

**Mv Calc, Ch 15B PT** Good luck to: \_\_\_\_\_ I ☐ the  $\iint_R \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) dA$  capacitor.

Directions: You may not use a calculator for the exam. Show convincing evidence to receive any partial credit. Part 1 will be taken over a 50 minute class meeting and is made of questions 1 – 4. Part 2 will be taken at the next class meeting, with maximum allotted time as 50 minutes, and will be made of questions 5 and 6.

Part 1 is made of questions 1 – 4.

1. Let  $\vec{F}(x, y, z) = \langle 2zx, y^2, -xz \rangle$ . Find  $\iint_{\sigma} (\vec{F} \cdot \vec{n}) dS$  where  $\sigma$  is that portion of the cube  $0 \leq x \leq 1$ ,

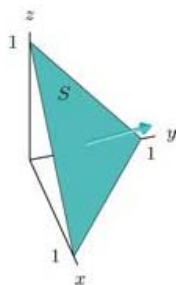
$0 \leq y \leq 1$ ,  $0 \leq z \leq 1$  with inward unit normal orientation, as follows:

a) without the use of the Divergence Theorem.                      b) use The Divergence Theorem.

2. Evaluate  $\iint_{\sigma} (x^2 + y^2 - z) dS$  where  $\sigma$  is that portion of the paraboloid  $z = 4 - x^2 - y^2$  between  $z = 1$  and  $z = 2$ .

3. Use Stokes' Theorem to evaluate  $\int_C y^2 dx + x^2 dy - (x + z) dz$  where  $C$  is a triangle in the  $xy$  plane with vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$ , and  $(1, 1, 0)$  with a counterclockwise orientation looking down the positive  $z$ -axis.

4. Compute the flux of  $\vec{F}(x, y, z) = \langle x^2, y^2, z^2 \rangle$  through the surface  $S$  shown below. Note: The arrow piercing through surface  $S$  shown below indicates the direction of the unit normal of  $S$ .



Part 2 is made of questions 5 and 6.

5. Use The Divergence Theorem to evaluate  $\iint_{\sigma} (\vec{F} \cdot \vec{n}) dS$  where  $\vec{F}(x, y, z) = yz\vec{i} + xy\vec{j} + xz\vec{k}$ ,  $\vec{n}$  is the outer unit normal to  $\sigma$ , and  $\sigma$  is the surface enclosed by the cylinder  $x^2 + y^2 = 1$  and the planes  $y = -1$  and  $y = 1$ .

6. Use Stokes' Theorem to evaluate  $\iint_{\sigma} ((\text{curl } \vec{F}) \cdot \vec{n}) dS$  where  $\vec{F}(x, y, z) = y\vec{k}$  and  $\sigma$  is that portion of the ellipsoid  $4x^2 + 4y^2 + z^2 = 4$  for which  $z \geq 0$ .