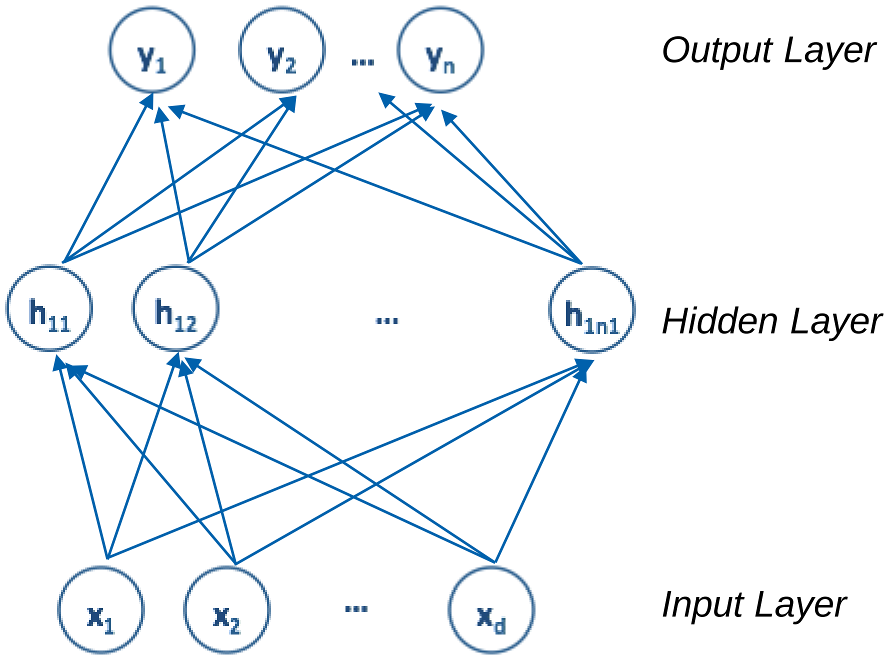
Deep Learning

Building Blocks

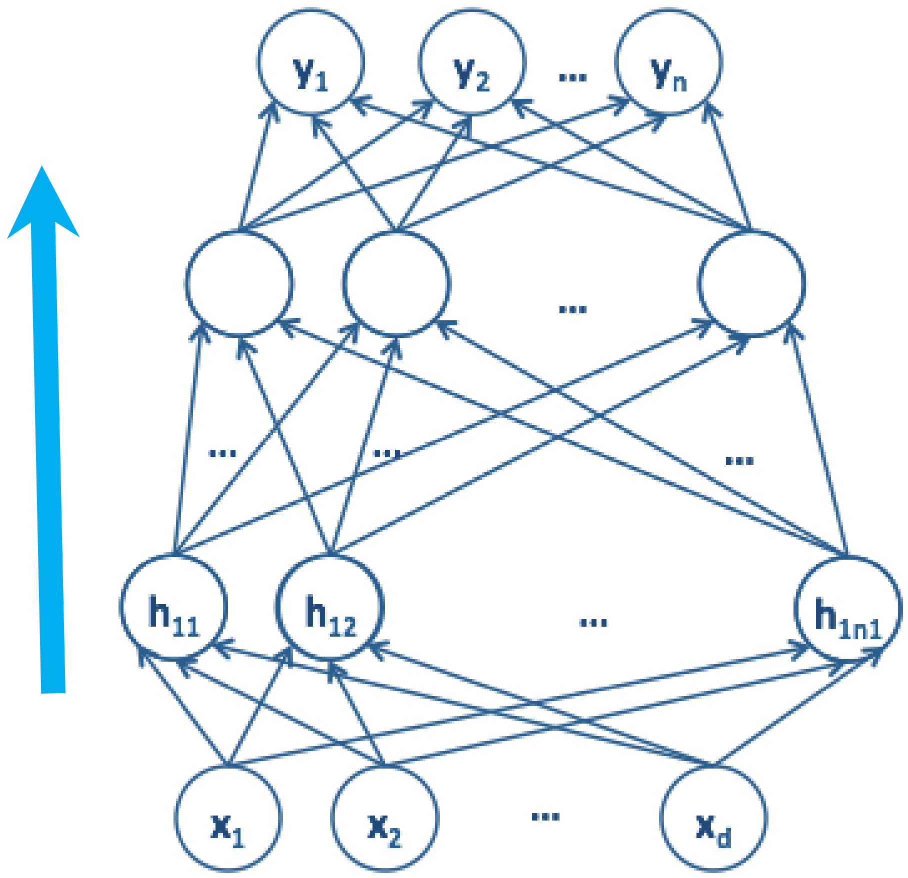
Arjun Jain

Refresher: Neural Network



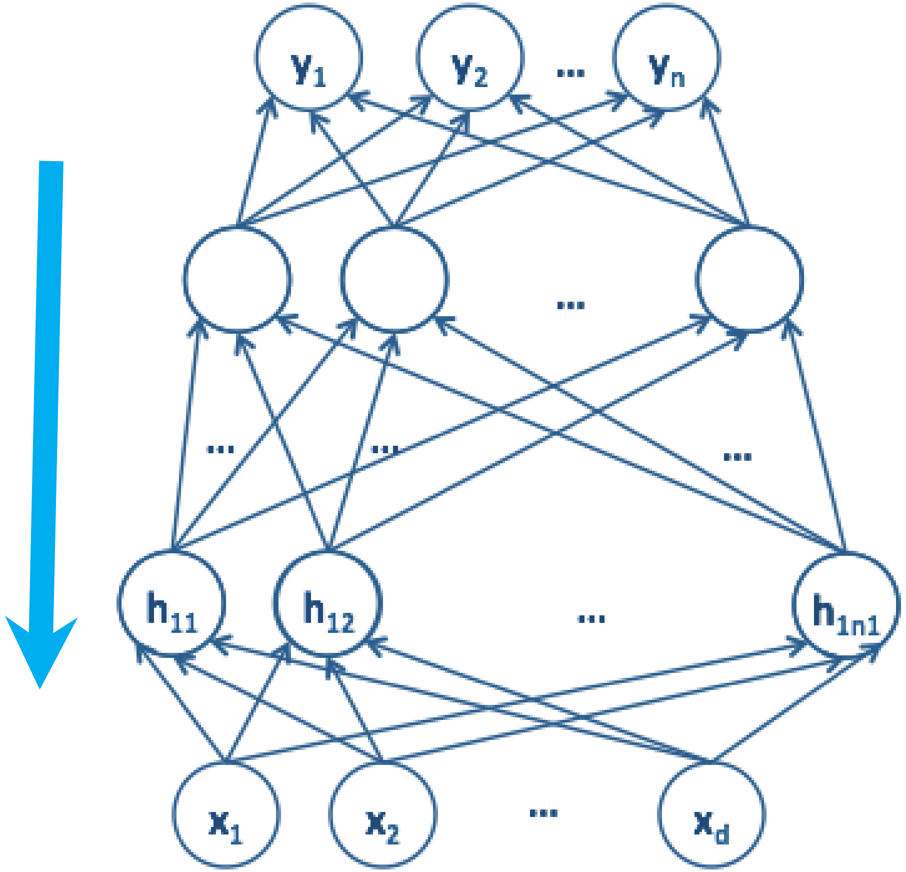
- Output layer represents the output classes (each mode corresponds to a class)
- Each node in the hidden layer has an activation function that acts on the input
- Each node in the hidden layer has a particular weight (towards a weighted sum)
- Feedforward refers to a unidirectional flow of information (and no lateral/intra-layer connections) from input to output
- Predicted output is compared to the actual output during the training process, and the difference (loss) determines how much the weights should be tweaked
- Tweaking the gradients and thereby the weights is done through back-propagation

Refresher: Deep Neural Networks - Forward



- More than one hidden layer in multi-layer (deep) neural networks, depth refers to the number of layers
- Each node in the hidden layer uses an activation function that acts on the input, and has a weight (towards a weighted sum)
- Forward pass refers to a unidirectional flow of information (and no lateral/intra-layer connections) from input to output
- When each input node is connected to all output nodes of the next layer, it is a fully connected network
- Output layer consists of nodes that represent a series of probabilities corresponding to the predicted likelihood of each class

Refresher: Deep Neural Networks - Backward



- Predicted output is typically incorrect when compared to actual value (training)
- <u>How much</u> the prediction is off from reality is measured using an error or loss function
- Backpropagation calculates the error gradient for all the weights and biases for all the layers (starting from output and working its way back)
- Forward pass with updated weights should lead to lower error, and over time a reduction in loss/error

Neural Network Constructed

Now, let’s review the following in turn:

- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

… and we’ll have a fully functioning network

Feed forward

Neural Network Constructed

Now, let's review the following in turn:

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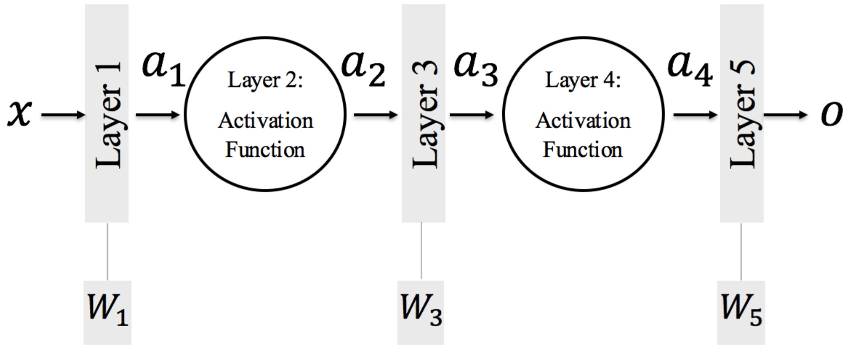
Multiple Layers – Feed Forward

- Process of calculating expected output
- Combines weights and activation functions with the inputs
- Iteratively performed over training set, and classifies test input

Multiple Layers – Feed Forward - Composition of Functions



$$a_1 = F(x, W_1),\ x \in \mathbb{R}^n$$

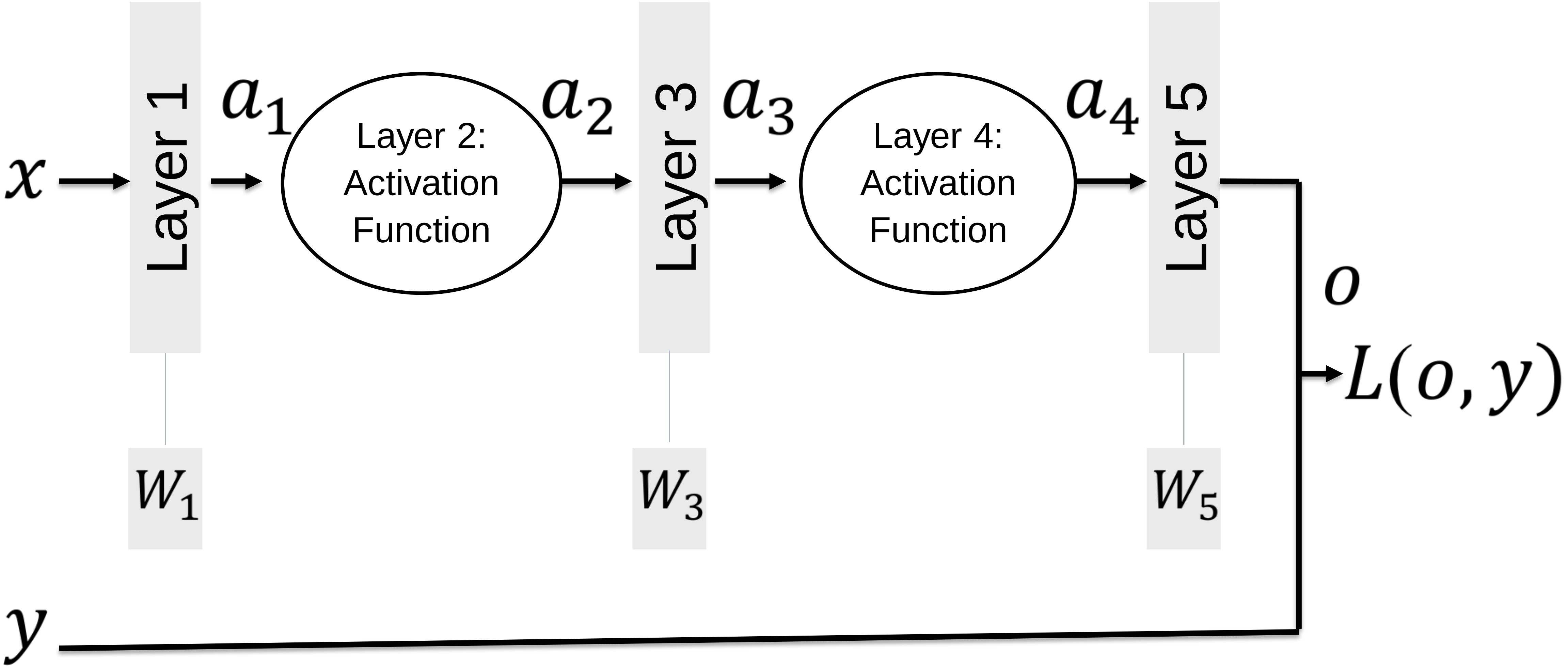
$$a_2 = G(a_1)$$

$$a_3 = H(a_2, W_3),$$

$$a_4 = J(a_3)$$

$$o = K(a_4, W_5) = K(J(H(G(F(x, W_1)), W_3)), W_5) \in \mathbb{R}^m$$

Multiple Layers – Feed Forward - Loss



Vector Calculus Refresher

Let $x \in R^n$ (a column vector) and let $f : R^n \to R$. The derivative of f with respect to x is the row vector:

$$\frac{\partial f}{\partial x} = (\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n})$$

$\frac{\partial f}{\partial x}$ is called the gradient of f.

Let $x \in R^n$ (a column vector) and let $f : R^n \to R^m$. The derivative of f with respect to x is the $m \times n$ matrix:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f(x)_1}{\partial x_1} & \ldots & \frac{\partial f(x)_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f(x)_m}{\partial x_1} & \ldots & \frac{\partial f(x)_m}{\partial x_n} \end{bmatrix}$$

$\frac{\partial f}{\partial x}$ is called the Jacobian matrix of f.

Sourced from: http://www.cs.huji.ac.il/~csip/tirgul3_derivatives.pdf

Back propagation

Neural Network Constructed

Now, let's review the following in turn:

- Feed forward
- Back propagation
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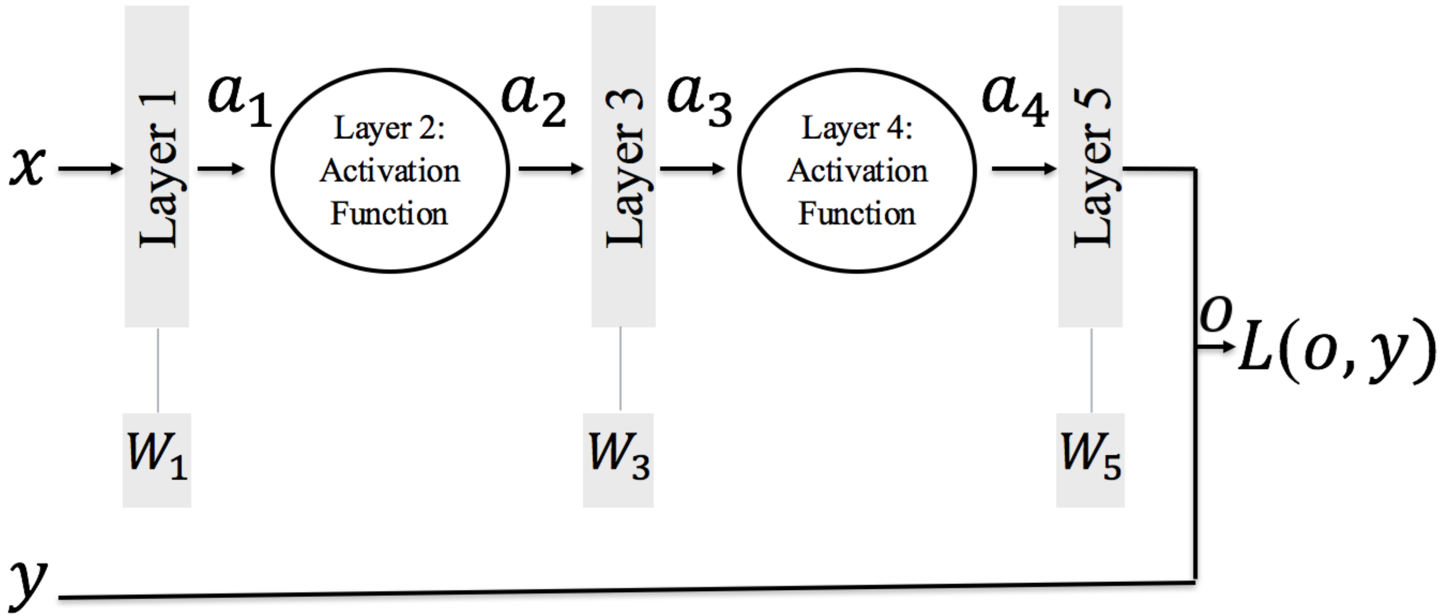
… and we'll have a fully functioning network

Multiple Layers – Back Prop

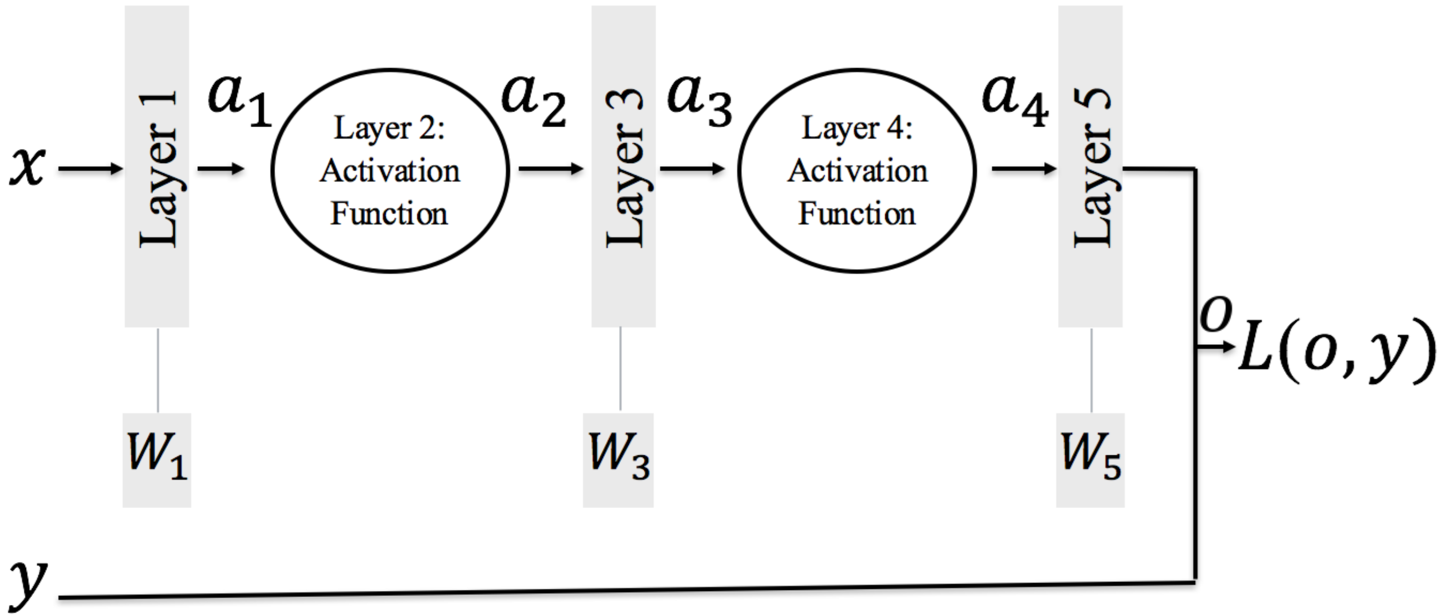
- At the end of each forward pass, we have a loss (difference between expected outcome and actual)
- The core of the back prop is a partial derivative of the Loss with respect to a weight - which tells us how quickly the Loss changes for any change in the weight
- Back Prop follows the chain rule of derivatives, i.e. the Loss can be computed for each and every weight in the network
- In practice, backward propagation is often abstracted away, because functions take care of it - but it’s important to know how it works

Multiple Layers – Back Prop: Chain Rule



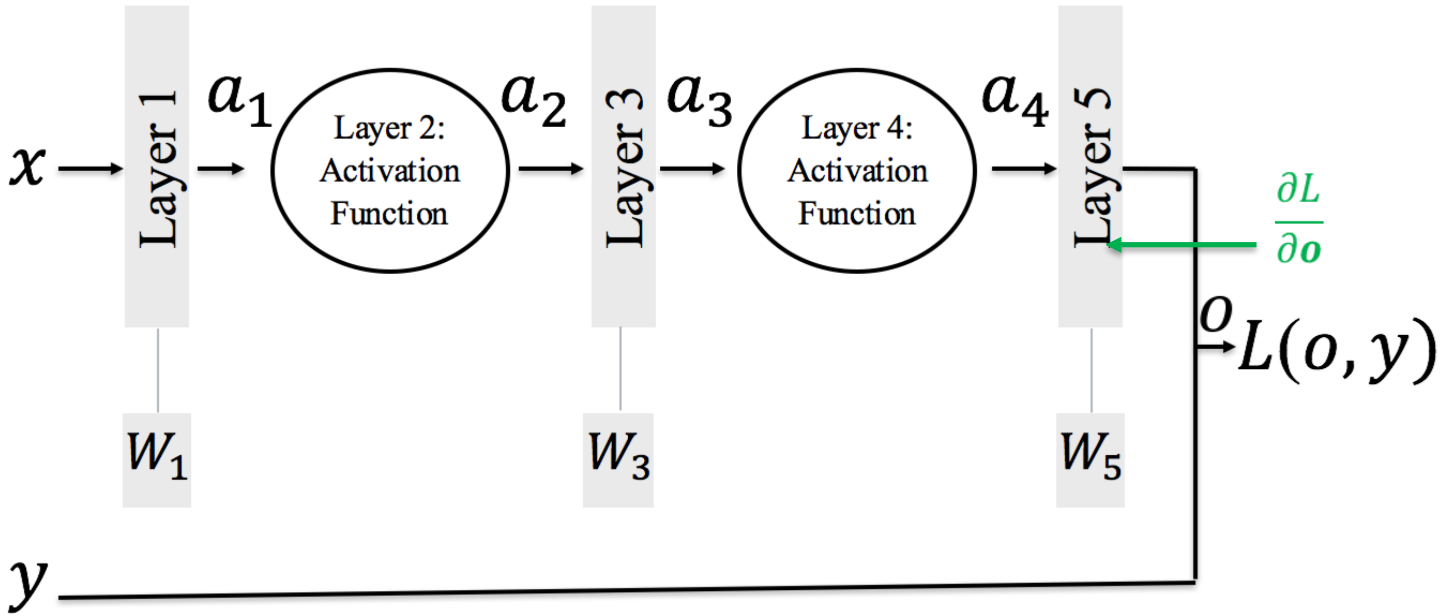
We want: $\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$

