علی بیجاری و علی موحدی

حل قسمت د:

نمایش کت ها برحسب $\frac{S_z}{\hbar}$ مشخص:

$2: \overset{|1\rangle}{|++++\rangle}$

$1: \overset{|1\rangle}{|+++-\rangle}, \overset{|2\rangle}{|++-+\rangle}, \overset{|3\rangle}{|+-++\rangle}, \overset{|4\rangle}{|-+++\rangle}$

$0: \overset{|1\rangle}{|++--\rangle}, \overset{|2\rangle}{|+-+-\rangle}, \overset{|3\rangle}{|+--+\rangle}, \overset{|4\rangle}{|-++-\rangle}, \overset{|5\rangle}{|-+-+\rangle}, \overset{|6\rangle}{|--++\rangle}$

$-1: \overset{|1\rangle}{|+---\rangle}, \overset{|2\rangle}{|-+--\rangle}, \overset{|3\rangle}{|--+-\rangle}, \overset{|4\rangle}{|---+\rangle}$

$-2: \overset{|1\rangle}{|----\rangle}$

transition operator: $T|1234\rangle = |4,1,2,3\rangle$

transition for $S_z = 2$:

$$\left(\langle ++++|T|++++\rangle\right) = \left(1\right) \begin{cases} \text{eigenvalue}: 1 \\ |k=0\rangle = 1|1\rangle \end{cases}$$

transition for $S_z = 1$:

$$\begin{bmatrix} \langle +++-|T|+++-\rangle & \langle +++-|T|++-+\rangle & \langle +++-|T|+-++\rangle & \langle +++-|T|-+++\rangle \\ \langle ++-+|T|+++-\rangle & \langle ++-+|T|++-+\rangle & \langle ++-+|T|+-++\rangle & \langle ++-+|T|-+++\rangle \\ \langle +-++|T|+++-\rangle & \langle +-++|T|++-+\rangle & \langle +-++|T|+-++\rangle & \langle +-++|T|-+++\rangle \\ \langle -+++|T|+++-\rangle & \langle -+++|T|++-+\rangle & \langle -+++|T|+-++\rangle & \langle -+++|T|-+++\rangle \end{bmatrix}$$

$$= \begin{bmatrix} \langle +++-|-+++\rangle & \langle +++-|+++-\rangle & \langle +++-|++-+\rangle & \langle +++-|+-++\rangle \\ \langle ++-+|-+++\rangle & \langle ++-+|+++-\rangle & \langle ++-+|++-+\rangle & \langle ++-+|+-++\rangle \\ \langle +-++|-+++\rangle & \langle +-++|+++-\rangle & \langle +-++|++-+\rangle & \langle +-++|+-++\rangle \\ \langle -+++|-+++\rangle & \langle -+++|+++-\rangle & \langle -+++|++-+\rangle & \langle -+++|+-++\rangle \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = S'_1$$

$\overline{\text{در نتیجه داریم}}$: $\det(S'_1 - \lambda \mathbb{1}) = 0$

$$\begin{cases} \text{eigenvalue 1}: -1 \quad |k=\pi\rangle = -|1\rangle + |2\rangle - |3\rangle + |4\rangle \\ \text{eigenvalue 2}: i \quad |k=\pi/2\rangle = i|1\rangle - |2\rangle - i|3\rangle + |4\rangle \\ \text{eigenvalue 3}: -i \quad |k=-\pi/2\rangle = -i|1\rangle - |2\rangle + i|3\rangle + |4\rangle \\ \text{eigenvalue 4}: 1 \quad |k=0\rangle = |1\rangle + |2\rangle + |3\rangle + |4\rangle \end{cases}$$

transition for $S_z=0$:

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$$\begin{array}{cccccc}
 & |-++-\rangle & |-+-+\rangle & |++--\rangle & |--++\rangle & |+-+-\rangle & |+--+\rangle \\
\end{array}$$

$$\begin{bmatrix}
\langle ++--|T|++--\rangle & \langle ++--|T|+-+-\rangle & \langle ++--|T|+--+\rangle & \langle ++--|T|-++-\rangle & \langle ++--|T|-+-+\rangle & \langle ++--|T|--++\rangle \\
\langle +-+-|T|++--\rangle & \langle +-+-|T|+-+-\rangle & \langle +-+-|T|+--+\rangle & \langle +-+-|T|-++-\rangle & \langle +-+-|T|-+-+\rangle & \langle +-+-|T|--++\rangle \\
\langle +--+|T|++--\rangle & \langle +--+|T|+-+-\rangle & \langle +--+|T|+--+\rangle & \langle +--+|T|-++-\rangle & \langle +--+|T|-+-+\rangle & \langle +--+|T|--++\rangle \\
\langle -++-|T|++--\rangle & \langle -++-|T|+-+-\rangle & \langle -++-|T|+--+\rangle & \langle -++-|T|-++-\rangle & \langle -++-|T|-+-+\rangle & \langle -++-|T|--++\rangle \\
\langle -+-+|T|++--\rangle & \langle -+-+|T|+-+-\rangle & \langle -+-+|T|+--+\rangle & \langle -+-+|T|-++-\rangle & \langle -+-+|T|-+-+\rangle & \langle -+-+|T|--++\rangle \\
\langle --++|T|++--\rangle & \langle --++|T|+-+-\rangle & \langle --++|T|+--+\rangle & \langle --++|T|-++-\rangle & \langle --++|T|-+-+\rangle & \langle --++|T|--++\rangle
\end{bmatrix}$$

$$= \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} = S'_0$$

نتیجه دارد : $\det(S'_0 - \lambda \mathbb{1}) = 0$

$$\begin{cases}
\text{eigenvalue 1}: -1, |k=\pi\rangle = |1\rangle - |3\rangle - |4\rangle + |6\rangle \\
\text{eigen value 2}: -1, |k=\pi\rangle = +|2\rangle - |5\rangle \\
\text{eigenvalue 3}: i, |k=\frac{\pi}{2}\rangle = -i|1\rangle + |3\rangle - |4\rangle + i|6\rangle \\
\text{eigen value 4}: -i, |k=-\frac{\pi}{2}\rangle = i|1\rangle + |3\rangle - |4\rangle - i|6\rangle \\
\text{eigen value 5}: 1, |k=0\rangle = |2\rangle + |5\rangle \\
\text{eigen value 6}: 1, |k=0\rangle = |1\rangle + |3\rangle + |4\rangle + |6\rangle
\end{cases}$$

transition for $S_z=2$:

$$\left(\langle ----|T|----\rangle\right) = \left(1\right) \Rightarrow \begin{cases} \text{eigen value: } 1 \\ |k=0\rangle = |----\rangle = |1\rangle \end{cases}$$

