8.) Find the area enclosed by the curve $y = e^{-x}$ and $y = 1 - x$ between the points of intersection.

$e^{-x} = 1 - x$ $\quad = -e^{-x} + 1 = 0$

$= e^{-x} + x - 1$ $\quad = e^{-x} = 1 \quad x = 0$

$e^{0} + 0 - 1 = 0$

$x, y = 0, 1$

Area = 0.

2

7.) $y = \cancel{x^2}\ x^3 - 3x^2 + 2x$ $\qquad x = 0, 2.$

Anti Deriv $= \frac{1}{4}x^4 - x^3 + x^2$

$\cancel{x(x^2 - 3x) + 2D}$

$x(x^2 - 3x + 2)$

$x^2 - 2x - x + 2$

$x(x-2) - 1(x-2)$

$S = -3$
$P = 2$ $\qquad (-2, -1)$

$(x-1)(x-2)$

$x = 1, 2, 0$

$$\int_0^1 (x^3 - 3x^2 + 2x) - \int_1^2 (\cancel{x}\, x^3 - 3x^2 + 2x)$$

$$\left[\frac{1}{4}(1)^4 - (1)^3 + (1)^2\right] - \left[\cancel{(2)^3} - B \cdot \left(\frac{1}{4}(2)^4 - (2)^3 + (2)^2\right) - \left(\frac{1}{4}(1)^4 - (1)^3 + (2)^2\right)\right]$$

$\frac{1}{4} - -\frac{1}{4} = \frac{1}{2}.$

$$\frac{x^2+3x+2}{x^3+2x^2} = \frac{1}{x+2} + \left(\frac{3/2}{x} - \frac{3/2}{x+2}\right) + \left(\frac{-1/2}{x} + \frac{1}{x^2} + \frac{1/2}{x+2}\right)$$

- $\frac{1}{x+2} - \frac{3/2}{x+2} + \frac{1/2}{x+2} = \frac{0}{x+2} = 0$

- $\frac{3/2}{x} + \frac{-1/2}{x} = \frac{1}{x}$

- $\frac{1}{x^2}$

$$\int 0 + \frac{1}{x} + \frac{1}{x^2}$$

$$= \int \frac{1}{x} + \int \frac{1}{x^2}$$

$$= \ln x + \int x^{-2} + C$$

$$= \ln x + -\frac{1}{x} + C$$

$$\int \frac{x^2 + 3x + 2}{x^3 + 2x^2}\,dx \qquad D = x^2(x+2)$$

$$\int \frac{x^2 + 3x + 2}{x^2(x+2)}\,dx$$

$$\frac{\cancel{x^2}}{\cancel{x^2}(x+2)} + \frac{3x}{x^2(x+2)} + \frac{2}{x^2(x+2)}$$

$$= \frac{1}{(x+2)} + \frac{3}{x(x+2)} + \frac{2}{x^2(x+2)}$$

•) $\frac{3}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2) + Bx}{x(x+2)}$ $\quad Ax + 2A + Bx$

$x(A + B) + 2A$

$A + B = 0$
$2A = 3$
$A = 3/2$

$B = -3/2$ $\quad = \frac{-3/2}{x+2} + \frac{3/2}{x}$

•) $\cancel{\frac{2}{x^2(x+2)} = \frac{A}{x^2} + \frac{B}{x+2} = \frac{Bx^2 + A(x+2)}{x^2(x+2)}}$ $\quad Ax + \cancel{2A + Bx^2}$

•) $\frac{2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} = \frac{Ax(x+2) + B(x+2) + Cx^2}{x^2(x+2)}$

$Ax^2 + 2Ax + Bx + 2B + Cx^2$
$x^2(A + C) + x(2A + B) + 2B$

$A + C = 0$
$2A + B = 0$
$2B = 2$

$2A + 1 = 0$ $\quad -1/2 + C = 0$

$B = 1$ $\quad A = -1/2$ $\quad C = 1/2$

$$= \frac{-1/2}{x} + \frac{1}{x^2} + \frac{1/2}{x+2}$$

$(x+1)(x-3)$

$x(x-3)+1(x-3)$

$x^2-3x+1x-3$

$x-2x-3$

$(x+2)(x-3)$ $\quad x(x-3)+2(x-3)$ $\quad x^2-x-6$

$x^2-3x+2x-6$

9. $\displaystyle\int \frac{x+5}{x^2+2x-3}$

$(x-1)(x+3) = x(x+3)-1(x+3)$

$= x^2+3x-1x-3$

$= x^2+2x-3$

$$\int \frac{x+5}{(x-1)(x-3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$= \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$= A(x+3) + B(x-1) = Ax + 3A + Bx - B$$