Multiple Layers – Back Prop: Chain Rule



We want: $\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$ Compute: $\frac{\partial L}{\partial \boldsymbol{o}}$

Multiple Layers – Back Prop: Chain Rule



We want:

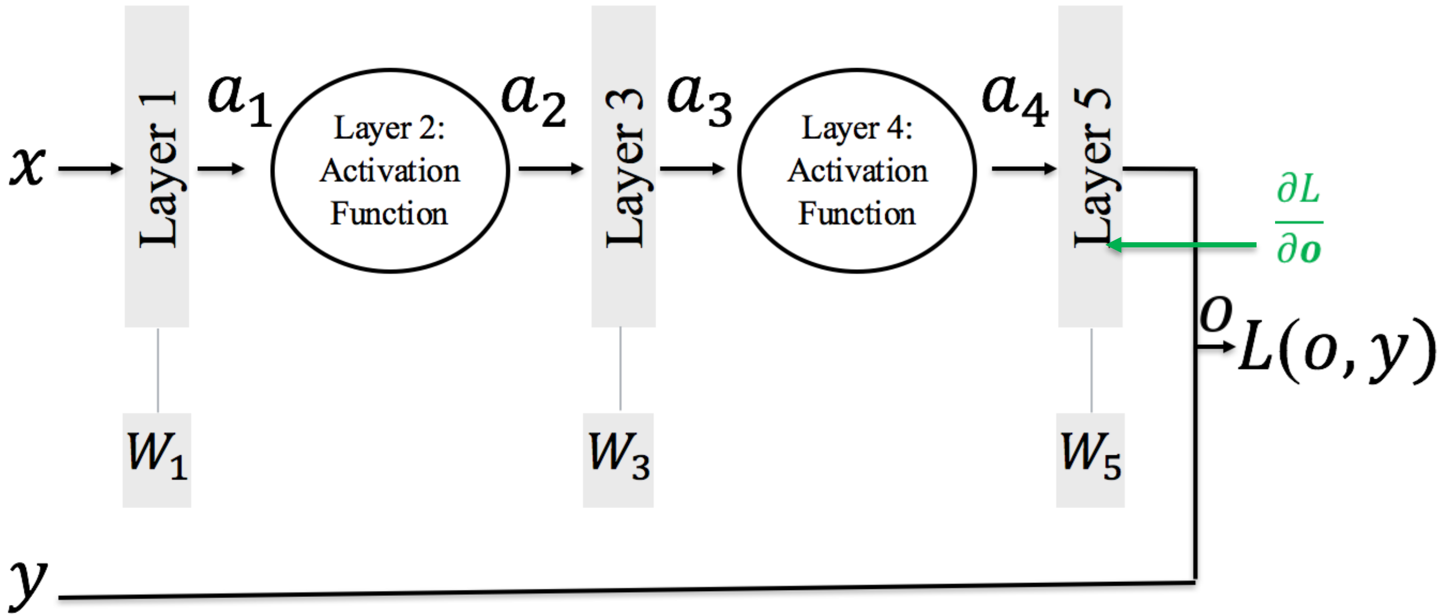
$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}$$

E.g: $L(\boldsymbol{o}, \boldsymbol{y}) = {}^1/_2 \parallel \boldsymbol{o} - \boldsymbol{y} \parallel^2$ then: $\frac{\partial L}{\partial \boldsymbol{o}}$

Multiple Layers – Back Prop: Chain Rule



We want:

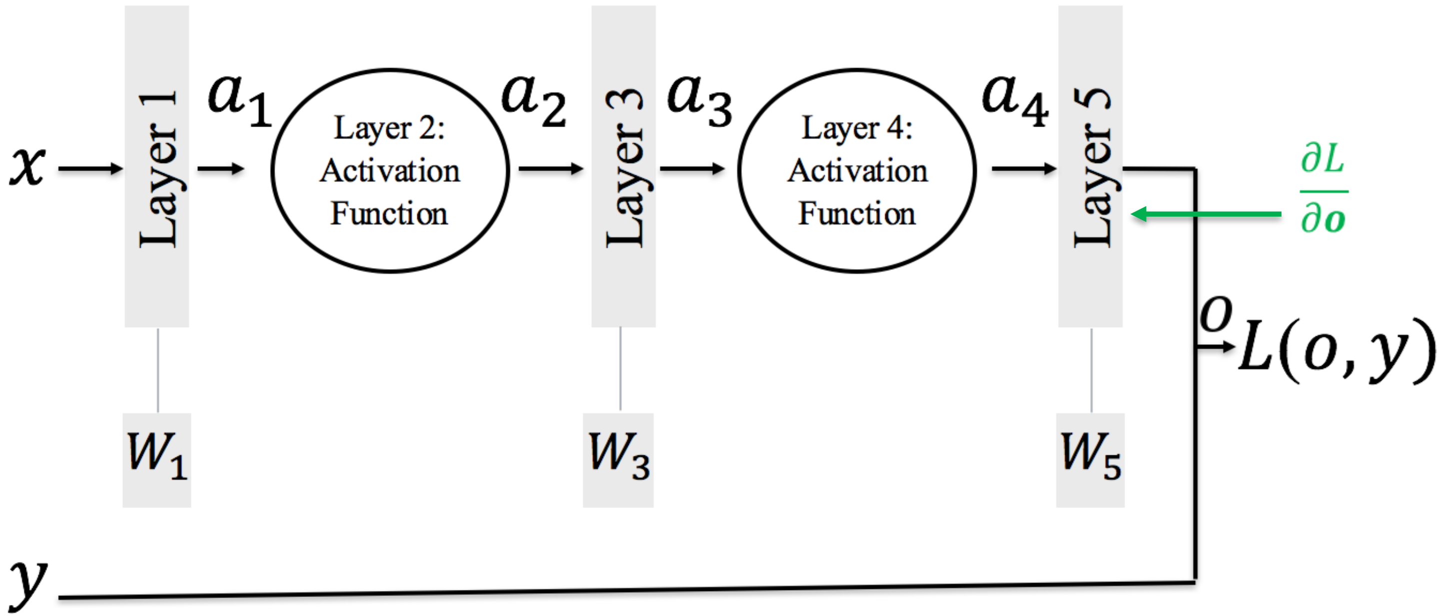
$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}$$

E.g: $L(\boldsymbol{o}, \boldsymbol{y}) = {}^{1}/_{2} \parallel \boldsymbol{o} - \boldsymbol{y} \parallel^2$ then: $\frac{\partial L}{\partial \boldsymbol{o}} = (\boldsymbol{o} - \boldsymbol{y})$

Multiple Layers – Back Prop: Chain Rule



$$\frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1\times m}$$

We want:

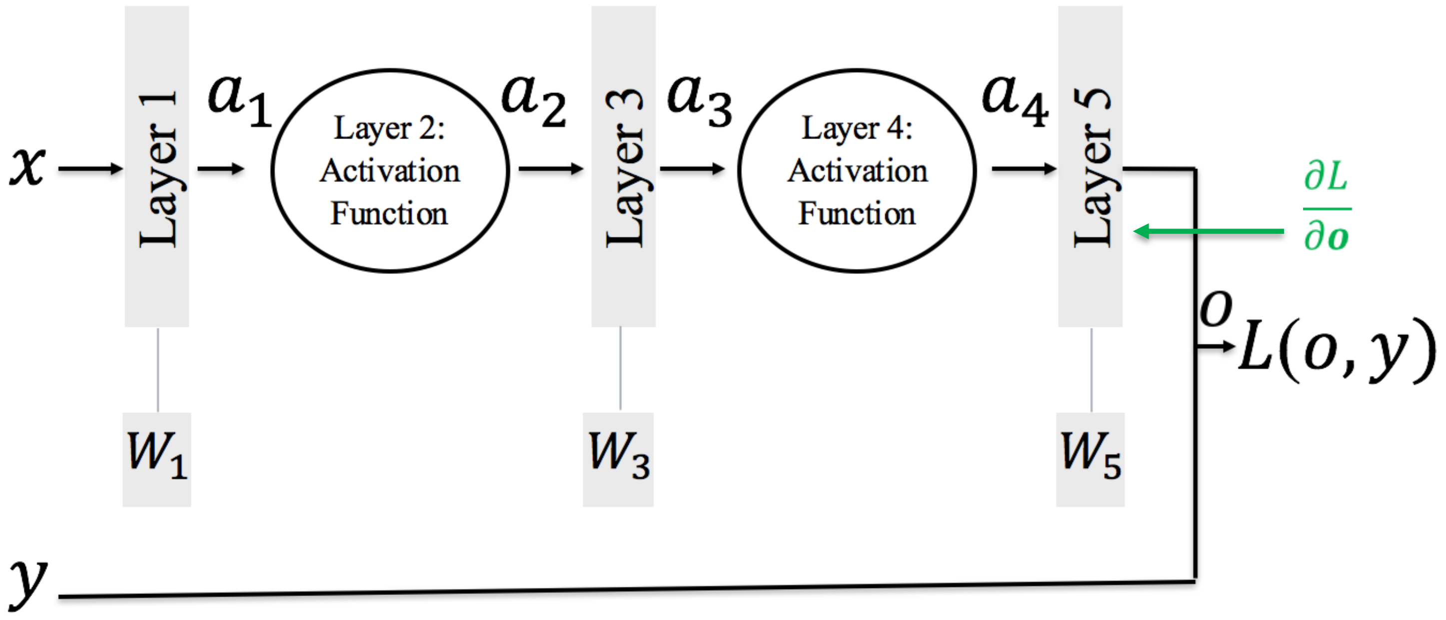
$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}$$

E.g: $L(\boldsymbol{o}, \boldsymbol{y}) = {}^1/_2 \parallel \boldsymbol{o} - \boldsymbol{y} \parallel^2$ then: $\frac{\partial L}{\partial \boldsymbol{o}} = (\boldsymbol{o} - \boldsymbol{y})$

Multiple Layers – Back Prop: Chain Rule



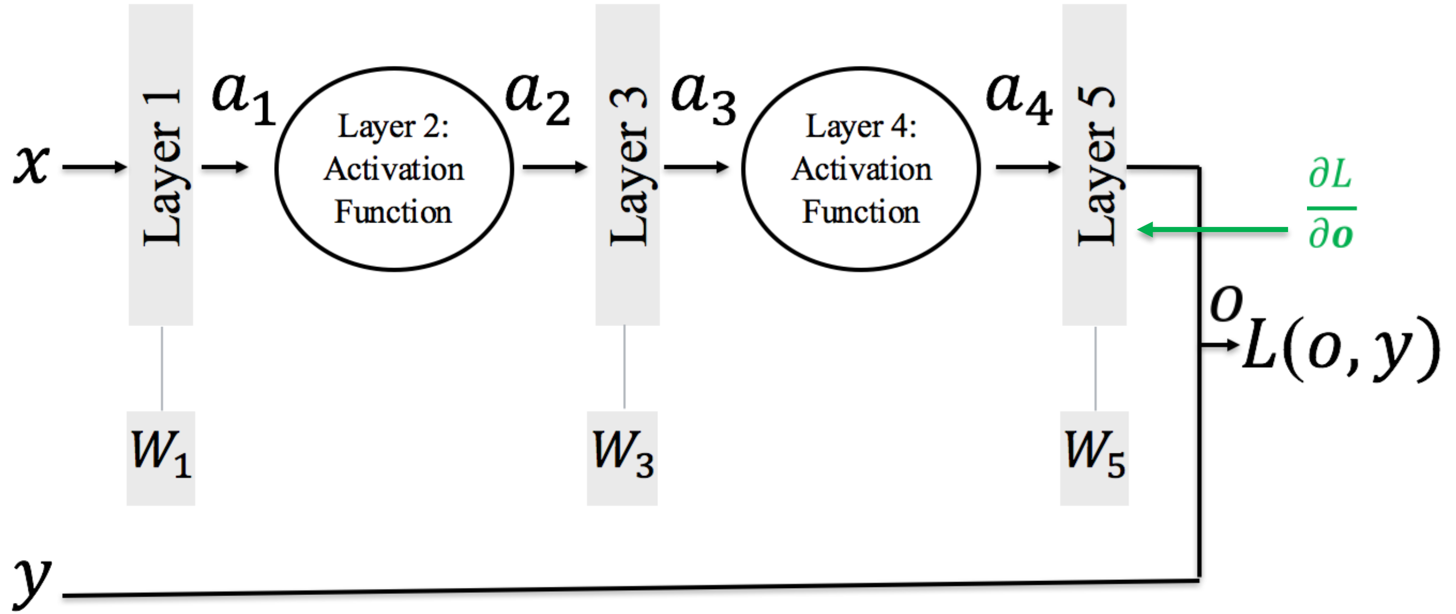
We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}, \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$$

Multiple Layers – Back Prop: Chain Rule



Some differentiable function with respect to a_4 and W_5, so some are also calling this differentiable programming

We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

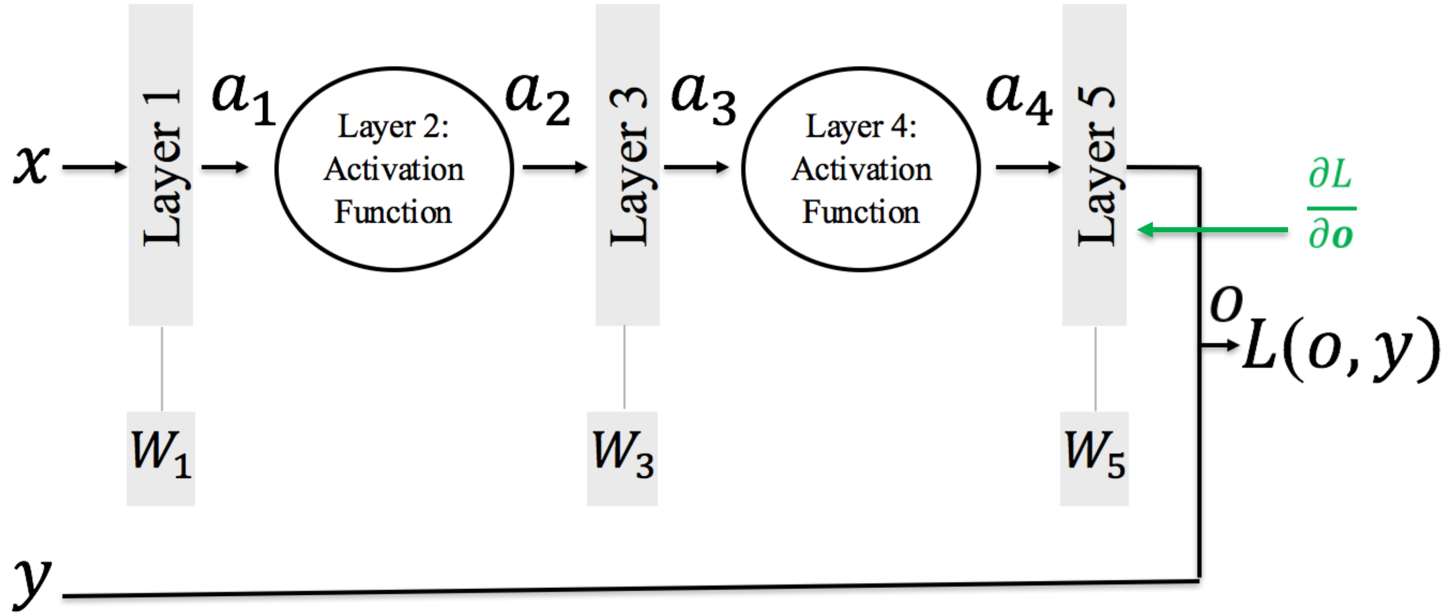
Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}, \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$$

since: $o = K(a_4, W_5)$

then: $\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$ is a Jacobian of size $\mathbb{R}^{m\times \dim(W_5)}$

Multiple Layers – Back Prop: Chain Rule



We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

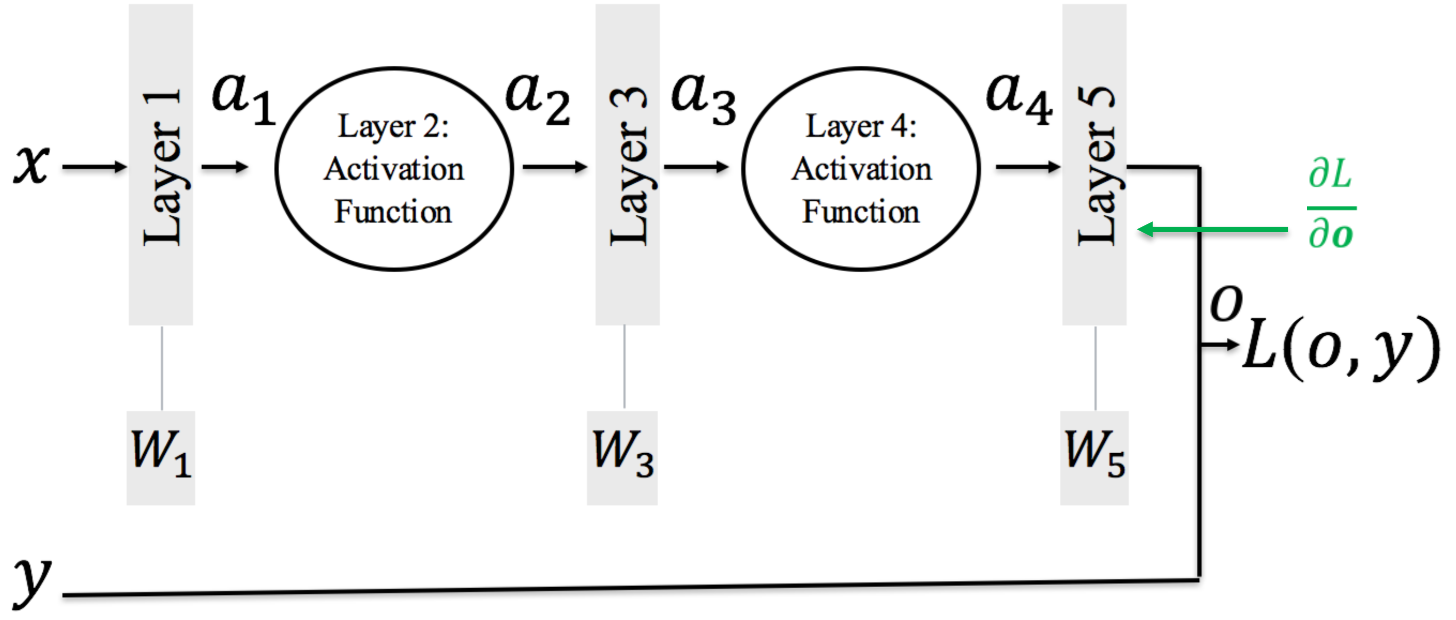
Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}, \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$$

since: $o = K(a_4, W_5)$

then: $$\left[\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}\right]_{kl} = \frac{\partial [K(\boldsymbol{a_4}, \boldsymbol{W_5})]_k}{\partial [\boldsymbol{W_5}]_l}$$

Multiple Layers – Back Prop: Chain Rule



Element (k, l) of the jacobian indicates how much the k-th output wiggles when we wiggle the l-th weight

We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

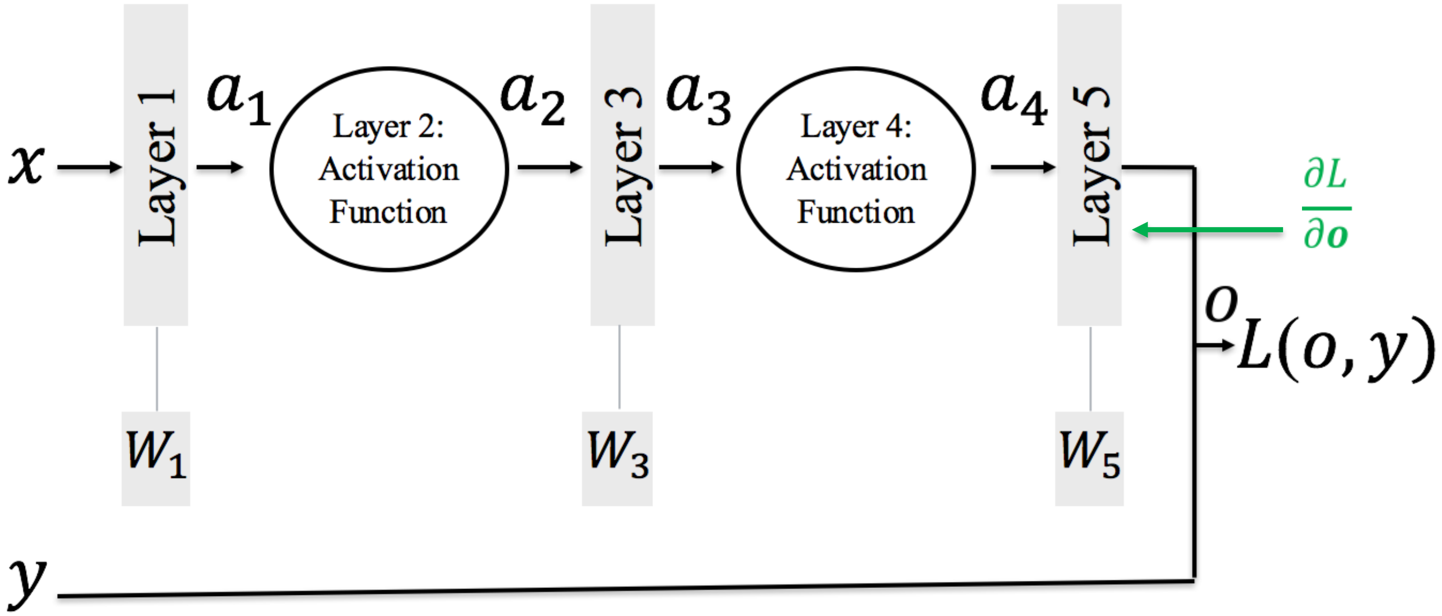
Compute:

$$\frac{\partial L}{\partial \boldsymbol{o}}, \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$$

since: $o = K(a_4, W_5)$

then: $$\left[\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}\right]_{kl} = \frac{\partial [K(\boldsymbol{a_4}, \boldsymbol{W_5})]_k}{\partial [\boldsymbol{W_5}]_l}$$