$\frac{2}{4}$ for $S_z=-1$ transition:

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$$\begin{bmatrix} \langle +---|\overset{\curvearrowright\,|-+--\rangle}{T|+---\rangle} & \langle +---|\overset{\curvearrowright\,|--+-\rangle}{T|-+--\rangle} & \langle +---|\overset{\curvearrowright\,|---+\rangle}{T|--+-\rangle} & \langle +---|\overset{\curvearrowright\,|+---\rangle}{T|---+\rangle} \\ \langle -+--|T|+---\rangle & \langle -+--|T|-+--\rangle & \langle -+--|T|--+-\rangle & \langle -+--|T|---+\rangle \\ \langle --+-|T|+---\rangle & \langle --+-|T|-+--\rangle & \langle --+-|T|--+-\rangle & \langle --+-|T|---+\rangle \\ \langle ---+|T|+---\rangle & \langle ---+|T|-+--\rangle & \langle ---+|T|--+-\rangle & \langle ---+|T|---+\rangle \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = S'_{-1}$$

$$\det(S'_{-1} - \lambda \mathbb{1}) = 0 \implies \begin{cases} \text{eigenvalue 1: } -1 \implies |k=\pi\rangle = |1\rangle - |2\rangle + |3\rangle - |4\rangle \\ \text{eigevalue 2: } i \implies |k=\frac{\pi}{2}\rangle = -i|1\rangle - |2\rangle + i|3\rangle + |4\rangle \\ \text{eigen value 3: } -i \implies |k=-\frac{\pi}{2}\rangle = i|1\rangle - |2\rangle - i|3\rangle + |4\rangle \\ \text{eigen value 4: } 1 \implies |k=0\rangle = |1\rangle + |2\rangle + |3\rangle + |4\rangle \end{cases}$$

حال که کت‌ها را برای هر یک از S_z با عمل دادن اپراتور انتقال ساختیم هامیلتونی را اثر دهیم، قبل از آن هر کت را بهنجار می‌کنیم:

for $S_z=2$:

$H|1\rangle = H|++++\rangle = |++++\rangle$

*اثر هامیلتونی بر روی ویژه‌کت‌های S_z را در بخش ب به طور تحلیلی حساب کردیم که در اینجا از آن نتایج بهره می‌بریم.

for $S_z=1$:

$*|k=\pi\rangle = \frac{1}{\sqrt{4}}(-|1\rangle + |2\rangle - |3\rangle + |4\rangle)$

$$H|k=\pi\rangle = H\left(\frac{1}{4}(-|1\rangle + |2\rangle - |3\rangle + |4\rangle)\right) = H\left\{\frac{1}{4}(-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle)\right\}$$

$$= \frac{1}{2}\left\{-\tfrac{1}{2}(|++-+\rangle + |-+++\rangle) + \tfrac{1}{2}(|+-++\rangle + |+++-\rangle) - \tfrac{1}{2}(|-+++\rangle + |++-+\rangle)\right.$$

$$\left. + \tfrac{1}{2}(|+-++\rangle + |++++-\rangle)\right\}$$

$$= \frac{1}{4}\left\{-2(|++-+\rangle + |-+++\rangle) + 2(|+-++\rangle + |+++-\rangle)\right\}$$

$$= \frac{1}{2}\left\{|+++-\rangle - |++-+\rangle + |+-++\rangle - |-+++\rangle\right\}$$

* $|K=\frac{\pi}{2}\rangle = i|1\rangle - |2\rangle - i|3\rangle + |4\rangle$

$$H|K=\tfrac{\pi}{2}\rangle = iH|1\rangle - H|2\rangle - iH|3\rangle + H|4\rangle = \frac{i}{2}\left(|{+}{+}{-}{+}\rangle + |{-}{+}{+}{+}\rangle\right) - \frac{1}{2}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) - \frac{i}{2}\left(|{-}{+}{+}{+}\rangle + |{+}{+}{-}{+}\rangle\right) + \frac{1}{2}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) = \underline{0}$$

* $|K=-\frac{\pi}{2}\rangle = -i|1\rangle - |2\rangle + i|3\rangle + |4\rangle$

$$H|K=-\tfrac{\pi}{2}\rangle = -iH|1\rangle - H|2\rangle + iH|3\rangle + H|4\rangle = \frac{-i}{2}\left(|{+}{+}{-}{+}\rangle + |{-}{+}{+}{+}\rangle\right) - \frac{1}{2}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) + \frac{i}{2}\left(|{-}{+}{+}{+}\rangle + |{+}{+}{-}{+}\rangle\right) + \frac{1}{2}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) = \underline{0}$$

* $|K=0\rangle = \frac{1}{2}\left(|1\rangle + |2\rangle + |3\rangle + |4\rangle\right)$

$$H|K=0\rangle = \frac{1}{4}\left(|{+}{+}{-}{+}\rangle + |{-}{+}{+}{+}\rangle\right) + \frac{1}{4}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) + \frac{1}{4}\left(|{-}{+}{+}{+}\rangle + |{+}{+}{-}{+}\rangle\right) + \frac{1}{4}\left(|{+}{-}{+}{+}\rangle + |{+}{+}{+}{-}\rangle\right) = \frac{1}{2}\left(|{+}{+}{+}{-}\rangle + |{+}{+}{-}{+}\rangle + |{+}{-}{+}{+}\rangle + |{-}{+}{+}{+}\rangle\right)$$

for $S_z = 0$:

* $|K=\pi\rangle = \frac{1}{2}\left(|1\rangle - |3\rangle - |4\rangle + |6\rangle\right)$

$$H|K=\pi\rangle = \frac{1}{4}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) - \frac{1}{4}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) - \frac{1}{4}\left(|{+}{-}{+}{-}\rangle + |{-}{+}{-}{+}\rangle\right) + \frac{1}{4}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) = \underline{0}$$

* $|K=\frac{\pi}{2}\rangle = -i|1\rangle + |3\rangle - |4\rangle + i|6\rangle$

$$H|K=\tfrac{\pi}{2}\rangle = \frac{-i}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) + \frac{1}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) - \frac{1}{2}\left(|{+}{-}{+}{-}\rangle + |{-}{+}{-}{+}\rangle\right) + \frac{i}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) = \underline{0}$$

* $|K=-\frac{\pi}{2}\rangle = +i|1\rangle + |3\rangle - |4\rangle - i|6\rangle$

$$H|K=-\tfrac{\pi}{2}\rangle = \frac{i}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) + \frac{1}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) - \frac{1}{2}\left(|{+}{-}{+}{-}\rangle + |{-}{+}{-}{+}\rangle\right) - \frac{i}{2}\left(|{-}{+}{-}{+}\rangle + |{+}{-}{+}{-}\rangle\right) = \underline{0}$$

* $|K_0=0\rangle = \frac{1}{2}(|1\rangle + |2\rangle + |3\rangle + |4\rangle)$

$H|K_0=0\rangle = \frac{1}{4}(|-+-+\rangle + |+-+-\rangle) + \frac{1}{4}(|-+-+\rangle + |+-+-\rangle) + \frac{1}{4}(|+-+-\rangle + |-+-+\rangle)$

$+ \frac{1}{4}(|-+-+\rangle + |+-+-\rangle) = \underline{|+-+-\rangle + |-+-+\rangle}$

* $|K=\pi\rangle = \frac{1}{\sqrt{2}}(|2\rangle - |5\rangle)$

$H|K=\pi\rangle = \frac{1}{\sqrt{2}}\left(-|+-+-\rangle + \frac{1}{2}(|-++-\rangle + |++--\rangle + |+--+\rangle + |--++\rangle)\right)$

$- \frac{1}{\sqrt{2}}\left(-|-+-+\rangle + \frac{1}{2}(|+--+\rangle + |--++\rangle + |-++-\rangle + |++--\rangle)\right)$

$= \frac{1}{\sqrt{2}}(-|+-+-\rangle + |-+-+\rangle)$

* $|K_0=0\rangle = \frac{1}{\sqrt{2}}(|2\rangle + |5\rangle)$

$H|K_0=0\rangle = \frac{1}{\sqrt{2}}\left(-|+-+-\rangle + \frac{1}{2}(|-++-\rangle + |++--\rangle + |+--+\rangle + |--++\rangle)\right)$

$+ \frac{1}{\sqrt{2}}\left(-|-+-+\rangle + \frac{1}{2}(|+--+\rangle + |--++\rangle + |-++-\rangle + |++--\rangle)\right)$

$= \frac{1}{\sqrt{2}}(|++--\rangle - |+-+-\rangle + |+--+\rangle + |-++-\rangle - |-+-+\rangle + |--++\rangle)$

for $S_z = -1$:

* $|K=\pi\rangle = \frac{1}{2}(|1\rangle - |2\rangle + |3\rangle - |4\rangle)$

$H|K=\pi\rangle = \frac{1}{4}(|-+--\rangle + |---+\rangle) - \frac{1}{4}(|+---\rangle + |--+-\rangle) + \frac{1}{4}(|-+--\rangle + |---+\rangle)$

$- \frac{1}{4}(|--+-\rangle + |+---\rangle) = \frac{1}{2}(-|+---\rangle + |-+--\rangle - |--+-\rangle + |---+\rangle)$

* $|K = \frac{\pi}{2}\rangle = -i|1\rangle - |2\rangle + i|3\rangle + |4\rangle$

$H|K = \frac{\pi}{2}\rangle = -\frac{i}{2}(|-+--\rangle + |---+\rangle) - \frac{1}{2}(|+---\rangle + |--+-\rangle) + \frac{i}{2}(|-+--\rangle + |---+\rangle)$

$+ \frac{1}{2}(|--+-\rangle + |+---\rangle) = \underline{0}$

* $|k=-\frac{\pi}{2}\rangle = i|1\rangle - |2\rangle - i|3\rangle + |4\rangle$

$$H|k=-\frac{\pi}{2}\rangle = \frac{i}{2}\left(|-+--\rangle + |---+\rangle\right) - \frac{1}{2}\left(|+---\rangle + |--+-\rangle\right) - \frac{i}{2}\left(|-+--\rangle + |---+\rangle\right)$$
$$+\frac{1}{2}\left(|--+-\rangle + |+---\rangle\right) = 0$$

* $|k=0\rangle = \frac{1}{2}\left(|1\rangle + |2\rangle + |3\rangle + |4\rangle\right)$

$$H|k=0\rangle = \frac{1}{4}\left(|-+--\rangle + |---+\rangle\right) + \frac{1}{4}\left(|+---\rangle + |--+-\rangle\right) + \frac{1}{4}\left(|-+--\rangle + |---+\rangle\right)$$
$$+\frac{1}{4}\left(|--+-\rangle + |+---\rangle\right) = \frac{1}{2}\left(|+---\rangle + |-+--\rangle + |--+-\rangle + |---+\rangle\right)$$

for $S_z=-2$:

