Wind Farm Sizing and Siting on IEEE-14 System

EEEL 4220 Fall 2025 Project 4

1 Introduction

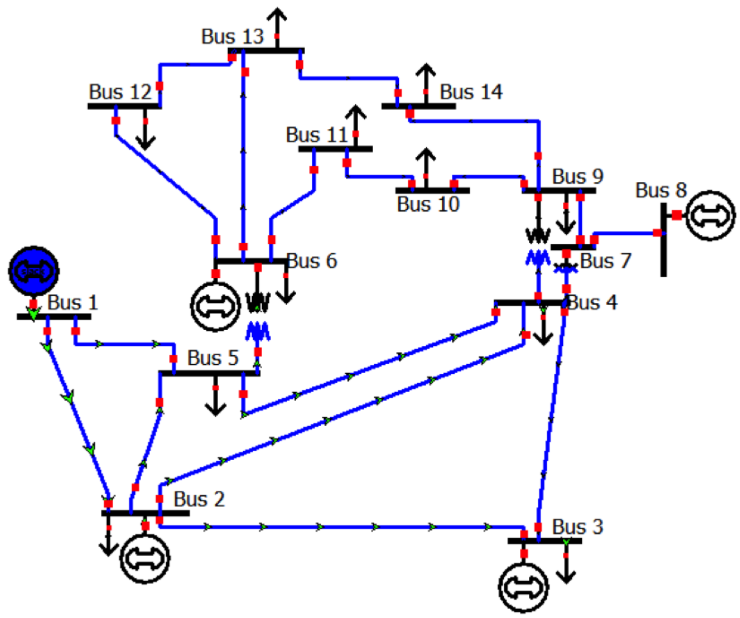

Figure 1: IEEE-14 bus system illustration.

In this project, you take the role of a system planner for a regional transmission operator tasked with integrating new wind generation capacity into an existing IEEE-14 bus power system. The system currently consists of multiple thermal generators, load centers, and transmission lines with finite ratings and cost functions.

Your goal is to determine where (siting) and how much (sizing) wind generation should be installed to minimize the total system cost while satisfying network constraints and meeting all system loads.

You are provided with:

- The IEEE-14 bus system data (buses, branches, generators, costs, and limits). Note that the original IEEE-14 bus system has a base unit of 100 MVA; for simplicity, we will just use MW in all units as in the given data.
- Five system scenarios with equal weights (probability) representing variations in demand and wind capacity factor.
- Candidate buses where wind farms can be built, each with a maximum feasible capacity.

You will use Python + CVXPY to model this as an optimization problem. You are recommended to use the HiGHS solver for this project and refer to Homework 7 for a starting example of economic dispatch with line constraints.

2 Project Objective

Formulate and solve an Optimal Siting and Sizing Problem for a single new wind project under demand and wind availability scenarios. The optimization should minimize the expected total system cost, which includes:

- Expected thermal generation cost (based on generator cost curves and scenario probabilities).
- Investment cost of wind capacity.

You must determine:

- The optimal wind capacity (MW) to install at each candidate bus under three wind farm cost cases: low cost: \$3.5/MW; medium cost: \$4/MW; and high cost: \$4.5/MW. Note that this number is prorated on a daily basis and averaged based on the system's power rating.
- Identify system total cost savings (including cost of installing the wind turbine) and emissions reductions in all cost cases. Use 500g per kWh of generated electricity for the emission intensity of all generators.
- If you can only install a wind farm at one location, what's the change in the result in the three wind farm cost cases?

3 Project Directions

This project will guide you through the step-by-step implementation of the *Wind Farm Siting and Sizing* model using Python and CVXPY. The process progresses from a simple economic dispatch validation to a full multi-scenario stochastic optimization.

