+\frac{1}{2}\left\{-\frac{1}{\sqrt{2}}|+---\rangle\right\}$$

$$= \frac{1}{4\sqrt{2}}\left(|--+-\rangle - |---+\rangle\right)$$

$$+\left(\frac{1}{4\sqrt{2}}\right)\left(-|--+-\rangle - |---+\rangle\right) + \frac{1}{2\sqrt{2}}|-+--\rangle$$

$$+\left(-\frac{3}{4\sqrt{2}}\right)\left(|--+-\rangle - |---+\rangle\right)$$

$$+\left(\frac{1}{4\sqrt{2}}\right)\left(|--+-\rangle + |---+\rangle\right) - \frac{1}{2\sqrt{2}}|+---\rangle$$

درنتیجه: $H|1,-1\rangle = -\frac{1}{2\sqrt{2}}|--+-\rangle + \frac{1}{2\sqrt{2}}|---+\rangle + \frac{1}{2\sqrt{2}}|-+--\rangle - \frac{1}{2\sqrt{2}}|+---\rangle$

$$H|1,+1\rangle = H\left(|1,+1\rangle \otimes |0,0\rangle\right) = \frac{1}{2}\left(2-\frac{3}{4}-\frac{3}{4}\right)|1,+1\rangle$$
(T⊗S)

$$+\left(\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|1,+1\rangle$$

$$+\frac{1}{2}\left\{\frac{1}{\sqrt{2}}|+-++\rangle\right\}$$

$$+\frac{1}{2}\left(0-\frac{3}{4}-\frac{3}{4}\right)|1,+1\rangle$$

$$+\left(\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right)|1,+1\rangle$$

$$+\frac{1}{2}\left(+\frac{1}{\sqrt{2}}|-+++\rangle\right)$$

$$=\frac{1}{4\sqrt{2}}\left\{|+++-\rangle-|++-+\rangle\right\}$$

$$+\left(\frac{1}{4\sqrt{2}}\right)\left\{|+++-\rangle+|++-+\rangle\right\}-\frac{1}{2\sqrt{2}}|+-++\rangle$$

$$+\left(-\frac{3}{4\sqrt{2}}\right)\left\{|+++-\rangle-|++-+\rangle\right\}$$

$$+\left(\frac{1}{4\sqrt{2}}\right)\left\{-|+++-\rangle-|++-+\rangle\right\}+\frac{1}{2\sqrt{2}}|-+++\rangle$$

در نتیجه: $$H|1,+1\rangle = \frac{-1}{2\sqrt{2}}|+++-\rangle+\frac{1}{2\sqrt{2}}|++-+\rangle-\frac{1}{2\sqrt{2}}|+-++\rangle+\frac{1}{2\sqrt{2}}|-+++\rangle$$

$$H|2,-1\rangle = H\left(\frac{\sqrt{2}}{2}\left\{|1,0\rangle\otimes|1,-1\rangle+|1,-1\rangle\otimes|1,0\rangle\right\}\right)$$
(T⊗T)

*هامیلتونی را به این شکل اثر دهیم

$$=\left\{S_1^zS_2^z+S_2^zS_3^z+S_3^zS_4^z+S_4^zS_1^z\right\}+\frac{1}{2}\left\{(S_1^+S_2^-+S^-S_2^+)+(S_2^+S_3^-+S_2^-S_3^+)+(S_3^+S_4^-+S_3^-S_4^+)+(S_4^+S_1^-+S_4^-S_1^+)\right\}\left(\frac{\sqrt{2}}{2}\left\{|1,0\rangle\otimes|1,-1\rangle+|1,-1\rangle\otimes|1,0\rangle\right\}\right)$$

$$=\left(\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(+\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right)|2,-1\rangle \quad \text{ترم} : S_1^zS_2^z$$

$$+\left(\frac{1}{4}-\frac{1}{4}-\frac{1}{4}+\frac{1}{4}\right)|2,-1\rangle \quad \text{ترم} : S_2^zS_3^z$$

$$+\left(\frac{1}{4}+\frac{1}{4}-\frac{1}{4}-\frac{1}{4}\right)|2,-1\rangle \quad \text{ترم} : S_3^zS_4^z$$

$$+\left(-\frac{1}{4}+\frac{1}{4}+\frac{1}{4}-\frac{1}{4}\right)|2,-1\rangle \quad \text{ترم} : S_4^zS_1^z$$

این ترم‌ها جمعشان صفر می‌شود

$$+\frac{1}{4}\left(|+---\rangle+|-+--\rangle\right) \quad \text{ترم} : \frac{1}{2}(S_1^+S_2^-+S_1^-S_2^+)$$

$$+\frac{1}{4}\left(|--+-\rangle+|-+--\rangle\right) \quad \text{ترم} : \frac{1}{2}(S_2^+S_3^-+S_2^-S_3^+)$$

$$+\frac{1}{4}\left(|---+\rangle+|--+-\rangle\right) \quad \text{ترم} : \frac{1}{2}(S_3^+S_4^-+S_3^-S_4^+)$$

$$+\frac{1}{4}\left(|---+\rangle+|+---\rangle\right) \quad \text{ترم} \; \frac{1}{2}(S_4^+S_1^-+S_4^-S_1^+)$$

$$=\frac{1}{2}\left\{|+---\rangle+|-+--\rangle+|--+-\rangle+|---+\rangle\right\}=|2,-1\rangle$$

$$\underset{(T\otimes T)}{H|2,2\rangle} = \left\{\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)\right\} + \tfrac{1}{2}\left\{(0+0)+(0+0)+(0+0)+(0+0)\right\}|2,2\rangle$$

$$= 1|2,2\rangle$$

$$\Rightarrow \boxed{H|2,2\rangle = 1|2,2\rangle}$$

$$\underset{(T\otimes T)}{H|2,-2\rangle} = \left\{\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)\right\} + \tfrac{1}{2}\left\{(0+0)+(0+0)+(0+0)+(0+0)\right\}|2,-2\rangle$$

$$= 1|2,-2\rangle$$

$$\Rightarrow \boxed{H|2,-2\rangle = 1|2,-2\rangle}$$

$$\underset{(T\otimes T)}{|2,1\rangle} = \tfrac{1}{2}\left\{|+-++\rangle + |-+++\rangle + |+++-\rangle + |++-+\rangle\right\} = \tfrac{\sqrt{2}}{2}\left\{|1,0\rangle|1,1\rangle + |1,1\rangle|1,0\rangle\right\}$$