* $|k=0\rangle = |----\rangle$

$H|k=0\rangle = |----\rangle$

حال هامیلتونی های بلوک قطری را نمایش می دهیم :

for $S_z=2$: $\begin{array}{cc} & H|k=0\rangle \\ \langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array}$

for $S_z=-2$: $\begin{array}{cc} & H|k=0\rangle \\ \langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array}$

for $S_z=+1$:

$$\begin{array}{cc} & \begin{array}{cccc} H|k=\pi\rangle & H|k=\frac{\pi}{2}\rangle & H|k=-\frac{\pi}{2}\rangle & H|k=0\rangle \end{array} \\ \begin{array}{c} \langle k=\pi| \\ \langle k=\frac{\pi}{2}| \\ \langle k=-\frac{\pi}{2}| \\ \langle k=0| \end{array} & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

for $S_z=-1$:

$$\begin{array}{cc} & \begin{array}{cccc} H|k=\pi\rangle & H|k=\frac{\pi}{2}\rangle & H|k=-\frac{\pi}{2}\rangle & H|k=0\rangle \end{array} \\ \begin{array}{c} \langle k=\pi| \\ \langle k=\frac{\pi}{2}| \\ \langle k=-\frac{\pi}{2}| \\ \langle k=0| \end{array} & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

for $S_z=0$:

$$
\begin{array}{c|cccccc}
 & H|k=\pi\rangle & H|k=\frac{\pi}{2}\rangle & H|k=-\frac{\pi}{2}\rangle & H|k=0\rangle & H|k=\pi\rangle & H|k=0\rangle \\
\hline
\langle k=\pi| & 0 & 0 & 0 & 0 & 0 & 0 \\
\langle k=\frac{\pi}{2}| & 0 & 0 & 0 & 0 & 0 & 0 \\
\langle k=-\frac{\pi}{2}| & 0 & 0 & 0 & 0 & 0 & 0 \\
\langle k=0| & 0 & 0 & 0 & 0 & 0 & \sqrt{2} \\
\langle k=\pi| & 0 & 0 & 0 & 0 & -1 & 0 \\
\langle k=0| & 0 & 0 & 0 & \sqrt{2} & 0 & -1
\end{array}
$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = S_0^1 \qquad S_0^2 = \begin{bmatrix} 0 & 0 & \sqrt{2} \\ 0 & -1 & 0 \\ \sqrt{2} & 0 & -1 \end{bmatrix}$$

ماتریس های مربوط به $S_z=2$، $S_z=-2$، $S_z=1$، $S_z=-1$ هم قطری بودند و عناصر روی قطر اصلی

ویژه مقادیر را به ما دادند که عبارتند از:

* $S_z=2$: eigenvalue $=+1$
* $S_z=-2$: eigen value $=+1$
* $S_z=+1$: eigen values $=-1, 0, 0, 1$
* $S_z=-1$: eigen values $=-1, 0, 0, 1$

حال برای $S_z=0$ که بلوک قطری است به شکل دو ماتریس 3×3 که ماتریس 3×3

بالایی قطری است اما پایینی خیر که آن را حل می کنیم:

$$\det(S_0^2-\lambda\mathbb{1})=0 \Rightarrow \begin{vmatrix} -\lambda & 0 & \sqrt{2} \\ 0 & -1-\lambda & 0 \\ \sqrt{2} & 0 & -1-\lambda \end{vmatrix} = \left(\left(-\lambda\times(-1-\lambda)\times(-1-\lambda)\right)+0+0\right)-\left(2\times(-1-\lambda)+0+0\right)$$

$$=-(\lambda^3+2\lambda^2+\lambda)-(-2-2\lambda)=-\lambda^3-2\lambda^2+\lambda+2=0$$

$$\Rightarrow \lambda_1=-2 \ , \ \lambda_2=-1 \ , \ \lambda_3=1$$

حال که ریشه های ماتریس بلوک قطری پایین را نیز یافتیم می توانیم ریشه های ماتریس $S_z=0$ که

ویژه مقادیر ما هستند را بگوییم:

* $S_z=0$: eigenvalues: $-2, -1, 0, 0, 0, 1$

* ویژه مقادیر با بخش های قبل مطابقت دارد.

①

حل بخش ه:

نمایش کت ها در پایه ی S_{tot} و S_z مشخص را از قسمت ج داریم حال در ادامه عملگر انتقال را بر روی هر کدام از کت های با S_{tot} و S_z مشخص اثر دهیم، در نتیجه خواهیم داشت:

for $S_{tot}=2$ transition we have:

$*\ |2,+2\rangle = |++++\rangle$

اثر عملگر انتقال خودش را می دهد

$$\left(\langle 2,+2|T|2,+2\rangle\right) = (1) \begin{cases} \text{eigenvalue} = 1 \\ |k=0\rangle = |2,+2\rangle \end{cases}$$

$*\ |2,+1\rangle = \frac{\sqrt{2}}{2}\left(|1,0\rangle \otimes |1,+1\rangle + |1,+1\rangle \otimes |1,0\rangle\right) = \frac{1}{2}\left(|+-++\rangle + |-+++\rangle + |+++-\rangle + |++-+\rangle\right)$

اثر عملگر انتقال خودش را می دهد

$$\left(\langle 2,+1|T|2,+1\rangle\right) = (1) \begin{cases} \text{eigen value} = 1 \\ |k=0\rangle = |2,+1\rangle \end{cases}$$

$*\ |2,0\rangle = \frac{1}{2}\left\{|+---\rangle + |-+--\rangle + |--+-\rangle + |---+\rangle\right\}$

اثر عملگر انتقال خودش را می دهد

$$\left(\langle 2,0|T|2,0\rangle\right) = (1) \begin{cases} \text{eigen value} = 1 \\ |k=0\rangle = |2,0\rangle \end{cases}$$

$*\ |2,-1\rangle = \frac{1}{2}\left(|+---\rangle + |-+--\rangle + |--+-\rangle + |---+\rangle\right)$

اثر عملگر انتقال خودش را می ده

$$\left(\langle 2,-1|T|2,-1\rangle\right) = (1) \begin{cases} \text{eigen Value} = 1 \\ |k=0\rangle = |2,-1\rangle \end{cases}$$

