Group ③.

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Qn1.)

Evaluate using substitution.

$$\int x\sqrt{x+3}\,dx$$

⇒ let $u = x+3$ $\quad x = u-3$

$du = dx$

$\frac{du}{dx} = 1$

$dx = du$

$$\int (u-3)\sqrt{u}\,du = \int (u^{3/2} - 3u^{1/2})\,du$$

$$\int u^{3/2}\,du = \frac{2}{5}u^{5/2}$$

$$\int 3u^{1/2}\,du = 3\cdot\frac{2}{3}u^{2/3} = 2u^{3/2}.$$

$$\int x\sqrt{x+3}\,dx = \frac{2}{5}u^{5/2} - 2u^{3/2} + C$$

ANS => $\int \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

4.).

$$\int x^2 e^x \, dx.$$

let $u = x^2$, $dv = e^x \, dx$.

$du = 2x \, dx$, $v = e^x$

Qn 1)

Qn 2.) $\int \frac{x^2}{(x^3+1)^2}\, dx$

let $u = x^3 + 1$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

$$\Rightarrow \int \frac{x^2}{u^2} \cdot \frac{du}{3x^2}$$

$$\Rightarrow 1/3 \int 1/u^2 \, du$$

$$\Rightarrow 1/3 \int u^{-2} \, du$$

$$= \frac{1}{3u} + C \quad \Rightarrow \frac{-1}{3(x^3+1)} + C$$

4.) $\int x^2 e^x \, dx$

Answer $\Rightarrow 0$.

Qn 3.)

show that $\int \frac{1}{1+e^x}\,dx = x - \ln(1+e^x) + c$.

$$\frac{1}{1+e^x} = 1 - \frac{e^x}{1+e^x}$$

$$\int \frac{1}{1+e^x}\,dx = \int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$$

$$= x - \int \frac{e^x}{1+e^x}\,dx.$$

let $u = 1+e^x$

$du = e^x dx$. $\Rightarrow \int \frac{1}{u}\,du = \ln|u| = \ln(1+e^x$

$\int \frac{e^x}{1+e^x}\,dx$

Ans. $\Rightarrow x - \ln(1+e^x) + c$

4. $\int x^2 e^x \, dx$

$u = x^2 \quad \frac{du}{dx} = 2x$

$du = 2x \, dx$

$dv = e^x \quad v = e^x$

$uv - \int v \, du$

$x^2 e^x - \int e^x \cdot 2x \, dx$

$x^2 e^x - 2\int e^x x \, dx$

$\int e^x x \, dx$

$u = x \quad \frac{du}{dx} = 1$

$du = dx$

$dv = e^x \quad v = e^x$

$x e^x - \int e^x \, dx$

$x e^x - e^x + c$

$x^2 e^x - 2(x e^x - e^x) + c$

$= x^2 e^x - 2x e^x + 2e^x + c$

$= e^x (x^2 - 2x + 2) + c$

5. Compute $\int x^2 \ln(x)\,dx$.

~~$u = x^2$~~ $u = \ln(x)$; $dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $v = \frac{x^3}{3}$

$\Rightarrow \int uv = uv - \int v\,du$

$\Rightarrow \int x^2 \ln(x)\,dx = \frac{x^3}{3}\ln(x) - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$

$\Rightarrow = \frac{x^3}{3}\ln(x) - \frac{1}{3}\int \frac{x^3}{x}\,dx$

$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2\,dx$

$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\cdot\frac{x^3}{3} + C$

$= \frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C$

10. Compute $\int \frac{x^2+3x+2}{x^3+2x^2}\,dx$

$\Rightarrow x^3+2x^2 = x^2(x+2)$

$\Rightarrow x^2+3x+2 \Rightarrow x^2+x+2x+2 = x(x+1)+2(x+1)$

$P=2\ S=3$ $\Rightarrow (x+2)(x+1)$

No. s 1 & 2

$\Rightarrow \int \frac{(x+1)(x+2)}{x^2(x+2)}\,dx \Rightarrow \int \frac{x+1}{x^2}\,dx$

$\Rightarrow \int \left(\frac{x}{x^2} + \frac{1}{x^2}\right)dx$

$\Rightarrow \int \left(\frac{1}{x} + \frac{1}{x^2}\right)dx$

$\Rightarrow \ln|x| + -\frac{1}{x} + C$

Date

Number 6

$u = e^x \quad dv = \cos x\, dx$

$du = e^x dx \quad v = \sin x$

$I = e^x \sin x - \int e^x \sin x\, dx$

$J = -e^x \cos x + \int e^x \cos x\, dx = -e^x \cos x + I$

$I = e^x \sin x - J = e^x \sin x - C - e^x \cos x + I)$:

$e^x \sin x + e^x \cos x - I$

$2I = e^x (\sin x + \cos x) \Rightarrow I = \frac{e^x}{2}(\sin x + \cos x) + C$

$\Rightarrow \int e^x \cos x\, dx = \frac{e^x}{2}(\sin x + \cos x) + C$

a. Evaluate

$$\int \frac{x+5}{x^2+2x-3}\,dx$$

x^2+2x-3

$s = 2$
$p = -3$ $(3, -1)$

$x^2+3x-x-3=0$

$x(x+3)-1(x+3)=0$

$(x-1)(x+3)$

$$\left[\frac{x+5}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}\right]^{(x-1)(x+3)}$$

$x+5 = A(x+3) + B(x-1)$

if $x = -3$

$-3+5 = 0 + (-3-1)B$

$2 = -4B$

$-\frac{1}{2} = B$

if $x = 1$

$1+5 = (1+3)A + 0$

$6 = 4A$

$\frac{6}{4} = A$

$\frac{3}{2} = A$

$$\int \frac{3}{2}\cdot\frac{1}{x-1} + -\frac{1}{2}\cdot\frac{1}{x+3}$$

let $x-1$ be u let $x+3$ be v

$\int \frac{1}{u} = \ln|u|$ $\int \frac{1}{v} = \ln|v|$

$$\frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+3| + C$$

Qn 8.) $y = e^{-x}$ and $y = 1 - x$.

$e^{-x} = 1 - x$.

Answer 8 O

let $F(x) = e^{-x} - (1 - x)$

$= e^{-x} + x - 1$

$F'(x) = 1 - e^{-x}$

$F''(x) = e^{-x} > 0$.

$x = 0$, $F'(x) = 0$ & $F''(0) = 1 > 0$.

$\therefore$ $x = 0$ is a global minimum.

Answer & $\Rightarrow 0$.

Qn 3.)

show that $\int \frac{1}{} \, dx = x - \ln(1 + e^x$