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NUMBERS 1 AND 2

$\int x\sqrt{x+3}\,dx.$

let; $u = x+3$

$du = dx.$

$\therefore x = u-3$

$\int (u-3)\sqrt{u}\,du \quad \int (u-3)(u^{1/2}$

$\int (u^{3/2} - 3u^{1/2})\,d$

$u^{3/2}du = \frac{2}{5}u^{5/2}, \quad \int 3u^{1/2}du$

$= 3 \cdot \frac{2}{5}u^{3/2} = 2u^{3}.$

$= \frac{2}{5}u^{5/2} - 2u^{3/2} + C.$

$u = x+3.$

$\Rightarrow \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C.$

2). $\int \frac{x^2}{(x^3+1)^2} dx$;

Let;

$u = x^3+1$

$du = 3x^2 dx \Rightarrow x^2 dx$.

$\frac{1}{u^2} \cdot \frac{du}{3} = \frac{1}{3}\int u^{2}\, du$

$\frac{1}{3}\int u^{-2} du = \frac{1}{3} \cdot \left(\frac{u^{-1}}{-1}\right) = -\frac{1}{3u}$

Sub; $u = x^3+1$

$\cdot -\frac{1}{3u}$; $-\frac{1}{3(x^3+1)} + C$;

ANS; $= -\frac{1}{3(x^3+1)} + C$

<u>Number 3 and 4</u>

$$\int \frac{1}{1+e^x}\,dx = x - \ln|1+e^x| + C$$

$\cancel{u = 1 + e^x}$

$\cancel{\frac{du}{dx} = e^x}$

$\cancel{du = e^x dx}$

$\cancel{\int \frac{1}{1+e^x}\,dx = \int \frac{1}{u} \cdot \frac{dx}{du}\,du}$

$$\frac{1}{1+e^x} = \frac{e^{-x}}{e^{-x}}$$

$$\int \frac{e^{-x}}{e^{-x}+1}\,dx$$

$u = e^{-x} + 1$

$\frac{du}{dx} = -e^{-x}$

$dx = \frac{du}{-e^{-x}}$

$$\int \frac{e^{-x}}{u} \cdot \frac{du}{-e^{-x}} = \int -\frac{1}{u}\,du$$

$= -\ln|u| + C$

$= -\ln|e^{-x}+1| + C$

but $\ln|e^{-x}+1| = \ln\left(\frac{1}{e^x}+1\right) = \ln\left(\frac{1+e^x}{e^x}\right|$

the same as $\ln|1+e^x| - \ln e^x$

$= \ln|1+e^x| - x$

$-[\ln|1+e^x| - x] + C$

$= x - \ln|1+e^x| + C$

$$\int x^2 e^x\,dx$$

$u = x^2 \qquad dv = e^x dx$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\Rightarrow uv - \int v\,du$

$x^2 e^x - \int e^x \cdot 2x\,dx$

$u = 2x \qquad dv = e^x dx$

$\frac{du}{dx} = 2 \qquad v = e^x$

$du = 2dx$

$\Rightarrow uv - \int v\,du$

$2x \cdot e^x - \int e^x\, 2dx$

$= 2x \cdot e^x - 2e^x + C$

$x^2 e^x - [2xe^x - 2e^x + C$

$x^2 e^x - 2xe^x + 2e^x + C$

$(x^2 - 2x + 2)e^x + C$

Number 5 and 6

Integral Calculus Worksheet Two (Breakout Room Group One) (ICS 1.2 A)

220950 - Edenmike Mwaura Ruhanga

5. $\int x^2 \ln(x)\, dx$

Let $u = \ln x$ $\quad dv = x^2\, dx$

$du = \frac{1}{x} \cdot dx$ $\quad v = \frac{x^3}{3}$

$\int u\, dv = uv - \int v\, du$

$$\int x^2 \ln(x)\, dx = \left[\ln x \cdot \frac{x^3}{3}\right] - \int \frac{x^3}{3} \cdot \frac{1}{x}\, dx$$

$$= \frac{x^3}{3} \ln(x) - \frac{1}{3} \int x^2\, dx$$

$$= \frac{x^3}{3} \ln(x) - \frac{x^3}{9} + c$$

6. $\int e^x \cos x\, dx$

$u = \cos x$ $\quad dv = e^x\, dx$

$du = -\sin x\, dx$ $\quad v = e^x$

$$\int e^x \cos x\, dx = [\cos x \cdot e^x] - \int e^x \cdot -\sin x\, dx$$

$$= e^x \cos x + \int e^x \sin x\, dx$$

$\int e^x \sin x\, dv$ Let $u = \sin x$ $\quad dv = e^x\, dx$

$du = \cos x\, dx$ & $v = e^x$

$$\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$$

$$\int e^x \cos x\, dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x\, dx\right)$$

$$2\int e^x \cos x\, dx = e^x \cos x + e^x \sin x$$

$$\int e^x \cos x\, dx = \frac{e^x \cos x + e^x \sin x}{2} + c$$

Number 7 and 8

7. $y = x^3 - 3x^2 + 2x$

$x(x^2 - 3x + 2) = 0$

$x = 0$ and $x^2 - 3x + 2 = 0$

P → 2

S → −3

N → −2 and −1

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2)$

$(x-1)(x-2)$

$x = 1, 2$ and 0.