$*\ |2,-2\rangle = |----\rangle$

اثر عملگر انتقال خودش را می دهد

$$\left(\langle 2,-2|T|2,-2\rangle\right) = (1) \begin{cases} \text{eigenvalue} = 1 \\ |k=0\rangle = |2,-2\rangle \end{cases}$$

for $S_{tot}=1$ transition we have:

$S_z=+1$: $[T\otimes T]: |1,+1\rangle = \frac{1}{2}\left(|+-++\rangle + |-+++\rangle - |+++-\rangle - |++-+\rangle\right)$

$[T\otimes S]: |1,+1\rangle = \frac{1}{\sqrt{2}}\left(|+++-\rangle - |++-+\rangle\right)$

$[S\otimes T]: |1,+1\rangle = \frac{1}{\sqrt{2}}\left(|+-++\rangle - |-+++\rangle\right)$

$$\begin{bmatrix} {}_{TT}\langle 1,1|T|1,1\rangle_{TT} & {}_{TT}\langle 1,1|T|1,1\rangle_{TS} & {}_{TT}\langle 1,1|T|1,1\rangle_{ST} \\ {}_{TS}\langle 1,1|T|1,1\rangle_{TT} & {}_{TS}\langle 1,1|T|1,1\rangle_{TS} & {}_{TS}\langle 1,1|T|1,1\rangle_{ST} \\ {}_{ST}\langle 1,1|T|1,1\rangle_{TT} & {}_{ST}\langle 1,1|T|1,1\rangle_{TS} & {}_{ST}\langle 1,1|T|1,1\rangle_{ST} \end{bmatrix} = \begin{bmatrix} 0 & +\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ +\frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} = T_{1,1}$$

* حالت های انتقال یافته را در پایه استاندارد کسب تا ماتریس را بیابیم.

* $T|1,1\rangle_{TT} = \frac{1}{2}\left(|++-+\rangle + |+-++\rangle - |-+++\rangle - |+++-\rangle\right)$

* $T|1,1\rangle_{TS} = \frac{1}{\sqrt{2}}\left(|-+++\rangle - |+++-\rangle\right)$

* $T|1,1\rangle_{ST} = \frac{1}{\sqrt{2}}\left(|++-+\rangle - |+-++\rangle\right)$

حل بردار: $\det(T_{11} - \lambda \mathbb{1}) = 0 \Longrightarrow$

$$\begin{cases} \text{eigen value 1: } i \Rightarrow |K=\frac{\pi}{2}\rangle = \frac{1}{\sqrt{2}}|1,1\rangle_{TT} - \frac{i}{2}|1,1\rangle_{TS} + \frac{i}{2}|1,1\rangle_{ST} \\ \text{eigen value 2: } -i \Rightarrow |K=-\frac{\pi}{2}\rangle = -\frac{1}{\sqrt{2}}|1,1\rangle_{TT} + \frac{i}{2}|1,1\rangle_{TS} - \frac{i}{2}|1,1\rangle_{ST} \\ \text{eigen value 3: } -1 \Rightarrow |K=+\pi\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle_{TS} + |1,1\rangle_{ST}\right) \end{cases}$$

$S_z=0$: $[T\otimes T]: |1,0\rangle = \frac{1}{\sqrt{2}}\left(|--++\rangle - |++--\rangle\right)$

$[T\otimes S]: |1,0\rangle = \frac{1}{2}\left(|+-+-\rangle + |-++-\rangle - |+--+\rangle - |-+-+\rangle\right)$

$[S\otimes T]: |1,0\rangle = \frac{1}{2}\left(|+-+-\rangle - |-++-\rangle + |+--+\rangle - |-+-+\rangle\right)$

$$\begin{bmatrix} {}_{TT}\langle 1,0|T|1,0\rangle_{TT} & {}_{TT}\langle 1,0|T|1,0\rangle_{TS} & {}_{TT}\langle 1,0|T|1,0\rangle_{ST} \\ {}_{TS}\langle 1,0|T|1,0\rangle_{TT} & {}_{TS}\langle 1,0|T|1,0\rangle_{TS} & {}_{TS}\langle 1,0|T|1,0\rangle_{ST} \\ {}_{ST}\langle 1,0|T|1,0\rangle_{TT} & {}_{ST}\langle 1,0|T|1,0\rangle_{TS} & {}_{ST}\langle 1,0|T|1,0\rangle_{ST} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} = T_{1,0}$$

* حالت های انتقال یافته را در پایه استاندارد کسب تا ماتریس را بیابیم.

* $T|1,0\rangle_{TT} = \frac{1}{\sqrt{2}}\left(|+--+\rangle - |-++-\rangle\right)$

* $T|1,0\rangle_{TS} = \frac{1}{2}\left(|-+-+\rangle + |--++\rangle - |++--\rangle - |+-+-\rangle\right)$

* $T|1,0\rangle_{ST} = \frac{1}{2}\left(|-+-+\rangle - |--++\rangle + |++--\rangle - |+-+-\rangle\right)$

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قطری‌سازی : $\det(T_{1,0}-\lambda \mathbb{1})=0 \Rightarrow \begin{cases} \text{eigen value 1} : i \Rightarrow |K=\frac{\pi}{2}\rangle = \frac{-1}{\sqrt{2}}|1,0\rangle_{TT} - \frac{i}{2}|1,0\rangle_{TS} + \frac{i}{2}|1,0\rangle_{ST} \\ \text{eigen value 2} : -i \Rightarrow |K=\frac{-\pi}{2}\rangle = \frac{-1}{\sqrt{2}}|1,0\rangle_{TT} + \frac{i}{2}|1,0\rangle_{TS} - \frac{i}{2}|1,0\rangle_{ST} \\ \text{eigen value 3} : -1 \Rightarrow |K=\pi\rangle = \frac{1}{\sqrt{2}}\left(|1,0\rangle_{TS} + |1,0\rangle_{ST}\right) \end{cases}$

