Counting

1. 5 unique letters · u, n, s, a, l

a) unusual - 1, 3, 5 are interchangable
1 2 3 4 5 6

b) other 5 letters

i) 5 different letters
$n = 5, \quad r = 5$

$$\frac{5!}{(5-5)!} = \boxed{120}$$

ii) 3 different letters, 2 same letters

$$\binom{1}{1} \binom{4}{3} \cdot \frac{5!}{2!} = 1 \cdot 4 \cdot 60 = \boxed{240}$$

iii) 2 different letters, 3 same letters

$$\binom{1}{1} \cdot \binom{4}{2} \cdot \frac{5!}{3!} = 1 \cdot 6 \cdot 20 = \boxed{120}$$

$120 + 240 + 120 = \boxed{480 \text{ strings total}}$

2. a) There are 13 choices to pick 2 ranks from

$\binom{13}{2} = \frac{13!}{2!(13-2)!} = \boxed{78}$

b) Pair of one value, 2 suits of 4

$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \boxed{6}$

c) Pair of two value

$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \boxed{6}$

d) Last card

$52 - 8 = 44$ cards

$78 \cdot 12 \cdot 12 \cdot 44 = \boxed{123,552 \text{ ways}}$

3. 16 songs / hr

1 couple at most one song

Case 1: The couple that's fighting gets x songs

$n = 16$, 6 other couples

1^6, bar = 7

$\binom{16+6-1}{6-1} = \binom{21}{5} = 20349$

Case 2: The couple fighting gets 1 songs

$n = 15$, 6 couples

$\binom{15+6-1}{6-1} = \binom{20}{5} = 15504$

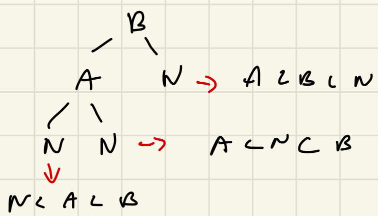
$20349 + 15504 = \boxed{35,853 \text{ ways}}$

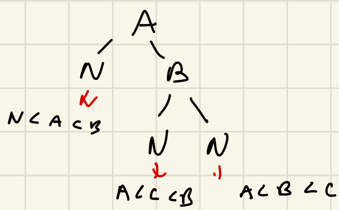
4. 12 nodes
1 - 12
root value = 3
right child = 9

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

① 2 nodes
A, C, B

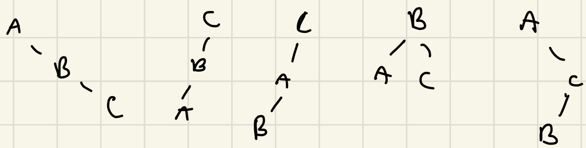
3 nodes
A, B, C





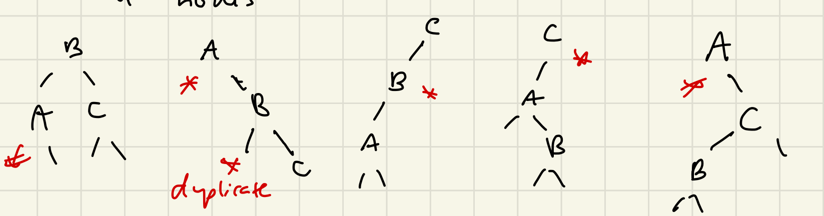
N 2 can either be the left or right child

② 3 nodes



= 5 orientation

4 nodes



= 4 orientation

5 nodes

A

one case

14 total

B

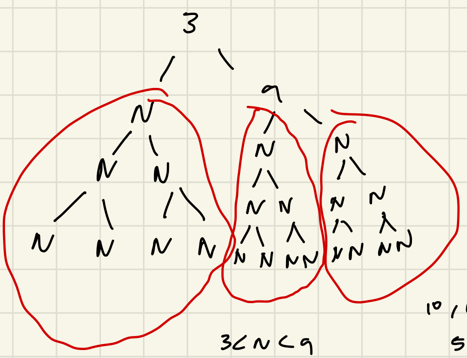
14 different orientations for 4 nodes

C

one case

14 total

③ Restrictions



1, 2

2 orientations

$3 < N < 9$

4, 5, 6, 7, 8

42 orientations

10, 11, 12

5 orientations

$2 \times 42 \times 5 = \boxed{420 \text{ possibilites}}$

5. 10 friends

4 identical nurses

1 nurses may /not scheduled for break

Case 1 : 4^{th} nurse on break

3 nurses and 10 friends

1. (8, 1, 1)
2. (7, 2, 1)
3. (6, 3, 1)
4. (6, 2, 2)
5. (5, 4, 1)
6. (5, 3, 2)
7. (4, 4, 2)
8. (4, 3, 3)

Case : 4^{th} nurse not on a break

1. (7, 1, 1, 1)
2. (6, 2, 1, 1)
3. (5, 3, 1, 1)
4. (5, 2, 2, 1)
5. (4, 4, 1, 2)
6. (4, 3, 2, 1)
7. (3, 3, 3, 1)
8. (2, 2, 2, 4)
9. (2, 2, 3, 3)

8 + 9 = $\boxed{\text{17 combinations}}$