

Quantitative Methods:

Exercises Set 1

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1. Linear demand is given by equation $Q = A - BP$, with $A > 0$ and $B > 0$.

- The price at which demand becomes zero is called “choke price.” Find the choke price.
- Plot that relationship (with P on horizontal axis and Q on vertical axis) for $A=48$, $B=2$.
- Find inverse demand (i.e., such function f that $P=f(Q)$) and plot it (with Q on horizontal axis and P on vertical axis) for $A=48$, $B=2$.

2. The cost of a patient’s stay in a hospital may be a function of the length of the stay (in days) and the amount of medical attention (in hours). Two patients had the following bills.

Patient	Days in Hospital (H)	Hours of Attention (A)	Total Bill (C)
1	4	10	\$ 500
2	7	30	\$ 1,125

If the cost is a linear function of length of stay, H , and hours of attention, A , such that $C(H,A) = aH + bA$, find the values of the constants a and b .

3. The prices of wheat and bread are P_w and P_b respectively. The demand (denoted by D) and supply (denoted by S) for each commodity are dependent on the two prices:

$$\begin{aligned} S_w &= 50 + 20P_w - 5P_b \\ S_b &= -10P_w + 10P_b \end{aligned}$$

$$\begin{aligned} D_w &= 50 - 10P_w + 5P_b \\ D_b &= 400 + 10P_w - 10P_b \end{aligned}$$

Find the values of P_w and P_b such that both markets are in equilibrium. That is, solve for P_w and P_b such that $D_w = S_w$ and $D_b = S_b$.

4. The inverse supply and demand functions for a particular market are given by the following equations.

$$\begin{aligned} P_d &= -(Q + 4)^2 + 100 \\ P_s &= (Q + 2)^2 \end{aligned}$$

Find the equilibrium price and quantity.

5. A company manufactures and sells a watch designed for sailboat racing. The financial analyst found that the company could sell 300 watches at a wholesale price of \$ 140 each, and 1,100 watches at \$ 92 each.

Assuming a linear relationship between price and demand, find a linear function that models the price-demand relationship.

What would be the price at a demand of 700 watches? 1200 watches?

6. A marketing manager wants to spend his entire budget, B , on advertising in two media, magazines and television. A unit of magazine costs \$ a and a unit of television costs \$ c . The manager wishes to buy twice as many television units as magazine units. In terms of a , c and B , how many magazine units should he buy?

7. **Linear relationship**

The Dundee Corporation has estimated that total production cost is 100,000 at a level of activity of 5,000 units and 240,000 at a level of 40,000 units.

Write a linear rule for this function and use it to estimate the level of fixed cost (cost incurred even though output is 0) of the Dundee company.

8. **Equilibrium price**

Given the following system of simultaneous equations for two substitute goods, beef b and pork p , find the equilibrium price and quantity for each market.

$$S_b = 15P_b - 5$$

$$S_p = 32P_p - 6$$

$$D_b = -3P_b + P_p + 82$$

$$D_p = 2P_b - 4P_p + 92$$

9. **Market equilibrium**

A used-car dealer in Fontainebleau found that when the price of a car is 1000€ there are 10 buyers who are interested in buying a car. When the price increases to 2000€ only 6 individuals are still interested.

The Supply function of used-cars is $P = 500 + 125 Q$

Assume that for Demand, price is a linear function of quantity.

What is the market equilibrium?

10. Price-Demand:

A company manufactures and sells a specialty watch. The financial research department, using statistical and analytical methods, determined that at a price of \$88 each, the demand would be 2000 watches, and at \$38 each, 12000 watches. Assuming a linear relationship between price and demand, find a linear function that models the price-demand relationship.

Consider:

1. Price as a function of Demand
2. Demand as a function of Price

What would be the price at a demand of 8000 watches, 15000 watches?

11. Simplify the following expressions:

a) $\left(\frac{x}{y}\right)^3 * \left(\frac{x}{z}\right)^{-2}$

b) $\frac{x^3 y^2}{x^5 y^{-2}}$

c) $\frac{24x^5 y^3 z^7}{6x^3 y^2 z^4}$

d) $\frac{120xy^3 z^7}{6x^3 y^2 z^4}$

12. Evaluate each of the following expressions.

a) $(9)^{3/2}$

b) $(8)^{4/3}$

c) $\left(\frac{1}{4}\right)^{5/2}$

d) $27^{-2/3}$

e) $(-4)^5$

f) $(-2)^6$

13. Find the future values (A) of the following amounts.

- a) $P = \$ 2000$, at $r = 5\%$ interest compounded annually for $t = 10$ years.

$$A = P(1 + r)^t$$

- b) $P = \$1,500,000$, at $r = 7\%$ interest compounded semi-annually ($n = 2$) for $t = 6$ years.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

- 14. If you borrowed \$ 800 for 4 years at simple yearly interest rate 4%, how much would the creditor receive at the termination of the contract? How much would the interest amount be?**

- 15. If a man invests for 10 years at 5% compounded quarterly, and receives \$ 16,436.20 at the end of the 10 years, what was his original investment?**

- 16. Suppose I put a sum of money in the bank at a rate of 3% compounded yearly. The total interest received after 2 years is \$ 300. How much was the original sum?**