$H|2,1\rangle$ Terms:

$S_1^z S_2^z$: $\left(-\tfrac{1}{4}\right)|+-++\rangle + \left(-\tfrac{1}{4}\right)|-+++\rangle + \left(\tfrac{1}{4}\right)|+++-\rangle + \left(\tfrac{1}{4}\right)|++-+\rangle$

$S_2^z S_3^z$: $\left(-\tfrac{1}{4}\right)|+-++\rangle + \left(\tfrac{1}{4}\right)|-+++\rangle + \left(\tfrac{1}{4}\right)|+++-\rangle + \left(-\tfrac{1}{4}\right)|++-+\rangle$

$S_3^z S_4^z$: $\left(\tfrac{1}{4}\right)|+-++\rangle + \left(\tfrac{1}{4}\right)|-+++\rangle + \left(-\tfrac{1}{4}\right)|+++-\rangle + \left(-\tfrac{1}{4}\right)|++-+\rangle$

$S_4^z S_1^z$: $\left(\tfrac{1}{4}\right)|+-++\rangle + \left(-\tfrac{1}{4}\right)|-+++\rangle + \left(-\tfrac{1}{4}\right)|+++-\rangle + \left(\tfrac{1}{4}\right)|++-+\rangle$

$\tfrac{1}{2}(S_1^+S_2^- + S_1^-S_2^+)$: $\tfrac{1}{2}\left\{|-+++\rangle + |+-++\rangle\right\}$

$\tfrac{1}{2}(S_2^+S_3^- + S_2^-S_3^+)$: $\tfrac{1}{2}\left\{|++-+\rangle + |+-++\rangle\right\}$

$\tfrac{1}{2}(S_3^+S_4^- + S_3^-S_4^+)$: $\tfrac{1}{2}\left\{|++-+\rangle + |+++-\rangle\right\}$

$\tfrac{1}{2}(S_4^+S_1^- + S_4^-S_1^+)$: $\tfrac{1}{2}\left\{|+++-\rangle + |-+++\rangle\right\}$

$$\rightarrow H|2,1\rangle = \tfrac{1}{2}\left\{\left(-\tfrac{1}{4}-\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{4}\right) + \tfrac{1}{2}(2)|+-++\rangle + |-+++\rangle + |+++-\rangle + |++-+\rangle\right\}$$

$$\Rightarrow \boxed{H|2,1\rangle = 1|2,1\rangle}$$

$$\underset{(\mathrm{TOT})}{|2,0\rangle} = \frac{1}{\sqrt{6}}\left\{|--++\rangle + |+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle + |++--\rangle\right\}$$

$\mathcal{H}|2,0\rangle$ Terms:

$$S_1^z S_2^z: \left(\tfrac{1}{4}\right)|--++\rangle + \left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(-\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|+--+\rangle + \left(\tfrac{1}{4}\right)|-+-+\rangle + \left(\tfrac{1}{4}\right)|++--\rangle$$

$$S_2^z S_3^z: \left(-\tfrac{1}{4}\right)|--++\rangle + \left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(\tfrac{1}{4}\right)|-++-\rangle + \left(\tfrac{1}{4}\right)|+--+\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle + \left(-\tfrac{1}{4}\right)|++--\rangle$$

$$S_3^z S_4^z: \left(\tfrac{1}{4}\right)|--++\rangle + \left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(-\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|+--+\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle + \left(\tfrac{1}{4}\right)|++--\rangle$$

$$S_4^z S_1^z: \left(-\tfrac{1}{4}\right)|--++\rangle + \left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(+\tfrac{1}{4}\right)|-++-\rangle + \left(\tfrac{1}{4}\right)|+--+\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle + \left(-\tfrac{1}{4}\right)|++--\rangle$$

$$\tfrac{1}{2}\left(S_1^+S_2^- + S_1^-S_2^+\right): \tfrac{1}{2}\left\{|-++-\rangle + |+-+-\rangle + |-+-+\rangle + |+--+\rangle\right\}$$

$$\tfrac{1}{2}\left(S_2^+S_3^- + S_2^-S_3^+\right): \tfrac{1}{2}\left\{|-+-+\rangle + |++--\rangle + |--++\rangle + |+-+-\rangle\right\}$$

$$\tfrac{1}{2}\left(S_3^+S_4^- + S_3^-S_4^+\right): \tfrac{1}{2}\left\{|+--+\rangle + |-+-+\rangle + |+-+-\rangle + |-++-\rangle\right\}$$

$$\tfrac{1}{2}\left(S_4^+S_1^- + S_4^-S_1^+\right): \tfrac{1}{2}\left\{|+-+-\rangle + |--++\rangle + |++--\rangle + |-+-+\rangle\right\}$$

$$\rightarrow \mathcal{H}|2,0\rangle = \frac{1}{\sqrt{6}}\left\{|--++\rangle + |+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle + |++--\rangle\right\}$$

$$\Longrightarrow \boxed{\mathcal{H}|2,0\rangle = 1|2,0\rangle}$$

$$\underset{(\mathrm{TOT})}{|1,1\rangle_{tr}} = \frac{1}{2}\left\{|+-++\rangle + |-+++\rangle - |+++-\rangle - |++-+\rangle\right\}$$

