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Commentary on Lara Buchak's risk and rationality

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Lara Buchak has written a splendid book that everyone interested in decision making should read. The mastery of technical material is impressive, and Buchak defends her views with ingenuity and insight. While I agree with her that the standard theory of risk is inadequate, I am not convinced by her alternative "risk weighted utility" (REU) because I do not find its "rank dependent" nature normatively appealing. After a sketch of REU, I will question certain aspects of its intuitive motivation, and raise some concerns about Buchak's attempts to avoid "Dutch books."

1 REU

In Buchak's picture a rational agent is represented by a triple $\langle u, p, r\rangle$ where u is a *utility* function that measures the agent's desires of outcomes, p is a *subjective probability* function that reflects her beliefs about the probabilities of these outcomes, and r is an increasing *risk function* that maps probabilities into real numbers. The first two elements are familiar from expected utility theory (EU), where the choice-worthiness of an act A that produces outcomes $o_1,\ldots, o_N$ with probabilities $p_1,\ldots, p_N$ is its expected utility $EU(A) = p_1{\cdot}u(o_1) + \cdots + p_N{\cdot}u(o_N)$. The risk function is the new twist. Adapting the formalism of "rank-dependent" utility theory, Buchak uses r to represent attitudes toward risk that cannot be captured in EU. In cases where A generates a better outcome o^+ with probability p and a worse outcome o^- with probability $1 - p$, Buchak proposes to measure A's choice-worthiness as $REU(A) = r(p){\cdot}u(o^+) + (1 - r(p)){\cdot}u(o^-)$. As in EU, this is a weighted average of utilities, but now the weights are the agent's probabilities

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transformed by r. This allows Buchak to capture attitudes toward risk that depend on "global" features of actions not reflected in their expected utilities, specifically features that concern the positions of outcomes in the ranking of all outcomes that the act might produce (e.g., the fact that o^+ is A's best outcome and that o^- is its worst). The r-function lets agents weight outcomes differentially depending on where in the ranking they appear. For example, if r is a convex function like $r(x) = x^2$, then $r(p) < p$ and the better (worse) outcome gets less (more) weight than it does in EU. This is what one might expect from a "risk avoidant" agent who is more concerned with avoiding bad outcomes than securing good ones. If r is concave, the agent will exhibit "risk-loving" behavior. And, when r is flat, $r(x) = x$, the agent will behave like an expected utility maximizer.[1]

In contrast with EU, when an outcome of utility $u(o)$ and probability $p(o)$ appears in some larger prospect its effect on the prospect's value in REU depends on where it is ranked relative to other outcomes. For example, suppose our agent is a miser[2] with $r(x) = x^2$, and consider:

	30%	20%	50%
A	\$10 (min)	\$100 (max)	\$20 (mid)
B	\$10 (mid)	\$150 (max)	\$0 (min)

The 30% chance at \$10 counts for \$5.10 of A's total REU value of \$18.10, but only \$2.10 of B's value of \$8.10. This difference is explained by the fact that, despite having the same probability, the \$10 is weighted more heavily in A, where it is a minimum, than in B, where it's a middle.

Despite this difference, REU and EU agree that each action has a value that remains fixed across different decisions problems with varying act menus. For example, to compute the REU value of act A above it suffices to know the agent's probabilities and utilities for A's outcomes, and her risk function (which is independent of the details of the decision in Buchak's picture). It does not matter, for example, that A's minimum is B's middle, or that A's maximum is not as big as B's. Indeed, if we erased B entirely we would still know enough to determine $REU(A)$. So, when Buchak talks of "global" properties she means properties of acts taken in isolation, not properties that acts have relative to some background of other possible acts.

[1] If A has three outcomes, ranked $u(o^+) > u(o) > u(o^-)$, with probabilities p^+, p and $p^- = 1 - (p^+ + p)$, then $REU(A) = w^+ \cdot u(o^+) + w \cdot u(o) + w^- \cdot u(o^-)$ where $w^+ = r(p^+)$, $w = r(p^+ + p) - r(p^+)$ and $w^- = 1 - r(p^+ + p)$.

[2] A *miser* is someone who values only money and who values it linearly at every wealth level, so that the difference in utility between gaining (losing) \$$n$ and gaining (losing) \$$n + 1$ stays fixed no matter how much money they have.

