

1. Evaluate with substitution

$$\int x \sqrt{x+3} \, dx$$

Let $u = x+3$ $\frac{du}{dx} = 1$ $du = dx$
 $x = u-3$

$$\begin{aligned} & \int (u-3) \sqrt{u} \, du \\ &= \int (u-3) u^{\frac{1}{2}} \, du \\ &= \int (u^{\frac{3}{2}} - 3u^{\frac{1}{2}}) \, du \\ &= \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 3 \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} \\ &= \frac{2}{5} u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + C \end{aligned}$$

2 Compute:

$$\int \frac{x^2}{(x^3+1)^2} \, dx$$

Let $u = (x^3+1)$ $\frac{du}{dx} = 3x^2$ $du = 3x^2 \, dx$

$$dx = \frac{1}{3} x^2 \, du$$

$$\begin{aligned} &= \frac{x^2}{(u)^2} \cdot \frac{1}{3} x^2 \, du \\ &= \int \frac{1}{(u)^2} \cdot \frac{1}{3} \, du \\ &= \frac{1}{3} \int u^{-2} \, du \end{aligned}$$

$$\int \frac{1}{u^3} du$$

$$\frac{1}{3} \int u^{-3} du$$

$$= \frac{1}{3} \cdot \frac{u^{-2}}{-2}$$

$$= \frac{1}{3} \cdot \frac{-1}{u^2} = -\frac{1}{3u^2} + C$$

$$= -\frac{1}{3(x^2 + 1)} + C$$