1. 15 students

8 questions

a) students with more than 1 question

8/15 for each student answering 1 question

6/15 1 question for each student + 1 student answer 2 questions

4/15 for 1 question each + 2 students and 2 questions each

$$\frac{\frac{15!}{7!}}{15^8} = \boxed{0.1012}$$

2. 00000 – 99999

2 odd digits

Generate 8 #'s in succession

$5 \times 4 \times 2 \times 6 \times 5 = 1200$

odd	odd			even
1	3	2	11	0
3	5	4	are	2
5	7	5	(es)	4
7	9	6	choices	6
9		7		8
		8		
		9		

$$P(\text{success}) = \frac{4200}{100{,}000} = \frac{21}{500} = 0.042$$

$$P(\text{failure}) = \frac{479}{500}$$

$$\binom{8}{5}\left(\frac{21}{500}\right)^5\left(\frac{479}{500}\right)^3 \approx \boxed{0.000006\%}$$

3. 3 six sided fair dice

A = 2^+ dice are 4^+
B = all 3 dice are the same

$P(A)$ $\quad P(1 \text{ dice being } 4+) = \frac{3}{6} = \frac{1}{2}$ $\quad P(F) = \frac{1}{2}$
$P(2 \text{ dice being } 4+) = \binom{3}{2}(0.5)^2(0.5) = 3/8$

$P(3 \text{ dice being } 4+) = \binom{3}{3}(0.5)^3(0.5)^0 = 1/8$
$P(A) = 1/8 + 3/8 = 1/2$

$P(B) = \frac{6}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

If independent, $P(B|A) = P(B)$ and $P(A|B) = P(A$

$P(A|B) = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2} = P(A)$
$P(B|A) = \frac{3}{3} \cdot \frac{1}{3} \cdot \frac{1}{6} = \frac{3}{36} = \frac{1}{12} \neq \frac{1}{36} = P(B)$

First one is 4+ $\qquad$ 2nd must be the same as 1st $\qquad$ 3rd must be the same

A is independent of B, but B is dependent to A

4. $P(\text{Flush}) \quad \dfrac{4 \cdot \binom{13}{5}}{\binom{52}{5}} = \dfrac{4 \cdot 1287}{2{,}598{,}960} \approx \boxed{0.001981}$

It's geometric as each hand follows the same odds as the last

$$E(x) = \frac{1}{p(\text{Flush})} = \frac{1}{0.001981} \approx 504.8485$$

$\boxed{505 \text{ hands}}$

5. E = team that won 4/5 games
F = superstar played

$P(F) = \frac{3}{4} = 0.75 \rightarrow$ played
$P(F^c) = 1/4 = 0.25 \rightarrow$ not played

$$P(F|E) = \frac{P(E|F)\,P(F)}{P(E|F)\,P(F) + P(F^c)\,P(E|F^c)}$$

1 game = 1 trial
$P(\text{winning}) = 0.7 \quad P(\text{loss}) = 0.3 = q$

Superstar is playing

$\therefore \quad P(\text{win } 4 \text{ games}) \quad P(\text{loss } 1 \text{ game})$

$$P(E|F) = \binom{5}{4}(0.5)^4(0.5)^1$$
$$= 0.15625$$
$= \text{prob. of team losing}$

$$P(F \mid E) = \frac{P(E \mid F)\ P(F)}{P(E \mid F)\ P(F) + P(F^c)\ P(E \mid F^c)}$$

$$= \frac{(0.36015)\ (0.75)}{(0.36015)\ (0.75) + (0.15625)\ (0.25)}$$

$P(F \mid E) = 0.87365$ = prob. team won 4/5 games when superstar played

$$P(F \mid E) = \boxed{87.365\ \%}$$