

علی بیجارو - علی موحیدی

حل قسمت ب پر دو:

(1)

$$H = \sum_{i=1}^4 S_i \cdot S_{i+1} = \sum_{i=1}^4 S_{z(i)} \cdot S_{z(i+1)} + \sum_{i=1}^4 \frac{1}{2} (S_{+i} S_{-(i+1)} + S_{-i} S_{+(i+1)})$$

عبارت بخش S_z :

$$2: |++++\rangle$$

$$1: |+++-\rangle \quad |+-++\rangle \quad |-+++\rangle$$

$$0: |++--\rangle \quad |--++\rangle \quad |+- -+\rangle \quad |-+-+\rangle \quad |+-+ -\rangle \quad |-+-+ -\rangle \quad |+-+ -\rangle$$

$$-1: |--+-\rangle \quad |--+ -\rangle \quad |-+ - -\rangle \quad |+- - -\rangle$$

$$-2: |----\rangle$$

$$H = (S_{1z} S_{2z} + S_{2z} S_{3z} + S_{3z} S_{4z} + S_{4z} S_{1z})$$

$$+ \frac{1}{2} \{ (S_1^+ S_2^- + S_1^- S_2^+) + (S_2^+ S_3^- + S_2^- S_3^+) + (S_3^+ S_4^- + S_3^- S_4^+) + (S_4^+ S_1^- + S_4^- S_1^+) \}$$

for $S_z = 2$:

$$H |++++\rangle = \left\{ \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right\} |++++\rangle$$

$$+ \frac{1}{2} \{ (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) \} = |++++\rangle + \frac{1}{2} |0\rangle$$

for $S_z = 1$:

$$H |+++-\rangle = \left\{ \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) \right\} |+++-\rangle$$

$$+ \frac{1}{2} \{ (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |+-+ -\rangle) + (|+-+ -\rangle + |0\rangle) \}$$

$$= 0 |+++-\rangle + \frac{1}{2} \{ |+++-\rangle + |+-+ -\rangle \}$$

$$H |+-+ -\rangle = \left\{ \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right\} |+-+ -\rangle$$

$$+ \frac{1}{2} \{ (|0\rangle + |0\rangle) + (|0\rangle + |+-+ -\rangle) + (|+-+ -\rangle + |0\rangle) + (|0\rangle + |0\rangle) \}$$

$$= 0 |+-+ -\rangle + \frac{1}{2} \{ |+-+ -\rangle + |+-+ -\rangle \}$$

$$\begin{aligned}
 H| - + + + \rangle &= \left\{ \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) \right\} | - + + + \rangle \\
 &+ \frac{1}{2} \left\{ (| + - + + \rangle + | 0 \rangle) + (| 0 \rangle + | 0 \rangle) + (| 0 \rangle + | 0 \rangle) + (| 0 \rangle + | + + + - \rangle) \right\} \\
 &= 0 | - + + + \rangle + \frac{1}{2} \{ | + - + + \rangle + | + + + - \rangle \}
 \end{aligned}$$

$$\begin{aligned}
 H| + - + + \rangle &= \left\{ \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right\} | + - + + \rangle \\
 &+ \frac{1}{2} \left\{ (| 0 \rangle + | - + + + \rangle) + (| + + - + \rangle + | 0 \rangle) + (| 0 \rangle + | 0 \rangle) + (| 0 \rangle + | 0 \rangle) \right\} \\
 &= 0 | + - + + \rangle + \frac{1}{2} \{ | - + + + \rangle + | + + - + \rangle \}
 \end{aligned}$$

for $S_z = 0$:

$$\begin{aligned}
 H| + - + - \rangle &= \left\{ \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) \right\} | + - + - \rangle \\
 &+ \frac{1}{2} \left\{ (| 0 \rangle + | - + + - \rangle) + (| + + - - \rangle + | 0 \rangle) + (| 0 \rangle + | + - + - \rangle) + (| 0 \rangle + | - - + + \rangle) \right\} \\
 &= -1 | + - + - \rangle + \frac{1}{2} \{ | - + + - \rangle + | + + - - \rangle + | + - + - \rangle + | - - + + \rangle \}
 \end{aligned}$$

$$\begin{aligned}
 H| - + + - \rangle &= \left\{ \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(-\frac{1}{2} \right) \right\} | - + + - \rangle \\
 &+ \frac{1}{2} \left\{ (| + - + - \rangle + | 0 \rangle) + (| 0 \rangle + | 0 \rangle) + (| 0 \rangle + | - + + - \rangle) + (| 0 \rangle + | 0 \rangle) \right\} \\
 &= 0 | - + + - \rangle + \frac{1}{2} \{ | + - + - \rangle + | - + + - \rangle \}
 \end{aligned}$$

$$\begin{aligned}
 H| - + - + \rangle &= \left\{ \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) + \left(-\frac{1}{2} \right) \left(\frac{1}{2} \right) + \left(\frac{1}{2} \right) \left(-\frac{1}{2} \right) \right\} | - + - + \rangle \\
 &+ \frac{1}{2} \left\{ (| + - - + \rangle + | 0 \rangle) + (| 0 \rangle + | - - + + \rangle) + (| - + + - \rangle + | 0 \rangle) + (| 0 \rangle + | + + - - \rangle) \right\} \\
 &= -1 | - + - + \rangle + \frac{1}{2} \{ | + - - + \rangle + | - - + + \rangle + | - + + - \rangle + | + + - - \rangle \}
 \end{aligned}$$

