$$H=\frac{-eg}{2m_ec}\vec{S}\cdot\vec{B}=\frac{|e|}{m_ec}\left(B_0S_z+B_1S_x\cos(\omega t)+B_1S_y\sin(\omega t)\right)$$

$$H=H_0+V(t)=\begin{cases}H_0=\dfrac{|e|B_0}{m_ec}\left(\dfrac{\hbar}{2}\right)\left\{|+\rangle\langle+|-|-\rangle\langle-|\right\}\\[2ex] V(t)=\underbrace{\dfrac{|e|B_1}{m_ec}\left(\dfrac{\hbar}{2}\right)}_{\gamma}\left\{e^{i\omega t}|-\rangle\langle+|+e^{-i\omega t}|+\rangle\langle-|\right\}\end{cases}$$

$\ast$ we assume: $\begin{cases}|+\rangle\Rightarrow|1\rangle\\ |-\rangle\Rightarrow|2\rangle\end{cases}$

Calculate ω_{21}: $\omega_{21}=\dfrac{E_2-E_1}{\hbar}=\dfrac{\left(\dfrac{-|e|B_0\hbar}{2m_ec}\right)-\left(\dfrac{|e|B_0\hbar}{2m_ec}\right)}{\hbar}=-\dfrac{|e|B_0}{m_ec}=\omega_{-+}$

$$\omega_{21}=-\omega_{12}\Rightarrow\omega_{12}=\frac{|e|B_0}{m_ec}=\omega_{+-}$$

$$\tilde{\gamma}=\frac{\gamma}{\hbar}=\frac{|e|B_1}{2m_ec}$$

Using past relationships, we have:

$$\left.\begin{array}{l} i\hbar\dfrac{\partial}{\partial t}|\alpha,t_0;t\rangle_S=H(t)|\alpha,t_0;t\rangle_S\\[1ex] |\alpha,t_0,t\rangle_S=e^{-\frac{i}{\hbar}H_0(t-t_0)}|\alpha;t_0;t\rangle_I\end{array}\right.\Rightarrow\begin{cases}i\hbar\dfrac{\partial}{\partial t}|\alpha,t_0;t\rangle_I=i\hbar\dfrac{\partial}{\partial t}\left(e^{\frac{i}{\hbar}H_0(t-t_0)}|\alpha,t_0;t\rangle\right)\\[1ex] V_I(t)=e^{\frac{i}{\hbar}H_0(t-t_0)}V_S(t)e^{-\frac{i}{\hbar}H_0(t-t_0)}\end{cases}$$

در نمایش دار: $i\hbar\,\partial/\partial t\,|\alpha,t_0;t\rangle_I=V_I(t)|\alpha,t_0;t\rangle_I$

$\ast$ $|\alpha,t_0;t\rangle_I=C_1(t)|+\rangle+C_2(t)|-\rangle$

$\ast$ $\langle\alpha,t_0;t|=C_1^*(t)\langle+|+C_2^*(t)\langle-|$

$$\Longrightarrow i\hbar\begin{pmatrix}\dot{C}_1\\ \dot{C}_2\end{pmatrix}=\begin{pmatrix}0 & \gamma e^{i\omega t}e^{i\omega_{12}t}\\ \gamma e^{-i\omega t}e^{i\omega_{21}t} & 0\end{pmatrix}\begin{pmatrix}C_1\\ C_2\end{pmatrix}$$

$$\Longrightarrow\begin{cases}i\hbar\dot{C}_1=\gamma e^{i(\overbrace{\omega-\omega_{21}}^{\tilde{\omega}})t}C_2=\gamma e^{i\tilde{\omega}t}C_2 & \text{(I)}\\[1ex] i\hbar\dot{C}_2=\gamma e^{-i(\overbrace{\omega-\omega_{21}}^{\tilde{\omega}})t}C_1=\gamma e^{-i\tilde{\omega}t}C_1 & \text{(II)}\end{cases}$$