$\mathcal{H}|1,1\rangle_{tr}$ Terms:

$$S_1^z S_2^z: \left(-\tfrac{1}{4}\right)|+-++\rangle + \left(-\tfrac{1}{4}\right)|-+++\rangle + \left(\tfrac{1}{4}\right)|+++-\rangle + \left(\tfrac{1}{4}\right)|++-+\rangle$$

$$S_2^z S_3^z: \left(-\tfrac{1}{4}\right)|+-++\rangle + \left(\tfrac{1}{4}\right)|-+++\rangle - \left(\tfrac{1}{4}\right)|+++-\rangle - \left(-\tfrac{1}{4}\right)|++-+\rangle$$

$$S_3^z S_4^z: \left(\tfrac{1}{4}\right)|+-++\rangle + \left(\tfrac{1}{4}\right)|-+++\rangle - \left(-\tfrac{1}{4}\right)|+++-\rangle - \left(-\tfrac{1}{4}\right)|++-+\rangle$$

$$S_4^z S_1^z: \left(\tfrac{1}{4}\right)|+-++\rangle + \left(-\tfrac{1}{4}\right)|-+++\rangle - \left(-\tfrac{1}{4}\right)|+++-\rangle - \left(\tfrac{1}{4}\right)|++-+\rangle$$

$$\tfrac{1}{2}\left(S_1^+S_2^- + S_1^-S_2^+\right) \quad \tfrac{1}{2}\left\{|-+++\rangle + |+-++\rangle\right\}$$

$$\tfrac{1}{2}\left(S_2^+S_3^- + S_2^-S_3^+\right): \tfrac{1}{2}\left\{|++-+\rangle - |+-++\rangle\right\}$$

$$\tfrac{1}{2}\left(S_3^+S_4^- + S_3^-S_4^+\right): \tfrac{1}{2}\left\{-|++-+\rangle - |+++-\rangle\right\}$$

$$\tfrac{1}{2}\left(S_4^+S_1^- + S_4^-S_1^+\right): \tfrac{1}{2}\left\{|+++-\rangle - |-+++\rangle\right\}$$

$$\Longrightarrow \boxed{\mathcal{H}|1,1\rangle_{tr} = 0|1,1\rangle_{tr}}$$

$$H|0,0\rangle = H\left(\frac{1}{\sqrt{3}}\left(|++--\rangle - \frac{1}{2}\left(|+-+-\rangle + |+--+\rangle + |-++-\rangle\right) + |--++\rangle\right)\right)$$

$(T\otimes T)$

$H|0,0\rangle$ terms:

$$S_1^z S_2^z = \left(\tfrac{1}{4}\right)|++--\rangle - \left(\tfrac{1}{2}\right)\left[\left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(-\tfrac{1}{4}\right)|+--+\rangle + \left(-\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle\right] + \left(\tfrac{1}{4}\right)|--++\rangle$$

$$S_2^z S_3^z = \left(-\tfrac{1}{4}\right)|++--\rangle - \left(\tfrac{1}{2}\right)\left[\left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(\tfrac{1}{4}\right)|+--+\rangle + \left(\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle\right] + \left(-\tfrac{1}{4}\right)|--++\rangle$$

$$S_3^z S_4^z = \left(\tfrac{1}{4}\right)|++--\rangle - \left(\tfrac{1}{2}\right)\left[\left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(-\tfrac{1}{4}\right)|+--+\rangle + \left(-\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle\right] + \left(\tfrac{1}{4}\right)|--++\rangle$$

$$S_4^z S_1^z = \left(-\tfrac{1}{4}\right)|++--\rangle - \left(\tfrac{1}{2}\right)\left[\left(-\tfrac{1}{4}\right)|+-+-\rangle + \left(\tfrac{1}{4}\right)|+--+\rangle + \left(\tfrac{1}{4}\right)|-++-\rangle + \left(-\tfrac{1}{4}\right)|-+-+\rangle\right] + \left(-\tfrac{1}{4}\right)|--++\rangle$$

$$\frac{1}{2}\left(S_1^+ S_2^- + S_1^- S_2^+\right) = \frac{1}{2}\left\{-\frac{1}{2}\left[|-++-\rangle + |-+-+\rangle + |+-+-\rangle + |+--+\rangle\right]\right\}$$

$$\frac{1}{2}\left(S_2^+ S_3^- + S_2^- S_3^+\right) = \frac{1}{2}\left\{|+-+-\rangle - \frac{1}{2}\left[|++--\rangle + |--++\rangle\right] + |-+-+\rangle\right\}$$

$$\frac{1}{2}\left(S_3^+ S_4^- + S_3^- S_4^+\right) = \frac{1}{2}\left\{-\frac{1}{2}\left[|+--+\rangle + |+-+-\rangle + |-+-+\rangle + |-++-\rangle\right]\right\}$$

$$\frac{1}{2}\left(S_4^+ S_1^- + S_4^- S_1^+\right) = \frac{1}{2}\left\{|-+-+\rangle - \frac{1}{2}\left[|--++\rangle + |++--\rangle\right] + |+-+-\rangle\right\}$$

$$\Longrightarrow H|0,0\rangle = \frac{1}{\sqrt{3}}\left\{|+-+-\rangle - \frac{1}{2}\left[|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right] + |-+-+\rangle\right\}$$

* حالت‌های کت‌ها و همچنین نتیجه اعمال هامیلتونی بر آن‌ها را یادداشت کنید:

1: $|2,2\rangle = |1,+1\rangle \otimes |1,+1\rangle = |++++\rangle$ & $H|2,2\rangle = |++++\rangle = |2,2\rangle$

2: $|2,1\rangle = \frac{1}{2}\{|+-++\rangle + |-+++\rangle + |+++-\rangle + |++-+\rangle\}$ & $H|2,1\rangle = |2,1\rangle$

3: $|2,0\rangle = \frac{1}{\sqrt{6}}\{|--++\rangle + |+-+-\rangle + |-++-\rangle + |+--+\rangle + |-+-+\rangle + |++--\rangle$ & $H|2,0\rangle = |2,0\rangle$

4: $|2,-1\rangle = \frac{1}{2}\{|+---\rangle + |-+--\rangle + |--+-\rangle + |---+\rangle\}$ & $H|2,-1\rangle = |2,-1\rangle$