$$\int_0^1 (x^3 - 3x^2 + 2x) + \int_1^2 (x^3 - 3x^2 + 2x)$$

$$= \left[\tfrac{1}{4}x^4 - \tfrac{3}{3}x^3 + \tfrac{2}{2}x^2 + c\right]$$

$$= \left[\tfrac{1}{4}x^4 - x^3 + x^2 + c\right]_0^1$$

$$\left(\tfrac{1}{4}(1)^4 - (1)^3 + (1)^2 + c\right) - \left(\tfrac{1}{4}(0)^4 - (0)^3 + (0)^2 + c\right)$$

$$= \left(\tfrac{1}{4} + c\right) \qquad = (0 + c)$$

$$= \tfrac{1}{4}$$

$$\left[\tfrac{1}{4}(x)^4 - x^3 + x^2 + c\right]_1^2$$

$$\left(\tfrac{1}{4}(2)^4 - (2)^3 + (2)^2 + c\right)$$

$$= (0 + c)$$

After substituting $x = 1$

$$= \left(\tfrac{1}{4} + c\right)$$

$$= \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}$$

$= \frac{1}{2}$ Square Meters

Bd

8. Area enclosed between points of intersections

$$e^{-x} = 1 - x$$

at $x = 0$

$$e^{-0} = 1 - 0 = 1.$$

$$A = \int_0^1 [e^{-x} - (1-x)\,dx = \int_0^1 e^{-x}\,dx - \int_0^1 (1-x)\,dx$$

$$= \int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1 = (-e^{-1}) - (-e^{-0}) = -e^{-1} + 1.$$

$$= 1 - e^{-1}$$

$$\int_0^1 (1-x)\,dx = \left[x - \frac{x^2}{2}\right]_0^1$$

$$\int (1-x) = \frac{1x^{0+1}}{1} - \frac{x^{1+1}}{2} + c \qquad = \left(1 - \frac{(1)^2}{2}\right) - \left(0 - \frac{(0)^2}{2}\right)$$

$$= x - \frac{x^2}{2} \qquad = \left(\tfrac{1}{2}\right) - 0 = \tfrac{1}{2}$$

$$= \cancel{\tfrac{1}{2} + (1 - e^{-1})} \qquad A = (1 - e^{-1}) - \tfrac{1}{2}$$

$$1 - \tfrac{1}{2} = \tfrac{1}{2}$$

$$= \tfrac{1}{2} - e^{-1} = \tfrac{1}{2} - \tfrac{1}{e}$$

$$= \tfrac{1}{2} - \tfrac{1}{e}$$

Number 9 and 10

9 $\int \frac{x+5}{x^2+2x-3}$

$S = 2$ $\quad 3, -1$

$P = -3$

$x^2 + 3x - x - 3 = 0$

$x(x+3) - 1(x+3) = 0$

$(x-1)(x+3) = 0$

$\int \frac{x+5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$

$Ax + 3A + Bx - B$

$x(A+B) + 3A - B$

$A + B = 1 \quad +$

$\underline{3A - B = 5}$

$\frac{4A}{4} = \frac{6}{4} \qquad A = \frac{3}{2} \qquad B = 1 - A$

$= 1 - \frac{3}{2} = -\frac{1}{2}$

$\int \frac{x+5}{x^2+2x-3}\,dx = \int \left(\frac{\frac{3}{2}}{x-1} - \frac{\frac{1}{2}}{x+3} \right) dx = \frac{3}{2} \ln|x-1| - \frac{1}{2} \ln|x+3| + c$

$= \frac{1}{2} \ln \frac{(x-1)^3}{x+3} + C$

10. $\int \frac{x^2+3x+2}{x^3+2x^2}\,dx = \int \frac{x^2+3x+2}{x^2(x+2)}\,dx$

$$\frac{x^2+3x+2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$$

$$A(x^2+2x) + B(x+2) + C(x^2)$$

$$\cancel{x^2(A+C}$$

$$Ax^2+2Ax+Bx+2B+Cx^2$$

$$x^2(A+C)+x(2A+B)+2B$$

$A+C=1$

$2A+B=3$ $\qquad \frac{\cancel{2}B}{\cancel{2}} = \frac{\overset{1}{\cancel{2}}}{2}$ $\qquad B=1$

$2B=2$

$2A+1=3$

$2A=2$ $\qquad A=1$ $\qquad 1+C=1$

$C=0$

$$\int \frac{x^2+3x+2}{x^2(x+2)} = \frac{1}{x} + \frac{1}{x^2}$$

$$= \ln|x| - \frac{1}{x} + C$$