$$\begin{aligned}
 H|+-+> &= \left\{ \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \right\} |+-+> \\
 &+ \frac{1}{2} \left\{ (|0> + |1>) + (|0> + |1>) + (|+-> + |0>) + (|0> + |1>) \right\} \\
 &= 0 |+-+> + \frac{1}{2} \left\{ |+-> + |+-> \right\}
 \end{aligned}$$

$$\begin{aligned}
 H|---+> &= \left\{ \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \right\} |---+> \\
 &+ \frac{1}{2} \left\{ (|0> + |0>) + (|+-> + |0>) + (|0> + |0>) + (|0> + |+->) \right\} \\
 &= 0 |---+> + \frac{1}{2} \left\{ |+-> + |+-> \right\}
 \end{aligned}$$

$$\begin{aligned}
 H|++--> &= \left\{ \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) \right\} |++--> \\
 &+ \frac{1}{2} \left\{ (|0> + |0>) + (|0> + |+->) + (|0> + |0>) + (|+-> + |0>) \right\} \\
 &= 0 |++--> + \frac{1}{2} \left\{ |+-> + |+-> \right\}
 \end{aligned}$$

for $S_z = -1$:

$$\begin{aligned}
 H|----> &= \left\{ \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) \right\} |----> \\
 &+ \frac{1}{2} \left\{ (|0> + |0>) + (|0> + |0>) + (|+-> + |0>) + (|0> + |+->) \right\} \\
 &= 0 |----> + \frac{1}{2} \left\{ |+-> + |+-> \right\}
 \end{aligned}$$

$$\begin{aligned}
 H|--+> &= \left\{ \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) \right\} |--+> \\
 &+ \frac{1}{2} \left\{ (|0> + |0>) + (|+-> + |0>) + (|0> + |---->) + (|0> + |0>) \right\} \\
 &= 0 |--+> + \frac{1}{2} \left\{ |+-> + |----> \right\}
 \end{aligned}$$

$$\begin{aligned}
 H|+-\rightarrow\rangle &= \left\{ \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) \right\} |+-\rightarrow\rangle \\
 &+ \frac{1}{2} \left\{ (|++\rightarrow\rangle + |0\rangle) + (|0\rangle + |--\rightarrow\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) \right\} \\
 &= 0|+-\rightarrow\rangle + \frac{1}{2} \{ |++\rightarrow\rangle + |--\rightarrow\rangle \}
 \end{aligned}$$

$$\begin{aligned}
 H|+---\rangle &= \left\{ \left(+\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) \right\} |+---\rangle \\
 &+ \frac{1}{2} \left\{ (|0\rangle + |-+-\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|---\rangle + |0\rangle) \right\} \\
 &= 0|+---\rangle + \frac{1}{2} \{ |-+-\rangle + |---\rangle \}
 \end{aligned}$$

for $S_Z = -2$:

$$\begin{aligned}
 H|----\rangle &= \left\{ \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) \right\} |----\rangle \\
 &+ \frac{1}{2} \left\{ (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) + (|0\rangle + |0\rangle) \right\} = |----\rangle + \frac{1}{2}|0\rangle
 \end{aligned}$$

$$\text{for } S_Z = 2 \Rightarrow \langle + + + + | \begin{matrix} | + + + + \rangle \\ \text{نوع} \end{matrix} = [1]$$

$$\text{for } S_Z = 1 \Rightarrow \begin{matrix} & | + + + - \rangle & | + + - + \rangle & | + - + + \rangle & | - + + + \rangle \\ \begin{matrix} \langle + + + - | \\ \langle + + - + | \\ \langle + - + + | \\ \langle - + + + | \end{matrix} & \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix} \end{matrix}$$

$$\text{for } S_Z = -2 \Rightarrow \langle - - - - | \begin{matrix} | - - - - \rangle \\ \text{نوع} \end{matrix} = [1]$$

for $S_Z = -1$: \Rightarrow

$$\begin{array}{c}
 |---+ \rangle \quad |--+ \rangle \quad |-+- \rangle \quad |+--- \rangle \\
 \begin{array}{c}
 \langle---+| \\
 \langle--+| \\
 \langle+-| \\
 \langle+---|
 \end{array}
 \begin{bmatrix}
 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
 \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
 \frac{1}{2} & 0 & \frac{1}{2} & 0
 \end{bmatrix}
 \end{array}
 \quad (3)$$

for $S_Z = 0$: \Rightarrow

$$\begin{array}{c}
 |++-- \rangle \quad |+-+ \rangle \quad |+--+ \rangle \quad |-++ \rangle \quad |-+-+ \rangle \quad |--++ \rangle \\
 \begin{array}{c}
 \langle++--| \\
 \langle+-+| \\
 \langle+--+| \\
 \langle-++| \\
 \langle+-+| \\
 \langle--++|
 \end{array}
 \begin{bmatrix}
 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\
 \frac{1}{2} & -1 & \frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\
 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\
 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\
 \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\
 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0
 \end{bmatrix}
 \end{array}$$