Lab Assignment 3, Hopfield networks

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Convergence and attractors

-1	-1	1	-1	1	-1	-1	1
-1	1	-1	-1	-1	1	-1	-1
-1	1	1	-1	-1	1	-1	1

-1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	1	-1	1
-1	-1	1	-1	1	-1	-1	1
-1	-1	1	-1	1	1	-1	1
-1	1	-1	-1	-1	1	-1	-1
-1	1	1	-1	-1	1	-1	1
-1	1	1	-1	1	-1	-1	1
1	-1	-1	1	1	-1	1	-1
1	1	-1	1	-1	1	1	-1
1	1	-1	1	1	-1	1	-1
1	1	-1	1	1	1	1	-1
1	1	1	1	-1	1	1	1
1	1	1	1	1	-1	1	1

Figure 1: Left: output from $x_1 d, x_2 d, x_3 d$. Right: all fixed points of the network.

- stored?
- $x_2 d$ converges to another closer fixed point
- 14 fixed points

Sequential Update

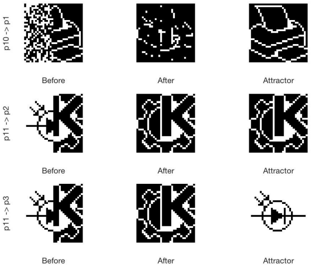

Figure 2: Middle column represents the results of the degraded patterns p10 and p11 after 7 iterations. The network succeeds to complete p11 into p2.

The number of incorrect states in each result, compared to the attractor it aimed for, turned out to be:
$[p10 \rightarrow p1, p11 \rightarrow p2, p11 \rightarrow p3] = [135, 0, 728]$.

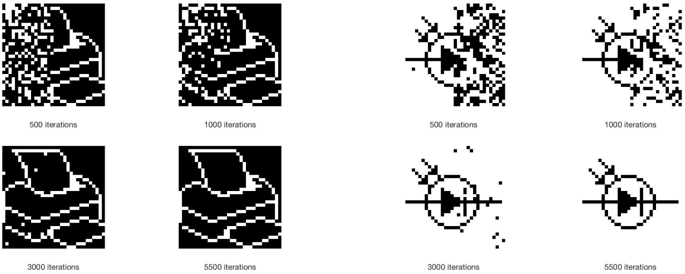

Figure 3: Units selected randomly when calculating the states for p10 and p11.

Energy

For the three different attractors `p1`, `p2` and `p3` the energies are:
$E_{attractors} = [-1439.4, -1365.6, -1462.3]$
And for the two distorted patterns, `p10` and `p11`, the energies are:
$E_{distorted} = [-415.9805, -173.5000]$

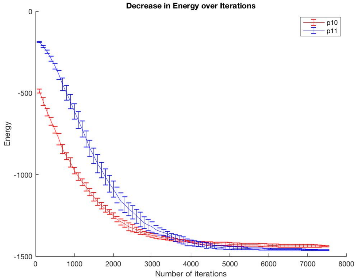

Figure 4: Decrease in energy for the patterns `p10` and `p11` as they approaches attractors. The results are obtained over 10 iterations.

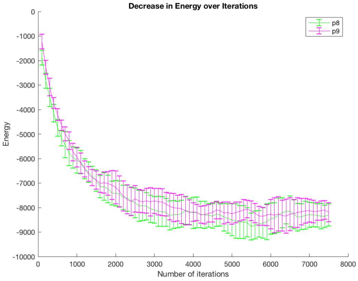

Figure 5: Decrease in energy for the patterns p8 and p9 when (1) W is normally distributed random numbers and (2) when W is a symmetric version of the normally distributed W. The results are obtained over 10 iterations.

Distortion Resistance

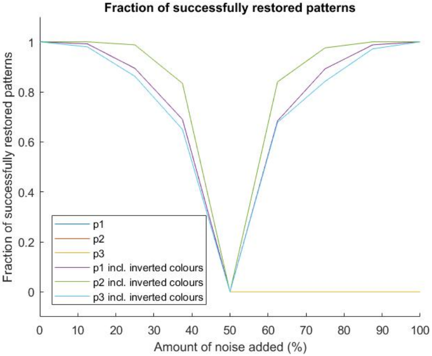

Figure 6: The fraction of restored pictures over 500 iterations when training with 3 pictures, with and without including inverted pictures.

- p2 has the best noise tolerance.
- The network can recover pictures when $< 50\%$ noise.
- The network can recover inverted pictures when $> 50\%$ noise.

Distortion Resistance - One example

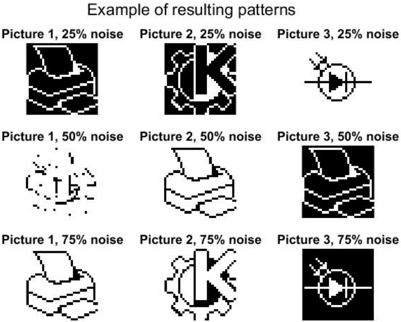

Figure 7: An example of the resulting patterns for different levels of noise, when training with 3 pictures. Title = input, picture = output.