2 Buchak's Treatment of the Allais paradox

It is well known that, as a descriptive matter, "rank dependent" preferences figure into human decision making. Buchak goes further and argues that they can *rationalize* risk attitudes that conflict with EU. I disagree. I think our attitudes toward risk are far less systematic and rational than Buchak suggests. What we think of as characteristic "risk averse" or "risk seeking" behaviors arise from an unruly hodgepodge of tendencies, some coherent and some not, that include: declining marginal utility, "global" effects of the type REU captures, regret, the status quo bias, loss aversion, and intrinsic desires to win, or not lose, that do not hinge on what might be won or lost. While Buchak recognizes and discusses the first three factors, she ignores most of the rest. She does not mention loss aversion at all, for example, which I see as a serious lacuna since I think it is at least as centrally implicated in our intuitive judgments about risk as the positional effects Buchak cites. As a result, some of the motivations Buchak offers for REU—e.g., her examples of choices that REU can rationalize but EU cannot—are undercut by a failure to consider alternative explanations. I'll illustrate this using the famous Allais Paradox.

Consider a choice between the following two acts, where dollar values indicate final *total* fortunes.

	1%	10%	89%
L_{ODD}	\$0	\$5,000,000	\$$x$
L_{EVEN}	\$1,000,000	\$1,000,000	\$$x$

According to a core principle of EU, the *Sure-thing Principle* (STP), one's preference between L_{ODD} and L_{EVEN} should *not* depend on x's value. So, agents asked to choose between L_1 and L_2 and then (independently) between L_3 and L_4 should prefer L_1 over L_2 iff they prefer L_3 over L_4.

	1%	10%	89%
L_1	\$0 (min)	\$5,000,000 (max)	\$0 (min)
L_2	\$1,000,000 (max)	\$1,000,000 (max)	\$0 (min)
L_3	\$0 (min)	\$5,000,000 (max)	\$1,000,000 (mid)
L_4	\$1,000,000 (only)	\$1,000,000 (only)	\$1,000,000 (only)

This is not what we observe: many thoughtful people prefer L_1 to L_2 but *dis*prefer L_3 to L_4. The usual rationale is that if one is probably going to end up with \$0 it makes sense to take a 10% chance at \$5M over an 11% chance of \$1M, but if one has a sure \$1M it is reasonable to keep it rather than risking the possibility of getting nothing for a mere 10% chance at an extra \$4M.

Faced with this "Allais paradox" proponents of EU must either say that many seemingly rational people are actually *ir*rational, or find some way to rationalize their preferences. Those taking the latter approach commonly argue that the outcomes have been *misdescribed*. The \$0 in L_3, they say, is really something like "getting \$0 and feeling regret because one could have had \$1M," or "getting \$0 after experiencing trepidation about assuming the risk of getting \$0." Once these negative emotions are written into the outcomes the paradoxical preferences are no longer a problem for STP because the "\$0"s that appear in the various L_n are not really the same outcome. Buchak criticizes such "redescription strategies" at length (114–147), and I largely agree with her objections.

We go different ways at this point. Whereas I think the $\{L_1 > L_2,\ L_3 < L_4\}$ pattern reflects systematic irrationality, Buchak thinks it is rationally permissible. She notes that there are two "local" considerations at play in each choice: L_{ODD} offers \$5M rather than \$1M with probability 0.1; L_{EVEN} offers \$1M rather than \$0 M with probability 0.01. She writes,

> The first consideration outweighs the second in the choice between L_1 and L_2, but not in the choice between L_3 and L_4... The weight that each consideration gets depends on where in the structure of the act the relevant outcomes appear. Considerations about outcomes 'higher up' in the order of outcomes get less weight for a person who is more concerned with securing a higher minimum that with securing some possibility of a much better outcome. (109)

Thus, Buchak rejects STP because it fails to recognize that different x-values generate different outcome rankings, and so different outcome weightings. When $x <$ \$1M the 1% outcome in L_{EVEN} is a *middle*. When $x >$ \$1M it is a *minimum*. At $x =$ \$1M it's an *only*. So, Buchak claims, STP fails to appreciate how the contribution that a 1% chance of \$1M outcome makes to the value of an act can depend on its position in the ranking of outcomes.

To rationalize $\{L_1 > L_2, L_3 < L_4\}$ within REU, Buchak notes that, even if the agent is a miser, if her risk function is of form $\boldsymbol{r}(p) = p^z$ where $2.26 \leq z \leq 16.88$ she will exhibit the preferences in question. With $r(p) = p^{2.26}$, REU(L_1) = \$28,000 > EU(L_2) = \$6,800, and REU(L_3) = \$999,523 < REU(L_4) = \$1,000,000. (One can make these numbers more plausible by allowing money to have declining marginal utility.) So, REU can rationalize the common Allais preferences.

