OR 7310-LOGISTICS WAREHOUSING AND SCHEDULING

FALL 2022

A Telecommunications Network Design Problem

Team Members:

Alican Yilmaz

Sai Charan Reddy Chavvareddy

Harsha sai Kanuru

Jay Walendra

Instructor: Ozlem Ergun

Outline of the Report

Problem Definition:

The problem in hand is a combination of facility location and minimum cost flow problem in the context of telecommunication networks. This is an NP-Hard problem, which means there is no polynomial time algorithm which exactly solves it. Thus, a heuristic approach must be developed to solve the problem within a good enough optimality gap. Both of the instances consist of cycles with different numbers of hubs which are denoted as capital letters. Each hub has potential ADM locations through which the data is transferred. The objective of the problem is satisfying the hub demands without violating the problem specific constraints. The details of the demands, costs, constraints as well as the mathematical model of the problem were given to us beforehand in the project description. The decision variables, objective and the constraints of the problem can be summarized as follows:

Variables:
Decision Variables
• One binary variable per Type 1 node (each valid hub/ring combination), to show whether an ADM should be installed there or not.
• One non-negative variable per commodity/arc pair to show how much of each commodity flows from the arc's start node to the arc's end node.

MODEL:
Minimize Costs
• ADM installation cost
• Flow cost on arcs and through ADMs and BBDXes

Constraints:
• The total bi-directional flow on an arc (the sum of the two directions) can be no more than the bidirectional capacity.
• No commodities may flow on a Type 0 arc unless their destination is at that hub.
• No commodities may flow on a Type 1 arc unless their origin is at that hub.
• No commodities may flow on a Type 0 arc unless an ADM is installed on that ring at that hub.
• No commodities may flow on a Type 1 arc unless an ADM is installed on that ring at that hub.
• No commodities may flow on a Type 3 arc unless an ADM is installed at that hub on both rings.
• The total flow of each commodity into its destination hub (i.e., into a Type 0 node on Type 0 arcs) must equal the demand for that commodity.
• The total flow of each commodity out of its origin hub (i.e., out of a Type 0 node on Type 1 arcs) must equal the demand for that commodity.
• The total flow of each commodity into each Type 1 node must equal the total flow of that commodity out of that Type 1 node.

Properties of the Problem Instances:

- Caldata Network

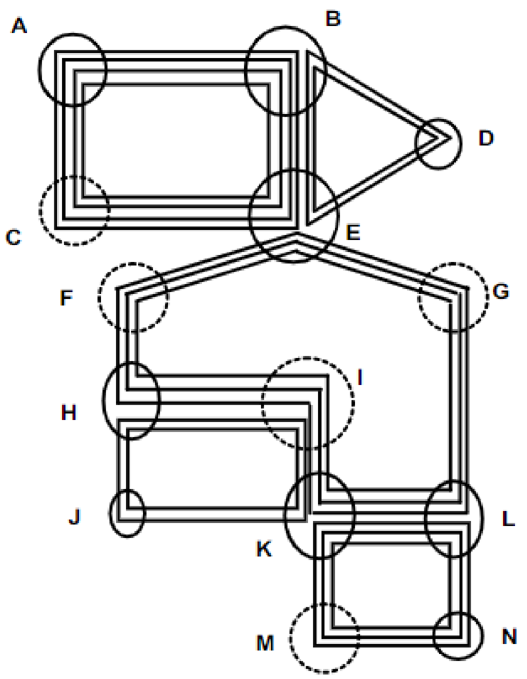

Figure 1: Plan for the company's Caldata network. Hubs with a solid circle (ABDEHJKLN) have a BBDX installed, and hubs with a dashed circle (CFGIM) do not.

First instance of the problem consists of 5 different cycles each having a different number of rings.(e.g., BDE cycle has 2 rings and ABEC cycle has 4 rings.) The total flow capacity at each arc is given as 24,000 units. BBDX's do not have capacities. However, due to arc capacity constraints, at most 48,000 units of data can flow through each ADM's on its rings. For the demand to be satisfied, there must be a data switch between the rings in two different cycles, in which case an ADM must be installed at both of the transfer rings. For instance, in order to satisfy the demand from C to G, data must flow from one ABEC ring to one EFHIKLG ring.

- Barry Network

Second instance of the problem consists of 3 different cycles. But this time, the total number of potential ADM locations is greater than the first problem instance. Data switch from one cycle to the other cycle can occur at hubs Q, M, K and I. Solid circles refers to the hubs where BBDX is installed. The arc capacity is given as 48,000 units. The unit flow costs and expected demand at each hub were given in the project description file.