We already have c_2 relation:

$$c_2(t) = -\frac{\tilde{\gamma}}{2\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t}\left(e^{i\Omega_1 t} - e^{-i\Omega_1 t}\right) = \frac{\tilde{\gamma}}{i\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t}\sin(\Omega_1 t)$$

* Use relation II and calculate c_1:

$$c_1(t) = \frac{i\hbar}{\gamma} e^{i\tilde{\omega}t}\dot{c}_2(t)$$

$$\dot{c}_2(t) = -\frac{\tilde{\gamma}}{2\Omega_1}\left\{\left(-i\frac{\tilde{\omega}}{2}\right)e^{-i\frac{\tilde{\omega}}{2}t}\left(e^{i\Omega_1 t} - e^{-i\Omega_1 t}\right) + (i\Omega_1)e^{-i\frac{\tilde{\omega}}{2}t}\left(e^{i\Omega_1 t} + e^{-i\Omega_1 t}\right)\right\}$$

$$= -\frac{\tilde{\gamma}}{2\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t}\left\{\left(i\Omega_1 - i\frac{\tilde{\omega}}{2}\right)e^{i\Omega_1 t} + \left(i\Omega_1 + i\frac{\tilde{\omega}}{2}\right)e^{-i\Omega_1 t}\right\}$$

$$\Longrightarrow c_1(t) = -\frac{\tilde{\gamma}}{2\Omega_1} e^{i\frac{\tilde{\omega}}{2}t}\left\{\overbrace{\left(\frac{-\Omega_1 + \frac{\tilde{\omega}}{2}}{\tilde{\gamma}}\right)}^{r_\beta} e^{i\Omega_1 t} - \overbrace{\left(\frac{\Omega_1 + \frac{\tilde{\omega}}{2}}{\tilde{\gamma}}\right)}^{r_\alpha} e^{-i\Omega_1 t}\right\}$$

$$= -\frac{\tilde{\gamma}}{2\Omega_1} e^{i\frac{\tilde{\omega}}{2}t}\left\{r_\beta e^{i\Omega_1 t} - r_\alpha e^{-i\Omega_1 t}\right\}$$

برای اینکه درستی رابطه $C_1(t)$ را اثبات کنیم داریم:

$$|c_1(t)|^2 + |c_2(t)|^2 = 1$$

$$|c_2(t)|^2 = \left(\frac{\tilde{\gamma}}{i\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t}\sin\Omega_1 t\right)\left(-\frac{\tilde{\gamma}}{i\Omega_1} e^{i\frac{\tilde{\omega}}{2}t}\sin\Omega_1 t\right) = \left|\left(\frac{\tilde{\gamma}}{\Omega_1}\right)^2\sin^2\Omega_1 t\right|$$

$$|c_1(t)|^2 = \left|\left(\frac{\tilde{\gamma}}{2\Omega_1}\right)^2\left\{\overbrace{r_\beta^2 + r_\alpha^2}^{2\left(\frac{\Omega_1^2 + \left(\frac{\tilde{\omega}}{2}\right)^2}{\tilde{\gamma}^2}\right)} - \overbrace{r_\alpha r_\beta}^{(-1)}\overbrace{\left(e^{2i\Omega_1 t} + e^{-2i\Omega_1 t}\right)}^{(2\cos(2\Omega_1 t))}\right\}\right|$$

$$= \left|\left(\frac{\tilde{\gamma}}{\cancel{2}\Omega_1}\right)^2\left\{\cancel{2}\left(\frac{\Omega_1^2 + \left(\frac{\tilde{\omega}}{2}\right)^2}{\tilde{\gamma}^2}\right) + \cancel{2}\overbrace{\cos(2\Omega_1 t)}^{1 - 2\sin^2(\Omega_1 t)}\right\}\right|$$

$$= \left|\left(\frac{\cancel{\tilde{\gamma}^2}}{2\Omega_1^2}\right)\frac{1}{\cancel{\tilde{\gamma}^2}}\left\{\Omega_1^2 + \overbrace{\frac{\tilde{\omega}^2}{4} + \tilde{\gamma}^2}^{\Omega_1^2} - 2\tilde{\gamma}^2\sin^2\Omega_1 t\right\}\right|$$

$$= \left|\left(\frac{1}{2\Omega_1^2}\right)\left\{2\Omega_1^2 - 2\tilde{\gamma}^2\sin^2(\Omega_1 t)\right\}\right| = \left|1 - \frac{\tilde{\gamma}^2}{\Omega_1^2}\sin^2\Omega_1 t\right|$$