To validate the system data and ensure solver setup is correct, first formulate a simple economic dispatch model that ignores wind generation and transmission constraints. You can start with the following steps

1. **Single-bus single-scenario, no wind planning.** Start assuming all buses one the same bus, minimize generation cost from a single scenario with one power balance constraint and the limit of each generator.
2. **14-bus single-scenario, no wind planning..** Then add transmission line constraints and introduce the bus angle. You can start by setting the line flow to 9999, which should produce the same result as the one bus example, then gradually adjust the flow limit.
3. **14-bus single-scenario, with wind planning.** Next, extend the model to include wind variables y_b and $w_{b,s}$ for a single scenario s^*. Use the following simplified formulation.
4. **14-bus 5-scenario, with wind planning.** Finally, assemble all scenarios together to produce the full planning model.

4 Project Deliverables

Each team must submit a complete project package consisting of a written report, a short presentation, and accompanying code and data. The written report (in PDF) serves as the main document summarizing your motivation, methods, results, and conclusions. The main text should focus on the key findings, including major results, important figures, and discussions, and should be no more than five pages using at least 11 pt font. A cover page listing the project title and team members should be included but does not count toward the page limit. Detailed mathematical formulations, algorithms, and any extended results should be provided in the appendix, which has no page limit.

All code and data used for the project should be submitted as a single compressed ZIP file along with the report and presentation. Code should be clearly commented and organized by function (e.g., deterministic optimization, demand prediction, and forecast-based optimization). A short README file should describe how to reproduce your main results.

All deliverables—the report, presentation, and ZIP file—must be uploaded to Coursework by the project deadline.

Model Formulation

This project formulates the *Optimal Siting and Sizing of Wind Generation* as a multi-scenario DC Optimal Power Flow (DC-OPF) problem to minimize total system cost. The model determines the optimal location and capacity of wind generation while ensuring network feasibility and demand satisfaction. The full formulation of the scenario-based planning problem is given as follows:

4.1 Sets and Indices

$b \in \mathcal{B}$	buses
$g \in \mathcal{G}$	thermal generators
$\ell \in \mathcal{L}$	transmission lines
$s \in \mathcal{S}$	scenarios (demand and wind capacity factors)

4.2 Parameters

c_{2g}, c_{1g}, c_{0g}	quadratic, linear, and fixed generation cost coefficients for generator g
$p_g^{\min}$, $p_g^{\max}$	minimum and maximum generation limits for generator g
x_ℓ	reactance of transmission line ℓ
$\bar{f}_\ell$	thermal limit of line ℓ
$D_{b,s}$	demand at bus b in scenario s
CF_s	wind capacity factor in scenario s
ω_s	probability (weight) of scenario s
$\bar{y}_b$	maximum installable wind capacity at bus b (0 if not candidate)
C_{inv}	investment cost per MW of installed wind capacity (\$/MW)
$A_{\ell b}$	incidence matrix element (+1 at from bus, -1 at to bus)

4.3 Decision Variables

$p_{g,s}$	power output of generator g under scenario s
$w_{b,s}$	wind generation at bus b under scenario s
y_b	installed wind capacity at bus b (MW)
$\theta_{b,s}$	voltage angle at bus b (radians)
$f_{\ell,s}$	power flow on line ℓ under scenario s

4.4 Objective Function

The objective minimizes the expected generation and investment cost:

$$\min_{p_{g,s},w_{b,s},y_b,\theta_{b,s},f_{\ell,s}} \left\{ \sum_{s\in\mathcal{S}} \omega_s \left(\sum_{g\in\mathcal{G}} (c_{2g}p_{g,s}^2 + c_{1g}p_{g,s} + c_{0g}) \right) + C_{\text{inv}} \sum_{b\in\mathcal{B}} y_b \right\} \tag{4.1}$$

4.5 Constraints

(a) DC Power Flow

$$f_{\ell,s} = \frac{1}{x_\ell} \sum_{b\in\mathcal{B}} A_{\ell b}\theta_{b,s}, \quad \forall \ell \in \mathcal{L},\, s \in \mathcal{S} \tag{4.2}$$

The *bus–branch incidence matrix* $A \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{B}|}$ describes how each transmission line (branch) connects pairs of buses in the power system. Each row of A corresponds to a transmission line $\ell \in \mathcal{L}$, and each column corresponds to a bus $b \in \mathcal{B}$.