$$= (A+B)x + 3A - B = x+5$$

$(A+B)x = 0$

$3A = 1$

$-B = 5$

$x+5 = A(x+3) + B(x-1)$

$x=1$; $1+5=6 = A(1+3) = 4A = A = 3/2$

$x=-3$; $-3+5=2 = B(-3-1) = -4B = B = -1/2$

$$\therefore \frac{x+5}{x^2+2x-3} = \frac{3/2}{x-1} - \frac{1/2}{x+3}$$

$$\int \frac{x+5}{x^2+2x-3}\,dx = \frac{3}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+3}\,dx$$

$$= \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+3| + C$$

5) $\int x^2 \ln(x)\, dx$ LIATE

$u = \ln x \qquad dv = x^2\, dx$

$du = dx \cdot \frac{1}{x} \qquad v = \frac{x^3}{3}$

$\frac{\ln x \cdot x^3}{3} - \int \frac{x^2}{3} \cdot dx$

$\frac{\ln x \cdot x^3}{3} - \frac{x^3}{9} + C$

6) $\int e^x \cos x\, dx$ LIATE

$u = \cos x \qquad dv = e^x dx$

$du = dx \cdot -\sin x \qquad v = e^x$

$\cos x \cdot e^x + \int e^x \cdot \sin x\, dx$ LIATE

$u = \sin x \qquad dv = e^x dx$

$du = +\cos x\, dx \qquad v = e^x$

$= +\sin x\, e^x - \int e^x \cos x$

$\cancel{\cos x \cdot e^x + \int e^x \sin x\, d}$

$+\sin x \cdot e^x - \int e^x \cos x + \cos x e^x = \int e^x \cos x\, dx$

$= +\sin x \cdot e^x + \cos x \cdot e^x = 2\int e^x \cos x$

$C + \frac{\sin x \cdot e^x + \cos x \cdot e^x}{2} = \int e^x \cos x$

Question 3

$$\int \frac{1}{1+e^x}\,dx = x - \ln(1+e^x) + c$$

$$\frac{d}{dx}\left[x - \ln(1+e^x)\right] = 1 - \frac{e^x}{1+e^x} = \frac{1+e^x-e^x}{1+e^x} = \frac{1}{1+e^x}$$

$$\int \frac{1}{1+e^x}\,dx = \int 1\,dx - \int \frac{e^x}{1+e^x}\,dx = x - \int \frac{e^x}{1+e^x}\,dx$$

$$du = e^x\,dx$$

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{du}{u} = \ln|u| + c = \ln(1+e^x) + c$$

4 $\int x^2 e^x dx$

$\int u\,dv = uv - \int v\,du$

Let

$u = x^2 \Rightarrow du = 2x\,dx$

$\int x^2 e^x dx = x^2 e^x - \int e^x (2x) dx$

We solve for parts again by apply Integration

$u = x$

$du = dx$

$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + c$

substitute back

$\int x^2 e^x dx = e^x x^2 - 2(x e^x - e^x)$

Step 6: factor

$\int x^2 e^x dx = e^x (x^2 - 2x + 2) + c$

Work sheet

1. Evaluate using substitution.

$$\int x\sqrt{x+3}\,dx$$

let $x+3 = u$ $\quad x = u-3$

$du = dx$

$$\int (u-3)\sqrt{u}\,du$$

$$\int (u^{3/2+1} - 3u^{1/2})\,du$$

$$2/5\,u^{5/2} - 2u^{3/2} + C$$

$$= 2/5\,(x+3)^{5/2} - 2(x+3)^{3/2} + C$$

2. Compute

$$\int \frac{x^2}{(x^3+1)^2}\,dx$$

$u = x^3+1$ $\quad x^2dx = du/3$

$du = 3x^2dx$

$$\int \frac{x^2}{(x^3+1)^2} = 1/3 \int \frac{u^{-2+1}}{-1}\,du$$

$$= 1/3\,\frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{3u} + C$$

$$= \frac{1}{3(x^3+1} + C$$