Multiple Layers – Back Prop: Chain Rule



We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Compute:

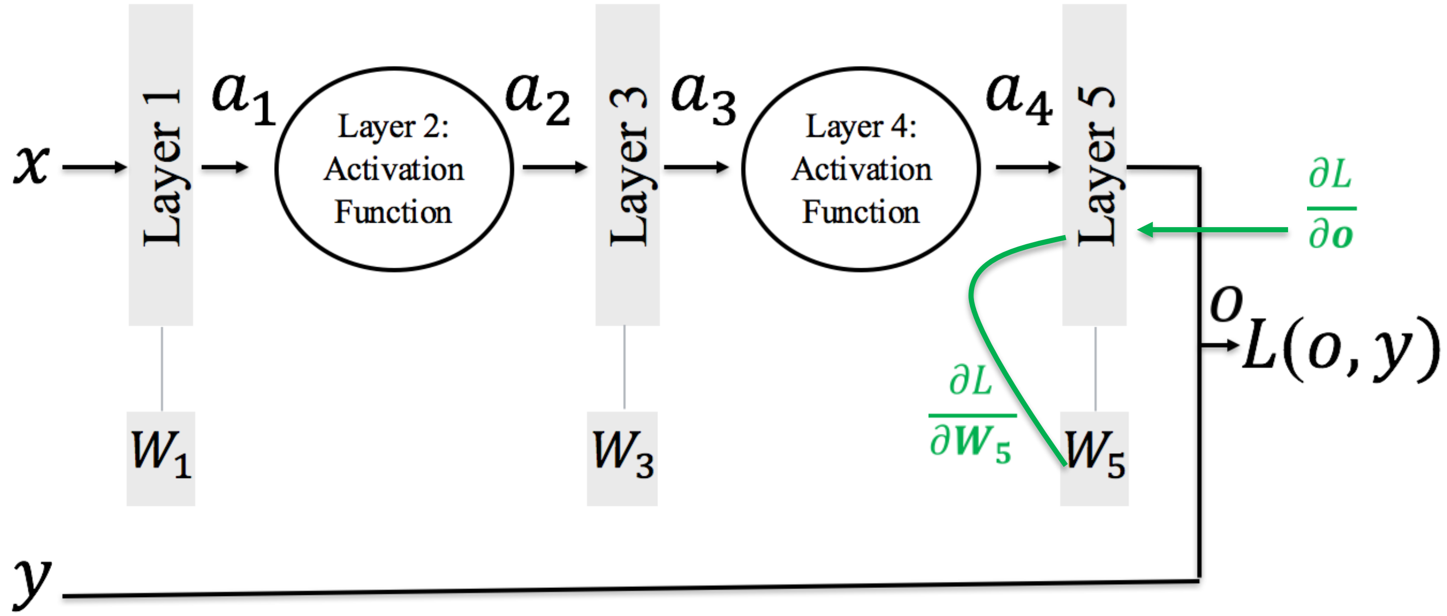
$$\frac{\partial L}{\partial \boldsymbol{W_5}} = \frac{\partial L}{\partial \boldsymbol{o}} \times \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}}$$

Remember:

$$\frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1 \times m}$$

$$\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}} \in \mathbb{R}^{1 \times \dim(W_5)}$$

Multiple Layers – Back Prop: Chain Rule



We want:

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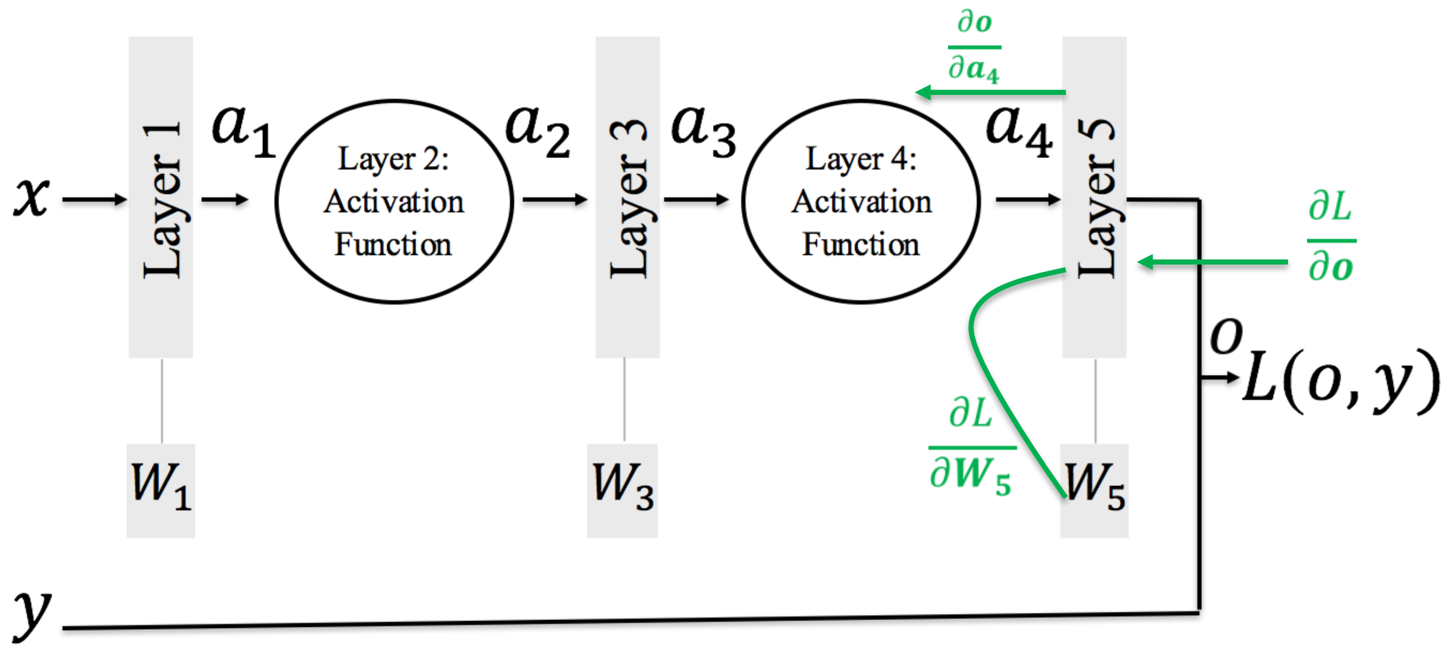
Compute:

$$\frac{\partial L}{\partial \boldsymbol{W_5}} = \frac{\partial L}{\partial \boldsymbol{o}} \times \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{W_5}} \quad \epsilon \ \mathbb{R}^{1 \times \dim(W_5)}$$

Multiple Layers – Back Prop: Chain Rule



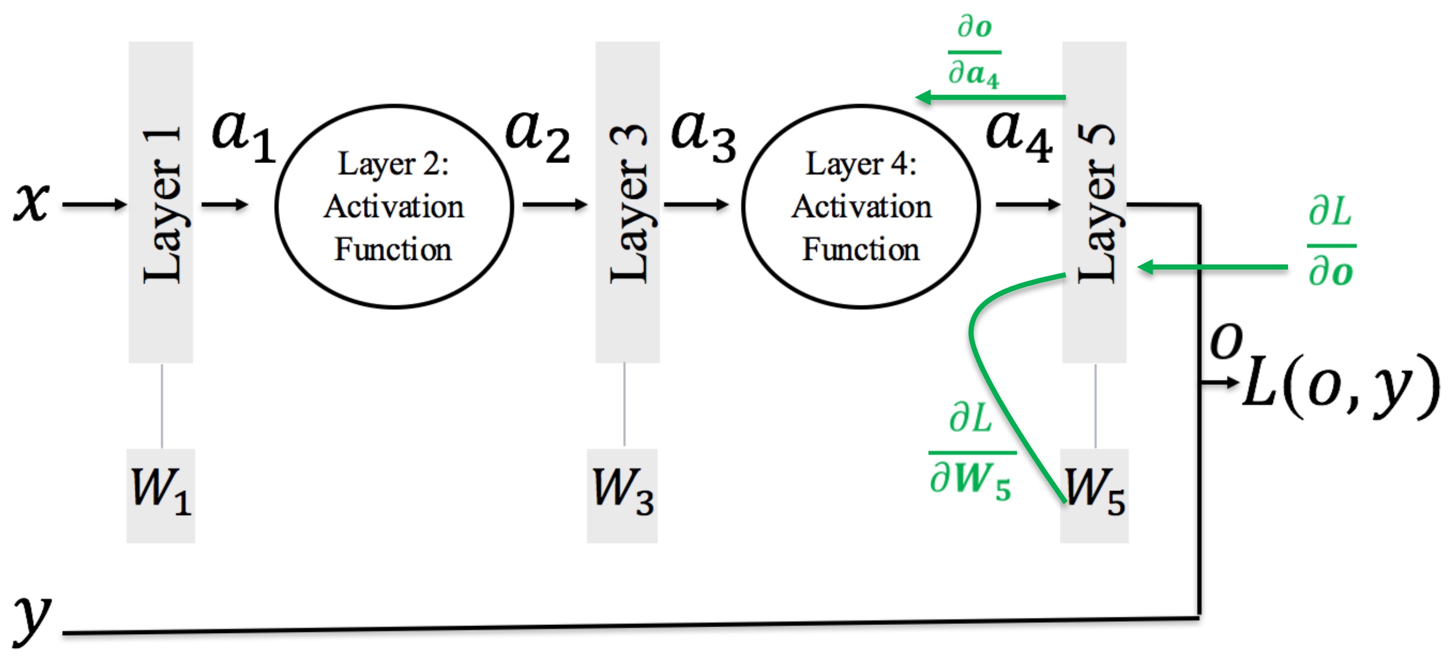
We want:

$$\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_3}, \frac{\partial L}{\partial W_5}$$

since: $\boldsymbol{o} = K(\boldsymbol{a_4}, \boldsymbol{W_5})$

then: $\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}}$ is a Jacobian of size $\mathbb{R}^{m \times \dim(a_4)}$

Multiple Layers – Back Prop: Chain Rule



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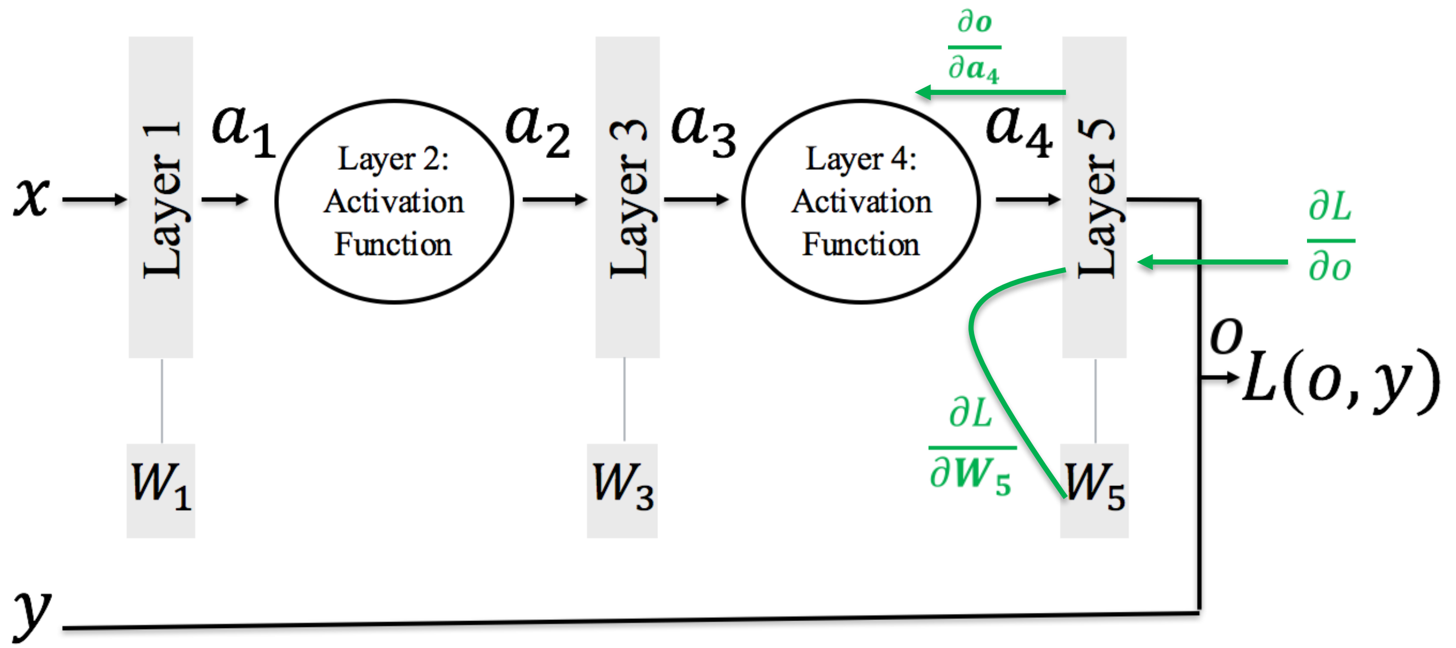
since: $\boldsymbol{o} = K(\boldsymbol{a_4}, \boldsymbol{W_5})$

then: $$\left[\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}}\right]_{kl} = \frac{\partial [K(\boldsymbol{a_4}, \boldsymbol{W_5})]_k}{\partial [\boldsymbol{a_4}]_l}$$

Multiple Layers – Back Prop: Chain Rule



Element (k, l) of the jacobian indicates how much the k-th output wiggles when we wiggle the l-th output of the previous layer (or input to this layer)

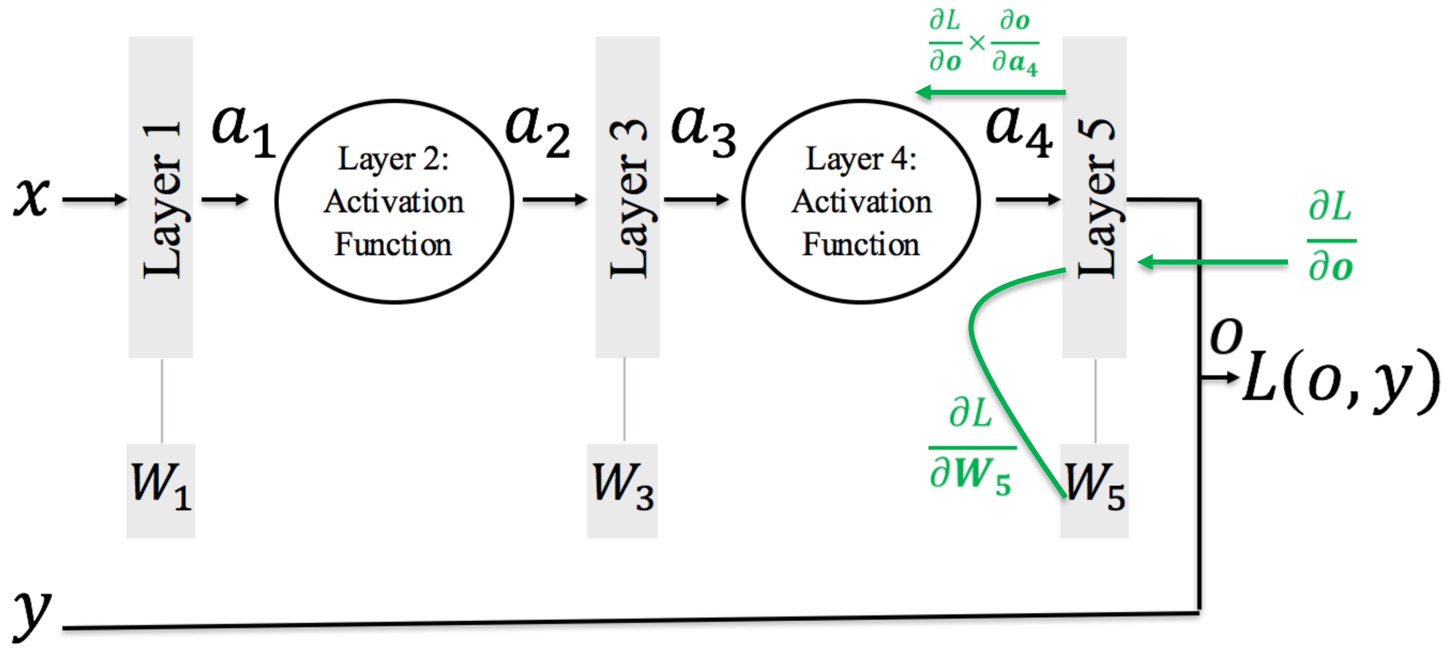
We want:

$$\frac{\partial L}{\partial \boldsymbol{W}_1}, \frac{\partial L}{\partial \boldsymbol{W}_3}, \frac{\partial L}{\partial \boldsymbol{W}_5}$$

since: $\boldsymbol{o} = K(\boldsymbol{a}_4, \boldsymbol{W}_5)$

then: $$\left[\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a}_4}\right]_{kl} = \frac{\partial [K(\boldsymbol{a}_4, \boldsymbol{W}_5)]_k}{\partial [\boldsymbol{a}_4]_l}$$

Multiple Layers – Back Prop: Chain Rule



Remember:

$$\frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1\times m}$$

$$\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \in \mathbb{R}^{m\times \dim(a_4)}$$

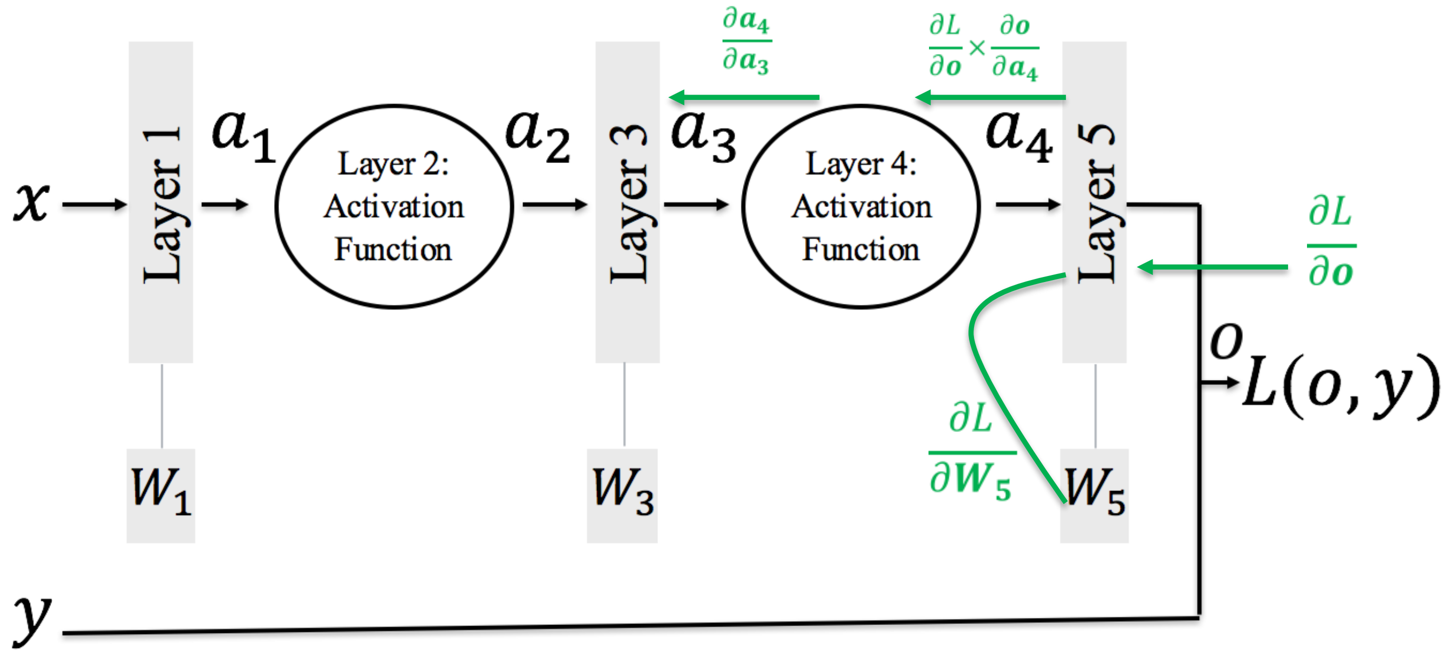
We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Backpropagate:

$$\frac{\partial L}{\partial \boldsymbol{o}} \times \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \in \mathbb{R}^{1\times \dim(a_4)}$$

Multiple Layers – Back Prop: Chain Rule



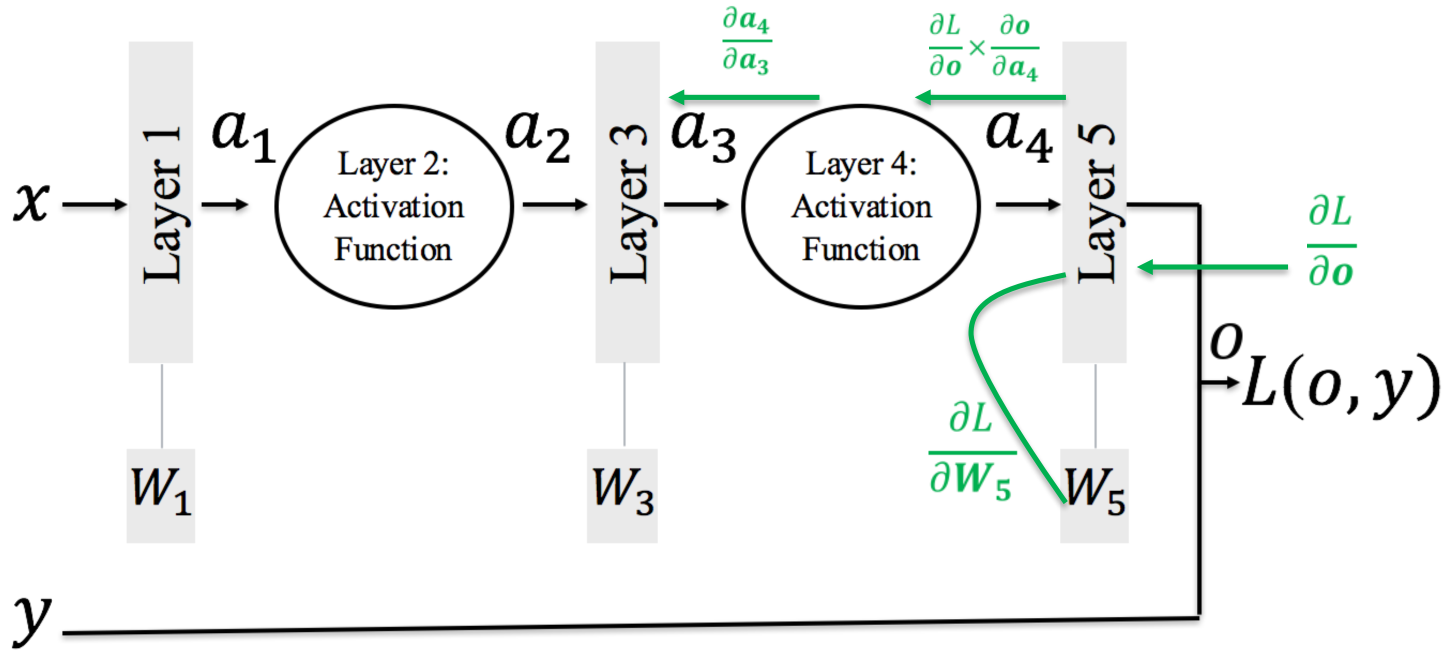
We want:

$$\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_3}, \frac{\partial L}{\partial W_5}$$

since: $a_4 = J(a_3)$

then: $\frac{\partial a_4}{\partial a_3}$ is a Jacobian of size $\mathbb{R}^{\dim(a_4) \times \dim(a_3)}$

