Discrete Structure Assignment 1 (Section 02)

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1. (a) $x^2 + x - 6 = 0$
$(x-2)(x+3) = 0$
$x = -3,\ x = 2$
$A = \{-3, 2\}$
$\therefore A' = \{x \in Z \mid x \neq -3, x \neq 2\}$

(b) $(B - A) \cap C = \{1,4\} \cap \{2,3,4\}$
$= \{4\}$

1. (c) $B \cap C = \{2, 4\}$
$n = 2$
$|P(B \cap C)| = 2^n = 2^2 = 4$

2. $(((P \cup Q) \cap R)' \cup Q')' = (((P \cup Q) \cap R)'')' \cap (Q')'$ → De Morgan's Law
$= ((P \cup Q) \cap R) \cap Q$ → Double Complement Law
$= Q \cap ((P \cup Q) \cap R)$ → Commutative Law
$= (Q \cap (P \cup Q)) \cap R$ → Associative law
$= Q \cap R$ → Absorption Law

3. (a) $|A|$ = number of students in art class.
$|S|$ = number of students in science class.
$|A \cup S| = |A| + |S| - |A \cap S|$
$= 35 + 57 - 12$
$= 80$
$\therefore$ 80 students

(b) $|A \cup S| = |A| + |S| - |A \cap S|$
$= 35 + 57 - 0$
$= 92$
$\therefore$ 92 students.

4. (a) p: you try hard
q: you have a talent
r: you will get rich

$\therefore (p \wedge q) \rightarrow r$

(b) The statement false implies that r must be false and $(p \wedge q)$ must be true, because a statement $X \rightarrow Y$ is false when X is true and Y is false. Since p is true (you try hard) and r is false, then q must be true as

$(p \wedge q) \rightarrow r = (T \wedge T) \rightarrow F = T \rightarrow F = F$.

$\therefore$ You try hard and have a talent but you do not get rich.

(c) r is true (you are rich). Since r is true, the statement $(p \wedge q) \rightarrow r$ must be true, because an $X \rightarrow Y$ statement is true whenever Y is true.
$\therefore$ The statement is true.

5.

p	q	r	$r \to p$	$q \wedge (r \to p)$	$p \vee (q \wedge (r \to p))$	A	$\sim p$	$q \to r$	B
F	F	F	T	F	F	T	T	T	T
F	F	T	F	F	F	T	T	T	T
F	T	F	T	T	T	F	T	F	F
F	T	T	F	F	F	T	T	T	T
T	F	F	T	F	T	F	F	T	F
T	F	T	T	F	T	F	F	T	F
T	T	F	T	T	T	F	F	F	F
T	T	T	T	T	T	F	F	T	F

$\therefore A \equiv B$.

6. Assume x is odd and y is even.

Since x is odd, $x = 2h+1$ for some integer h, and since y is even, $y = 2k$ for some integer k.

$$
\begin{aligned}
x^2 - 2y &= (2h+1)^2 - 2(2k) \\
&= 4h^2 + 4h + 1 - 4k \\
&= 2(2h^2 + 2h - 2k) + 1 \\
&= 2r + 1, \text{ where } r = 2h^2 + 2h - 2k \text{ for some integer } r.
\end{aligned}
$$

$\therefore$ since $x^2 - 2y$ satiesfies the form of $2(\text{integer}) + 1$, therefore $x^2 - 2y$ is odd. The statement is true and is proven.