5: $|2,2\rangle = |++++\rangle$ & $H|2,2\rangle = |2,-2\rangle$

6: $|1,+1\rangle_{(T\otimes T)} = \frac{1}{2}\{|+-++\rangle + |-+++\rangle - |+++-\rangle - |++-+\rangle\}$ & $H|1,+1\rangle = 0$

7: $|1,+1\rangle_{(T\otimes S)} = \frac{1}{\sqrt{2}}\{|+++-\rangle - |++-+\rangle\}$ & $H|1,+1\rangle = \frac{1}{\sqrt{8}}\{-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\}$

8: $|1,+1\rangle_{(S\otimes T)} = \frac{1}{\sqrt{2}}\{|+-++\rangle - |-+++\rangle\}$ & $H|1,+1\rangle = \frac{1}{\sqrt{8}}\{-|+-++\rangle + |-+++\rangle + |++-+\rangle - |+++-\rangle\}$

9: $|1,-1\rangle_{(T\otimes T)} = \frac{1}{2}\{|--+-\rangle + |---+\rangle - |+---\rangle - |-+--\rangle\}$ & $H|1,-1\rangle = 0$

10: $|1,-1\rangle_{(T\otimes S)} = \frac{1}{\sqrt{2}}\{|--+-\rangle - |---+\rangle\}$ & $H|1,-1\rangle = \frac{1}{\sqrt{8}}\{-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\}$

11: $|1,-1\rangle_{(S\otimes T)} = \frac{1}{\sqrt{2}}\{|+---\rangle - |-+--\rangle\}$ & $H|1,-1\rangle = \frac{1}{\sqrt{8}}\{-|+---\rangle + |-+--\rangle - |--+-\rangle + |---+\rangle\}$

12: $|1,0\rangle_{(T\otimes T)} = \frac{1}{\sqrt{2}}\{|--++\rangle - |++--\rangle\}$ & $H|1,0\rangle = 0$

13: $|1,0\rangle_{(T\otimes S)} = \frac{1}{2}\{|+-+-\rangle + |-++-\rangle - |+--+\rangle - |-+-+\rangle\}$ & $H|1,0\rangle = -\frac{1}{2}\{|-+-+\rangle - |+-+-\rangle\}$

14: $|1,0\rangle_{(S\otimes T)} = \frac{1}{2}\{|+-+-\rangle - |-++-\rangle + |+--+\rangle - |-+-+\rangle\}$ & $H|1,0\rangle = \frac{1}{2}\{|-+-+\rangle - |+-+-\rangle\}$

15: $|0,0\rangle_{(T\otimes T)} = \frac{1}{\sqrt{3}}\{|++--\rangle - \frac{1}{2}(|+-+-\rangle + |+--+\rangle + |-+-+\rangle + |-++-\rangle) + |--++\rangle\}$

& $H|0,0\rangle = \frac{1}{\sqrt{3}}\{|+-+-\rangle - \frac{1}{2}(|+--+\rangle + |++--\rangle + |-++-\rangle + |--++\rangle) + |-+-+\rangle\}$

16: $|0,0\rangle_{S\otimes S} = \frac{1}{2}\{|+-+-\rangle - |-++-\rangle - |+--+\rangle + |-+-+\rangle\}$

& $H|0,0\rangle = \{-|+-+-\rangle + \frac{1}{2}(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle) - |-+-+\rangle\}$

(8) حال به محاسبه ی عناصر ماتریسی بلوک قطری با S_{tot} مشخص و S_Z مشخص می پردازیم:

$S_{tot}=2\ ,\ S_Z=2$:

$\langle 2,+2|H|2,+2\rangle = 1\ ,\ \langle 2,1|H|2,+2\rangle=0\ ,\ \langle 2,0|H|2,2\rangle=0\ ,\ \langle 2,-1|H|2,2\rangle=0$

$\langle 2,-2|H|2,2\rangle=0\ ,\ {}_{TT}\langle 1,+1|H|2,2\rangle=0\ ,\ {}_{TS}\langle 1,+1|H|2,2\rangle=0\ ,\ {}_{ST}\langle 1,+1|H|2,2\rangle=0$

${}_{TT}\langle 1,0|H|2,2\rangle=0\ ,\ {}_{TS}\langle 1,0|H|2,2\rangle=0\ ,\ {}_{ST}\langle 1,0|H|1,0\rangle=0\ ,\ {}_{TT}\langle 1,-1|H|2,2\rangle=0$

${}_{TS}\langle 1,-1|H|2,2\rangle=0\ ,\ {}_{ST}\langle 1,-1|H|2,2\rangle=0\ ,\ {}_{TT}\langle 0,0|H|2,2\rangle=0\ ,\ {}_{SS}\langle 0,0|H|2,2\rangle=0$

درنتیجه یک ماتریس 1x1 داریم : $[1]$ for $S_{tot}=2$ & $S_Z=+2$

$S_{tot}=2\ ,\ S_Z=1$:

$\langle 2,2|H|2,1\rangle=0\ ,\ \langle 2,1|H|2,1\rangle=1\ ,\ \langle 2,0|H|2,1\rangle=0\ ,\ \langle 2,-1|H|2,1\rangle=0$

$\langle 2,-2|H|2,1\rangle=0\ ,\ {}_{TT}\langle 1,+1|H|2,1\rangle=0\ ,\ {}_{TS}\langle 1,+1|H|2,1\rangle=0\ ,\ {}_{ST}\langle 1,+1|H|2,1\rangle=0$

${}_{TT}\langle 1,0|H|2,1\rangle=0\ ,\ {}_{TS}\langle 1,0|H|2,1\rangle=0\ ,\ {}_{ST}\langle 1,0|H|2,1\rangle=0\ ,\ {}_{TT}\langle 1,-1|H|2,1\rangle=0$