Multiple Layers – Back Prop: Chain Rule



Remember:

$$\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \times \frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1\times \dim(a_4)}$$

$$\frac{\partial \boldsymbol{a_4}}{\partial \boldsymbol{a_3}} \in \mathbb{R}^{\dim(a_4)\times \dim(a_3)}$$

We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

since: $\boldsymbol{a_4} = J(\boldsymbol{a_3})$

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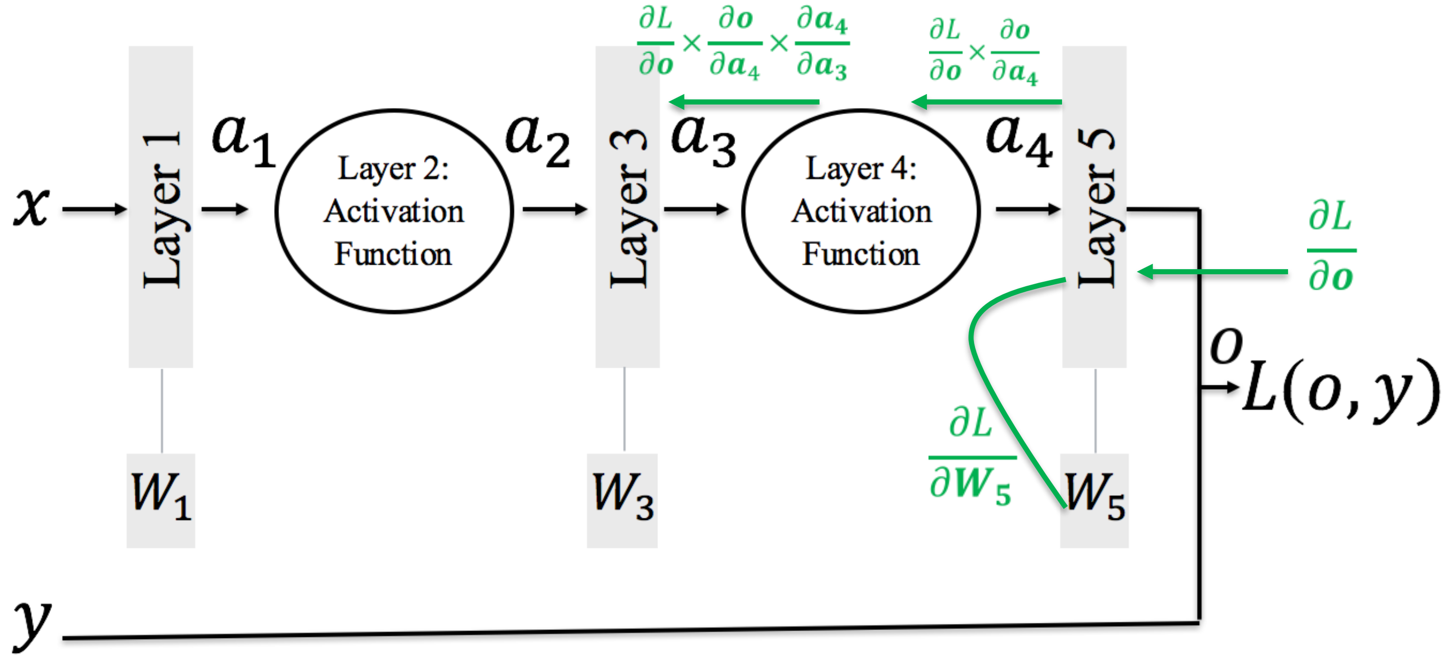
Multiple Layers – Back Prop: Chain Rule



Remember:

$$\frac{\partial o}{\partial a_4} \times \frac{\partial L}{\partial o} \in \mathbb{R}^{1\times \dim(a_4)}$$

$$\frac{\partial a_4}{\partial a_3} \in \mathbb{R}^{\dim(a_4)\times \dim(a_3)}$$

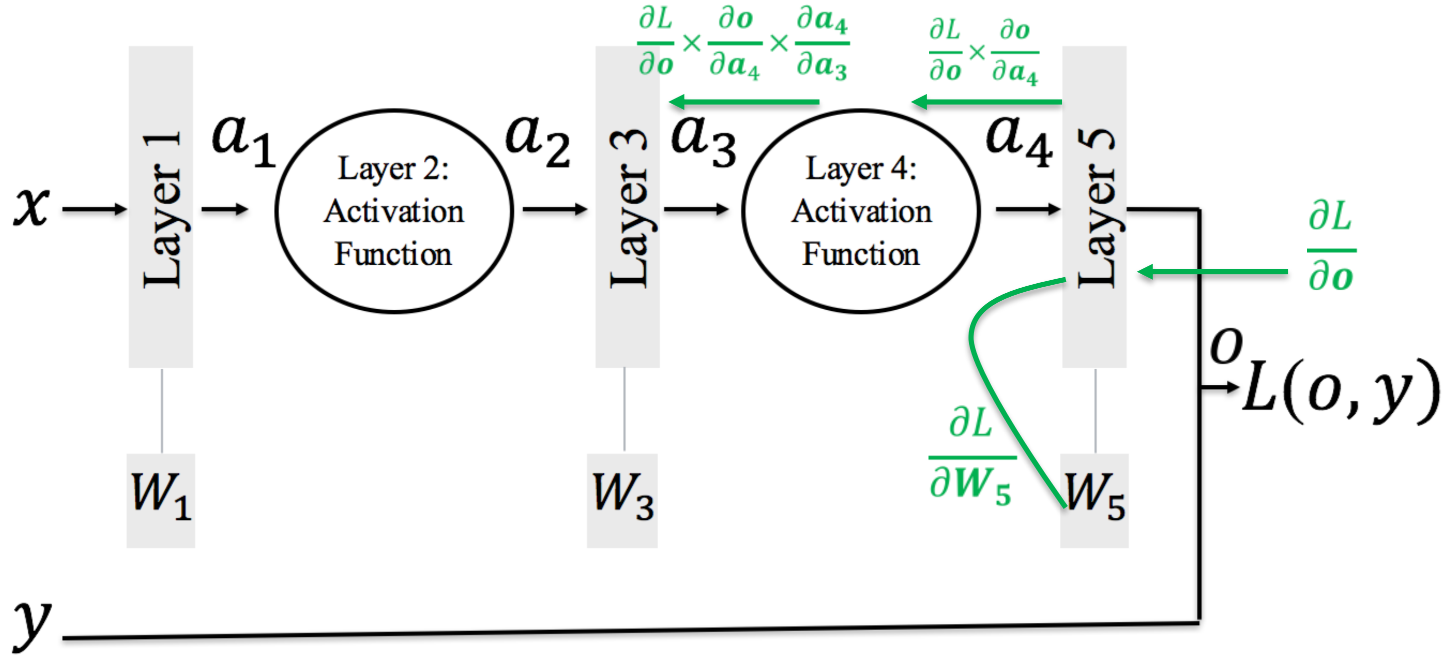
We want:

$$\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_3}, \frac{\partial L}{\partial W_5}$$

Backpropagate:

$$\frac{\partial L}{\partial o} \times \frac{\partial o}{\partial a_4} \times \frac{\partial a_4}{\partial a_3} \in \mathbb{R}^{1\times \dim(a_3)}$$

Multiple Layers – Back Prop: Chain Rule



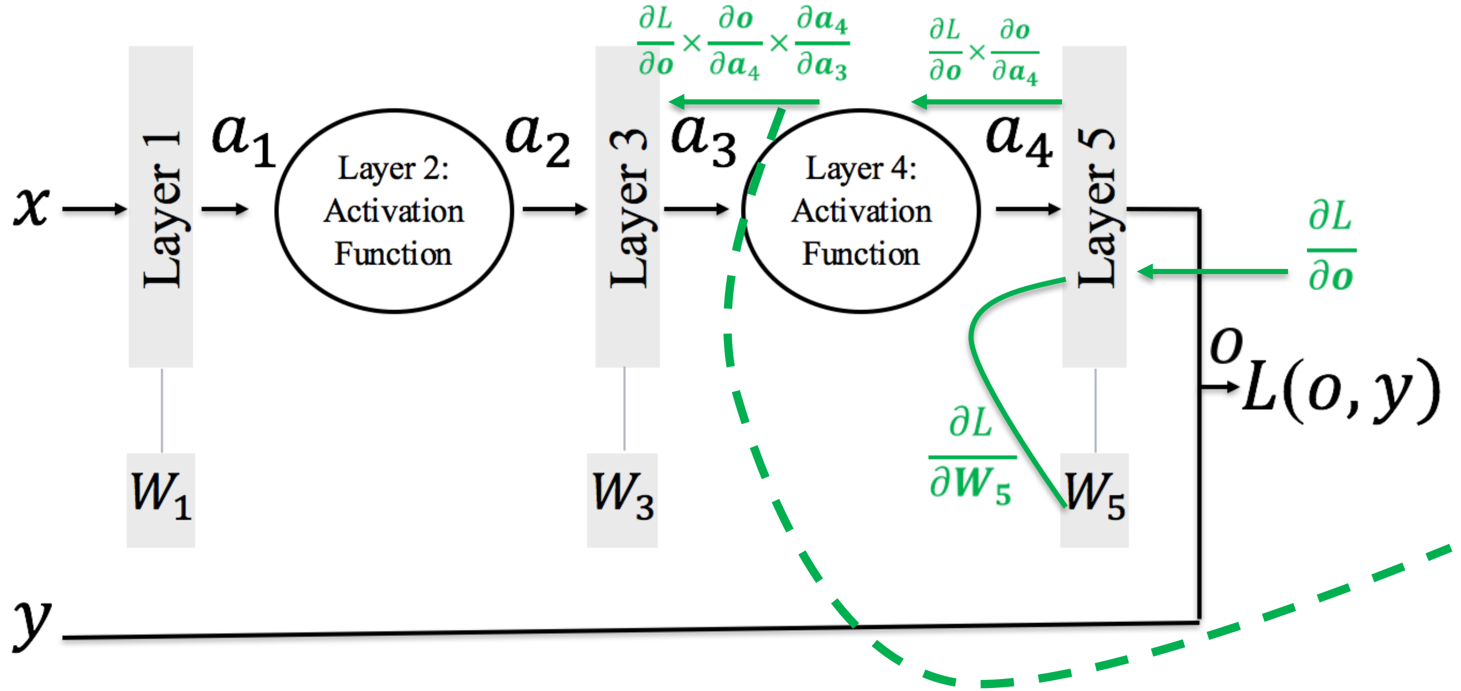
We want:

$$\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_3}, \frac{\partial L}{\partial W_5}$$

since: $\boldsymbol{a_3 = K(a_2, W_3)}$

then: $\frac{\partial \boldsymbol{a_3}}{\partial \boldsymbol{W_3}}$ is a Jacobian of size $\mathbb{R}^{\dim(a_3)\times\dim(W_3)}$

Multiple Layers – Back Prop: Chain Rule



Remember:

$$\frac{\partial L}{\partial \boldsymbol{o}} \times \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \times \frac{\partial \boldsymbol{a_4}}{\partial \boldsymbol{a_3}} \in \mathbb{R}^{1\times \dim(a_3)}$$

$$\frac{\partial \boldsymbol{a_3}}{\partial \boldsymbol{W_3}} \in \mathbb{R}^{\dim(a_3)\times \dim(W_3)}$$

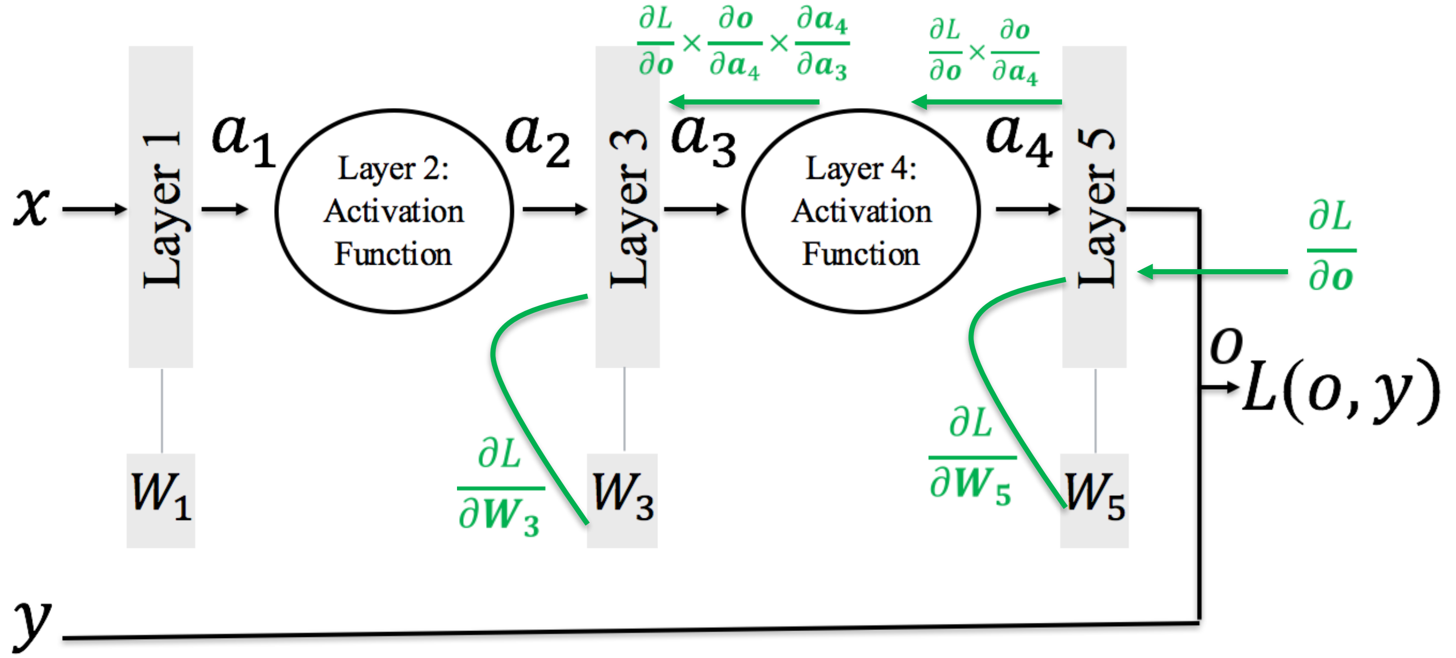
We want:

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}}$$

Now we can compute:

$$\frac{\partial L}{\partial \boldsymbol{W_3}} = \frac{\partial L}{\partial \boldsymbol{o}} \times \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \times \frac{\partial \boldsymbol{a_4}}{\partial \boldsymbol{a_3}} \times \frac{\partial \boldsymbol{a_3}}{\partial \boldsymbol{W_3}}$$

Multiple Layers – Back Prop: Chain Rule



Remember:

$$\frac{\partial L}{\partial o} \times \frac{\partial o}{\partial a_4} \times \frac{\partial a_4}{\partial a_3} \in \mathbb{R}^{1 \times \dim(a_3)}$$

$$\frac{\partial a_3}{\partial W_3} \in \mathbb{R}^{\dim(a_3) \times \dim(W_3)}$$

We want:

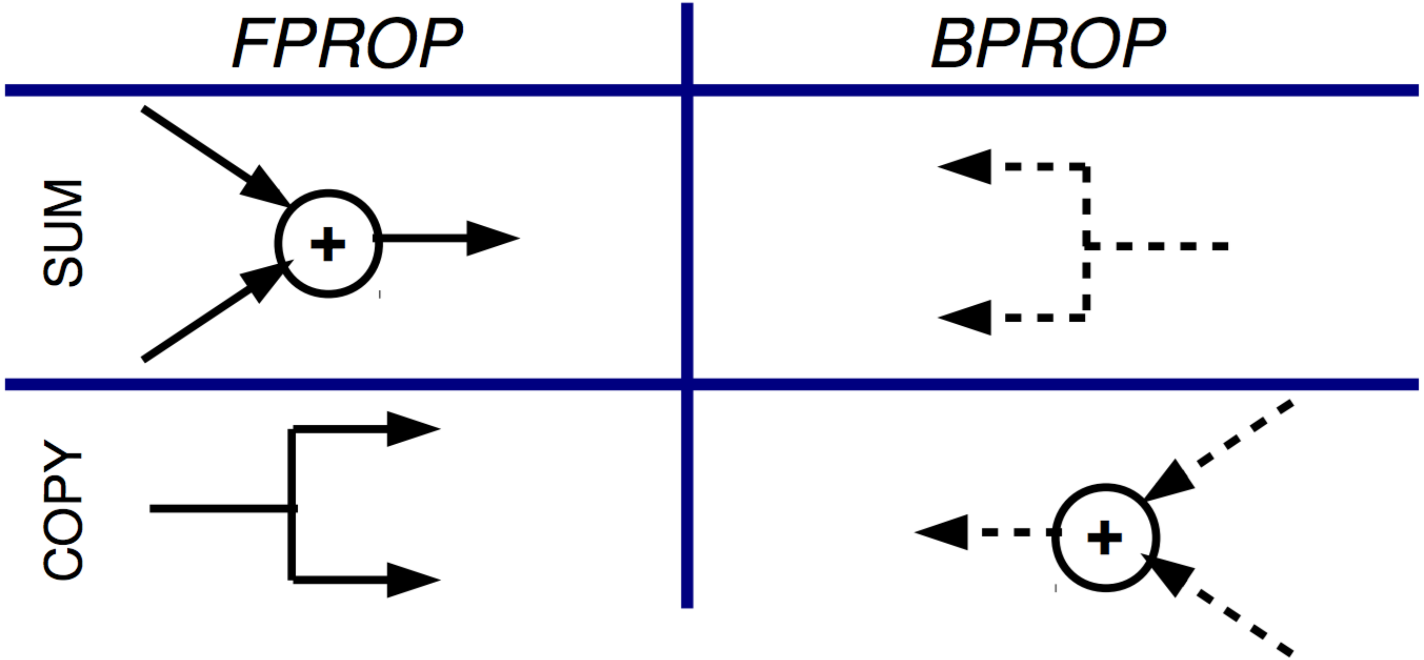
$$\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_3}, \frac{\partial L}{\partial W_5}$$

Now we can compute:

$$\frac{\partial L}{\partial W_3} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial a_4} \times \frac{\partial a_4}{\partial a_3} \times \frac{\partial a_3}{\partial W_3}$$

Multiple Layers – Back Prop: Chain Rule

FPROP and BPROP are duals of each other:



Fully Connected

Neural Network Constructed

Now, let's review the following in turn:

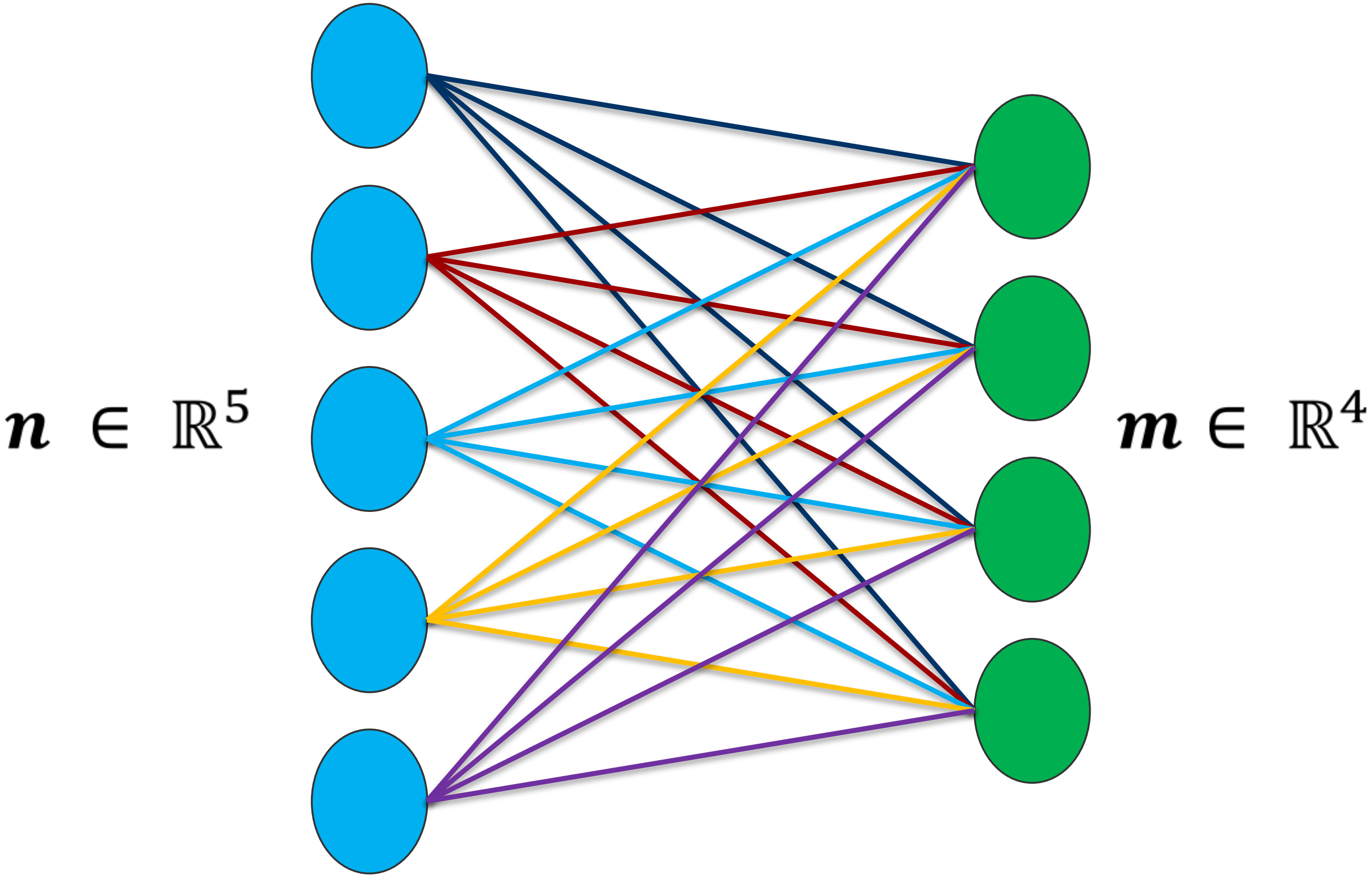
- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

