9) Evaluate using partial fractions

$$\int \frac{x+5}{x^2+2x-3}\,dx.$$

x^2+2x-3

$(x-1)(x+3)$

$$\int \frac{x+5}{(x-1)(x+3)}\,dx = \frac{A}{x-1} + \frac{B}{x+3}$$

$$= \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$= Ax + 3A + Bx - B$$

$$Ax + Bx + 3A - B$$

$$(A+B)x + 3A - B$$

$A+B=1$ $\quad 3A-B=5$

$A = 1-B$

$3(1-B) - B = 5$

$3 - 3B - B = 5$

$3 - 4B = 5$

$3 - 5 = 4B$

$\frac{-2}{4} = \frac{4B}{4}$

$B = -\frac{1}{2}$

$A = 1 - \left(-\frac{1}{2}\right) = 1 + \frac{1}{2} = \frac{3}{2}$

$A = \frac{3}{2}, \quad B = -\frac{1}{2}$

$$\int \frac{x+5}{x^2+2x-3}\,dx = \frac{3}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+3}\,dx$$

$$= \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+3| + c$$

10. Compute

$$\int \frac{x^2+3x+2}{x^3+2x^2}\,dx$$

$x^3+2x^2 = x^2(x+2)$

$$\int \frac{x^2+3x+2}{x^2(x+2)}\,dx$$

$$= \int \frac{x^2+3x+2}{x^2(x+2)}\,dx = \frac{A}{x} + \frac{B}{x}$$

$x^2+3x+2 = (x+1)(x+2)$

$$\int \frac{x^2+3x+2}{x^3+2x^2}\,dx = \int \frac{(x+1)(x+2)}{x^2(x+2)}\,dx$$

$$= \int \frac{x+1}{x^2}\,dx = \frac{x}{x^2} + \frac{1}{x^2} = \frac{1}{x} + x^{-2}$$

$$\int \left(\frac{1}{x} + x^{-2}\right)dx = \ln|x| - \frac{1}{x} + C$$

$$= \ln|x| - \frac{1}{x} + C$$