

Quantitative Methods:

Exercises Set 5

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These exercises were prepared by Michèle Hibon, Emeritus Lecturer, and Ilia Tsetlin, Associate Professor of Decision Sciences, both at INSEAD.

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Normal distribution

1. Random variable Z is normally distributed with mean 0 and standard deviation 1.

- a) What is the probability that $Z > 1.645$?
- b) What is the probability that $Z < -1.645$?
- c) Find such a value of A that probability that Z is between $-A$ and A is 0.95 (i.e., that $\text{Prob}(-A < Z < A) = 0.95$)

2. Ages of the MBA students at INSEAD tend to be normally distributed with a mean of 28.7 years and a standard deviation of 2.4 years.

- a) What is the probability that a student randomly selected is less than 27?
- b) What is the probability that a student randomly selected is more than 31?

3. The weight of babies born nowadays has a mean of 3.25 kilograms and a standard deviation of 0.35 kilograms. These weights tend to be normally distributed.

- a) What is the percentage of babies that weigh more than 4 kilograms?
- b) What is the probability that a baby will weigh between 2.5 and 3.5 kilograms?
- c) It is estimated that of all babies born, 22% need additional care because of their low weight. What is the maximum a baby could weigh and still be in the 'additional care' group?

4. Monthly percentage changes in the Dow-Jones stock price index is normally distributed with a mean of 0.65 and a standard deviation of 3.5.

Answer each question if a month is selected at random.

- a) What is the probability that the index rose during the month?
- b) What is the probability of an increase exceeding 5 %?
- c) What percentage increase is exceeded with a probability of 0.05?

5. **The service lives of Sultania light bulbs are normally distributed with a mean of 1200 hours and a standard deviation of 36 hours.**

- a) What percentage of the bulbs will last for longer than 1272 hours?
- b) What percentage of bulbs will last for less than 1146 hours?
- c) The 10% of the bulbs with the longest service life will last more than how many hours?
- d) The 20% of the bulbs with the shortest service life will last less than how many hours?

6. **Executive Airlines operates small luxury jets on commuter flights from Orly airport to Heathrow (London). It has found that its flight time from Orly to Heathrow is normally distributed with a mean of one hour and a standard deviation of 6 minutes.**

To promote its service, the airline management is considering offering customers a money-back guarantee: if any flight takes more than 70 minutes to get to London, each passenger riding that plane will receive a full refund of the €400 (one-way) fare. Thus, sometimes the airline will pay out the refunds, but usually it will not.

What is the probability that a plane will take more than 70 minutes to get to London.

What is the expected loss of revenue per passenger because of this policy?

7. **Footloose is a disco-club, where the “best-seller” is the tequila sunrise.**

The number of tequila sunrises sold, Q , is normally distributed with mean 250. It is known that $Prob(Q > 275) = 0.15$.

What is the variance?

8. **The weekly consumption of laundry detergent per household in a given city is normally distributed.**

What are the mean and the standard deviation of this distribution if it is known that 99% of the households in the city use between one and ten liters?

9. **The time it takes a student to go from her house to school is normally distributed with a mean of 20 minutes and a standard deviation of 5.**

Estimate the percentage of time she will be late for class if she leaves her house 30 minutes before the start of class. If you were her, how much time would you allow to go to school each day?

- 10. An architect designing the men's gymnasium at a university wants to make the interior doors high enough, so that 95% of the men will have at least a 1-foot clearance.**

Assuming that the heights will be normally distributed, with a mean of 70 inches and a standard deviation of 3 inches, how high must the architect make the doors?

- 11. An important quality characteristic for soft-drink bottlers is the amount of soft drink in the bottle (just think what Pepsi could do if Coke bottles were sometimes only half full). In a particular filling process (when working as usual), the number of ounces injected into a 12-ounce bottle is normally distributed with a mean of 12.00 ounces and a standard deviation of 0.04 ounces. Bottles that contain less than 11.90 ounces do not meet the bottler's quality standard and are sold at a substantial discount.**

What is the probability that a randomly selected bottle will fail to meet the quality standard?

- 12. Accurate sales forecast are critical in ensuring efficient production and distribution of a product. However, you have found that your analyst's sales forecast are often off the mark. Reviewing her forecasts for the past few years, it appears to you that her errors (in percentage terms) are normally distributed with a mean of 0 percent and a standard deviation of 10 percent.**

- (a) If you obtain one sales forecast from the analyst, what is the probability that her forecast is inaccurate by more than 15 percent?
- (b) As part of an incentive scheme for encouraging accurate forecasts, you decide to offer the analyst a \$1000 bonus when she gives accurate forecasts: she will receive the bonus if her error, in any direction (+ or -), is at most c percent. You want to choose this cutoff c so that, at the current accuracy levels, only 5 percent of her forecasts will earn the bonus. What value of c should you use?