… and we'll have a fully functioning network

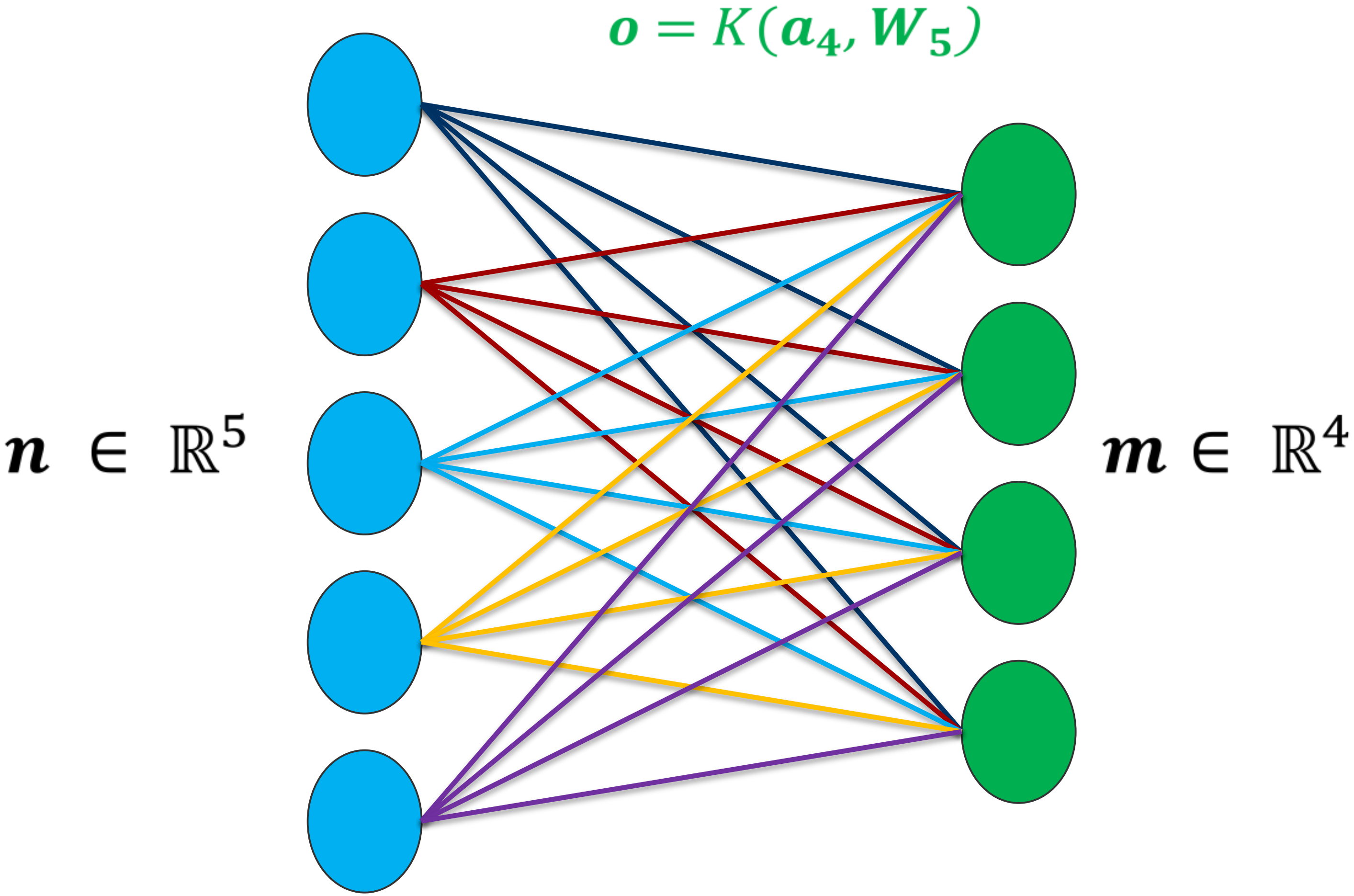
Fully Connected

- Neurons have connections to all activations of the previous layer
- Number of connections add up very quickly due to all the combinations
- Forward pass of a fully connected layer -> one matrix multiplication followed by a bias offset and activation function (you'll soon see what we mean)

Fully Connected



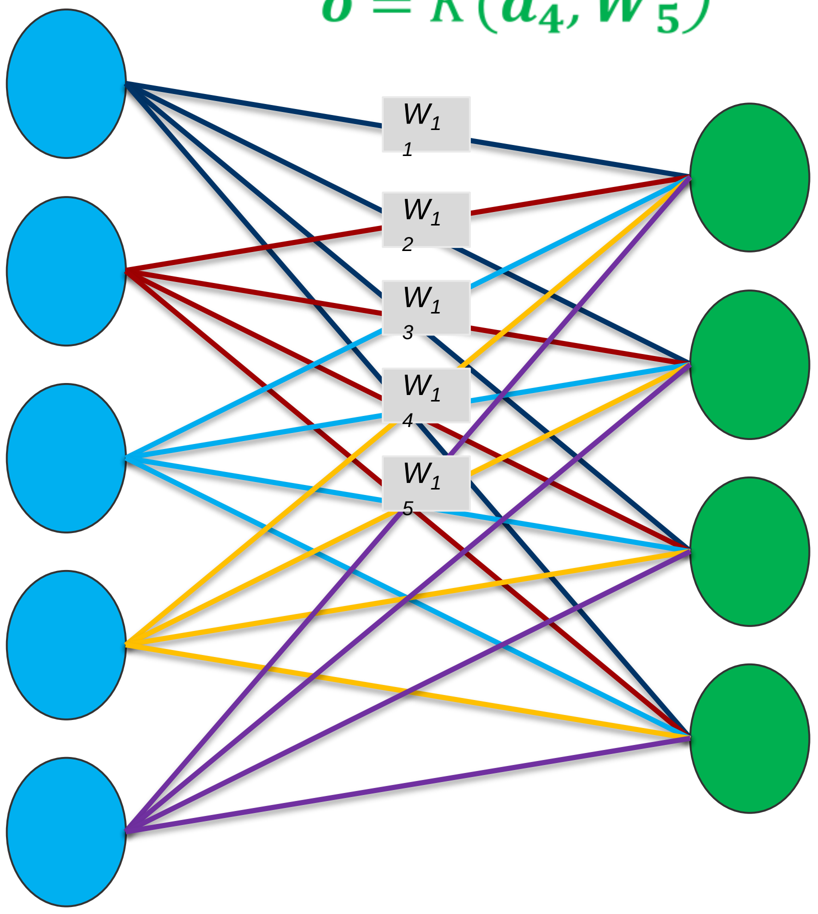
Fully Connected



Fully Connected

$$\boldsymbol{o} = K(\boldsymbol{a}_4, \boldsymbol{W}_5)$$

$\boldsymbol{n} \in \mathbb{R}^5$

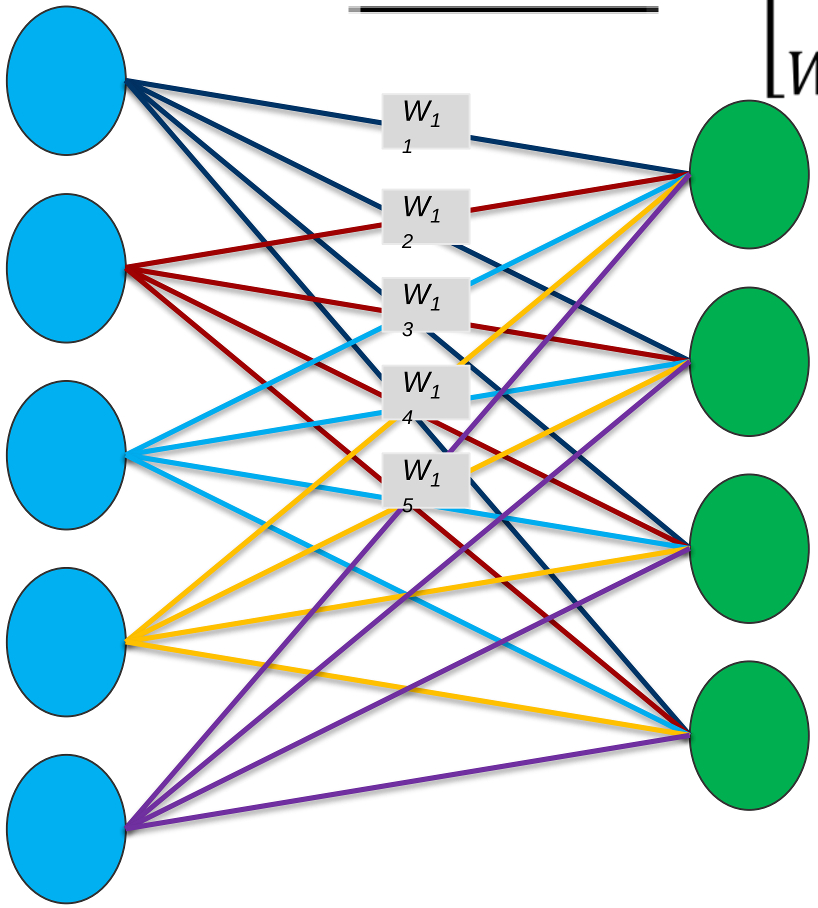


$\boldsymbol{m} \in \mathbb{R}^4$

Fully Connected

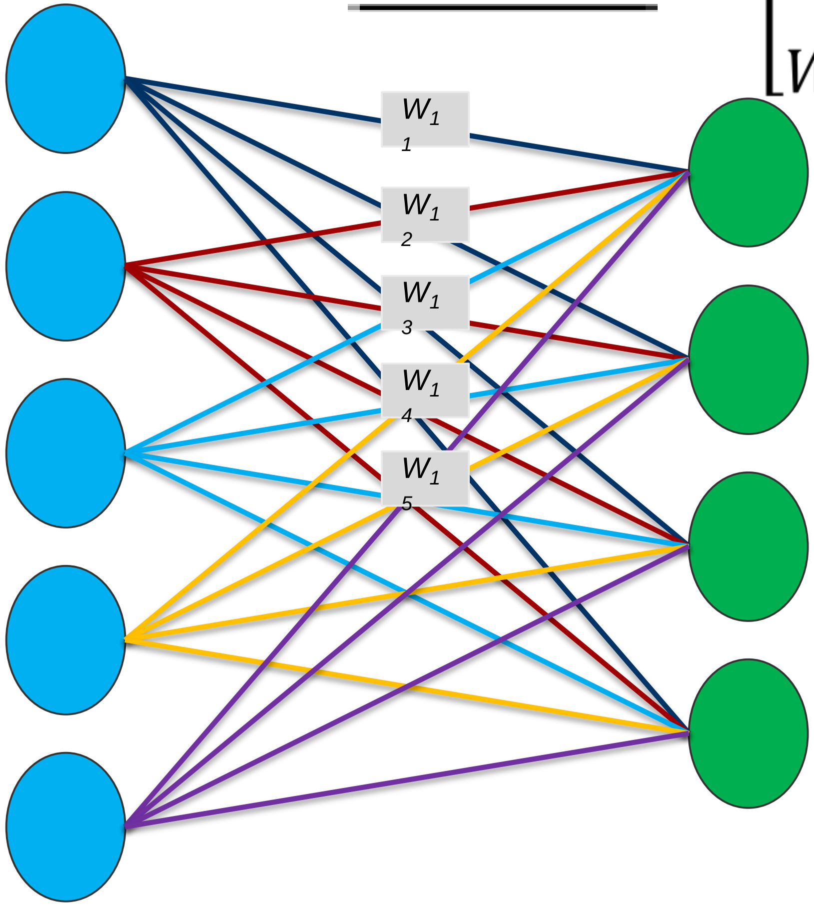
$$= \begin{bmatrix} W_{11} & \cdots & W_{15} \\ \vdots & \ddots & \vdots \\ W_{41} & \cdots & W_{45} \end{bmatrix}$$

$$\boldsymbol{n} \in \mathbb{R}^5$$



$$\boldsymbol{m} \in \mathbb{R}^4$$

Fully Connected



$n \in \mathbb{R}^5$

$m \in \mathbb{R}^4$

$$= \begin{bmatrix} W_{11} & \cdots & W_{15} \\ \vdots & \ddots & \vdots \\ W_{41} & \cdots & W_{45} \end{bmatrix}$$

$m_1 = n_1 \times W_{11} + n_2 \times W_{12} + .. + n_5 \times W_{15}$

Fully Connected

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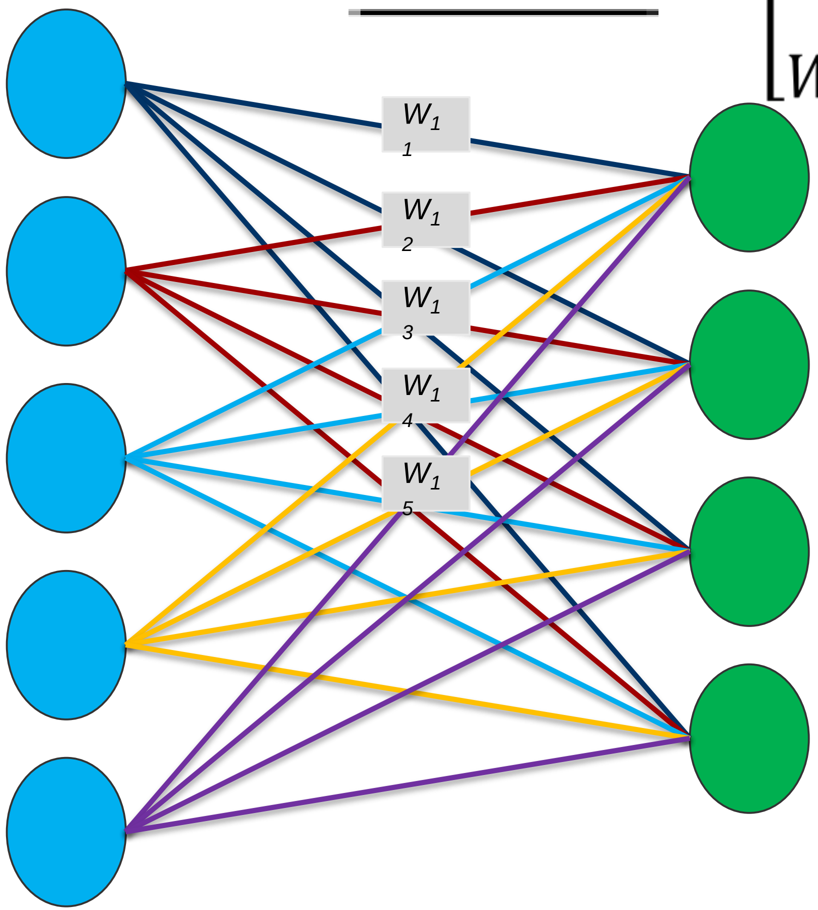
$$\begin{bmatrix} W_{11} & \cdots & W_{15} \\ \vdots & \ddots & \vdots \\ W_{41} & \cdots & W_{45} \end{bmatrix}$$

$\boldsymbol{n} \in \mathbb{R}^5$



$\boldsymbol{m} \in \mathbb{R}^4$

$$m_1 = n_1 \times W_{11} + n_2 \times W_{12} + .. + n_5 \times W_{15}$$

$$m_2 = n_1 \times W_{21} + n_2 \times W_{22} + .. + n_5 \times W_{25}$$

$$m_3 = n_1 \times W_{31} + n_2 \times W_{32} + .. + n_5 \times W_{35}$$

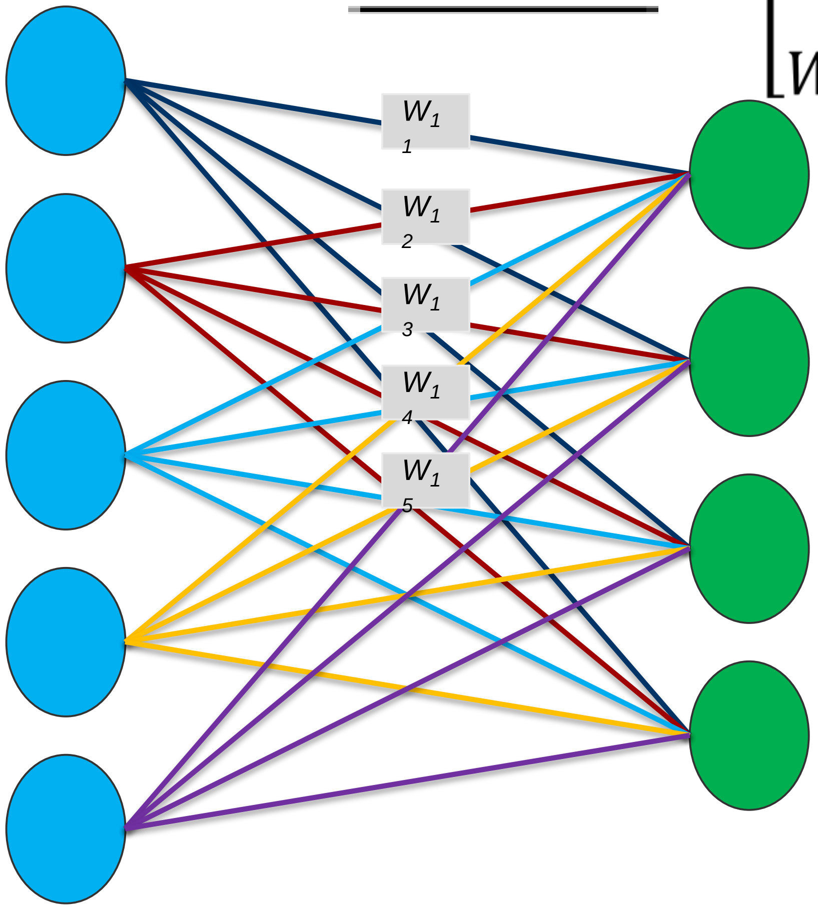
$$m_4 = n_1 \times W_{41} + n_2 \times W_{24} + .. + n_5 \times W_{45}$$

Fully Connected

Q:
Why fully connected?

$$\begin{bmatrix} W_{11} & \cdots & W_{15} \\ \vdots & \ddots & \vdots \\ W_{41} & \cdots & W_{45} \end{bmatrix}$$