While Buchak sees this as a major advantage, I count it as a strike *against* REU. I see the $\{L_1 > L_2,\ L_3 < L_4\}$ preferences as irrational, and would explain them as being the result of two well-established facts about irrationalities in human behavior—the *status quo bias* and *loss aversion*—and a plausible hypothesis about the way people shift their views about the status quo. These three considerations provide a compelling *causal* explanation of the common Allais preferences, but do not *rationalize* them. Rather, they show how the preferences arise from features of human psychology that are rationally defective.

The status quo bias is the tendency to think of outcomes not in absolute terms, but as gains or losses relative to some perceived status quo. When Bill Gates is offered a bet that pays \$100 if it rains and costs \$50 otherwise, he probably does not think, "that's \$79,467,302,243 if it rains and \$79,467,302,093 if not." If Gates is like most people, he merely thinks, "I gain \$100 if it rains and lose \$50 if not." Evaluating acts by looking only at how they change one's fortune (and ignoring the fortune's size) makes sense if values of outcomes do not vary with changes in baseline wealth. But, none of us has such a utility function.

Loss aversion is the tendency to prefer avoiding losses to securing equivalent gains, where gains and losses are evaluated relative to the perceived status quo. For example, the loss of, say, $10 relative to the status quo has a negative value that exceeds the positive value of a $10 gain. Because of this, agents are willing to assume more risk to avoid a sure loss than to improve on an equivalent sure gain. One might risk a $2000 loss with a probability slightly greater than one-half to avoid a sure $1000 loss.

The combination of status quo bias and loss aversion leaves people open to irrational framing effects. Here's a famous example:

> *Choice-1* I give you $500. Then, you choose between (a) giving back $100 for sure, or (b) giving back either $200 or $0 depending on the toss of a fair coin.
> *Choice-2* I give you $300. Then you choose between (a*) getting another $100 for sure, or (b*) getting either $200 or $0 depending on the toss of a fair coin.

When these options are presented independently, many people prefer (b) to (a), and (a*) to (b*). In Choice-1 the status quo is "I have $500", which frames (a) and (b) as losses, and agents prefer a 50–50 chance of losing $200 to a sure $100 loss. The status quo in Choice-2 is "I have $300" and (a*) are (b*) are gains, which makes the agent averse to risking the sure $100 to secure a 50–50 chance at gaining $200. This is irrational since (a) and (a*), and (b) and (b*), are the same options differently described. Moral: the status quo bias and loss aversion can lead us to make irrational decisions. I think Buchak will agree with me thus far (66).

My last hypothesis concerns the psychological mechanisms by which a person's picture of the status quo might *change*. We know the status quo can change when the agent comes to regard some item as her own (the *endowment effect*, Thaler 1980), or when she makes a decision (Kőszegi and Rabin 2007). I think a related mechanism is at work in the Allais problem. People sometimes shift their view of the status quo to include *highly desirable sure things* they have the power to choose. We tend to assume that agents assess options relative to whatever status quo was in place when they were initially presented with them, but this is wrong. When an agent faces a decision in which one option offers a prize that is *certain* to *improve* her situation, she will sometimes revise her picture of the status quo before choosing. She will factor in the sure prize (or best such prize if there is more than one), and reevaluate her other options relative to this new baseline. I do not claim that this always occurs (I don't think status quo shifts obey *any* general laws), but I think it often does, particularly when the sure prizes involved are large.

Consider the second Allais choice. On one natural (but false) view, the chooser will see herself in this situation:

	1%	10%	89%
L_3	$0	$5,000,000	$1,000,000
L_4	$1,000,000	$1,000,000	$1,000,000
			SQ = $0

The "SQ = \$0" at the bottom indicates a feature of the *entire* decision problem, though not of any *individual* option (which means that it is *not* recognized by REU). It says that outcomes of more than \$0 are seen as gains while those less than \$0 are seen as losses. Since there are no losses, there is no loss aversion. At worst, the agent ends up with SQ, and her decision boils down to whether a 10% chance of *improving* by \$5M is better than an 11% chance of *improving* by \$1M.