${}_{TS}\langle 1,-1|H|2,1\rangle=0\ ,\ {}_{ST}\langle 1,-1|H|2,1\rangle=0\ ,\ {}_{TT}\langle 0,0|H|2,1\rangle=0\ ,\ {}_{SS}\langle 0,0|H|2,1\rangle=0$

درنتیجه یک ماتریس 1x1 داریم : $[1]$ for $S_{tot}=2$ & $S_Z=+1$

$S_{tot}=2\ ,\ S_Z=0$:

$\langle 2,2|H|2,0\rangle=0\ ,\ \langle 2,1|H|2,0\rangle=0\ ,\ \langle 2,0|H|2,0\rangle=1\ ,\ \langle 2,-1|H|2,0\rangle=0$

$\langle 2,-2|H|2,0\rangle=0\ ,\ {}_{TT}\langle 1,+1|H|2,0\rangle=0\ ,\ {}_{TS}\langle 1,+1|H|2,0\rangle=0\ ,\ {}_{ST}\langle 1,+1|H|2,0\rangle=0$

${}_{TT}\langle 1,0|H|2,0\rangle=0\ ,\ {}_{TS}\langle 1,0|H|2,0\rangle=0\ ,\ {}_{ST}\langle 1,0|H|2,0\rangle=0\ ,\ {}_{TT}\langle 1,-1|H|2,0\rangle=0$

${}_{TS}\langle 1,-1|H|2,0\rangle=0\ ,\ {}_{ST}\langle 1,-1|H|2,0\rangle=0\ ,\ {}_{TT}\langle 0,0|H|2,0\rangle=0\ ,\ {}_{SS}\langle 0,0|H|2,0\rangle=0$

درنتیجه یک ماتریس 1x1 داریم : $[1]$ for $S_{tot}=2$ & $S_Z=0$

$S_{tot}=2\ ,\ S_z=-1$:

$\langle 2,2|H|2,-1\rangle=0\ ,\ \langle 2,1|H|2,-1\rangle=0\ ,\ \langle 2,0|H|2,-1\rangle=0\ ,\ \langle 2,-1|H|2,-1\rangle=1$

$\langle 2,-2|H|2,-1\rangle=0\ ,\ {}_{TT}\langle 1,1|H|2,-1\rangle=0\ ,\ {}_{TS}\langle 1,+1|H|2,-1\rangle=0\ ,\ {}_{ST}\langle 1,+1|H|2,-1\rangle=0$

${}_{TT}\langle 1,0|H|2,-1\rangle=0\ ,\ {}_{TS}\langle 1,0|H|2,-1\rangle=0\ ,\ {}_{ST}\langle 1,0|H|2,-1\rangle=0\ ,\ {}_{TT}\langle 1,-1|H|2,-1\rangle=0$

${}_{TS}\langle 1,-1|H|2,-1\rangle=0\ ,\ {}_{ST}\langle 1,-1|H|2,-1\rangle=0\ ,\ {}_{TT}\langle 0,0|H|2,-1\rangle=0\ ,\ {}_{SS}\langle 0,0|H|2,-1\rangle=0$

درنتیجه یک ماتریس 1x1 داریم : $[1]$ for $S_{tot}=2$ & $S_z=-1$

$S_{tot}=2\ ,\ S_z=-2$:

$\langle 2,2|H|2,-2\rangle=0\ ,\ \langle 2,1|H|2,-2\rangle=0\ ,\ \langle 2,0|H|2,-2\rangle=0\ ,\ \langle 2,-1|H|2,-2\rangle=0$

$\langle 2,-2|H|2,-2\rangle=1\ ,\ {}_{TT}\langle 1,+1|H|2,-2\rangle=0\ ,\ {}_{TS}\langle 1,+1|H|2,-2\rangle=0\ ,\ {}_{ST}\langle 1,+1|H|2,-2\rangle=0$

${}_{TT}\langle 1,0|H|2,-2\rangle=0\ ,\ {}_{TS}\langle 1,0|H|2,-2\rangle=0\ ,\ {}_{ST}\langle 1,0|H|2,-2\rangle=0\ ,\ {}_{TT}\langle 1,-1|H|2,-2\rangle=0$

${}_{TS}\langle 1,-1|H|2,-2\rangle=0\ ,\ {}_{ST}\langle 1,-1|H|2,-2\rangle=0\ ,\ {}_{T}\langle 0,0|H|2,-2\rangle=0\ ,\ {}_{SS}\langle 0,0|H|2,-2\rangle=0$

درنتیجه یک ماتریس 1x1 داریم : $[1]$ for $S_{tot}\,2$ & $S_z=-2$

$S_{tot}=1\ ,\ S_z=+1$:

$\langle 2,2|H|1,1\rangle_{TT}=0\ ,\ \langle 2,1|H|1,1\rangle_{TT}=0\ ,\ \langle 2,0|H|1,1\rangle_{TT}=0\ ,\ \langle 2,-1|H|1,1\rangle_{TT}=0$

$\langle 2,-2|H|1,1\rangle_{TT}=0\ ,\ {}_{TT}\langle 1,1|H|1,+1\rangle_{TT}=0\ ,\ {}_{TS}\langle 1,+1|H|1,1\rangle_{TT}=0\ ,\ {}_{ST}\langle 1,+1|H|1,1\rangle_{TT}=0$

${}_{TT}\langle 1,0|H|1,1\rangle_{TT}=0\ ,\ {}_{TS}\langle 1,0|H|1,1\rangle_{TT}=0\ ,\ {}_{ST}\langle 1,0|H|1,1\rangle_{TT}=0\ ,\ {}_{TT}\langle 1,-1|H|1,1\rangle_{TT}=0$

${}_{TS}\langle 1,-1|H|1,1\rangle_{TT}=0\ ,\ {}_{ST}\langle 1,-1|H|1,1\rangle_{TT}=0\ ,\ {}_{TT}\langle 0,0|H|1,1\rangle_{TT}=0\ ,\ {}_{SS}\langle 0,0|H|1,1\rangle_{TT}=0$