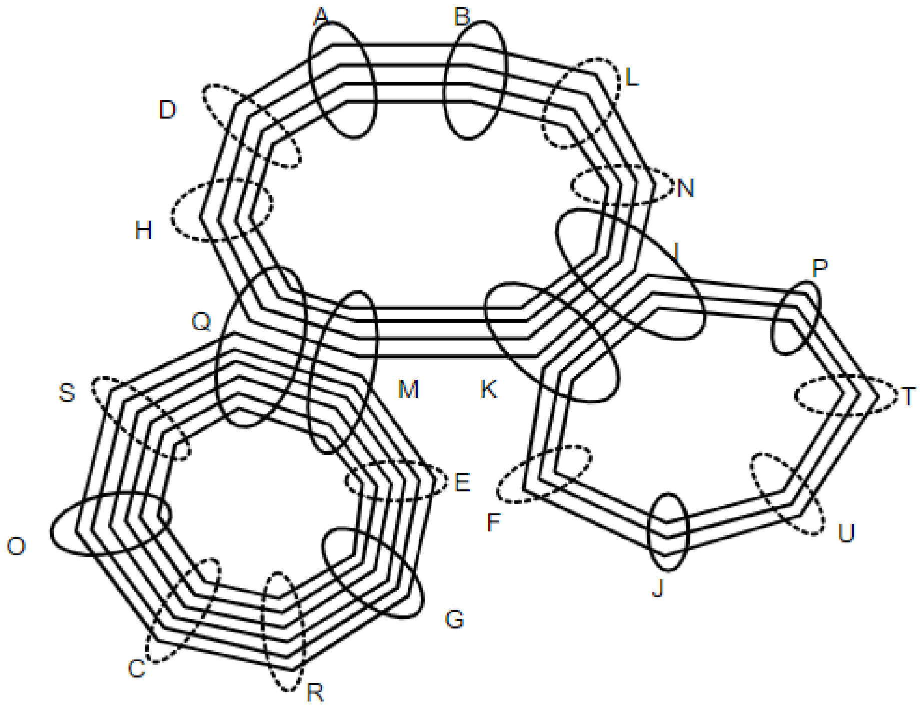

Figure 2: Plan for the company's Barry network. Hubs with solid oval have a BBDX installed, and hubs with a dashed oval do not.

Our Approach:

To understand the structure of the problem better, first, we divided the given problem into two distinct sub-problems. More specifically, for the caldata network, the first sub-problem is the upper part of the problem cut by hub E, i.e. the two cycles ABEC and BEC, and the second sub-problem is the lower side of the main problem cut by E. For the Barry network, the first sub-problem contains the cycles ABLNIKMQHD and PIKFJUT, and the second sub-problem isolates the QMEGRCOS cycle. As the size of the problem gets smaller, each of the sub-problems can easily be solved by CPLEX in negligible times. The total number of ADMs installed at each hub in the optimal solution of the given sub-problems are recorded. This result is used as a lower bound on the total number of ADMs required for the whole problem instance for the following steps. Total number of ADMs installed in the optimal solutions can be seen below, in Table-1:

Table 1: total number of ADMs installed in the optimal solution at each sub-problem

hubs	sub-prob1 adm #	sub-prob2 adm #
a	3	
b	4	
c	1	
d	1	
e	1	1
f		1
g		1
h		2
i		1
j		1
k		3
l		1
m		1
n		2

In the second step, we used the demand and capacity information as another lower bound on the total number of ADMs needed at each hub. More specifically, for each hub, all of the incoming and outgoing demands are summed up and that summation is divided by 48,000.(For Barry Network the sum is divided by 96,000). The round-up value of this division gives the minimum number of ADMs required at each hub:

$$\text{min \# of ADMs at hub i} = \left\lceil \frac{\sum_{\substack{I \in \text{incoming demand to hub } i, \\ O \in \text{outgoing demand from hub } i}} d_i(I) + d_i(O)}{48{,}000} \right\rceil$$

Below, minimum number of ADMs required at each hub can be seen, in Table 2:

Table 2: minimum number of required ADMs to be installed at each hub

Total_demand	Hub	ADM	min adm needed
144000.00	a	3	3
168000.00	b	3.5	4
3000.00	c	0.1	1
42000.00	d	0.9	1
15000.00	e	0.4	1
8000.00	f	0.2	1
3000.00	g	0.1	1
12000.00	h	0.3	1
1000.00	i	0.1	1
55000.00	j	1.2	2
85000.00	k	1.8	2
19000.00	l	0.4	1
3000.00	m	0.1	1
122000.00	n	2.6	3

In the third step, the nodes that connect two separate cycles are investigated further. The flow that goes through that intersection hubs is ignored when the problem is separated into isolated cycles. For Caldata Network, the total flow that goes through E includes all the demands that are coming from upper cycle(instance of sub-problem 1) to the lower cycles.(instance of sub-problem 2) ,and vice versa. Let that flow be f(E). The round-up value of the total flow divided by 24,000 gives the new minimum number of ADMs required at that intersection hub for Caldata Network:

$$\text{min \# of ADMs at hub E} = \left\lceil \frac{F(E)}{24{,}000} \right\rceil$$

Similar idea is applied to the Barry Network. However, in this setting, there are two potential intersection hubs through which data transfer might occur. Total flow that goes through either Q or M, and K or I is calculated and divided by 48,000 to find the minimum number of ADMs required at those combined hubs.