$S_z=-1$: $[T\otimes T] \Rightarrow |1,-1\rangle = \frac{1}{2}\left(|--+-\rangle + |---+\rangle - |+---\rangle - |-+--\rangle\right)$

$[T\otimes S] \Rightarrow |1,-1\rangle = \frac{1}{\sqrt{2}}\left(|--+-\rangle - |---+\rangle\right)$

$[S\otimes T] \Rightarrow |1,-1\rangle = \frac{1}{\sqrt{2}}\left(|+---\rangle - |-+--\rangle\right)$

$$\begin{bmatrix} {}_{TT}\langle 1,-1|T|1,-1\rangle_{TT} & {}_{TT}\langle 1,-1|T|1,-1\rangle_{TS} & {}_{TT}\langle 1,-1|T|1,-1\rangle_{ST} \\ {}_{TS}\langle 1,-1|T|1,-1\rangle_{TT} & {}_{TS}\langle 1,-1|T|1,-1\rangle_{TS} & {}_{TS}\langle 1,-1|T|1,-1\rangle_{ST} \\ {}_{ST}\langle 1,-1|T|1,-1\rangle_{TT} & {}_{ST}\langle 1,-1|T|1,-1\rangle_{TS} & {}_{ST}\langle 1,-1|T|1,-1\rangle_{ST} \end{bmatrix} = \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & -1/2 \end{bmatrix} = T_{1,-1}$$

حالت‌های انتقال یافته را در تابل قرار دهیم تا ماتریس را بیابیم.

$* \ T|1,-1\rangle_{TT} = \frac{1}{2}\left(|---+\rangle + |+---\rangle - |-+--\rangle - |--+-\rangle\right)$

$* \ T|1,-1\rangle_{TS} = \frac{1}{\sqrt{2}}\left(|---+\rangle - |+---\rangle\right)$

$* \ T|1,-1\rangle_{ST} = \frac{1}{\sqrt{2}}\left(|-+--\rangle - |--+-\rangle\right)$

قطری‌سازی : $\det(T_{1,-1}-\lambda \mathbb{1})=0 \quad \begin{cases} \text{eigenvalue1} : i \Rightarrow |K=\pi/2\rangle = \frac{-1}{\sqrt{2}}|1,-1\rangle_{TT} - \frac{i}{2}|1,-1\rangle_{TS} + \frac{i}{2}|1,-1\rangle_{ST} \\ \text{eigenvalue2} : -i \Rightarrow |K=-\pi/2\rangle = \frac{-1}{\sqrt{2}}|1,-1\rangle_{TT} + \frac{i}{2}|1,-1\rangle_{TS} - \frac{i}{2}|1,-1\rangle_{ST} \\ \text{eigen value 3} : -1 \Rightarrow |K=+\pi\rangle = \frac{1}{\sqrt{2}}\left(|1,-1\rangle_{TS} + |1,-1\rangle_{ST}\right) \end{cases}$

for $S_{tot}=0$ transition we have:

$S_z=0$: $[T\otimes T] : |0,0\rangle = \frac{1}{\sqrt{3}}\left\{|++--\rangle - \frac{1}{2}\left(|+-+-\rangle + |+--+\rangle + |-++-\rangle + |-+-+\rangle\right) + |--++\rangle\right\}$

$[S\otimes S] : |0,0\rangle = \frac{1}{2}\left\{|+-+-\rangle - |-++-\rangle - |+--+\rangle + |-+-+\rangle\right\}$

$$
\begin{bmatrix}
{}_{TT}\langle 0,0|T|0,0\rangle_{TT} & {}_{TT}\langle 0,0|T|0,0\rangle_{SS} \\
{}_{SS}\langle 0,0|T|0,0\rangle_{TT} & {}_{SS}\langle 0,0|T|0,0\rangle_{SS}
\end{bmatrix}
=
\begin{bmatrix}
-\frac{1}{2} & -\frac{\sqrt{3}}{2} \\
-\frac{\sqrt{3}}{2} & \frac{1}{2}
\end{bmatrix}
= T_{0,0}
$$

$$* T|0,0\rangle_{TT} = \frac{1}{\sqrt{3}}\left\{|-++-\rangle - \frac{1}{2}\left(|-+-+\rangle + |++--\rangle + |--++\rangle + |+-+-\rangle\right) + |+--+\rangle\right\}$$

$$* T|0,0\rangle_{SS} = \frac{1}{2}\left(|-+-+\rangle - |--++\rangle - |++--\rangle + |+-+-\rangle\right)$$

ویژه‌حالات: $\det(T_{0,0} - \lambda \mathbb{1}) = 0 \Rightarrow \begin{cases} \text{eigen value 1: } -1 \Rightarrow |K=\pi\rangle = \frac{\sqrt{3}}{2}|0,0\rangle_{TT} + \frac{1}{2}|0,0\rangle_{SS} \\ \text{eigen value 2: } +1 \Rightarrow |K=0\rangle = -\frac{\sqrt{3}}{2}|0,0\rangle_{SS} + \frac{1}{2}|0,0\rangle_{TT} \end{cases}$

حال که تمامی کت های انتقال یافته را محاسبه کردیم می‌توانیم هامیلتونی را بر آن ها اثر دهیم:

for $S_{tot} = 2$:

1: $H|K=0\rangle_{2,2} = |2,+2\rangle$

2: $H|K=0\rangle_{2,1} = |2,+1\rangle$

3: $H|K=0\rangle_{2,0} = |2,0\rangle$

4: $H|K=0\rangle_{2,-1} = |2,-1\rangle$

5: $H|K=0\rangle_{2,-2} = |2,-2\rangle$

for $S_{tot} = 1$:

1: $H\left|K=\frac{\pi}{2}\right\rangle_{1,1} = \underbrace{\frac{-1}{\sqrt{2}} \times 0}_{0} - \frac{i}{2}\left(\frac{-1}{2\sqrt{2}}|+++-\rangle + \frac{1}{2\sqrt{2}}|++-+\rangle - \frac{1}{2\sqrt{2}}|+-++\rangle + \frac{1}{2\sqrt{2}}|-+++\rangle\right)$