2)

درستی دارد : $|C_1(t)|^2 + |C_2(t)|^2 = 1 - \left(\frac{\gamma}{\Omega_1}\right)^2 \sin^2 \Omega_1 t + \left(\frac{\gamma}{\Omega_1}\right)^2 \sin^2 \Omega_1 t = 1$ ✓

$* \quad |C_1(t)|^2 = 1 - |C_2(t)|^2$

حال به محاسبه‌ی $\langle S \rangle$ می‌پردازیم :

$$\langle S_z \rangle = \langle \alpha, t_0; t | S_z | \alpha, t_0; t \rangle = \langle \alpha, t_0; t | \left(\frac{\hbar}{2}\right) \left[|+\rangle\langle +| - |-\rangle\langle -| \right] | \alpha, t_0; t \rangle$$

$$= \left(\frac{\hbar}{2}\right)\left(-|C_2|^2 + |C_1|^2\right) = \left(\frac{\hbar}{2}\right)\left(1 - 2|C_2(t)|^2\right)$$

$$\langle S_y \rangle = \langle \alpha, t_0; t | \left(\frac{\hbar}{2}\right) \left[i|-\rangle\langle +| - i|+\rangle\langle -| \right] | \alpha, t_0; t \rangle$$

$$= \langle \alpha, t_0; t | \left(\frac{\hbar}{2}\right) \left(i|-\rangle C_1(t) - i|+\rangle C_2(t) \right) = \left(\frac{\hbar}{2}\right)\left(i C_1(t) C_2^*(t) - i C_2(t) C_1^*(t) \right)$$

$$\langle S_x \rangle = \langle \alpha, t_0; t | \left(\frac{\hbar}{2}\right) \left[|+\rangle\langle -| + |-\rangle\langle +| \right] | \alpha, t_0; t \rangle$$

$$= \langle \alpha, t_0; t | \left(\frac{\hbar}{2}\right) \left(C_2(t)|+\rangle + C_1(t)|-\rangle \right) = \left(\frac{\hbar}{2}\right)\left(C_1(t) C_2^*(t) + C_2(t) C_1^*(t) \right)$$

حال به محاسبه‌ی $C_1(t)C_2^*(t)$ و $C_2(t)C_1^*(t)$ می‌پردازیم :

$$C_2(t) = \frac{\gamma}{i\hbar\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t} \sin \Omega_1 t \qquad \& \qquad C_2^*(t) = -\frac{\gamma}{i\hbar\Omega_1} e^{i\frac{\tilde{\omega}}{2}t} \sin \Omega_1 t$$

$$C_1(t) = \frac{-\gamma}{2\hbar\Omega_1} e^{i\frac{\tilde{\omega}}{2}t} \left(r_\beta e^{i\Omega_1 t} - r_\alpha e^{-i\Omega_1 t} \right) \quad \& \quad C_1^*(t) = -\frac{\gamma}{2\hbar\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t} \left(r_\beta e^{-i\Omega_1 t} - r_\alpha e^{i\Omega_1 t} \right)$$

$$C_2(t) C_1^*(t) = \left\{ \frac{\gamma}{i\hbar\Omega_1} e^{-i\frac{\tilde{\omega}}{2}t} \sin \Omega_1 t \right\} \left\{ -\frac{\gamma}{2\hbar\Omega_1} \left(r_\beta e^{-i\Omega_1 t} - r_\alpha e^{i\Omega_1 t} \right) \times e^{-i\frac{\tilde{\omega}}{2}t} \right\}$$

$$= \underbrace{-\frac{\gamma^2}{2i(\hbar\Omega_1)^2}}_{K} \underbrace{\sin(\Omega_1 t) \left(r_\beta e^{-i\Omega_1 t} - r_\alpha e^{i\Omega_1 t} \right) e^{-i\tilde{\omega} t}}_{B(t)} = -K B(t)$$

