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Quantitative Methods
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Quantitative Methods:

Exercises Set 3

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1. For each of the total revenue *R*, total cost *C* and profit *P* functions, find the derivative, called the marginal function, and evaluate it at *x* = 8. Is the original function increasing or decreasing at *x* = 8?

a) $R(x) = 50x - x^2$

b) $C(x) = x^2 + 10x + 48$

c) $P(x) = 3x^2 - 28x + 132$

d) $C(x) = 3x^3 - 21x^2 + 11x + 65$

2. Find the marginal revenue functions associated with each of the following supply functions

$P =$ price and $Q =$ quantity. Evaluate them at $Q = 5$

a) $P = Q^2 + 4Q + 9$

b) $P = \frac{1}{2}Q^2 + 3Q + 8$

c) $P = \frac{1}{4}Q + 60$

Hint: To find a marginal revenue function R', given a supply or demand function, first find the total revenue function

$R = P * Q$ and then take the derivative of R with respect to Q.

3. Differentiate each of the following:

a) $f(x) = 13$
b) $f(x) = -27$

c) $f(x) = 7x - 12$

d) $f(x) = 25 - 6x$

e) $f(x) = 9x^4$

f) $f(x) = -5x^7$

g) $f(x) = 4x^{-3}$

4. The average waiting time, *T*, in a hospital emergency room depends on the number of people on the staff, *x*.

The form of the relationship is:
$T = x^2 - 30x + 240$ for $x \geq 1$

Find the level of staff that will minimize the average waiting time.

5. A manager has a linear demand function in units of the form

demand = $50 - 4p$ $\quad p$ = price
The cost of each unit is \$5, and the company has a longstanding policy that requires $p \leq 10$.
How much should they charge to maximize profit?
If $p \leq 8$ is required, what prices should they charge?

6 Revenue and cost. The price-demand equation and the cost function for the production of items are given, respectively, by

$$x = 6000 - 30p \qquad \text{and} \qquad C(x) = 72000 + 60x$$

where *x* is the number of items that can be sold at a price of \$*p* per item and *C*(*x*) is the total cost (in dollars) of producing *x* items.

a) Express the price p as a function of the demand.

b) Find the marginal cost.

c) Find the revenue function.

d) Find the marginal revenue for $x = 1500$ and $x = 4500$ and interpret the results.

7. Find the level of output at which profit P is maximized for the firm in each of the following cases, given the total revenue *R* and a cost *C* functions.

Consider only $Q > 0$ and check if it is a maximum.

a) $R = 600Q - 5Q^2$, $\quad C = 320 + 20Q$

b) $R = 1300Q - 4Q^2$, $\quad C = 2000 + 100Q$

8. **If price per unit of product (i.e., inverse demand) is *P*(*Q*), total revenue is given by *TR*(*Q*)=*P*(*Q*)*Q. Marginal revenue *MR*(*Q*) is the derivative of *TR*(*Q*).**

For a linear inverse demand $P(Q)=A-B*Q$, find $MR(Q)$.

9. **A monopolist faces the following demand and total cost conditions:**

$Q=120-2P$, $TC(Q)=Q^2$.

What is the profit-maximizing price and quantity, and what are the resulting monopoly profits?

10. **A company's profit at a point in time is given by $f(x)=20+3x+x^2$, where x is the number of years the company has been in business ($x=0$ in 1970) and $f(x)$ is in millions of dollars.**

a) At what rate are company profits growing after 3 years?

b) Predict the level of profits when $x = 3$

11. **The demand function for a good is $P=125-Q^{1.5}$**

a) Write the expressions for Total Revenue (TR) and Marginal revenue (MR)

b) Evaluate TR and MR at $Q=10$
Explain in words the meaning of each function

c) Calculate the value of Q for which $MR=0$
At what value of Q does the sale of further units first start to reduce total revenue?