Boğaziçi University
Department of Industrial Engineering
IE 313 Supply Chain Management
Assignment 3

There are various formulations of the travelling salesman problem. When $x_{ij} = 1$ if the salesperson goes from city i directly to city j; 0 otherwise (x_{ii} are not defined or are very large numbers), then the formulation can be developed as:

$$\begin{array}{llll} (1) & \min & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} & \\ (2) & \text{s.t.} & \sum_{j:j \neq i} x_{ij} = 1 & j = 1, 2, \cdots, n \\ (3) & & \sum_{i:i \neq j} x_{ij} = 1 & i = 1, 2, \cdots, n \\ & & x_{ij} \in \{0, 1\} & \forall i, j \end{array}$$

This is an assignment model. The solution may lead to *sub-tours.* To avoid sub tours let S be a non-empty strict subset of $N = \{1, 2, \cdots, n\}$ and add the following constraint.

$$(4) \qquad \sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1 \text{ for } S \subset N, 2 \leq |S| \leq n - 1$$

Constraints (2), (3), (4) (the sub tour elimination constraint), together with binary restrictions on the variables, and objective (1) is a valid formulation of the TSP. The trouble with this formulation is there are too many sub tour elimination constraints to write. A strategy that is usually employed is to solve the assignment model, detect sub tours, add only relevant sub tour elimination constraints and continue in this manner until you get a solution without a sub tour.

An alternative formulation is the sequential formulation of Miller, Tucker, Zemlin (1960) (MTZ). They define a new decision variable u_i to indicate the sequence in which city i is visited ($i = 2, \cdots, n$). They replace the sub tour elimination constraints (4) by:

$$(5) \qquad u_i - u_j + n x_{ij} \leq n - 1, \qquad i \neq j, i, j = 2, 3, \cdots n$$

(1) subject to (2), (3), (5) together with binary restrictions on x_{ij} variables, nonnegative u_i's, is also a valid formulation of the TSP. A slightly stronger version of the MTZ constraints is given by Desrochers and Laporte (1991) as:

$$(6) \qquad u_i - u_j + (n-1)x_{ij} + (n-3)x_{ji} \leq n - 2, \qquad i \neq j, i, j = 2, 3, \cdots n$$

which yields (1) subject to (2), (3), (6). These formulations solve small instances of the TSP efficiently but may not be able to give a solution for large instances (since its linear programming relaxation is weak and it may take a long time for an IP solver to find an integer solution).

Given this background,

(1) Choose a formulation and a strategy to solve TSP. Download TR81 cities data from the course web page.
(2) Solve the TSP for randomly chosen 15 cities. Report your findings.
(3) Solve the TSP for 81 cities. Report your findings.
(4) There is a 5-point bonus if you find the optimal TSP tour and plot it on the map of Turkey.