I do not believe Allais choosers typically think this way. They see L_4 as a large, sure gain and reason like this: "I've been given a million dollars! [This puts the new SQ at the old SQ bumped up by \$1M.] So, my decision is whether to protect my new millionaire status, or to put it at risk by paying a million to buy L_3." So, she sees her choice like this:

	1%	10%	89%
L_3	\$0	\$5,000,000	\$1,000,000
L_4	\$1,000,000	\$1,000,000	\$1,000,000
			SQ = \$1,000,000

Here everything is assessed relative to a baseline in which the agent *already has* a \$1M. The 1% outcome under L_3 is now a million dollar *loss*. Loss aversion kicks in, and the question becomes whether a 1% risk of *losing* \$1M is worth a 10% chance of *gaining* \$4M. For an agent thinking of incremental gains and losses, and not factoring in assets brought to the table, it might be that losing \$1M is perceived as being more than ten times worse than gaining \$4M.[3] Of course, if she takes L_3 and the bad 1% event happens, she will be back to the *old* SQ. Unfortunately, this now seems undesirable since it is assessed in a context in which she sees herself as a millionaire. This is why L_4 seem worse than L_3.

In contrast, in the choice between L_1 and L_2 there's no *sure* way for the agent to improve her lot. In fact, the most probable result (89%) is that she stays where she is. In this context, we would not expect the status quo to change:

	1%	10%	89%
L_1	\$0	\$5,000,000	\$0
L_2	\$1,000,000	\$1,000,000	\$0
			SQ = \$0

Now, she is comparing a 10% chance of a \$5M *improvement* against an 11% chance of a \$1M improvement. Since there is no loss aversion, the agent is likely to choose the "riskier" L_1.

[3] If the agent has a declining marginal utility function for money, say $u(x) = ln(x)$, her utility for losing a million would only have to be about 1.1 times greater than her utility for gaining four million.

So, I claim, the reasons for the $\{L_1 > L_2, L_3 < L_4\}$ preferences have less to do with stable attitudes toward risk (as Buchak suggests), and more to do with transient assessments of *relative value*. They result from the fact that offering a choice between L_3 and L_4 typically *changes* the perceived status quo, while offering a choice between L_1 and L_2 typically does not.

Let me emphasize that his explanation of the Allais preferences is not a version of the 'redescription' strategy. The goal of that strategy is to *rationalize* the preferences by arguing that the outcomes in the problem are *incorrectly described*. I am not aiming either to redescribe or rationalize—I see the $\{L_1 > L_2, L_3 < L_4\}$ preferences as plainly irrational—and I don't think considerations of regret or trepidation enter in at all. On my interpretation, if an Allais chooser is asked what her situation will be if she chooses L_3 and the 1% event happens, she might answer "I'll suffer a loss of \$1M." She *might* also believe that she will experience regret or trepidation, but, even if she does, I doubt this is what drives their preferences (since the amount of regret or trepidation has to be implausibly large to make the difference). Loss averse agents do not dislike losses because they will regret making the choices that led to them; they dislike losses because they see them as demotions from where they are.

So, I am not convinced by Buchak's attempt to rationalize the Allais preferences within REU. It ignores a more plausible explanation that involves the status quo bias, loss aversion, and the fact that being offered sure things can cause agents to rethink the status quo.

3 Can REU avoid Dutch books?

Now, let's consider Buchak's handling of "Dutch book" arguments. Here is the setup: Jacob is a miser with a fortune of \$$f$. He is also a Buchakian risk-avoider with $r(x) = x^2$. He will encounter bookies who offer him bets on the outcome of a horse race involving Stewball. $\langle a, b\rangle$ will denote a bet that adds \$$a$ to Jacob's fortune if Stewball wins and adds \$$b$ otherwise, where a and b are any real numbers. If Jacob buys $\langle a, b\rangle$ for \$$k$ he ends up with \$$(f - k + a)$ if Stewball wins, and with \$$(f - k + b)$ if Stewball loses. Since negative numbers are losses, *selling* $\langle a, b\rangle$ is equivalent to *buying* $\langle -a, -b\rangle$. Finally, for the sake of definiteness, suppose that Jacob is 30% confident that Stewball will win.