$\boldsymbol{n} \in \mathbb{R}^5$



$\boldsymbol{m} \in \mathbb{R}^4$

$$m_1 = n_1 \times W_{11} + n_2 \times W_{12} + .. + n_5 \times W_{15}$$

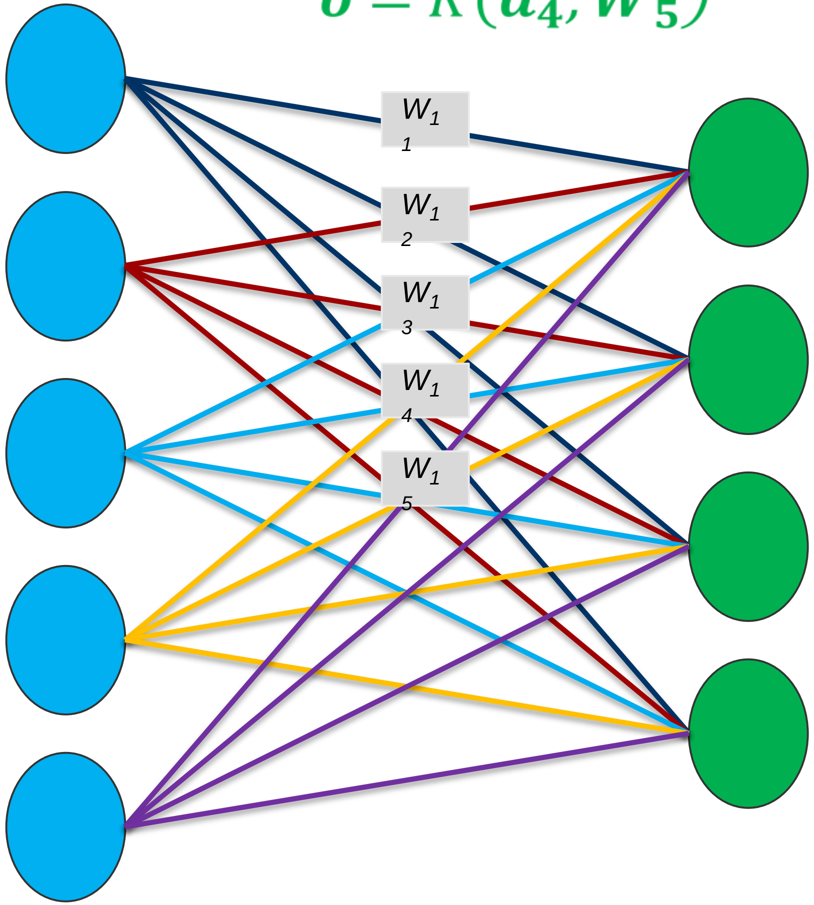
$$m_2 = n_1 \times W_{21} + n_2 \times W_{22} + .. + n_5 \times W_{25}$$

$$m_3 = n_1 \times W_{31} + n_2 \times W_{32} + .. + n_5 \times W_{35}$$

$$m_4 = n_1 \times W_{41} + n_2 \times W_{24} + .. + n_5 \times W_{45}$$

Fully Connected

$$\boldsymbol{o} = K(\boldsymbol{a}_4, \boldsymbol{W}_5)$$



$\boldsymbol{n} \in \mathbb{R}^5$

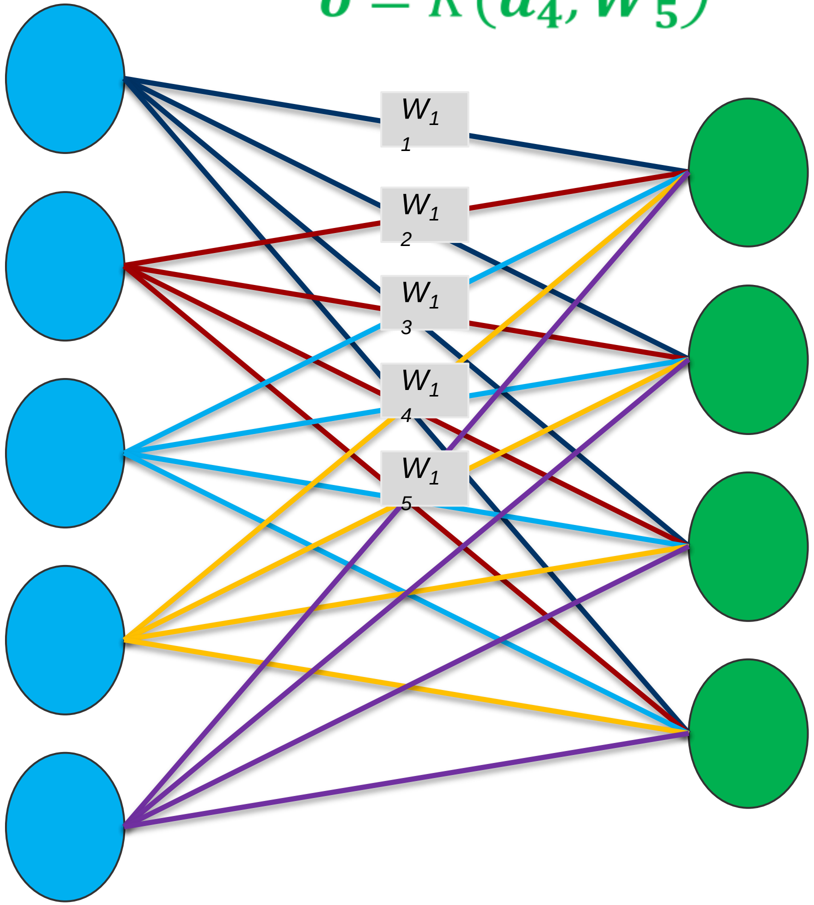
$\boldsymbol{m} \in \mathbb{R}^4$

$$\boxed{\boldsymbol{m} = W^{4\times5} \cdot \boldsymbol{n}}$$

Fully Connected

$$o = K(a_4, W_5)$$

$n \in \mathbb{R}^5$

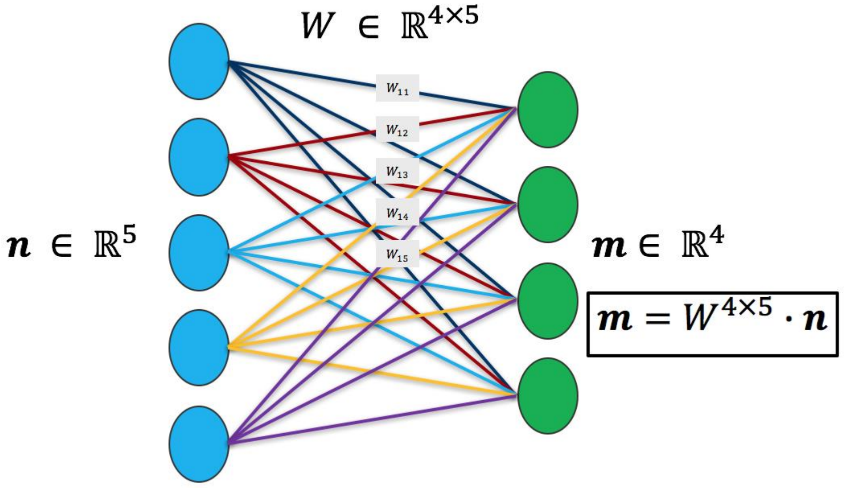


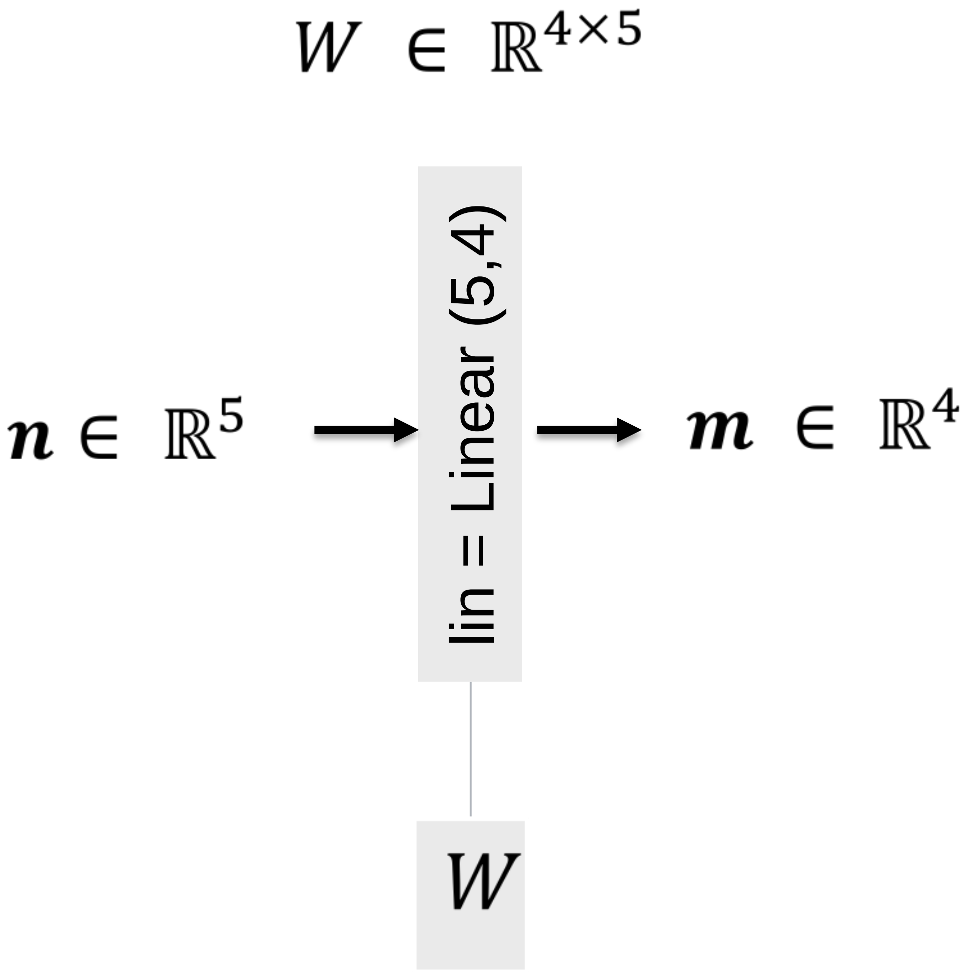
$m \in \mathbb{R}^4$

forward()

$$\boxed{m = W^{4\times5} \cdot n} + b^{4\times1}$$

Fully Connected





Fully Connected: Forward pass

$$W \in \mathbb{R}^{4\times 5}$$

```
n
```

```
array([[0.68637243],
       [0.97851301],
       [0.37695692],
       [0.09355431],
       [0.92320742]])
```

$$\boldsymbol{n} \in \mathbb{R}^5 \longrightarrow \text{lin = Linear (5,4)} \longrightarrow \boldsymbol{m} \in \mathbb{R}^4$$

W

Fully Connected: Forward pass

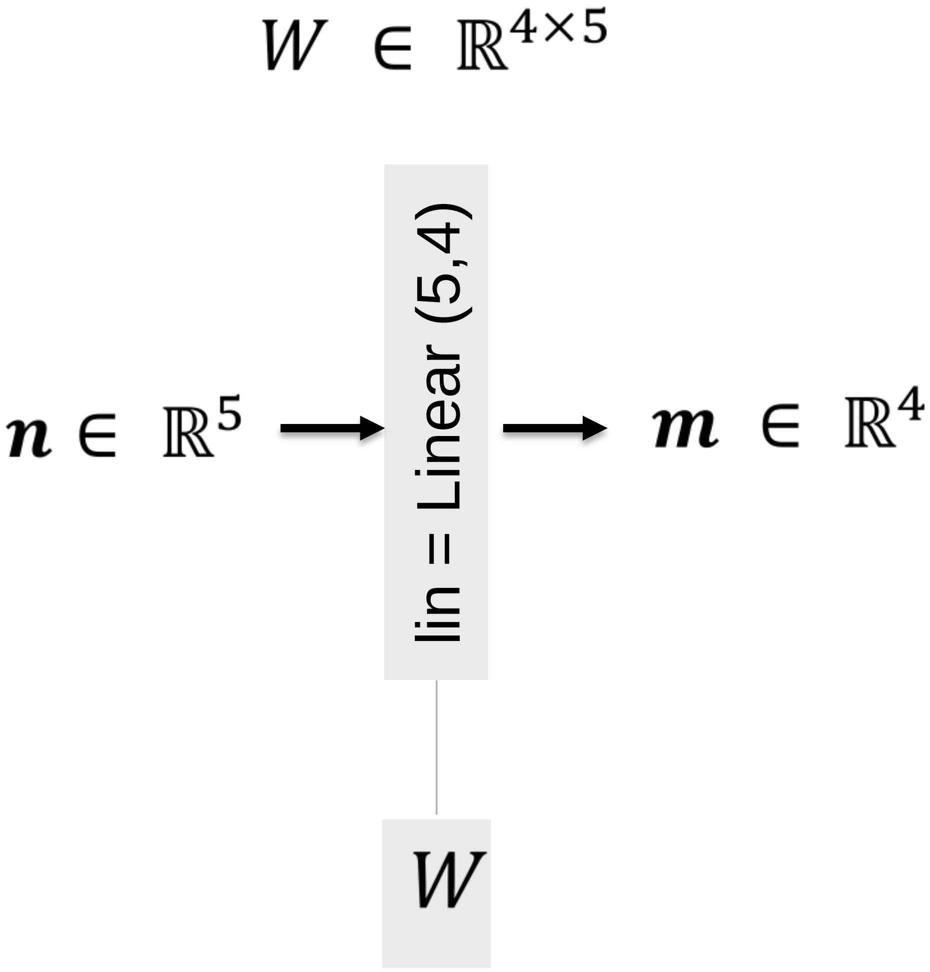
```
n = np.random.rand(5,1)
n
```

```
array([[0.68637243],
       [0.97851301],
       [0.37695692],
       [0.09355431],
       [0.92320742]])
```

```
class Linear():
    def __init__(self, in_size, out_size):
        self.W = np.random.randn(in_size, out_size) * 0.01
        self.b = np.zeros((1, out_size))
        self.params = [self.W, self.b]
        self.gradW = None
        self.gradB = None
        self.gradInput = None

    def forward(self, X):
        self.X = X
        output = np.dot(self.X, self.W) + self.b
        return output
```

```
lin = Linear(5,4)
m = lin.forward(n)
```



Fully Connected: Forward pass

$$W \in \mathbb{R}^{4\times 5}$$

```
n = np.random.rand(5,1)
n
```

```
array([[0.68637243],
       [0.97851301],
       [0.37695692],
       [0.09355431],
       [0.92320742]])
```

```
class Linear():
    def __init__(self, in_size, out_size):
        self.W = np.random.randn(in_size, out_size) * 0.01
        self.b = np.zeros((1, out_size))
        self.params = [self.W, self.b]
        self.gradW = None
        self.gradB = None
        self.gradInput = None

    def forward(self, X):
        self.X = X
        output = np.dot(self.X, self.W) + self.b
        return output
```

```
lin = Linear(5,4)
m = lin.forward(n)
```

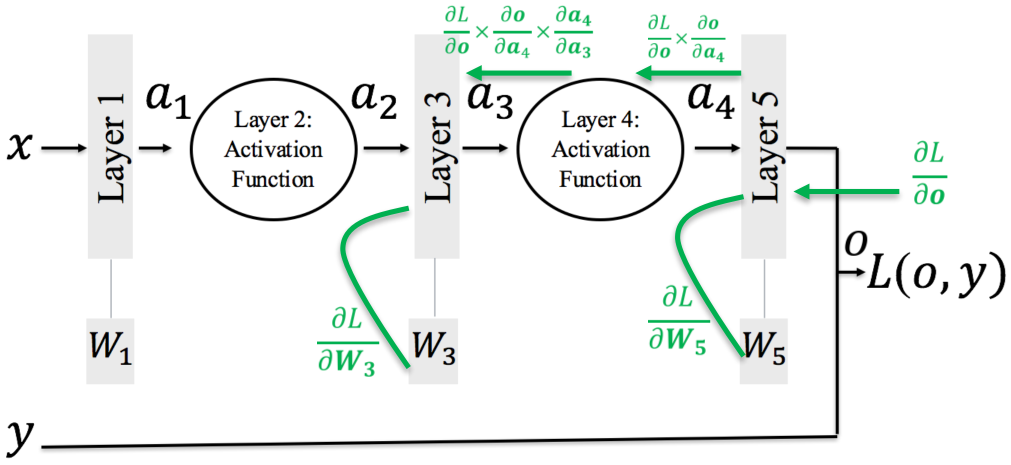
$$\boldsymbol{n} \in \mathbb{R}^5 \longrightarrow \text{lin = Linear (5,4)} \longrightarrow \boldsymbol{m} \in \mathbb{R}^4$$

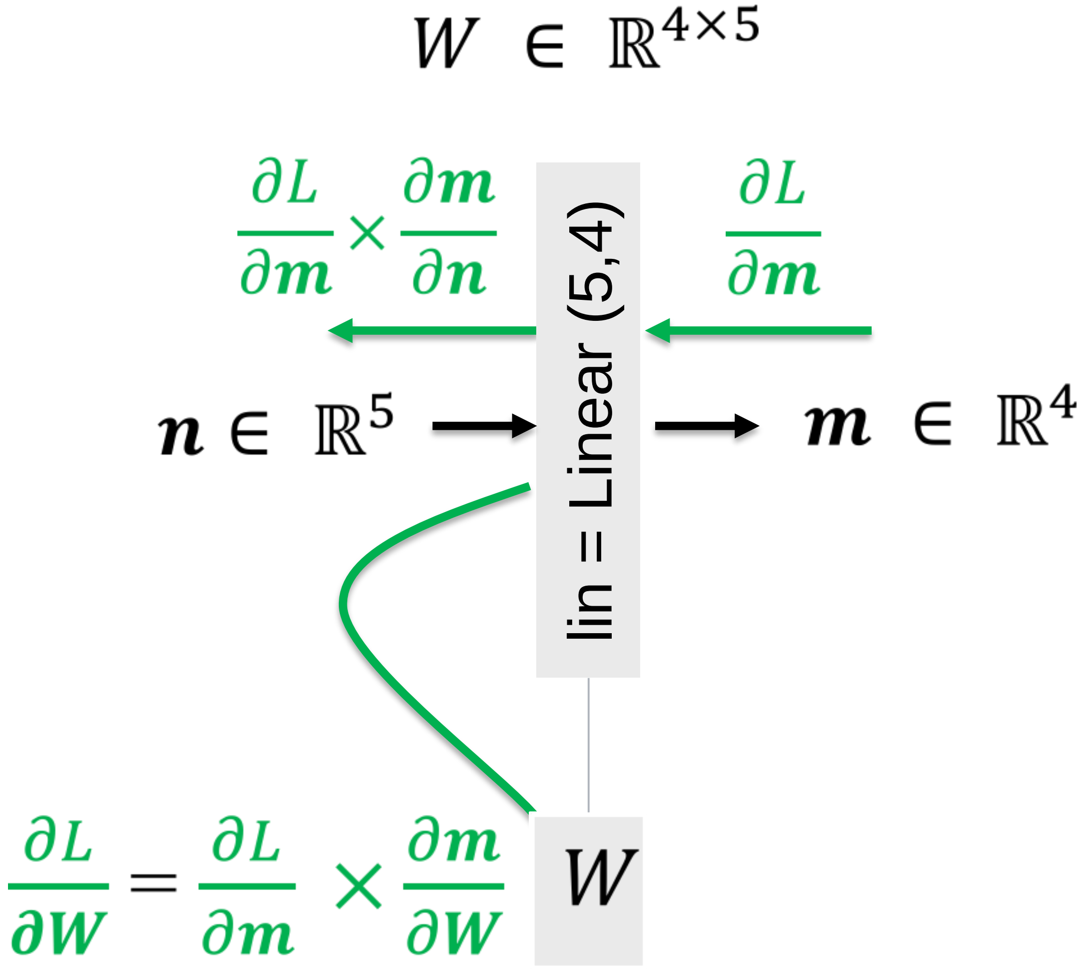
$$W$$

```
m
```

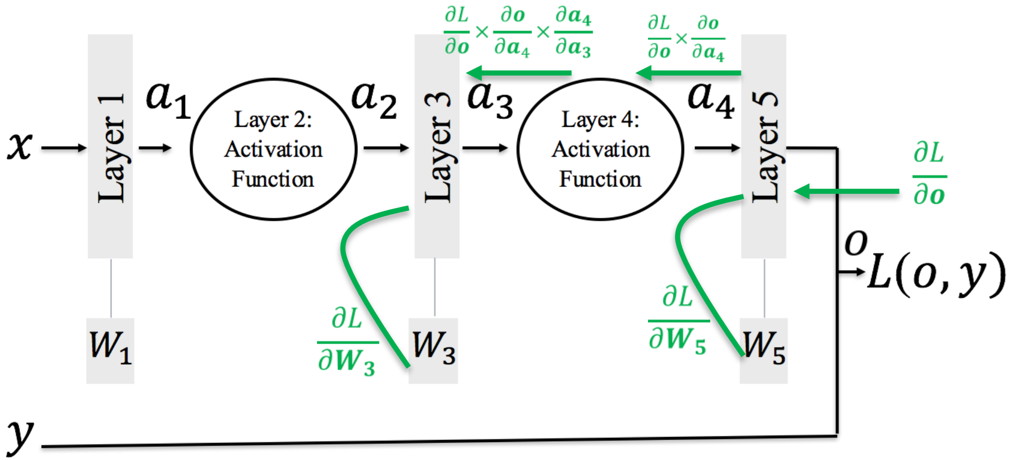
```
array([[ 0.00652516],
       [-0.02650564],
       [ 0.00077862],
       [-0.00458161]])
```

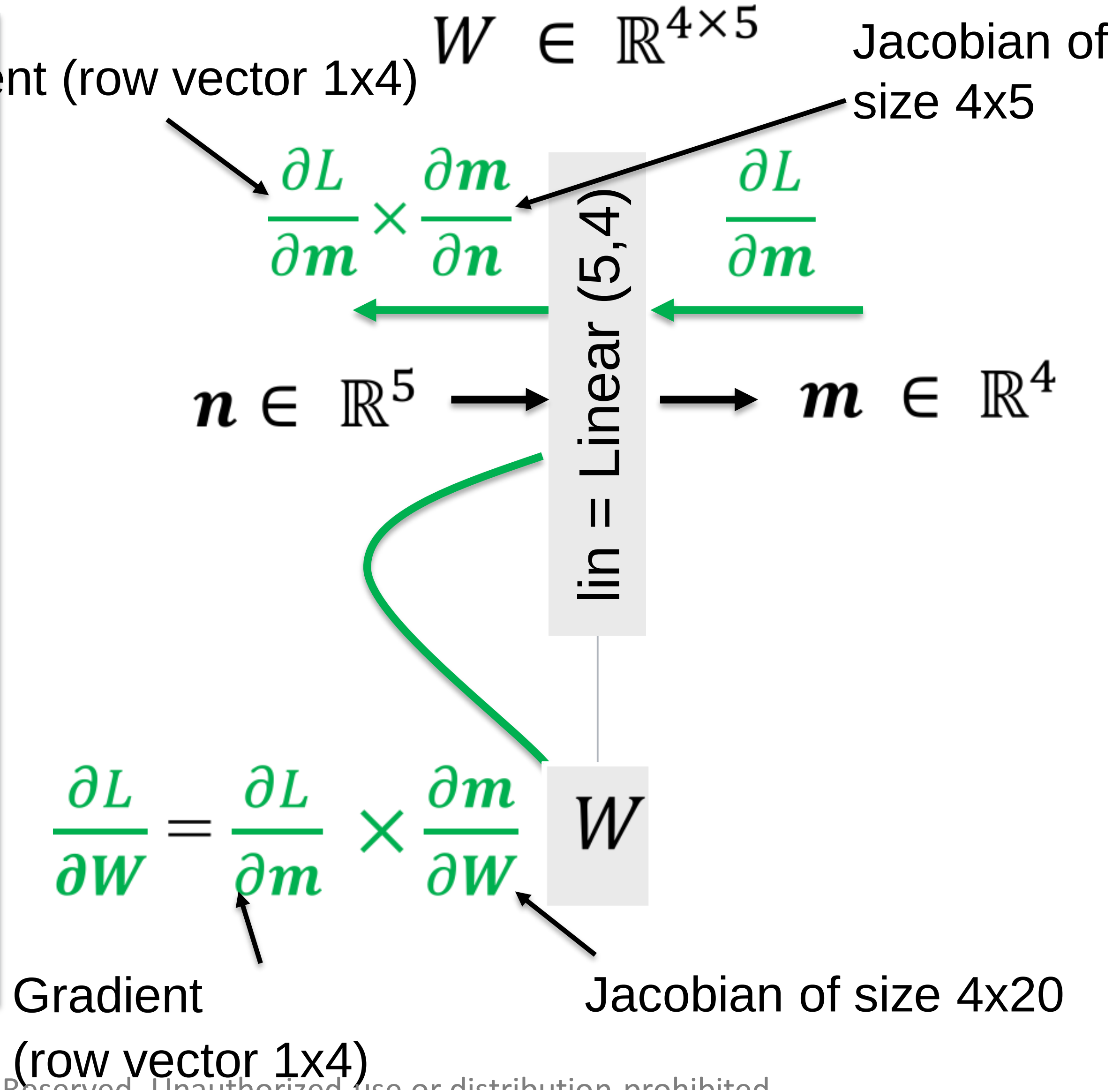
Fully Connected: Backward pass





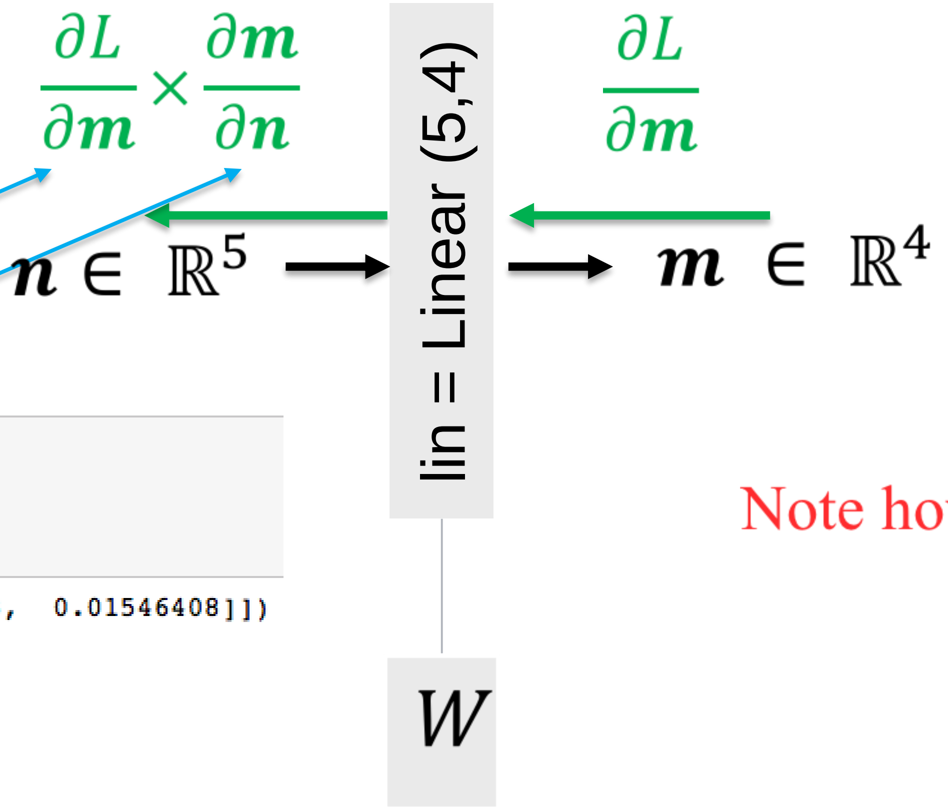
Fully Connected: Backward pass





Fully Connected: Backward pass

$$W \in \mathbb{R}^{4\times 5}$$




```python
# upstream gradient * dm/dn
# upstream gradient = dL/dm (which is nextgrad)
# dm/dn = d(W*n)/dn = W
np.dot(nextgrad, W)
```

```
array([[-0.01586083, -0.02361688, -0.00616945, -0.01881653,  0.01546408]])
```