$\langle 2,2|H|1,1\rangle_{TS}=0\ ,\ \langle 2,1|H|1,1\rangle_{TS}=0\ ,\ \langle 2,0|H|1,1\rangle_{TS}=0\ ,\ \langle 2,-1|H|1,1\rangle_{TS}=0$

$\langle 2,-2|H|1,1\rangle_{TS}=0\ ,\ {}_{TT}\langle 1,1|H|1,1\rangle_{TS}=0\ ,\ {}_{TS}\langle 1,1|H|1,1\rangle_{TS}=-,5\ ,\ {}_{ST}\langle 1,+1|H|1,+1\rangle_{TS}=-,5$

${}_{TT}\langle 1,0|H|1,1\rangle_{TS}=0\ ,\ {}_{TS}\langle 1,0|H|1,1\rangle_{TS}=0\ ,\ {}_{ST}\langle 1,0|H|1,1\rangle_{TS}=0\ ,\ {}_{TT}\langle 1,-1|H|1,+1\rangle_{TS}=0$

${}_{TS}\langle 1,-1|H|1,1\rangle_{TS}=0\ ,\ {}_{ST}\langle 1,-1|H|1,1\rangle_{TS}=0\ ,\ {}_{TT}\langle 0,0|H|1,1\rangle_{TS}=0\ ,\ {}_{SS}\langle 0,0|H|1,+1\rangle_{TS}=0$

$\langle 2,2|H|1,1\rangle_{ST}=0$, $\langle 2,1|H|1,1\rangle_{ST}=0$, $\langle 2,0|H|1,1\rangle_{ST}=0$, $\langle 2,-1|H|1,1\rangle_{ST}=0$

$\langle 2,-2|H|1,1\rangle_{ST}=0$, ${}_{TT}\langle 1,1|H|1,1\rangle_{ST}=0$, ${}_{TS}\langle 1,1|H|1,1\rangle_{ST}=-0{,}5$, ${}_{ST}\langle 1,+1|H|1,1\rangle_{ST}=-0{,}5$

${}_{TT}\langle 1,0|H|1,1\rangle_{ST}=0$, ${}_{TS}\langle 1,0|H|1,1\rangle_{ST}=0$, ${}_{ST}\langle 1,0|H|1,1\rangle_{ST}=0$, ${}_{TT}\langle 1,-1|H|1,1\rangle_{ST}=0$

${}_{TS}\langle 1,-1|H|1,1\rangle_{ST}=0$, ${}_{ST}\langle 1,-1|H|1,1\rangle_{ST}=0$, ${}_{TT}\langle 0,0|H|1,1\rangle_{ST}=0$, ${}_{SS}\langle 0,0|H|1,1\rangle_{ST}=0$

درنتیجه یک ماتریس 3x3 داریم :

$$\begin{array}{ccc} |1,1\rangle_{TT} & |1,1\rangle_{TS} & |1,1\rangle_{ST} \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -0{,}5 & -0{,}5 \\ 0 & -0{,}5 & -0{,}5 \end{bmatrix}$$

for $S_{tot}=1$ & $S_z=+1$

$S_{tot}=1$, $S_z=0$:

$\langle 2,2|H|1,0\rangle_{TT}=0$, $\langle 2,1|H|1,0\rangle_{TT}=0$, $\langle 2,0|H|1,0\rangle_{TT}=0$, $\langle 2,-1|H|1,0\rangle_{TT}=0$

$\langle 2,-2|H|1,0\rangle_{TT}=0$, ${}_{TT}\langle 1,1|H|1,0\rangle_{TT}=0$, ${}_{TS}\langle 1,1|H|1,0\rangle_{TT}=0$, ${}_{ST}\langle 1,+1|H|1,0\rangle_{TT}=0$

${}_{TT}\langle 1,0|H|1,0\rangle_{TT}=0$, ${}_{TS}\langle 1,0|H|1,0\rangle_{TT}=0$, ${}_{ST}\langle 1,0|H|1,0\rangle_{TT}=0$, ${}_{TT}\langle 1,-1|H|1,0\rangle_{TT}=0$

${}_{TS}\langle 1,-1|H|1,0\rangle_{TT}=0$, ${}_{ST}\langle 1,-1|H|1,0\rangle_{TT}=0$, ${}_{TT}\langle 0,0|H|1,0\rangle_{TT}=0$, ${}_{SS}\langle 0,0|H|1,0\rangle_{TT}=0$

$\langle 2,2|H|1,0\rangle_{TS}=0$, $\langle 2,1|H|1,0\rangle_{TS}=0$, $\langle 2,0|H|1,0\rangle_{TS}=0$, $\langle 2,-1|H|1,0\rangle_{TS}=0$

$\langle 2,-2|H|1,0\rangle_{TS}=0$, ${}_{TT}\langle 1,1|H|1,0\rangle_{TS}=0$, ${}_{TS}\langle 1,1|H|1,0\rangle_{TS}=0$, ${}_{ST}\langle 1,+1|H|1,0\rangle_{TS}=0$

${}_{TT}\langle 1,0|H|1,0\rangle_{TS}=0$, ${}_{TS}\langle 1,0|H|1,0\rangle_{TS}=-0{,}5$, ${}_{ST}\langle 1,0|H|1,0\rangle_{TS}=-0{,}5$, ${}_{TT}\langle 1,-1|H|1,0\rangle_{TS}=0$

${}_{TS}\langle 1,-1|H|1,0\rangle_{TS}=0$, ${}_{ST}\langle 1,-1|H|1,0\rangle_{TS}=0$, ${}_{TT}\langle 0,0|H|1,0\rangle_{TS}=0$, ${}_{SS}\langle 0,0|H|1,0\rangle_{TS}=0$

$\langle 2,2|H|1,0\rangle_{ST}=0$, $\langle 2,1|H|1,0\rangle_{ST}=0$, $\langle 2,0|H|1,0\rangle_{ST}=0$, $\langle 2,-1|H|1,0\rangle_{ST}=0$

