$$\Rightarrow (x^2+1)^{1/2}$$

$$\Rightarrow \sqrt{x^2+1}$$

Group Six

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Questions:

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1. $\int x\sqrt{x+3}\,dx$

let $x+3$ be u

$u = x+3$

$x = u-3$

$dx = du$

$\int u-3(u^{\frac{1}{2}})\,du \quad \int\left(u^{3/2} - 3u^{1/2}\right)du$

$\int u^{3/2}\,du = \left[\frac{2}{5}u^{5/2}\right] + c \quad \int 3u^{1/2}\,du = 3\cdot\frac{2}{3}u^{3/2} = \left[2u^{3/2}\right] + c$

$\text{Ans} = \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + c$

2. $\int \frac{x^2}{(x^3+1)^2}\,dx$

Let $u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{1}{3}\,du$

$\int \frac{x^2}{(x^3+1)^2}\,dx = \frac{1}{3}\int u^{-2}\,du = \frac{1}{3}(-u^{-1}) + c$

$= \frac{1}{3(x^3+1)} + c$

Number 3.

Show that $\int \frac{1}{1+e^x} dx = x - \ln(1+e^x) + C$

$\int \frac{1}{1+e^x} dx = \int \frac{1+e^x-e^x}{1+e^x} dx$

$= \int \frac{1+e^x}{1+e^x} - \int \frac{e^x}{1+e^x} dx$

$= \int 1 \, dx - \int \frac{e^x}{1+e^x} dx$

$\int 1 \, dx = x$

$\frac{e^x}{1+e^x} dx$

Let $1+e^x = u$

$\frac{du}{dx} = e^x$

$dx = \frac{du}{e^x}$

$\int \frac{\cancel{e^x}}{u} \frac{du}{\cancel{e^x}} = \frac{1}{u} du = \ln|u| + c$

but $u = 1+e^x$

$\frac{1}{u} du = \ln|1+e^x| + c$

$\int 1 dx - \int e^x/_{1+e^x} dx = x - \ln|1+e^x| + c$

Therefore:

$\int \frac{1}{1+e^x} dx = x - \ln|1+e^x| + c.$

Number 4.

$\int x^2 e^x dx$

LIATE

Let $x^2 = u$

$\frac{du}{dx} = 2x \quad du = 2x\,dx$

$dv = e^x dx$

$v = e^x$

$uv - \int v du$

$x^2 \cdot e^x - \int e^x \cdot 2x\,dx$

$x^2 e^x - 2\int e^x \cdot x\,dx$

$\Downarrow$

Integrate by parts:

Let $u = x$

$du/dx = 1 \quad du = dx$

$e^x dx = dv$

$v = e^x$

$x^2 e^x - 2[x \cdot e^x - \int e^x du]$

$x^2 e^x - 2[xe^x - e^x]$

$= x^2 e^x - 2xe^x + 2e^x + c$

Factor e^x out

$e^x[x^2 - 2x + 2] + c$

5. $\int x^2 \ln x \, dx$

$u = \ln x$ $\qquad$ $dv = x^2 \, dx$

$du = \frac{1}{x} \, dx$ $\qquad$ $v = \frac{x^3}{3}$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^2 \ln x \, dx = \ln x \, \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \cancel{\frac{1}{3} \ln x} \quad \int \frac{1}{3} x^2 \, dx$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C$$

Find $\int e^x \cos x \, dx$

$u = \cos x \qquad dv = e^x \, dx$

$du = -\sin x \, dx \qquad v = e^x$

$$\int e^x \cos x \, dx = \cos x \, e^x - \int e^x - \sin x \, dx$$

$$= e^x \cos x + \int e^x \sin x \, dx$$

$u = \sin x \qquad dv = e^x \, dx$

$du = - \cos x \, dx \qquad v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$\overset{\text{same}}{\boxed{\int e^x \cos x \, dx}} = e^x \cos x + e^x \sin x - \overset{\text{same}}{\boxed{\int e^x \cos x \, dx}}$$

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \cos x \, dx + \int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$2\int e^x \cos x \, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x \, dx = \frac{e^x \cos x + e^x \sin x}{2}$$

Q7. Find the area enclosed by the curve $y = x^3 - 3x^2 + 2x$ and the x-axis between $x = 0$ and $x = 2$

$y = 0$:

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x(x-1)(x-2) = 0$

$x = 0, 1, 2$

$x \in (0, 1)$:

$x = 0.5$: $y = (0.5)^3 - 3(0.5)^2 + 2(0.5) = 0.125 - 0.75 + 1 = 0.375 > 0$

$x \in (1, 2)$:

$x = 1.5$: $y = 3.375 - 6.75 + 3 = -0.375 < 0$

$$A = \int_0^1 (x^3 - 3x^2 + 2x)\,dx - \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$\int (x^3 - 3x^2 + 2x)\,dx = \frac{x^4}{4} - x^3 + x^2 + C$$

$$\int_1^2 (x^3 - 3x^2 + 2x^2 + 2x)\,dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$$

$\frac{16}{4} - 8 + 4 = 4 - 8 + 4 = 0$

$\frac{1}{4} - 1 + 1 = \frac{1}{4}$

$$\int_1^2 (\;)\,dx = 0 - \frac{1}{4} = -\frac{1}{4}$$

$A = \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{2}$

$A = \frac{1}{2}$ square units

Q8. Find the area enclosed by the curves $y = e^{-x}$ and $y = 1$ between their points of intersection.

set $e^{-x} = 1 - x$

$1 = (1 - x)e^{x}$

x	$e^{\{-x\}}$	$1 - x$
0	1	1
1	0.3679	0
-1	2.718	2

$x = 1.5$:

$e^{-1.5} = 0.2231$, $1 - 1.5 = 0.5$

$x = 0.5$: $e^{-0.5} = 0.6065$, $1 - 0.5 = 0.5$

0.6: $e^{-0.6} = 0.5488$, $1 - 0.6 = 0.4$

0.7: $e^{-0.7} = 0.4966$, $1 - 0.7 = 0.3$

$x \approx 0.56$

$x_1 = 0, \quad x_2 \approx 1.841$

$e^{-x} = 1 - x$ (s $x \approx 1.8414$)

At $x = 0$: $e^{-0} = 1 - 0 = 1$ ⇒ same

$x = 1$: $e^{-1} = 0.3679$, $1 - 1 = 0$ ⇒ e^{-x} is above

$e^{-x} > 1 - x$ btn 0 & 1.841

$$A = \int_0^{1.841} \left[e^{-x} - (1 - x) \right] dx$$

$$A = \int_0^{1.841} \left[e^{-x} - 1 + x \right] dx$$

$$\int (e^{-x} - 1 + x)\, dx = -e^{-x} - x + \frac{x^2}{2} + C$$

$$A = \left[-e^{-x} - x + \frac{x^2}{2} \right]_0^{1.841}$$

$$A_1 = -e^{-1.841} - 1.841 + \frac{(1.841)^2}{2}$$

$$A_1 = -0.158 - 1.841 + 1.695 = -0.304$$

$x = 0$: $A_0 = -1 - 0 + 0 = -1$

$$A = (-0.304) - (-1) = 0.696$$

$A \approx 0.696$ square units

9) $\int \frac{x+5}{x^2+2x\mp 3}\,dx$

x^2-2x+3

$x = \frac{-1\cdot -2 \pm \sqrt{(-2)^2-4\cdot 1\cdot -3}}{2\cdot 1}$

$x = \frac{2\pm 4}{2} = 3 \text{ or } -1$

Root Factors $= (x-3)(x+1)$

$\int \frac{x+5}{(x-3)(x+1)}\,dx$

$\Rightarrow \frac{A}{x-3} + \frac{B}{x+1} = \frac{x+5}{(x-3)(x+1)}$

$A(x+1) + B(x-3) = x+5$

Let $x=3$; $A(3+1) = 3+5$

$4A = 8$

$A = 2$

Let $x=-1$; $B(-1-3) = -1+5$

$-4B = 4$

$B = -1$

$\int \frac{2}{x-3}\,dx + \int \frac{-1}{x+1}\,dx$

$\Rightarrow 2\int \frac{1}{x-3}\,dx - \int \frac{1}{x+1}\,dx$

$\Rightarrow 2\ln|x-3| - \ln|x+1| + c$

$\Rightarrow \ln|x-3|^2 - \ln|x+1| + c$

$\Rightarrow \ln\left|\frac{(x-3)^2}{(x+1)}\right| + c$

$$A(\cancel{x^2+2x}) + B(x+2) + Cx^2 = x^2+3x$$

10) $$\int \frac{x^2+3x+2}{x^3+2x^2} \, \partial x$$

$$\Rightarrow \int \frac{(x+1)\cancel{(x+2)}}{x^2\cancel{(x+2)}} \, \partial x \Rightarrow \int \frac{x+1}{x^2} \, \partial x$$

$$\Rightarrow \frac{A}{x} + \frac{B}{x^2} = \frac{x+1}{x^2}$$

$$Ax + B = x+1$$

$$x = Ax \quad \therefore A = 1$$

$$B = 1$$

$$\int \frac{1}{x} \, \partial x + \int \frac{1}{x^2} \, \partial x$$

$$\Rightarrow \ln|x| + \left[\frac{x^{-2+1}}{-2+1} \right] + C$$

$$\Rightarrow \ln|x| + \left[\frac{-1}{x} \right] + C$$

$$\Rightarrow \ln|x| - \frac{1}{x} + C$$