$$C_1(t) C_2^*(t) = \left\{ \frac{-\gamma}{i\hbar\Omega_1} \sin(\Omega_1 t) e^{i\frac{\tilde{\omega}}{2}t} \right\} \left\{ \frac{-\gamma}{2\hbar\Omega_1} \left(r_\beta e^{i\Omega_1 t} - r_\alpha e^{-i\Omega_1 t} \right) e^{i\frac{\tilde{\omega}}{2}t} \right\}$$

$$= \underbrace{\frac{\gamma^2}{2i(\hbar\Omega_1)^2}\sin(\Omega_1 t)}_{K}\underbrace{\left(r_\beta e^{i\Omega_1 t} - r_\alpha e^{-i\Omega_1 t}\right)e^{i\tilde{\omega} t}}_{A(t)} = K\,A(t)$$

می توان نتیجه گیری کرد که :

$$* \left(C_2(t)\,C_1^*(t)\right)^* = C_1(t)\,C_2^*(t)$$

حال به ادامه ی محاسبه ی $\langle S_y\rangle$ و $\langle S_x\rangle$ می پردازیم :

$$\langle S_y\rangle = \left(\frac{\hbar}{2}\right) i\left(C_1(t)C_2^*(t) - C_2(t)C_1^*(t)\right) = \left(\frac{\hbar}{2}\right) i\left(K(A(t)+B(t))\right) = \left(\frac{\hbar}{2}\right) iK\left[A(t)+B(t)\right]$$

$$= \sin(\Omega_1 t)\frac{\gamma^2}{2(\hbar\Omega_1)^2}\left(\frac{\hbar}{2}\right)\left[r_\alpha\underbrace{\left(e^{i(\Omega_1-\tilde{\omega})t} - e^{-i(\Omega_1-\tilde{\omega})t}\right)}_{-2\cos(\Omega_1-\tilde{\omega})t} + r_\beta\underbrace{\left(e^{-i(\Omega_1+\tilde{\omega})t} + e^{i(\Omega_1+\tilde{\omega})t}\right)}_{2\cos(\Omega_1+\tilde{\omega})t}\right]$$

$$= \left(\frac{\hbar}{2}\right)\sin(\Omega_1 t)\frac{\tilde{\gamma}^2}{\Omega_1^2}\left(r_\beta\cos(\Omega_1+\tilde{\omega})t - r_\alpha\cos(\Omega_1-\tilde{\omega})t\right) \quad (\mathrm{I})$$

می توان رابطه ی I را ساده تر کرد که خواهیم داشت :

$$(\mathrm{II})\; r_\beta\cos(\Omega_1+\tilde{\omega})t - r_\alpha\cos(\Omega_1-\tilde{\omega})t = \frac{1}{\tilde{\gamma}}\left\{\overbrace{\left(-\Omega_1+\frac{\tilde{\omega}}{2}\right)\cos(\Omega_1+\tilde{\omega})t}^{(i)} - \overbrace{\left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\cos(\Omega_1-\tilde{\omega})t}^{(ii)}\right\}$$

$$(i):\; -\left(\Omega_1-\frac{\tilde{\omega}}{2}\right)\cos(\Omega_1+\tilde{\omega})t = -\left(\Omega_1-\frac{\tilde{\omega}}{2}\right)\left\{\cos\Omega_1 t\cos\tilde{\omega}t - \sin\Omega_1 t\sin\tilde{\omega}t\right\}$$

$$(ii):\; \left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\cos(\Omega_1-\tilde{\omega})t = \left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\left\{\cos\Omega_1 t\cos\tilde{\omega}t + \sin\Omega_1 t\sin\tilde{\omega}t\right\}$$

$$\Longrightarrow\; (i - ii) = -2\Omega_1\cos\Omega_1 t\cos\tilde{\omega}t - \tilde{\omega}\sin\Omega_1 t\sin\tilde{\omega}t$$

$$\langle S_y\rangle = \left(\frac{\hbar}{2}\right)\frac{\tilde{\gamma}}{\Omega_1^2}\sin\Omega_1 t\left(-2\Omega_1\cos\Omega_1 t\cos\tilde{\omega}t - \tilde{\omega}\sin\Omega_1 t\sin\tilde{\omega}t\right)$$