$\langle 2,-2|H|1,0\rangle_{ST}=0$, ${}_{TT}\langle 1,1|H|1,0\rangle_{ST}=0$, ${}_{TS}\langle 1,1|H|1,0\rangle_{ST}=0$, ${}_{ST}\langle 1,+1|H|1,0\rangle_{ST}=0$

${}_{TT}\langle 1,0|H|1,0\rangle_{ST}=0$, ${}_{TS}\langle 1,0|H|1,0\rangle_{ST}=-0{,}5$, ${}_{ST}\langle 1,0|H|1,0\rangle_{ST}=-0{,}5$, ${}_{TT}\langle 1,-1|H|1,0\rangle_{ST}=0$

${}_{TS}\langle 1,-1|H|1,0\rangle_{ST}=0$, ${}_{ST}\langle 1,-1|H|1,0\rangle_{ST}=0$, ${}_{TT}\langle 0,0|H|1,0\rangle_{ST}=0$, ${}_{SS}\langle 0,0|H|1,0\rangle_{ST}=0$

درنتیجه یک ماتریس 3x3 داریم :

$$\begin{array}{ccc} |1,0\rangle_{TT} & |1,0\rangle_{TS} & |1,0\rangle_{ST} \\ \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -0{,}5 & -0{,}5 \\ 0 & -0{,}5 & -0{,}5 \end{bmatrix}$$

for $S_{tot}=1$ & $S_z=0$

$S_{tot}=1 \;,\; S_z=-1 :$

$\langle 2,2|H|1,-1\rangle_{TT}=0 \;,\; \langle 2,1|H|1,-1\rangle_{TT}=0 \;,\; \langle 2,0|H|1,-1\rangle_{TT}=0 \;,\; \langle 2,-1|H|1,-1\rangle_{TT}=0$

$\langle 2,-2|H|1,-1\rangle_{TT}=0 \;,\; {}_{TT}\langle 1,+1|H|1,-1\rangle_{TT}=0 \;,\; {}_{TS}\langle 1,+1|H|1,-1\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,+1|H|1,-1\rangle_{TT}=0$

${}_{TT}\langle 1,0|H|1,-1\rangle_{TT}=0 \;,\; {}_{TS}\langle 1,0|H|1,-1\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,0|H|1,-1\rangle_{TT}=0 \;,\; {}_{TT}\langle 1,-1|H|1,-1\rangle_{TT}=0$

${}_{TS}\langle 1,-1|H|1,-1\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,-1|H|1,-1\rangle_{TT}=0 \;,\; {}_{TT}\langle 0,0|H|1,-1\rangle_{TT}=0 \;,\; {}_{SS}\langle 0,0|H|1,-1\rangle_{TT}=0$

$\langle 2,2|H|1,-1\rangle_{TS}=0 \;,\; \langle 2,1|H|1,-1\rangle_{TS}=0 \;,\; \langle 2,0|H|1,-1\rangle_{TS}=0 \;,\; \langle 2,-1|H|1,-1\rangle_{TS}=0$

$\langle 2,-2|H|1,-1\rangle_{TS}=0 \;,\; {}_{TT}\langle 1,+1|H|1,-1\rangle_{TS}=0 \;,\; {}_{TS}\langle 1,+1|H|1,-1\rangle_{TS}=0 \;,\; {}_{ST}\langle 1,+1|H|1,-1\rangle_{TS}=0$

${}_{TT}\langle 1,0|H|1,-1\rangle_{TS}=0 \;,\; {}_{TS}\langle 1,0|H|1,-1\rangle_{TS}=0 \;,\; {}_{ST}\langle 1,0|H|1,-1\rangle_{TS}=0 \;,\; {}_{TT}\langle 1,-1|H|1,-1\rangle_{TS}=0$

${}_{TS}\langle 1,-1|H|1,-1\rangle_{TS}=-0{,}5 \;,\; {}_{ST}\langle 1,-1|H|1,-1\rangle_{TS}=-0{,}5 \;,\; {}_{TT}\langle 0,0|H|1,-1\rangle_{TS}=0 \;,\; {}_{SS}\langle 0,0|H|1,-1\rangle_{TS}=0$

$\langle 2,2|H|1,-1\rangle_{ST}=0 \;,\; \langle 2,+1|H|1,-1\rangle_{ST}=0 \;,\; \langle 2,0|H|1,-1\rangle_{ST}=0 \;,\; \langle 2,-1|H|1,-1\rangle_{ST}=0$

$\langle 2,-2|H|1,-1\rangle_{ST}=0 \;,\; {}_{TT}\langle 1,+1|H|1,-1\rangle_{ST}=0 \;,\; {}_{TS}\langle 1,+1|H|1,-1\rangle_{ST}=0 \;,\; {}_{ST}\langle 1,+1|H|1,-1\rangle_{ST}=0$

${}_{TT}\langle 1,0|H|1,-1\rangle_{ST}=0 \;,\; {}_{TS}\langle 1,0|H|1,-1\rangle_{ST}=0 \;,\; {}_{ST}\langle 1,0|H|1,-1\rangle_{ST}=0 \;,\; {}_{TT}\langle 1,-1|H|1,-1\rangle_{ST}=0$

${}_{TS}\langle 1,-1|H|1,-1\rangle_{ST}=-0{,}5 \;,\; {}_{ST}\langle 1,-1|H|1,-1\rangle_{ST}=-0{,}5 \;,\; {}_{TT}\langle 0,0|H|1,-1\rangle_{ST}=0 \;,\; {}_{SS}\langle 0,0|H|1,-1\rangle_{ST}=0$

درنتیجه یک ماتریس 3x3 داریم :

$$\begin{array}{ccc} |1,-1\rangle_{TT} & |1,-1\rangle_{TS} & |1,-1\rangle_{ST} \end{array}$$
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -0{,}5 & -0{,}5 \\ 0 & -0{,}5 & -0{,}5 \end{bmatrix} \quad \text{for } S_{tot}=1 \;\&\; S_z=-1$$