Below, minimum number of ADMs required at hub intersection hub E can be seen, in Table 3:

Table 3: Minimum number of required ADM to be installed based on total flow calculation at hub E for Caldata Network

Total Flow	Intersection Hub E	min adm required at hub E
118000.00	A,B,C,D<->F,G,H,I,J,K,L,M,N	4.916666667

At the final step, based on the results obtained from the previous steps, a lower bound on the minimum number of ADMs is obtained for each hub. In the case where the minimum ADM number required is different, the maximum of those is chosen. For instance, total demand based calculation requires at least 3 ADMs at hub N, whereas optimal solution of the subproblem results in 2 ADMs total. In this case, 3 is taken as the lower bound on the number of ADMs for hub N. Then, those lower bounds at each hub are added to the mathematical MIP model as a new constraint and the problem is updated. With the added constraints, the problem search space gets significantly smaller, thus the CPLEX model could find satisfactory solutions with acceptable optimality gaps. Below, four new constraints on the number of ADMs at locations A,B,L and N in the Barry Network can be found, in Table 4.

Table 4: New constraints are added to the mathematical model after the heuristic is applied. Lower bounds on the total number of ADMs after heuristic is applied is shown here at locations A,B,L and N for Barry Network.

```
    # new constraint based on witthin cycle optimized and total demand
subject to ai1:
    adm["ABLNIKMQHD_A_1"]+adm["ABLNIKMQHD_A_2"]+adm["ABLNIKMQHD_A_3"]+adm["ABLNIKMQHD_A_4"]>=4;
subject to ai2:
    adm["ABLNIKMQHD_B_1"]+adm["ABLNIKMQHD_B_2"]+adm["ABLNIKMQHD_B_3"]+adm["ABLNIKMQHD_B_4"]>=1;
subject to ai3:
    adm["ABLNIKMQHD_L_1"]+adm["ABLNIKMQHD_L_2"]+adm["ABLNIKMQHD_L_3"]+adm["ABLNIKMQHD_L_4"]>=1;
subject to ai4:
    adm["ABLNIKMQHD_N_1"]+adm["ABLNIKMQHD_N_2"]+adm["ABLNIKMQHD_N_3"]+adm["ABLNIKMQHD_N_4"]>=1;
```

Model Results:

We conducted the experiments on the two given problem instances to test the efficiency of our heuristic approach. Computational experiments were carried out using AMPL with CPLEX Optimization Studio on 16 GB of RAM and M1-chip processor. For the Caldata Network, the global optimal solution has been found after a 7-hour run time with the mathematical model. Our approach was run with a 10 minute time limit, and it was able to find a feasible solution with 0.05% optimality gap. The summary of the results can be seen in Table 5 below:

Table 5: Experimental Results of the heuristic on the two problem instances

Caldata Network		objective	total time(seconds)	optimality gap(%)
	global optimal	37731000	>3 hours	
	heuristic	37753000	600	0.058307493
Barry Network				
	lower bound	57022461	>3 hours	
	heuristic	63764600	>3 hours	11.82365489

For Barry Network, a lower bound needs to be calculated since the mathematical model can not find the optimal solution within the allowable time limits. For that, we updated the mathematical model adding new constraints based on the total demand at each hub and arc capacity. Basically, we implemented the second step in our heuristic approach to add new constraints. It is important to highlight that those new constraints do not alter the problem in hand. It only limits the search space that the MIP model searches for. After running the model for >3 hours, we interrupted the run when the optimality gap is *%20.43* with objective value of *"68672150"*. By solving the equation: (*68672150*-LB)/LB=20.43/100, we obtained the LB value as *"57022461"*. With using the best lower bound we have found by using the efficiently modified mathematical model, we then calculated the optimality gap as %11.82.

Further Remarks:

Our heuristic approach gives very good results on the Caldata Network within a short time limit. (>0.05% optimality gap.) However, more research needs to be done specifically for the Barry Network, as the structure of it is more complex ,and a more efficient algorithm is needed. Although we have developed "flow-path" based improvement heuristic ideas to improve the solution building upon the results of the sub-problems, due to time limitation and possible intricacies that needs to be addressed in implementation, we decided to omit it. Secondly, we conclude that our approach might work very efficiently on specific types of networks, but more research needs to be done to be able to make a comment on the generalizability of our heuristic.