For a transmission line ℓ that connects a "from" bus i and a "to" bus j, the elements of the incidence matrix A are defined as:

$$A_{\ell b} = \begin{cases} +1, & \text{if bus } b = i \text{ (the sending bus)}, \\ -1, & \text{if bus } b = j \text{ (the receiving bus)}, \\ 0, & \text{otherwise.} \end{cases} \tag{4.3}$$

Thus, each row of A contains exactly one $+1$ and one -1, representing the connection between two buses by a single transmission line.

(b) Line Flow Limits

$$-\bar{f}_\ell \le f_{\ell,s} \le \bar{f}_\ell, \quad \forall \ell \in \mathcal{L},\, s \in \mathcal{S} \tag{4.4}$$

(c) Generator Output Limits - we assume all generators are on so there is no unit commitment in this project

$$p_g^{\min} \le p_{g,s} \le p_g^{\max}, \quad \forall g \in \mathcal{G},\, s \in \mathcal{S} \tag{4.5}$$

(d) Wind Generation Limits

$$0 \le w_{b,s} \le CF_s \cdot y_b, \quad \forall b \in \mathcal{B},\, s \in \mathcal{S} \tag{4.6}$$

(e) Wind Capacity Limits (0 for buses without wind capacity potential)

$$0 \leq y_b \leq \bar{y}_b, \quad \forall b \in \mathcal{B} \tag{4.7}$$

(f) Power Balance at Each Bus

$$\sum_{g \in \mathcal{G}_b} p_{g,s} + w_{b,s} - D_{b,s} = \sum_{\ell \in \mathcal{L}} A_{\ell b} f_{\ell,s}, \quad \forall b \in \mathcal{B},\ s \in \mathcal{S} \tag{4.8}$$

The nodal power balance constraint enforces power conservation at every bus in the network.

Although generation, demand, and line flows are indexed by different sets ($\mathcal{G}$, $\mathcal{B}$, and $\mathcal{L}$), they can be combined consistently using incidence matrices that map each quantity to its corresponding bus.

- **Generator–bus incidence matrix:**

$$G \in \mathbb{R}^{|\mathcal{G}| \times |\mathcal{B}|}, \quad G_{gb} = \begin{cases} 1, & \text{if generator } g \text{ is connected to bus } b, \\ 0, & \text{otherwise.} \end{cases}$$

  Each row corresponds to a generator, and each column corresponds to a bus. For example, if two generators are connected to bus 2, the corresponding column in G will have two entries equal to 1.

- **Line–bus incidence matrix:**

$$A \in \mathbb{R}^{|\mathcal{L}| \times |\mathcal{B}|}, \quad A_{\ell b} = \begin{cases} +1, & \text{if bus } b \text{ is the sending (“from”) end of line } \ell, \\ -1, & \text{if bus } b \text{ is the receiving (“to”) end of line } \ell, \\ 0, & \text{otherwise.} \end{cases}$$

  Each row of A thus contains one $+1$ and one -1, indicating the connectivity of line ℓ.

(g) Reference Bus Angle

$$\theta_{b_{\text{ref}},s} = 0, \quad \forall s \in \mathcal{S} \tag{4.9}$$

(h) (Optional) Discrete Siting Constraint - this would lead to a mixed-integer quadratic programming

$$y_k \leq \bar{y}_k z_k, \quad \forall k \in \mathcal{W} \tag{4.10}$$

$$\sum_{k \in \mathcal{W}} z_k \leq 1, \quad z_k \in \{0, 1\} \tag{4.11}$$