According to REU, Jacob's buying and selling prices for $\langle a, b\rangle$ depend on the relative sizes of a and b. If $a \geq b$, he will buy at any price at or below \$$(a{\cdot}0.09 + b{\cdot}0.91)$, and sell at any price at or above \$$(a{\cdot}0.51 + b{\cdot}0.49)$. If $a \leq b$, he will buy at or below \$$(a{\cdot}0.51 + b{\cdot}0.49)$, and sell at or above \$$(a{\cdot}0.09 + b{\cdot}0.91)$. For example, Jacob will buy $\langle 1, 0\rangle$ for no more than 9¢, and sell it for no less than 51¢. In EU "max buy" and "min sell" prices always coincide: each agent has a "fair price" for any bet, a price at which he is equally happy to buy or sell. Not so in REU. In general, risk-avoidant agents like Jacob exhibit a MaxBuy < MinSell pattern, while risk-lovers exhibit a MaxBuy > MinSell pattern. Though Buchak does not speak this way, one can think of a REU-risk avoider (seeker) as assigning a higher (lower) value to $\langle a, b\rangle$ when it is an *asset* that he can sell than when it is a potential *acquisition* that he can buy.

This disparity between asset and acquisition prices leaves REU maximizers susceptible to “Dutch books”. Risk-avoiders refuse to buy “books” of bets that are sure to make them money, while risk-lovers buy “books” that are sure to lose them money. As Buchak and I agree, both things are irrational. I will focus on the risk-avoidant agent, but everything I say could be adapted to a risk-lover (though instead of passing up gains, she’d be incurring losses).

To see how Jacob’s preferences expose him to Dutch books, suppose he is walking past two bookmaking shops. The first is selling $\langle 1, 0\rangle$ for 10¢, while the second is buying it (i.e., selling $\langle -1, 0\rangle$) for 50¢. I hope you will agree with Buchak and me that Jacob errs if he walks by both shops without buying something (assuming no transaction costs). By declining both offers he forgoes an arbitrage opportunity in which he buys $\langle 1, 0\rangle$ at 10¢ and sell it back at 50¢, thereby passing up a sure 40¢ gain and getting nothing to show for it. Of course, as Buchak and I agree, there is nothing irrational about foregoing a sure gain if one expects to do better by taking a risk. For example, an EU maximizer with a 5% probability for Stewball winning should decline the first offer but accept the second, thereby eschewing a *sure* 40¢ to get an *expected* 47.5¢. What is irrational (again we agree) is passing up a sure 40¢ without getting something in return, either $\langle 0.9, -0.1\rangle$ (the result of buying $\langle 1, 0\rangle$ for 10¢) or $\langle -0.5, 0.5\rangle$ (the result of selling $\langle 1, 0\rangle$ for 50¢). Unfortunately, the only way, in general, for Jacob to ensure that he buys at least one bet, and avoids Dutch book, is by having MaxBuy($\langle 1, 0\rangle$) = MinSell($\langle 1, 0\rangle$). This seems bad for REU, which sets MaxBuy($\langle 1, 0\rangle$) = 9¢ and MinSell($\langle 1, 0\rangle$) = 51¢. At these prices, Jacob will refuse both bookies’ offers.

Buchak aims to defuse this worry by arguing that, properly applied, REU does *not* allow Jacob to decline both offers. (201–212) Following a well-worn path, she rejects the *Package Principle* (PP), an implicit premise of the Dutch book argument which, in this context, says that Jacob’s maximum selling price for $\langle 1, 0\rangle$ at the second bookie’s shop should not depend on what happens at the first shop. Buchak claims, to the contrary, that, while Jacob’s minimum selling price for $\langle 1, 0\rangle$ should be 51¢ when he has no opportunity to buy it, it should fall to 9¢ once he declines to buy it for 10¢. (211) More generally, this *repricing strategy* says that, upon declining to buy $\langle 1, 0\rangle$ from the first bookie, Jacob should drop his minimum selling price for the bet from 51¢ to 9¢, which ensures that he will always accept at least one of the two bookies’ offers.[4]

Buchak spends less time justifying the repricing strategy than one would like, and it comes off as a bit of a *deus ex machina*. Before considering what I take to be her justification, let me dismiss a common but misguided alternative, which Buchak too rejects. (207) This is the idea that after Jacob declines the first bookie he is justified

[4] I am assuming that Jacob will pay the REU recommended buying price of 9¢ at the first bookie’s shop, and that the repricing strategy has him compensate by lowering his sale price to 9¢ at the second bookie’s. It would be possible to run things the other way, so that Jacob sells at the REU recommended 51¢ price at the second bookie’s and compensates by raising his buying price to 51¢ at the first bookie’s. This does not affect my argument at all. The reasoning all goes through, but with a lot of *mutatis mutandis*.