* با جایگذاری رابطه II در I داریم

$$= \left(\frac{\hbar}{2}\right)\left(-\frac{2\tilde{\gamma}}{\Omega_1}\overbrace{\sin\Omega_1 t\cos\Omega_1 t}^{\frac{1}{2}\sin(2\Omega_1 t)}\cos\tilde{\omega}t - \frac{\tilde{\omega}\tilde{\gamma}}{\Omega_1^2}\sin^2\Omega_1 t\sin\tilde{\omega}t\right)$$

در نتیجه داریم : $\langle S_y\rangle = \left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_1}\sin(2\Omega_1 t)\cos\tilde{\omega}t - \frac{\tilde{\omega}\tilde{\gamma}}{\Omega_1^2}\sin^2\Omega_1 t\sin\tilde{\omega}t\right\}$

$$\langle S_x\rangle=\left(\frac{\hbar}{2}\right)\left(c_1c_2^*+c_2c_1^*\right)=\left(\frac{\hbar}{2}\right)K_1\left[A(t)-B(t)\right]=K_1\left(\frac{\hbar}{2}\right)\Bigg\{\underbrace{r_\beta\left(e^{i(\Omega_1+\tilde{\omega})t}-e^{-i(\Omega_1+\tilde{\omega})t}\right)}_{r_\beta\,2i\sin(\Omega_1+\tilde{\omega})t}$$

$$-\underbrace{r_\alpha\left(e^{-i(\Omega_1-\tilde{\omega})t}-e^{i(\Omega_1-\tilde{\omega})t}\right)}_{-r_\alpha\,2i\sin(\Omega_1-\tilde{\omega})t}\Bigg\}$$

$$=\frac{\tilde{\gamma}}{\Omega_1^2}\sin(\Omega_1 t)\left\{r_\beta\sin(\Omega_1+\tilde{\omega})t+r_\alpha\sin(\Omega_1-\tilde{\omega})t\right\}\quad \mathrm{III}$$

می‌توان رابطه‌ی III را ساده‌تر کرد که خواهیم داشت:

$$\mathrm{IV}\quad r_\beta\sin(\Omega_1+\tilde{\omega})t+r_\alpha\sin(\Omega_1-\tilde{\omega})t=\frac{1}{\tilde{\gamma}}\Bigg\{\overbrace{\left(-\Omega_1+\frac{\tilde{\omega}}{2}\right)\sin(\Omega_1+\tilde{\omega})t}^{(i)}+\overbrace{\left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\sin(\Omega_1-\tilde{\omega})t}^{(ii)}\Bigg\}$$

$$(i):\ -\left(\Omega_1-\frac{\tilde{\omega}}{2}\right)\sin(\Omega_1+\tilde{\omega})t=-\left(\Omega_1-\frac{\tilde{\omega}}{2}\right)\left\{\sin\Omega_1 t\cos\tilde{\omega}t+\cos\Omega_1 t\sin\tilde{\omega}t\right\}$$

$$(ii):\ \left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\sin(\Omega_1-\tilde{\omega})t=\left(\Omega_1+\frac{\tilde{\omega}}{2}\right)\left\{\sin\Omega_1 t\cos\tilde{\omega}t-\cos\Omega_1 t\sin\tilde{\omega}t\right\}$$

$$\Longrightarrow (i+ii)=-2\Omega_1\cos\Omega_1 t\sin\tilde{\omega}t+\tilde{\omega}\sin\Omega_1 t\cos\tilde{\omega}t$$

$$\langle S_x\rangle=\left(\frac{\hbar}{2}\right)\frac{\tilde{\gamma}}{\Omega_1^2}\sin(\Omega_1 t)\left(-2\Omega_1\cos\Omega_1 t\sin\tilde{\omega}t+\tilde{\omega}\sin\Omega_1 t\cos\tilde{\omega}t\right)$$

