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1. $\int x\sqrt{x+3}\,dx$

let $u = x+3 \Rightarrow \frac{du}{dx} = 1$; $du = dx$

but also $x = u-3$. $[u = x+3]$

$\therefore \int x\sqrt{x+3} \Rightarrow \int (u-3)\sqrt{u}\,dx$

$= \int (u^{3/2} - 3u^{1/2})\,dx$

$\therefore \frac{2}{5}u^{5/2} - 2u^{3/2} + c.$

$\therefore \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + c.$

2. $\int \frac{x^2}{(x^3+1)^2}\,dx.$

let $u = (x^3+1)$; $\frac{du}{dx} = 3x^2$; $dx = \frac{du}{3x^2}$

$\therefore \int \frac{\cancel{x^2}}{(x^3+1)^2} \cdot \frac{du}{3\cancel{x^2}} \quad \therefore \int \frac{1}{u^2} \cdot \frac{1}{3} \Rightarrow \int \frac{1}{3} u^{-2}$

$= \cancel{\frac{1}{3} \ln u}$

$\Rightarrow -\frac{1}{3}u + c$

$\therefore \frac{-1}{3(x^3+1)} + c.$

3. $\int \frac{1}{1+e^x}$

$\frac{1}{1+e^x} \times \frac{1-e^x}{1-e^x} \Rightarrow 1 - \frac{e^x}{1+e^x}$

$\therefore \int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$

↳ let $u = (1+e^x)$; $\frac{du}{dx} = e^x$; $dx = \frac{du}{e^x}$.

$\therefore \int 1\,dx - \int \frac{\cancel{e^x}}{\cancel{1+e^x}\,u} \cdot \frac{du}{\cancel{e^x}}$

$\therefore \int 1\, dx - \int \frac{1}{u} \cdot du$

$(u = 1 + e^{x})$

$\therefore x - \ln|1+e^{x}| + c.$

4) $\int x^2 e^x \, dx$

let $u = x^2$; $\frac{dy}{dx}\, du = 2x$

let $dv = e^x$; $dv = e^x$

$\therefore x^2 e^x - \int 2x e^x \, dx.$

$\int 2x e^x \, dx$

let $u = 2x$; $du = 2$

let $v = e^x$; $dv = e^x$

$\therefore 2xe^x - \int 2e^x \, dx$

$\therefore 2x\cancel{^2}e^x - 2e^x$

$\Rightarrow x^2 e^x - (2xe^x - 2e^x) + c.$

$\therefore e^x (x^2 - 2x + 2) + c.$

5) $\int x^2 \ln(x) \, dx.$

~~let u = ln(x) ; du = 1/x .dx ; dx = x~~

let $u = \ln x$; $du = 1/x$

let $dv = x^2$; $v = x^3/3$

$\therefore \ln x \left(x^3/3\right) - \int \frac{\cancel{x^3}\, x^2}{3} \cdot \frac{1}{\cancel{x}} \, dx$

$\therefore \frac{x^3}{3} \ln(x) - \frac{1}{3} \int \frac{1}{\cancel{x}} \cancel{x} x^2 \, dx$

$\therefore \frac{x^3}{3} \ln(x) - \left[\frac{1}{3} \cdot \frac{x^3}{3}\right] + c$

$\therefore \frac{x^3}{3} \ln(x) - \frac{x^3}{9} + c.$

5. $\int x^2 \ln(x)\,dx$

$u = \ln(x)$ $\quad dv = x^2$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = \frac{1}{3}x^3$

$uv - \int v\,du$

$\ln x \cdot \frac{1}{3}x^3 - \int\left(\frac{1}{3}x^2 \cdot \frac{1}{x}\right)$

$\ln x \cdot \frac{1}{3}x^3 - \int \frac{1}{3}x^2$

$\frac{1}{3}x^3 \cdot \ln x - \frac{1}{9}x^3 + C$

6. $\int e^x \cos x\,dx$

$u = \cos x$ $\quad dv = e^x\,dx$

$du = -\sin x$ $\quad v = e^x$

$uv - \int v\,du$

$e^x \cdot \cos x + \underbrace{\int e^x \cdot \sin x}$

$u = \sin x$ $\quad dv = e^x\,dx$

$du = \cos x$ $\quad v = e^x$

$e^x \cdot \sin x - \int e^x \cdot \cos x$

$I = \int e^x \cos x$

$I = e^x \cos x + \int e^x \sin x$

$I = e^x \cos x + \left(e^x \sin x - \int e^x \cos x\right)$

$I = e^x \cos x + e^x \sin x - I$

$2I = e^x \cos x + e^x \sin x$

$\frac{2I}{2} = \frac{e^x}{2}(\cos x + \sin x)$

$= \frac{e^x}{2}(\cos x + \sin x)$

7.

$\int_0^1 x^3 - 3x^2 + 2x + \int_1^2 x^3 - 3x^2 + 2x$

$\left[\frac{1}{4}x^4 - x^3 + x^2\right]_0^1$

$\frac{1}{4}(1)^4 - (1)^3 + (1)^2$

$= 0.25$

$\frac{1}{4}(2)^4 - (2)^3 + (2)^2$

$= 0$

$0 - 0.25 = -0.25$

$|0.25| + |-0.25|$

$= 0.5$ sq units

8. $\int e^{-x} - \int 1 - x$

$-e^{-x} - \left(x - \frac{1}{2}x^2\right)$

$-e^{-x} - x + \frac{1}{2}x^2$

$e^{-x} = 1 - x$

$e^{-x} + x - 1 = 0$

$9. \int \frac{x+5}{x^2+2x-3}$

$P = -3$
$S = 2$
$N = 3, -1$

$x^2 - x + 3x - 3$

$x(x-1) + 3(x-1)$

$\frac{x+5}{(x-1)(x+3)}$

$\frac{A}{x-1} + \frac{B}{x+3}$

$\frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$

$Ax + 3A + Bx - B$

$x(A+B) + 3A - B$

$A+B = 1$ $\quad B = -A$

$3A - B = 5$

$3A + A = 5$

$4A = 5$ $\quad A = 5/4$

$B = -5/4$

$\int \frac{5}{4(x-1)} - \int \frac{5}{4(x+3)}$

$\frac{5}{4} \int \frac{1}{x-1} - \frac{5}{4} \int \frac{1}{x+3}$

$\frac{5}{4} \ln|x-1| - \frac{5}{4} \ln|x+3| + C$

$10. \int \frac{x^2+3x+2}{x^3+2x^2}$

$\frac{x^2+3x-2}{x^2(x+2)}$

$P = 0$
$S = 2$
$N = 2, 0$

$x + 2x + 0$

$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

$\frac{A(x^2+2x) + B(x+2x) + C(x^2)}{x^2(x+2)}$

$\cancel{Ax^2 + 2Ax + Bx^2 + 2Bx + Cx^2}$

$\cancel{x^2(A+B+C) + x(2A+2B)}$

$\cancel{2A + 2B = 3}$

$\cancel{A+B = 3/2}$

$\cancel{C + \frac{3}{2} = 1}$

$\cancel{C = -0.5}$

$\cancel{A+B = 0.5}$

$Ax(x+2) + B(x+2) + Cx^2$

$(A+C)x^2 + (2A+B)x + 2B$

$2B = 2 \quad B = 1$

$1 + C = 1$

$2A + B = 3 \quad \underline{\underline{A = 1}} \quad C = 0$

$2A + 1 = 3$

$$\int \frac{1}{x} + \int \frac{1}{x^2} + \frac{0}{x+2}$$

$$\ln x + \frac{1}{x} + c$$