Note how $\frac{\partial \boldsymbol{m}}{\partial \boldsymbol{n}} = \boldsymbol{W}$

Fully Connected: Backward pass

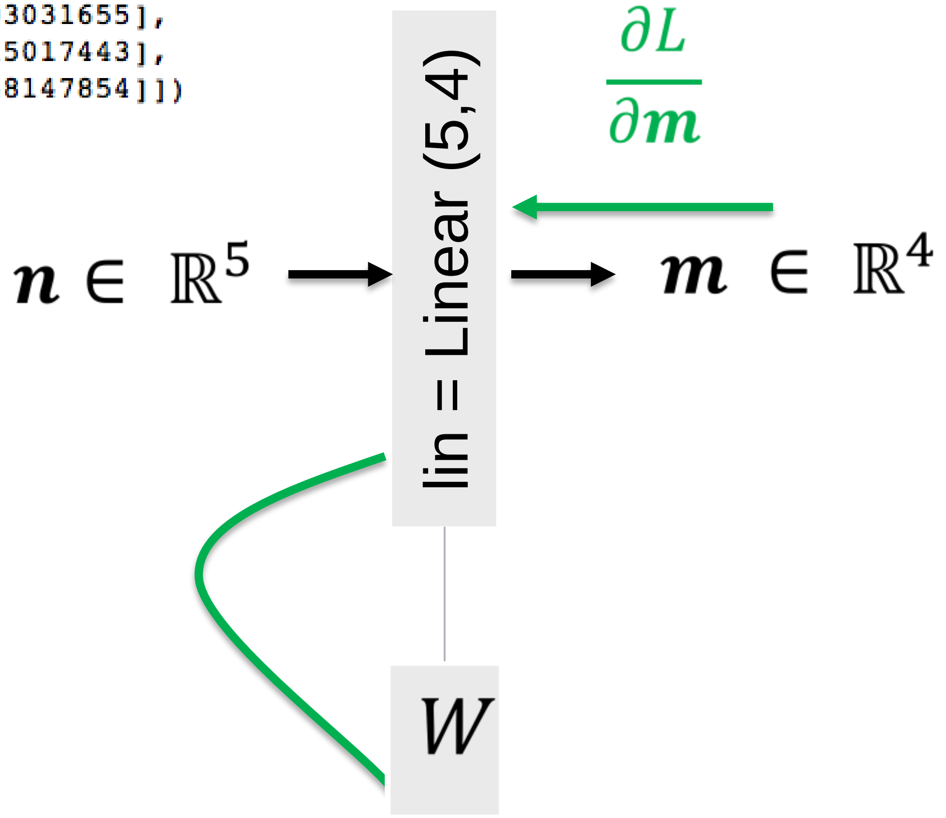
```
# dL/dW = dL/dm * dm/dW
np.dot(nextgrad, dodw).reshape(4,5)
```

```
array([[-0.93909512, -1.33880201, -0.51575265, -0.12800105, -1.26313287],
       [ 0.02253929,  0.03213269,  0.01237862,  0.00307216,  0.03031655],
       [-0.11164944, -0.15917077, -0.06131806, -0.0152181 , -0.15017443],
       [-0.35796246, -0.51032195, -0.19659359, -0.04879119, -0.48147854]])
```

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial m} \times \frac{\partial m}{\partial W}$$

```
# Jacobian of dm/dw (4x20)
dodw = np.zeros((4,20))
st = 0
for i in range(4):
    for j in range(5):
        dodw[i][st] = n[j]
        st = st + 1
```

$$W \in \mathbb{R}^{4\times 5}$$



Activation Functions

Neural Network Constructed

Now, let's review the following in turn:

- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

… and we'll have a fully functioning network

Activation Functions

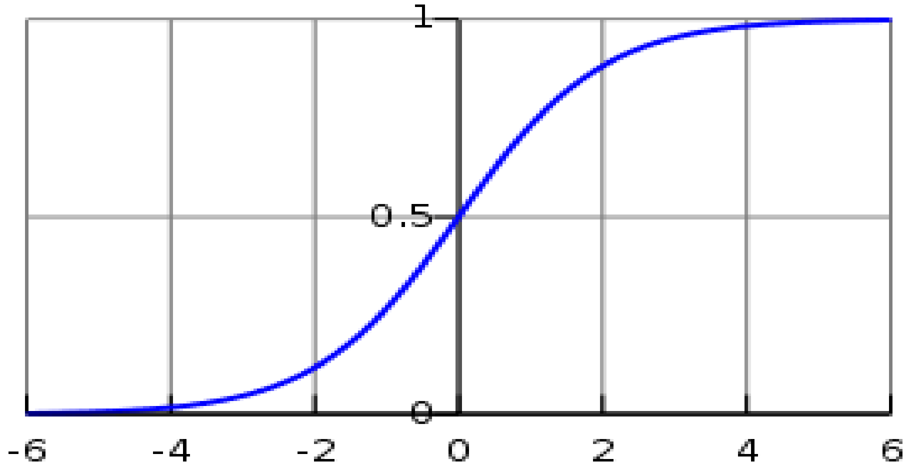
- An activation function takes a single input value and applies a function to it - typically a ‘non-linearity’
- Why non-linearity: converts a linear function (sum of weights) into a polynomial of higher degree, this is what allows non-linear decision boundaries
- Activation functions decide whether a neuron is ‘switched on’, i.e. it acts as a gate, by applying a non-linear function on the input
- Many different types of activation functions exist

Activation Functions: **Sigmoid**

- Activation function of form **f(x) = 1 / 1 + exp(-x)**
- Ranges from 0-1
- S-shaped curve
- Historically popular
  - Interpretation as a saturating “firing rate” of a neuron

Drawbacks:

1. Its output is not zero centered.
   - Hence, make the gradient go too far in different directions
2. Vanishing Gradient Problem
3. Slow convergence



Sigmoid

$$\sigma(x) = \frac{1}{(1 + e^{-x})}$$

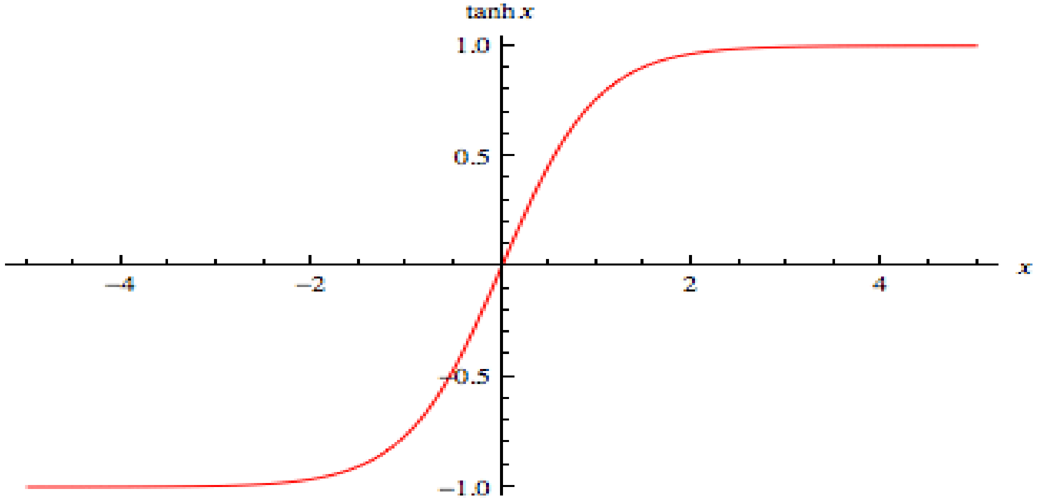
Image sourced from: DeepLearning.net

Activation Functions: **tanh(x)**

- Ranges between -1 to +1
- Output is zero centered
- Generally preferred over Sigmoid function

Drawback:

Though optimisation is easier, it still suffers from the Vanishing Gradient Problem



tanh

$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

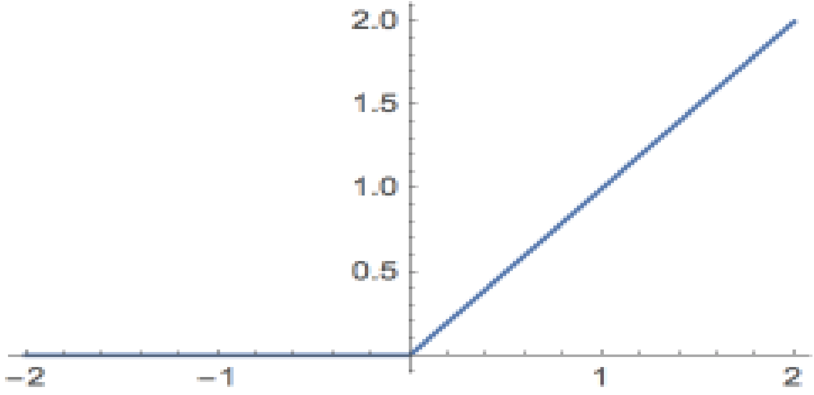
Image sourced from: Wolfram Alpha

Activation Functions: **ReLU**

- Simple
- Much better convergence than tanh and sigmoid function.
- Very efficient in computation

Drawbacks:

- Output is not necessarily zero centered.
- Should only be used within hidden layers of a NN model



ReLU

$$f(x) = \max(0, x)$$

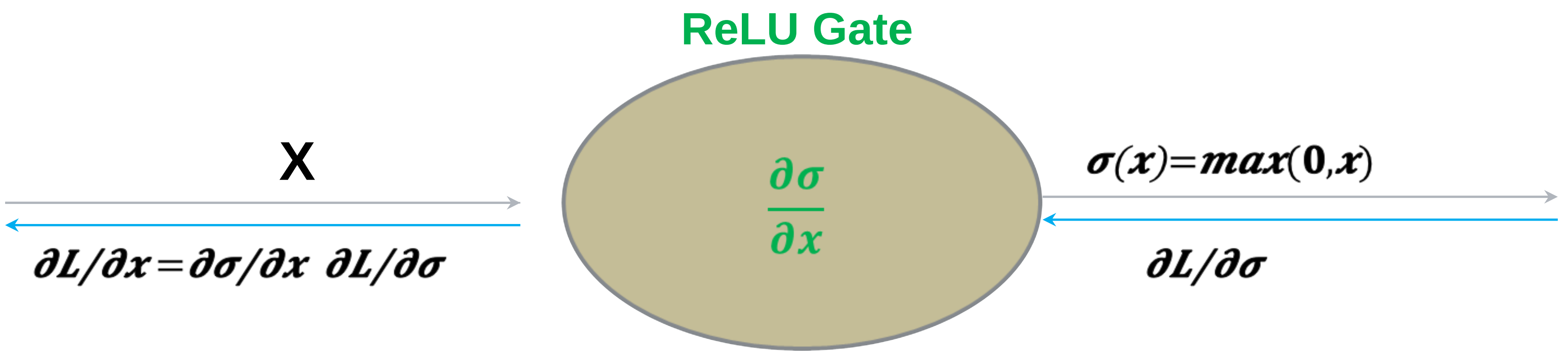
Image sourced from: Wolfram Alpha Documentation

Activation Functions: Problems with **ReLU**

- Some gradients can be fragile during training and can 'die'
- Results in weight update, and possibly never activating again
- 'Dead neurons'

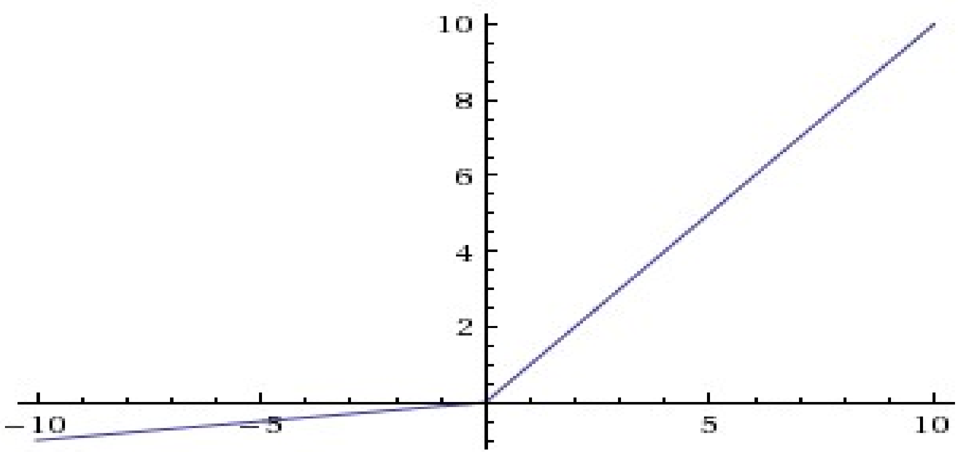
Experiment:

- What happens when x = -10?
- What happens when x = 0?
- What happens when x = 10?



Activation Functions: **Leaky ReLU**

- Introduced to overcome the problem of dying neurons.
- Introduces a small slope to keep the neurons alive
- Does not saturate in the positive region



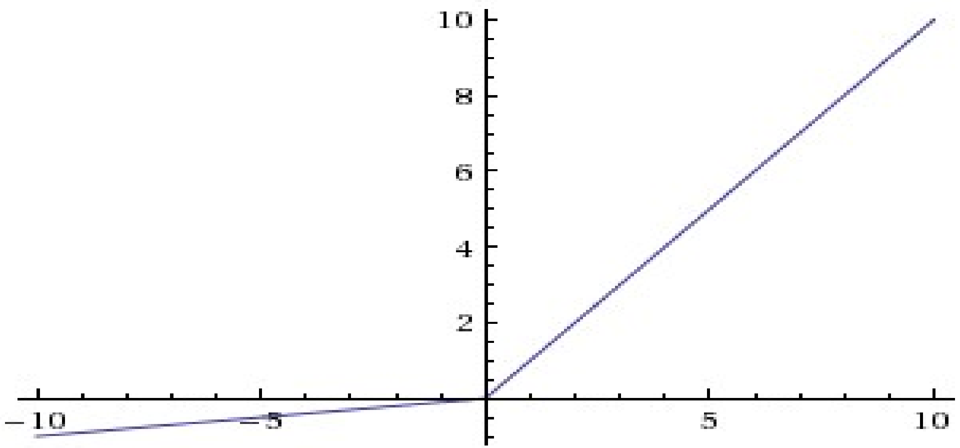
Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Image sourced from: Wolfram Alpha Documentation

Activation Functions: **Leaky ReLU**

- Introduced to overcome the problem of dying neurons.
- Introduces a small slope to keep the neurons alive
- Does not saturate in the positive region



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Back Propagate into α
(learnable parameter)

$$f(x) = \max(\alpha x, x)$$

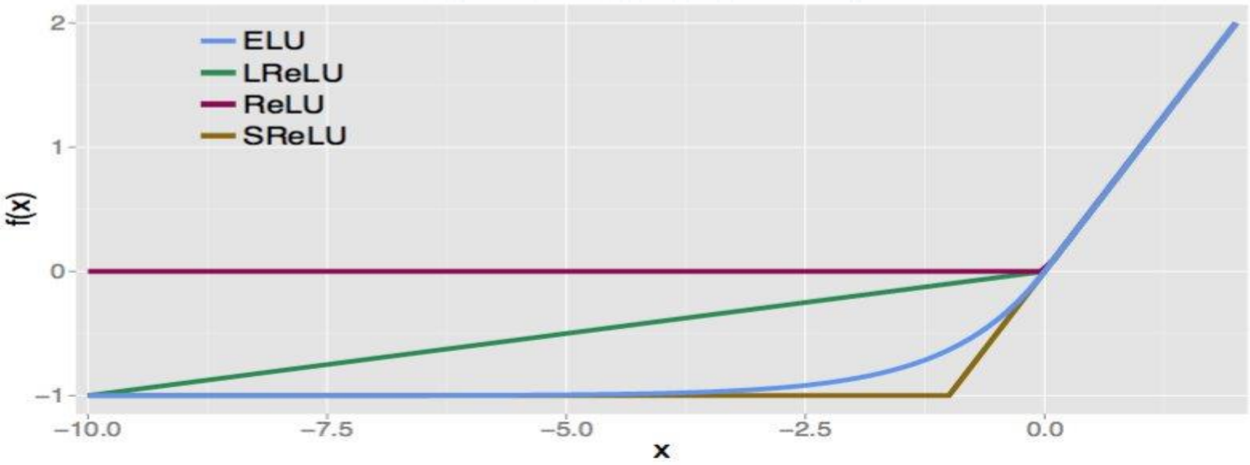
Parametric Rectifier PReLU

Image sourced from: Wolfram Alpha Documentation

Activation Functions: **ELU**

- Converge cost to zero faster and produce more accurate results
- Nearly zero mean outputs
- ELU is very similar to RELU except when inputs are negative
- All advantages of ReLU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$



Drawback:
Computation requires exp()

Exponential Linear Units (ELU)

Image sourced from: Fast and Accurate Deep Network Learning by Exponential Linear Units (ELU), Clevert et al., 2015

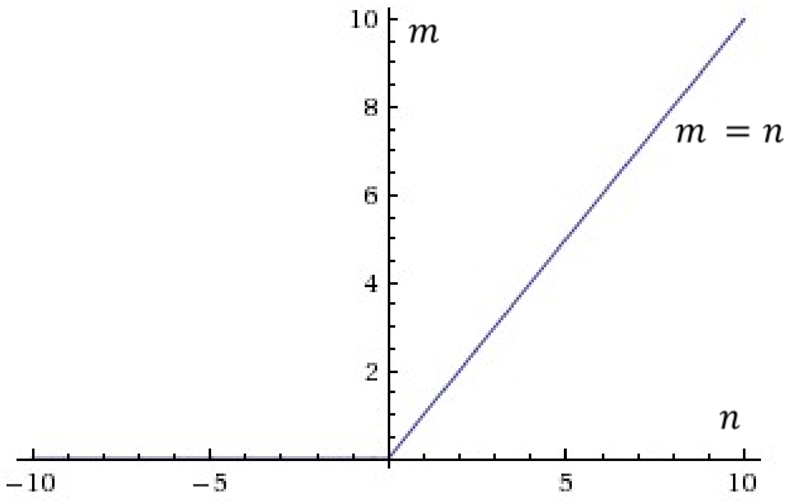
Activation Functions: **In practice...**

- **ReLU** is preferred
  - *Note:* Be careful with learning rates + Monitor the fraction of "**dead**" units in network
- Possibly try Leaky ReLU or Maxout
- Never use Sigmoid
- Try tanh
  - Typically perform worse than ReLU though

Activation Functions: **ReLU**

$$\frac{\partial L}{\partial \boldsymbol{W_1}}, \frac{\partial L}{\partial \boldsymbol{W_3}}, \frac{\partial L}{\partial \boldsymbol{W_5}} \rightarrow \text{ReLU} \rightarrow \boldsymbol{m} \in \mathbb{R}^d$$

$$\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \times \frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1\times \dim(a_4)}$$



