5. $\int x^2 \ln(x)\, dx$

$$\int u\, dv = uv - \int v\, du$$

Let $u = \ln(x) \rightarrow du = \frac{1}{x}\, dx$

$dv = x^2 dx \rightarrow v = \frac{x^3}{3}$

$$\int x^2 \ln(x)\, dx = \frac{x^3}{3}\ln(x) - \int \frac{x^3}{3} \cdot \frac{1}{x}\, dx$$

$$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2\, dx$$

$$\int x^2 dx = \frac{x^3}{3}$$

$$\int x^2 \ln(x)\, dx = \frac{x^3}{3}\ln(x) - \frac{1}{3}\cdot\frac{x^3}{3} + C$$

$$\text{Ans} = \int x^2 \ln(x)\, dx - \frac{x^3}{9} + C$$

8. $e^{-x} = 1 - x$

$x = 0$, $e^0 = 1$, $1 - 0 = 1 \Rightarrow$ equal
$x = 1$, $e^{-1} = \frac{1}{e} = 0.368$, $1 - 1 = 0 \Rightarrow$ not equal
$x = -1$, $e^1 = 2.718$, $1 - (-1) = 2 \Rightarrow$ close

$x = -0.6$ and $x = 0$

At $x = -0.5$

$e^{-(-0.5)} = e^{0.5} = 1.65$ $\quad 1 - (-0.5) = 1.5$

So $e^{-x} > 1 - x$

Top curve $\Rightarrow y = e^{-x}$

$$A = \int_{-0.6}^{0} \left[e^{-x} - (1 - x)\right] dx$$

$$\int e^{-x}\, dx = -e^{-x} \qquad \int (1 - x)\, dx = x - \frac{x^2}{2}$$

$$A = \left[-e^{-x} - \left(x - \frac{x^2}{2}\right)\right]_{-0.6}^{0}$$

At $x = 0$: $-e^0 - (0 - 0) = -1$
At $x = -0.6$: $-e^{0.6} - (-0.6 - 0.18) = -1.822 + 0.78 = -1.042$

$A = (-1) - (-1.042) = 0.042$

Ans = 0.042 sq. units.