* با جایگذاری IV در III داریم

$$=\left(\frac{\hbar}{2}\right)\left(-\frac{2\tilde{\gamma}}{\Omega_1}\overbrace{\cos\Omega_1 t\sin\Omega_1 t}^{\frac{1}{2}\sin(2\Omega_1 t)}\sin\tilde{\omega}t+\frac{\tilde{\gamma}\tilde{\omega}}{\Omega_1^2}\sin^2\Omega_1 t\cos\tilde{\omega}t\right)$$

در نتیجه داریم: $\langle S_x\rangle=\left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_1}\left(\sin(2\Omega_1 t)\sin\tilde{\omega}t\right)+\frac{\tilde{\omega}\tilde{\gamma}}{\Omega_1^2}\left(\sin^2\Omega_1 t\cos\tilde{\omega}t\right)\right\}$

روابطی که در کد برای رسم استفاده کردیم عبارتند از:

$$\langle S_x\rangle=\left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_1}\left(\sin(2\Omega_1 t)\sin\tilde{\omega}t\right)+\frac{\tilde{\omega}\tilde{\gamma}}{\Omega_1^2}\left(\sin^2\Omega_1 t\cos\tilde{\omega}t\right)\right\}$$

$$\langle S_y\rangle=\left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_1}\left(\sin(2\Omega_1 t)\cos\tilde{\omega}t\right)-\frac{\tilde{\omega}\tilde{\gamma}}{\Omega_1^2}\left(\sin^2\Omega_1 t\sin\tilde{\omega}t\right)\right\}$$

$$\langle S_z\rangle=\left(\frac{\hbar}{2}\right)\left\{1-2\left(\frac{\tilde{\gamma}}{\Omega_1}\right)^2\sin^2\Omega_1 t\right\}$$

$$\langle S^2\rangle=\left(\langle S_x^2\rangle+\langle S_y^2\rangle+\langle S_z^2\rangle\right)\Rightarrow|\langle S\rangle|=\sqrt{\langle S_x\rangle^2+\langle S_y\rangle^2+\langle S_z\rangle^2}$$

$$x_1 = \frac{\tilde{\gamma}}{\omega_{21}} = 0.1 \qquad \& \qquad x_2 = \frac{\omega}{\omega_{21}} = 0.6,\ 0.8,\ \underbrace{1}_{\text{resonance}}$$

* $\omega = x_2\,\omega_{21}$
* $\tilde{\omega} = \omega - \omega_{21} = \omega_{21}\left(\frac{\omega}{\omega_{21}} - 1\right) = \omega_{21}(x_2 - 1)$
* $\tilde{\gamma} = x_1\,\omega_{21}$
* $\Omega = \sqrt{\tilde{\gamma}^2 + \left(\frac{\omega - \omega_{21}}{2}\right)^2} = \omega_{21}\sqrt{\left(\frac{\tilde{\gamma}}{\omega_{21}}\right)^2 + \left(\frac{\frac{\omega}{\omega_{21}} - 1}{2}\right)^2} = \omega_{21}\left(x_1^2 + \left(\frac{x_2 - 1}{2}\right)^2\right)^{1/2}$

* از روابط گفته شده در این قسمت برای رسم $\frac{\langle S\rangle}{\frac{\hbar}{2}}$ برحسب زمان برای حالت مختلف ω_{21} استفاده می‌کنیم

بدون بعد می‌شود

* مقادیر پیش‌فرضی که برای رسم استفاده می‌کنیم عبارتند از:

* $t = 30\,s$
* $x_1 = 0.1$
* $x_2 = 0.6,\ 0.8,\ 1$

نکته‌ای که هنگام ترسیم $\frac{\langle S\rangle}{(\frac{\hbar}{2})}$ برحسب t باید به آن توجه کرد عبارت است از:

$$H = \overbrace{\frac{\hbar}{2}\omega_0\,\sigma_z}^{H_0} + \overbrace{\frac{\hbar}{2}\tilde{\gamma}(\sigma_x \cos\omega t + \sigma_y \sin\omega t)}^{V(t)}$$

$$* \begin{cases} \omega_0 = \alpha B_0 \\ \tilde{\gamma} = \alpha B_1 \end{cases} ;\ \alpha = \frac{-e g}{2 m_e c}$$