Activation Functions: **ReLU**

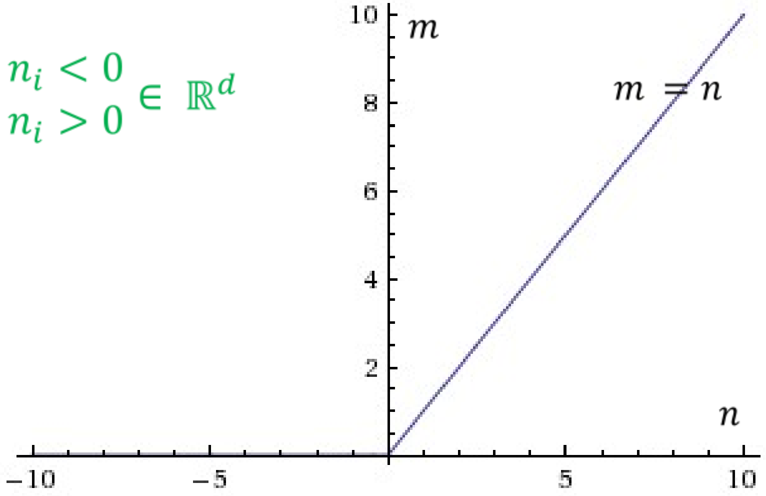
$$\frac{\partial L}{\partial \boldsymbol{m}} \cdot \frac{\partial \boldsymbol{m}}{\partial \boldsymbol{n}} \in \mathbb{R}^{1 \times \dim(n)}$$

$$\frac{\partial L}{\partial \boldsymbol{m}} \in \mathbb{R}^{1 \times \dim(m)}$$

$$\boldsymbol{n} \in \mathbb{R}^d \longrightarrow \boxed{\text{ReLU}} \longrightarrow \boldsymbol{m} \in \mathbb{R}^d$$

$$\frac{\partial \boldsymbol{o}}{\partial \boldsymbol{a_4}} \times \frac{\partial L}{\partial \boldsymbol{o}} \in \mathbb{R}^{1 \times \dim(a_4)}$$

$$\frac{\partial m_i}{\partial n_i} = \frac{\partial max(0, n_i)}{\partial n_i} = \begin{cases} 0 & if \;\; n_i < 0 \\ 1 & if \;\; n_i > 0 \end{cases} \in \mathbb{R}^d$$



Activation Functions: **ReLU (forward)**

```python
class ReLU:
    def forward(self, X):
        self.output = np.maximum(X, 0)
        return self.output
```

$$\boldsymbol{n} \in \mathbb{R}^d \longrightarrow \boxed{\text{ReLU}} \longrightarrow \boldsymbol{m} \in \mathbb{R}^d$$

```python
n
```

```
array([[ 0.58439657],
       [ 0.6157001 ],
       [-1.24037245],
       [-0.93783622],
       [-1.41779859]])
```

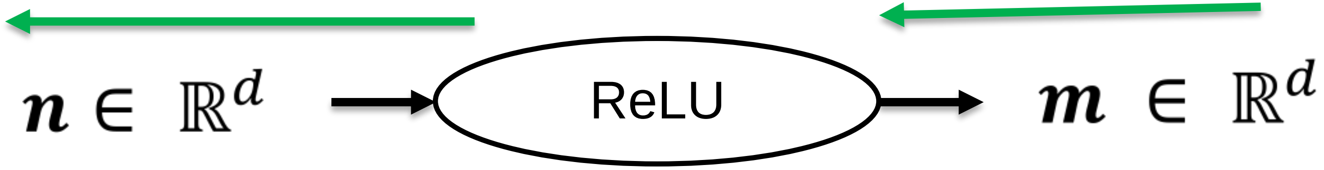
```python
relu = ReLU()
m = relu.forward(n)
```

```python
m
```

```
array([[0.58439657],
       [0.6157001 ],
       [0.        ],
       [0.        ],
       [0.        ]])
```

Activation Functions: **ReLU (backward)**

$$\frac{\partial L}{\partial \boldsymbol{m}} \cdot \frac{\partial \boldsymbol{m}}{\partial \boldsymbol{n}} \in \mathbb{R}^{1\times \dim(n)} \qquad \frac{\partial L}{\partial \boldsymbol{m}} \in \mathbb{R}^{1\times \dim(m)}$$



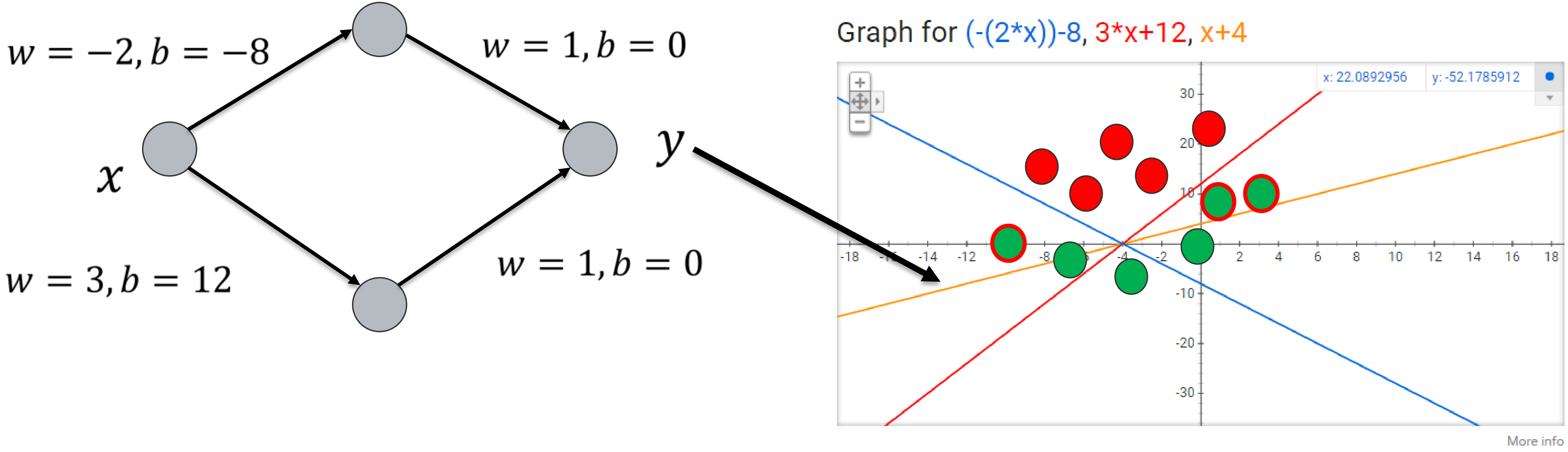

```python
class ReLU:
    def forward(self, X):
        self.output = np.maximum(X, 0)
        return self.output

   def backward(self, nextgrad):
       self.gradInput = nextgrad.copy()
       self.gradInput[self.output <=0] = 0
       return self.gradInput, []
```

```
relu.backward(m)
```

```
array([[1.],
       [1.],
       [0.],
       [0.],
       [0.]])
```

ReLU


$w = -2, b = -8$
$w = 1, b = 0$
x
y
$w = 3, b = 12$
$w = 1, b = 0$
Graph for (-(2*x))-8, 3*x+12, x+4
x: 22.0892956
y: -52.1785912
30
20
10
-10
-20
-30
-18
-14
-12
-8
-4
-2
2
4
6
8
10
12
14
16
18
More info

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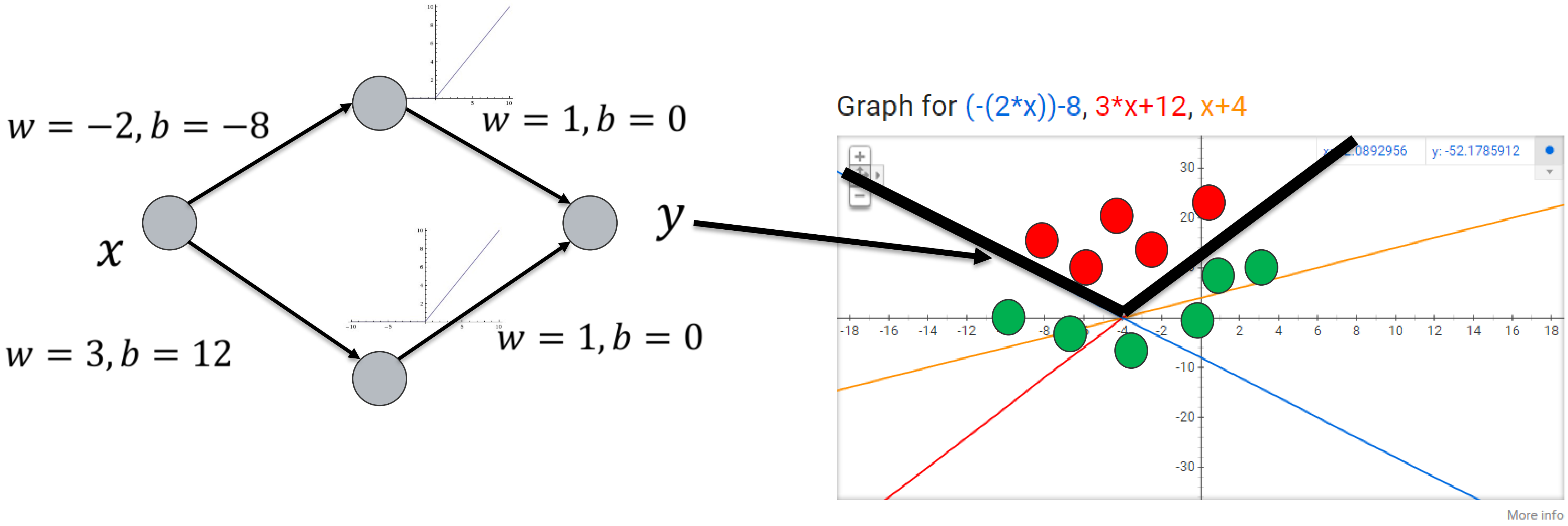
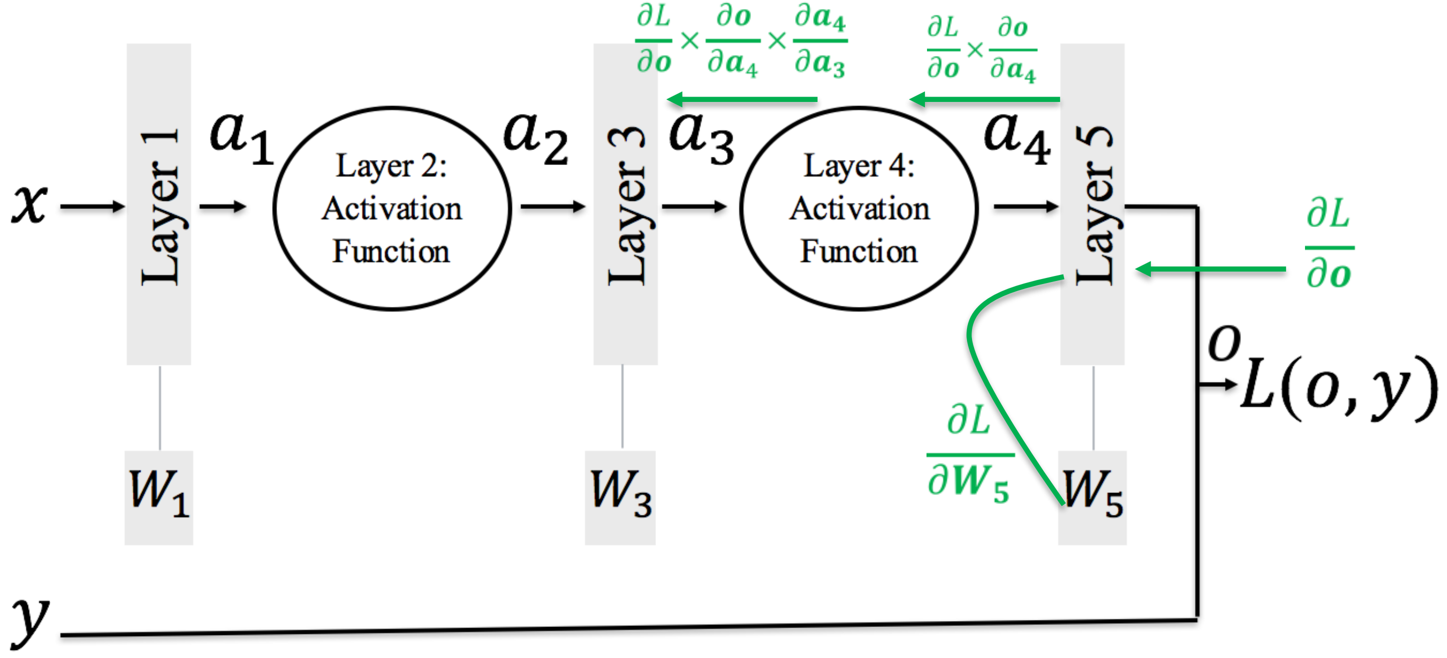
ReLU



Vanishing/Exploding Gradients



$$\frac{\partial L}{\partial W_3} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial a_4} \times \frac{\partial a_4}{\partial a_3} \times \frac{\partial a_3}{\partial a_2} \times \frac{\partial a_2}{\partial a_1} \times \frac{\partial a_1}{\partial W_1}$$

Vanishing/Exploding Gradients

$$\frac{\partial L}{\partial W_3} = \frac{\partial L}{\partial o} \times \frac{\partial o}{\partial a_4} \times \frac{\partial a_4}{\partial a_3} \times \frac{\partial a_3}{\partial a_2} \times \frac{\partial a_2}{\partial a_1} \times \frac{\partial a_1}{\partial W_1}$$

Computing gradients involves many factors of W (and the repeated activation functions a):

- Largest singular value of the matrices > 1: Exploding gradients
- Largest singular value of the matrices< 1: Vanishing gradients

(Think of single neuron and a single weight. Then this becomes a geometric series and either goes to 0 or infinity.)

SoftMax

Neural Network Constructed

Now, let's review the following in turn:

- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

… and we'll have a fully functioning network

Softmax Layer

- Softmax function is a multinomial logistic classifier, i.e. it can handle multiple classes
- Softmax typically the last layer of a neural network based classifier
- Softmax function is itself an activation function, so doesn’t need to be combined with an activation function

Softmax

$S \in \mathbb{R}^d$ → SoftMax → $p \in \mathbb{R}^d$ $\quad p_i = \frac{e^{s_i}}{\sum_{j=1}^{d} e^{s_j}}$

Softmax

$$S \in \mathbb{R}^d \longrightarrow \boxed{\text{SoftMax}} \longrightarrow p \in \mathbb{R}^d \quad p_i = \frac{e^{s_i}}{\sum_{j=1}^{d} e^{s_j}}$$

	unnormalized log probabilities	$\xrightarrow{\text{exp}}$	*unnormalized probabilities*	$\xrightarrow{\text{normalize}}$	probabilities
one	**3.15**		**23.33**		**0.104**
two	5.3		200.3		0.895
three	-2.1		.122		0.00

Softmax

$$S \in \mathbb{R}^d \longrightarrow \text{SoftMax} \longrightarrow p \in \mathbb{R}^d \quad p_i = \frac{e^{s_i}}{\sum_{j=1}^{d} e^{s_j}}$$

```
n = np.array([3.15,5.3,-2.1]).reshape(1,3)
m = softmax(n)
```

```
def softmax(x):
    exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
    return exp_x / np.sum(exp_x, axis=1, keepdims=True)
```

```
n
```

```
array([[ 3.15,  5.3 , -2.1 ]])
```

```
m
```

```
array([[1.04274135e-01, 8.95178684e-01, 5.47180443e-04]])
```

Cross Entropy

Neural Network Constructed

Now, let’s review the following in turn:

- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

… and we’ll have a fully functioning network

Cross-entropy loss

- Cross-entropy loss (often called Log loss) quantifies our unhappiness for the predicted output based on its deviation from the desired output
- Perfect prediction would have a loss of 0 (we will see how)
- With gradient descent, we try to reduce this (cross-entropy) loss for a classification problem

Cross Entropy

$S \in \mathbb{R}^d$ → Cross Entropy → $\boldsymbol{cost} \in \mathbb{R}^1$

y_i is 1 (and 0 otherwise) if and only if sample belongs to class i

$$s = f(x_i; W)$$

$$L_i = -y_i \cdot log\left(\frac{e^{s_i}}{\sum_j e^{s_j}}\right)$$

$$L = \sum_i L_i$$

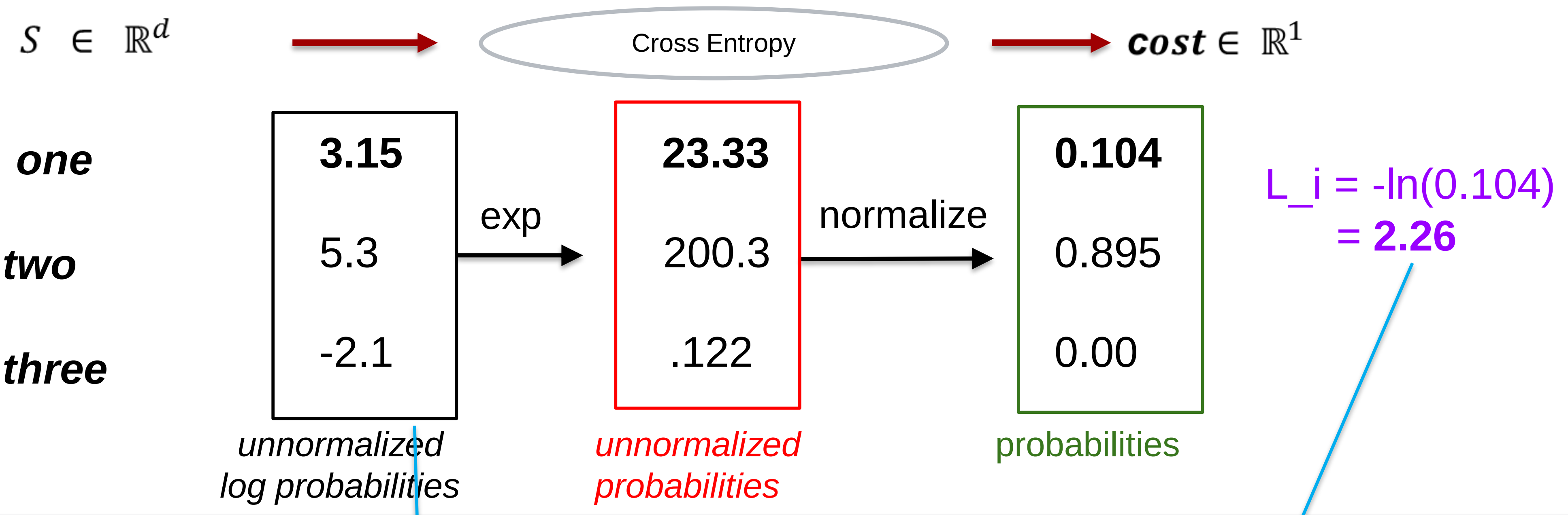
Cross Entropy

$S \in \mathbb{R}^d \rightarrow$ Cross Entropy $\rightarrow \boldsymbol{cost} \in \mathbb{R}^1$

	unnormalized log probabilities		unnormalized probabilities		probabilities
one	**3.15**	exp →	**23.33**	normalize →	**0.104**
two	5.3		200.3		0.895
three	-2.1		12		0.00

L_i = -ln(0.104)
= **2.26**

Cross Entropy




```
n1 = np.array([3.15, 5.3, -2.1]).reshape(1,3)
n2 = np.array([1,0,0]).reshape(1,3)
```

```
ce = CrossEntropy()
ce.forward(n1,n2)
```

```
2.2607319332115297
```

Cross Entropy

```
class CrossEntropy:
    def forward(self, X, y):
        y_idx = y.argmax()
        self.p = softmax(X)
        cross_entropy = -np.log(self.p[y_idx])
        loss = cross_entropy
        return loss

    def backward(self, X, y):
        y_idx = y.argmax()
        grad = softmax(X)
        grad[y_idx] -= 1
        return grad
```

```
n1 = np.array([3.15, 5.3, -2.1])
n2 = np.array([1,0,0])
```

```
ce = CrossEntropy()
ce.forward(n1,n2)
```

```
2.2607319332115297
```

Cross Entropy

$$\frac{\partial L}{\partial s} \in \mathbb{R}^{1\times d}$$

$$loss \in \mathbb{R}^1$$

$$S \in \mathbb{R}^d \longrightarrow \text{Cross Entropy} \longrightarrow \boldsymbol{cost} \in \mathbb{R}^1$$

$$\frac{\partial p_j}{\partial s_k} = \begin{cases} p_j(1-p_j)\ if\ j=k \\ -p_j p_k\ \ if\ j \neq k \end{cases}$$

y_i is 1 (and 0 otherwise) if and only if sample belongs to class i

$$L_i = -y_i \cdot log\left(\frac{e^{s_i}}{\sum_j e^{s_j}}\right)$$

$$L = \sum_i L_i$$

Cross Entropy

$$\frac{\partial L}{\partial \boldsymbol{s}} \in \mathbb{R}^{1\times d}$$

$$S \in \mathbb{R}^d$$

$$S \in \mathbb{R}^d \longrightarrow \text{Cross Entropy} \longrightarrow p \in \mathbb{R}^d$$

$$S \in \mathbb{R}^d$$

$$p_i = \frac{e^{s_i}}{\sum_j e^{s_j}}$$

$$L_i = -y_i \cdot log\left(\frac{e^{s_i}}{\sum_j e^{s_j}}\right)$$

$$L = \sum_i L_i$$

Cross Entropy

$$\frac{\partial L}{\partial s} \in \mathbb{R}^{1\times d}$$

$$S \in \mathbb{R}^d$$

$$S \in \mathbb{R}^d \rightarrow \boxed{\text{Cross Entropy}} \rightarrow p \in \mathbb{R}^d$$

$$p_i = \frac{e^{s_i}}{\sum_j e^{s_j}}$$

$$s = f(x_i; W)$$

$$p_i = \frac{e^{s_i}}{\sum_j e^{s_j}}$$

$$L_i = -y_i \cdot log(p_i)$$

$$L = \sum_i L_i$$

$$\frac{\partial L}{\partial s_k} = p_k - y_k$$

Cross Entropy

```python
class CrossEntropy:
    def forward(self, X, y):
        y_idx = y.argmax()
        self.p = softmax(X)
        cross_entropy = -np.log(self.p[y_idx])
        loss = cross_entropy
        return loss

    def backward(self, X, y):
        y_idx = y.argmax()
        grad = softmax(X)
        grad[y_idx] -= 1
        return grad
```

```python
n1 = np.array([3.15, 5.3, -2.1])
n2 = np.array([1,0,0])
```

```python
ohat = softmax(n1)
```

```python
print ohat
```

```
[1.04274135e-01 8.95178684e-01 5.47180443e-04]
```

```python
ce.backward(n1,n2)
```

```
array([-8.95725865e-01,  8.95178684e-01,  5.47180443e-04])
```

```python
ohat - n2
```

```
array([-8.95725865e-01,  8.95178684e-01,  5.47180443e-04])
```

In summary

We saw how each of the components of a deep neural network contributes to its functioning.

- Feed forward
- Back propagation
- Fully connected layer
- Activation functions
- Softmax function
- Cross-entropy loss

Next, we will see how to make it all work.

Thank you!