$+ \frac{i}{2}\left(-\frac{1}{2\sqrt{2}}|+-++\rangle + \frac{1}{2\sqrt{2}}|-+++\rangle + \frac{1}{2\sqrt{2}}|++-+\rangle - \frac{1}{2\sqrt{2}}|+++-\rangle\right)$

$= \underline{0}$

$$2: H|k=-\tfrac{\pi}{2}\rangle_{1,1} = \underbrace{\tfrac{-1}{\sqrt{2}}\times 0}_{0} + \tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\right]$$

$$-\tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\right]$$

$$= \underline{\underline{0}}$$

$$3: H|k=+\pi\rangle_{1,1} = \tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\right]$$

$$+\tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\right]$$

$$= \tfrac{1}{2}\left\{-|+++-\rangle + |++-+\rangle - |+-++\rangle + |-+++\rangle\right\}$$

$$4: H|k=\tfrac{\pi}{2}\rangle_{1,0} = \left(\underbrace{\tfrac{-1}{\sqrt{2}}\times 0}_{0}\right) - \tfrac{i}{2}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$+\tfrac{i}{2}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$= \underline{\underline{0}}$$

$$5: H|k=-\tfrac{\pi}{2}\rangle_{1,0} = \left(\underbrace{\tfrac{-1}{\sqrt{2}}\times 0}_{0}\right) + \tfrac{i}{2}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$-\tfrac{i}{2}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$= \underline{\underline{0}}$$

$$6: H|k=+\pi\rangle_{1,0} = \tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$+\tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2}\right)\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$= \tfrac{1}{\sqrt{2}}\left[-|+-+-\rangle + |-+-+\rangle\right]$$

$$7: H|k=\tfrac{\pi}{2}\rangle_{1,-1} = \left(\underbrace{\tfrac{-1}{\sqrt{2}}\times 0}_{0}\right) - \tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\right]$$

$$+\tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+---\rangle + |-+--\rangle - |--+-\rangle + |---+\rangle\right]$$

$$= \underline{\underline{0}}$$

$$8:\ H|k=-\tfrac{\pi}{2}\rangle_{1,-1} = \underbrace{\left(\tfrac{-1}{\sqrt{2}}\times 0\right)}_{0} + \tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\right]$$

$$-\tfrac{i}{2}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|+---\rangle + |-+--\rangle - |--+-\rangle + |---+\rangle\right]$$

$$= 0$$

$$9:\ H|k=+\pi\rangle_{1,-1} = \tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\right]$$

$$+\tfrac{1}{\sqrt{2}}\left(\tfrac{1}{2\sqrt{2}}\right)\left[-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\right]$$

$$= \tfrac{1}{2}\left\{-|--+-\rangle + |---+\rangle + |-+--\rangle - |+---\rangle\right\}$$

for $S_{tot} = 0$:

$$1:\ H|k=\pi\rangle_{0,0} = \tfrac{\sqrt{3}}{2}\times\tfrac{1}{\sqrt{3}}\left\{|+-+-\rangle - \tfrac{1}{2}\left(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right) + |-+-+\rangle\right\}$$

$$+\tfrac{1}{2}\times 1\left\{-|+-+-\rangle + \tfrac{1}{2}\left(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right) - |-+-+\rangle\right\}$$

$$= 0$$

$$2:\ H|k=0\rangle_{0,0} = -\tfrac{\sqrt{3}}{2}\times 1\left\{-|+-+-\rangle + \tfrac{1}{2}\left(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right) - |-+-+\rangle\right\}$$

$$\underbrace{+\tfrac{1}{2}\times\tfrac{1}{\sqrt{3}}}_{+\frac{\sqrt{3}}{6}}\left\{|+-+-\rangle - \tfrac{1}{2}\left(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right) + |-+-+\rangle\right\}$$

$$= \left(\tfrac{3\sqrt{3}}{6} + \tfrac{\sqrt{3}}{6}\right)|+-+-\rangle + \left(\tfrac{3\sqrt{3}}{6} - \tfrac{\sqrt{3}}{6}\right)\tfrac{1}{2}\left(|+--+\rangle + |-++-\rangle + |++--\rangle\right.$$

$$\left. + |--++\rangle\right) + \left(+\tfrac{3\sqrt{3}}{6} + \tfrac{\sqrt{3}}{6}\right)|-+-+\rangle$$

$$= +\tfrac{2}{\sqrt{3}}|+-+-\rangle - \tfrac{1}{\sqrt{3}}\left(|+--+\rangle + |-++-\rangle + |++--\rangle + |--++\rangle\right)$$

$$+\tfrac{2}{\sqrt{3}}|-+-+\rangle = -|0,0\rangle_{T,T} + \sqrt{3}\,|0,0\rangle_{S,S}$$

for $S_{tot}=1$: (1) $|k=\frac{\pi}{2}\rangle_{1,1} = -\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left[|+-++\rangle + |-+++\rangle - |+++-\rangle - |++-+\rangle\right]$

$$-\frac{i}{2}\left(\frac{1}{\sqrt{2}}\right)\left[|+++-\rangle - |++-+\rangle\right]$$

$$+\frac{i}{2}\left(\frac{1}{\sqrt{2}}\right)\left[|+-++\rangle - |-+++\rangle\right]$$

$$= \left(\frac{i-1}{2\sqrt{2}}\right)|+-++\rangle - \left(\frac{i+1}{2\sqrt{2}}\right)|-+++\rangle - \left(\frac{i-1}{2\sqrt{2}}\right)|+++-\rangle + \left(\frac{i+1}{2\sqrt{2}}\right)|++-+\rangle$$

