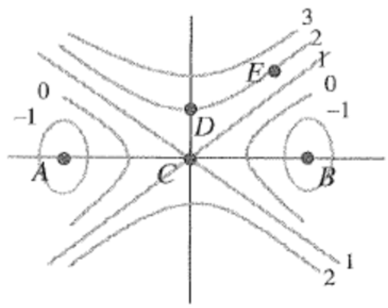
Good Luck to: ____

Directions: Directions: A calculator is not allowed. To receive any partial credit convincing evidence based on concepts from Calculus must be shown; otherwise minimal, if any, partial credit will be awarded.



1. The contour diagram of f is shown above.
    a. Which of the points A, B, C, D and E appear to be critical points? Classify each critical points as local extreme values of saddle points. Briefly explain your answer.
    b. Describe possible gradient vectors of f at points C, D and E.

2. Verify that (0, 0) , (0, ½) , (0, - ½) , (1, 0) , and (-1, 0) are critical points of the surface $z = f(x,y) = (x^2 + y^2)e^{-x^2-4y^2}$ and classify them as local extreme values or saddle points.

3. Let $f(x,y) = 3x^2 + 3y^2 - 12x - 6y + 18$ and let T be the triangle with vertices (-2, 1) , (0, 1), and (-1, 2). Find the maximum and minimum values of f with triangle T as the constraint.

4. A company manufactures a product using *x*, *y*, and *z* units of three different raw materials. The quantity produced is given by the function $Q = 60x^{1/3}y^{1/4}z^{2/5}$. Suppose the cost of the materials per unit is \$20, \$15, and \$24 respectively.
    a. Find the maximum production if the budget is limited to \$5900. What is the meaning of λ in this case?
    b. Find the cheapest way to produce 6000 units of the product. What is the meaning of λ in this case?

5. Let $\vec{v} = 3\hat{i} + a\hat{j} + b\hat{k}$ be a vector in space where a and b are unknown constants.
    a. Compute the cross product, $\vec{v} \times (3\hat{j} + \hat{k})$. Leave your answer in terms of a and b .
    b. Use the answer in part a with The Method of Lagrange Multipliers to find the values of a and b such that the magnitude of the cross product $\left\|\vec{v} \times (3\hat{j} + \hat{k})\right\|$ is the largest with $\|\vec{v}\| = 13$. (Hint: Maximize the square of the magnitude instead of the magnitude)
    c. Use the geometrical definition of the Cross Product to find a condition on a and b such that $\left\|\vec{v} \times (3\hat{j} + \hat{k})\right\|$ is at its maximum. Then find the values of a , b that satisfy the condition $\|\vec{v}\| = 13$. Compare to your answer in part b.

END OF PT