- When restoring x, the network sometimes need more than one update for convergence, but the number of restored patterns is equally good regardless of one or more updates.

Distortion Resistance - 9 attractors

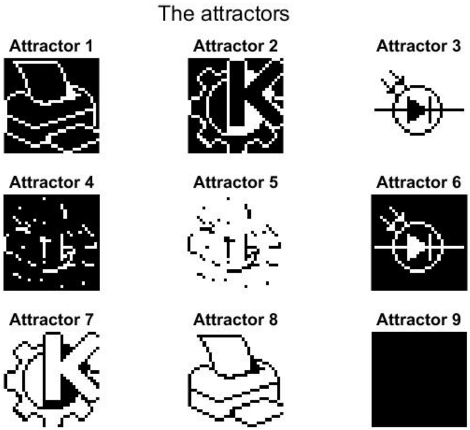

Figure 8: The attractors, when training with 3 pictures.

Capacity

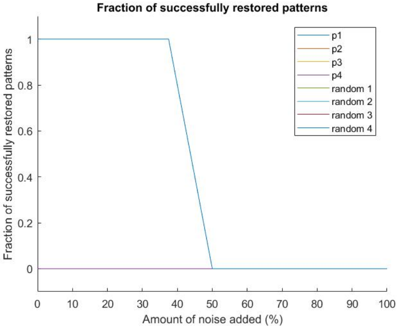

Figure 9: The fraction of successfully restored pictures over 500 iterations, for picture and random data, when training with 4 patterns.

- By adding p4 to the weight matrix, the network abruptly loses its ability to restore any picture.
- When replacing the pictures with random data of the same size, the network can restore all data with noise less than 50%.

Capacity - Adding more memory

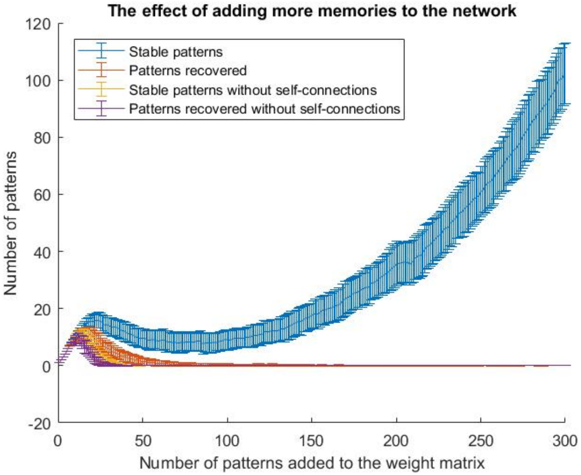

Figure 10: The effect of removing self-connections when adding more memories to the network. A 5% noise was used and the test was repeated 100 times.

Capacity - Removing self-connections

- The number of stable patterns increase with number of patterns added to the network.
- The number of successfully recovered patterns from 5% noise quickly reaches a maximum and then decreases down to zero again.
- This implies that training on a large number of patterns leads to poor generalization performance.
- By removing the self-connections, and thus the spurious patterns, the stable patterns exhibit the same behaviour.

Capacity - Biased patterns

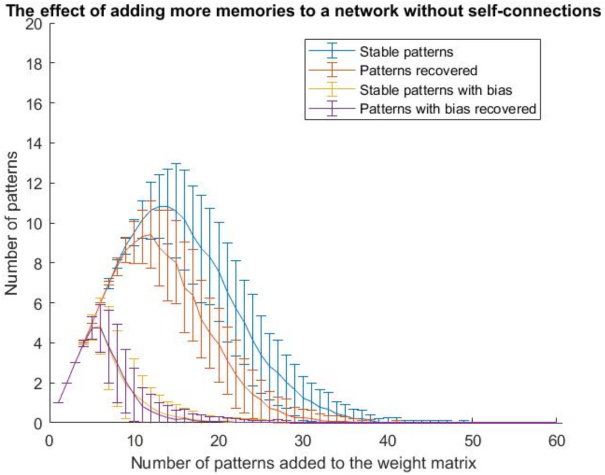

Figure 11: The effect of having biased patterns when adding more memories to a network without self-connections. A 5% noise was used and the test was repeated 100 times.

Capacity - Biased patterns

- The mean maximum number of retriavable patterns is 9.42, which is not too far from the theoretical value of 13.8.
- Having biased patterns decreases the network's performance regarding both stable and retrievable patterns.
- This can explain the previous difference between picture data and random data. The pictures are biased towards one colour, and are thus harder to recover.
- p2, which was the most noise tolerant, has a more even distribution between 1 and -1 than p1 and p3.

Sparse Patterns

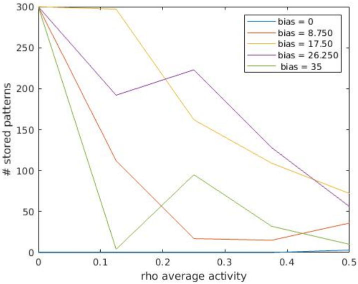

Figure 12: Number of stored patterns with varying bias and average activity.

- bias $\approx$ 17.50 best results
- more active neurons to store $\rightarrow$ less stored patterns