in lowering his selling price for ⟨1, 0⟩ from 51 to 9¢ simply *because* this prevents him from being Dutch booked. The problem with this is that Jacob does not have "avoiding Dutch books" among his goals. As a miser, his sole goal is to amass money, and he is interested in securing sure gains (avoiding sure losses) *only* to the extent that it helps him do this. This is *not* automatic: eschewing sure gains is often the best way to pursue larger, uncertain gains, and taking sure losses is often the best way to avoid larger potential losses. So, when considering the second bookie's offer, Jacob's question is not "What act will let me avoid Dutch book given that I did not buy ⟨1, 0⟩?" it's "What act should I expect to make me the most money given that I did not buy ⟨1, 0⟩?" It would be nice if these questions always got the same answer (as in EU), but we cannot presuppose this here. Without assuming the repricing strategy (whose legitimacy is at issue), REU's 51¢ selling price seems to mean that, sure gains notwithstanding, Jacob does better by declining the second bookie's offer. Unless proponents of repricing can show Jacob that lowering his price from 51¢ to 9¢ is his best way of making money, he will not do it. The fact that not lowering his price will complete a Dutch book is immaterial: having turned down the first bookie's offer, his best option might be to complete the Dutch book. Indeed, REU seems to suggest this. So, proponents of REU cannot justify the repricing strategy *merely* by noting that it prevents Jacob from forgoing sure gains.

A second potential justification for repricing proposes that ⟨1, 0⟩ *becomes less desirable* to Jacob once he declines to buy it. The idea is that by declining the first bookie's offer Jacob somehow alters the bet's global properties in a way that makes owning it less appealing. It is, of course, familiar that an act's desirability can wax or wane with changes in the desirer's assets on account of the declining marginal utility of money, but this is not at play here. Still, there might be other "interactions" that explain why the price for ⟨1, 0⟩ should change. Buchak sometimes speaks this way (e.g., her talk of "changing structural properties," 211), but it is hard to square with her theory. In REU the lowest price at which Jacob should sell ⟨1, 0⟩ is a function of just three things: the bet's payoffs, his estimates of the probabilities of these payoffs, and his risk function. Declining to buy the bet from the first bookie will not affect any of these things, and so will not alter any global property of ⟨1, 0⟩ that affects its REU-value. If Jacob calculates MinSell(⟨1, 0⟩) before and after refusing to buy ⟨1, 0⟩ he gets the same answer! So, consistent with REU, Jacob's selling price for ⟨1, 0⟩ cannot *change* as a result of what he does with the first bookie's offer.

A final rationale for repricing, more in line with Buchak's thinking, is based on the idea that, rather than altering the price at which Jacob sells ⟨1, 0⟩, his refusal to buy it from the first bookie alters the *options* the second bookie can offer. Buchak writes, "we cannot even state the 'full' outcomes of [buying or selling] a bet without knowing which other bets are in effect." (211) The thought here is that each bet should be assessed against the background "status quo" of assets the agent already holds, a background that may itself include risks. "But," she writes, "what the status quo is, and whether [it] should be thought of as a gamble itself, depends on what other bets are in effect." If I understand her correctly, Buchak is suggesting that, when assessing the advisability of buying (selling) a bet, Jacob should integrate it into (subtract it from) his existing portfolio of assets, and set a price that reflects the desirability of the resulting *augmented* (reduced) portfolio.

Let me suggest one way to do this (there are probably others). I will use MaxBuy and MinSell to denote the prices REU recommends for bets when Jacob otherwise has only riskless (constant) assets in his portfolio, i.e., when he is in the state $\langle 0, 0\rangle$. The goal is to find a coherent pricing strategy that will capture the idea that Jacob's prices for buying (selling) a bet $\langle a^*, b^*\rangle$ may be different when it is offered in state $\langle 0, 0\rangle$ than when it is offered in a state where when Jacob owns another risky prospect $\langle a, b\rangle$. Here are two suggestions for how such a pricing strategy might work:

[1] If Jacob's only risky asset is a bet $\langle a, b\rangle$ *with a maximum buying price of 0*, then he should pay no more than MaxBuy($\langle a + a^*, b + b^*\rangle$) to add $\langle a^*, b^*\rangle$ to his portfolio.

[2] If Jacob's only risky asset is a bet $\langle a, b\rangle$ whose maximum buying price is $m > 0$, then his MaxBuy price for $\langle a^*, b^*\rangle$ in this context should coincide with his MaxBuy price for $\langle a^*, b^*\rangle$ when his only non-risky asset is $\langle a - m, b - m\rangle$. (This last bet has a maximum buying price of 0.)