(2) $|k=-\frac{\pi}{2}\rangle_{1,1} = -\left(\frac{i+1}{2\sqrt{2}}\right)|+-++\rangle + \left(\frac{i-1}{2\sqrt{2}}\right)|-+++\rangle + \left(\frac{i+1}{2\sqrt{2}}\right)|+++-\rangle - \left(\frac{i-1}{2\sqrt{2}}\right)|++-+\rangle$

(3) $|k=\pi\rangle_{1,1} = \frac{1}{2}\left(|+++-\rangle - |++-+\rangle + |+-++\rangle - |-+++\rangle\right)$

(4) $|k=\frac{\pi}{2}\rangle_{1,0} = -\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\left[|--++\rangle - |++--\rangle\right]$

$$-\frac{i}{2}\left(\frac{1}{2}\right)\left[|+-+-\rangle + |-++-\rangle - |+--+\rangle - |-+-+\rangle\right)$$

$$+\frac{i}{2}\left(\frac{1}{2}\right)\left[|+-+-\rangle - |-++-\rangle + |+--+\rangle - |-+-+\rangle\right)$$

$$= \frac{1}{2}\left[-|--++\rangle + |++--\rangle - i|-++-\rangle + i|+--+\rangle\right]$$

(5) $|k=-\frac{\pi}{2}\rangle_{1,0} = \frac{1}{2}\left[-|--++\rangle + |++--\rangle + i|-++-\rangle - i|+--+\rangle\right]$

(6) $|k=\pi\rangle_{1,0} = \frac{1}{\sqrt{2}}\left[|+-+-\rangle - |-+-+\rangle\right]$

(7) $|k=\frac{\pi}{2}\rangle_{1,-1} = -\frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)\left[|--+-\rangle + |---+\rangle - |+---\rangle - |-+--\rangle\right]$

$$-\frac{i}{2}\left(\frac{1}{\sqrt{2}}\right)\left[|--+-\rangle - |---+\rangle\right]$$

$$+\frac{i}{2}\left(\frac{1}{\sqrt{2}}\right)\left[|+---\rangle - |-+--\rangle\right]$$

$$= -\left(\frac{1+i}{2\sqrt{2}}\right)|--+-\rangle + \left(\frac{i-1}{2\sqrt{2}}\right)|---+\rangle + \left(\frac{i+1}{2\sqrt{2}}\right)|+---\rangle - \left(\frac{i-1}{2\sqrt{2}}\right)|-+--\rangle$$

(8) $|k=\frac{-\pi}{2}\rangle_{1,-1} = \left(\frac{i-1}{2\sqrt{2}}\right)|--+-\rangle - \left(\frac{i+1}{2\sqrt{2}}\right)|---+\rangle - \left(\frac{i-1}{2\sqrt{2}}\right)|+---\rangle + \left(\frac{i+1}{2\sqrt{2}}\right)|-+--\rangle$

(9) $|k=\pi\rangle_{1,-1} = \left(\frac{1}{2}\right)\left[|+---\rangle + |--+-\rangle - |---+\rangle - |-+--\rangle\right]$

اکنون ماتریس های مربوط به S_{tot} و S_z با k های مشخص را تشکیل می دهیم:

for $S_{tot}=2$:

$$\begin{array}{cc} & H|k=0\rangle_{2,2} \\ {}_{2,2}\langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & H|k=0\rangle_{2,1} \\ {}_{2,1}\langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & H|k=0\rangle_{2,0} \\ {}_{2,0}\langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & H|k=0\rangle_{2,-1} \\ {}_{2,-1}\langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & H|k=0\rangle_{2,-2} \\ {}_{2,-2}\langle k=0| & \begin{bmatrix} 1 \end{bmatrix} \end{array}$$

for $S_{tot}=1$:

$$\begin{array}{cc} & \begin{array}{ccc} H|k=\frac{\pi}{2}\rangle_{1,1} & H|k=-\frac{\pi}{2}\rangle_{1,1} & H|k=\pi\rangle_{1,1} \end{array} \\ \begin{array}{c} {}_{1,1}\langle k=\frac{\pi}{2}| \\ {}_{1,1}\langle k=-\frac{\pi}{2}| \\ {}_{1,1}\langle k=\pi| \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{ccc} H|k=\frac{\pi}{2}\rangle_{1,0} & H|k=-\frac{\pi}{2}\rangle_{1,0} & H|k=\pi\rangle_{1,1} \end{array} \\ \begin{array}{c} {}_{1,0}\langle k=\frac{\pi}{2}| \\ {}_{1,0}\langle k=-\frac{\pi}{2}| \\ {}_{1,0}\langle k=\pi| \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} H|k=\frac{\pi}{2}\rangle_{1,-1} & H|k=-\frac{\pi}{2}\rangle_{1,-1} & H|k=\pi\rangle_{1,-1} \end{array} \\ \begin{array}{c} {}_{1,-1}\langle k=\frac{\pi}{2}| \\ {}_{1,-1}\langle k=-\frac{\pi}{2}| \\ {}_{1,-1}\langle k=\pi| \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

for $S_{tot}=0$:

$$|k=\pi\rangle_{0,0} = \frac{1}{2}\left[|++--\rangle - |-++-\rangle - |+--+\rangle + |--++\rangle \right]$$

$$|k=0\rangle_{0,0} = \frac{1}{2\sqrt{3}}\left[|++--\rangle + |-++-\rangle + |+--+\rangle + |--++\rangle \right] - \frac{1}{\sqrt{3}}\left[|+-+-\rangle + |-+-+\rangle \right]$$

$$\begin{array}{cc} & \begin{array}{cc} H|k=\pi\rangle_{0,0} & H|k=0\rangle_{0,0} \end{array} \\ \begin{array}{c} {}_{0,0}\langle k=\pi| \\ {}_{0,0}\langle k=0| \end{array} & \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} \end{array}$$

همانطور که از محاسبات پیداست تمامی ماتریس ها قطری هستند و عناصر روی قطر ویژه مقادیر ما هستند
که عبارتند از: $1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, -1, -1, -1, -2$ (با تبهگنی های قبل یکسان است)