Integral Group

1) $\int x\sqrt{x+3}\,dx$

let $x+3 = u$

$\frac{du}{dx} = 1$

$du = dx$

$\int x u^{1/2} \cdot du$

$x = u - 3$

$\int x\sqrt{x+3}\,dx = \int (u-3)\, u^{1/2}\, du$

$= \int u^{3/2} - 3u^{1/2}\, du$

$= \frac{u^{3/2+1}}{3/2+1} - 3\int \frac{u^{1/2+1}}{1/2+1} + c$

$= \frac{u^{5/2}}{5/2} - \frac{3u^{3/2}}{3/2}$

$= \frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + c$

2) $\int \frac{x^2}{(x^3+1)^2}\,dx$

let $x^3+1=u$ $\quad \frac{dy}{dx}=3x^2$ $\quad dx=\frac{dy}{3x^2}$

$\int \frac{x^2}{u^2}\cdot\frac{du}{3x^2}$

$\frac{1}{3}\int \frac{du}{u^2} = \frac{1}{3}\int \frac{u^{-2+1}}{-2+1} = \frac{1}{3}\int \frac{u^{-1}}{-1}$

$= \frac{-1}{3(x^3+1)}$

3 $\int \frac{1}{1+e^x}\,dx = x - \ln(1+e^x)+C$

Let $u = 1+e^x$

$du = e^x\,dx$

$dx = \frac{du}{e^x}$

If $u = 1+e^x$ then $e^x = u-1$

$dx = \frac{du}{u-1}$

$\int \frac{1}{1+e^x}\,dx = \int \frac{1}{u}\cdot\frac{du}{u-1}$

$= \int \frac{1}{u(u-1)}\cdot du$

$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$

$1 = A(u-1) + B\cdot u$

~~Let~~ If $u=0$: $1 = -A \Rightarrow A = -1$

If $u=1$: $1 = B \cdot 1 \Rightarrow B = 1$

$$\int \frac{1}{u(u-1)} = \int \left(\frac{-1}{u} + \frac{1}{u-1} \right)$$

$$= -\ln|u| + \ln|u-1| + C$$

$$= -\ln|1+e^x| + \ln|1+e^x - 1| + C$$

$$= -\ln|1+e^x| + \ln|e^x| + C$$

$$= x - \ln|1+e^x| + C$$

4 $\int x^2 e^x dx$ Division by parts

$u = x^2$, $dv = e^x dx \Rightarrow du = 2x dx$, $v = e^x$

$= x^2 e^x - \int 2x e^x dx$

Division by parts for $\int 2x e^x dx$

$2(xe^x - \int e^x dx) = 2(xe^x - e^x)$

$= x^2 e^x - (2xe^x - 2e^x)$

$= e^x(x^2 - 2x + 2) + C$

5 $\int x^2 \ln(x)\, dx$

By parts $u = \ln x$, $dv = x^2 dx$

$du = \frac{1}{x} dx$, $v = \frac{x^3}{3}$

$= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$

$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$

6 $\int e^x \cos x \, dx$

$u = \cos x, \quad dv = e^x dx$

$[e^x \cos x + (\int e^x \sin x \, dx)]$ Let this be I

Let this be i

Integrate J

$u = \sin x, \quad dv = e^x dx$

$= e^x \sin x - \int e^x \cos x \, dx = e^x \sin x - I$

Substitute J

$I = e^x \cos x + (e^x \sin x - I) \Rightarrow 2I = e^x (\sin x + \cos x)$

$I = \frac{e^x}{2} (\sin x + \cos x) + C$

$= \frac{e^x}{2} (\sin x + \cos x) + C$

7

$$\int_0^2 [x^3 - 3x^2 + 2x] =$$

$$\frac{x^4}{4} - \frac{3x^3}{3} + \frac{2x^2}{2}$$

$$\left| \frac{1}{4}x^4 - x^3 + x^2 \right|_0^1 + \left| \right|_1^2$$

$$\frac{1}{4}(1)^4 - 1^3 + 1^2$$

$$= \frac{1}{4}$$

$$\frac{1}{4}(0)^4 - 0^3 + 0^2$$

$$= 0$$

$$0 - \frac{1}{4}$$

$$= -\frac{1}{4}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} = 0.5 \text{ sq. units}$$

8. Find the area of the curves $y = e^{-x}$ and $y = 1 - x$ between their points of intersection.

$e^{-x} = 1 - x$ 2x

$x = 1 - e^{-x}$

$x + e^{-x} = 1$

9 $\int \frac{x+5}{x^2+2x-3}$

$x^2 + 2x - 3$ $x^2 + 3x - x - 3$

prod $= -3$

sum $= 2 (3, -1)$ $x(x+3) - 1(x+3)$

$(x-1)(x+3)$

$$\frac{x+5}{(x-1)(x+3)}$$

$$= \frac{A}{x-1} + \frac{B}{x+3}$$

$A(x+3) + B(x-1) = (x+5)$

$Ax + 3A + Bx - B = (x+5)$

$(Ax + Bx) + (3A - B) = (x+5)$

$x(A+B) = 1$

$(3A - B) = 5$

$4A = 6$ $A = 1.5 + B = 1$

$A = \frac{6}{4} \frac{3}{2}$ y ? $B = 1 - 1.5$

$= \frac{9}{2} : 4.5 - (-0.5) = 5$ $B = -0.5$

$$\int \frac{1.5}{x-1} + \int \frac{-0.5}{x+3}$$

$$1.5 \int \frac{1}{x-1} dx + -0.5 \int \frac{1}{x+3}$$

$$\frac{3}{2} \ln|x-1| + -\frac{1}{2} \ln|x+3| + c$$

10. $\int \frac{x^2+3x+2}{x^3+2x^2}$

$x(x^2+2x^1)$
$= x^2(x+2)$

x^2+3x+2 $\quad$ $x^2+2x+x+2$
prod = 2
sum = 3 (2, 1)
$x(x+2)1(x+2)$
$(x+1)(x+2)$

$\frac{(x+1)(x+2)}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

$A\left[x^2(x+2)\right] = x^0+2x = Ax^0+2Ax$

$B\left[(x+2)\right] = Bx+2B$

$C\ [x^0] \quad Cx^2$

$Ax^2+2Ax+Bx+2B+Cx^0$

$x^0(A+C)+x(2A+B)+2B$

$2B = 2$
$B = 1$

$1+C=1$
$C = 1-1$
$C = 0$

$2A+B = 3$
$2A+1 = 3$
$2A = 3-1$
$2A = 2$
$A = 1$

$\int \frac{1}{x}dx + \int \frac{1}{x^2}dx + \int \frac{0}{(x+2)}dx$

$\ln|x| + \ln|x^2|$

$\int \frac{x^{-2+1}}{-2+1}dx = \frac{x^{-1}}{-1} = \frac{-1}{x}$

$= \ln|x| + \frac{-1}{x} + c$

Group members

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