According to [2], if Jacob owns $\langle a, b\rangle$ and is buying $\langle a^*, b^*\rangle$, he should view the purchase of the second bet as completing the purchase of the "book" $\{\langle a, b\rangle, \langle a^*, b^*\rangle\}$, where $\langle a, b\rangle$ has already been bought for its maximum price. [1] then gives Jacob a way of pricing $\langle a^*, b^*\rangle$'s contribution to the book's overall value.

To see how it works, suppose we want to know how much $\langle -1, 0\rangle$ is worth to Jacob as an acquisition when he owns $\langle 1, 0\rangle$. [2] has us imagine him starting in the riskless state and buying $\langle 1, 0\rangle$ for 9¢, and so becoming the owner of $\langle 0.91, -0.09\rangle$, with its buying price of zero. Then, by [1] Jacob is supposed to pay anything up to MaxBuy($\langle 0.91 -1, -0.09\rangle$) = −9¢ to buy $\langle -1, 0\rangle$. Since the profit secured in buying $\langle -1, 0\rangle$ cancels out the cost incurred in buying $\langle 1, 0\rangle$, Jacob ends up paying \$0 for $\langle 0, 0\rangle$, as desired.[5] The disparity in the prices that Jacob pays for $\langle -1, 0\rangle$ is explained by the disparate effects that buying $\langle -1, 0\rangle$ has on his risk profile. He *takes on* risk when he goes from the risk-free $\langle 0, 0\rangle$ state to the risky $\langle -1, 0\rangle$. As a risk-avoider, he will want to be compensated for adding this risk, which is why he insists on 51¢ to cover a bet whose expected loss is only 30¢. But, when he has already purchased $\langle 1, 0\rangle$ for 9¢, Jacob *sheds* risk by buying $\langle -1, 0\rangle$: he goes from the risky $\langle 0.91, -0.09\rangle$ to the risk-free $\langle -0.09, -0.09\rangle$. Now, he will take as little as 9¢ to cover $\langle 1, 0\rangle$ because, in addition to getting 9¢, he is divesting himself of unwanted risk. So, what seems like the single act "buy $\langle -1, 0\rangle$" is best assessed as two acts with different global features.

Buchak might hope to use [1] and [2] to explain why Jacob, as a risk-averter, should sell $\langle 1, 0\rangle$ for a little as 9¢ after he turns down the first bookie's offer. She does not give us much to go on, but let me offer the following in the spirit of a "rational reconstruction":

[5] Order does not matter. If we first have Jacob buy $\langle -1, 0\rangle$ for −51¢, and then buy $\langle 1, 0\rangle$ for MaxBuy($\langle 1 -0.49, 51\rangle$), he again ends up paying \$0 for $\langle 0, 0\rangle$.

i. In a context where Jacob *never entertains* the first bookie's offer (perhaps her shop is closed), he should evaluate the second bookie's offer as he would if he were in the risk *risk-free* context $\langle 0, 0 \rangle$.
ii. In a context where Jacob *entertains and declines* the first bookie's offer, he should evaluate the second bookie's offer as he would if he held the *risk-laden* asset $\langle 0.91, -0.09 \rangle$, i.e., as if he had bought $\langle 1, 0 \rangle$ for 9¢.

If both claims are true, then [1] has Jacob sell $\langle 1, 0 \rangle$ for at least 51¢ in (i) and as little as 9¢ in (ii). The Dutch book is avoided in (ii) because, even though Jacob will pass up the first bookie's offer of $\langle 1, 0 \rangle$ for 10¢, he will happily buy $\langle -1, 0 \rangle$ from the second bookie for as little as 9¢.

This would solve the problem by providing a rationale for repricing, but I am skeptical that either (ii) or [1] can be squared with REU. Start with (ii). Why should being offered $\langle 1, 0 \rangle$ *at a price he rejects* leave Jacob in a state different from the one he would be in if he had not heard the offered at all? He owns $\langle 0, 0 \rangle$ either way! I am not sure what to say here, but perhaps the idea is that refusing to buy $\langle 1, 0 \rangle$ (at any price) amounts to getting $\langle 0, 0 \rangle$ for free, in which case Jacob's choices at the first bookie's shop will all have the form "$\langle 1, 0 \rangle$ for *y*¢ or $\langle 0, 0 \rangle$ for 0¢." One might then reason as follows:

> Since Jacob's maximum buying prices for $\langle 0.91, -0.09 \rangle$ and $\langle 0, 0 \rangle$ are both 0¢, he will be indifferent between buying one or the other for free. Though getting $\langle 0, 0 \rangle$ for free carries no risk while $\langle 1, 0 \rangle$ for 9¢ is risky, this won't phase Jacob since he will see the risk involved in buying the latter bet as being fully compensated by the expected gains it promises (he gets a bet with a 30¢ expected payoff for only 9¢). So, since he does not care which bet he buys, nothing relevant changes if we portray the first bookie offering Jacob the choice "either buy $\langle 1, 0 \rangle$ for 10¢ or buy $\langle 0.91, -0.09 \rangle$ for 0¢." So, (ii) is true because if Jacob refuses the first bookie's offer (by taking $\langle 0, 0 \rangle$ for 0¢ over $\langle 1, 0 \rangle$ for 10¢), then he will be in the very same position as he would have been in had he bought $\langle 1, 0 \rangle$ for 9¢.

This reasoning, albeit sound in EU, is fallacious in REU, where bets with the same buy- price can have different sell-prices. This happens here. Jacob will demand more compensation to sell $\langle 0.91, -0.09 \rangle$ than to sell $\langle 0, 0 \rangle$ because, while selling $\langle 0, 0 \rangle$ for 0¢ is no-risk/no-reward, selling $\langle 0.91, -0.09 \rangle$ exposes him to a rather large risk of loss (30% chance of losing 91¢) with little potential for gain (70% chance of winning 9¢). Thus, while Jacob is indifferent between $\langle 0.91, -0.09 \rangle$ and $\langle 0, 0 \rangle$ when *buying*, he is not indifferent when *selling*. This undercuts the stated rationale for (ii). It might accurately describe Jacob's situation when he is a buyer at the first shop, but it cannot justify (ii), which concerns his situation as a seller at the second shop.

Turning now to [1], assume that, for whatever reason, Jacob is at the second shop holding $\langle 0.91, -0.09 \rangle$. [1] tells him to buy $\langle -1, 0 \rangle$ at the price at which he would buy the sum of it and $\langle 0.91, -0.09 \rangle$. This seems to invite Jacob to reason like this: "if the bookie offers me $\langle -1, 0 \rangle$ then I can pay her by first *giving* her $\langle 0.91, -0.09 \rangle$, which is no skin off my nose since this is a bet that I would not pay anything to buy.

I can then sell her ⟨0.09, 0.09⟩ for 9¢ to make up the balance. She gets ⟨1, 0⟩, I spend 9¢, and dump a bet that has no value to me. So, if she offers more than 9¢ for ⟨−1, 0⟩, I should take it."

This reasoning is inconsistent with REU. It has Jacob divesting himself of an *asset* and receiving nothing in return. This would only make sense if the asset were *worthless*, by which I man worthless *when sold* not *when bought*. It might seem that ⟨0.91, −0.09⟩ is worthless because Jacob will *buy* it for at most 0¢, but in REU this is not the relevant price when he holds the bet as an *asset.* The bet is valuable to Jacob at the second bookie's shop because of what he can sell it for, and should be valued accordingly. While giving it away does eliminate the risk in Jacob's portfolio, it also costs him in expected payoff. When he weighs benefits against costs, Jacob judges that giving ⟨0.91, −0.09⟩ away will cost him 42¢, and so sets that as his selling price. This price may strike us as high, but it's what REU requires of a Buchakian risk avoidant miser with $r(x) = x^2$ and Jacob's probabilities. So, according to REU itself, it would be a mistake for Jacob to follow [1] and price ⟨1, 0⟩ at only 9¢, since that would amount to giving something worth 42¢ away for nothing. Consistent with REU, Jacob can sell ⟨1, 0⟩ by *selling* ⟨0.91, −0.09⟩ for 42¢ and then selling ⟨0.09, 0.09⟩ for 9¢. What he cannot do is sell ⟨1, 0⟩ by *giving away* ⟨0.91, −0.09⟩ and selling ⟨0.09, 0.09⟩. Doing so would misprice an *asset* as if it were an *acquisition*. Again, this contradicts a core tenet of REU: selling and buying prices can differ because the risk incurred when adding a bet to one's portfolio need not coincide with the risk incurred when subtracting it.

In the end, I do not see how the repricing strategy can be squared with REU. It may be that I have just not hit on the right approach, or it may be that there is another way for REU-maximizers to avoid Dutch books. But, unless some way can be found, REU will lose much of its appeal an account of rational choice.

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