1) تصویر شرودینگر (چارچوب آزمایشگاه)

$$|\psi_s(t)\rangle = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} \qquad \langle S\rangle_{lab} = \frac{\hbar}{2}\langle\psi_s|\sigma|\psi_s\rangle$$

مشاهده‌پذیرهای عرضی که از این دامنه به دست می‌آیند به سمت در فرکانس z حول محور (ω_0) می‌چرخند.

2) تصویر دیراک (چارچوب چرخان)

$$|\psi_I(t)\rangle = e^{iH_0 t/\hbar}|\psi_s(t)\rangle = e^{i\omega_0 t\frac{\sigma_z}{2}}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

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$$V_I(t) = e^{-iH_0 t/\hbar}\, V(t)\, e^{iH_0 t/\hbar} = \frac{\hbar\tilde{\gamma}}{2}\left(\sigma_x \cos\tilde{\omega} t + \sigma_y \sin\tilde{\omega} t\right)$$

$* \ \tilde{\omega} = \omega - \omega_0$ $\qquad\qquad * \ \Omega_R = \left(\tilde{\gamma}^2 + \left(\frac{\tilde{\omega}}{2}\right)^2\right)^{1/2}$

حال عامل سریع $e^{\pm i\omega_0 t}$ حذف شده و بردار بلوخ در این چارچوب با اختلاف فرکانس $\tilde{\omega}$ و فرکانس رابی Ω_R در حال چرخش است.

منظور چارچوب چرخان است.

درنتیجه مسئله به این شکل حل می‌شود و مقدار میانگین $\langle S_x\rangle^R$، $\langle S_y\rangle^R$، $\langle S_z\rangle^R$ که قبلاً به دست آوردیم در تصویر دیراک مشخص می‌شود.

برای اینکه بتوان رسم را در چارچوب ثابت یعنی آزمایشگاه انجام داد باید بین چارچوب چرخان و آزمایشگاه تبدیل زد که داریم:

یک چرخش به اندازه $\omega_0 t$ حول محور $+\hat{z}$، محورهای چرخان را به حالت محور آزمایشگاه بازمی‌گرداند

$$\langle S_x\rangle_{lab} = \cos\omega_0 t\, S_x^R - \sin\omega_0 t\, S_y^R$$

$$\langle S_y\rangle_{lab} = \sin\omega_0 t\, S_x^R + \cos\omega_0 t\, S_y^R$$

$$\langle S_z\rangle_{lab} = S_z^R$$

درنتیجه داریم:

$$\omega t = \tilde{\omega} t + \omega_0 t$$

این دقیقاً همان فازی است که در رسم محورهای ثابت x و y و z ظاهر می‌شود.

درنتیجه برای اینکه که درست دوران کند باید در انتها این تبدیل را انجام دهیم:

$$\langle S_x\rangle_{lab} = \left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_R}\sin(2\Omega_R t)\sin\omega t + \frac{\tilde{\gamma}\tilde{\omega}}{\Omega_R^2}\sin^2(\Omega_R t)\cos\omega t\right\}$$

$$\langle S_y\rangle_{lab} = \left(\frac{\hbar}{2}\right)\left\{-\frac{\tilde{\gamma}}{\Omega_R}\sin(2\Omega_R t)\cos\omega t - \frac{\tilde{\gamma}\tilde{\omega}}{\Omega_R^2}\sin^2(\Omega_R t)\cos\omega t\right\}$$

$$\langle S_z\rangle_{lab} = \left(\frac{\hbar}{2}\right)\left\{1 - 2\left(\frac{\gamma}{\Omega_R}\right)^2 \sin^2(\Omega_R t)\right\}$$

* منظور از ω_0 همان ω_{21} می‌باشد.