$S_{tot}=0 \;,\; S_z=0 :$

$\langle 2,2|H|0,0\rangle_{TT}=0 \;,\; \langle 2,+1|H|0,0\rangle_{TT}=0 \;,\; \langle 2,0|H|0,0\rangle_{TT}=0 \;,\; \langle 2,-1|H|0,0\rangle_{TT}=0$

$\langle 2,-2|H|0,0\rangle_{TT}=0 \;,\; {}_{TT}\langle 1,+1|H|0,0\rangle_{TT}=0 \;,\; {}_{TS}\langle 1,+1|H|0,0\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,+1|H|0,0\rangle_{TT}=0$

${}_{TT}\langle 1,0|H|0,0\rangle_{TT}=0 \;,\; {}_{TS}\langle 1,0|H|0,0\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,0|H|0,0\rangle_{TT}=0 \;,\; {}_{TT}\langle 1,-1|H|0,0\rangle_{TT}=0$

${}_{TS}\langle 1,-1|H|0,0\rangle_{TT}=0 \;,\; {}_{ST}\langle 1,-1|H|0,0\rangle_{TT}=0 \;,\; {}_{TT}\langle 0,0|H|0,0\rangle_{TT}=-0{,}5 \;,\; {}_{SS}\langle 0,0|H|0,0\rangle_{TT}=\frac{\sqrt{3}}{2}$

$\langle 2,2|H|0,0\rangle_{SS}=0 \;,\; \langle 2,+1|H|0,0\rangle_{SS}=0 \;,\; \langle 2,0|H|0,0\rangle_{SS}=0 \;,\; \langle 2,-1|H|0,0\rangle_{SS}=0$

$\langle 2,-2|H|0,0\rangle_{SS}=0 \;,\; {}_{TT}\langle 1,+1|H|0,0\rangle_{SS}=0 \;,\; {}_{TS}\langle 1,+1|H|0,0\rangle_{SS}=0 \;,\; {}_{ST}\langle 1,+1|H|0,0\rangle_{SS}=0$

${}_{TT}\langle 1,0|H|0,0\rangle_{SS}=0 \;,\; {}_{TS}\langle 1,0|H|0,0\rangle_{SS}=0 \;,\; {}_{ST}\langle 1,0|H|0,0\rangle_{SS}=0 \;,\; {}_{TT}\langle 1,-1|H|0,0\rangle_{SS}=0$

${}_{TS}\langle 1,-1|H|0,0\rangle_{SS}=0 \;,\; {}_{ST}\langle 1,-1|H|0,0\rangle_{SS}=0 \;,\; {}_{TT}\langle 0,0|H|0,0\rangle_{SS}=\frac{\sqrt{3}}{2} \;,\; {}_{SS}\langle 0,0|H|0,0\rangle_{SS}=-1{,}5$

درنتیجه یک ماتریس 2x2 خواهیم داشت:

$$\begin{array}{cc} |0,0\rangle_{TT} & |0,0\rangle_{SS} \\ \end{array}$$
$$\begin{bmatrix} -0,5 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -1,5 \end{bmatrix}$$

for $S_{tot}=0$ & $S_z=0$

در نهایت ماتریس 16x16 ما خواهد شد:

$$\begin{array}{cccccccccccccccc}
|2,2\rangle & |2,1\rangle & |2,0\rangle & |2,-1\rangle & |2,-2\rangle & |1,+1\rangle & |1,+1\rangle & |1,+1\rangle & |1,0\rangle & |1,0\rangle & |1,0\rangle & |1,-1\rangle & |1,-1\rangle & |1,-1\rangle & |0,0\rangle & |0,0\rangle
\end{array}$$
$$\begin{bmatrix}
\boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0,5 & -0,5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0,5 & \frac{\sqrt{3}}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -1,5
\end{bmatrix}$$

حال به محاسبه ی ویژه مقادیر می پردازیم:

* برای 5 ماتریس اول که 1x1 هستند ویژه مقدار برابر 1 است و نیاز به محاسبه ندارد.

* برای 3 ماتریس 3x3 چون دارای مقادیر یکسان هستند پس برای یکی ویژه مقدار را می یابیم.

$\det(U - \lambda I) = 0$:

$$\begin{vmatrix} 0-\lambda & 0 & 0 \\ 0 & -0{,}5-\lambda & -0{,}5 \\ 0 & -0{,}5 & -0{,}5-\lambda \end{vmatrix} = \left(-\lambda \times (-0{,}5-\lambda) \times (-0{,}5-\lambda)\right) - (-0{,}25\lambda) = -\lambda^3 + \lambda^2 = 0 \Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = 0 \\ \lambda_3 = -1 \end{cases}$$

درنتیجه 6 ویژه مقدار صفر و 3 ویژه مقدار 1- داریم.

$\det(U - \lambda I) = 0$:

$$\begin{vmatrix} -0{,}5-\lambda & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -1{,}5-\lambda \end{vmatrix} = \underbrace{(-0{,}5-\lambda) \times (-1{,}5-\lambda)}_{\cancel{0{,}75} + 0{,}5\lambda + 1{,}5\lambda + \lambda^2} - \frac{\cancel{3}}{\cancel{4}} = \lambda^2 + 2\lambda$$

$$\Delta = \pm\sqrt{4} \Rightarrow \lambda_{1,2} = -1 \pm \frac{\sqrt{4}}{2} = 0 \ \& \ -2$$ ویژه مقادیر ماتریس اندازه:

درنتیجه ویژه مقادیر ماتریس اندازه:

$$\underbrace{1, 1, 1, 1, 1}_{\text{مربوط به 5 ماتریس 1x1}} \ \& \ \underbrace{0, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1, -1}_{\text{مربوط به 3 ماتریس 3x3}} \ \& \ \underbrace{-2, 0}_{\text{مربوط به ماتریس 2x2}}$$

ویژه مقادیر مستخرج با نسبت با واقعیت یکسان است که نشان دهنده درست بودن حل مستخرج است.